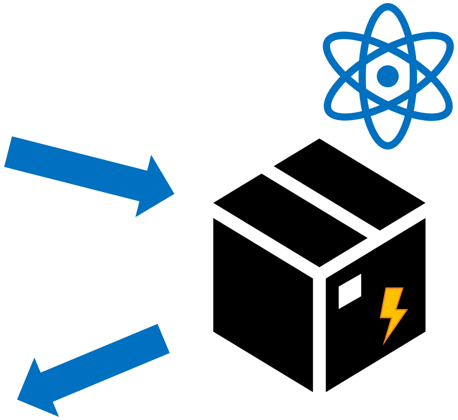
Channel **tomography** for quantum noise **characterization** and mitigation



Simone Roncallo

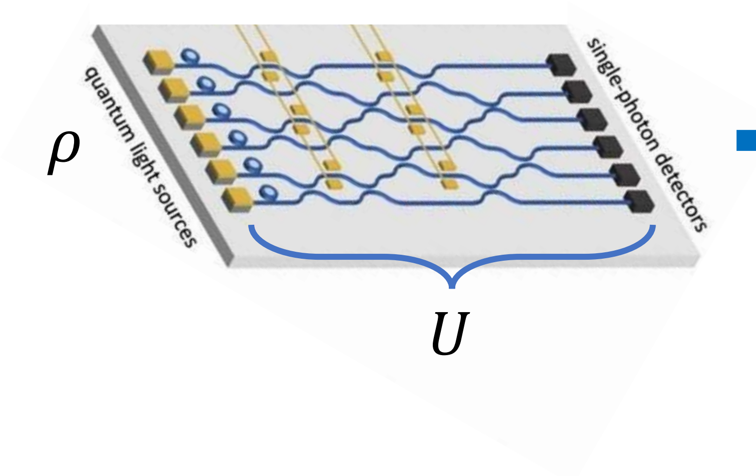
Department of Physics @ University of Pavia

Joint work with L. Maccone and C. Macchiavello

QTML 2023 at CERN

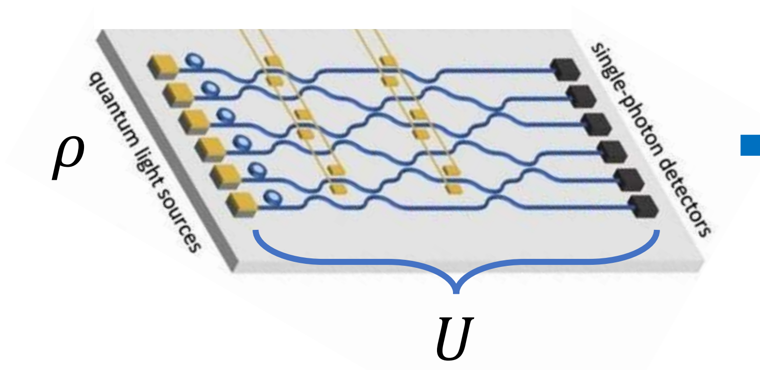
22 November 2023

Noisy computations

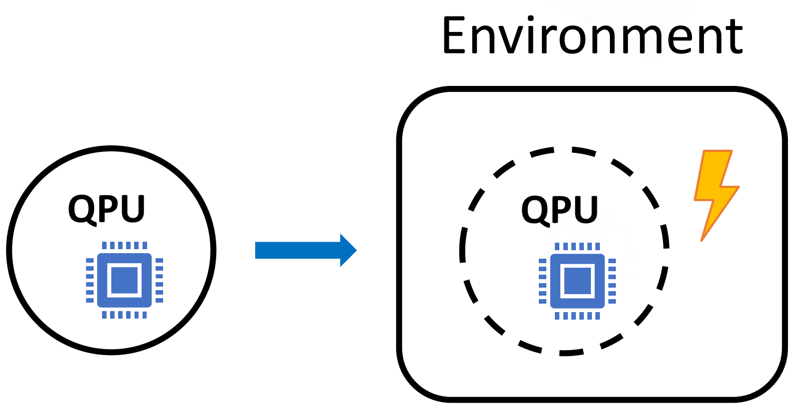


POTENTIAL
ADVANTAGE

Noisy computations



POTENTIAL
ADVANTAGE



NOISE

$$U\rho_S U^\dagger \rightarrow U_{SE}(\rho_S \otimes \rho_E)U_{SE}^\dagger$$

Noisy computations

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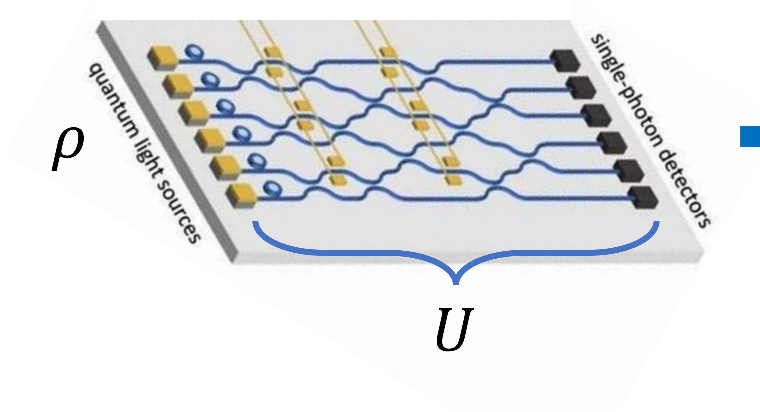
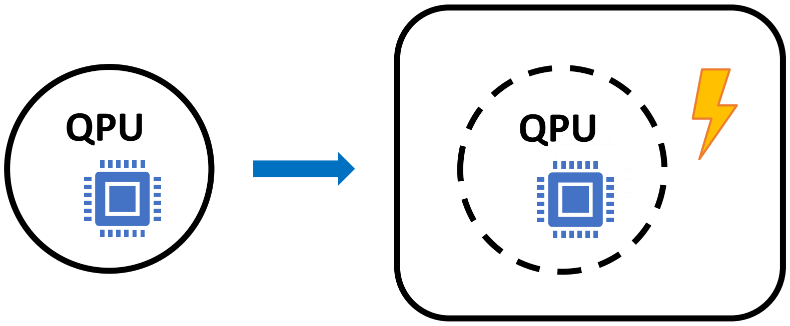


POTENTIAL ADVANTAGE

Environment



NOISE

$$U\rho_S U^\dagger \rightarrow U_{SE}(\rho_S \otimes \rho_E)U_{SE}^\dagger$$

Quantum channels

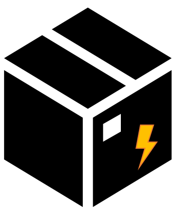
Quantum channels

Noise modelled by completely-positive trace-preserving (CPTP) maps

[V. Jagadish et al.] Quanta 7: 54-67 (2018)

Quantum channels

Noise modelled by completely-positive trace-preserving (CPTP) maps



$$\rho \rightarrow N(\rho)$$

[V. Jagadish et al.] Quanta 7: 54-67 (2018)

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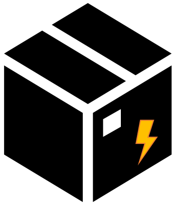
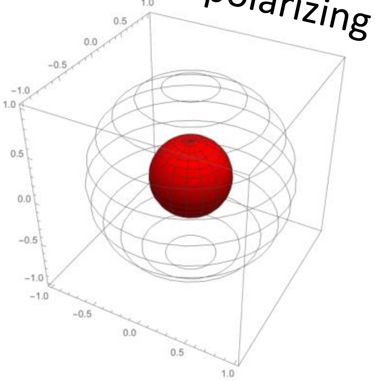
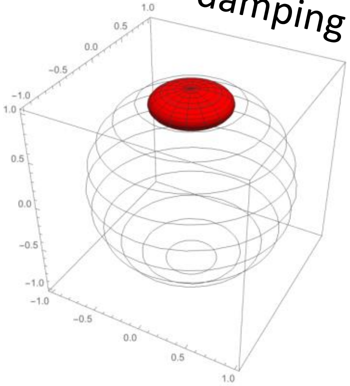


$$\rho \to N(\rho)$$





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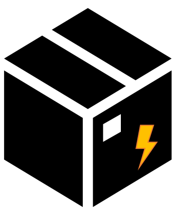

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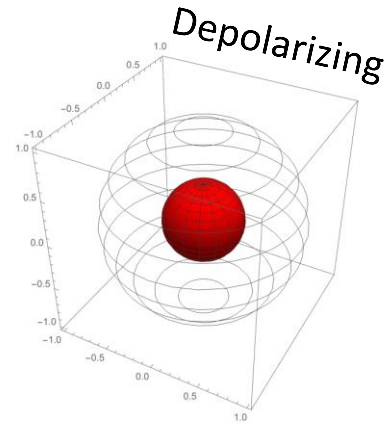

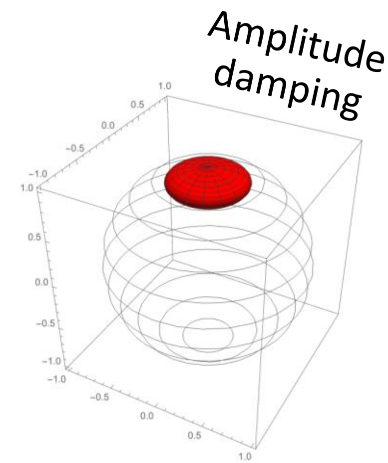

Several representations available…

[V. Jagadish et al.] Quanta 7: 54-67 (2018)

Pauli transfer matrix

n-qubit

Pauli basis $\{P_i\}$

H Pauli transfer matrix

$2^{2n} \times 2^{2n}$ matrix

n-qubit

Pauli basis $\{P_i\}$ ➡

Pauli transfer matrix (PTM)

$$\Gamma_{ij} = \frac{1}{2^n} \mathrm{Tr}[P_i N(P_j)]$$

Pauli transfer matrix

$2^{2n} \times 2^{2n}$ matrix

n-qubit

Pauli basis $\{P_i\}$

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PTMs act by matrix multiplication!

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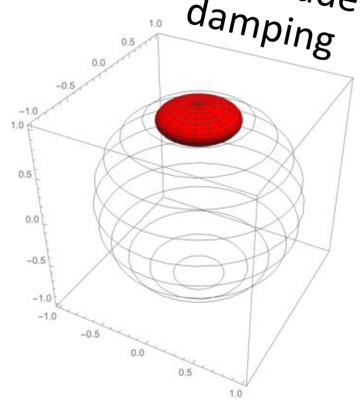
Pauli basis $\{P_i\}$

Pauli transfer matrix (PTM)

$$\Gamma_{ij} = \frac{1}{2^n} \mathrm{Tr}[P_i N(P_j)]$$

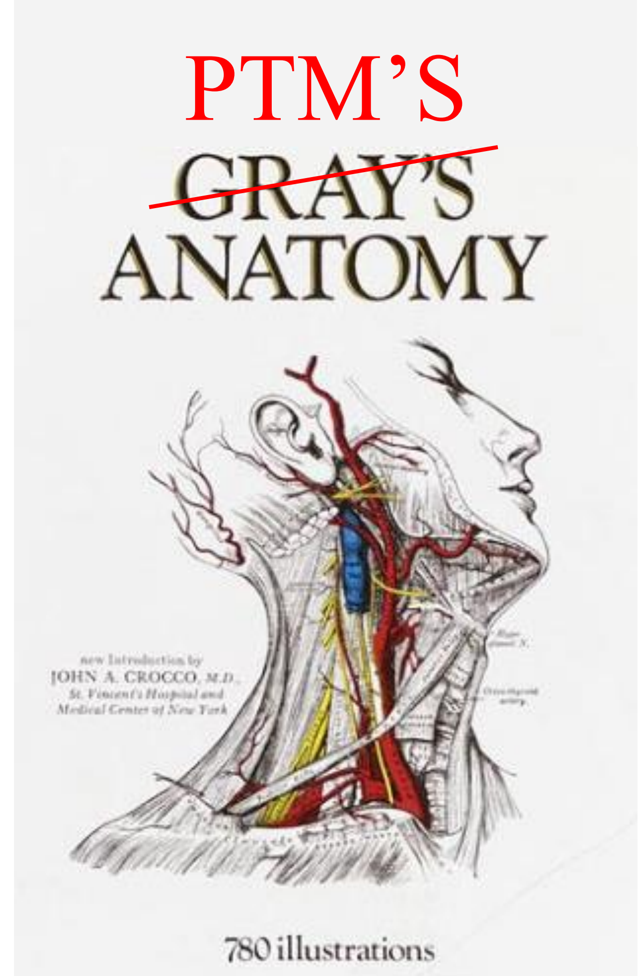
PTMs act by matrix multiplication!

$$\Gamma_{\mathrm{amp}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-p} & 0 & 0 \\ 0 & 0 & \sqrt{1-p} & 0 \\ p & 0 & 0 & 1-p \end{bmatrix}$$



Pauli transfer matrix

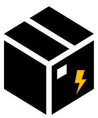
$$\text{PTM} = \left[\begin{array}{c|c} 1 & \vec{0} \\ \hline \vec{v} & M \end{array}\right]$$



Pauli transfer matrix

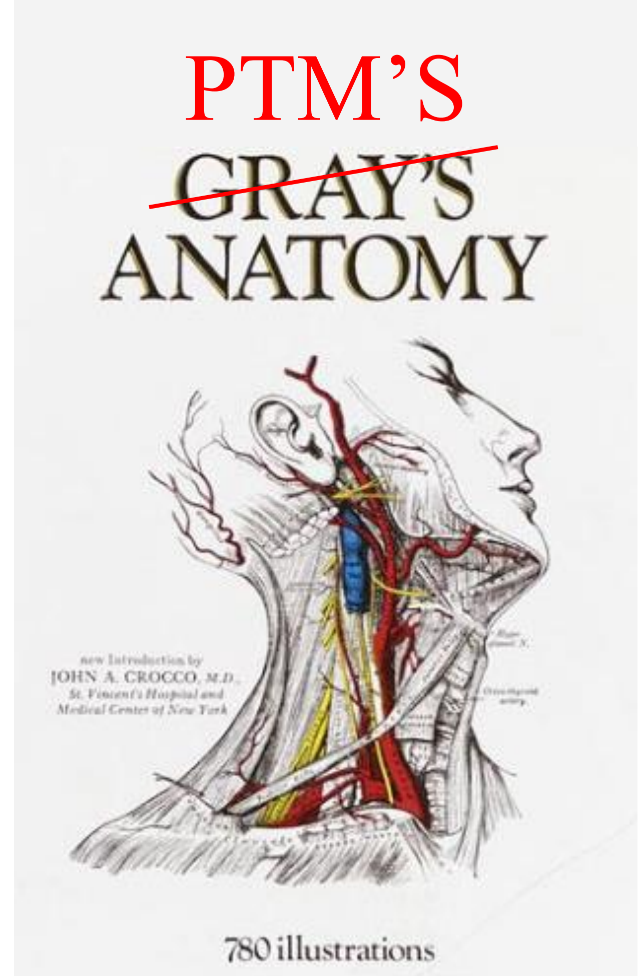
$2^{2n} \times 2^{2n}$ matrix

Trace-preserving



PTM

$$= \left[\begin{array}{c|c} 1 & \vec{0} \\ \hline \vec{v} & M \end{array} \right]$$



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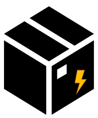
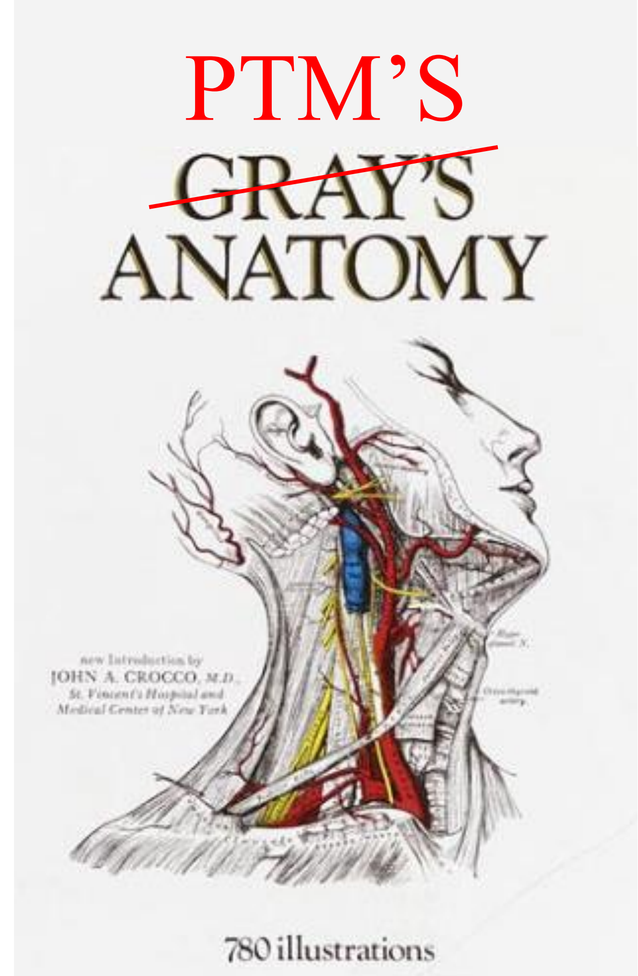


PTM

$$
= \left[\begin{array}{c|c} 1 & \vec{0} \\ \hline \vec{v} & M \end{array}\right]
$$

Non-unitality

$N(I) \neq I$



Pauli transfer matrix

$2^{2n} \times 2^{2n}$ matrix

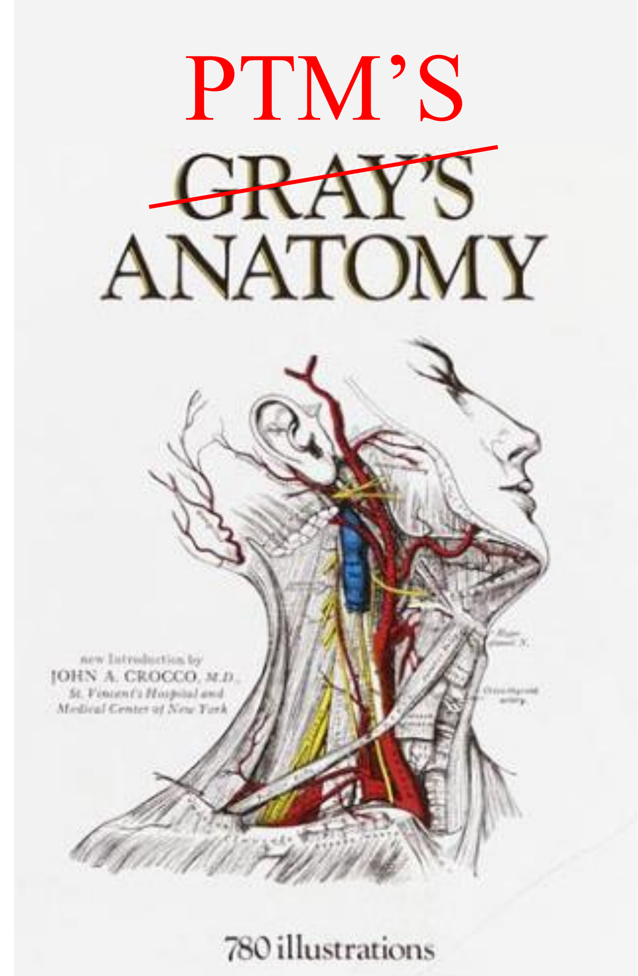
Trace-preserving


PTM

$$= \left[\begin{array}{c|c} 1 & \vec{0} \\ \hline \vec{v} & M \end{array}\right]$$

Non-unitality
$N(I) \neq I$

$(2^{2n} - 1)^2$
entries



$-\boxed{H}-$ Pauli transfer matrix

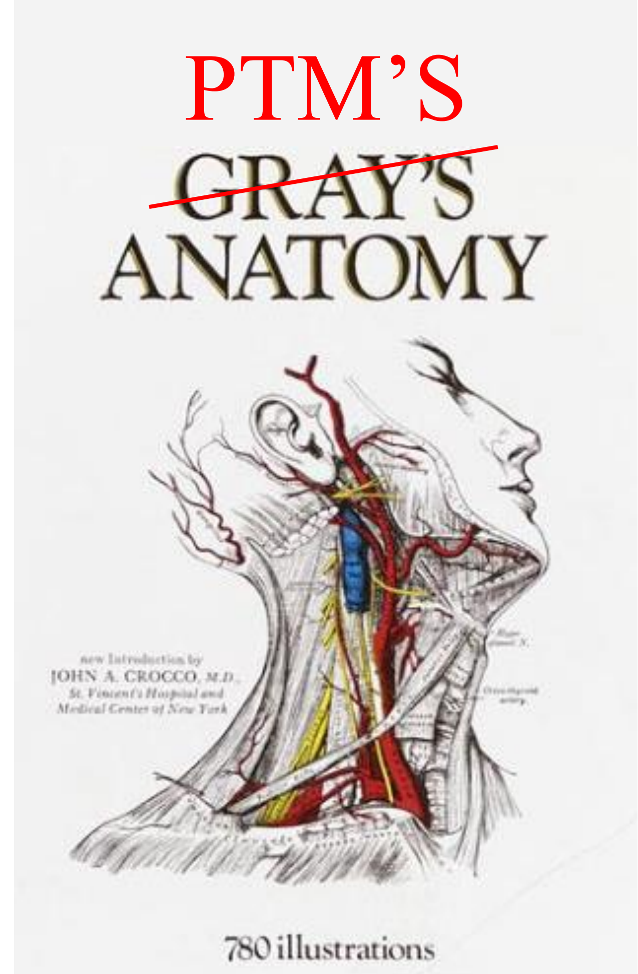
$2^{2n} \times 2^{2n}$ matrix

Trace-preserving

$$\text{PTM} = \left[\begin{array}{c|c} 1 & \vec{0} \\ \hline \vec{v} & M \end{array}\right]$$

Non-unitality $N(I) \neq I$ → $\vec{v}$

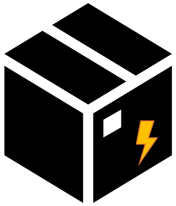
$(2^{2n}-1)^2$ entries → M



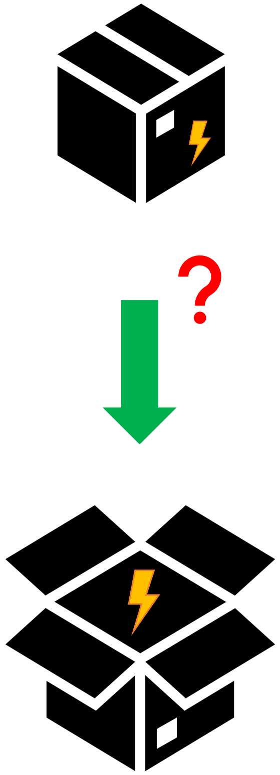
Diagonal for Pauli channels!

$2^{2n} - 1$ entries

Channel tomography



Channel tomography



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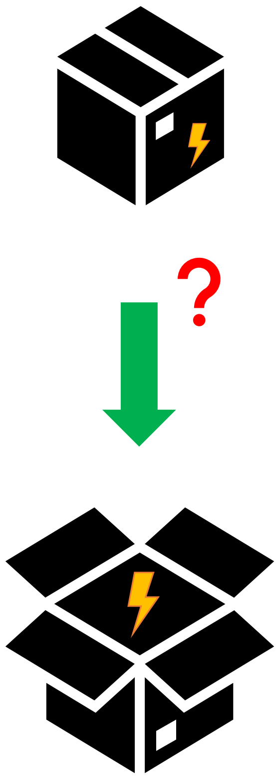
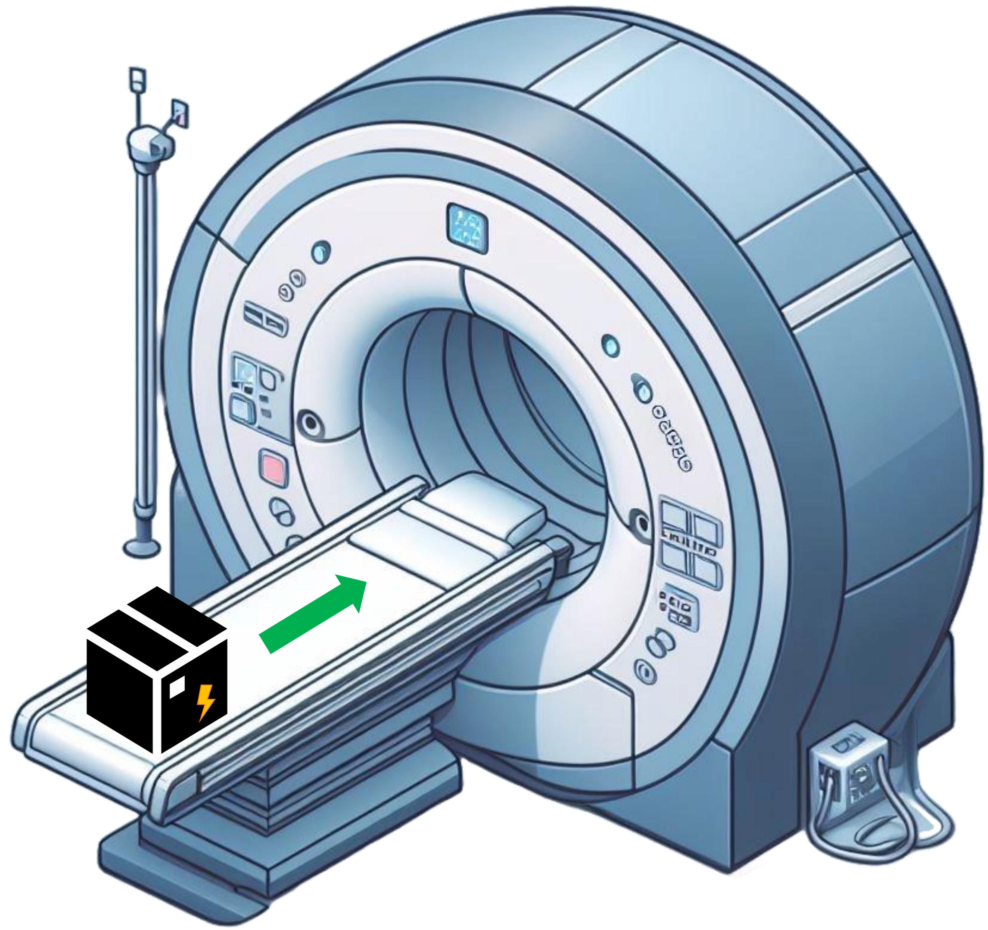




Quantum process tomography

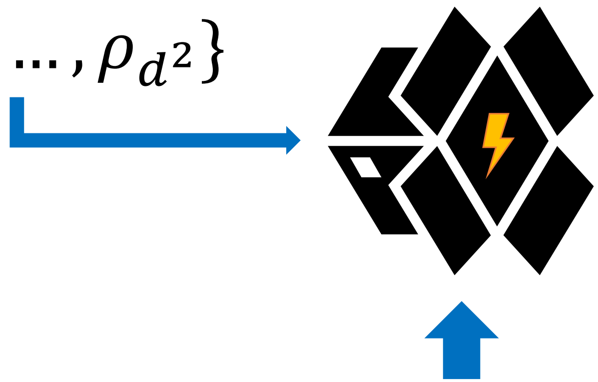
$$\{\rho_1, \rho_2, \dots, \rho_{d^2}\}$$

$$\{O_1, O_2, \dots, O_{d^2}\}$$

Quantum process tomography

QPT

$\{\rho_1, \rho_2, \dots, \rho_{d^2}\}$



$\{O_1, O_2, \dots, O_{d^2}\}$

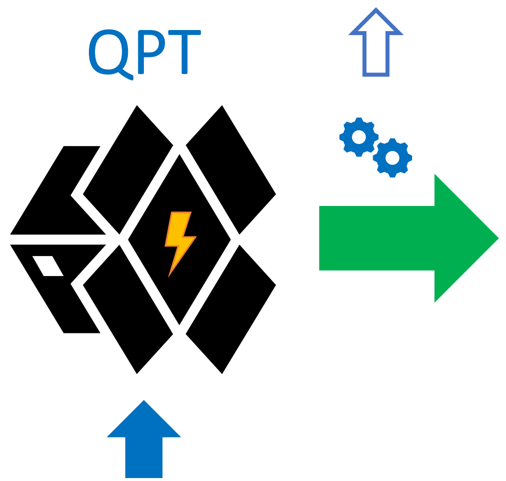
Quantum process tomography

post-processing

linear inversion, maximum likelihood, …

QPT

$\{\rho_1, \rho_2, \dots, \rho_{d^2}\}$



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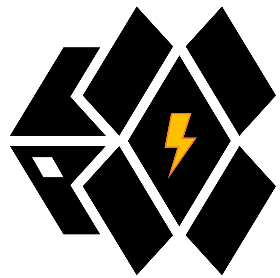

$$\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \Gamma_{d^2 0} & \cdots & \Gamma_{d^2 d^2} \end{bmatrix}$$

$2^{2n} \times 2^{2n}$ matrix

$\{O_1, O_2, \dots, O_{d^2}\}$

Quantum process tomography

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linear inversion, maximum likelihood, ...

QPT

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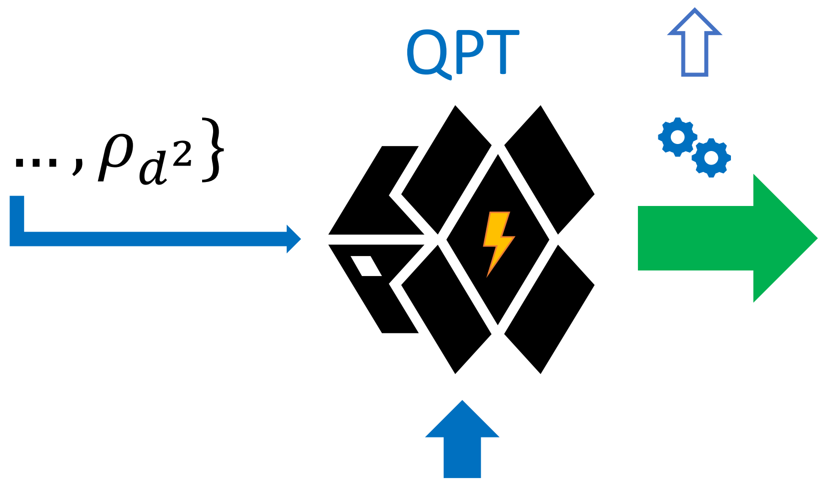

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$2^{2n} \times 2^{2n}$ matrix

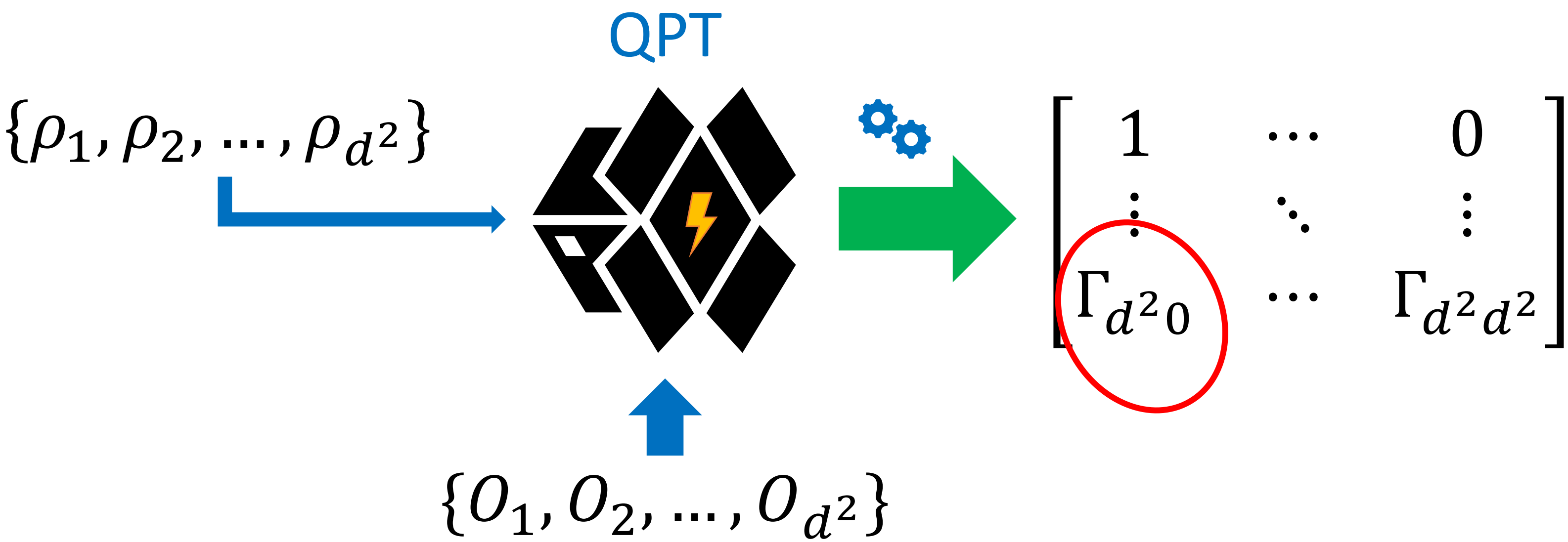
$\{O_1, O_2, \dots, O_{d^2}\}$

for n qubits

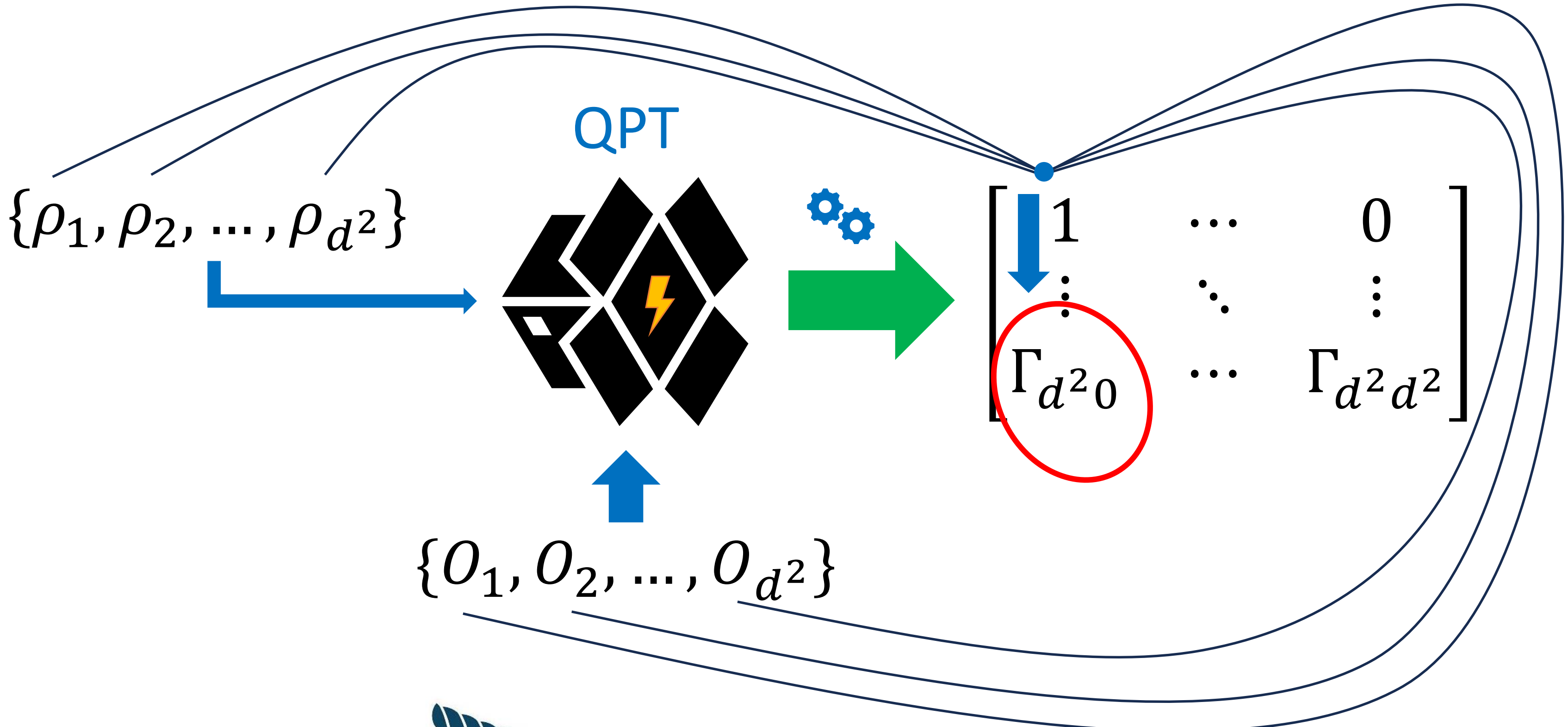
2^{4n} experiments

Resource-hungry!

Selective characterizations



Selective characterizations

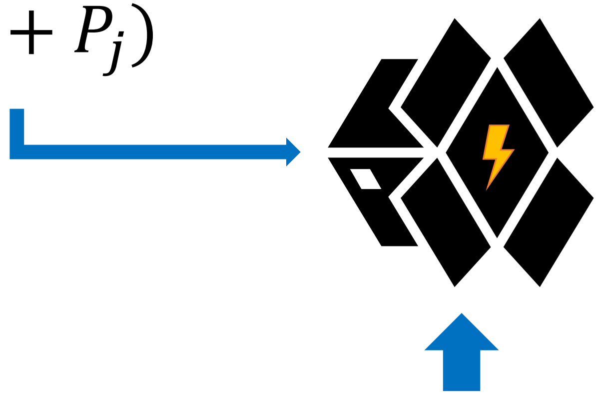


Lacks flexibility!

Pauli transfer matrix direct reconstruction

$$\frac{1}{d}(1 + P_j)$$

DPTM



P_i

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Pauli transfer matrix direct reconstruction

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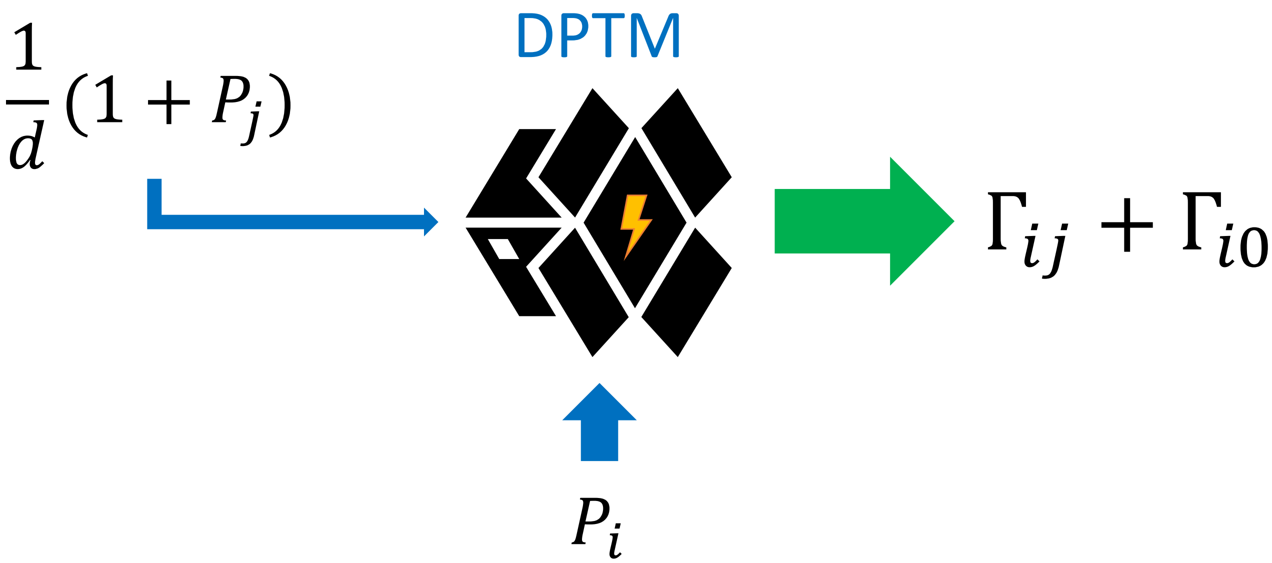

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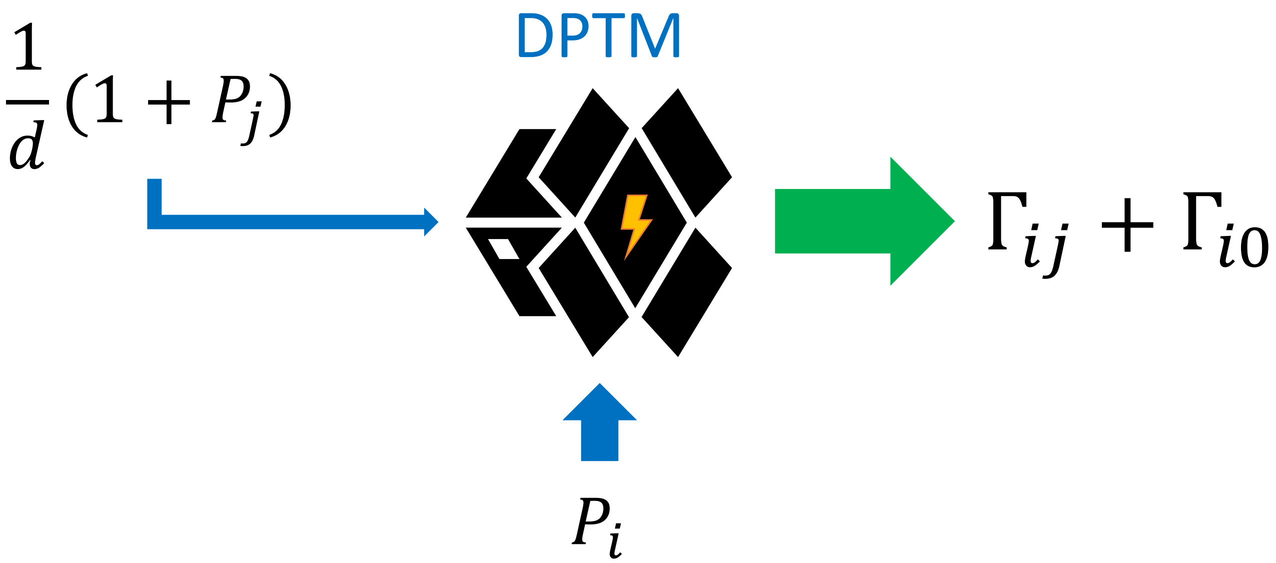

Direct selective characterizations!

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Pauli transfer matrix direct reconstruction

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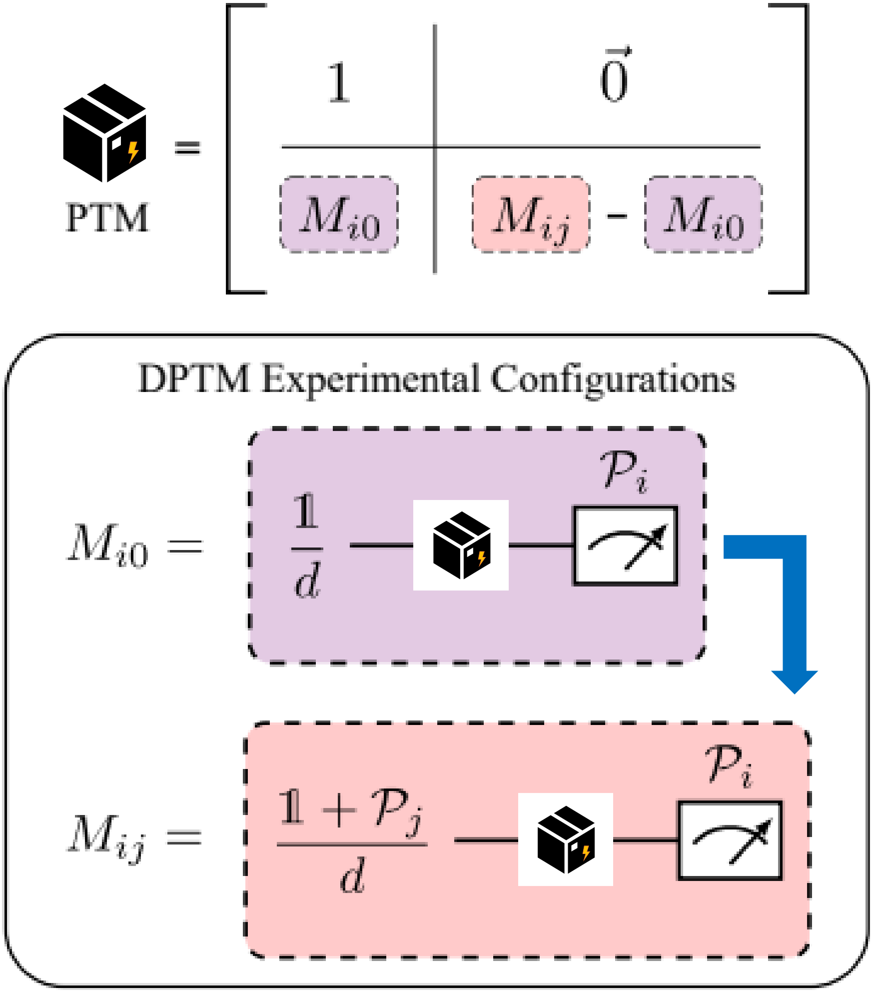
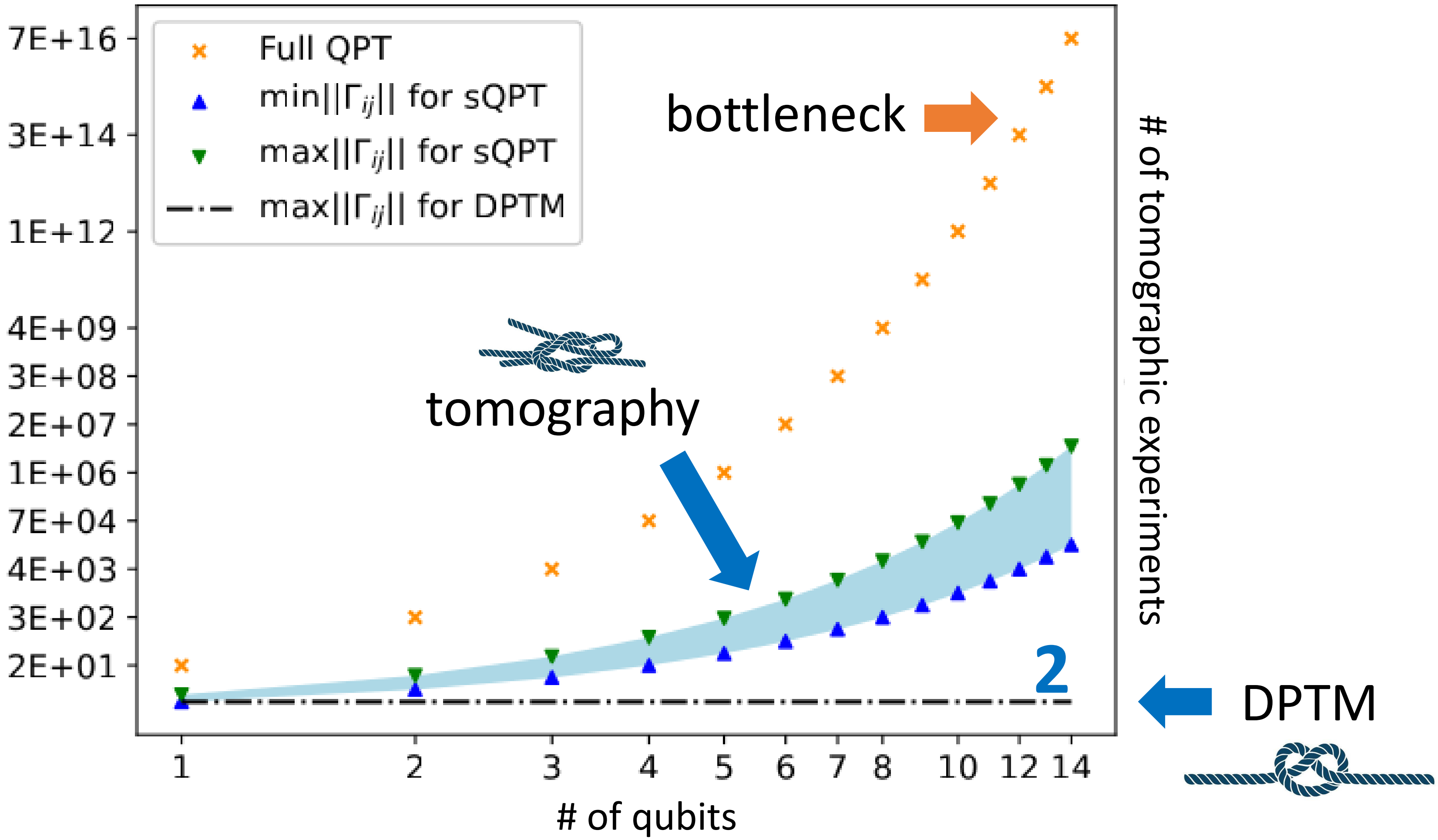


Single PTM component

single Γ_{ij} extraction

Statistics unchanged!



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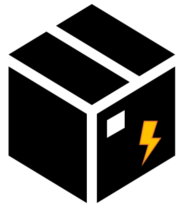
Noisy expectations

Noise modelled by completely-positive trace-preserving (CPTP) maps



$$\rho \to N(\rho)$$

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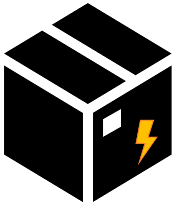

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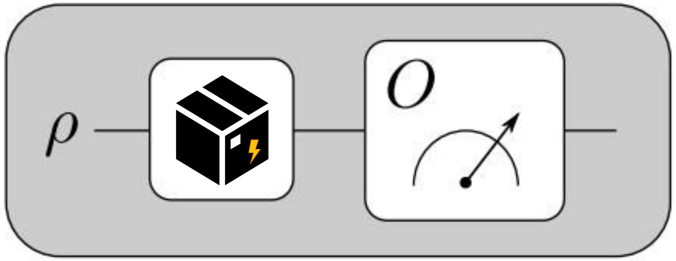
Expectation values becomes noisy!



$$\langle O \rangle_{N(\rho)}$$

Noisy expectations

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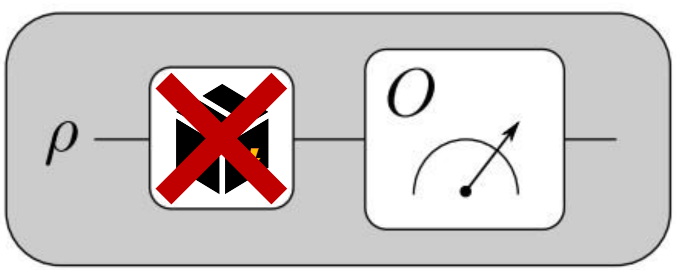
$$\rho \to N(\rho)$$

Expectation values becomes noisy!



$$\langle O \rangle_{\cancel{N}(\rho)}$$

Noise deconvolution

Generic mitigation by post-processing correction

$$\langle O\rangle_{\rho} = \sum_{k,j=0}^{d^2-1} (\Gamma^{-1})_{kj} \mathrm{Tr}[O P_j] \langle P_k\rangle_{N(\rho)}$$

[S. Mangini, L. Maccone and C. Macchiavello] EPJ Quantum Technol. **9**, 29 (2022)

[**SR**, L. Maccone and C. Macchiavello] PRA **107**, 022419 (2023)

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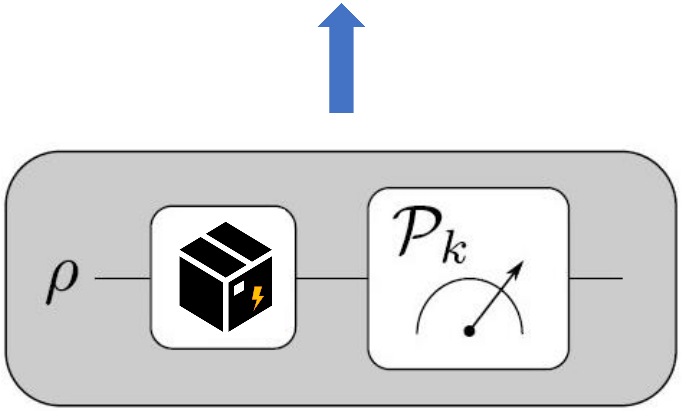

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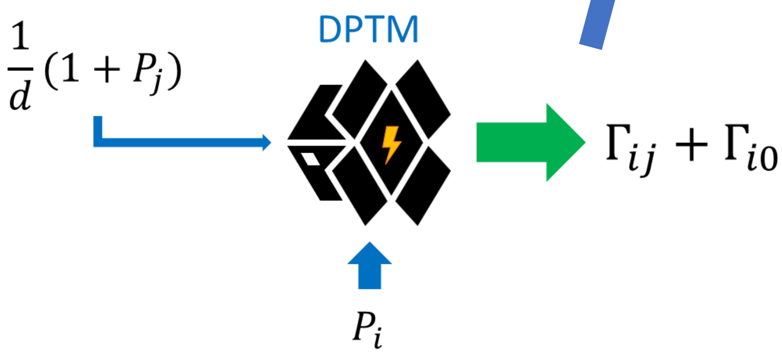

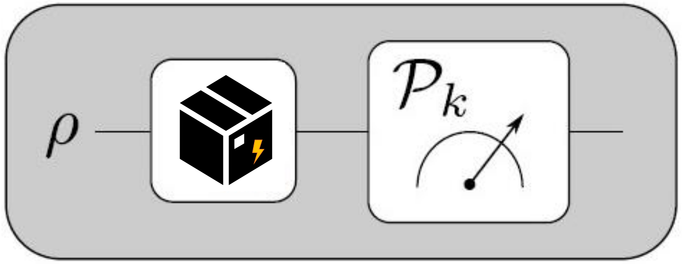

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Noise deconvolution

~~Generic~~ mitigation by post-processing correction
Pauli

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Depolarizing channel

Deconvolution of $\langle X\otimes Y\rangle$:

[S. Mangini, L. Maccone and C. Macchiavello] EPJ Quantum Technol. **9**, 29 (2022)

[**SR**, L. Maccone and C. Macchiavello] PRA **107**, 022419 (2023)

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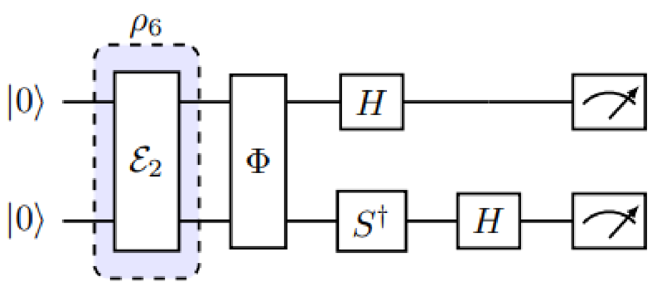

Γ_{66}

Depolarizing channel

Deconvolution of $\langle X \otimes Y \rangle$:
1 (DPTM) tomographic experiment instead of 6 (QPT)

[S. Mangini, L. Maccone and C. Macchiavello] EPJ Quantum Technol. **9**, 29 (2022)
[**SR**, L. Maccone and C. Macchiavello] PRA **107**, 022419 (2023)

Summary

$$\Phi_{\text{PTM}} = \left[\begin{array}{c|c} 1 & \vec{0} \\ \hline M_{i0} & M_{ij} - M_{i0} \end{array}\right]$$

- Direct PTM reconstruction vs standard QPT

Single-entry exponentially cheaper

H Summary

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SR, L. Maccone and C. Macchiavello

1. "Pauli transfer matrix direct reconstruction: channel characterization without full process tomography" Quantum Sci. Technol. **9** 015010 (2024)
2. "Multiqubit noise deconvolution and characterization" Phys. Rev. A **107**, 022419 (2023)

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