QTML 2023

Splitting and Parallelizing of QCNNs for Learning Translationally Symmetric Data

arXiv:2306.07331

Koki Chinzei

Fujitsu Research, Japan

Collaborators

Quoc Hoan Tran

Kazunori Maruyama

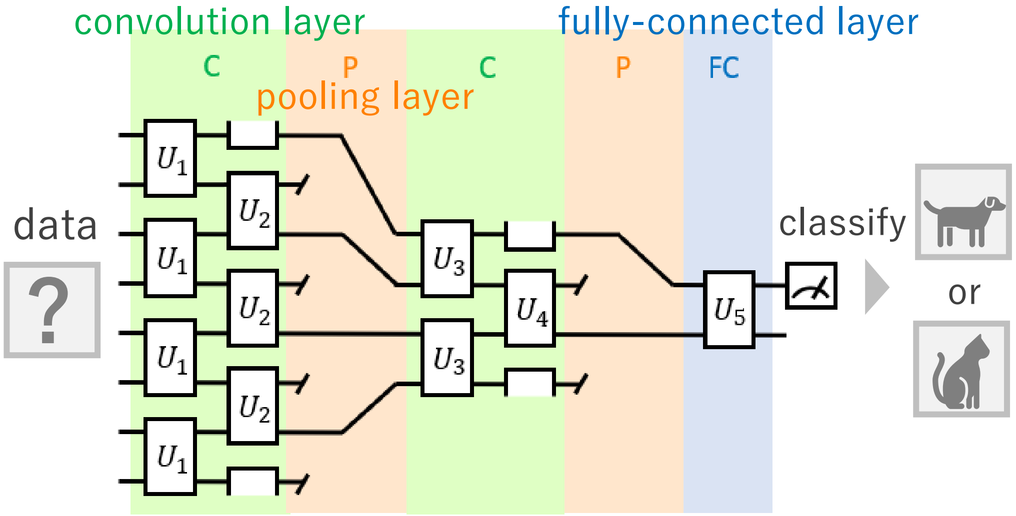
Hirotaka Oshima

Shintaro Sato

Background

◆ **Quantum Convolutional Neural Networks（QCNN）** Cong et al, Nat. Phys. (2019)

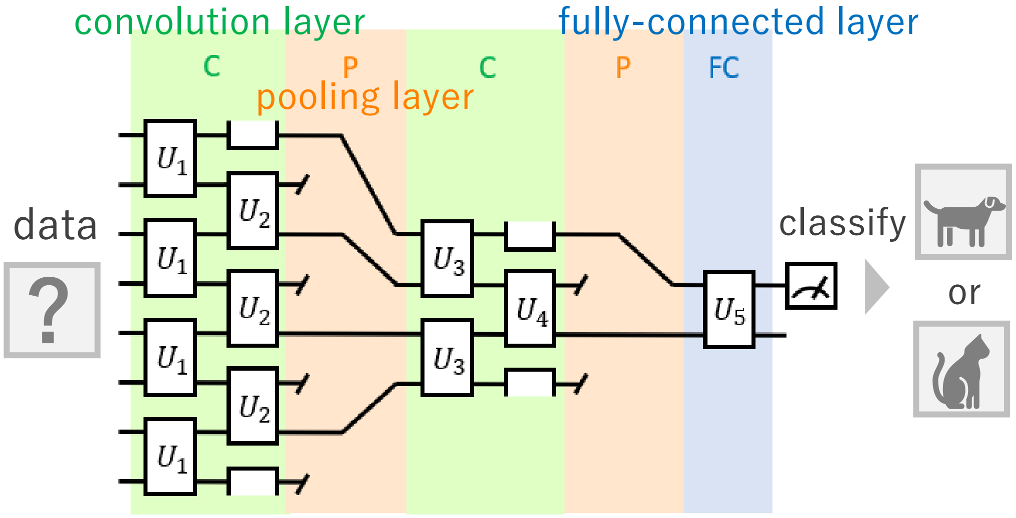
➢ Classify quantum data
based on parametrized quantum circuit



Background

◆ Quantum Convolutional Neural Networks (QCNN) Cong et al, Nat. Phys. (2019)

➢ Classify quantum data
based on parametrized quantum circuit

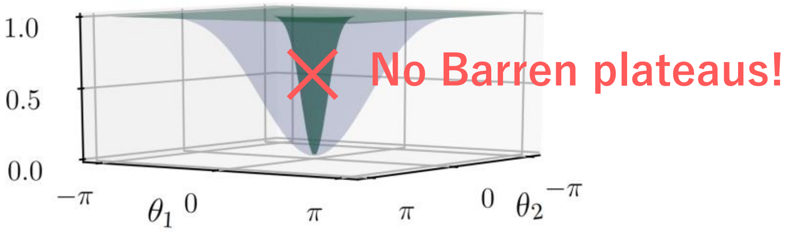


High feasibility and trainability

- ✓ Logarithmic circuit depth
- ✓ Absence of barren plateau

▶ A promising QML model

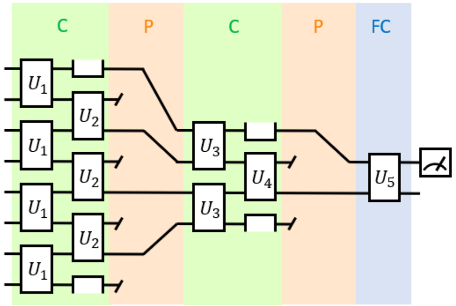
Pesah et al, PRX (2021)



Huge measurement cost in QCNN

Huge measurement costs hinder solving large-scale problems in practice

Measurement cost ∼ O(#parameters × #training data × #maximum epoch × #shots/obs.)



$$N_{\text{parameter}} \sim \mathcal{O}(n)$$

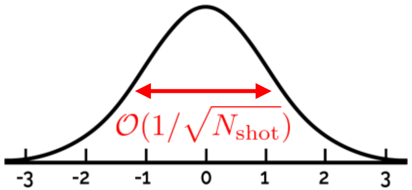
Caro et al,
Nat. Comm. (2022)

Generalization

$$\text{gen} \in \mathcal{O}\left(\sqrt{\frac{T\log(MT)}{N_{\text{data}}}}\right)$$

Data size

$$N_{\text{data}} \sim T\log(MT) \sim \mathcal{O}(\text{polylog}(n))$$



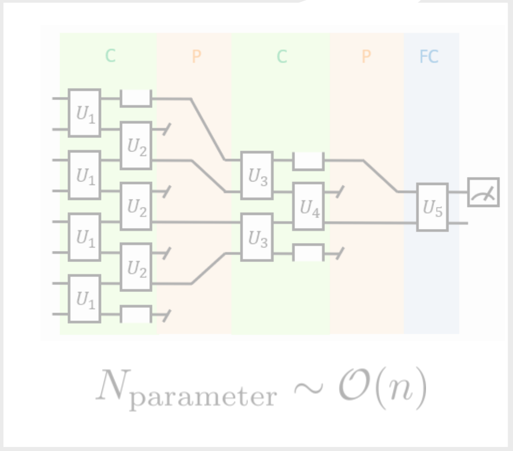
Estimated expectation value

$$N_{\text{shot}} \sim \mathcal{O}(\epsilon^{-2})$$

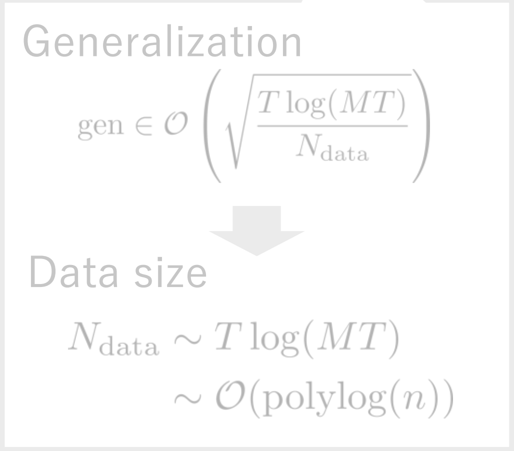
Huge measurement cost in QCNN

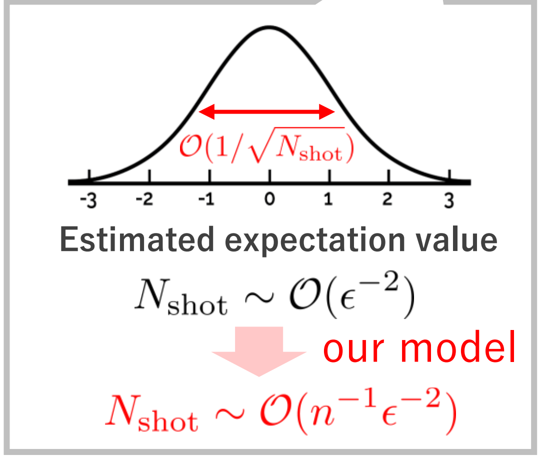
Huge measurement costs hinder solving large-scale problems in practice

Measurement cost ∼ O(#parameters × #training data × #maximum epoch × #shots/obs.)



Caro et al, Nat. Comm. (2022)





This talk

- Propose a new QCNN model with high measurement efficiency
- It reduces the required number of shots by a factor of $O(1/n)$

Basic idea for efficient QCNN

Leverage the prior symmetry knowledge of data for an efficient model

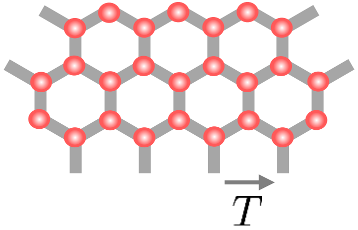
Basic idea for efficient QCNN

Leverage the prior symmetry knowledge of data for an efficient model

This study focuses on translational symmetry

$$T\rho_{\mathrm{in}}T^{\dagger} = \rho_{\mathrm{in}}$$

translation op.



Solids

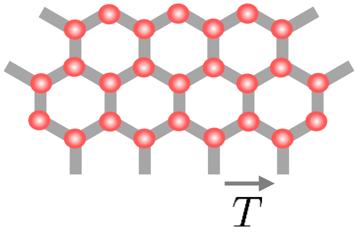
Basic idea for efficient QCNN

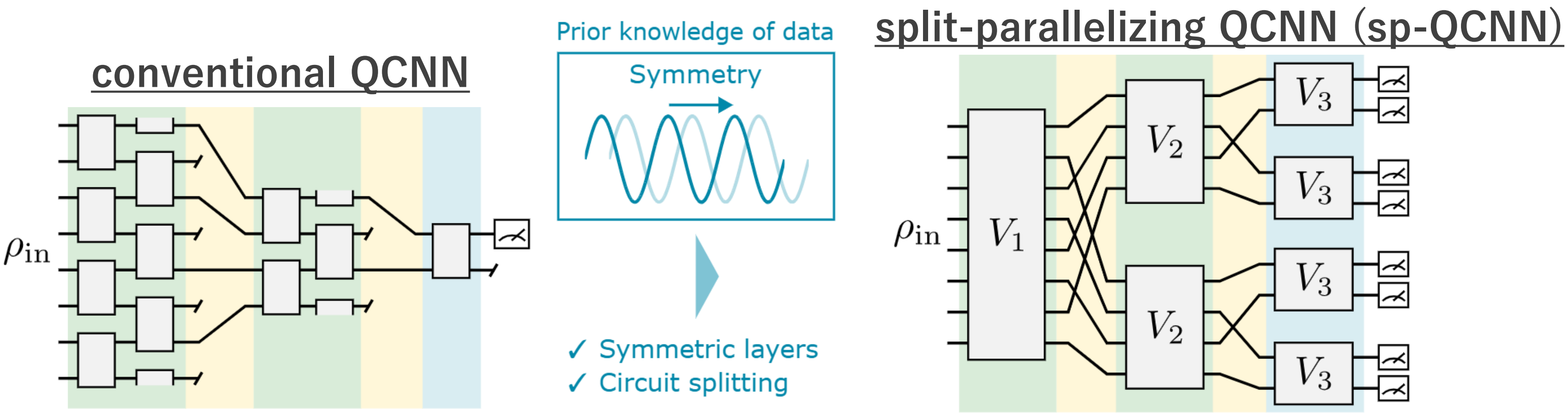
Leverage the prior symmetry knowledge of data for an efficient model

This study focuses on translational symmetry

$$T\rho_{\text{in}}T^\dagger = \rho_{\text{in}}$$

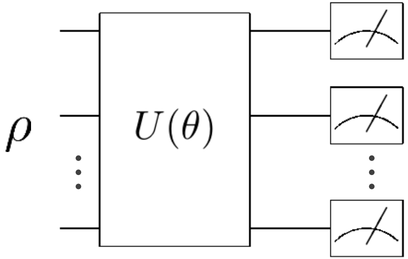
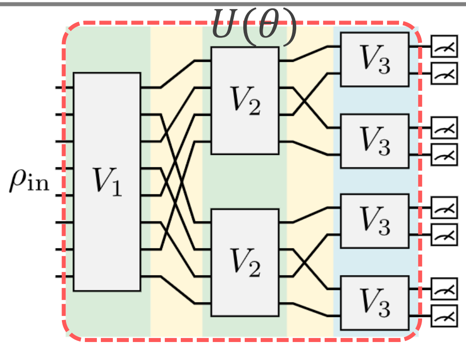
translation op.



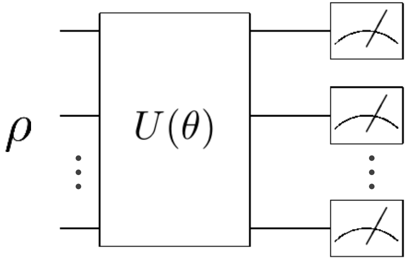
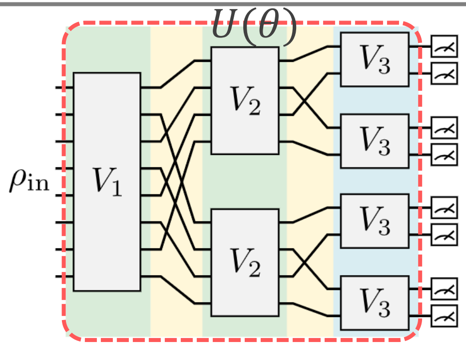
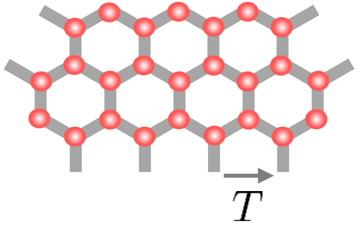


Effective parallelization of conventional QCNN

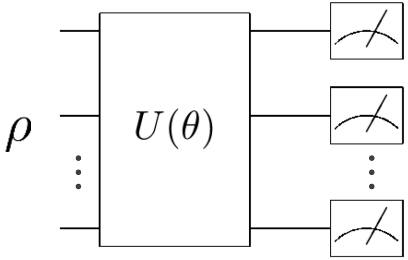
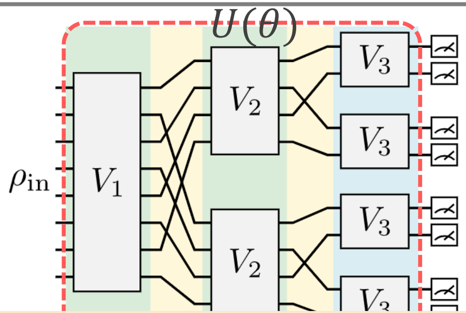
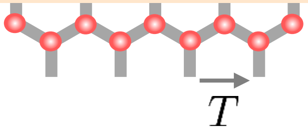
Relation with geometric QML (GQML)

	GQML Larocca2022, etc		This work
model	 Equivariance $[U(\theta), S] = 0$	$\supseteq$	 Equivariance $[U(\theta), T] = 0$

Relation with geometric QML (GQML)

	GQML Larocca2022, etc		This work
model	 Equivariance $[U(\theta), S] = 0$	$\ni$	 Equivariance $[U(\theta), T] = 0$
data	If this is a cat, this is a cat too. $f($  $)$ $=$ $f($  $)$ Data is **not symmetric** (Function is **symmetric**)	$\neq$	 $T \rho_{\text{in}} T^\dagger = \rho_{\text{in}}$ translation op. Data is **symmetric**

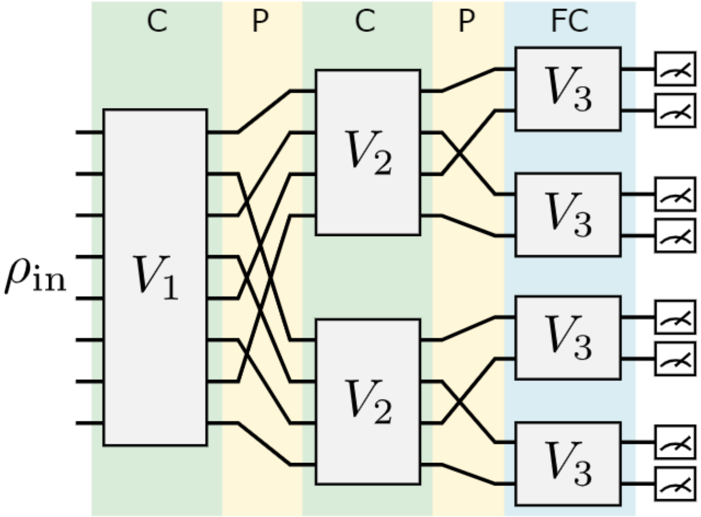
Relation with geometric QML (GQML)

	GQML Larocca2022, etc		This work
model	 Equivariance $[U(\theta), S] = 0$	$\supset$	
data	If this is a cat, this is a cat too. $f($  $) = f($  $)$ **Data is not symmetric** (Function is **symmetric**)	$\neq$	 $T \rho_{\mathrm{in}} T^{\dagger} = \rho_{\mathrm{in}}$ translation op. Data is **symmetric**

(New idea!)
The combination of equivariance and data symmetry results in high measurement efficiency!

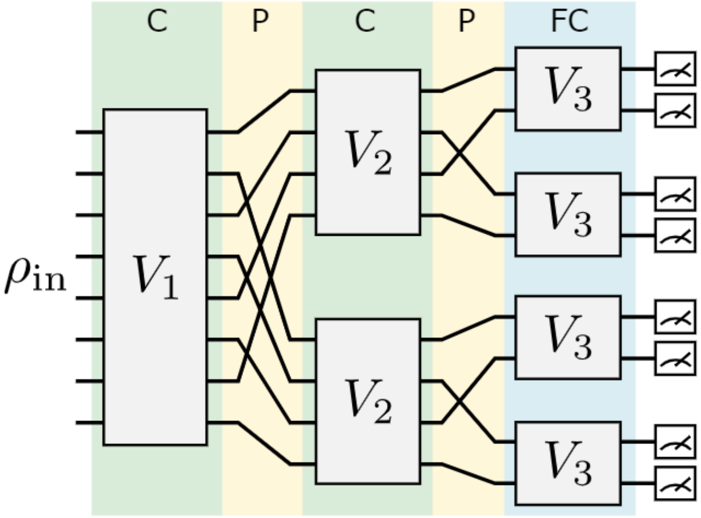
Two building blocks

split-parallelizing QCNN (sp-QCNN)

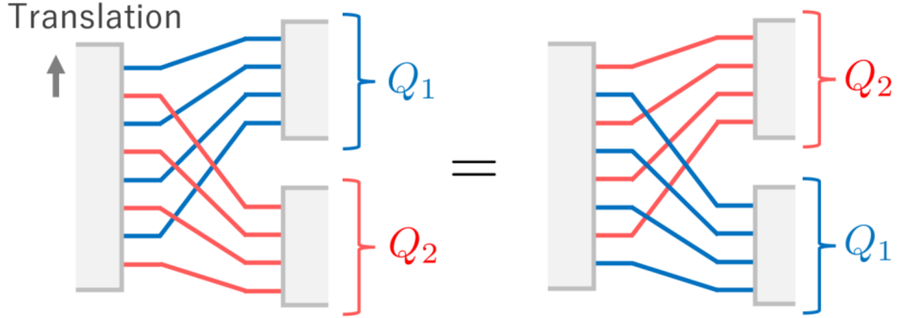


Two building blocks

split-parallelizing QCNN (sp-QCNN)



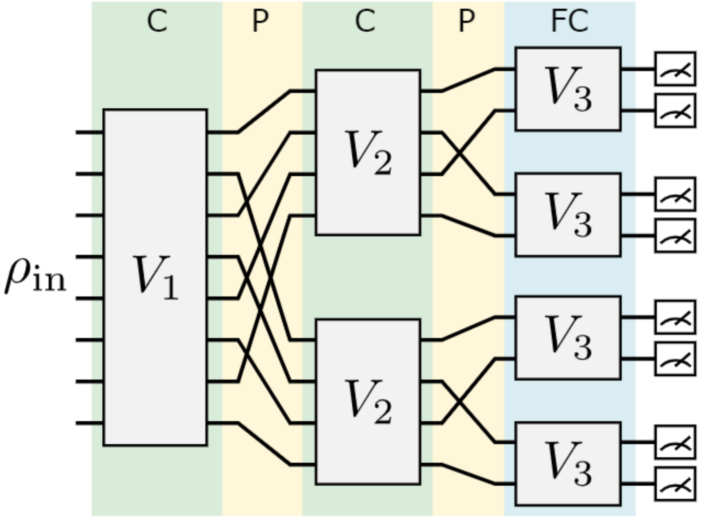
1. T-symmetric circuit splitting

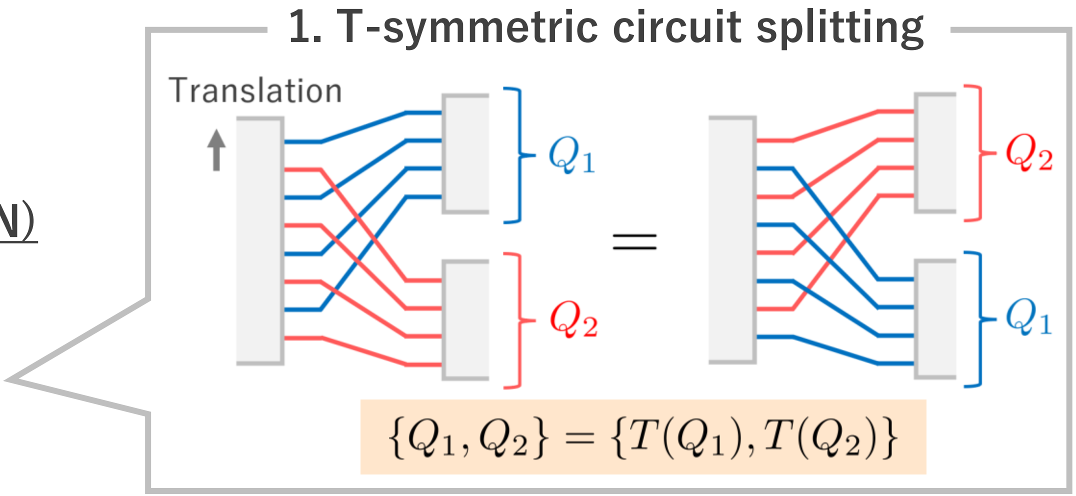


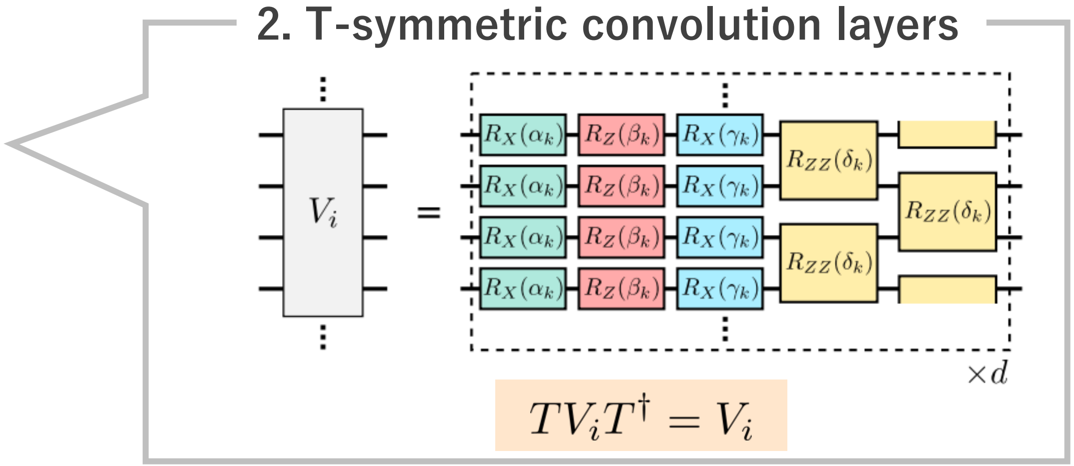
$$\{Q_1, Q_2\} = \{T(Q_1), T(Q_2)\}$$

Two building blocks

split-parallelizing QCNN (sp-QCNN)

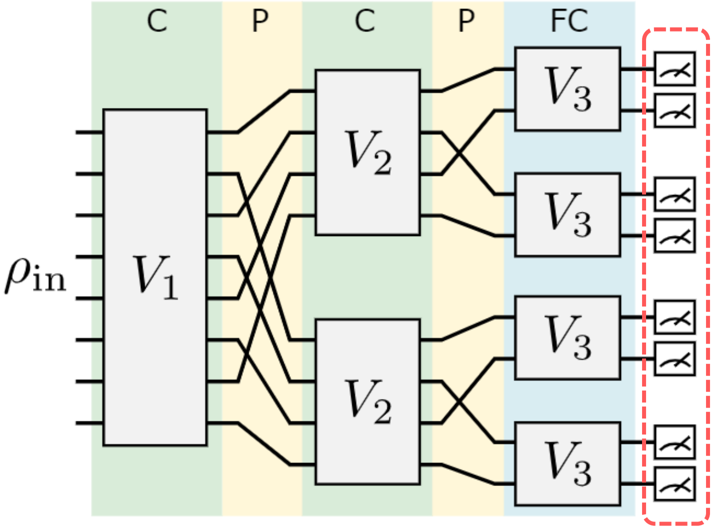






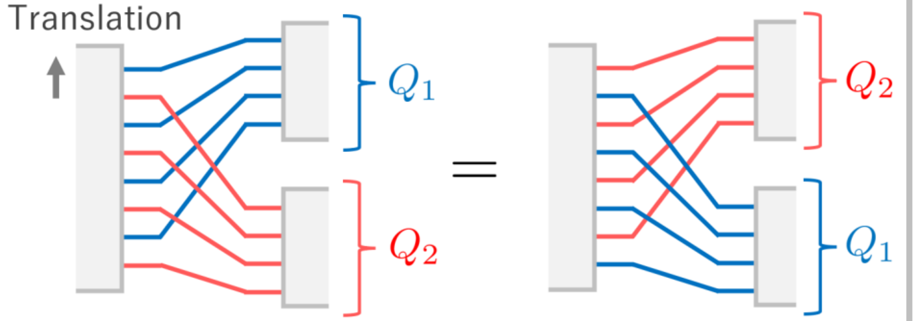
Two building blocks

split-parallelizing QCNN (sp-QCNN)



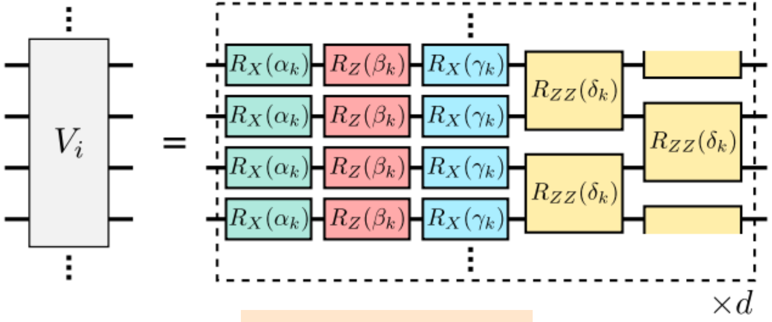
output $= \sum_j \langle Z_j \rangle / n$

1. T-symmetric circuit splitting



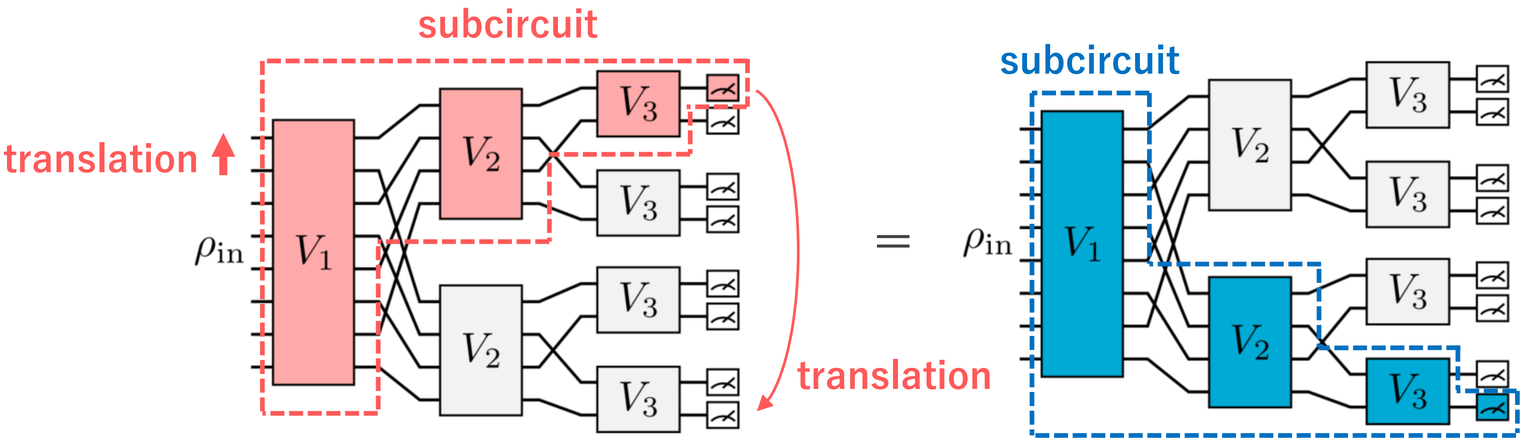
$\{Q_1, Q_2\} = \{T(Q_1), T(Q_2)\}$

2. T-symmetric convolution layers



$TV_iT^\dagger = V_i$

Mechanism for efficient measurement



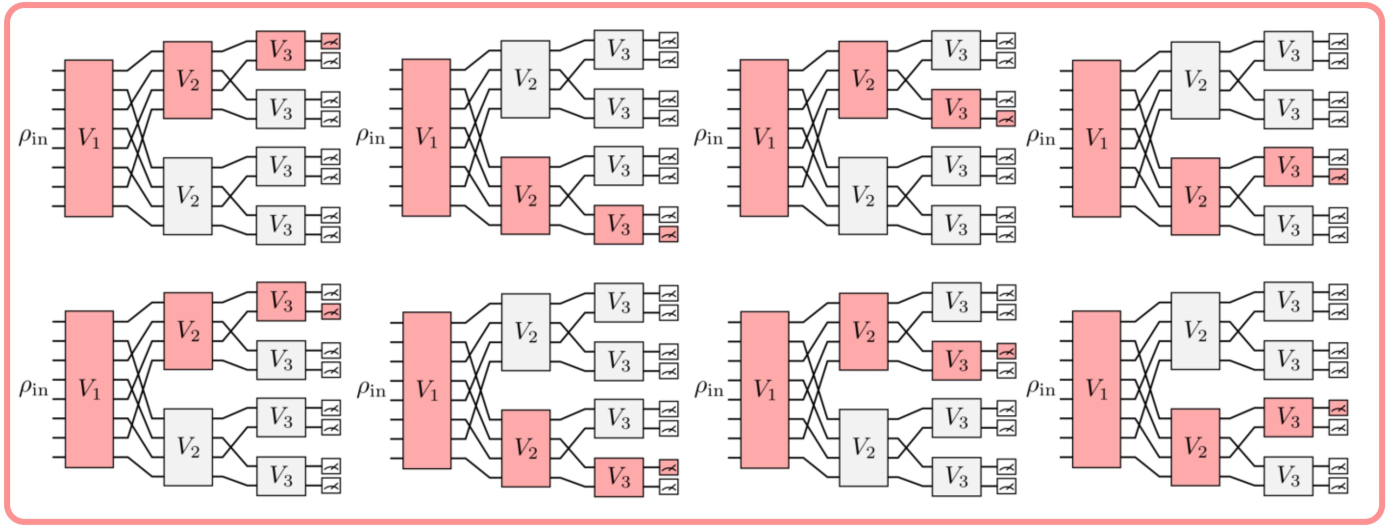
$$\mathrm{tr}\left[U\rho U^\dagger Z_1\right] = \mathrm{tr}\left[(TUT^\dagger)(T\rho T^\dagger)(TU^\dagger T^\dagger)Z_1\right] = \mathrm{tr}\left[U\rho U^\dagger Z_n\right]$$

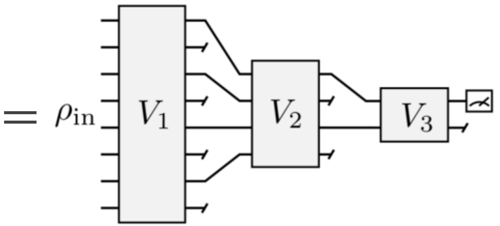
(T: translation operator, U: unitary operator for the entire circuit)

T-symmetric data × T-symmetric
- circuit splitting
- unitary operations

→ **The two subcircuits are equivalent**

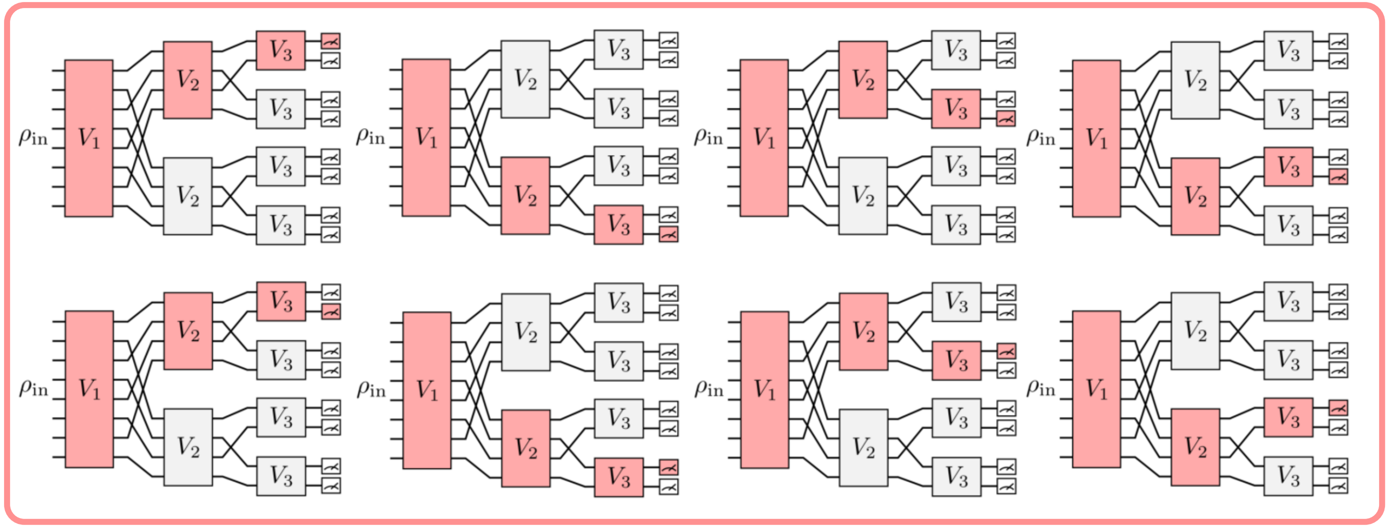
Mechanism for efficient measurement





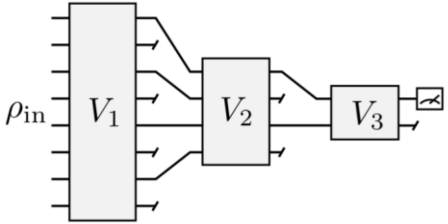
The n subcircuits are all equivalent to conventional QCNN
（Expectation values on all qubits are equal）

Mechanism for efficient measurement



conventional QCNN

$=$



The n subcircuits are all equivalent to conventional QCNN
（Expectation values on all qubits are equal）

The sp-QCNN can effectively parallelize n QCNNs.

Measurement efficiency is improved by a factor of $O(n)$

Performance verification

FUJITSU

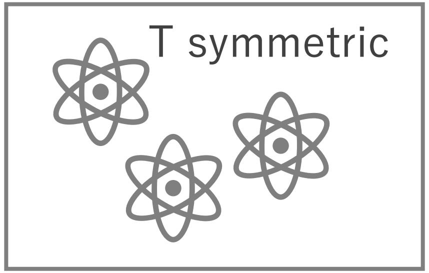
◆ Quantum phase recognition

Cong et al, Nat. Phys. (2019)

$$H = -\sum_{j=1}^{L} Z_j X_{j+1} Z_{j+2} - h_1 \sum_{j=1}^{L} X_j - h_2 \sum_{j=1}^{L} X_j X_{j+1}.$$

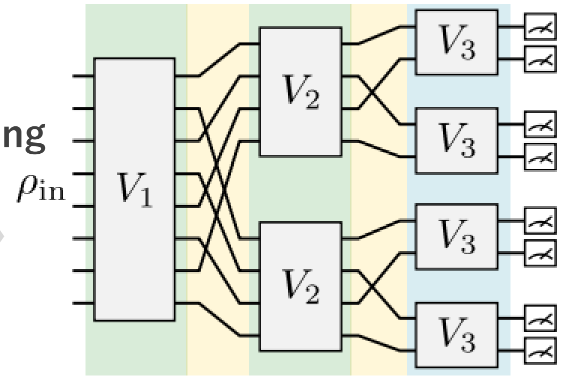
Ground state belongs to topological phase?

Training data
(some ground states)

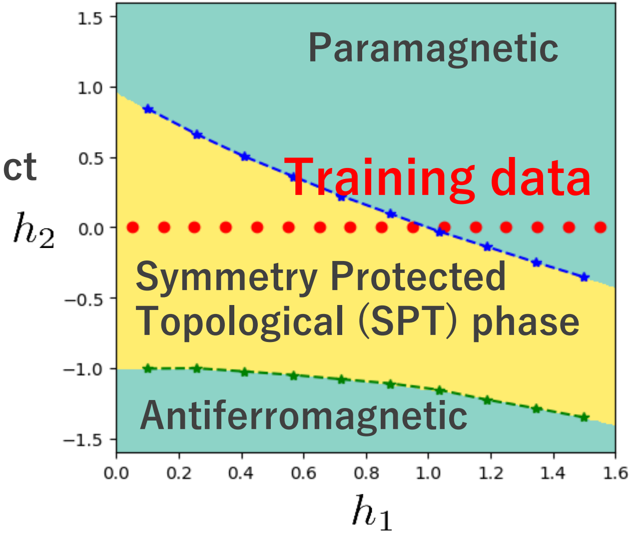


learning

sp-QCNN



predict



$$\text{Loss} = \frac{1}{2M} \sum_{i}^{M} \left(y_i - \sum_{j} \langle \phi_i | U^\dagger Z_j U | \phi_i \rangle / n \right)^2$$

The sp-QCNN accelerates the learning process

◆ Classical simulation (Qulacs)

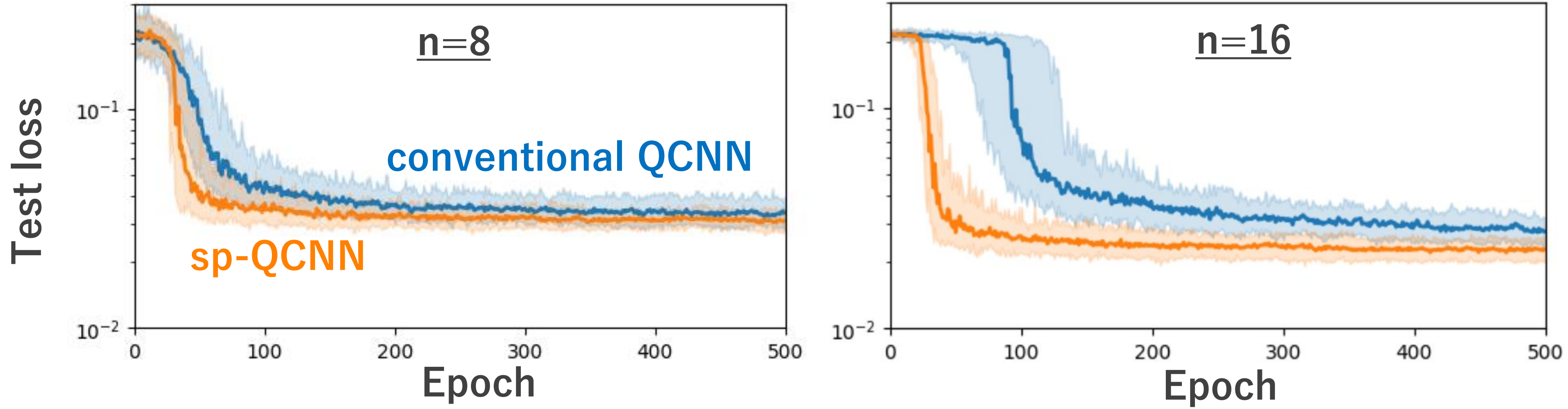
- Used a small number of measurement shots to estimate the gradient
  → Statistical errors can disturb the learning process

The sp-QCNN accelerates the learning process

FUJITSU

◆ Classical simulation (Qulacs)

➢ Used a small number of measurement shots to estimate the gradient
→ Statistical errors can disturb the learning process



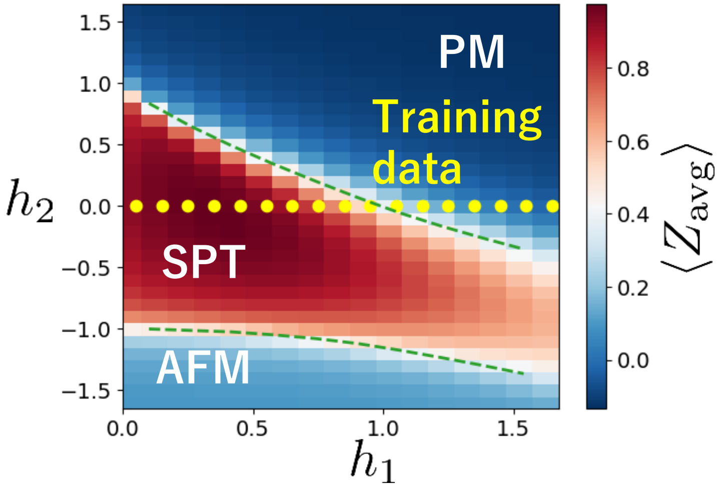
➢ High measurement efficiency suppresses statistical errors to stabilize and accelerate the learning process.
➢ Improvement becomes more obvious as the number of qubits increases

The sp-QCNN accelerates the learning process

◆ Classical simulation (Qulacs)

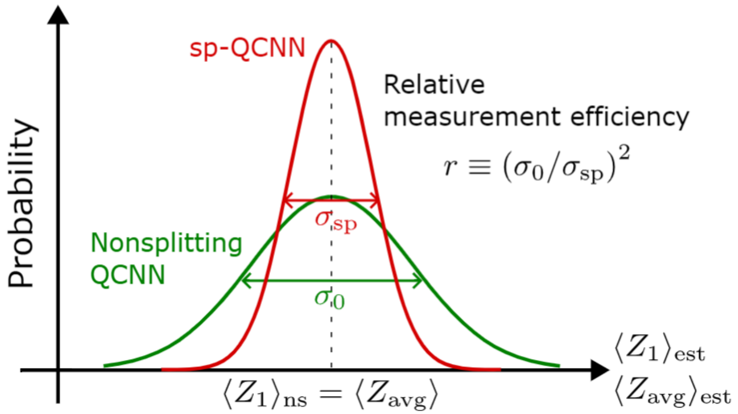
- Used a small number of measurement shots to estimate the gradient
  → Statistical errors can disturb the learning process

Phase diagram predicted by sp-QCNN



- Can predict the entire phase diagram
  = Good generalization

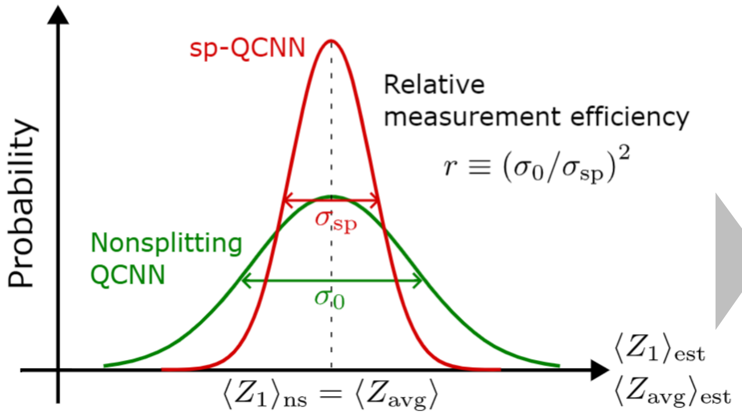
Quantifying measurement efficiency

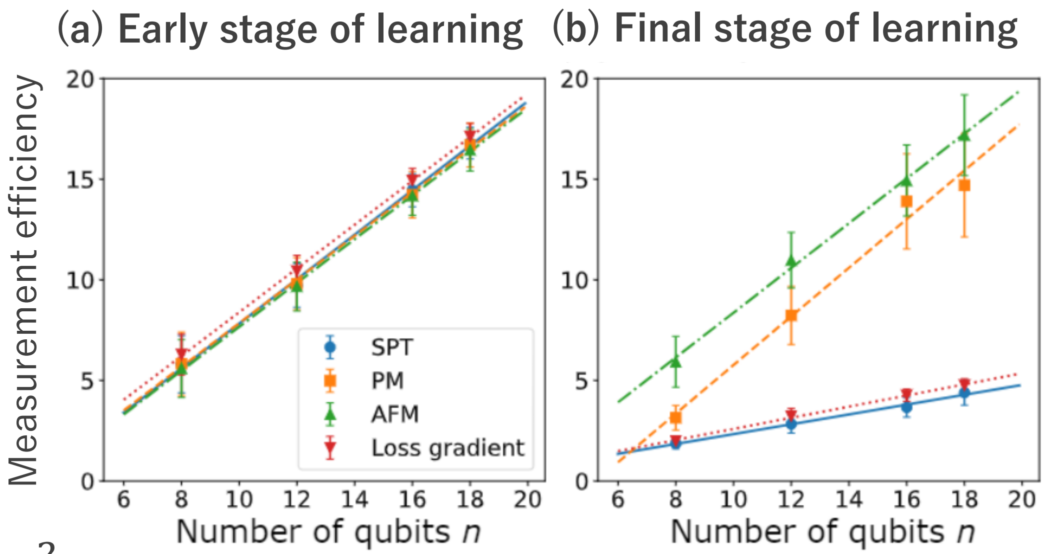


Measurement efficiency

$$= \left(\frac{\text{Estimation error in conventional QCNN}}{\text{Estimation error in sp−QCNN}}\right)^2$$

Quantifying measurement efficiency





Measurement efficiency

$$= \left(\frac{\text{Estimation error in conventional QCNN}}{\text{Estimation error in sp–QCNN}} \right)^2$$

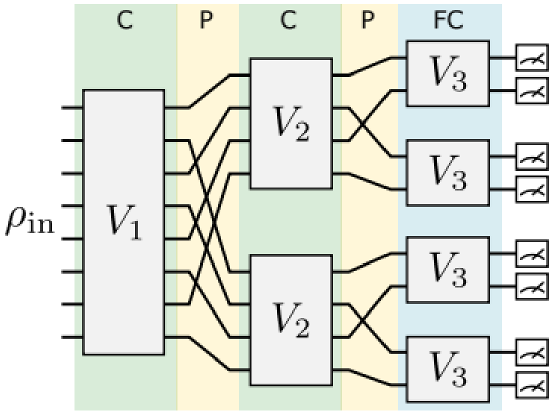
Measurement efficiency increases linearly as n
→ $\boldsymbol{O(n)}$ times improvement

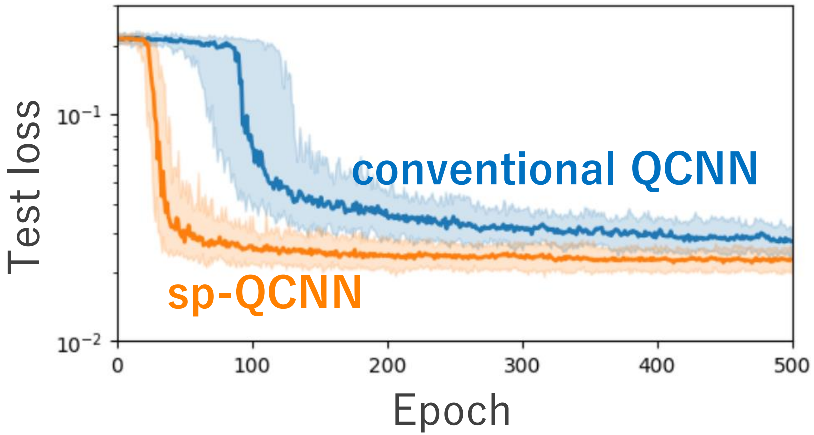
Summary

FUJITSU

Proposed an efficient model, sp-QCNN, based on prior symmetry knowledge

- It improves measurement efficiency by a factor of O(n) for translationally symmetric data





arXiv:2306.07331