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Quantum Distance Calculation for ε-Graph Construction

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Overview

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Introduction

- topological data analysis (TDA) can characterise features of the manifold from which the data is sampled
- TDA tends to be computationally inefficient (exponential space complexity)
- in 2022, several papers on quantum TDA algorithms came out (Akhalwaya et al. 2022) [1]
- could quantum algorithms speed up parts of the TDA process?

Overview

1. Introduction
2. ε-Graph Construction for Topological Data Analysis
3. SWAP-Test For Quantum Distance Calculation
4. Circuit $\mathcal{U}_n$ For Parallel Distance Calculation by Gitiaux et al. 2022
5. Query Complexity Of $\mathcal{U}_n$ for ε-Graph Construction
6. Discussion

ε-Graph Construction for Topological Data Analysis

- many classical TDA algorithms are based on the ε-Graph:

Definition (ε-graph)

Given a finite set of ℓ-dimensional points $S \subseteq \mathbb{R}^\ell$ of size $|S| = n$ and scale $\varepsilon > 0$, the ε-graph is an undirected graph $G_\varepsilon = (V, E_\varepsilon)$ where $V = S$ and

$$E_\varepsilon = \{\{u, v\} \,|\, \delta(u, v) < \varepsilon, u \neq v \in S\}$$

where δ is the euclidean metric.

- ε-graph construction takes time $\mathcal{O}(n^2)$ (better algorithms like the kd-tree take on average $\mathcal{O}(n \log n)$)
- Gitiaux et al. 2022 [3] proposed a circuit that could potentially speed up this process

Overview

SWAP-Test For Quantum Distance Calculation

- A common way to estimate the distance between two quantum states

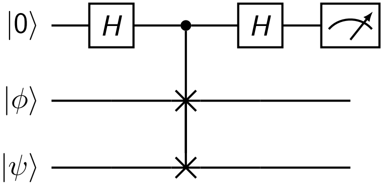

Figure 1: Quantum circuit representing the standard SWAP test

The probability of measuring 0 is

$$\frac{1}{2}(\langle\phi|\langle\psi| + \langle\psi|\langle\phi|)\frac{1}{2}(|\phi\rangle|\psi\rangle + |\psi\rangle|\phi\rangle)$$
$$= \frac{1}{2} + \frac{1}{2}|\langle\psi|\phi\rangle|^2$$

SWAP-Test For Quantum Distance Calculation

- the distance is estimated by noting that for normalised vectors, we have

  $$|\phi - \psi| = \sqrt{2(1 - |\langle\phi|\psi\rangle|)} = \sqrt{2(1 - \sqrt{2p - 1})}$$

  where $p \equiv P(0)$ is the probability of measuring 0 in the ancilla
- repeat the circuit in Figure 1 N times and use $\hat{p} = 1 - \frac{1}{N}\sum_{i=1}^{N} X_i$ as the estimate for p, where X_i is the measurement oucome of the ith experiment
- the number of repetitions determines the accuracy of the distance estimate

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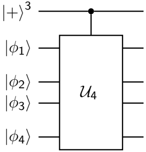
Circuit $\mathcal{U}_n$ For Parallel Distance Calculation by Gitiaux et al. 2022

- start with a base circuit $\mathcal{U}_4$
- 3 ancilla qubits
- 4 inputs are encoded in 4 qubit states (access for e.g. QRAM)
- 3 successive applications of CSWAP gates put allpossible pairs between the 4 input states into the first two input registers in superposition
- write $\mathcal{U}_4$ to abbreviate the circuit



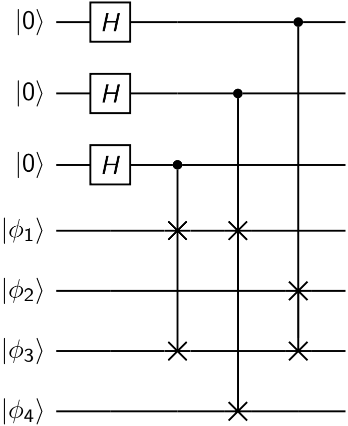


Figure 2: Quantum circuit representing $\mathcal{U}_4$ (Gitiaux et al. 2022)

Circuit $\mathcal{U}_n$ For Parallel Distance Calculation by Gitiaux et al. 2022

- for $n = 2^k$ input states, the circuit $\mathcal{U}_n$ that moves all possible pairs into the first two input registers is constructed recursively from the circuit $\mathcal{U}_4$
- $\mathcal{U}_n$ uses $3n/2 - 3$ CSWAP gates and $d_n = 3\log_2(n/2)$ ancillary qubits
- this is not a uniform superposition

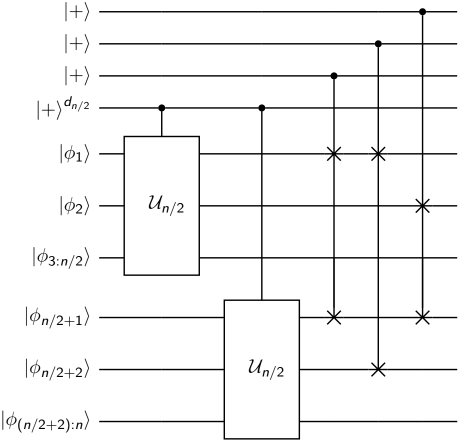


Figure 3: Quantum circuit representing $\mathcal{U}_n$ (Gitiaux et al. 2022)

Circuit $\mathcal{U}_n$ For Parallel Distance Calculation by Gitiaux et al. 2022

- like in the SWAP test, add one ancilla and a final CSWAP and H
- measure the top ancilla qubit and the d_n ancilla qubits
- estimate p_{0ij} for $i, j \in [\![1, n]\!]$ where p_{0ij} is the probability of measuring the ancillary state $|0ij\rangle$, where $|ij\rangle$ is shorthand for the d_n-dimensional ancillary basis state associated with $(|\phi_i\rangle, |\phi_j\rangle)$
- we have for d_n ancilla qubits

$$p_{0ij} = \frac{1 + |\langle\phi_i|\phi_j\rangle|^2}{2^{d_n}}$$

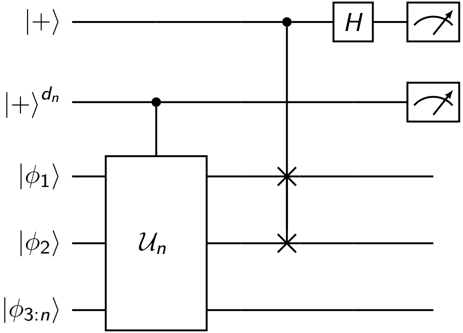


Figure 4: Final SWAP test and measurement (Gitiaux et al. 2022)

Overview

Query Complexity Of $\mathcal{U}_n$ for ε-Graph Construction

Proposition ([2])

The number of circuit repetitions necessary to correctly identify with arbitrary precision that two points are not ε-neighbours is at least $\mathcal{O}\left(\frac{n^3}{\ln n}\right)$. In particular, this implies a total number of CSWAP gates of at least $\mathcal{O}\left(\frac{n^4}{\ln n}\right)$.

Query Complexity Of $\mathcal{U}_n$ for ε-Graph Construction

- *Remark 1*: This is a sharp bound
- *Remark 2*: The complexities of $\mathcal{U}_n$ and the classically exact results are not directly comparable
- *Remark 3*: such complexity comparison does not necessarily immediately translate to a comparison in real execution time, as different quantum hardware will execute gates at different speeds

Overview

Discussion

- the complexity analysis suggest that there is no immediate quantum advantage in using $\mathcal{U}_n$ rather than classical algorithms for the construction of ε-graphs
- $\mathcal{U}_n$ is a very interesting algorithm if one wants to prepare all possible quantum state pairs in superposition in only two registers (the original intent of the paper)
- one might look for other quantum algorithms for fast ε-graph construction with lower complexity

References I

[1] Akhalwaya, I. Y., Ubaru, S., Clarkson, K. L., Squillante, M. S., Jejjala, V., He, Y.-H., Naidoo, K., Kalantzis, V., and Horesh, L. (2022). Towards quantum advantage on noisy quantum computers. *arXiv preprint*.

[2] Chmielewski, N. M., Amini, N., Jacquot, P., and Mikael, J. (2023). Quantum distance calculation for epsilon-graph construction. *arXiv preprint*.

[3] Gitiaux, X., Morris, I., Emelianenko, M., and Tian, M. (2022). Swap test for an arbitrary number of quantum states. *Quantum Information Processing*, 21(10).

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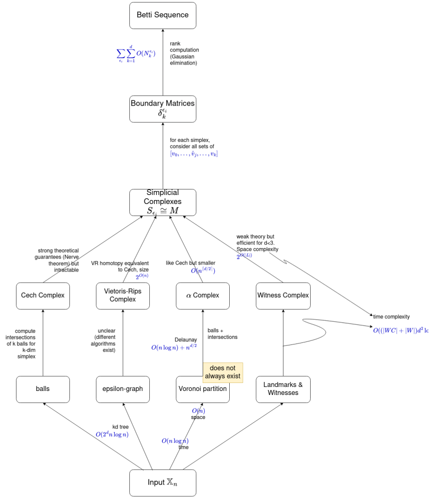

Betti Sequence

$\sum_{\epsilon_i} \sum_{k=1}^{d} O(N_k^{\epsilon_i})$

rank computation (Gaussian elimination)

Boundary Matrices $\delta_k^{\epsilon_i}$

for each simplex, consider all sets of $[v_0, \dots, \hat{v}_j, \dots, v_k]$

Simplicial Complexes $S_{\epsilon_i} \cong M$

strong theoretical guarantees (Nerve theorem) but intractable

VR homotopy equivalent to Cech, size $2^{O(n)}$

like Cech but smaller $O(n^{\lceil d/2 \rceil})$

weak theory but efficient for d<3. Space complexity $2^{O(|L|)}$

time complexity

$O((|WC| + |W|)d^2 \,\mathrm{lc}$

Cech Complex

Vietoris-Rips Complex

α Complex

Witness Complex

compute intersections of k balls for k-dim simplex

unclear (different algorithms exist)

Delaunay $O(n \log n) + n^{d/2}$

balls + intersections

does not always exist

balls

epsilon-graph

Voronoi partition

Landmarks & Witnesses

kd tree $O(2^d n \log n)$

$O(n)$ space

$O(n \log n)$ time

Input $\mathbb{X}_n$

Figure 5: Different Approaches To Calculate The Betti Sequence