Overhead-constrained circuit knitting for variational quantum dynamics

QTML 2023 – CERN

Gian Gentinetta

Computational Quantum Science Laboratory

Hybrid quantum-classical quantum simulations

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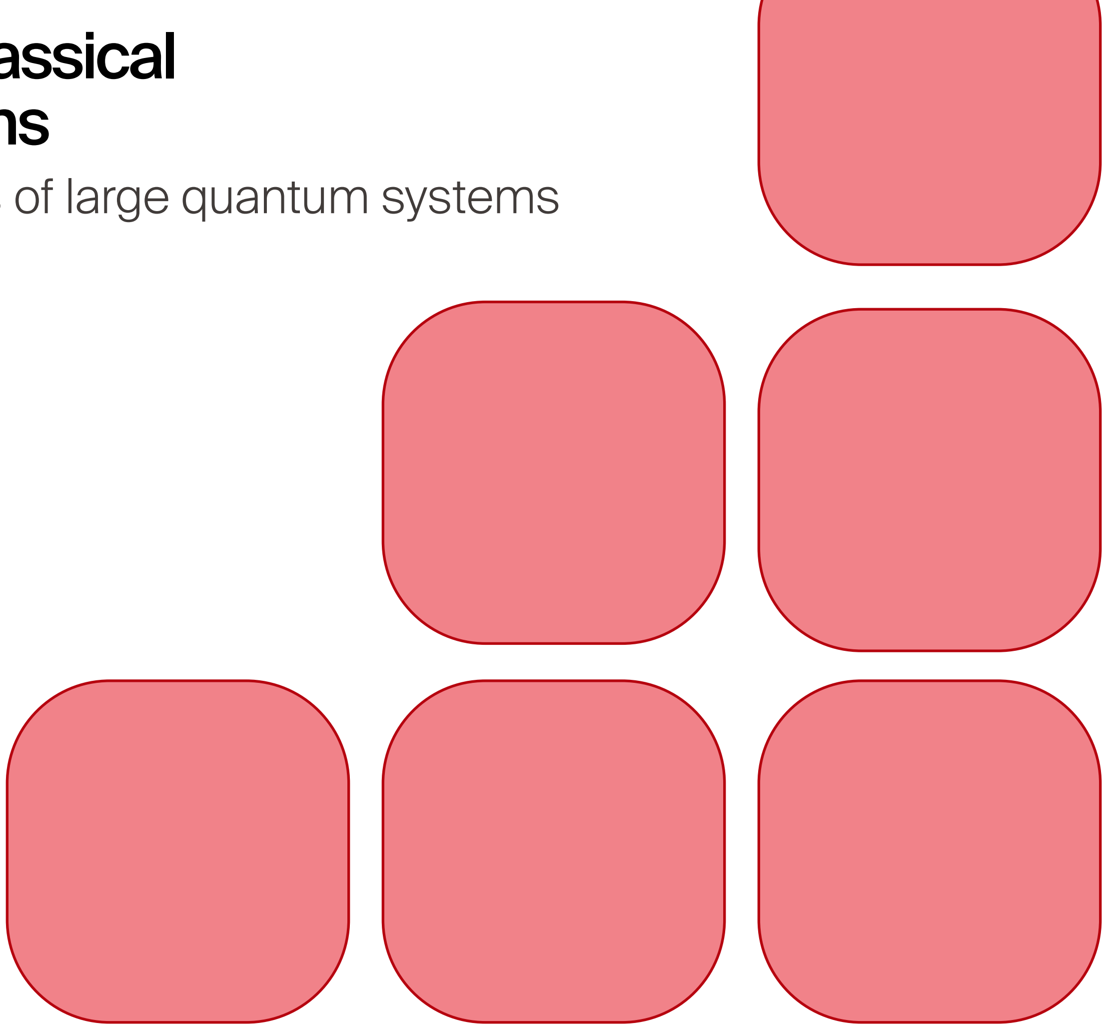
- Goal: Simulate dynamics of large quantum systems



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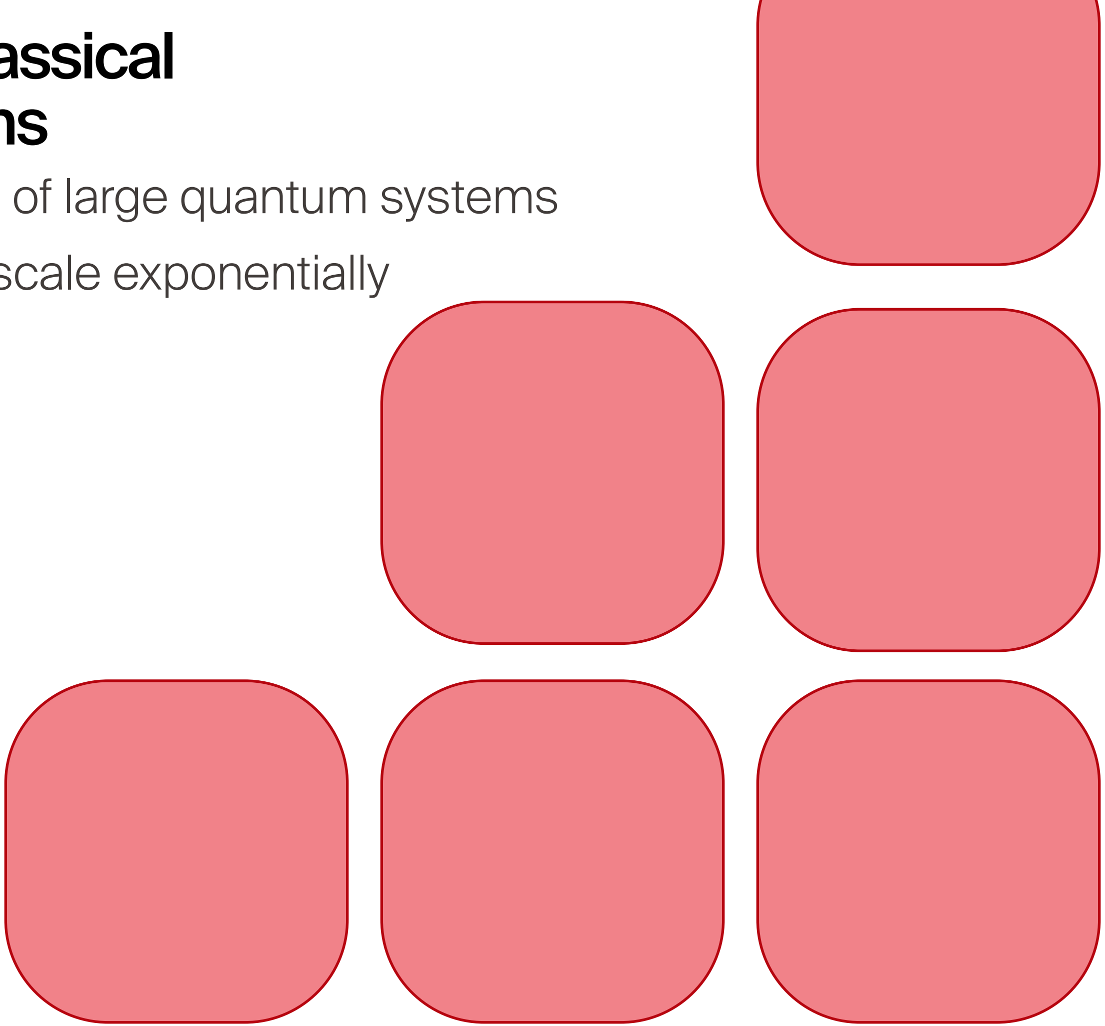

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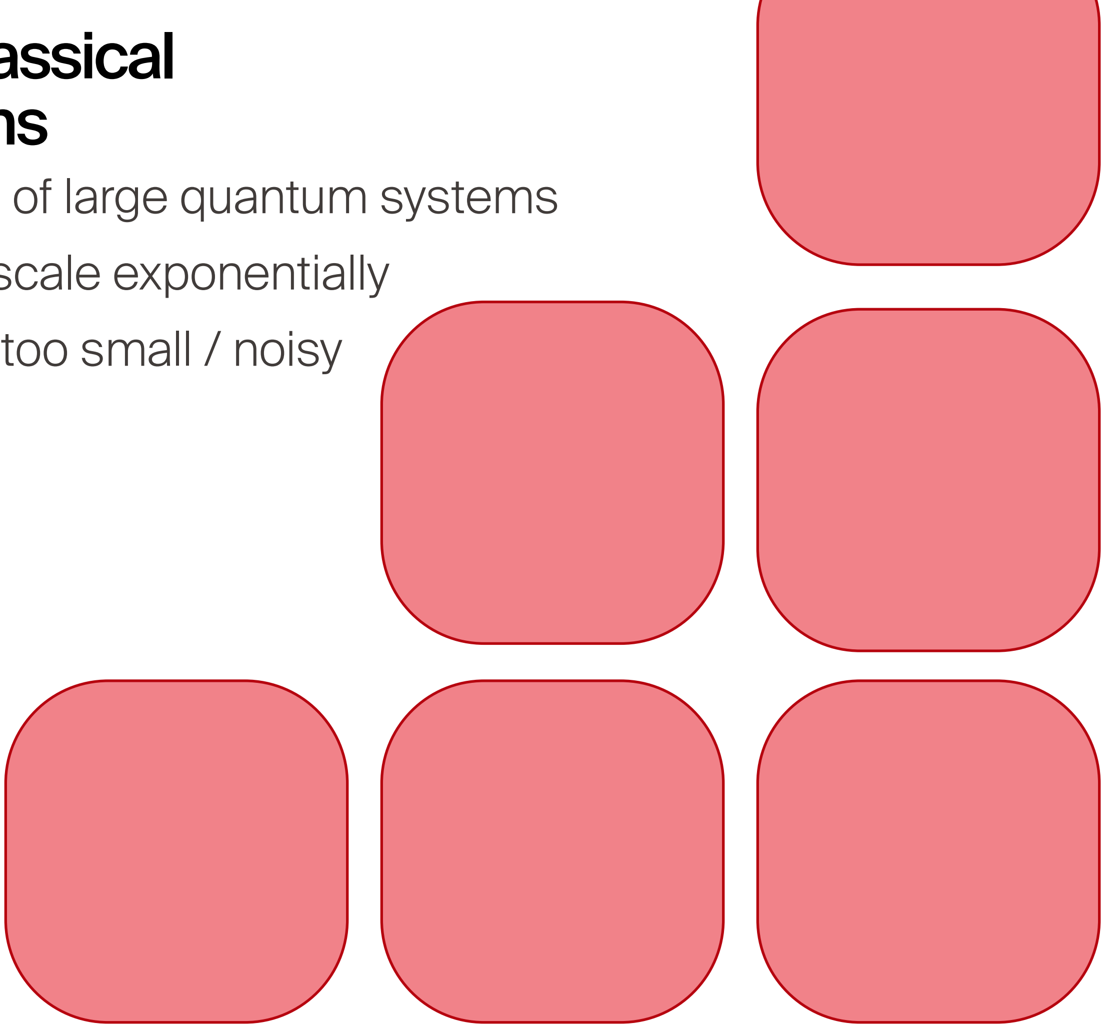

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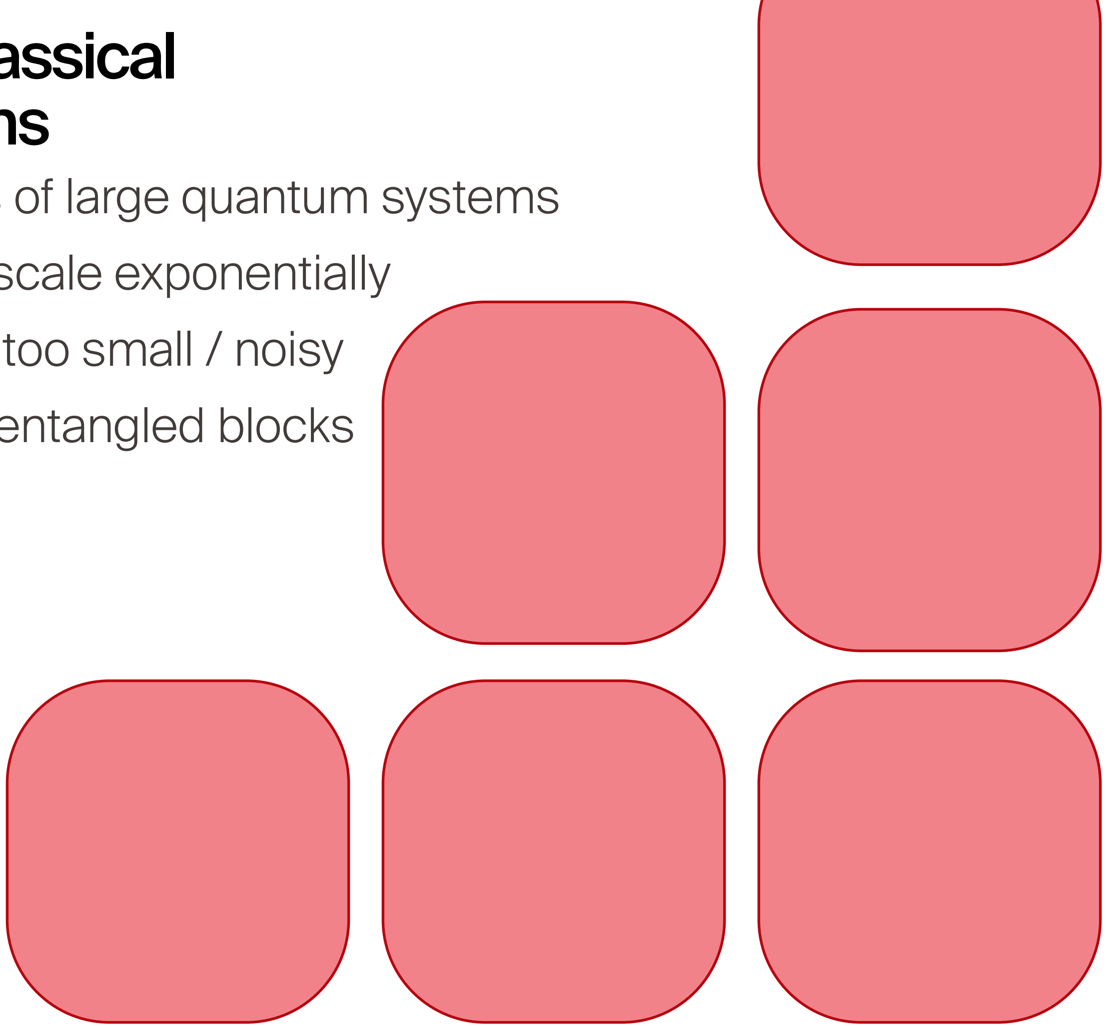

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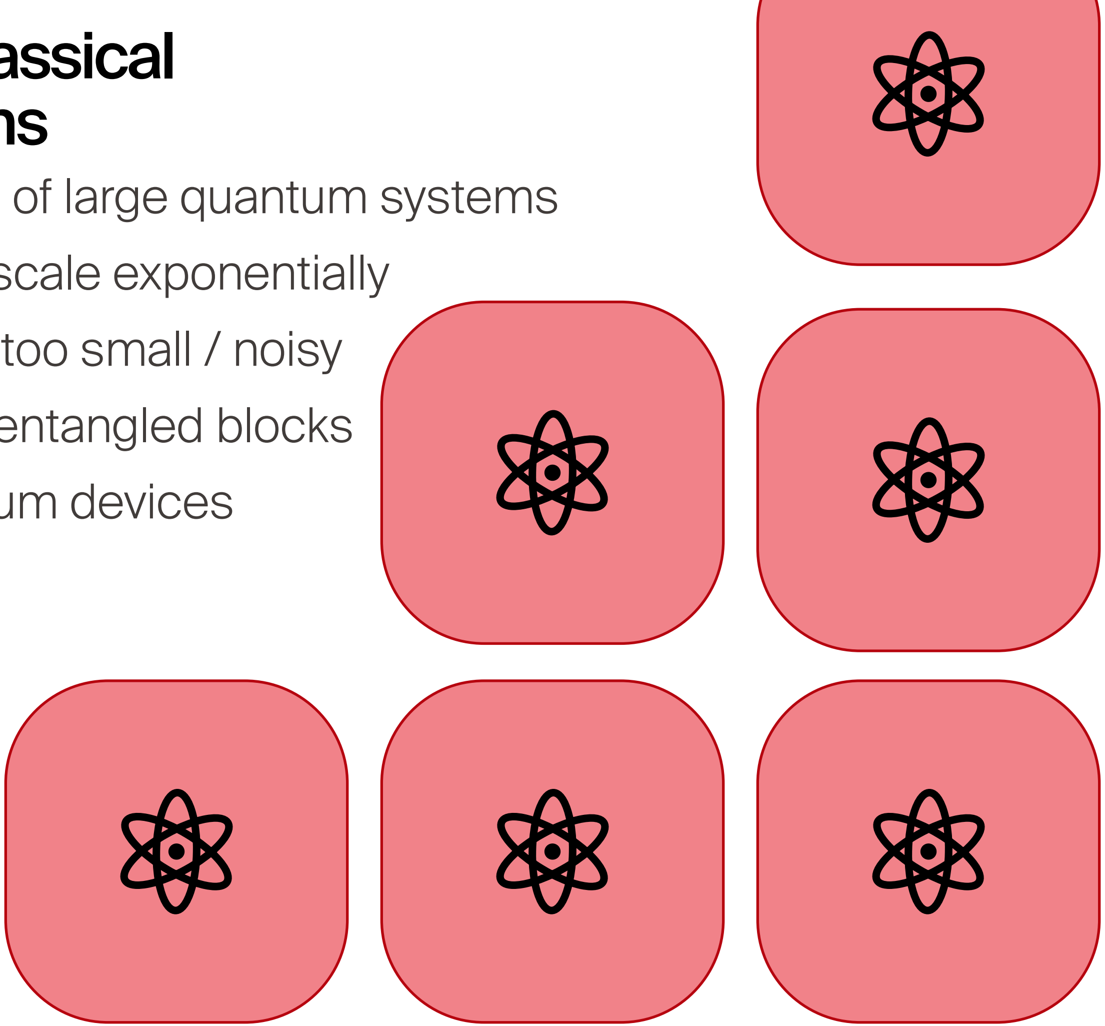

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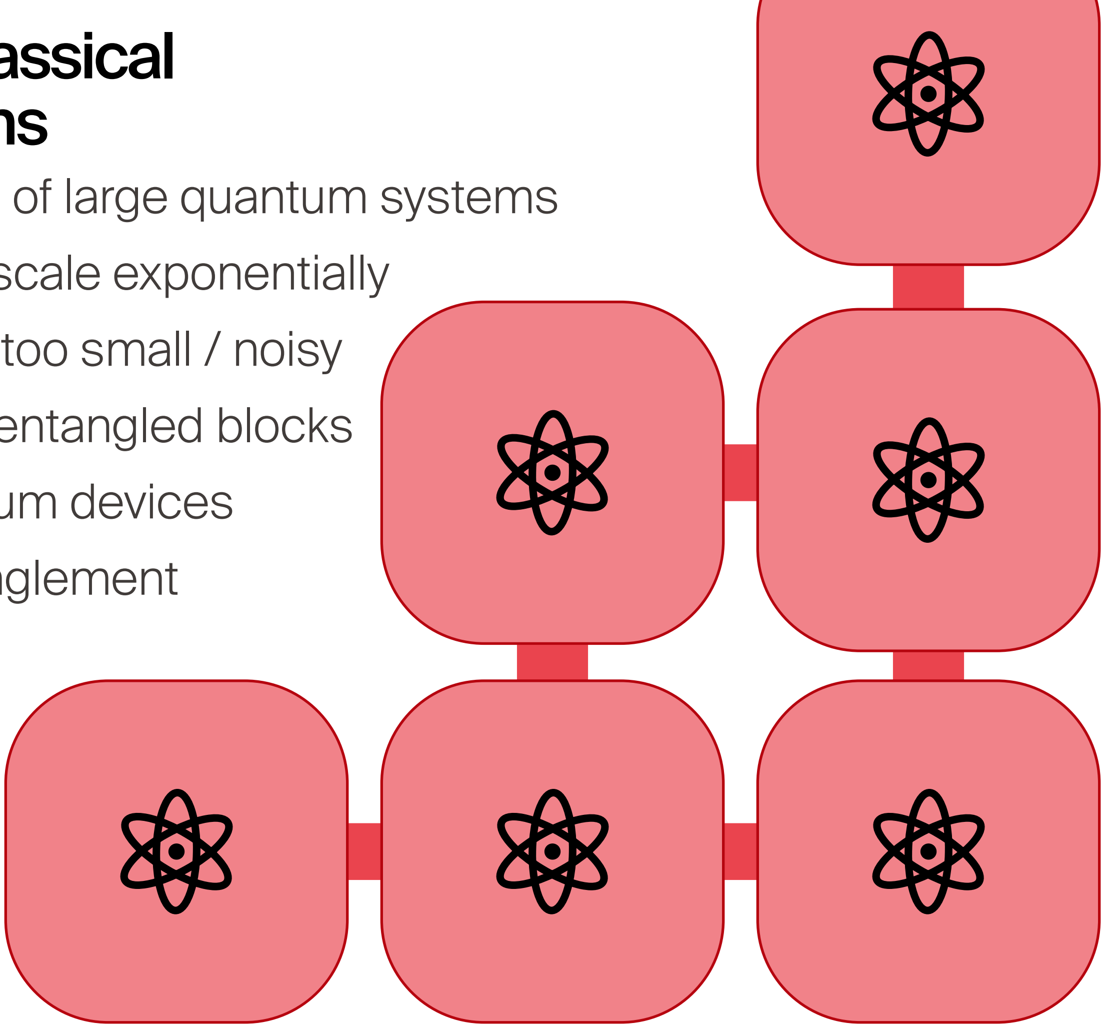

Hybrid quantum-classical quantum simulations

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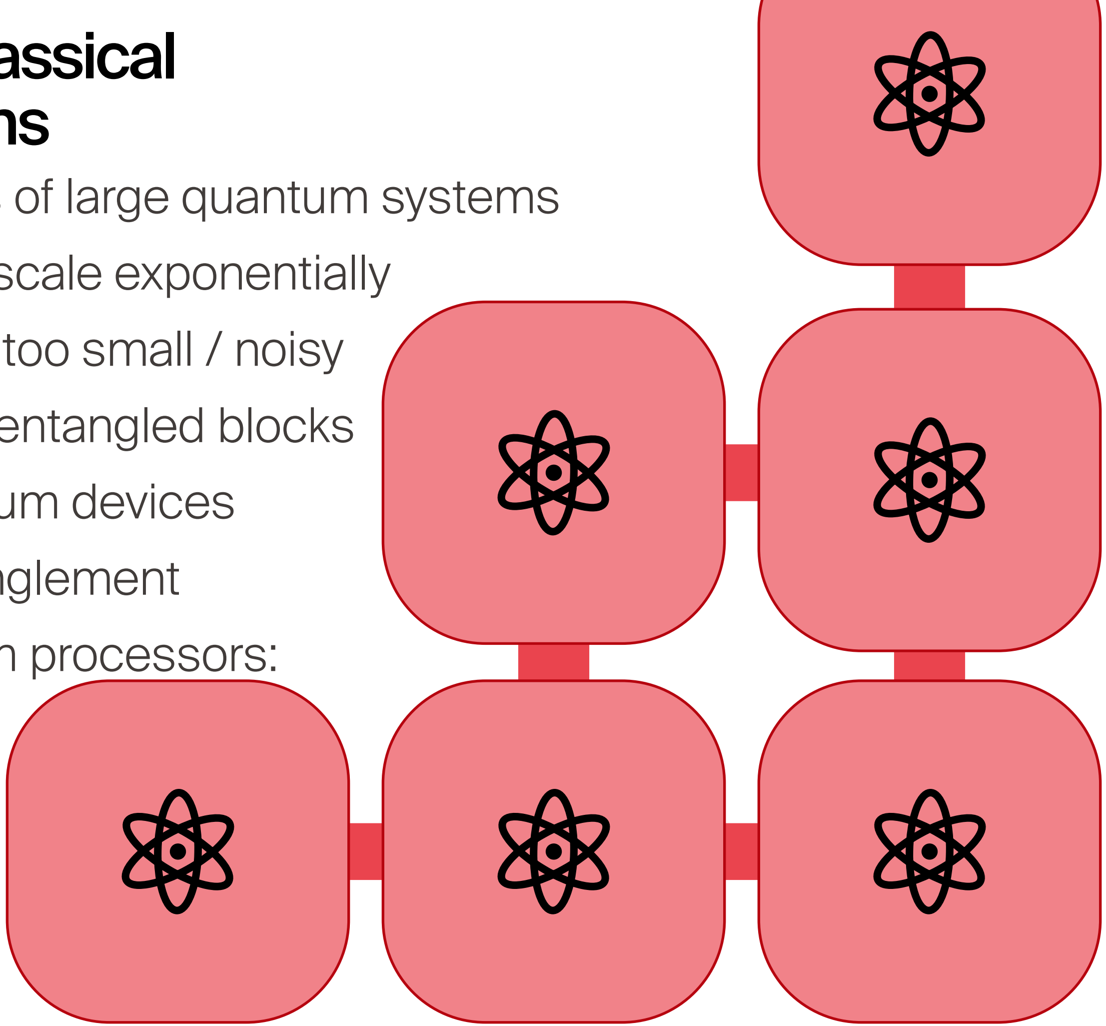
- Next generation quantum processors:



IBM Quantum Roadmap (2022)

Divide and conquer

- Simulate dynamics of spins in a transverse field Ising model

$$H = \sum_{\langle ij \rangle} J_{ij} Z_i Z_j + \sum_i X_i$$

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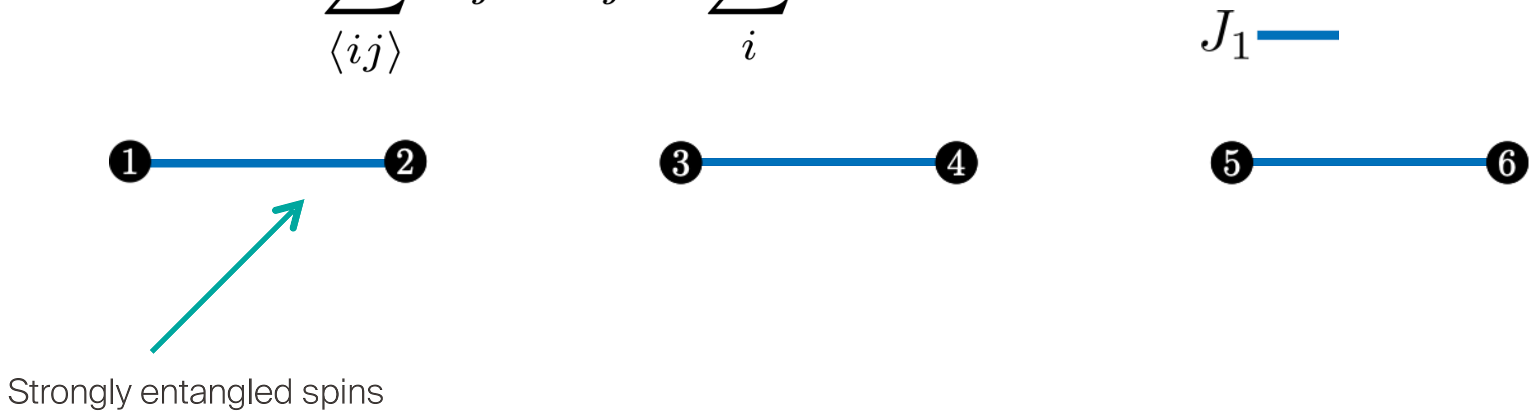

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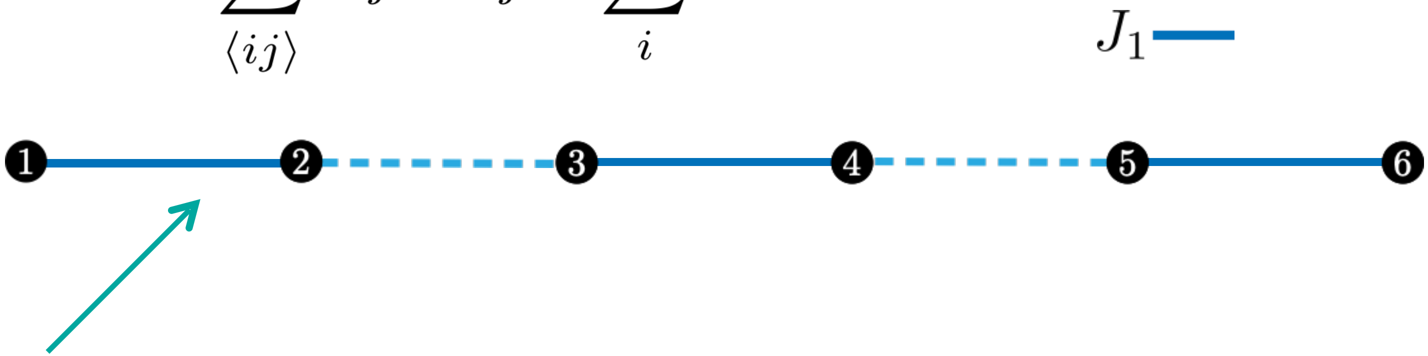

Strongly entangled spins

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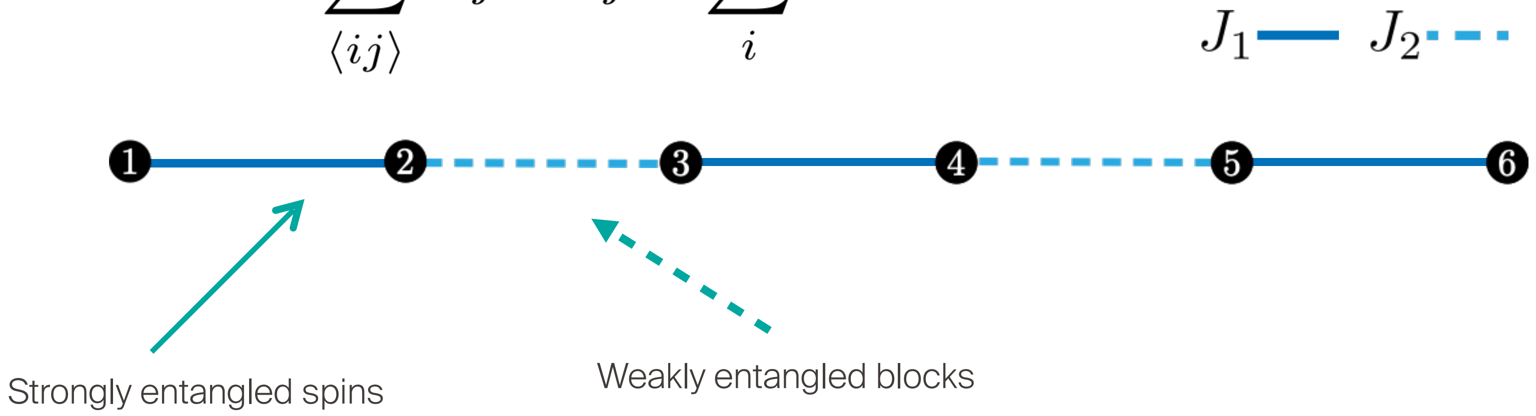

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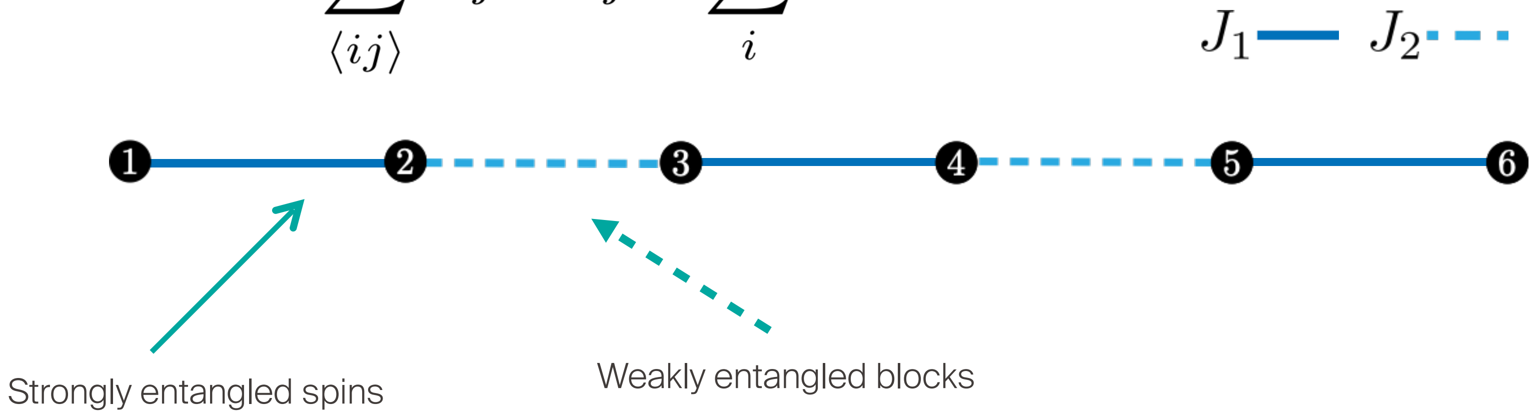

$\Longrightarrow$Cut system into blocks

Projected variational quantum dynamics (PVQD)

Barison, Vicentini, Carleo (2021)

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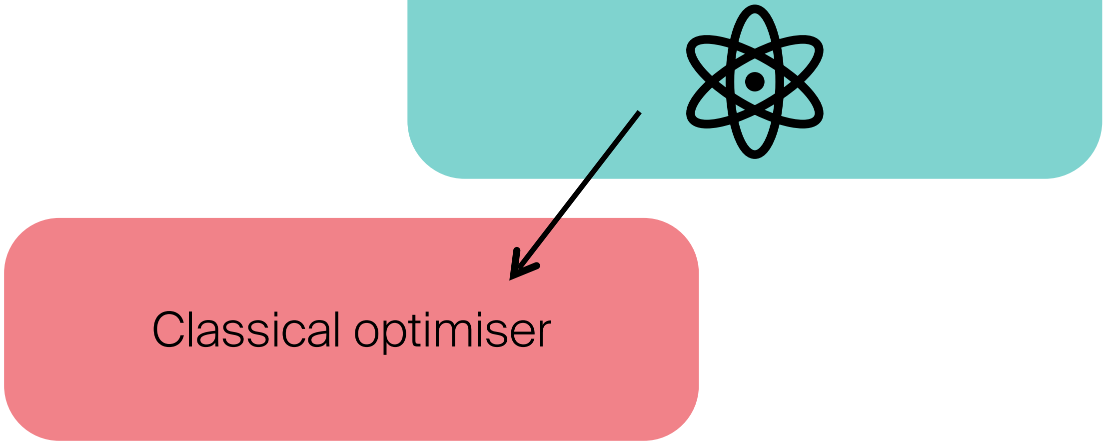

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Classical optimiser

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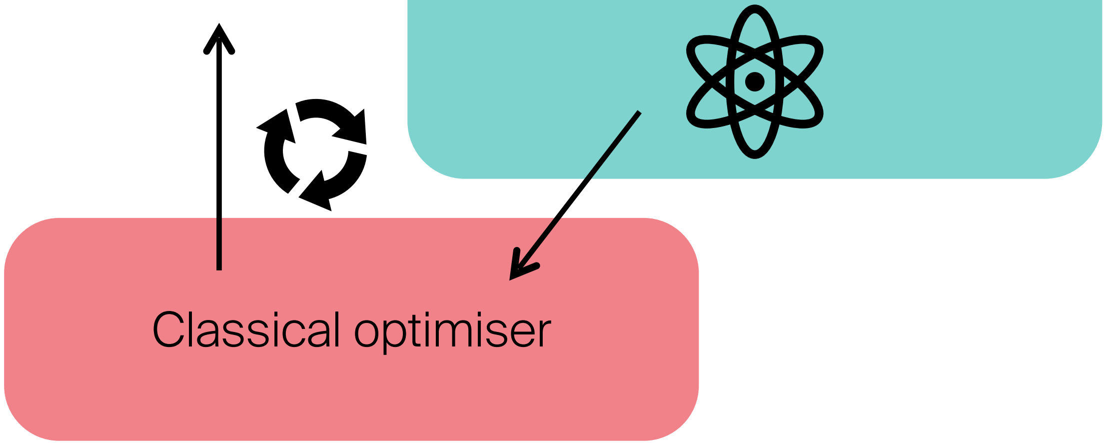

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Classical optimiser

$\Longrightarrow$ Only requires measuring fidelities

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Circuit knitting

Bravyi et al. (2016); Peng et al. (2020); Mitarai, Fuji (2021); Piveteau, Sutter (2023)

Circuit knitting

- Need to measure $\left|\langle\psi(\theta)|e^{-i\Delta t H}|\psi(\theta_{t-1})\rangle\right|^2$ across blocks

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- Sampled according to $p_k \propto |\alpha_k|$

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Global quantum channel

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Cost: Sampling overhead $\omega = \left(\sum_k |\alpha_k|\right)^2$

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Choosing gates for circuit knitting

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- How to construct the ansatz?

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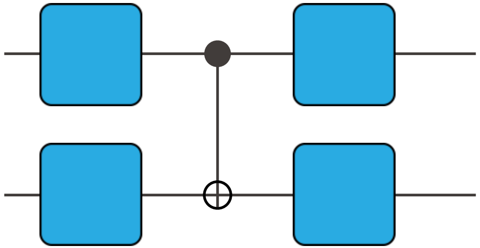

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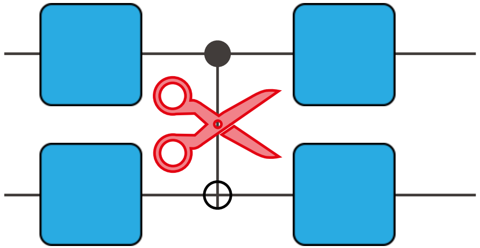

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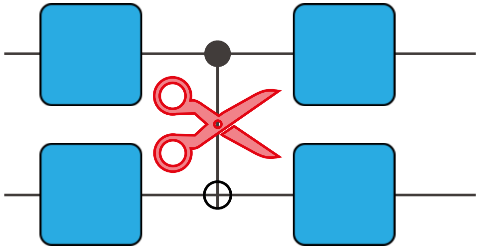

$\Longrightarrow \quad \omega = 9$

Cut multiple gates $\Longrightarrow$ Multiply overhead

2 CNOTs $\Longrightarrow \omega = 81$

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L CNOTs $\Longrightarrow \omega = 9^L$

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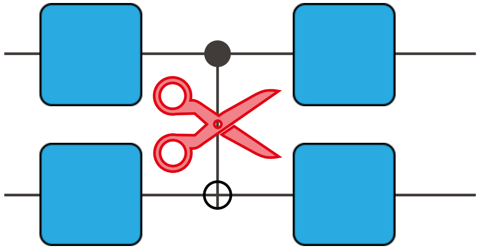

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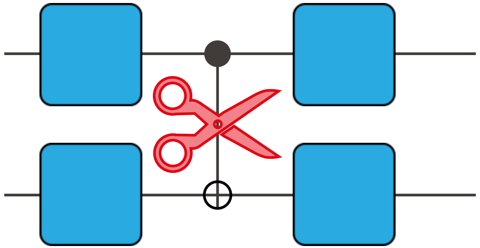

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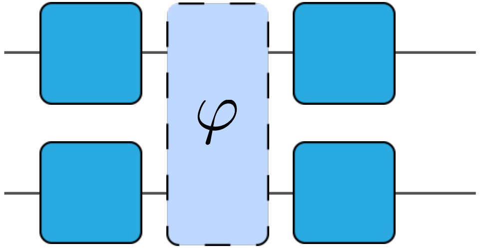

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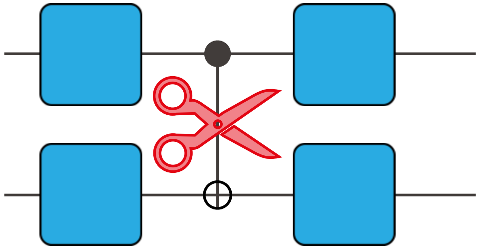

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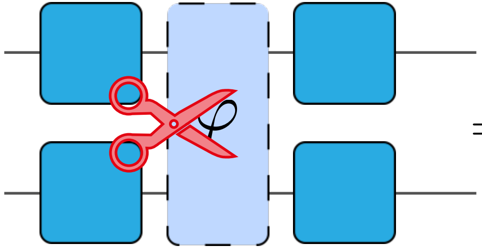

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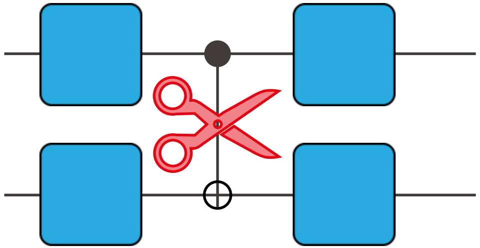

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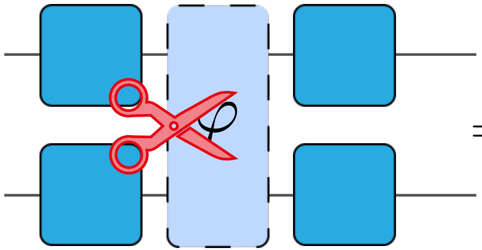


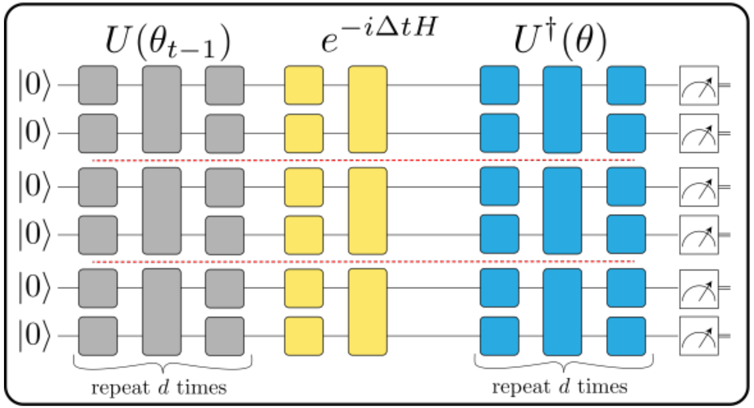
$$\implies \omega(\varphi) = (1 + 2|\sin(\varphi)|)^2$$

Small angles
$\implies$ small overhead

Mitarai, Fuji (2021)

Circuit Knitting Ansatz (CKA)

BPA



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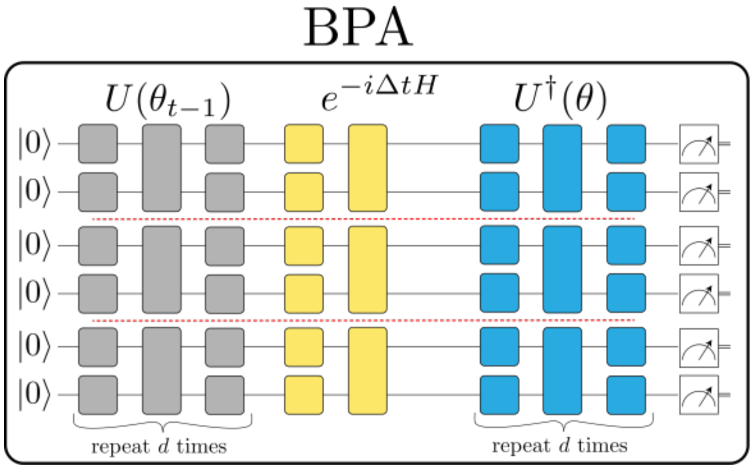

- Block product approximation (BPA): No entangling gates

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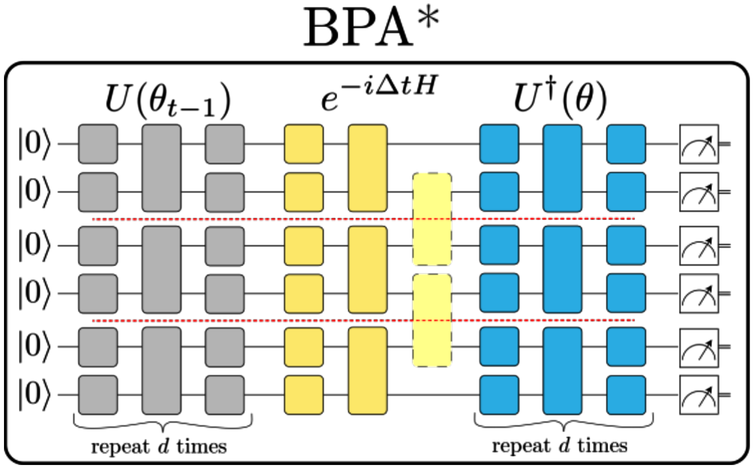

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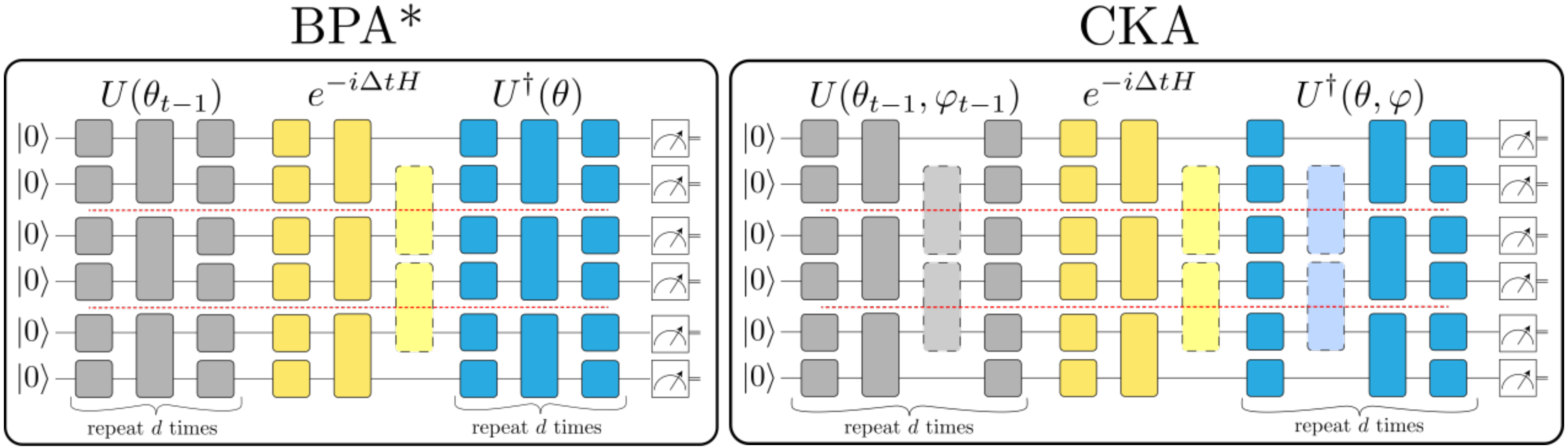
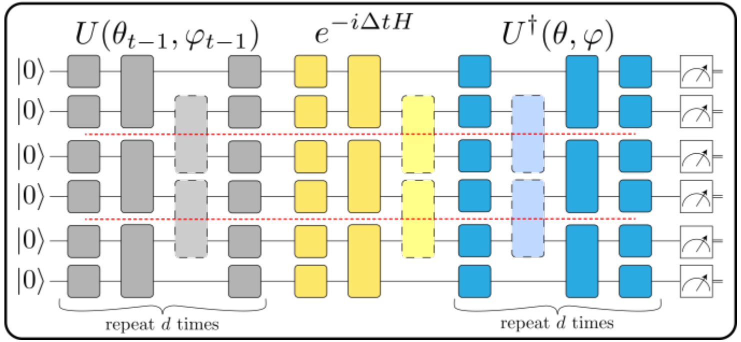


- Block product approximation (BPA): No entangling gates
- BPA*: Only Trotter overhead $\omega_{\Delta t} = (1 + 2|\sin(2J\Delta t)|)^{2L}$
- Circuit knitting ansatz (CKA): Introduce entangling gates between blocks

Circuit Knitting Ansatz (CKA)

CKA



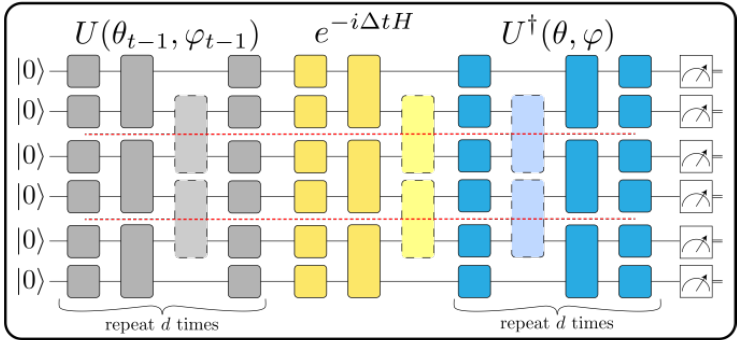
Circuit Knitting Ansatz (CKA)

- Overhead is multiplicative

$$\omega(\varphi) = \omega_{\Delta t} \cdot \left(\prod_{i=1}^{M} \left(1 + 2|\sin(\varphi_i)|\right)^2 \right)^2$$

- Need to constrain the overhead!

CKA



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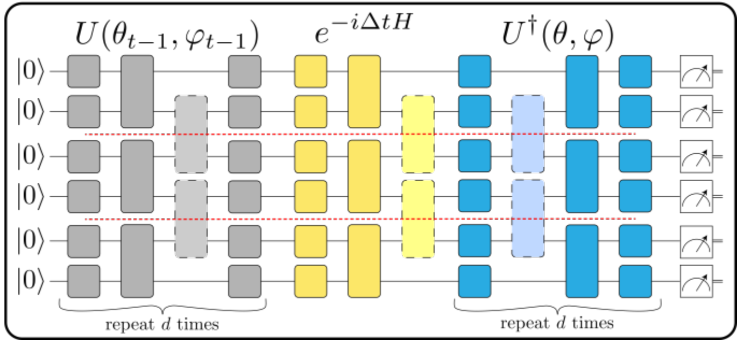

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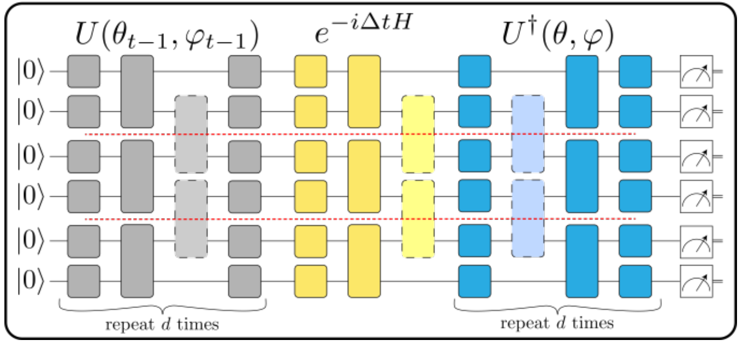

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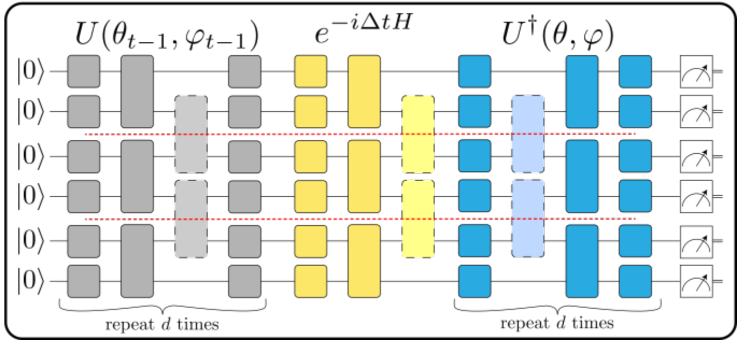

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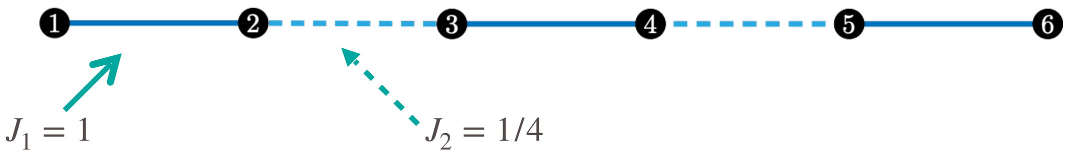
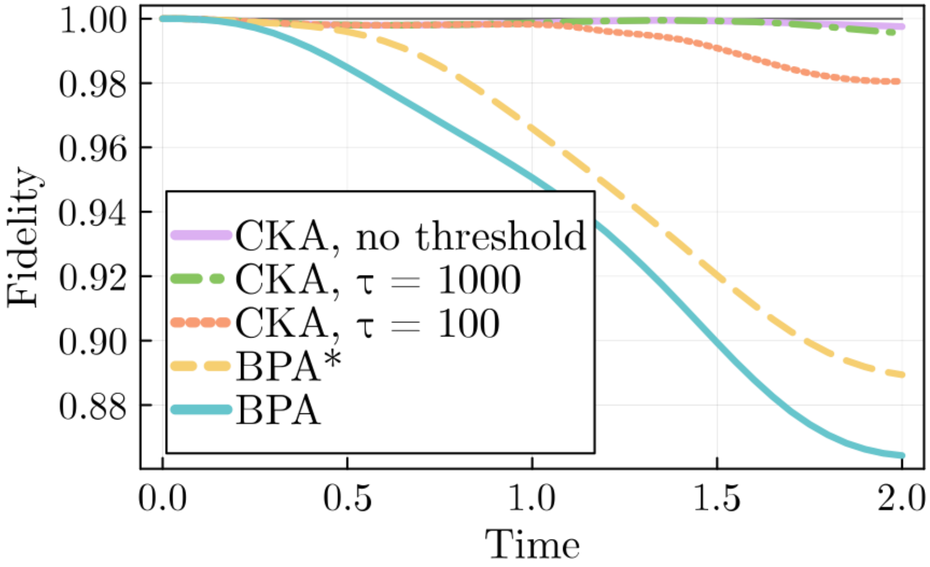
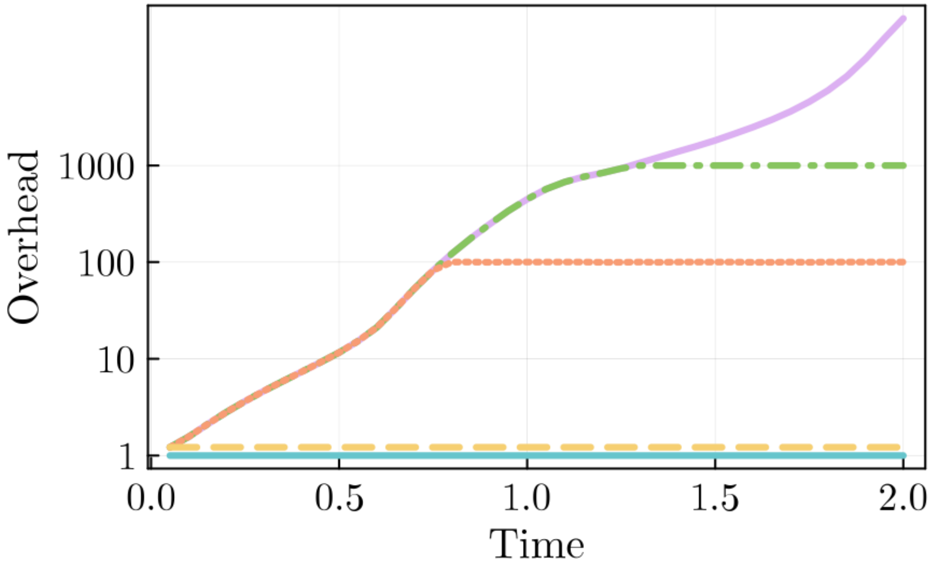
Higher threshold $\tau \Longrightarrow$ higher expressibility

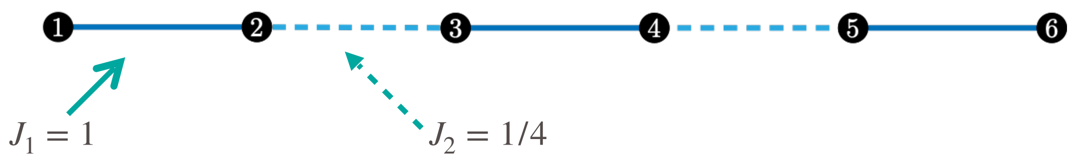
Results

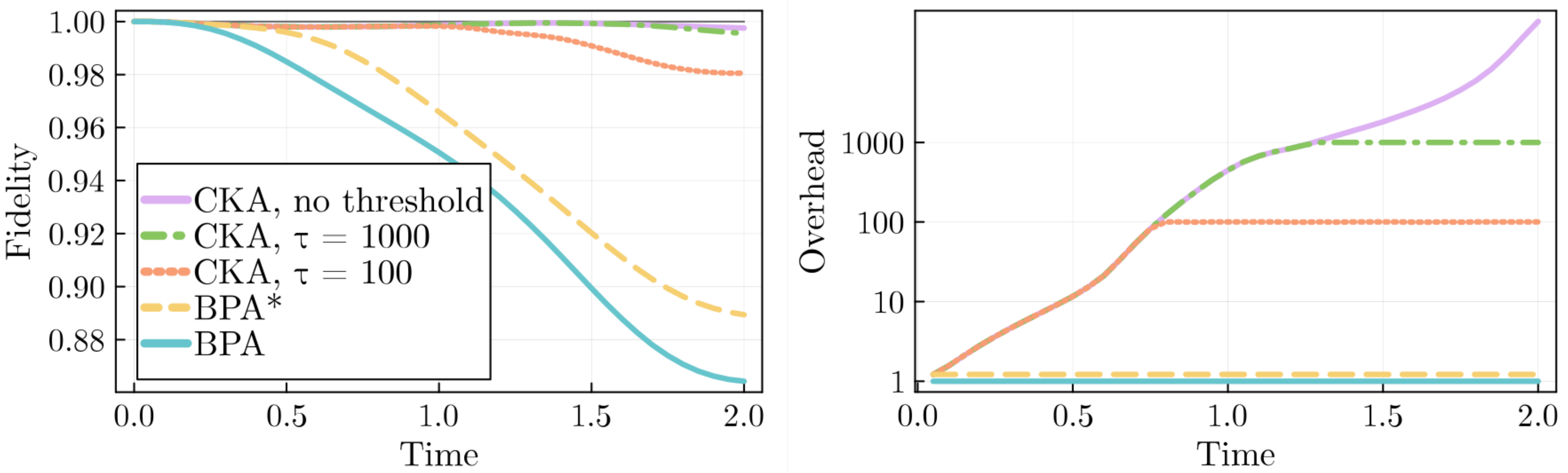

1
2
3
4
5
6
$J_1 = 1$
$J_2 = 1/4$


Fidelity
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.0
0.5
1.0
1.5
2.0
Time
CKA, no threshold
CKA, τ = 1000
CKA, τ = 100
BPA*
BPA


Overhead
1000
100
10
1
0.0
0.5
1.0
1.5
2.0
Time

Results





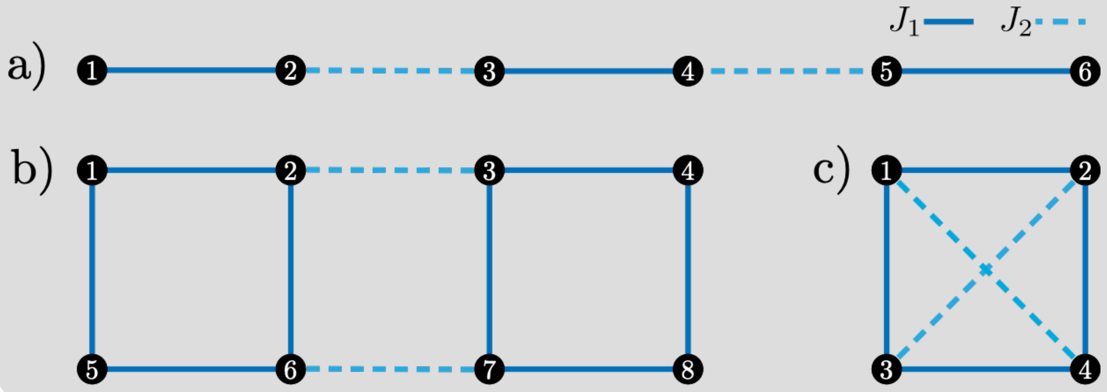
Higher threshold ⟹ Higher fidelity

Additional results

Shot noise

Entanglement entropy

More systems



GG, Friederike Metz, Giuseppe Carleo (2023), arXiv:2309.07857

Conclusion and Outlook

- Simulate dynamics of quantum systems across multiple devices
- Control total budget of shots using overhead threshold

- Can we find cheaper decompositions for the circuit cutting scheme?
- Hardware implementation?
- Application to fermionic systems?

Thank you!

More on the arXiv : GG, Friederike Metz, Giuseppe Carleo (2023)