Agency for Science, Technology and Research
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Homogeneous space expressibility of parametric quantum circuits

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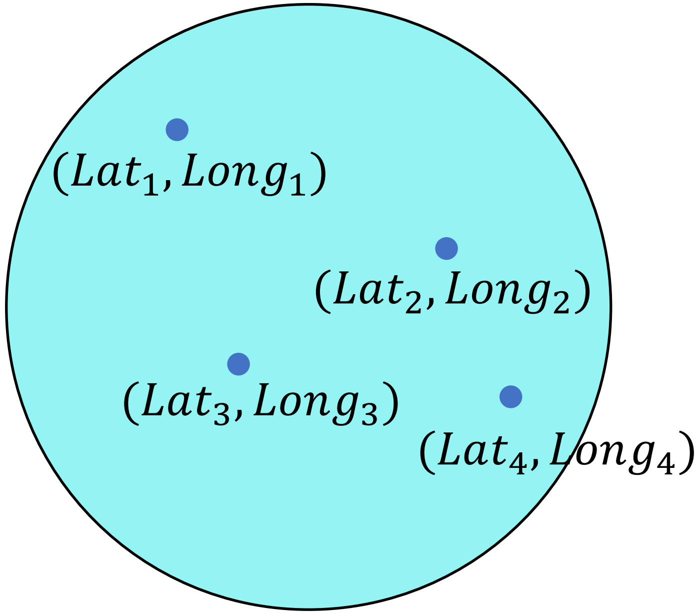
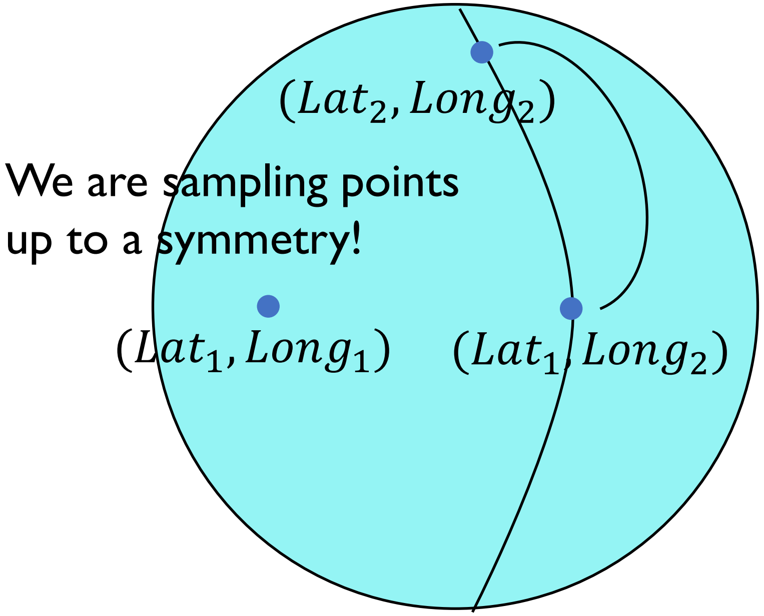
arXiv:2309.05253

Task: I give you a globe and ask you to throw darts at it. I want a uniform sample of longitudes.

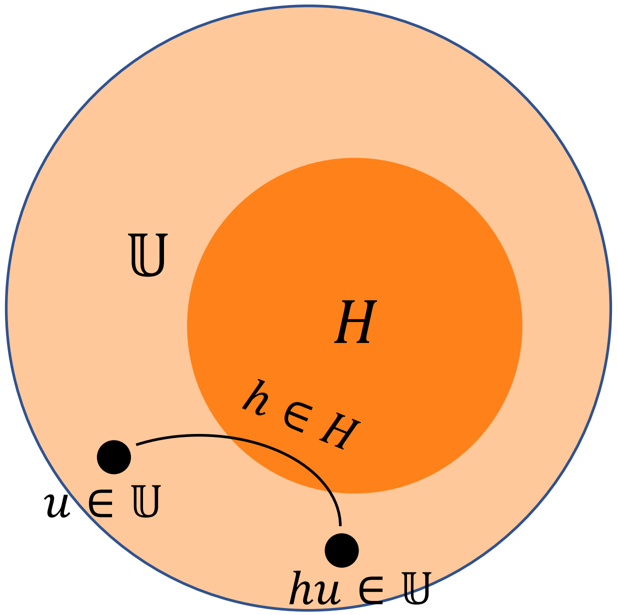


Throw darts across the globe.

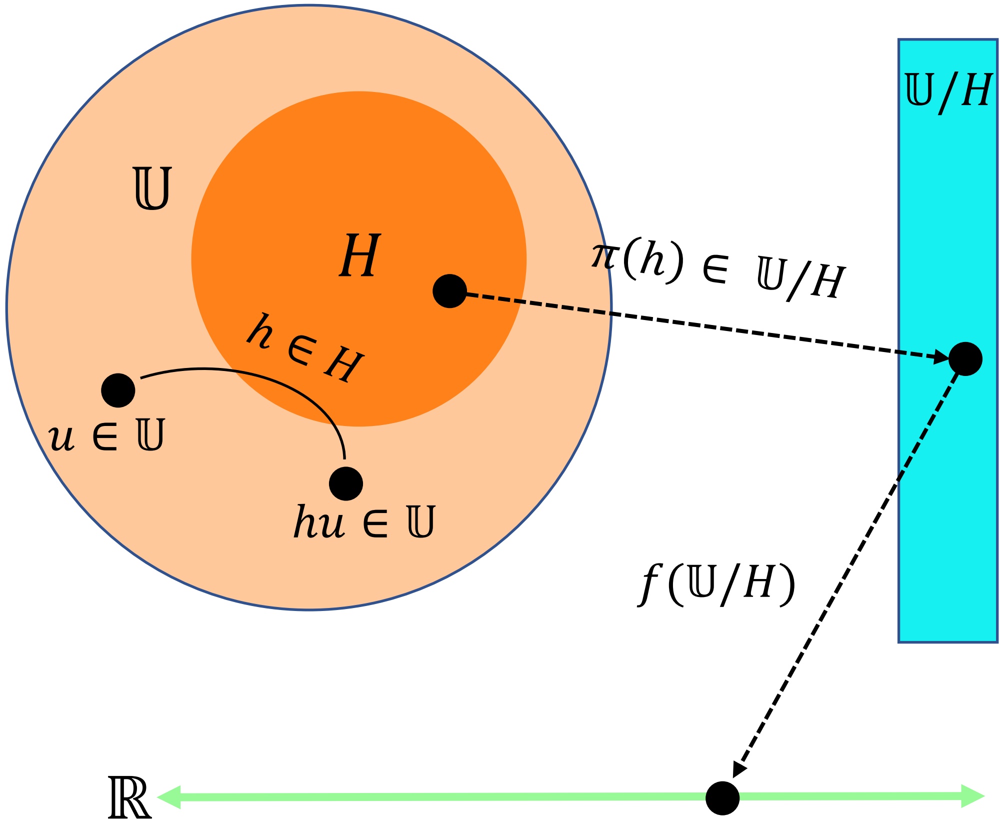


Fix the equator, and only throw darts here!

Homogeneous Spaces of the unitary group



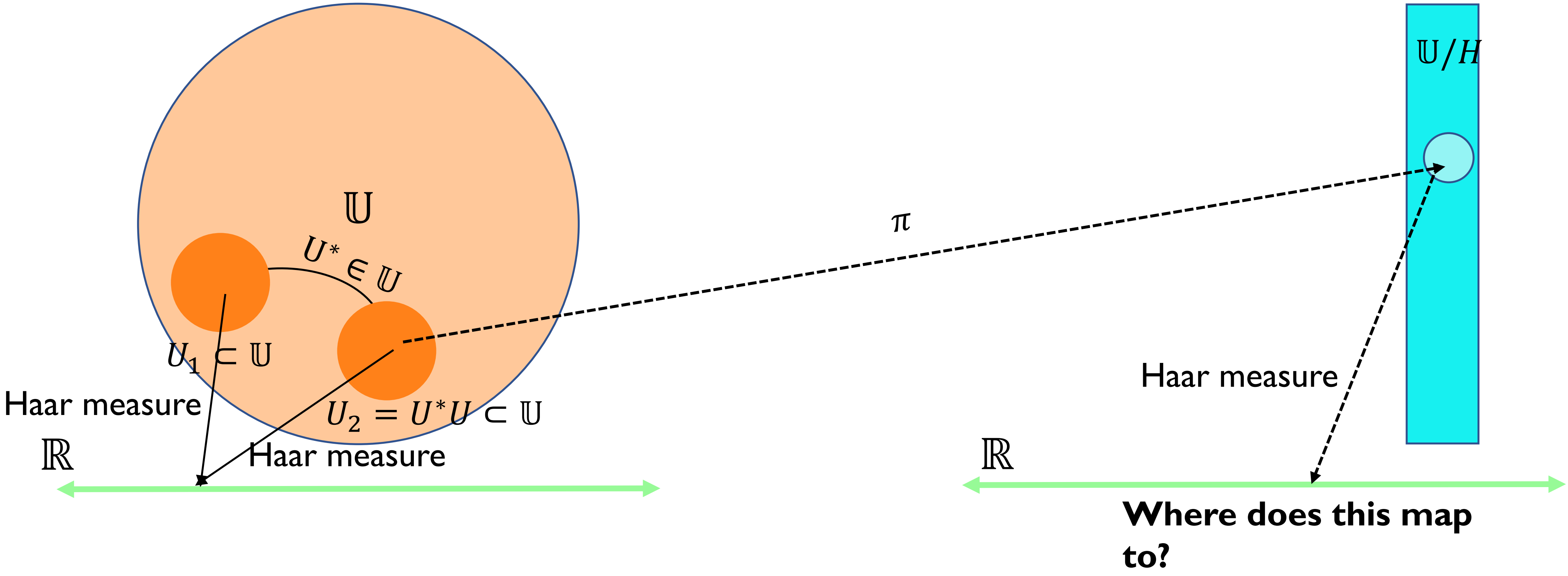
- $u_1 \sim u_2 \Rightarrow u_1 = hu_2$ for some $h \in H$.
- Equivalence relations $\Rightarrow$ Equivalence classes $\Rightarrow \mathbb{U}/H$.



- Functions from $\mathbb{U}/H \rightarrow \mathbb{R}$ map every $H -$ related element in $\mathbb{U}$ to the same point in $\mathbb{R}$.

We are dividing out the symmetry!

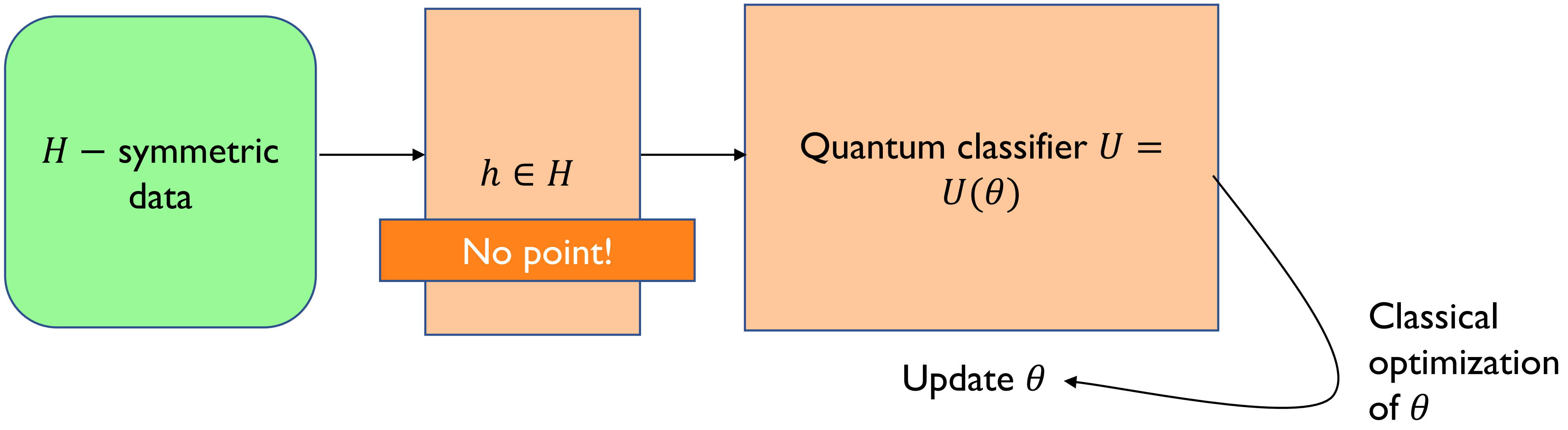
Sampling $\mathbb{U}$ up to a symmetry H: Sampling on $\mathbb{U}/H$



$$P(X \subset \mathbb{U}/H) = P(\pi^{-1}(X) \subset \mathbb{U}).$$

Sampling on $\mathbb{U}/H$: just sample on $\mathbb{U}$, send to equivalence class.

Quantum machine learning up to a symmetry



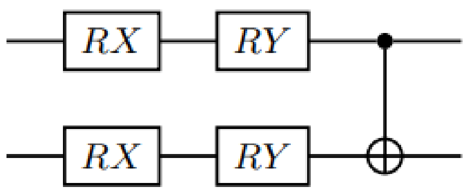
Key Idea

Start with a state $|\psi\rangle$ and want to learn the unitary $\mathrm{U}(\theta_{opt})$ --- if your data is H symmetric, we could require that $|\psi\rangle$ and $h|\psi\rangle$ should be treated the same way.

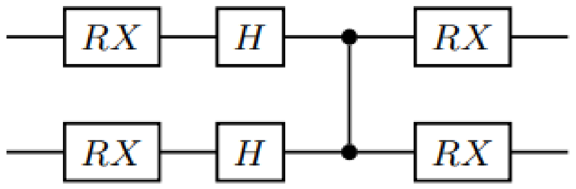
Expressibility, up to a symmetry

$$E^{\mathbb{U}/H} = \lim_{N\to\infty} N^{-1} \sum_{i=1}^{N} \min_{\theta} \mathcal{D}_{\mathbb{U}/H}(|i\rangle_{\mathbb{U}/H}, |\psi(\theta)\rangle)$$

$H = \{I, SWAP\}$



(a) Circuit 1



(b) Circuit 2

Features:

- Adding parametrized gates does not worsen the expressibility.
- If a symmetry is `stronger' than another, it is `easier' to be highly expressible.
- Appending a symmetry creating element to the circuit makes it expressible in $\mathbb{U}$

Circuit #	$E_{\mathscr{A}}^{\mathbb{U}(4)/H}$	Expr from [29]
Circuit 1	0.070 ± 0.001	0.095 ± 0.002
Circuit 2	$0.204 \pm < 10^{-3}$	0.216 ± 0.012

FIG. 3: The table shows the values of the expressibilities calculated using the homogeneous expressibility scheme as well as the expressibility in [29].

29: arXiv:1905.10876, Expressibility and Entangling Capability of Parameterized Quantum Circuits for Hybrid Quantum-Classical Algorithms, Sim, Sukin and Johnson, Peter D. and Aspuru-Guzik, Alán

Outlook (up to a symmetry)

- If you have a system where you need unitary operators, up to a symmetry --- homogeneous spaces provide a natural setting. **Our paper: t −designs and pseudo-randomness as well (happy to discuss afterwards!)**
- Random sampling on homogeneous spaces with the tools that we have.
- Expressibility up to a symmetry --- realistic!

- **Future work: anywhere where you can think of the words up to a symmetry …. ☺**