Progress of Muon Collider Lattice Design v0.6

Kyriacos Skoufaris and **Christian Carli**

With special thanks to K. Oide, P. Raimondi, D. Schulte and R. Tomas

25-May-2023

Outline

- 10TeV Muon Collider v0.6
  - Final Focusing Scheme
  - Chromatic Correction & Matching Schemes
  - Arc
- Tracking studies
- Discussion

10TeV Muon Collider

TABLE I. 10 TeV center of mass energy muon collider.

Parameters	Symbol	Unit	10TeV com mc
Particle energy	E	GeV	5000
Particle momentum	P_0	GeV c^{-1}	5000
Luminosity per IP	$\mathcal{L}$	10^{34} cm^{-2} s^{-1}	20
Bunch population	N_p	10^{12}	1.8
Transverse normalized rms emittance	$\varepsilon_{nx} = \varepsilon_{ny}$	μm	25
Transverse geometric rms emittance	$\varepsilon_{gx} = \varepsilon_{gy}$	nm	0.528
Longitudinal emittance ($4\pi\ \sigma_E\ \sigma_T$)	ε_l	eVs	0.314
Longitudinal geometric emittance ($\frac{\varepsilon_l c}{4\pi E_{0\mu}}$)	ε_{lg}	mm	70
Rms bunch length	σ_z	mm	1.5
Relative rms energy spread	δ	%	0.1
Beta function at IP	$\beta_x^\star = \beta_y^\star$	mm	1.5
Power per beam with 5 Hz repetition rate	$P_{\rm beam}$	MW	7.2

10TeV Muon Collider - In a nutshell

1.5mm β^*

=> ~500Km βs in the Final Focusing (FF) scheme (also large δ=0.1%).

=> Enormous chromatic aberrations at the optical functions (described by Montague functions).

=> Necessity for a local Chromatic Correction (CC) scheme right after the FF quads.

=> Use of dipole-sextupol kicks at areas with large betas and dispersion.

=> The CC generate significant positive momentum compaction factor (α_p) and should be controlled (keep the bunch length short) in the arcs among other parameters.

Muon decay (short lifetime τ_0~2.2μs or τ_{5TeV}~0.1s)

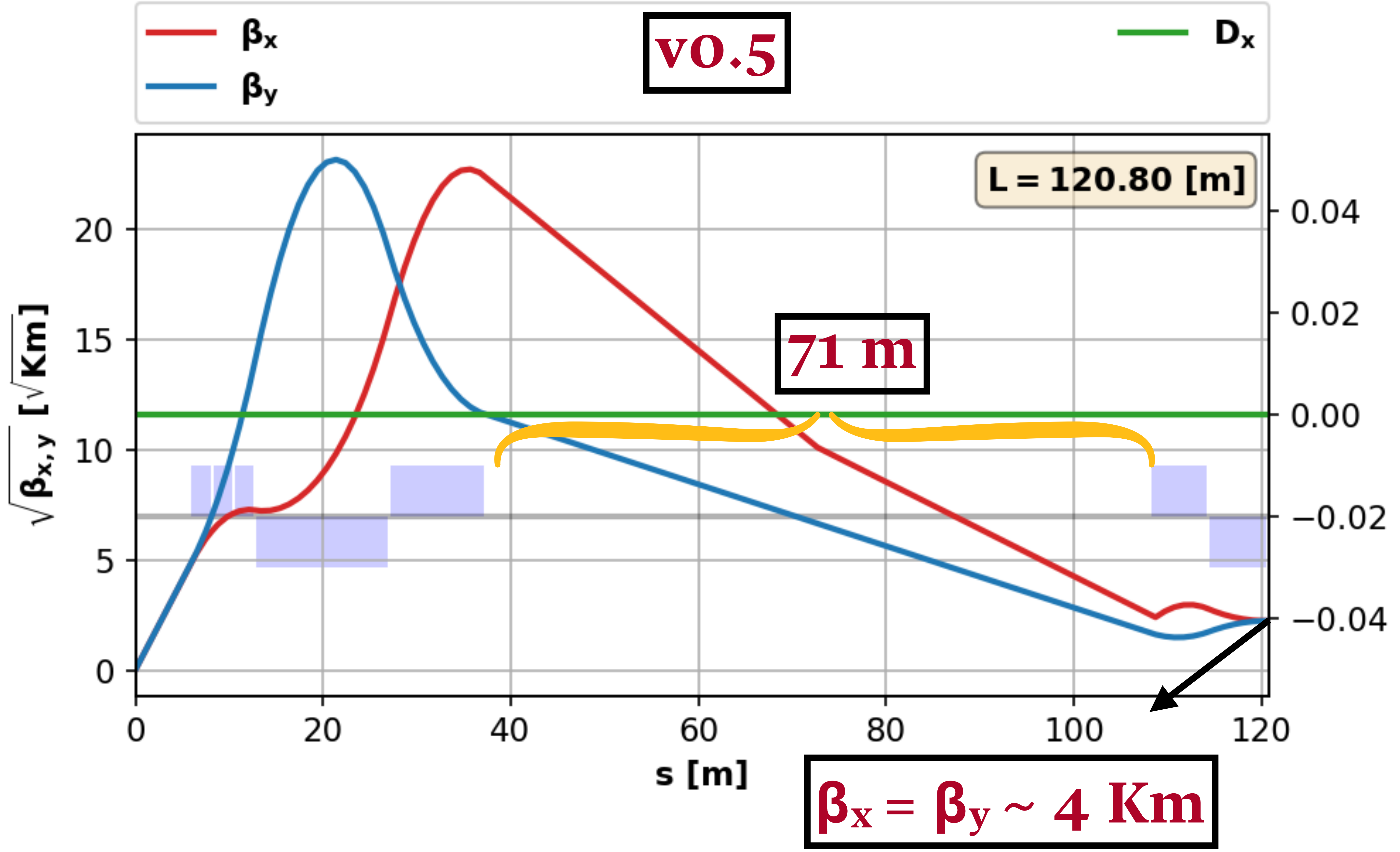
=> The resulting neutrinos even from a short straight piece of collider generate a narrow "radiation cone" that is an issue at the location, where they reach the earth surface.

=> The planned shape of the collider is like a race track (2 straight sections for IPs).

=> Extensive use of dipoles and combined function magnets.

10TeV Muon Collider **v0.5**
(Recap)

10TeV Muon Collider - Final Focusing Scheme

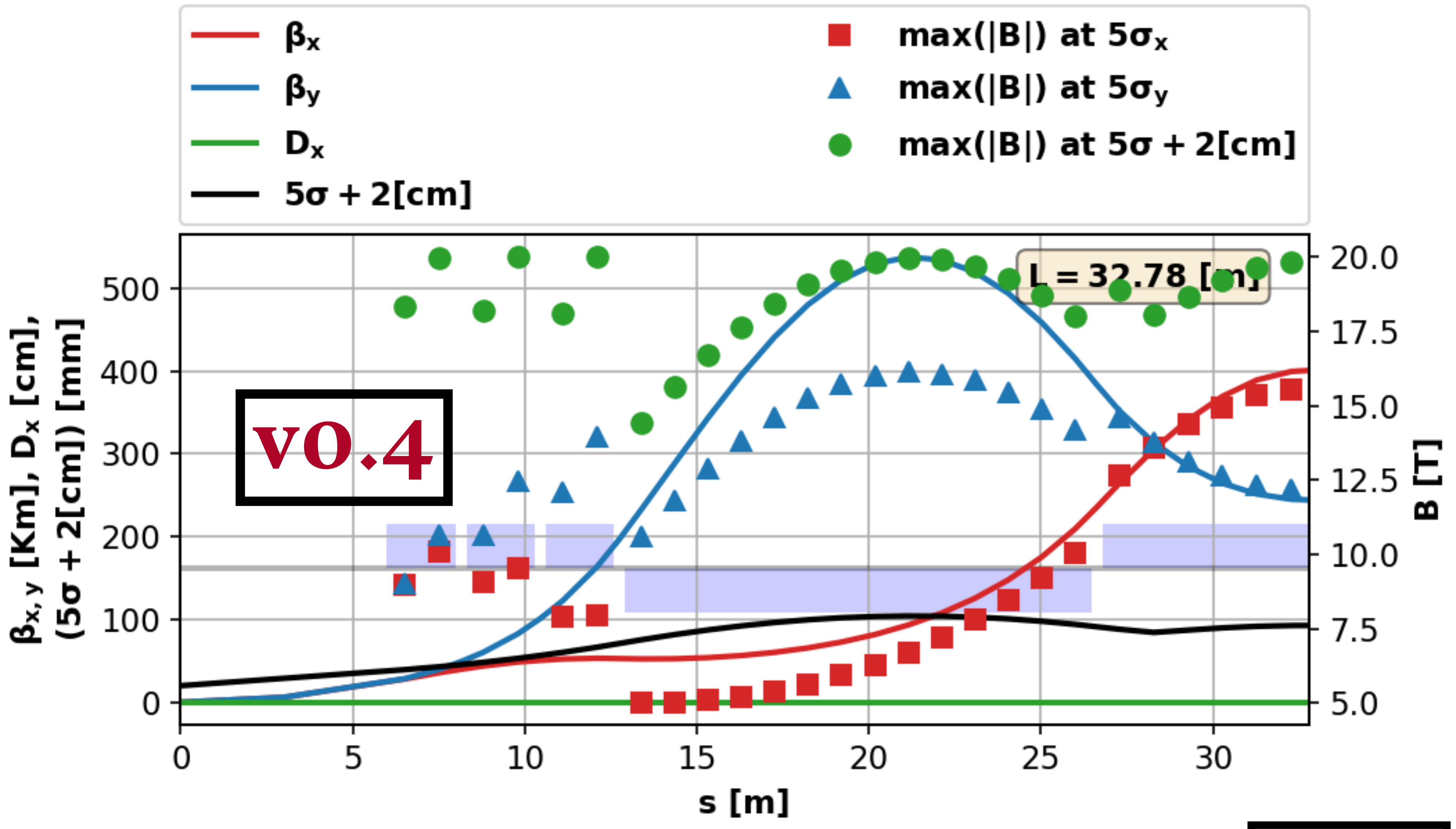


$$B\rho = 16678.205\ [Tm]$$

$$Aperture = 2(5\sigma + 0.02)\ [m]$$

$$\sigma_j = \sqrt{\frac{\beta_j \varepsilon_{nj}}{\beta_r \gamma_r} + (D_j \delta_p)^2}\ [m]\ with\ j = x, y$$

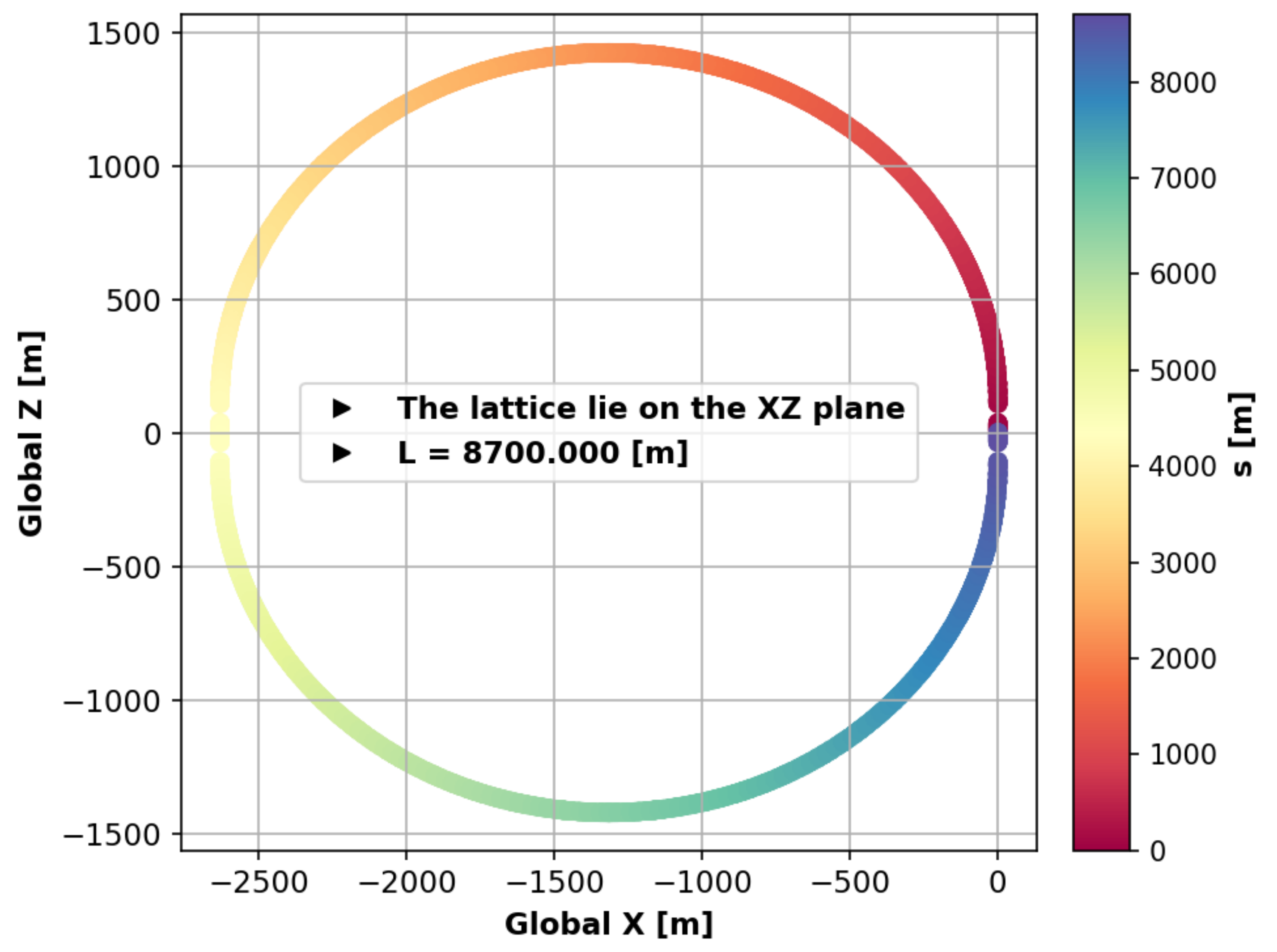
$$\sigma = max(\sigma_x, \sigma_y)\ [m]$$

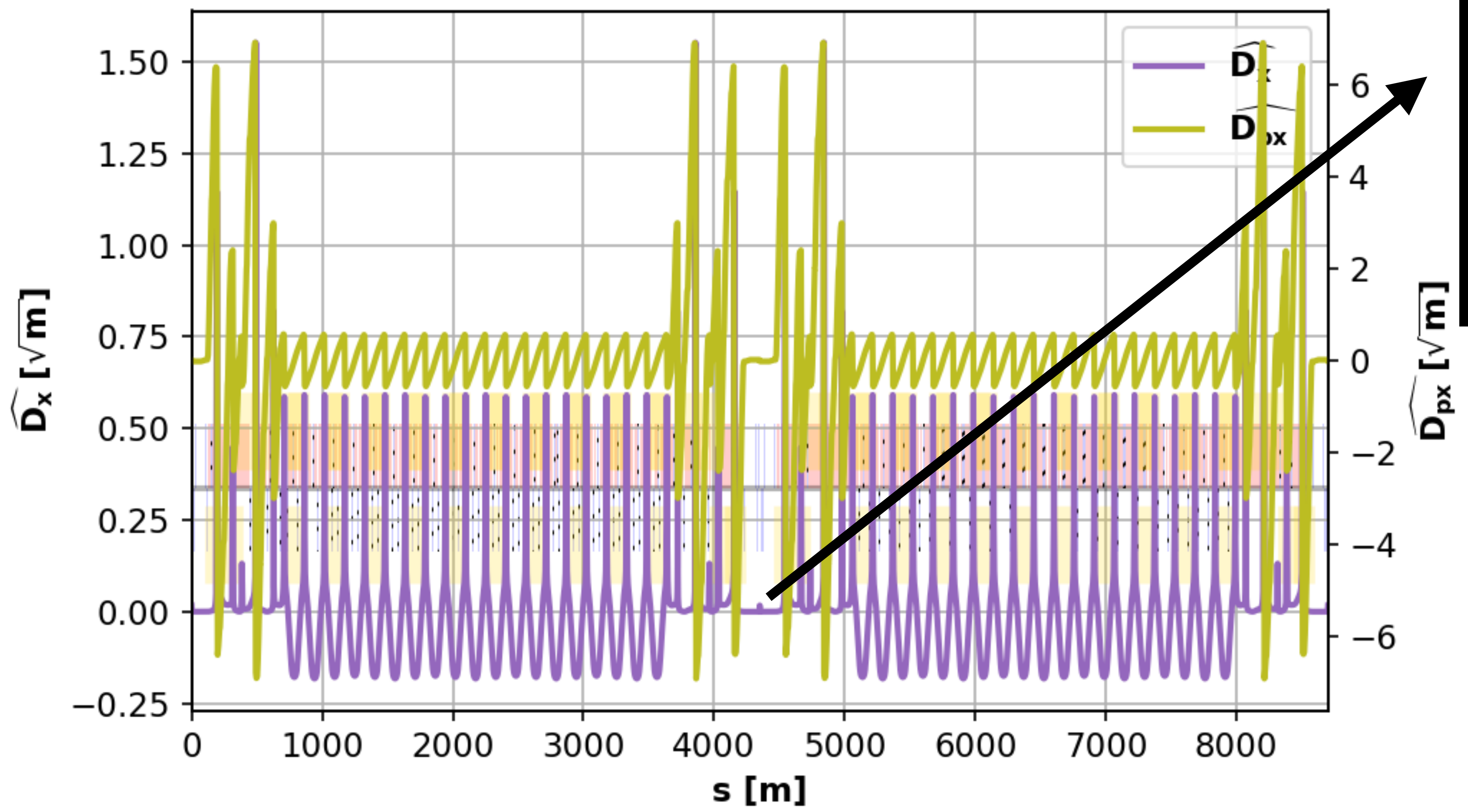


Entering the CC with small βs resulted in:

- Smaller aperture
- Smaller Ws
- Less impact from unwanted multipolar components
- Easier control of βs

10TeV Muon Collider - Full Lattice

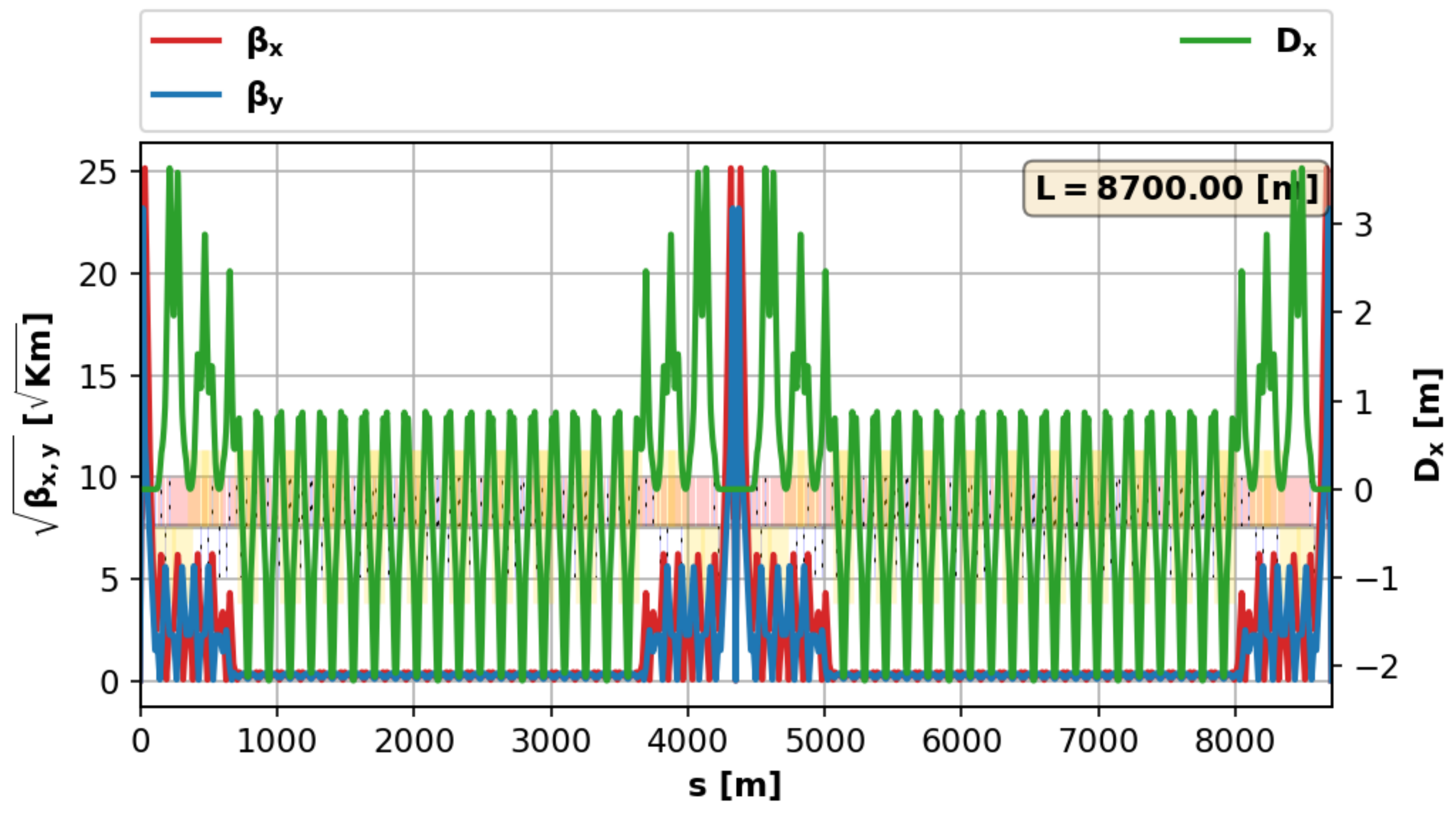


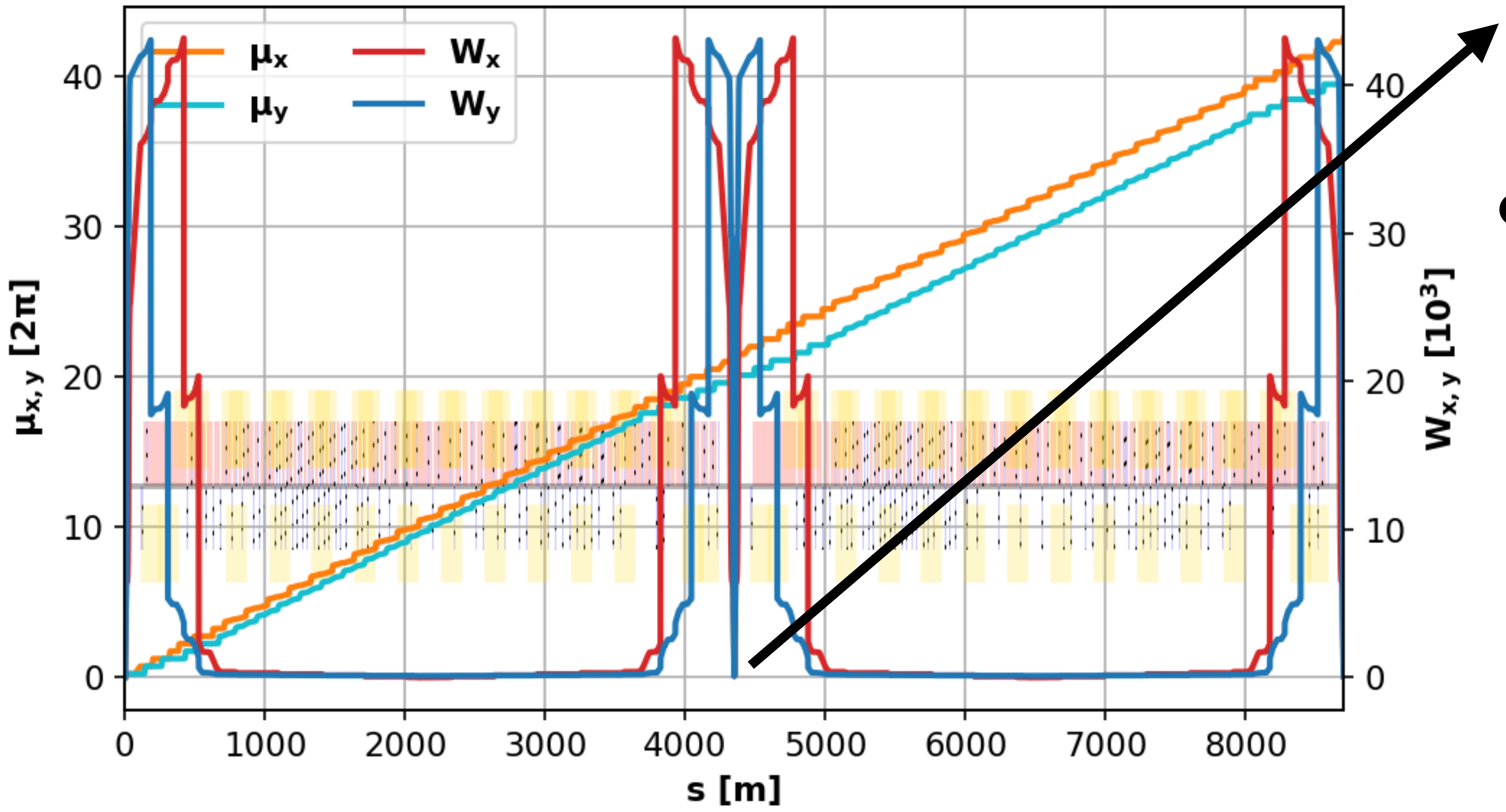


$D_{nx}, D_{npx} \neq 0$

$(D_x{*}\delta)^2 \simeq 4{*}(\varepsilon_g{*}\beta_x)$

$\sigma_x \simeq \sqrt{5}\sigma_{\beta x}$

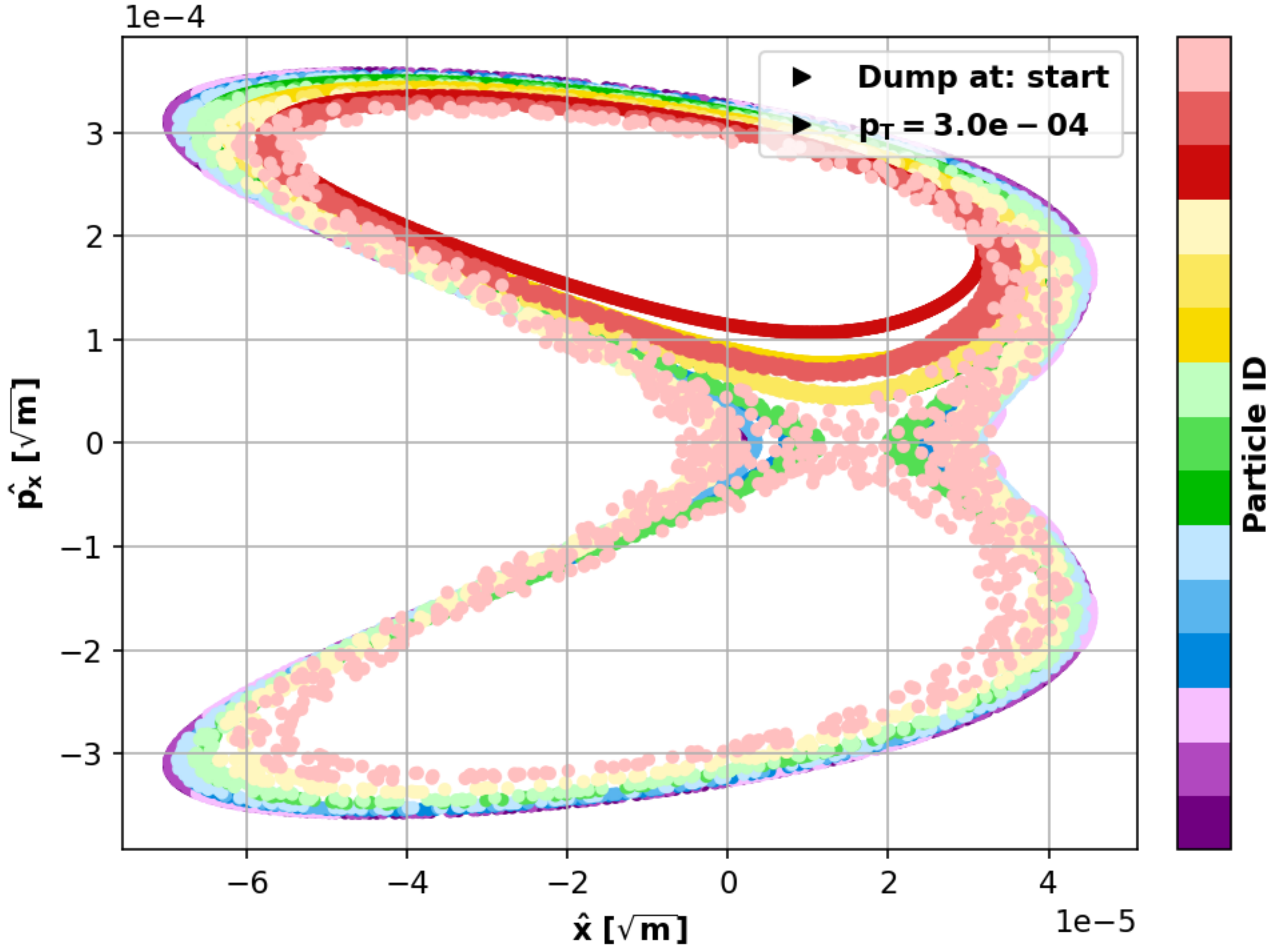


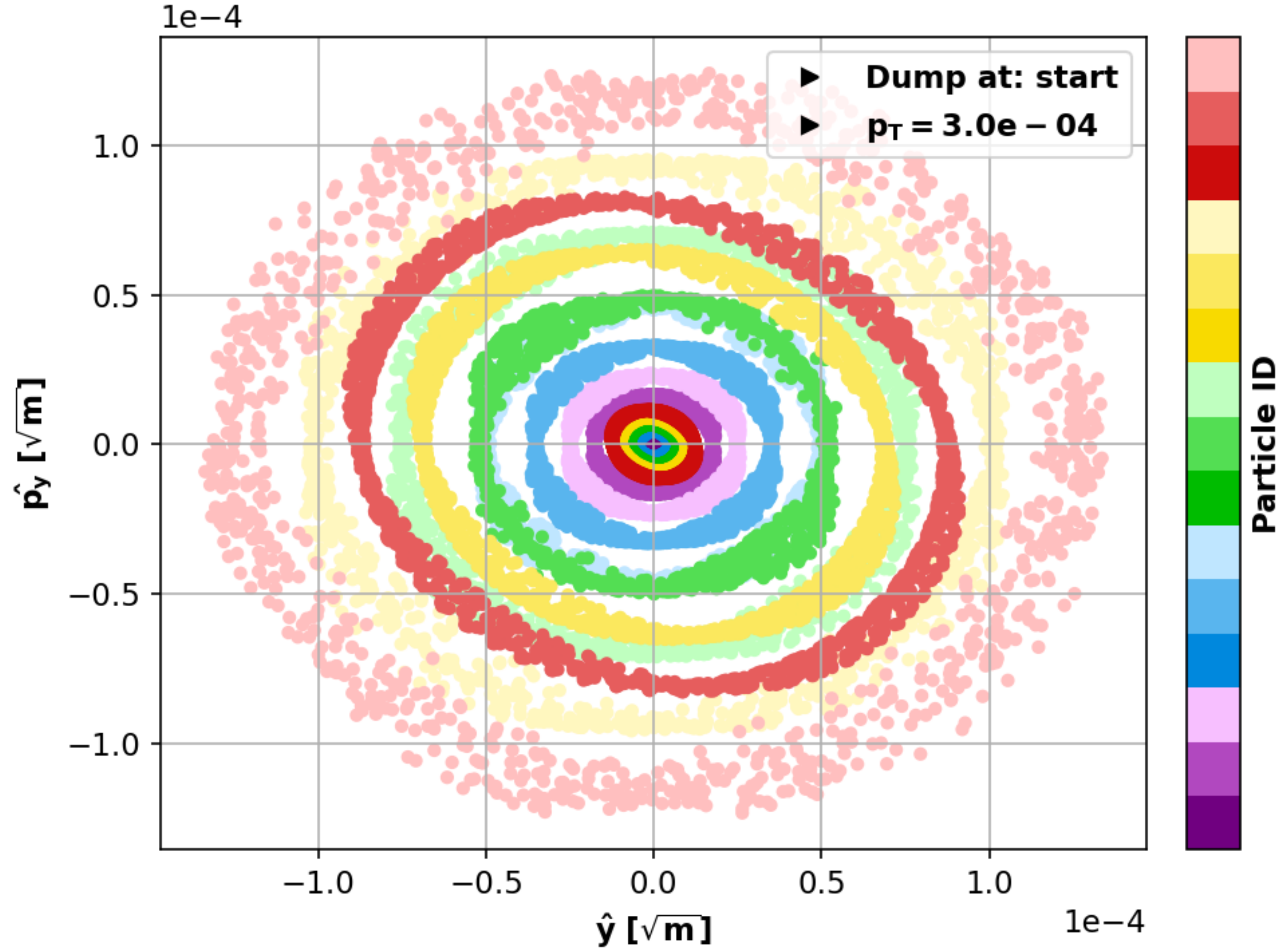


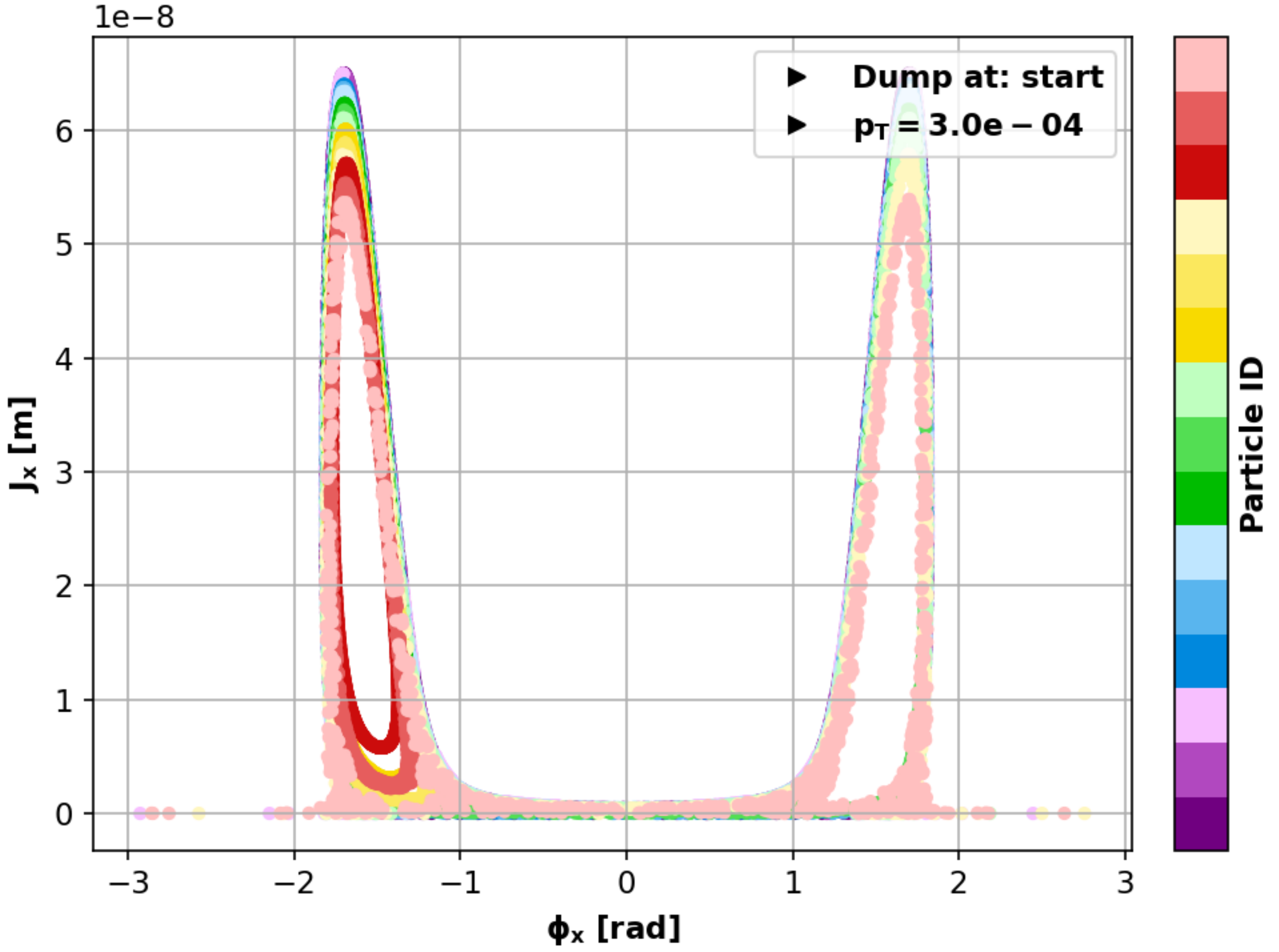
$\delta{*}W_{x,y} \simeq 0.1$

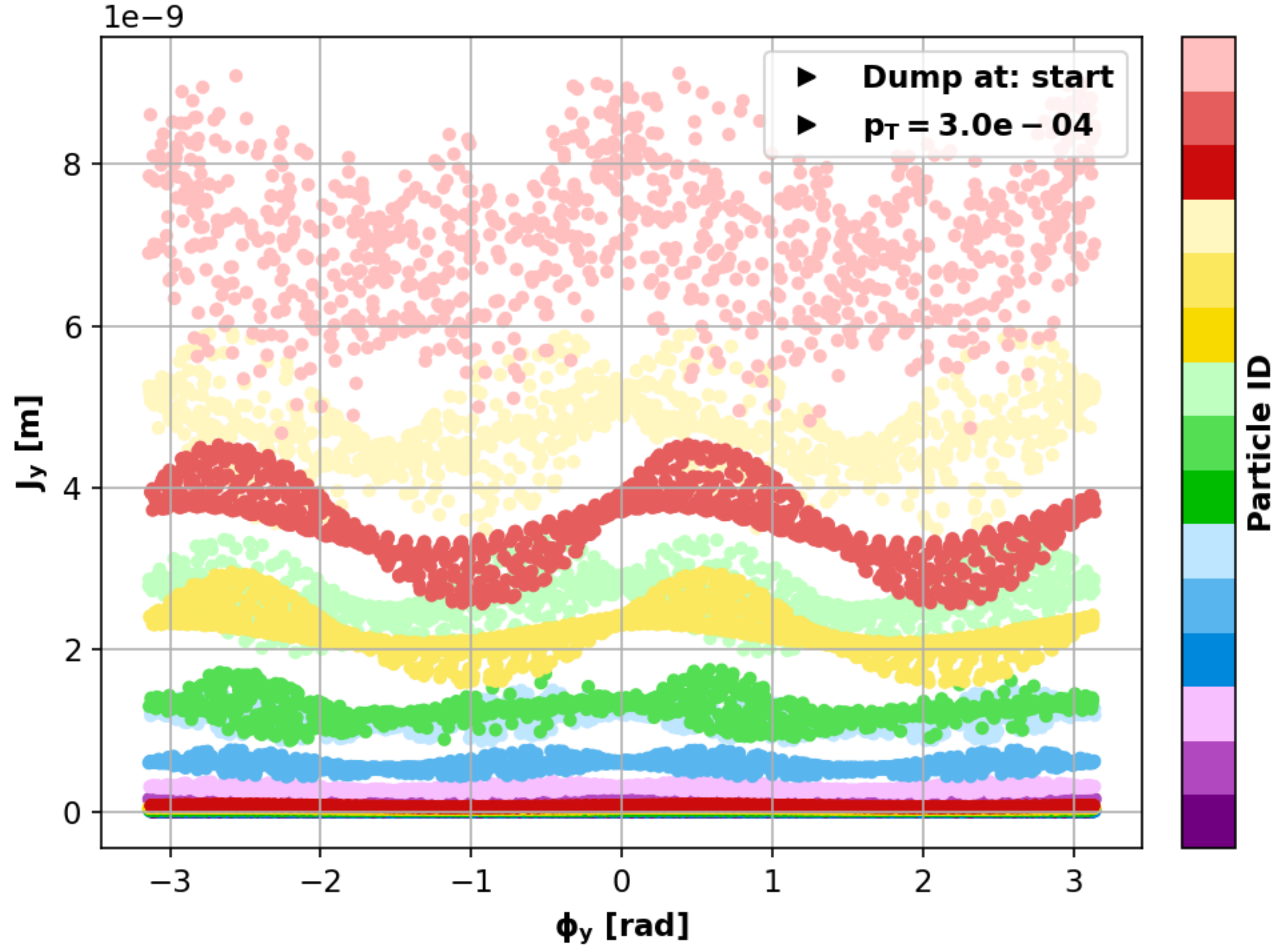
- The impact of the energy depended beta-beating, due to non zero δ*W, on the Luminosity should be assessed.

10TeV Muon Collider - Tracking Studies







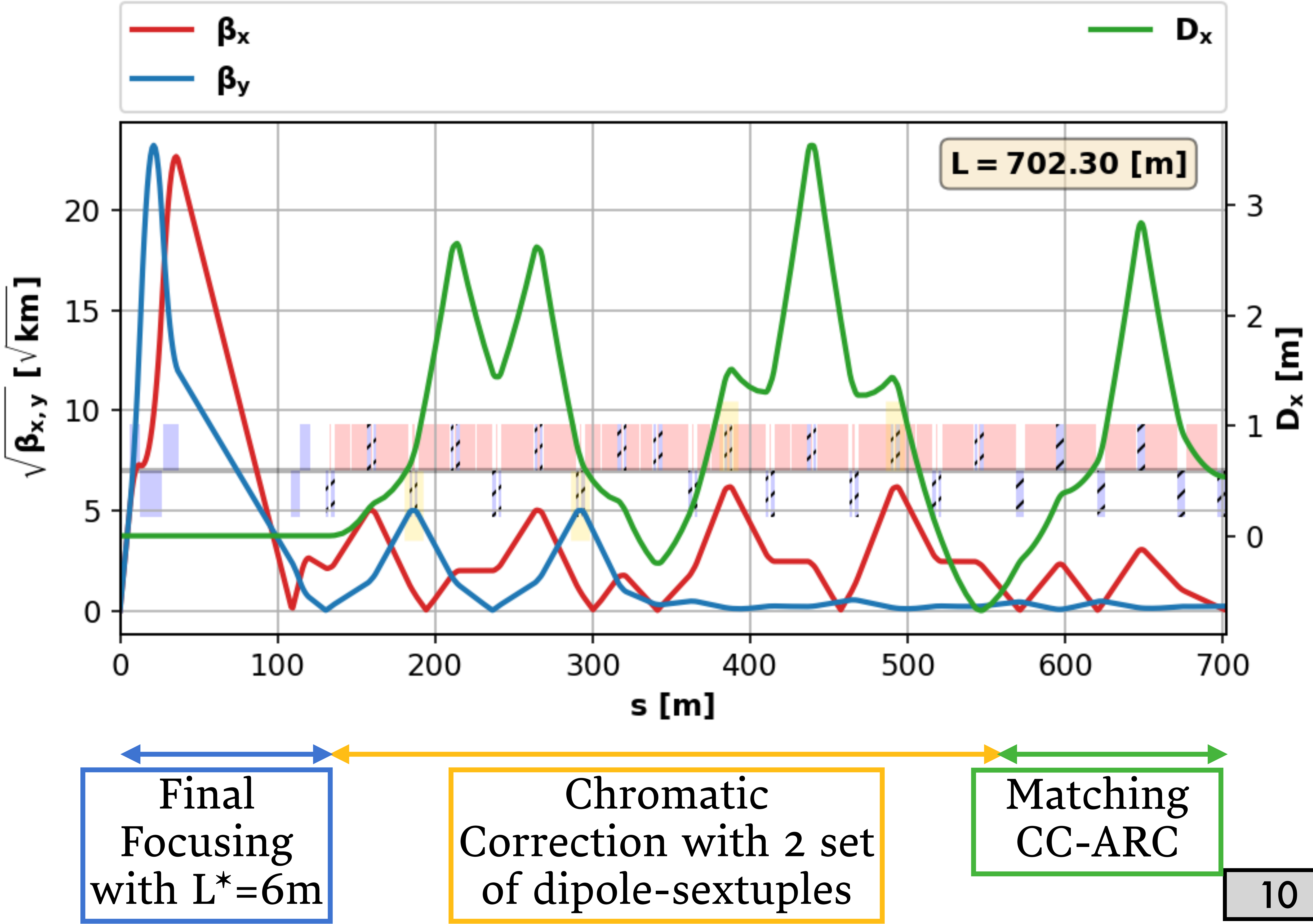


10TeV Muon Collider **v0.6**
(New design)

10TeV Muon Collider - Extended Final Focusing Schemes

Colour code for lattice elements:

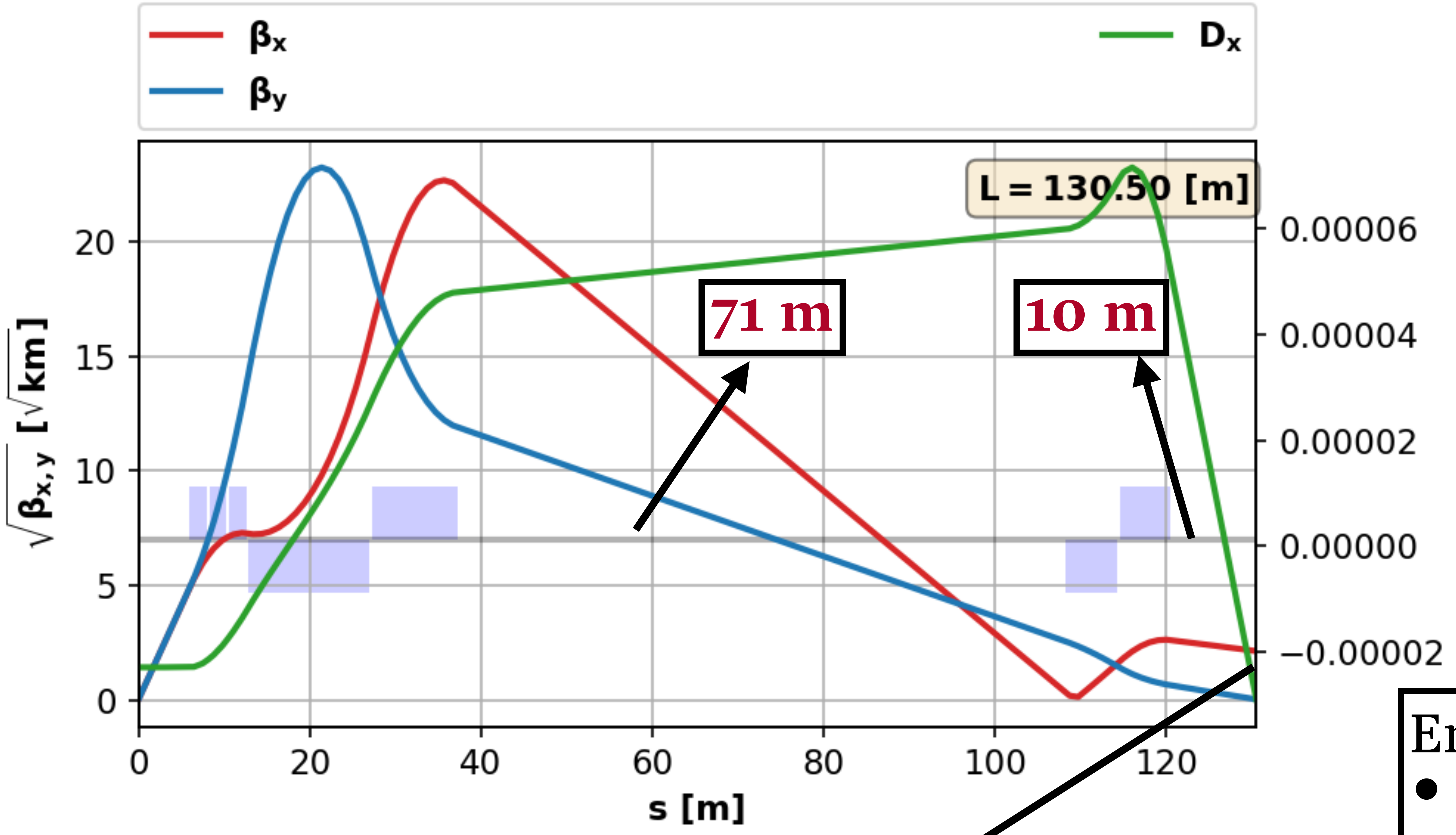
- Red dipoles
- Blue quadrupoles
- Hashed blue dipole-quadrupoles
- Red + Gold dipole-sextupoles (all 1m long)



10TeV Muon Collider - Final Focusing Quads

- $L^* = 6$m and five quadrupoles are used.
- The maximum magnetic field at the magnet aperture is set to 20T.
- Due to the fast increase (decrease) of the β functions right after the IP, the first magnet is split in shorter ones with different gradient, reducing that way the length of the FF scheme.
- The $\beta_{x,y}$ are reduced by two order of magnitude at the end of the FF quads while the last four quadrupoles are used to control the $\beta_{x,y}$ and $\alpha_{x,y}$ in the chromatic correction section.
- Inclusion of a drift section for a smoother reduction/control of the beta values (β_x,β_y) at the end of the FF scheme. This help to keep the Montague chromatic functions at smaller values in the chromatic correction section.

10TeV Muon Collider - Final Focusing Quads



$$B\rho = 16678.205\ [Tm]$$

$$Aperture = 2(5\sigma + 0.02)\ [m]$$

$$\sigma_j = \sqrt{\frac{\beta_j \varepsilon_{nj}}{\beta_r \gamma_r} + (D_j \delta_p)^2}\ [m]\ with\ j = x, y$$

$$\sigma = max(\sigma_x, \sigma_y)\ [m]$$

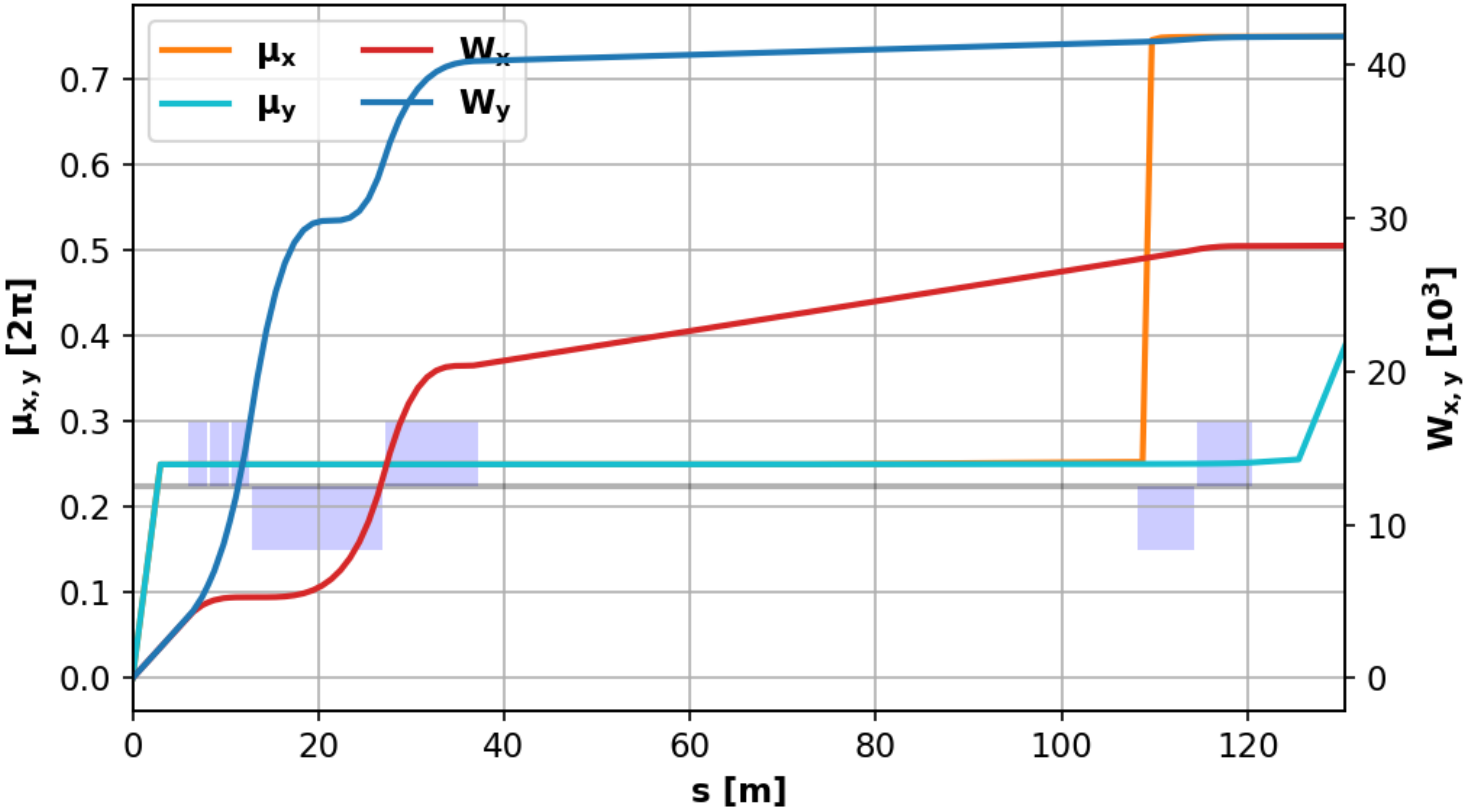
β_x ~ 4.5 km
β_y ~ 43 cm

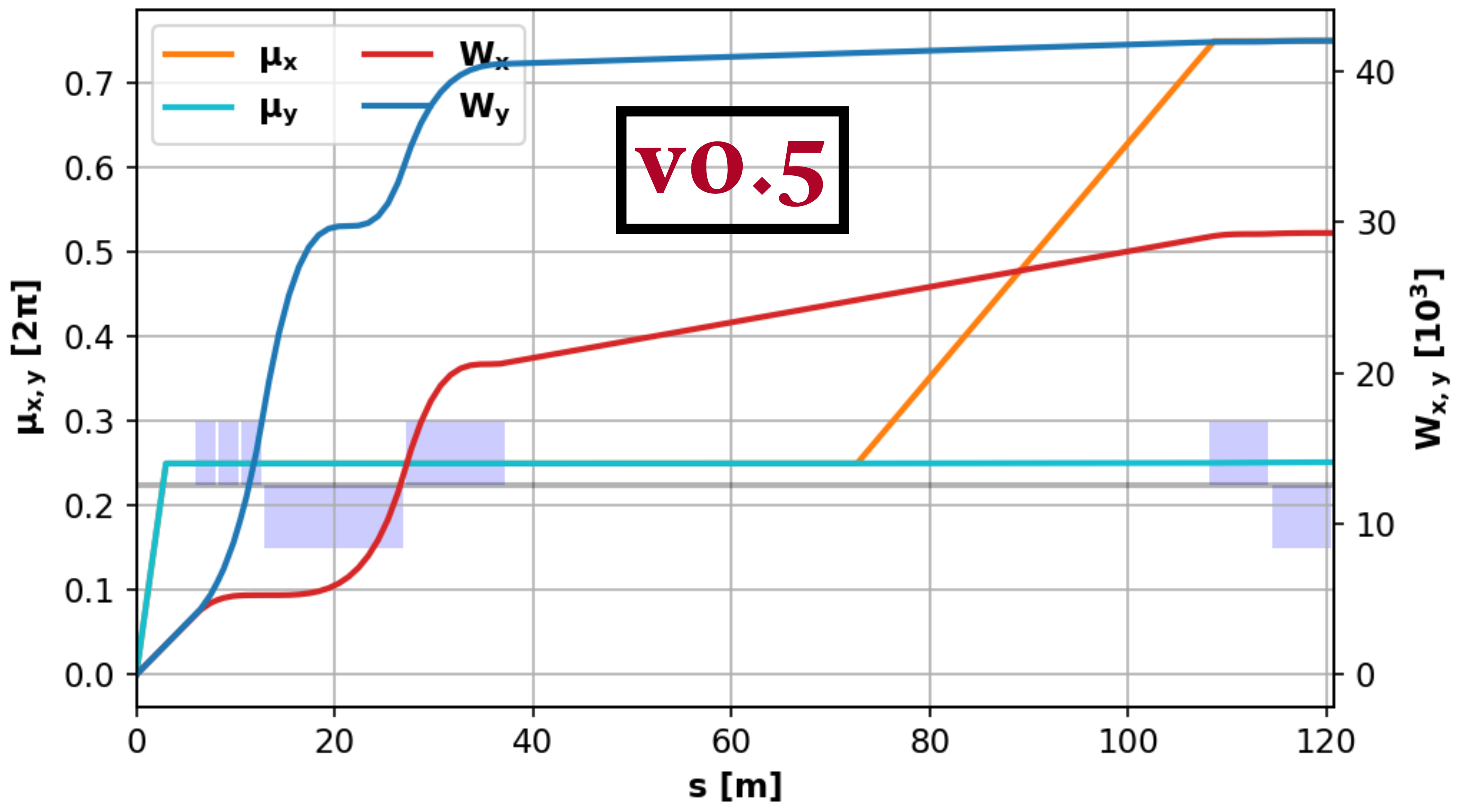
Entering the CC with small βs:

- Smaller aperture
- Smaller Ws
- Smaller unwanted multipolar kicks
- Easier control of βs

10TeV Muon Collider - Final Focusing Quads

- Due to strong focusing quadrupoles (β^*=1.5mm), the Montague chromatic functions ($W_{x,y}$) that describe the optics perturbation for off-momentum particles w.r.t on-momentum one become very large.
- Together with the large momentum spread ($\delta=10^{-3}$), these W values indicate enormous chromatic effects that should be compensated in order to avoid performance degradation.





10TeV Muon Collider - Chromatic Correction & Matching Schemes

- The maximum allowed magnetic field is assumed to be the 16T.

Chromatic Correction (CC) scheme

- The CC scheme include 2 sets (doublets) of combined function dipole-sextupole magnets and each set is placed at positions with large β_q, where q=x or y, for the correction of the W_q at the end of CC scheme.

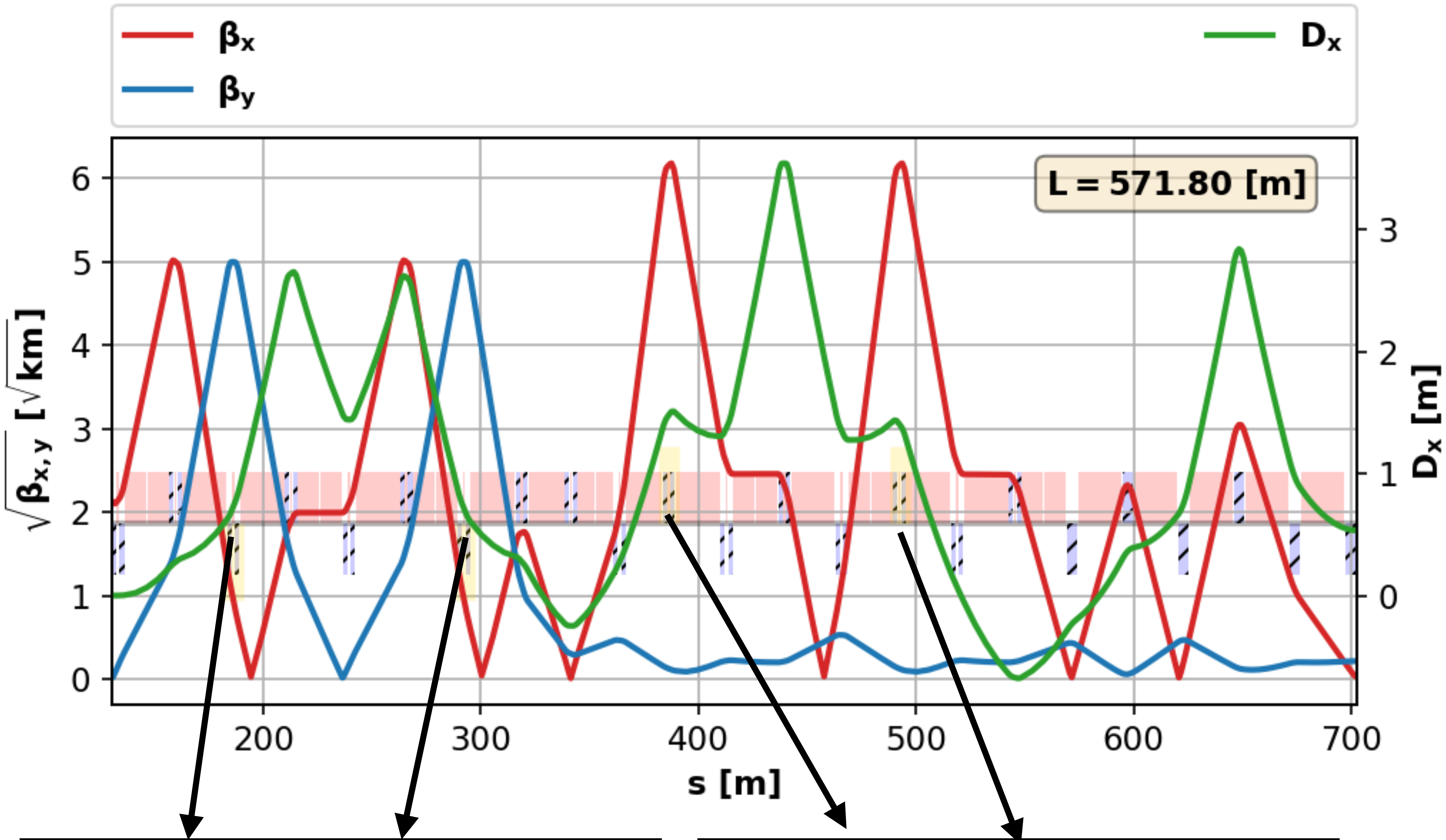
- Each set include a pair of dipole-sextupole magnets with the same k_2 and are separated by -I transform at x and y planes for the compensation of the RDTs excited by the sextupolar component.

Matching scheme (CC-Arc)

- The $\beta_{x,y}$, $\alpha_{x,y}$, D_x and D_{px} are matched by controlling the strength of six dipole-quadrupole and the dipole length separating the dipole-quadrupole magnets.

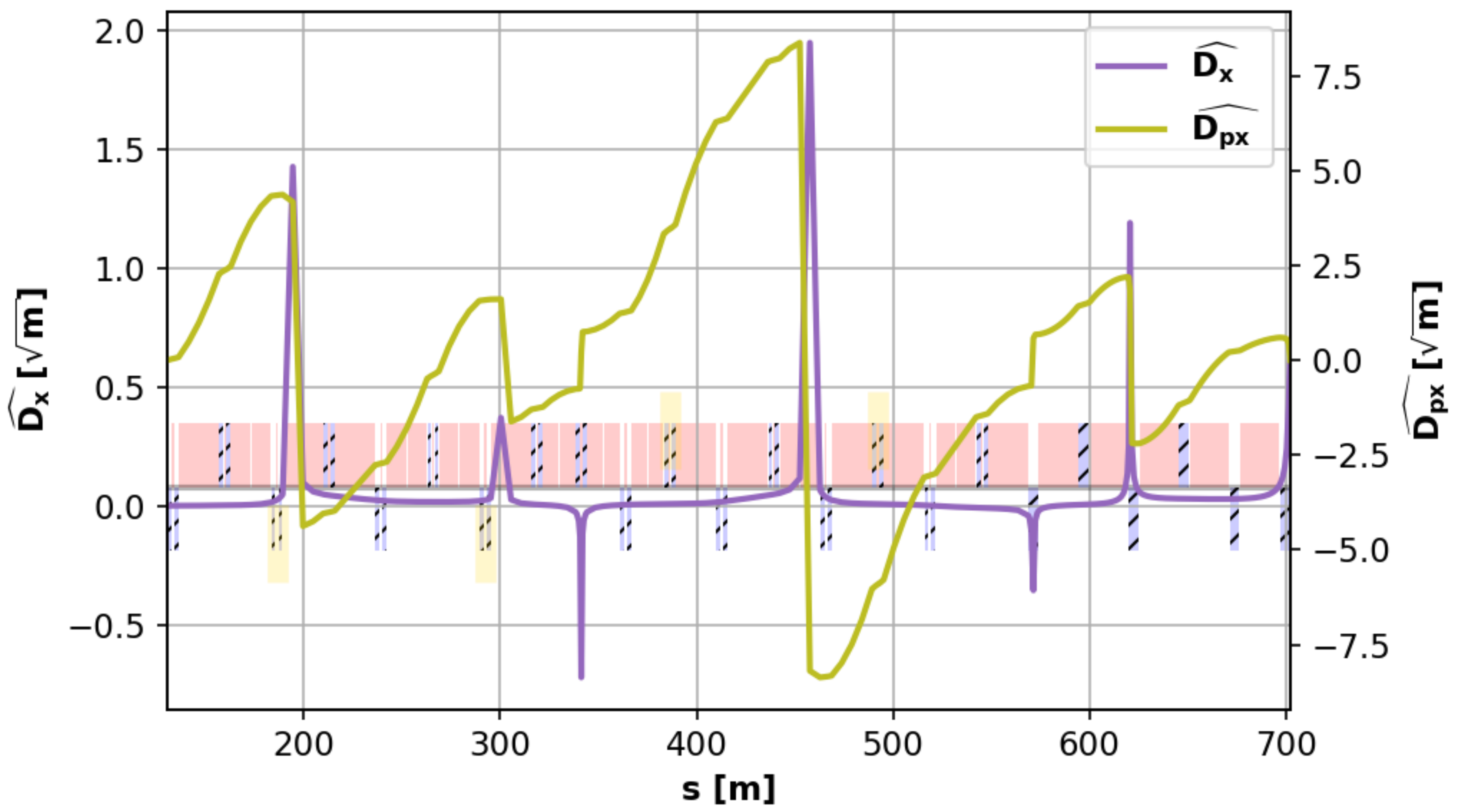
- The matching of the optical functions is facilitated by controlling its value at the end of the CC scheme (keeping it to small values).

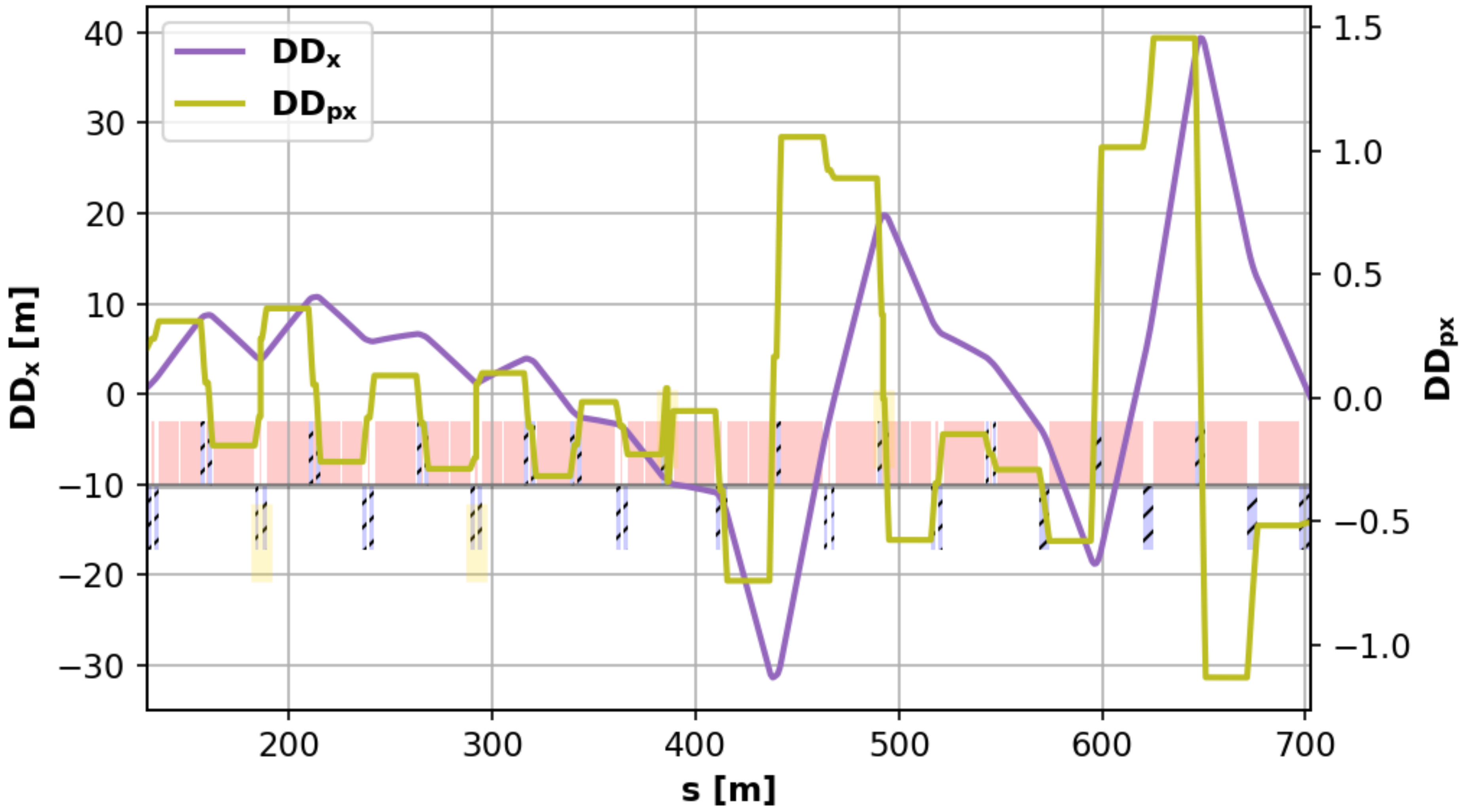
10TeV Muon Collider - Chromatic Correction & Matching Schemes



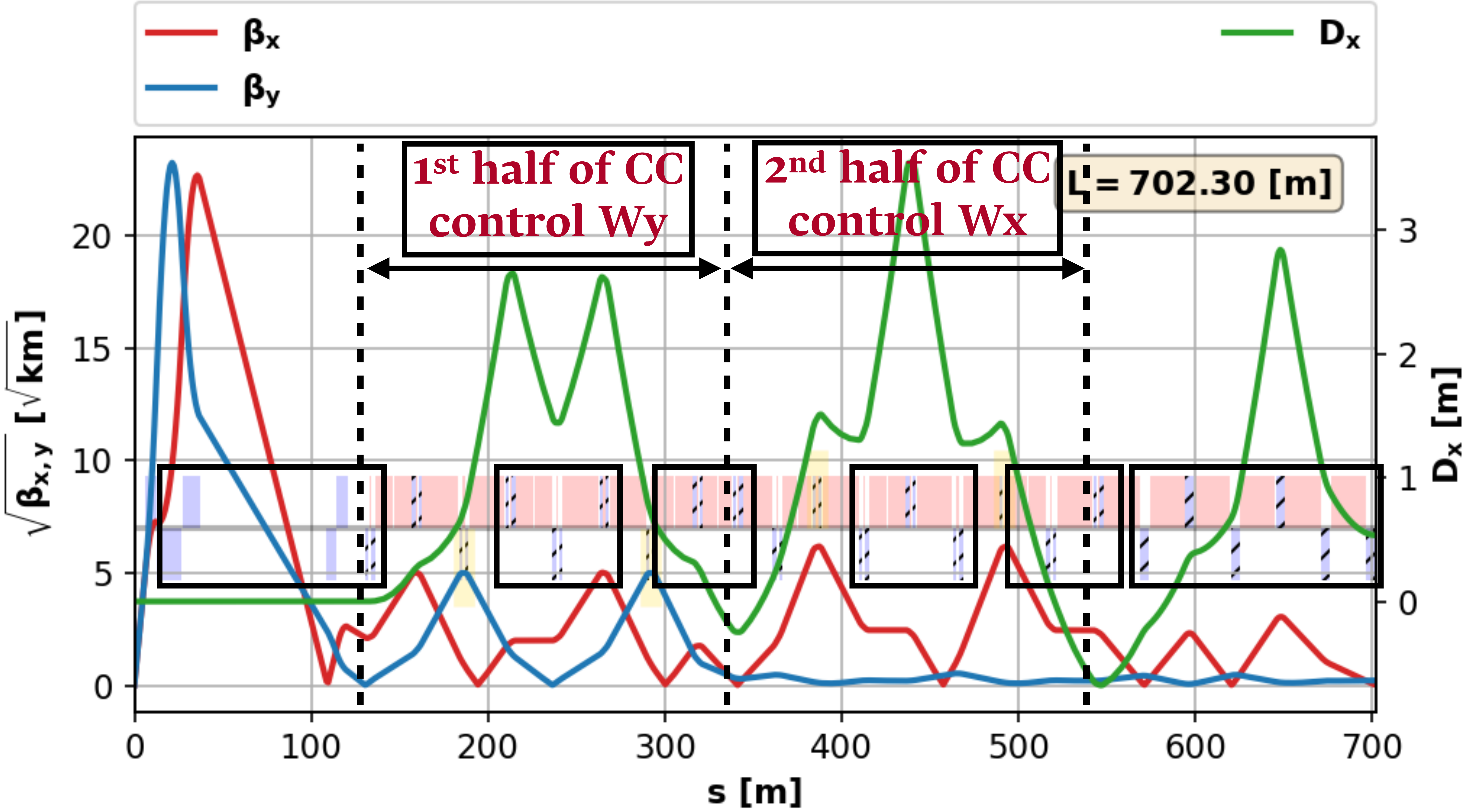
1st DS doublet for W_y
Large D_x, β_y and small β_x

2nd DS doublet for W_x
Large D_x, β_x and small β_y

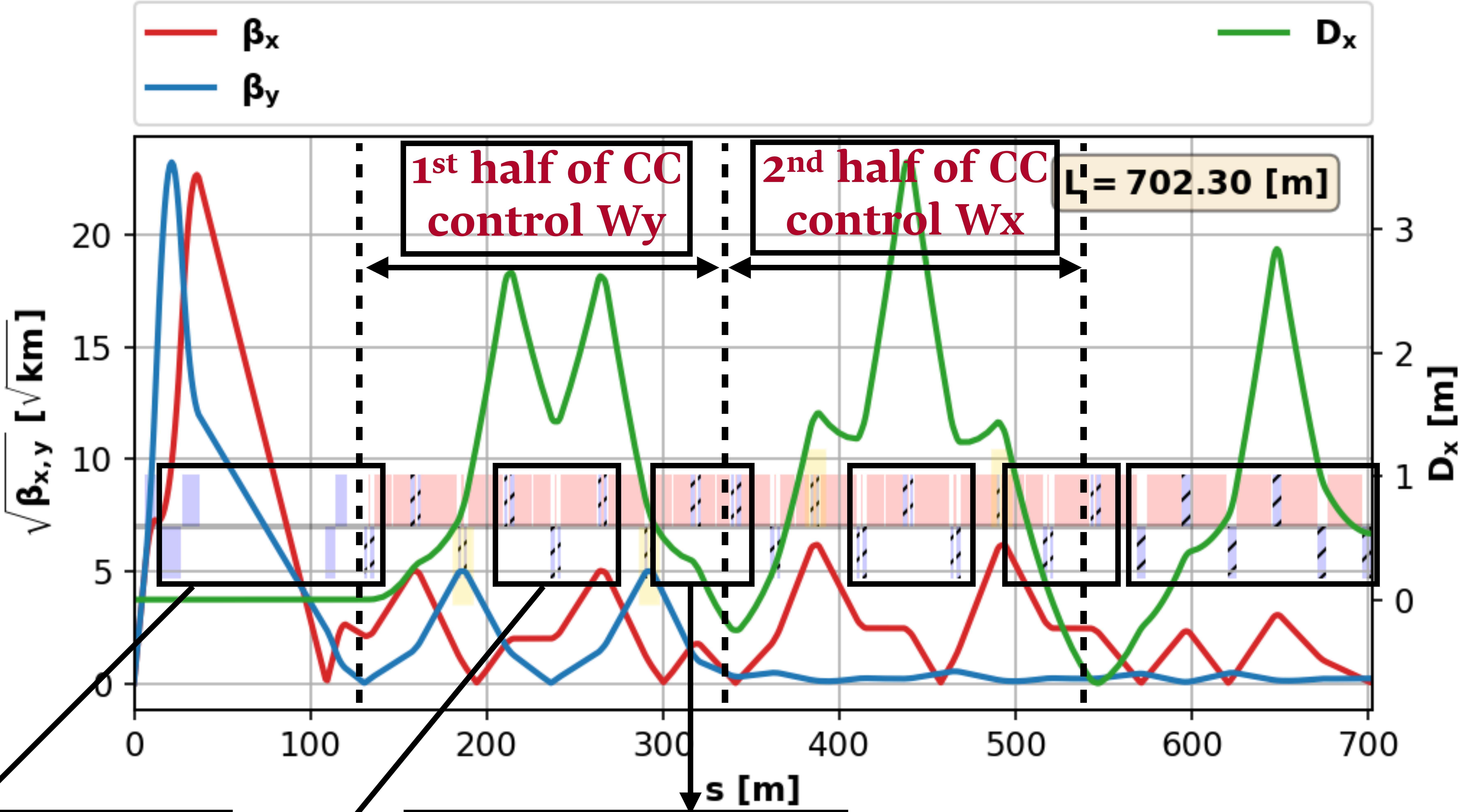




10TeV Muon Collider - Extended Final Focusing Schemes



10TeV Muon Collider - Extended Final Focusing Schemes

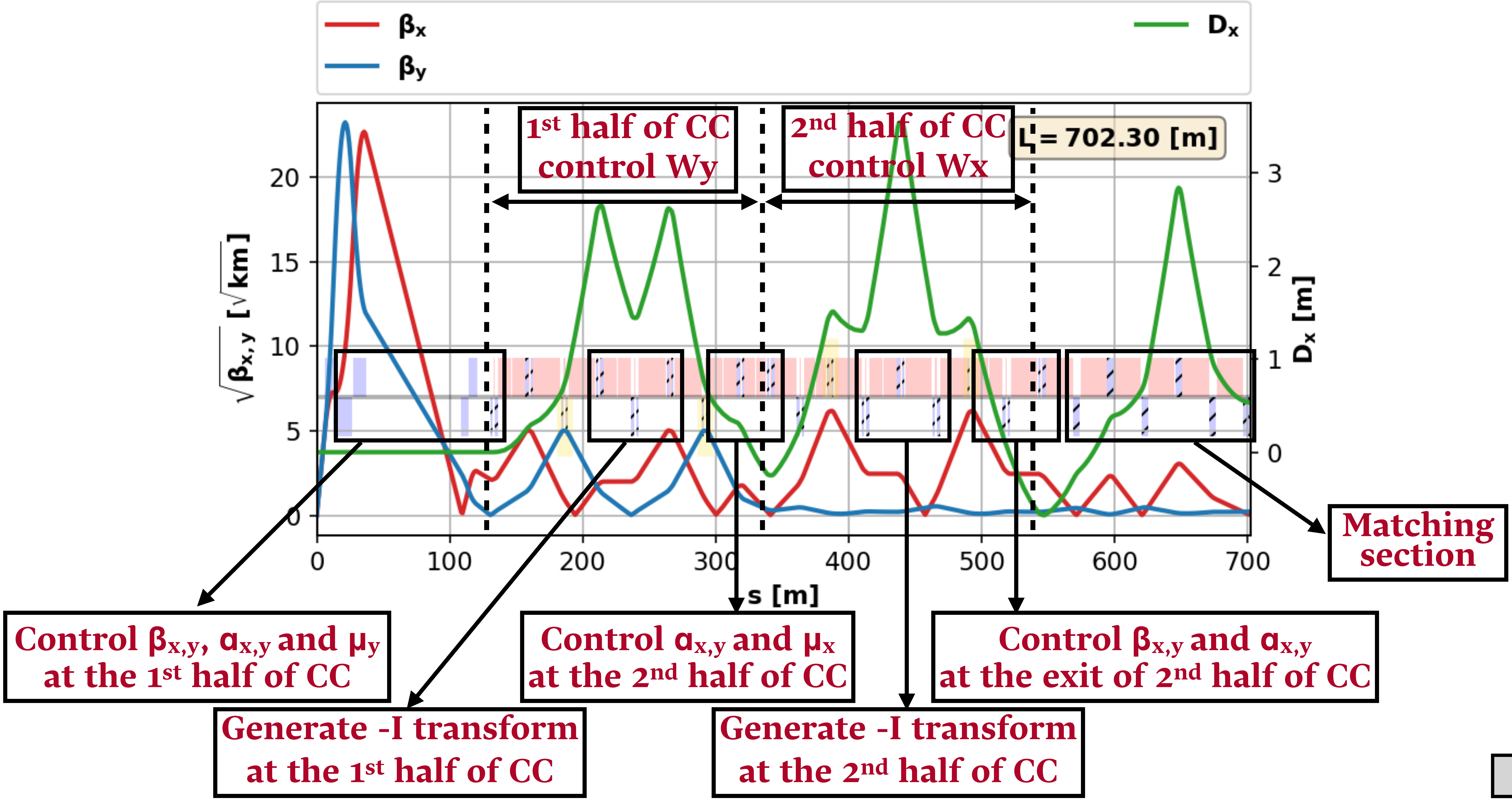


Control $\beta_{x,y}$, $\alpha_{x,y}$ and μ_y at the 1st half of CC

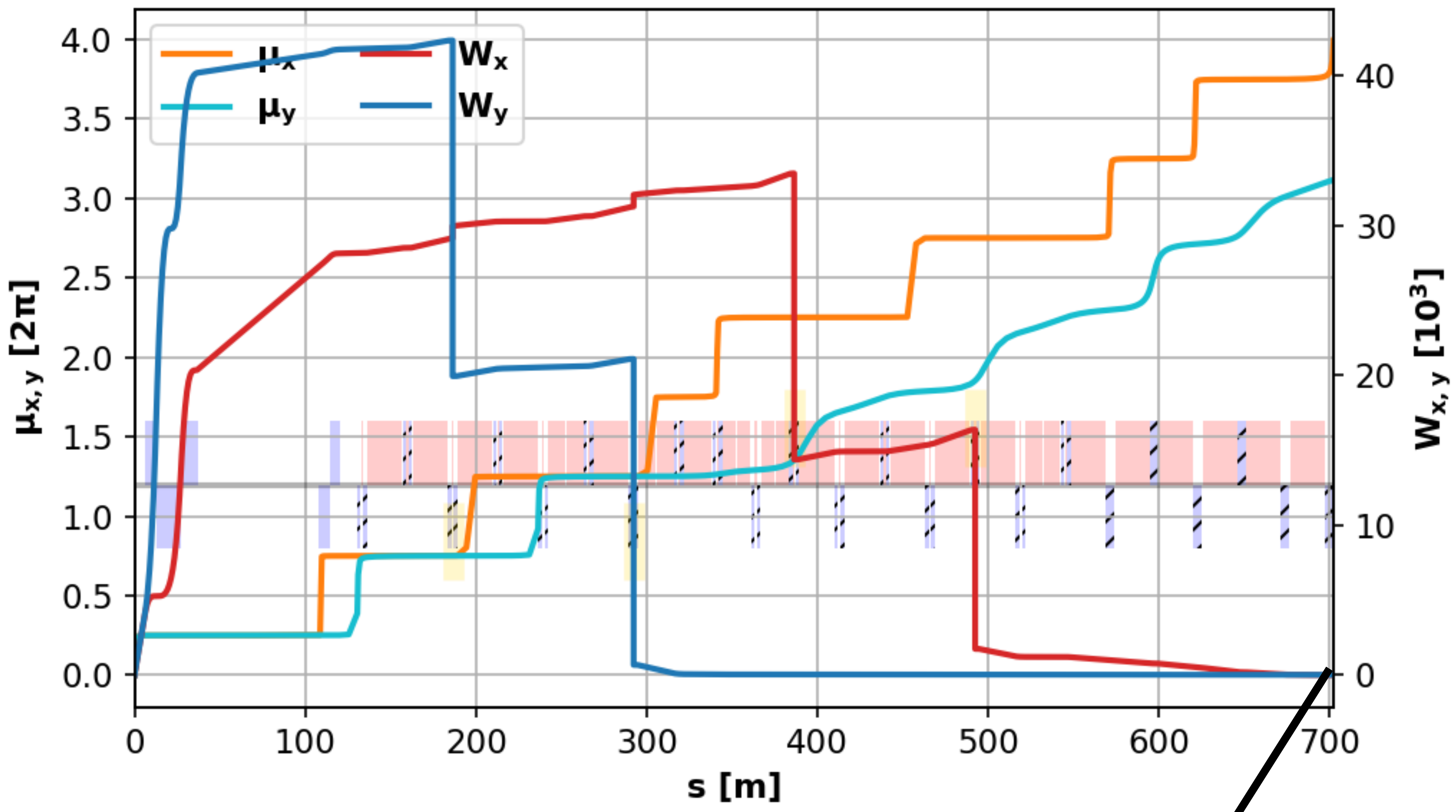
Control $\alpha_{x,y}$ and μ_x at the 2nd half of CC

Generate -I transform at the 1st half of CC

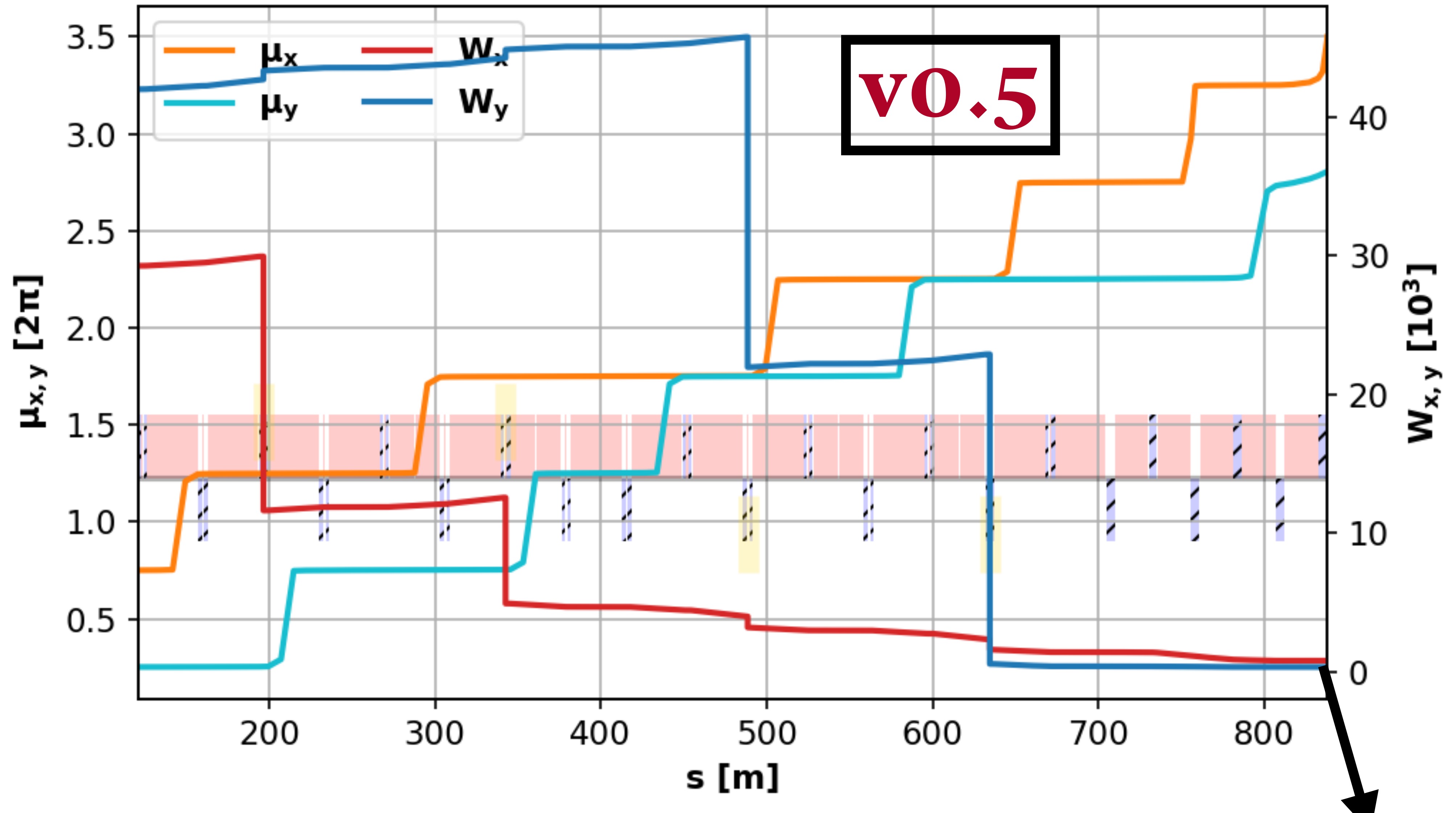
10TeV Muon Collider - Extended Final Focusing Schemes



10TeV Muon Collider - Chromatic Correction & Matching Schemes



$\delta^* W_{x,y} < 0.02$

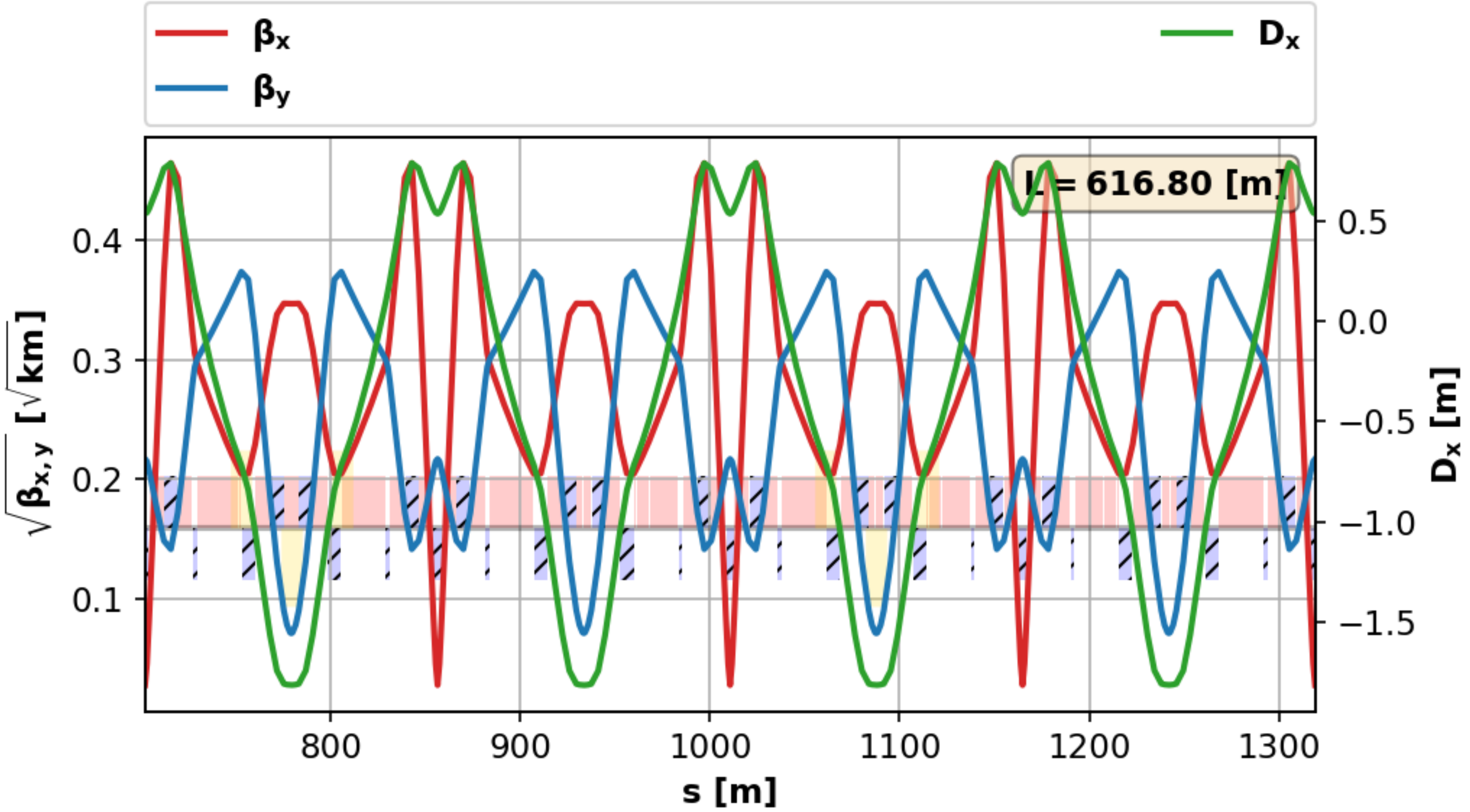


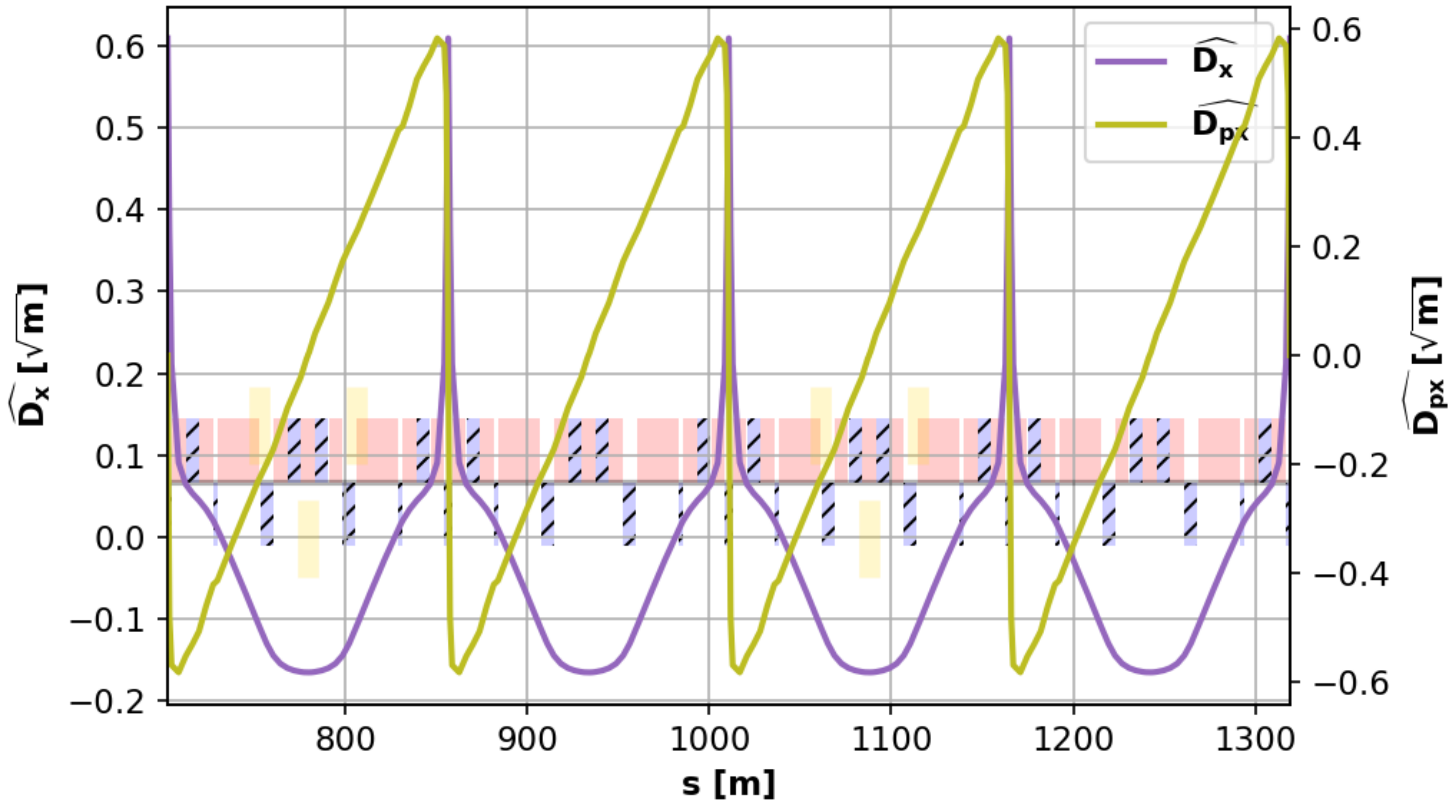
$\delta^* W_{x,y} < 1$

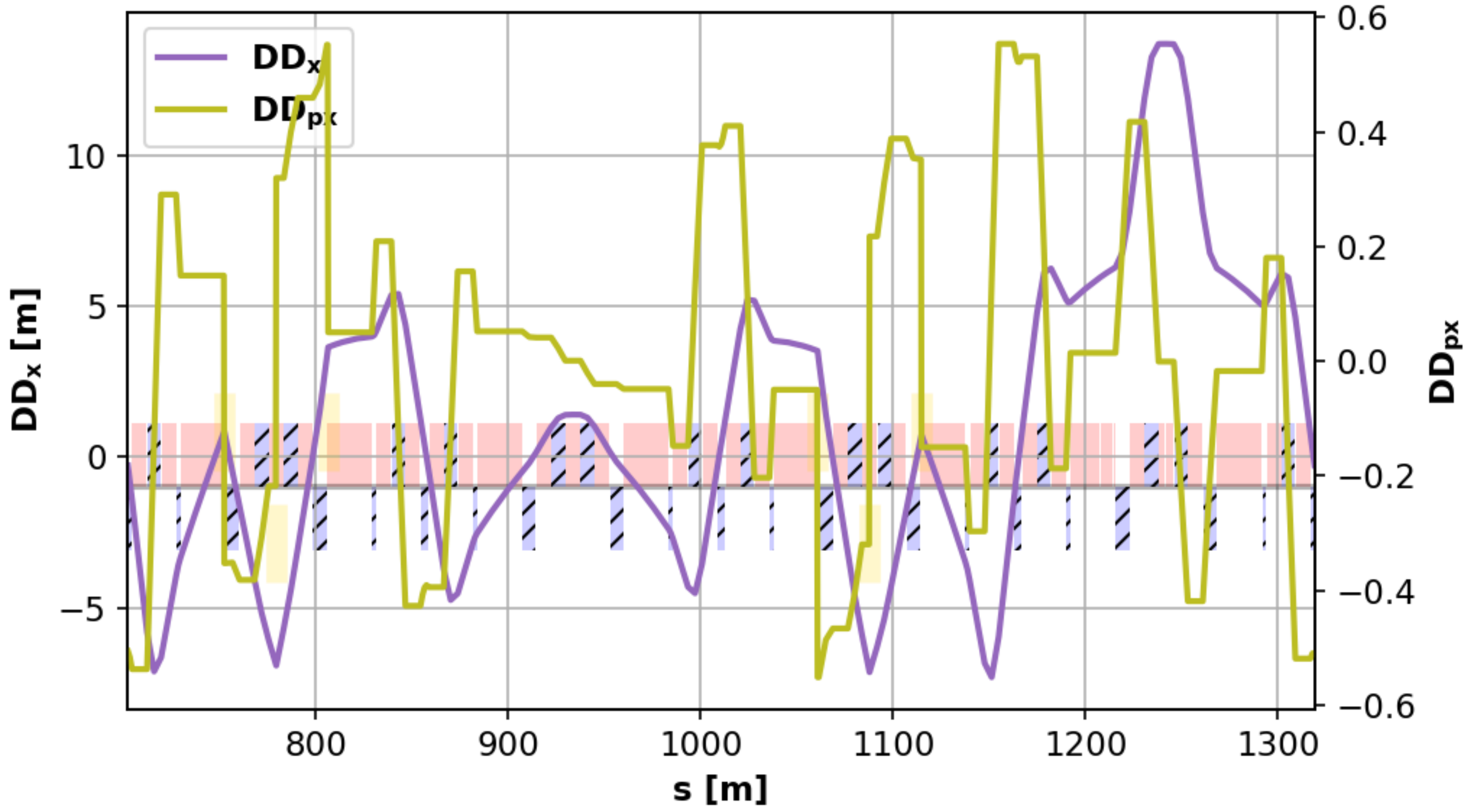
10TeV Muon Collider - Arc

- The CC scheme produces a large positive contribution to the momentum compaction factor (α_p) and phase slip ($\eta_p \sim \alpha_p - 4.5 \times 10^{-10}$) thus, a negative contribution from the arcs is generated in order to keep η_p small and stay below transition ($\eta_p, \alpha_p < 0$).
- The maximum allowed magnetic field is assumed to be the 16T.
- Each arc section consist of repeated Flexible Momentum Compaction (FMC) cells (each one is made out of 2 FODO cells).
- The integrated strength of a set of dipoles located at areas with negative dispersion controls the α_p while with another set of dipoles, the 2π closing of the trajectory is controlled.
- The linear chromaticity at x and y planes is controlled with a set of combined function dipole-sextupole magnets separated by a -I transform.
- The phase advance per FMC cell is $3\pi/2$ (-I transform every second cell).

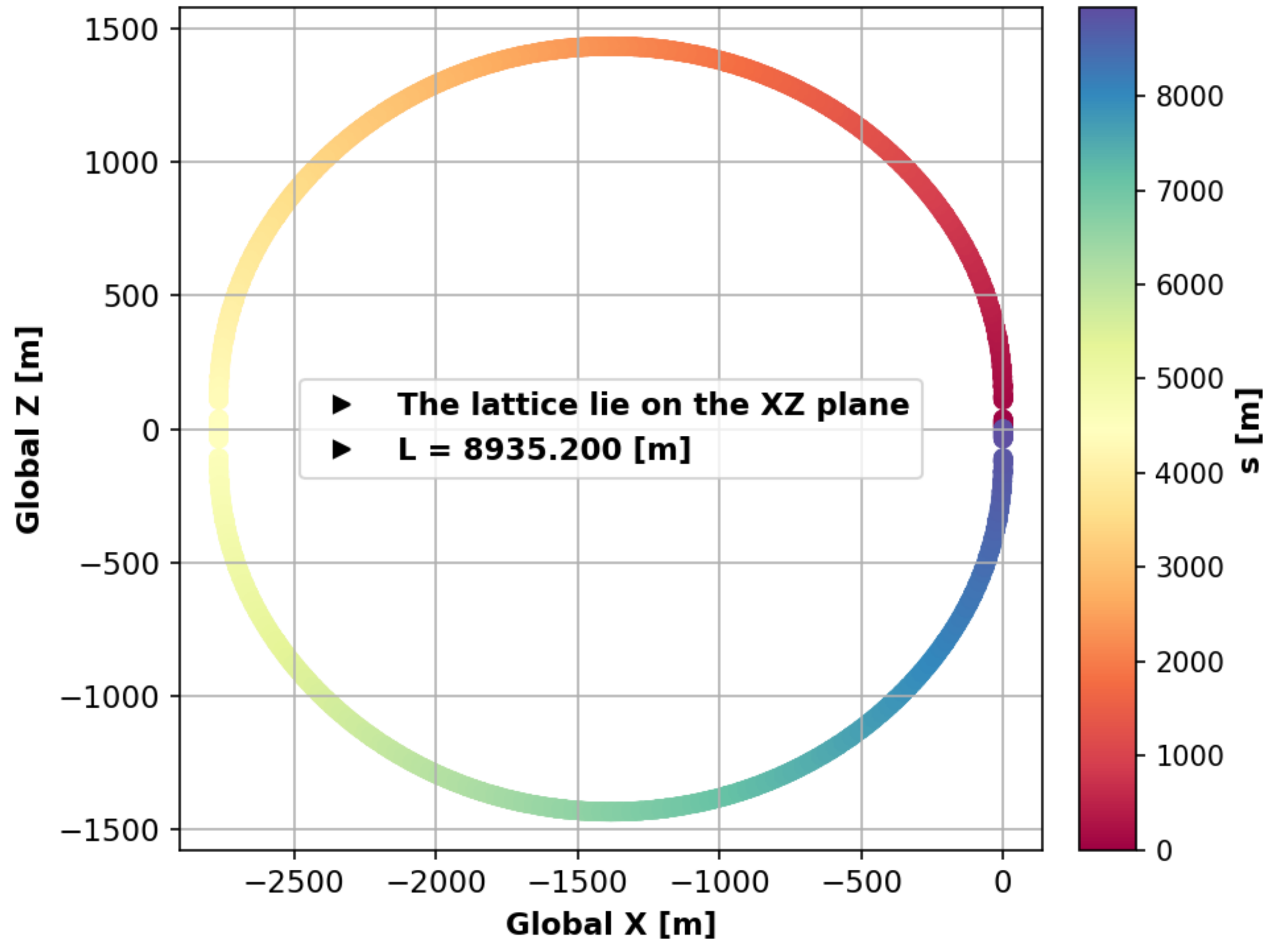
10TeV Muon Collider - Arc

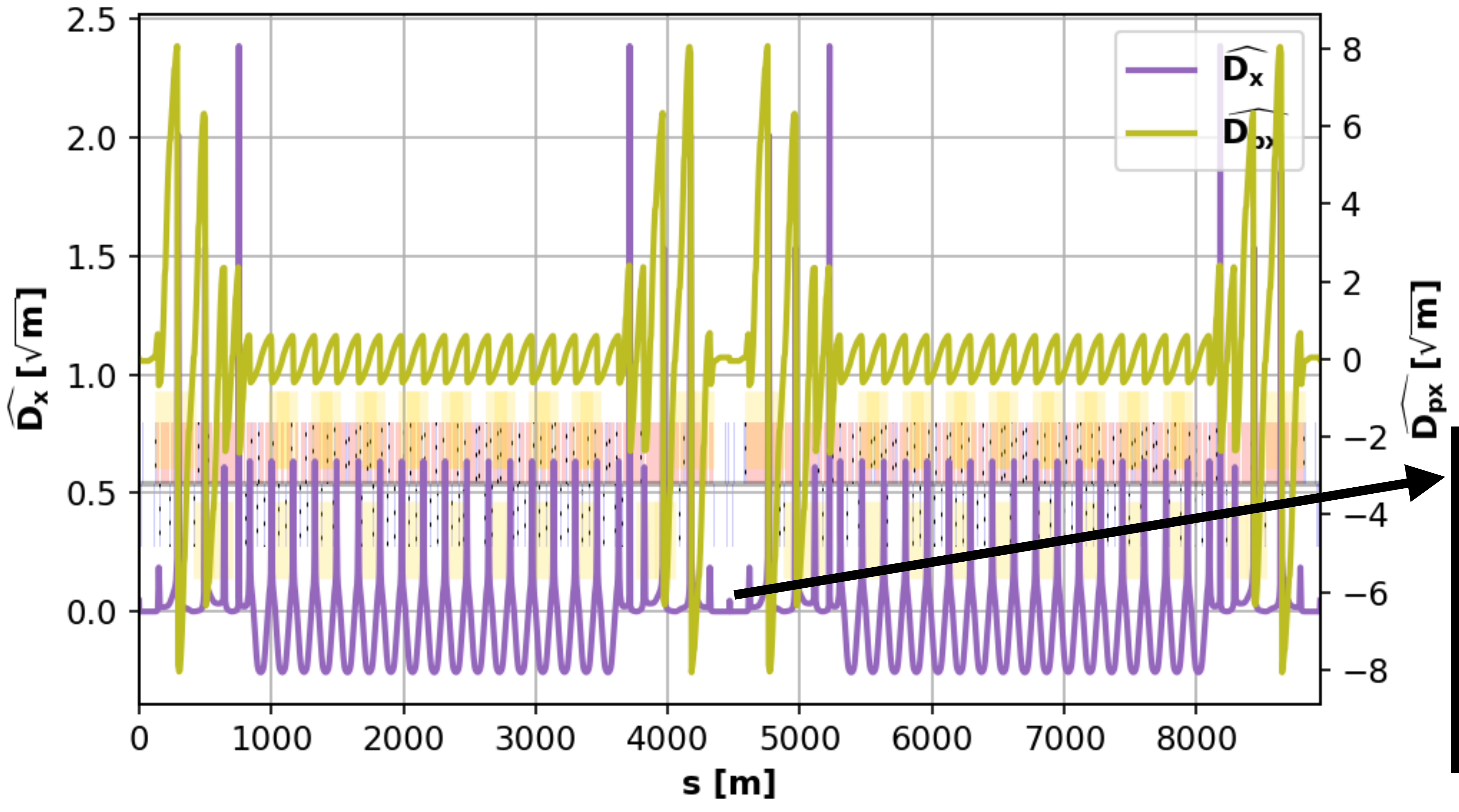






10TeV Muon Collider - Full Lattice v0.5

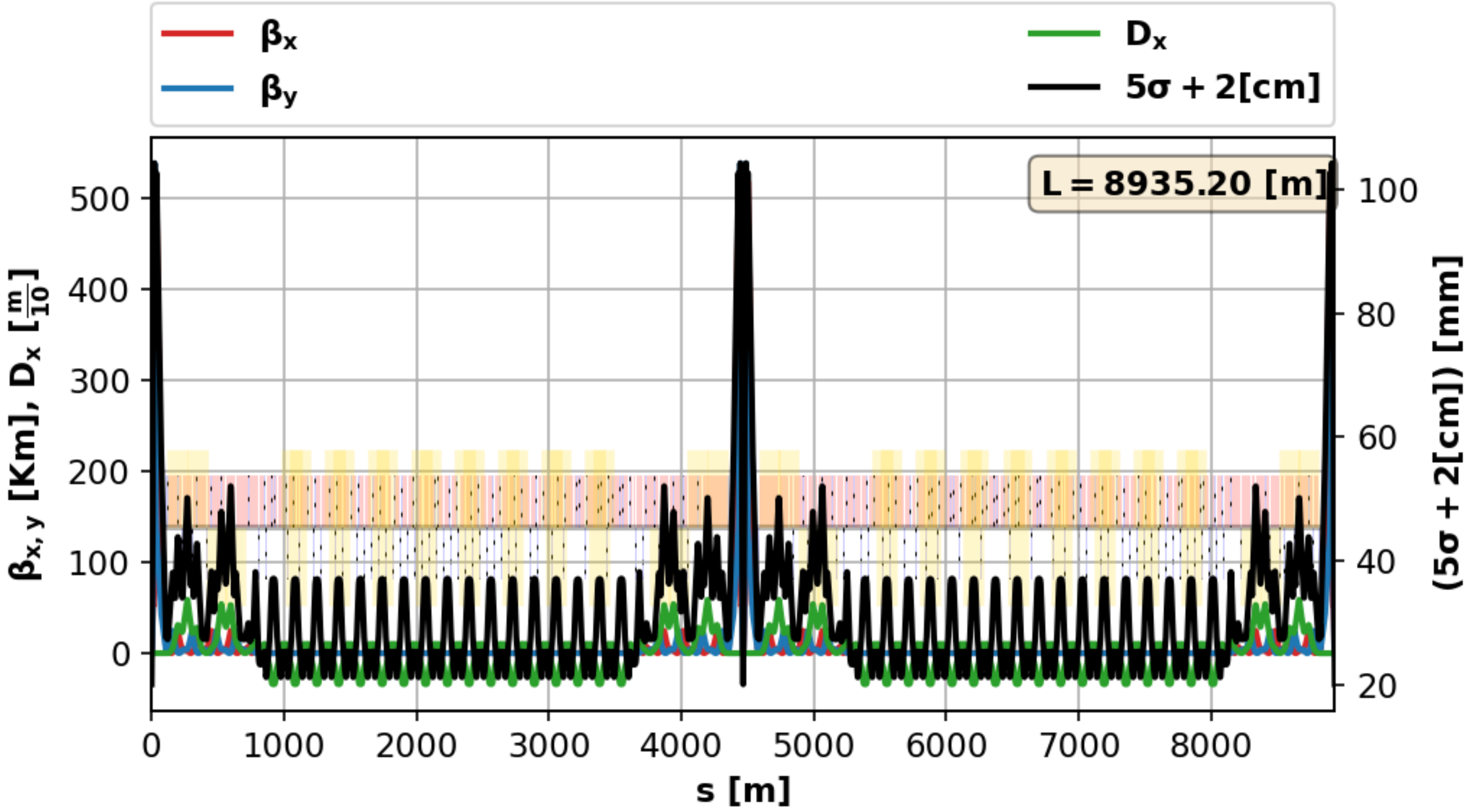


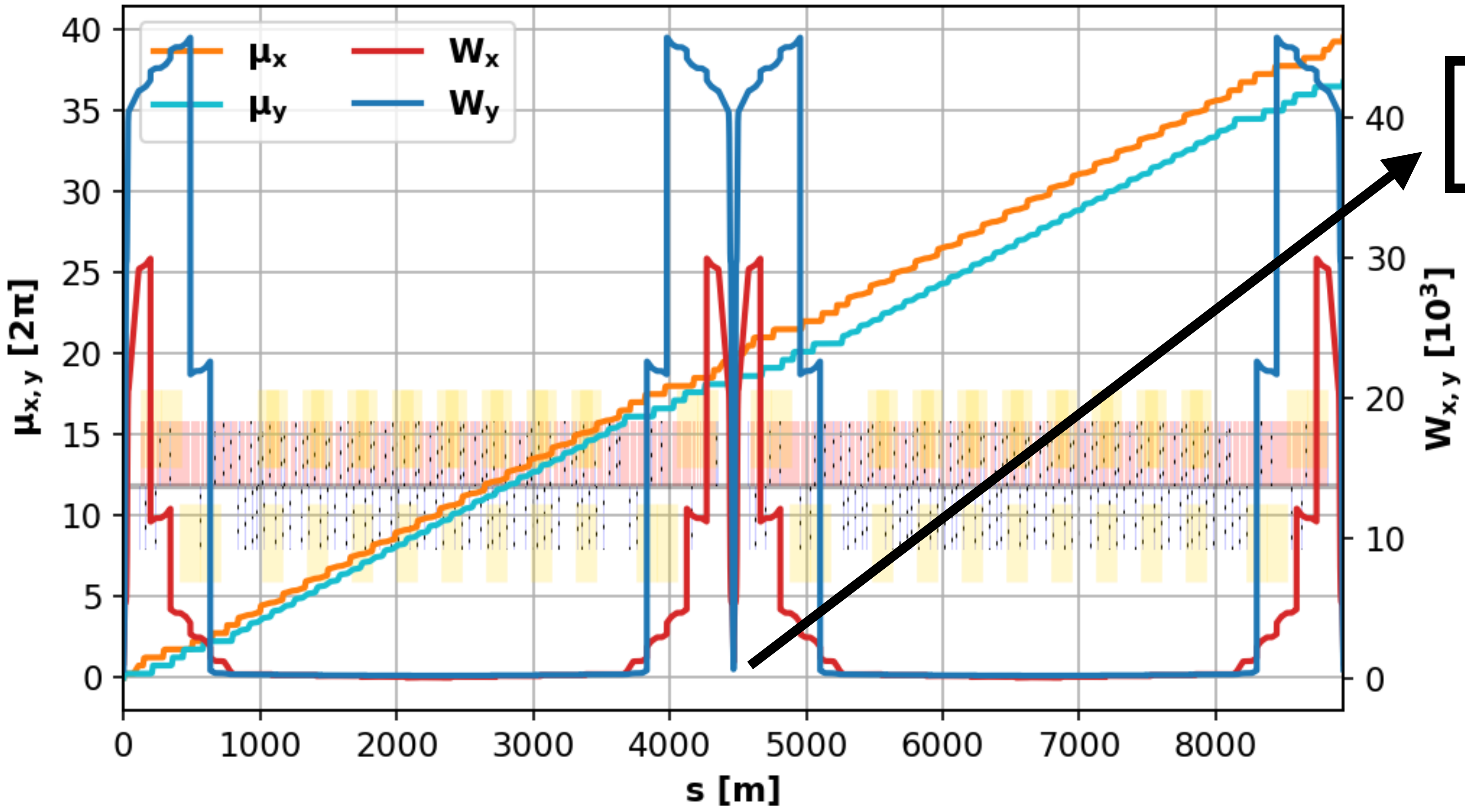


$D_{nx}, D_{npx} \neq 0$

$(D_x * \delta)^2 \simeq 4*(\varepsilon_g * \beta_x)$

$\sigma_x \simeq \sqrt{5}\sigma_{\beta x}$

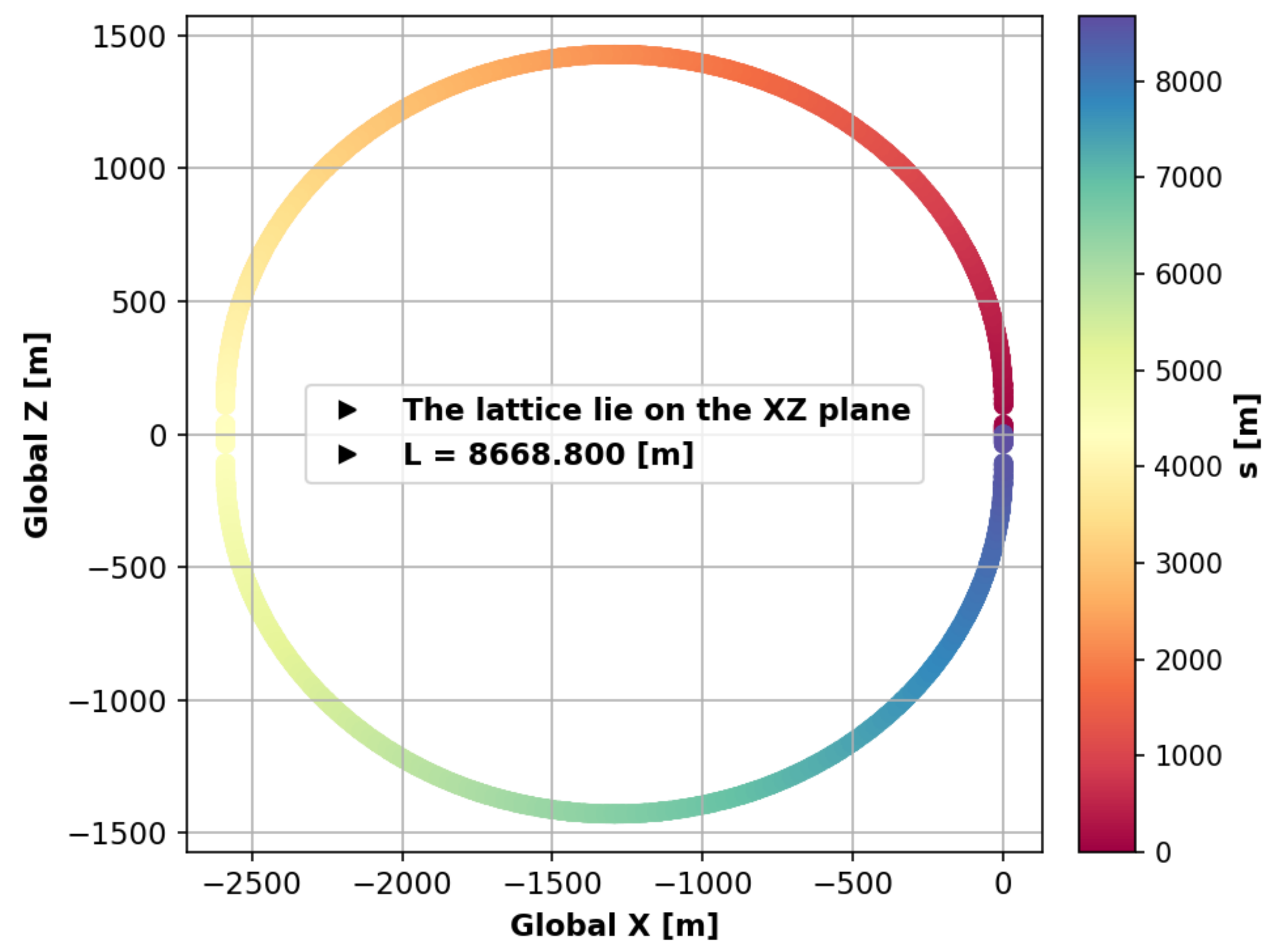


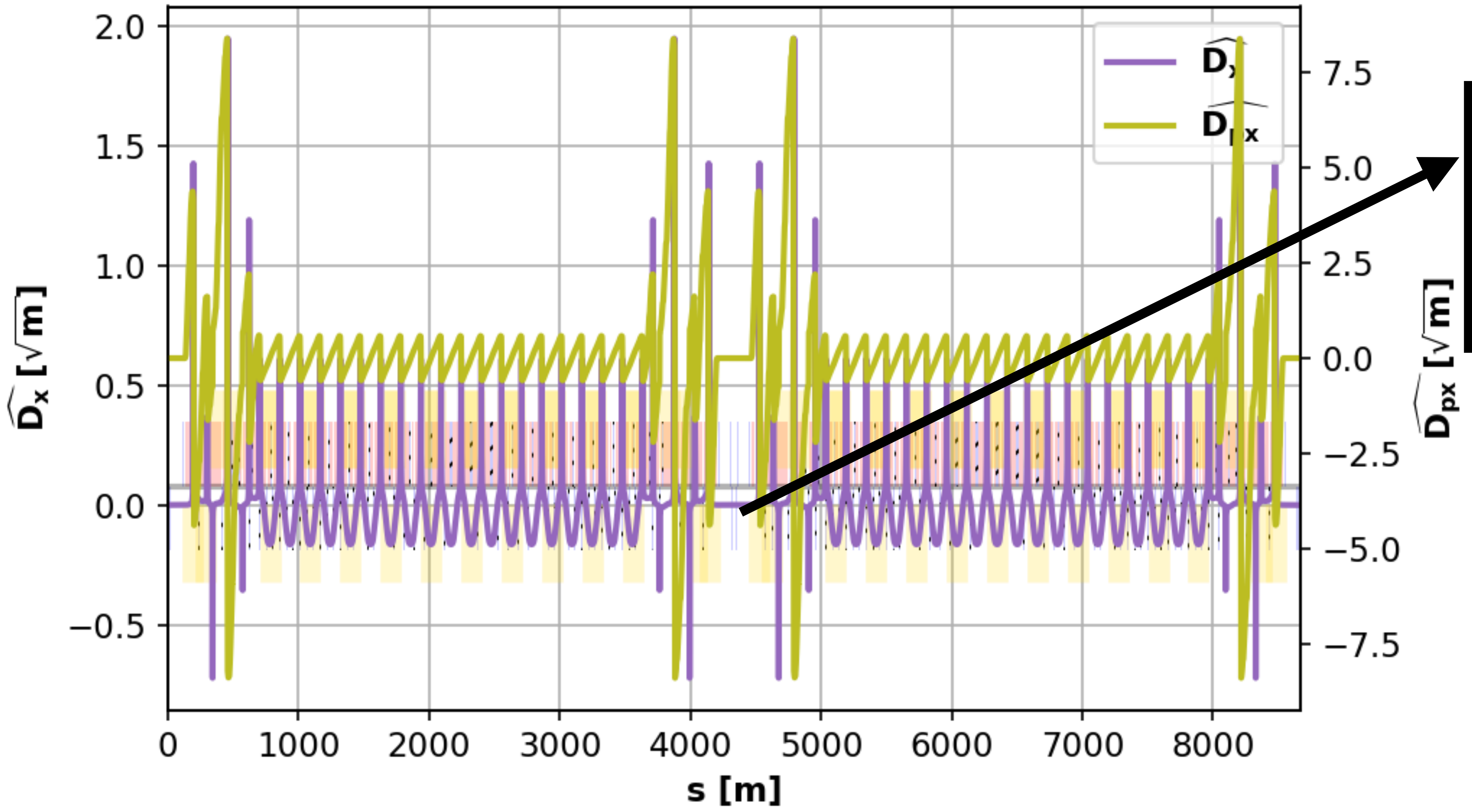


$\delta * W_{x,y} \simeq 1$

Matching issues?

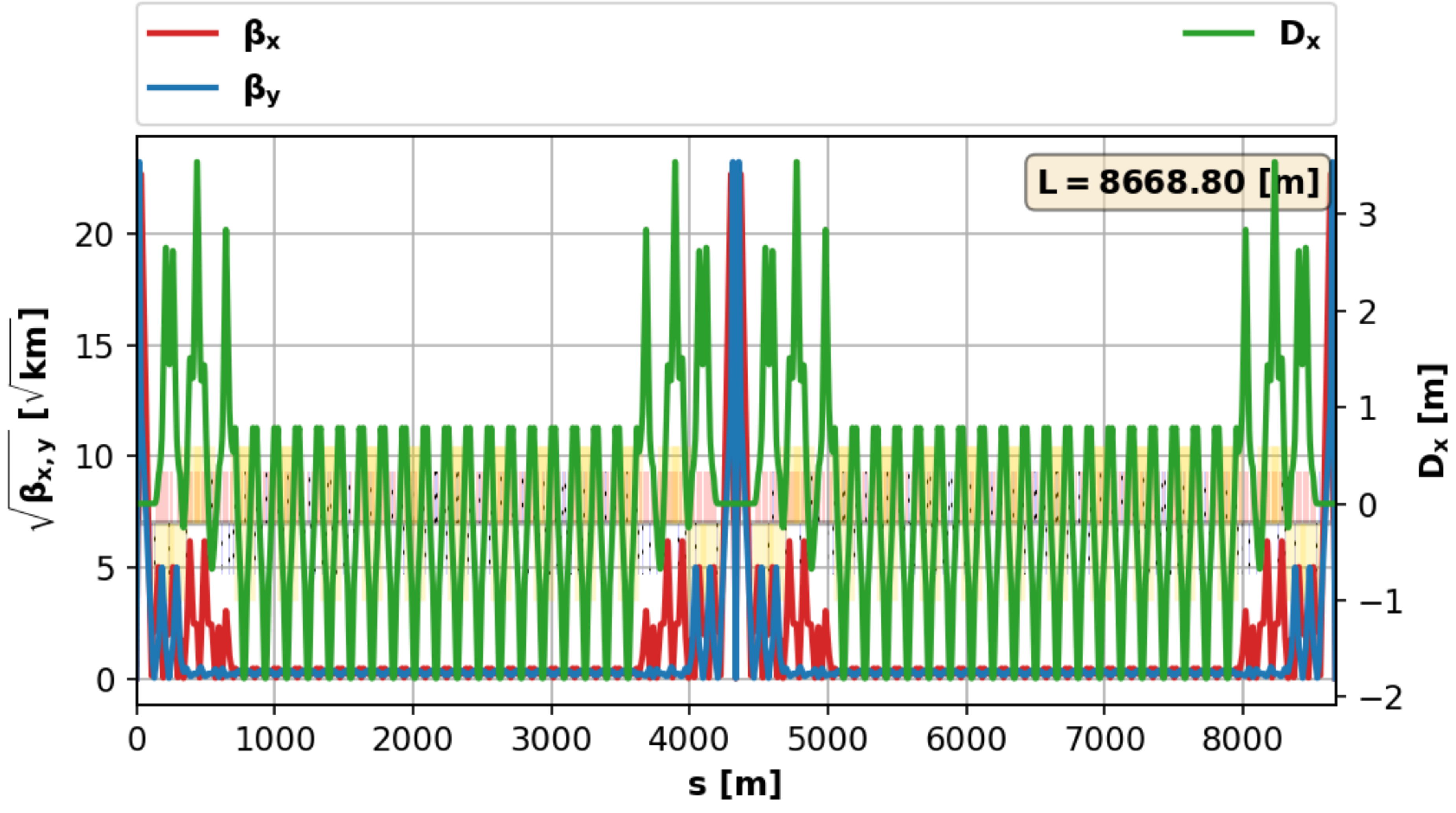
10TeV Muon Collider - Full Lattice

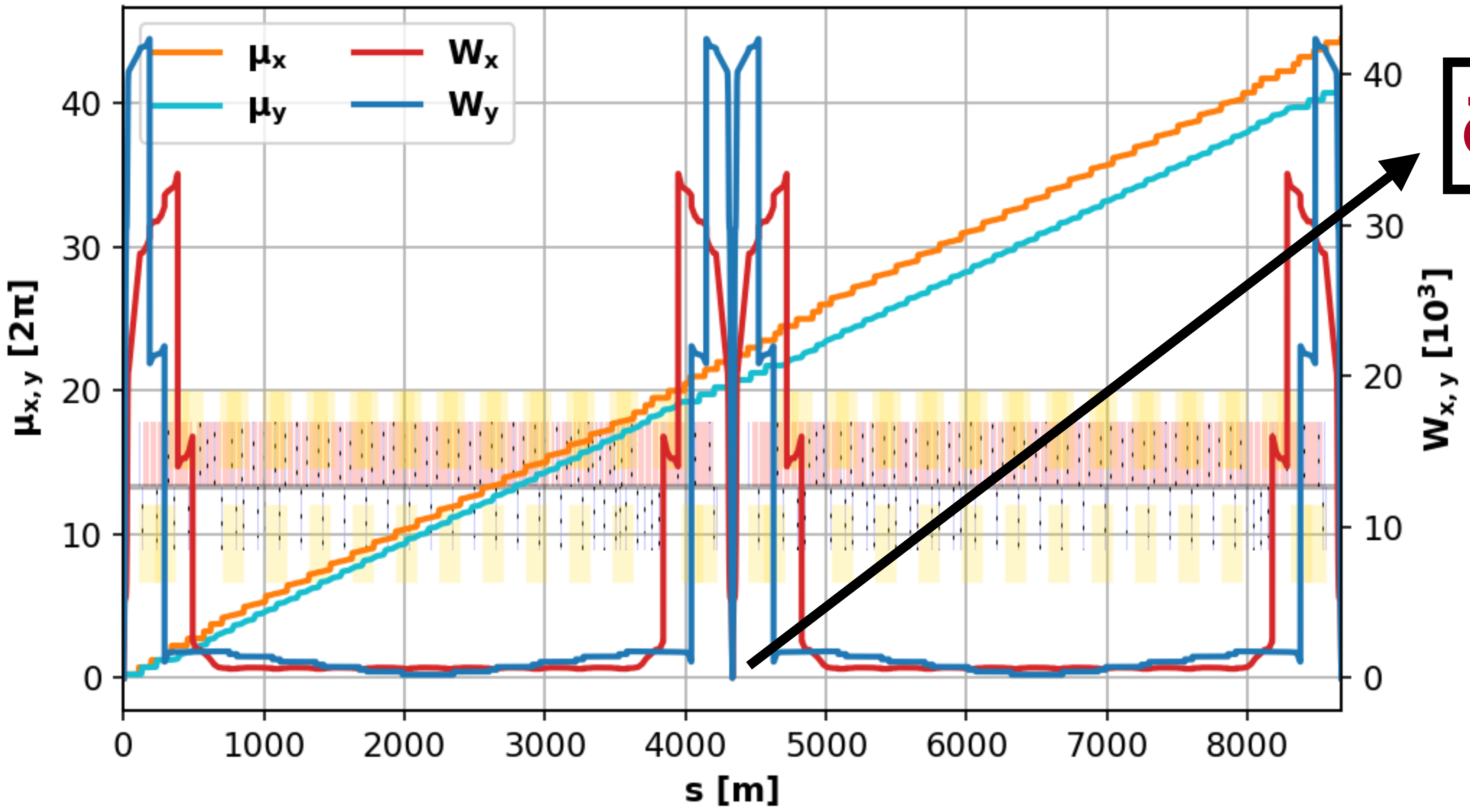




D_{nx}, D_{npx} ~ 0

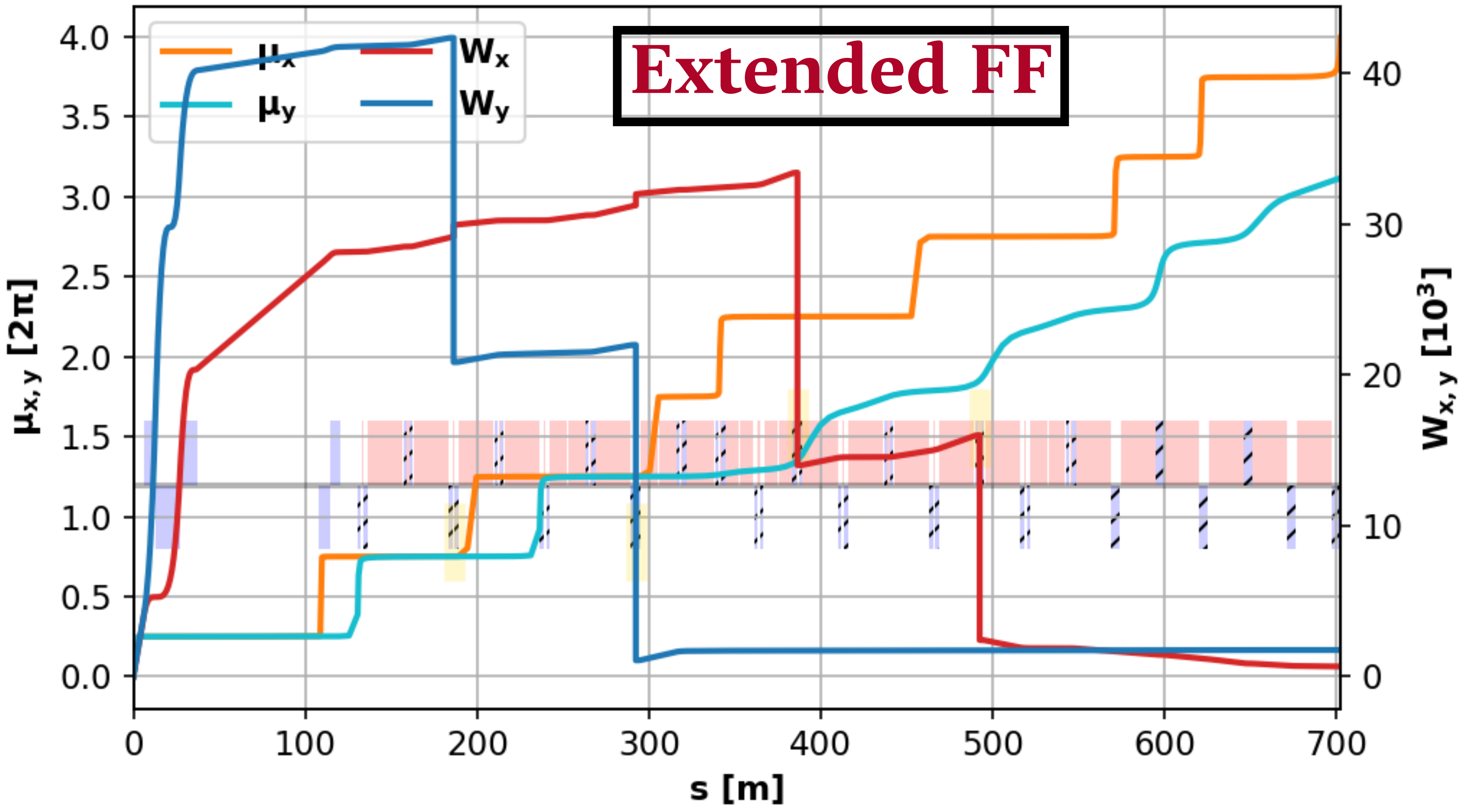
$\sigma_x \simeq \sigma_{\beta x}$

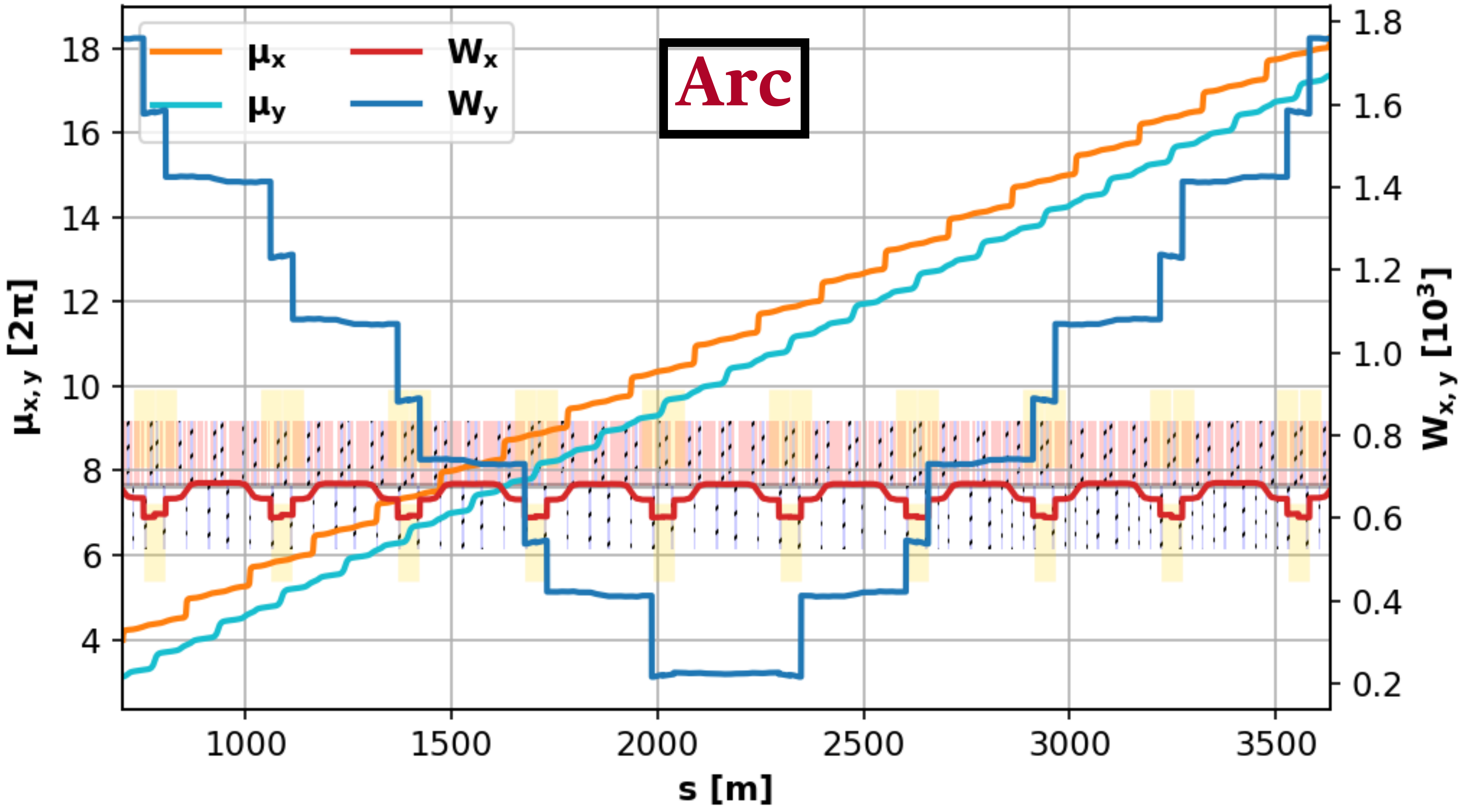




$\delta^* W_{x,y}$ ~0

10TeV Muon Collider - Full Lattice





Match of linear chromaticity with arc dipole-sextuples.

↓

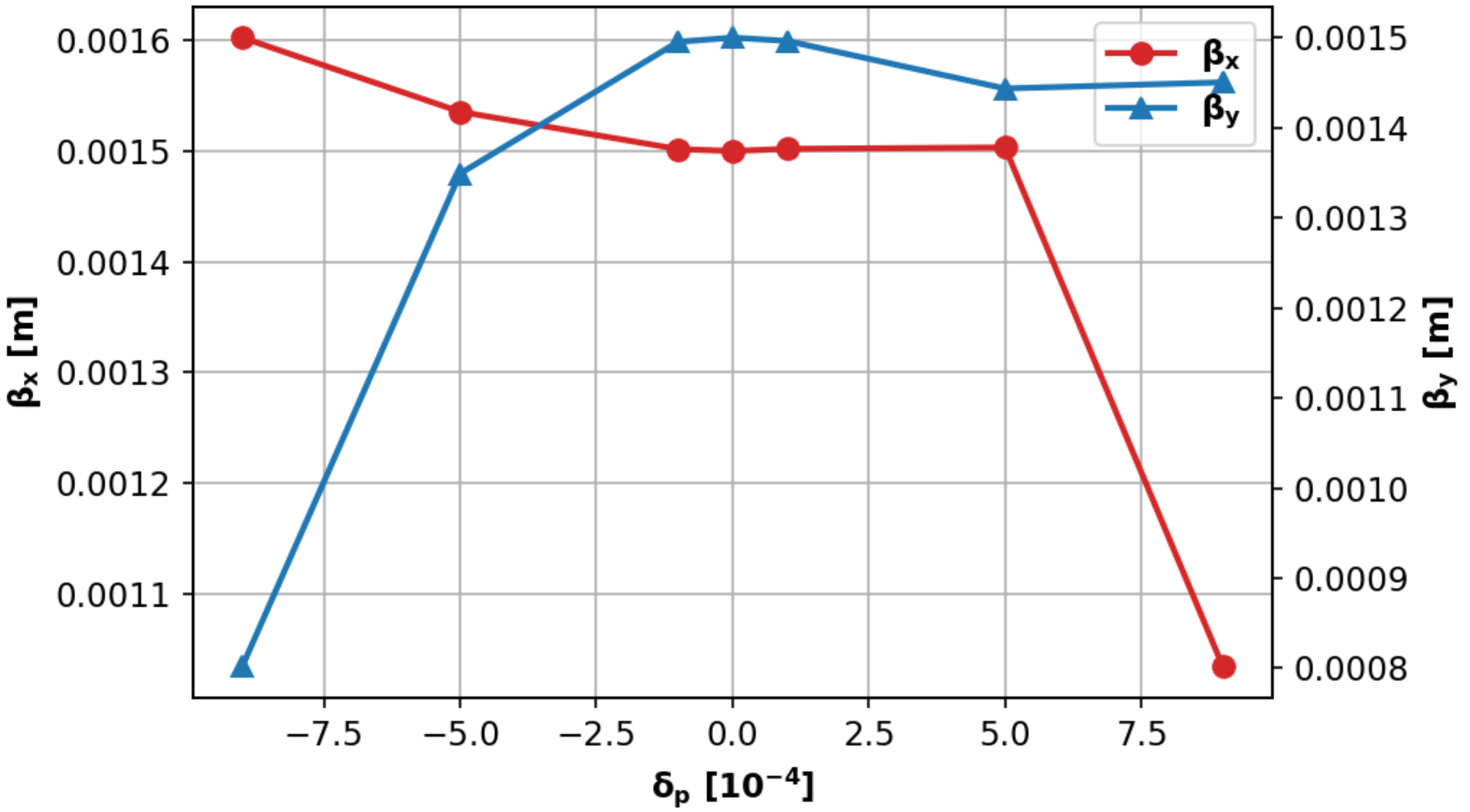
The $W_{x,y}$ at the IP are perturbed thus, the dipole-sextuples at the CC are know used to correct $W_{x,y}$ at IP.

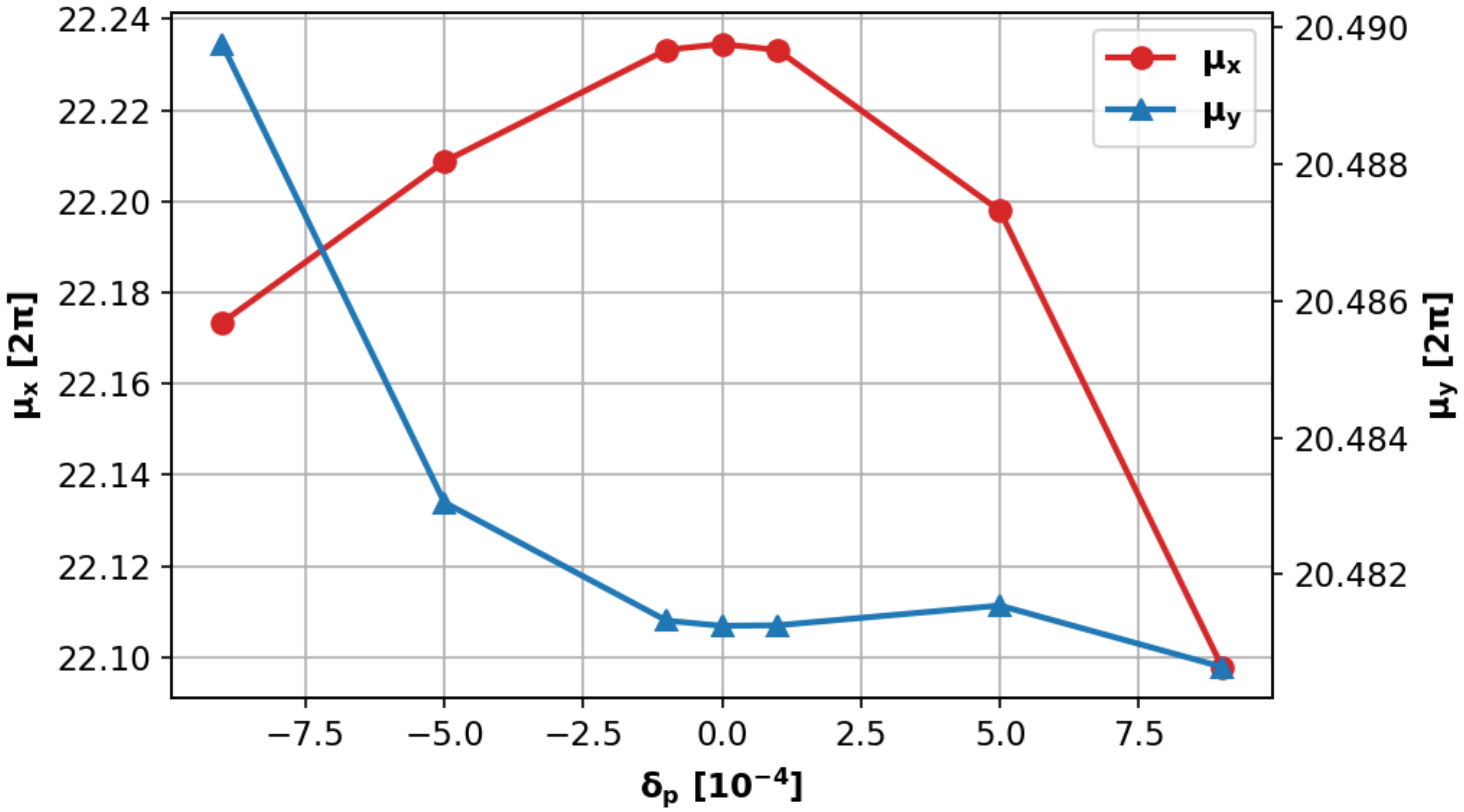
↓

The $W_{x,y}$ at the end of the extended FF is not zero.

10TeV Muon Collider - Full Lattice

- **In order to keep the $\beta^{\star}_{x,y}$ unchanged for different δs, the μ_y from the IP to the 1st dipole-sextupole of the CC as well as the μ_x from the IP to the 3rd dipole-sextupole of the CC are controlled.**

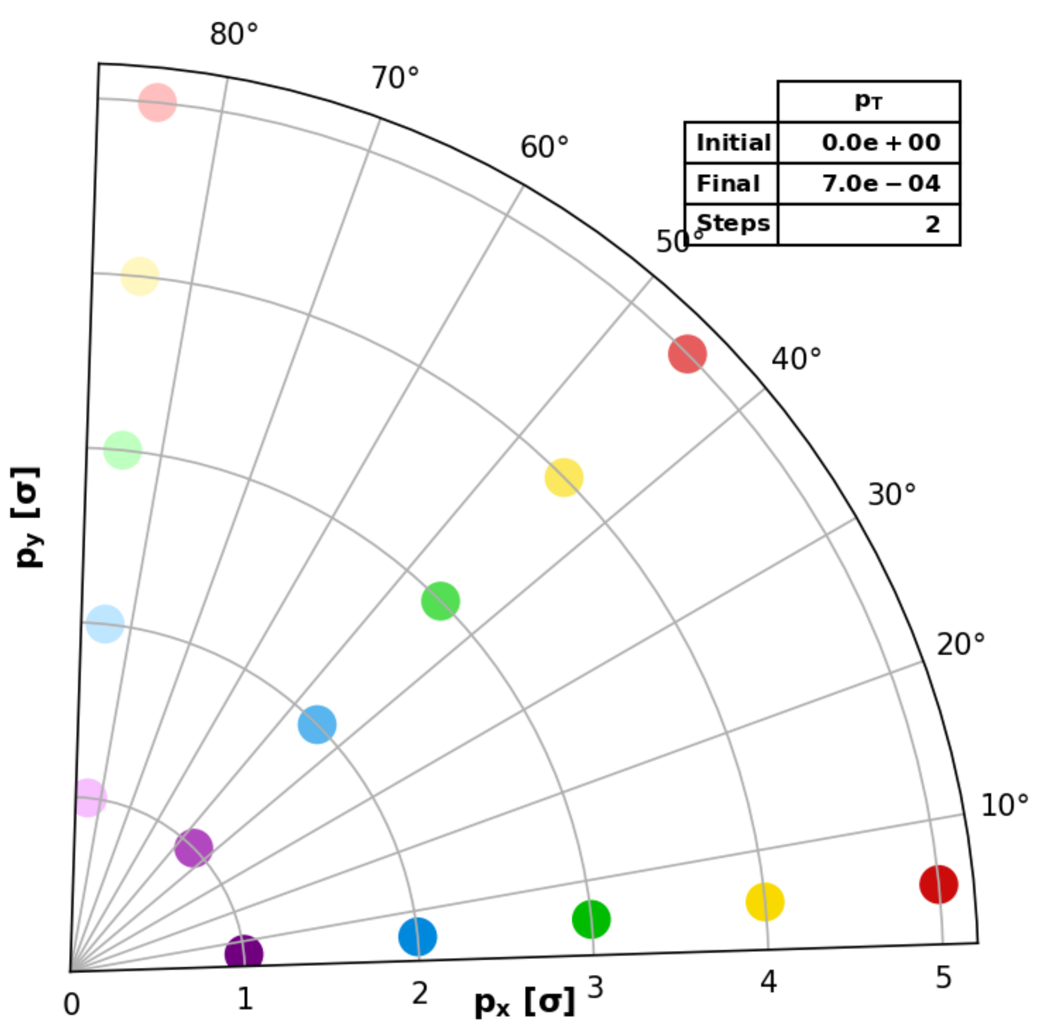


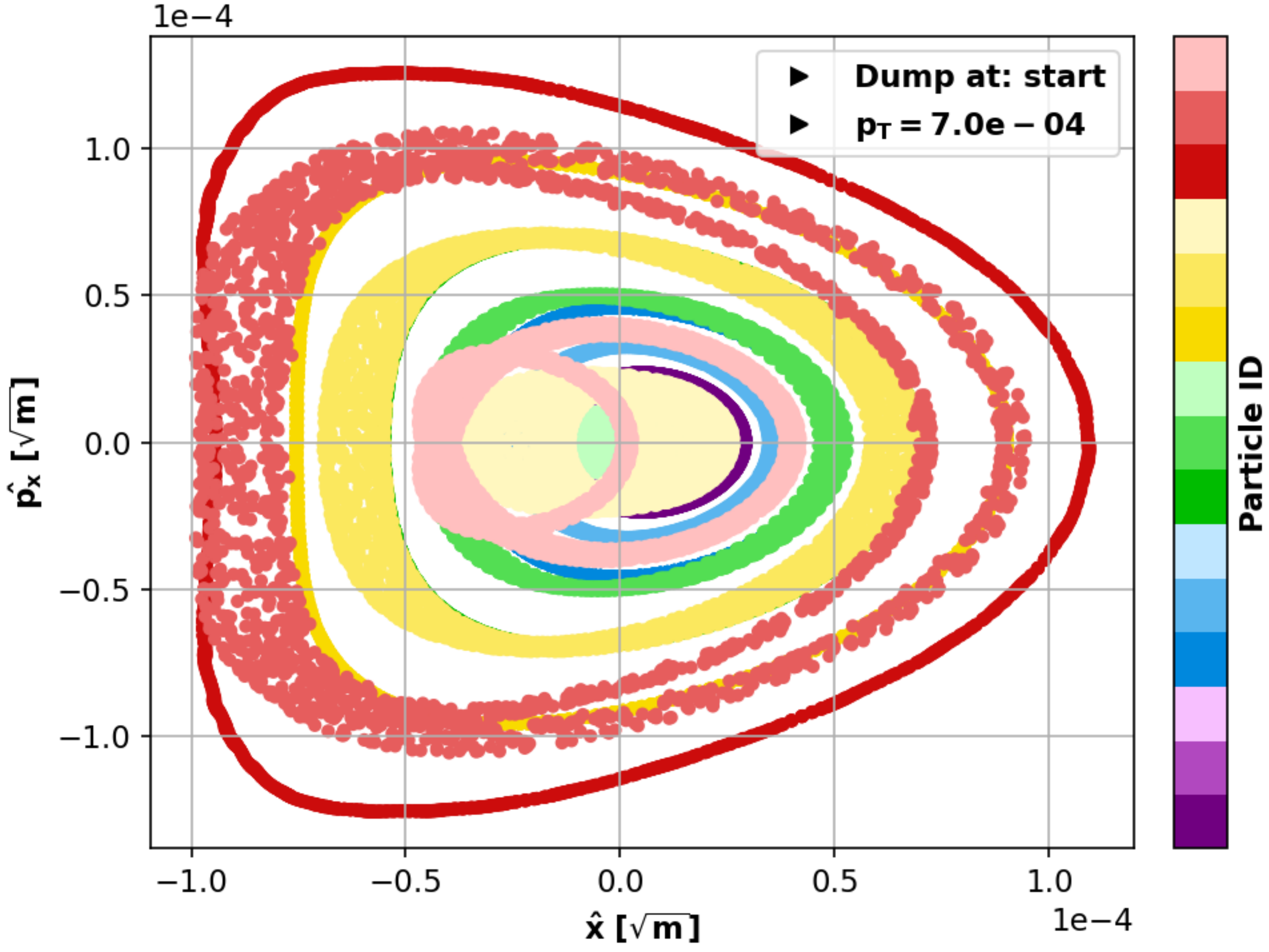


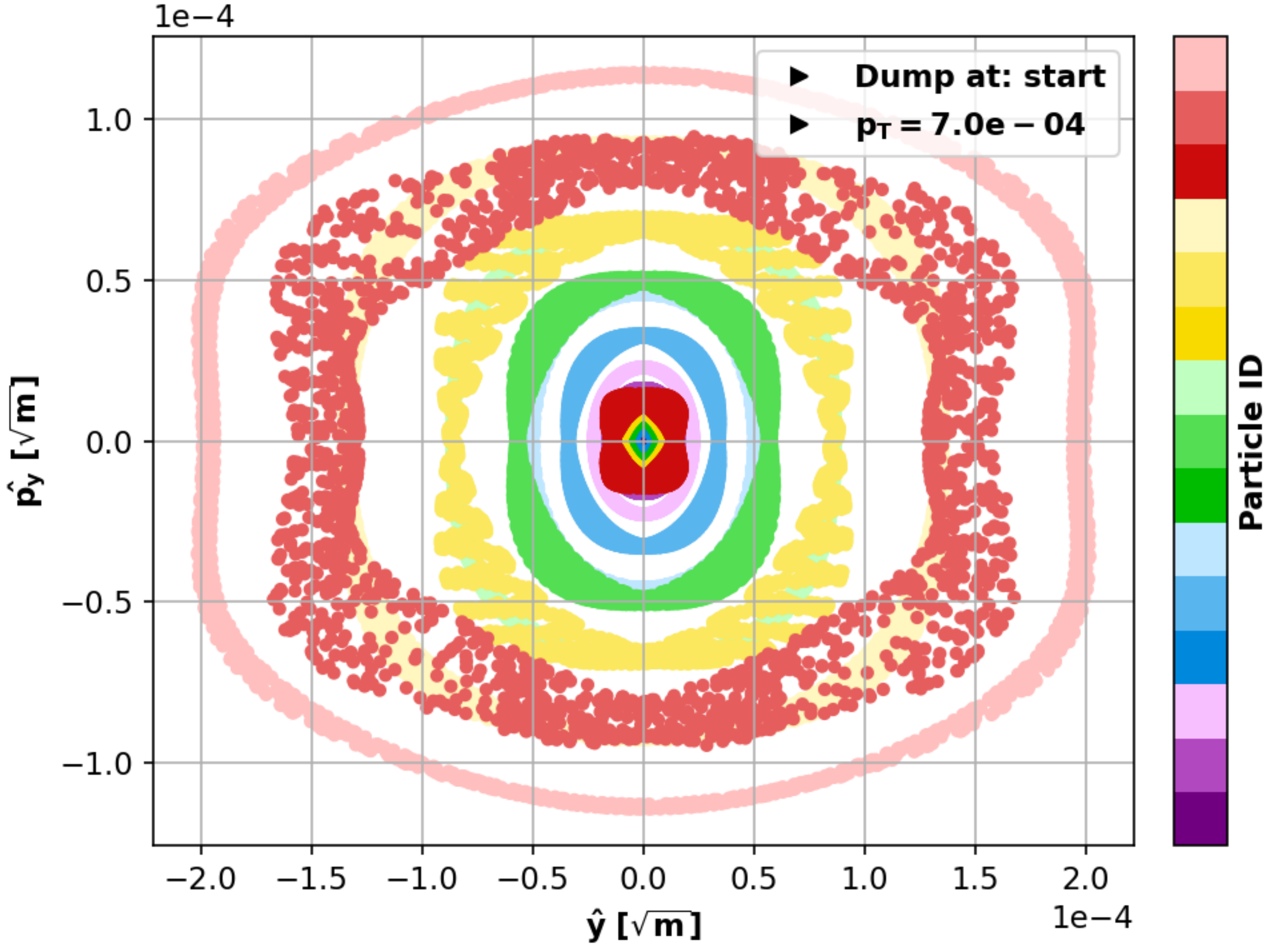
10TeV Muon Collider - Tracking Studies

- Off momentum transverse DA is significantly improved since earlier iterations (v0.4) and is getting closer to required performances.

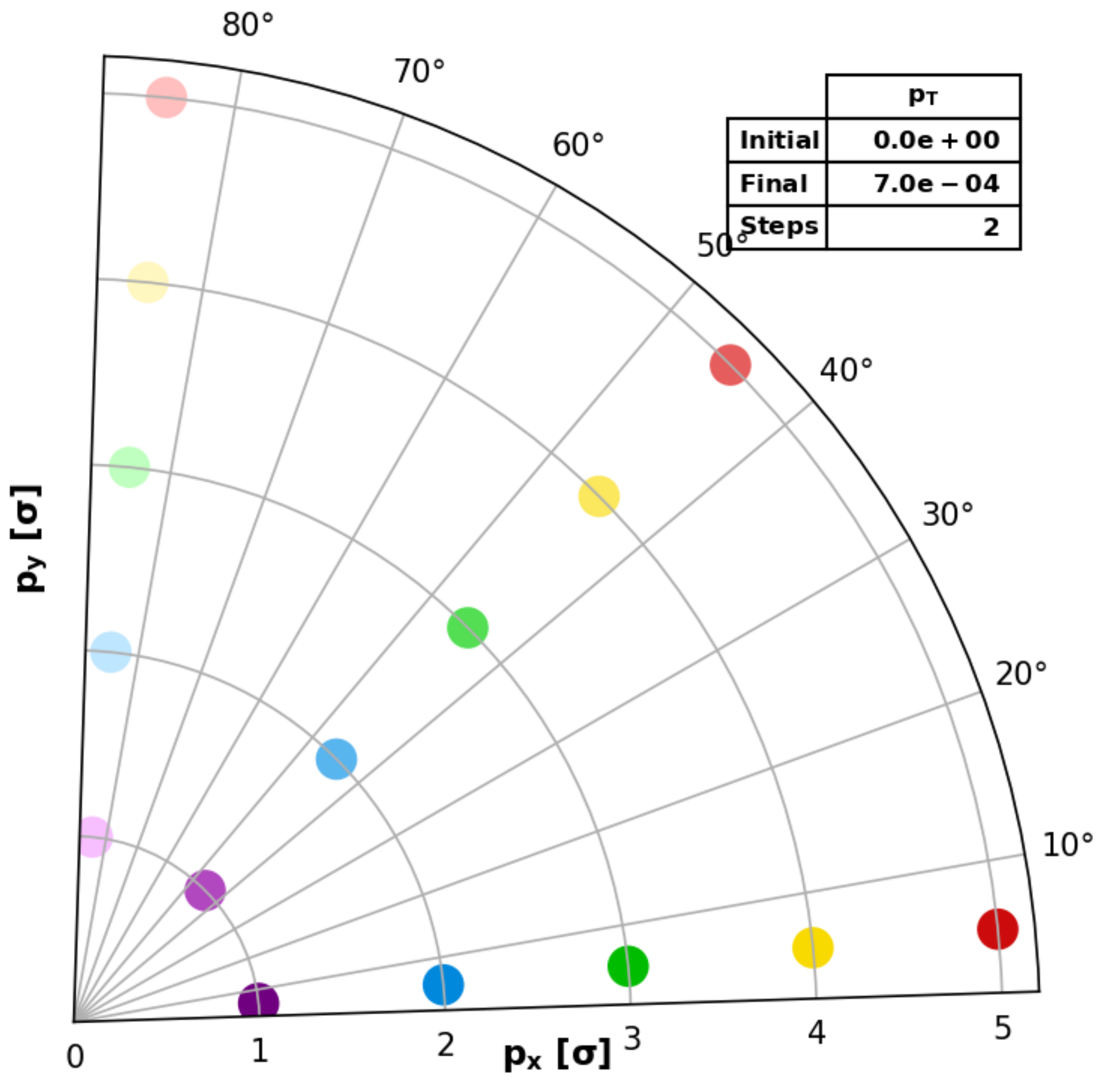
p_T [%]	DA_{min} [σ]
0.07	5
0.08	4
0.09	3
0.1	<1

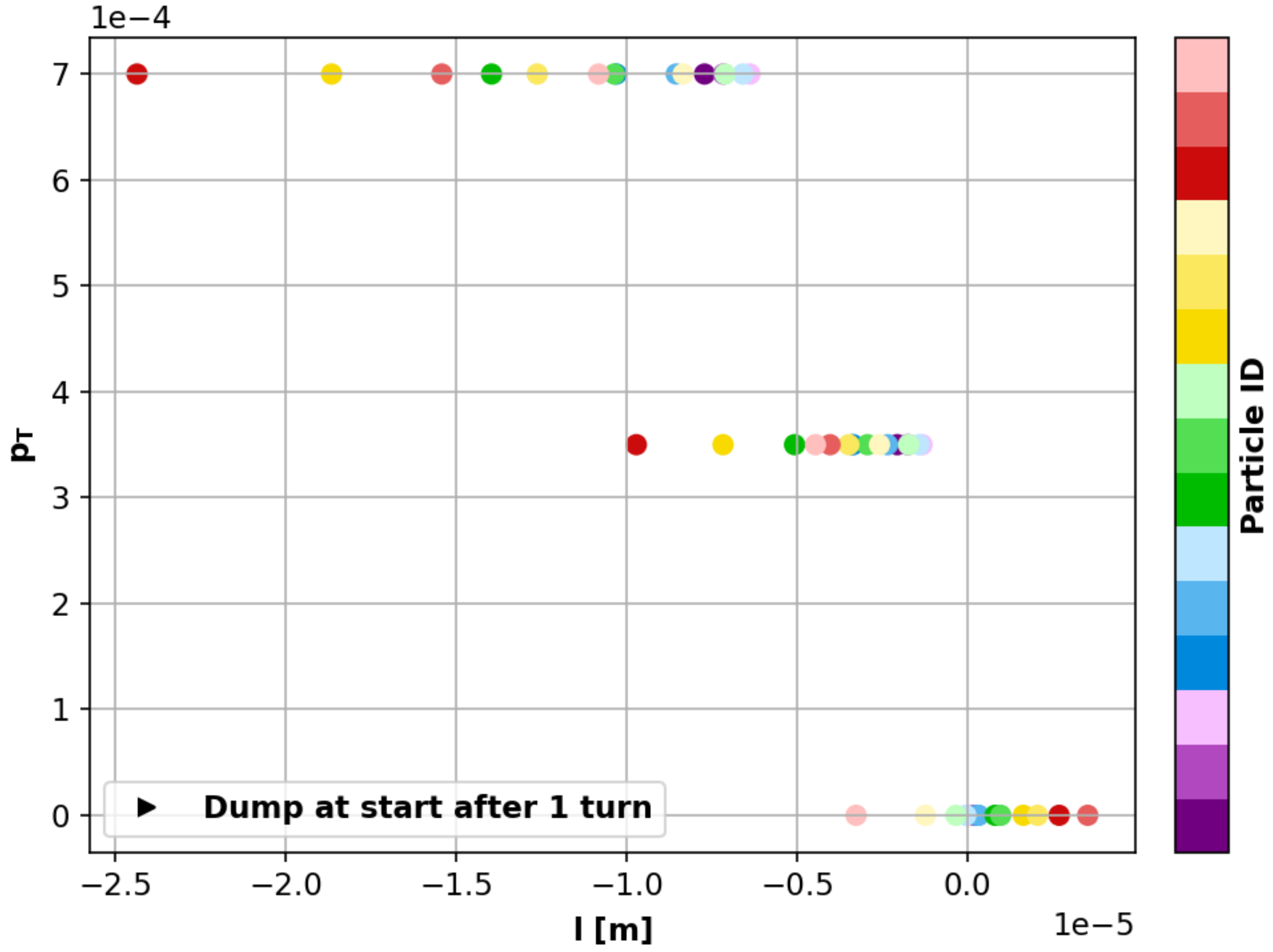






10TeV Muon Collider - Tracking Studies





Q_x=44.46902

ξ_x=0.02452

Q_y=40.96249

ξ_y=-0.57673

α_p=-1.97061e-07

$$\frac{C}{C_0} = 1 + \alpha_p \delta_p + \alpha_p^{(2)} \delta_p^2 + \alpha_p^{(3)} \delta_p^3 + \cdots$$

C/C_0 = 1 - 1.5e-5 / 8668.5 = 1 - 1.73e-9

$1+\alpha_p\delta_p$ = 1 - 1.97e-7 * 7e-4 = 1 - 1.38e-10

Thank you for your time!

All the **presented studies** are **work in progress** thus, any input is very welcome.

International
MUON Collider
Collaboration

CERN

home.cern

International
MUON Collider
Collaboration

Backup