“Gradient flow scale setting with tree-level improvement”

Christian Schneider

Funded by
the European Union

AQTIVATE

The *AQTIVATE* project receives funding from the European Union’s HORIZON MSCA Doctoral Networks programme, under Grant Agreement No. 101072344.

Outline

- Scale setting
- Gradient flow and tree-level improvement
- Gradient flow measurements
- Analysis
- Conclusion and Outlook

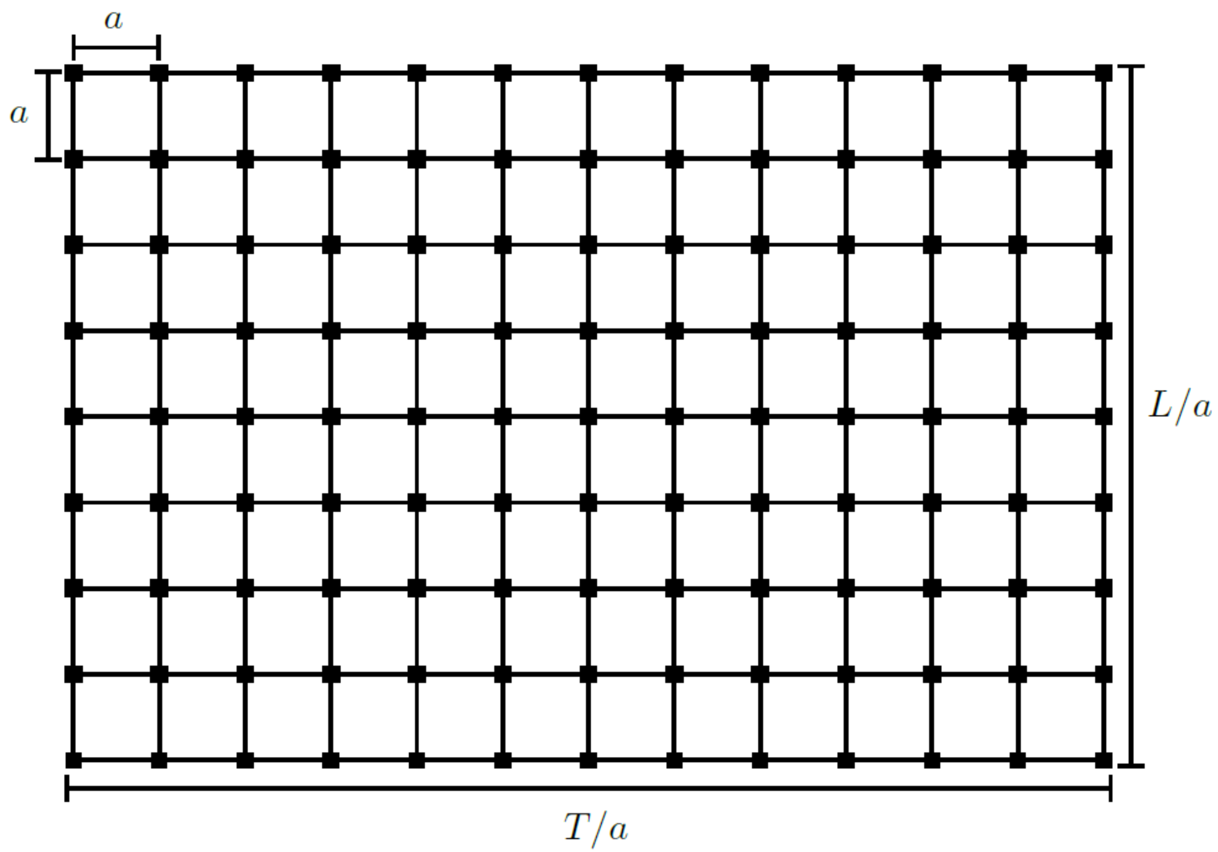
Scale setting

- Matching lattice observables to their physical value.

- Example: $$a = \frac{am_p}{m_{p,phys}}$$

- High precision is required!

- Gradient flow scales :
  - Efficient to calculate and usually very precise.
  - Subject to cutoff effects!



Gradient flow and tree-level improvement

$$\partial_t B_\mu(x,t) = D_\nu G_{\mu\nu}(x,t), \qquad B_\mu(x,t)\Big|_{t=0} = A_\mu(x)$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \qquad D_\mu = \partial_\mu + [B_\mu, \cdot]$$

- Use two different versions: Wilson & Zeuthen flow.
- Zeuthen flow already $O(a^2)$ improved!

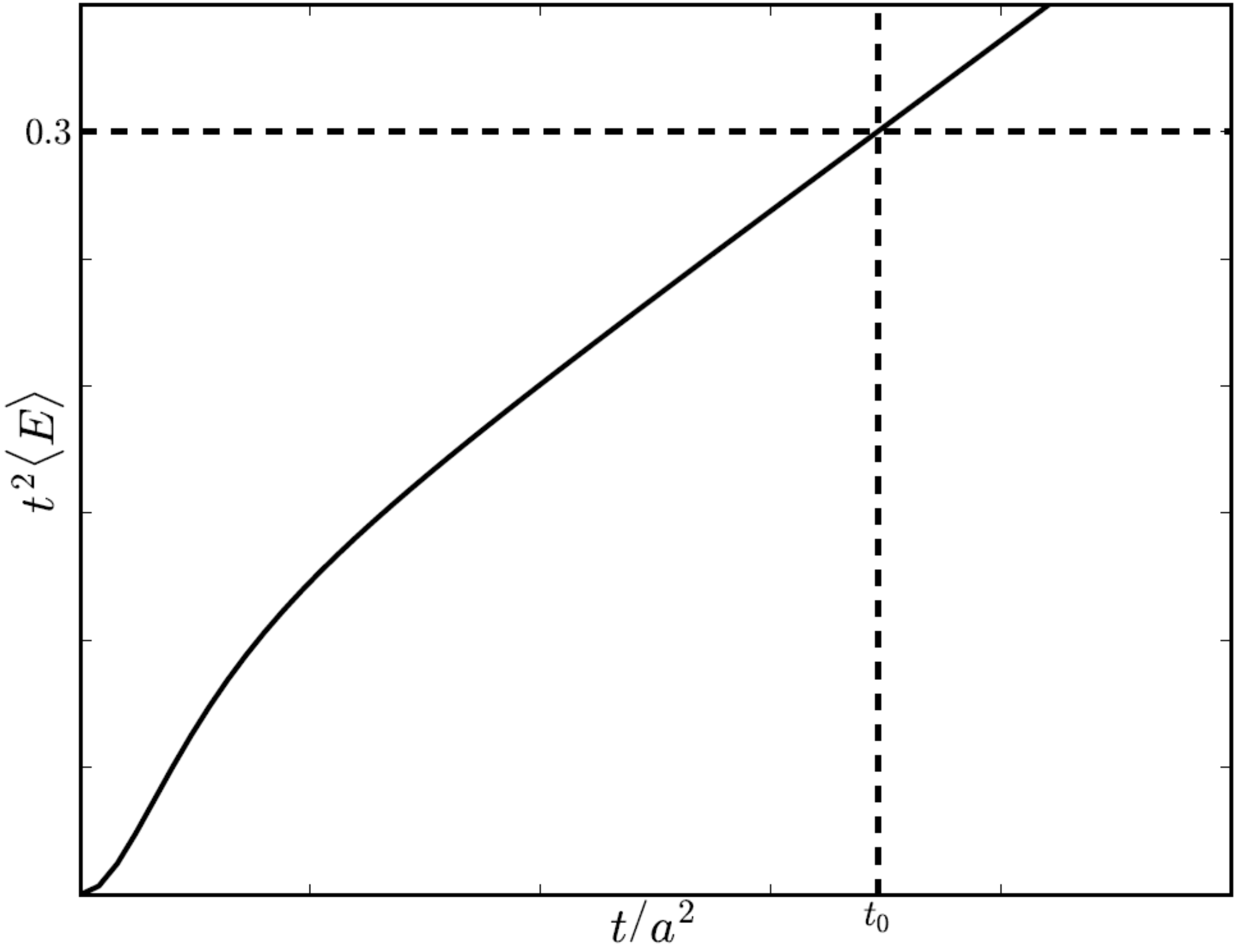
Gradient flow and tree-level improvement

$$\mathcal{E}(t) = t^2 \langle E(x,t) \rangle = 0.3$$

[Lüscher 2010]

$$t \frac{d}{dt} \mathcal{E}(t) = W(t/a^2) = 0.3$$

[Borsanyi, Fodor 2012]



Gradient flow and tree-level improvement

- Perturbative expansion:

$$t^2\langle E(x,t)\rangle = Ng_0^2(C(a^2/t) + O(g_0^2))$$ [Fodor, Nogradi 2014]

- Replace $t^2\langle E(x,t)\rangle \ \rightarrow \ t^2\langle E(x,t)\rangle / C(a^2/t)$

- Determine C(t, L, T) perturbatively for different [action][flow][operator] combinations.

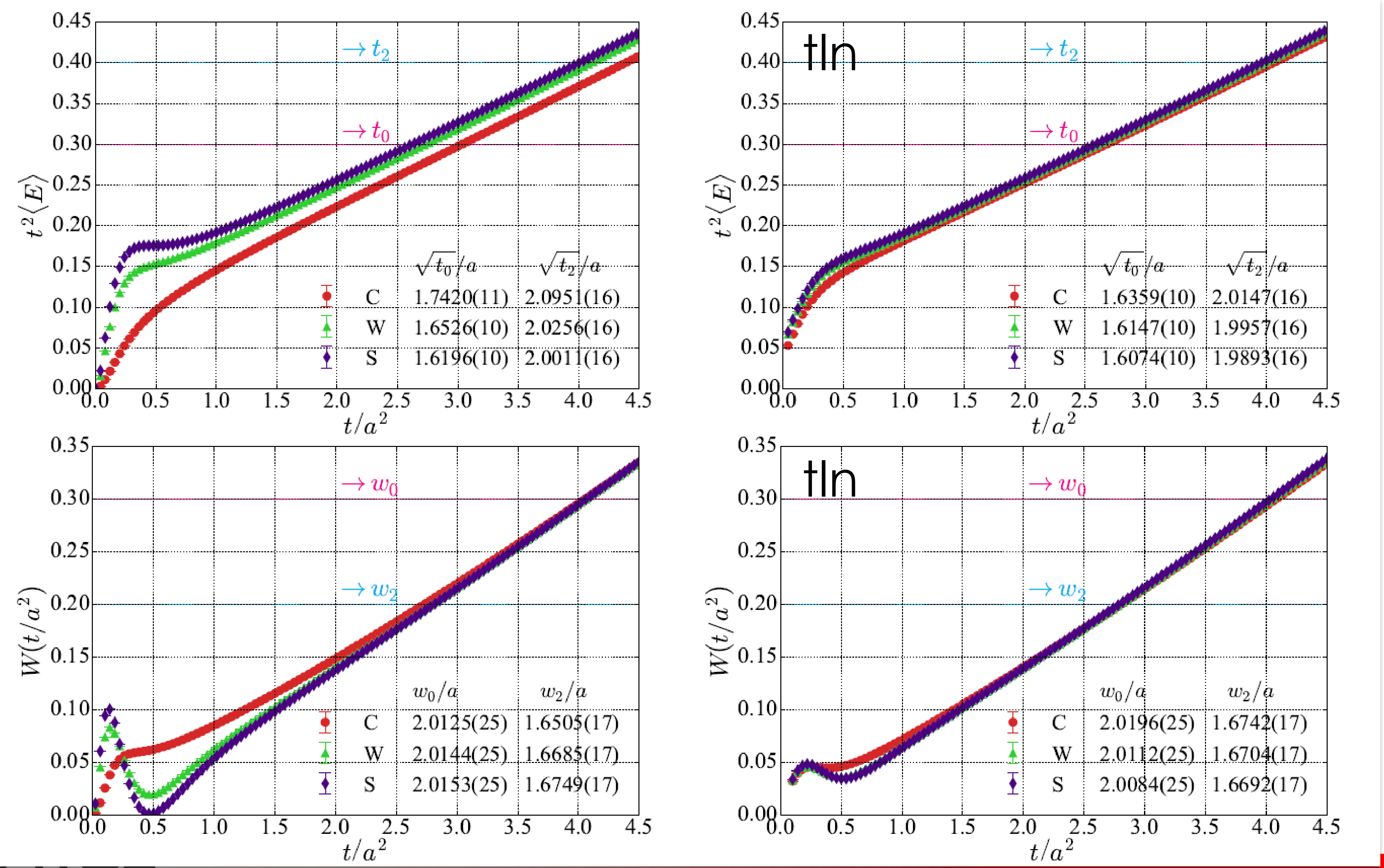
- Notable improvement for determinations of the step-scaling β function for SU(3) with $N_f = 4, 6, 8, 10, 12.$ [Hasenfratz, Rebbi, Witzel 2019 & 2020]

Gradient flow measurements

- RBC-UKQCD's 2+1 flavor Shamir domain-wall fermion and Iwasaki gauge field ensembles.

ensemble	β_b	L/a	T/a	am_ℓ	am_s^{sea}	am_{res}	N_{cfg}	trajectories	step
C1	2.13	24	64	0.005	0.040	0.003154(15)	1636	495-8670	5
C2	2.13	24	64	0.010	0.040	0.003154(15)	1419	1455-8545	5
M1	2.25	32	64	0.004	0.030	0.0006697(34)	628	290-3425	5
M2	2.25	32	64	0.006	0.030	0.0006697(34)	889	272-3824	4
M3	2.25	32	64	0.008	0.030	0.0006697(34)	544	250-2965	5
F1	2.31	48	96	0.002144	0.02144	0.0009679(21)	98	4000-7880	40
X1	2.37	32	64	0.0047	0.0186	0.0006296(58)	119	1000-6900	50

Anc



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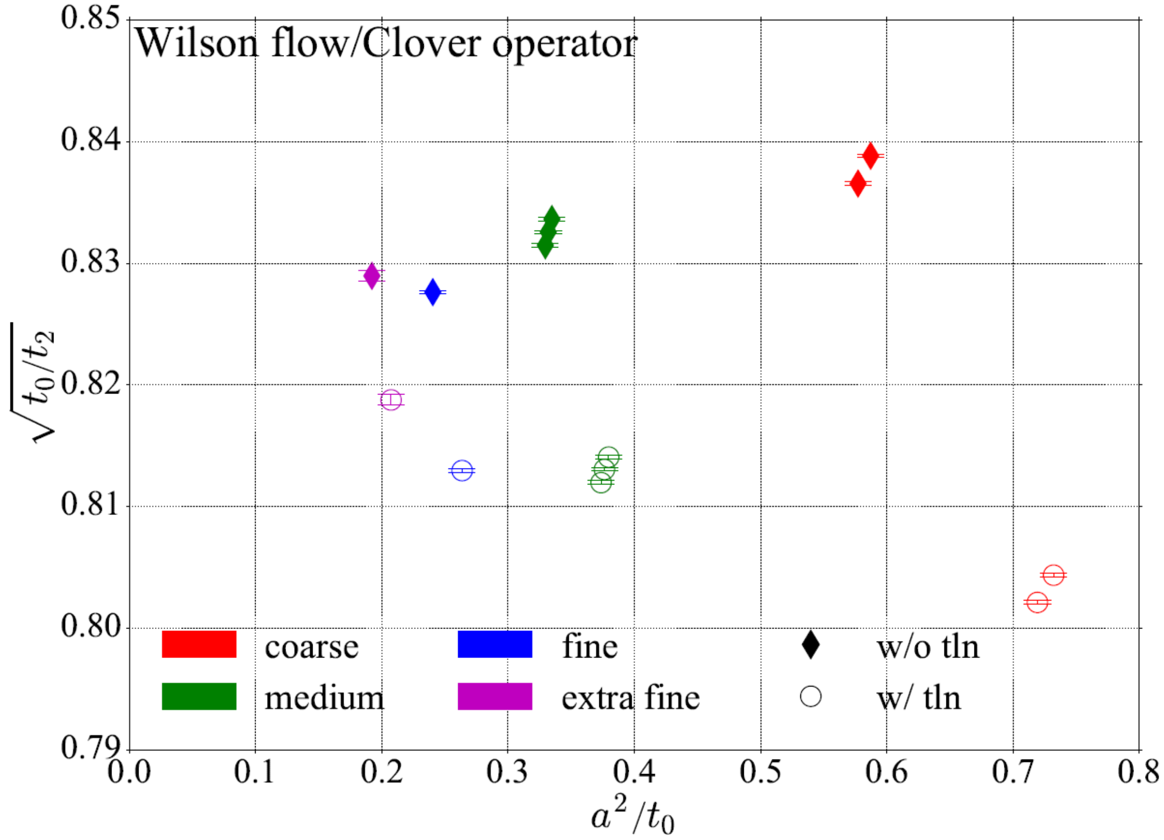
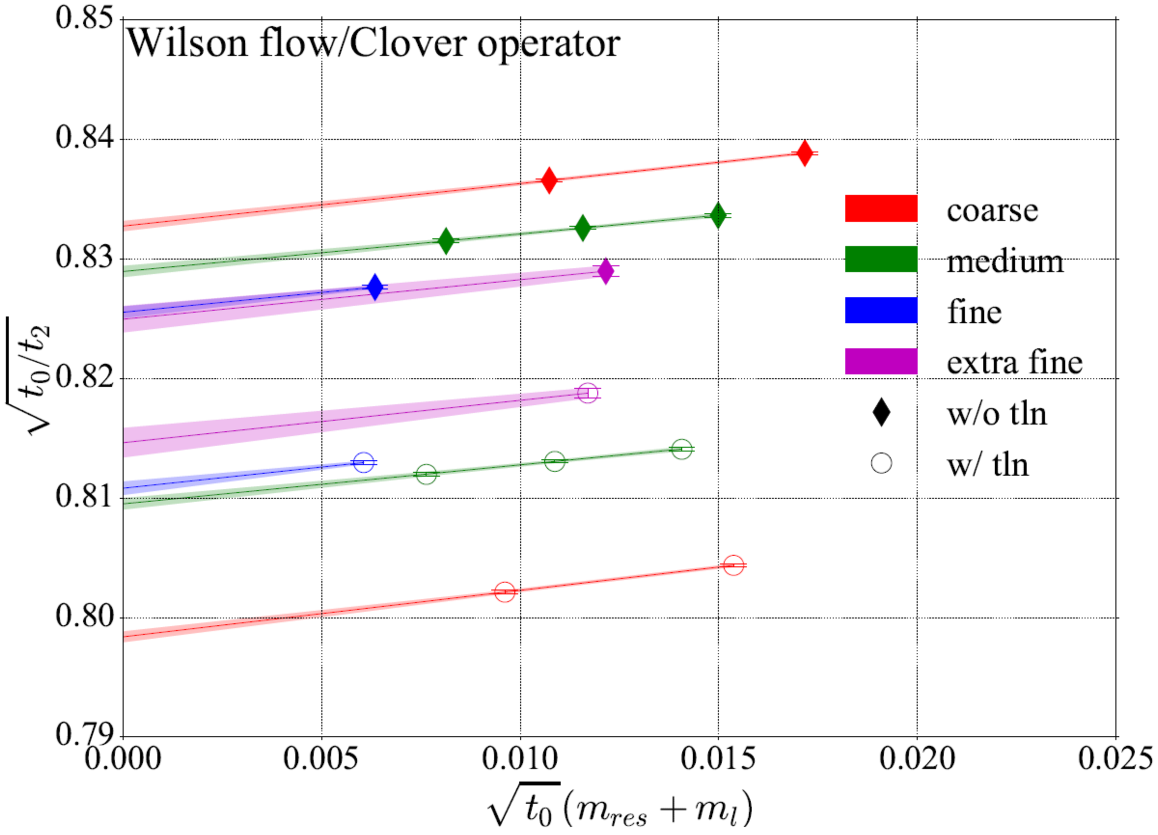
Analysis

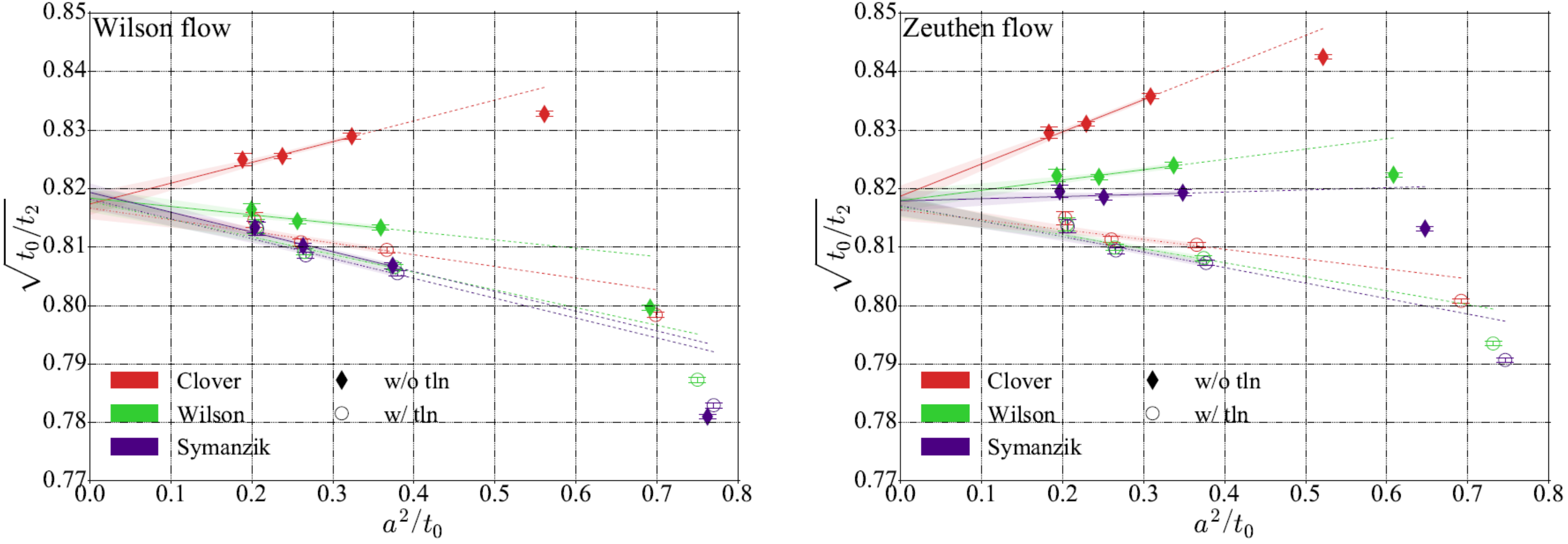
Comparison of ratio w/ and w/o tln



Chiral extrapolation



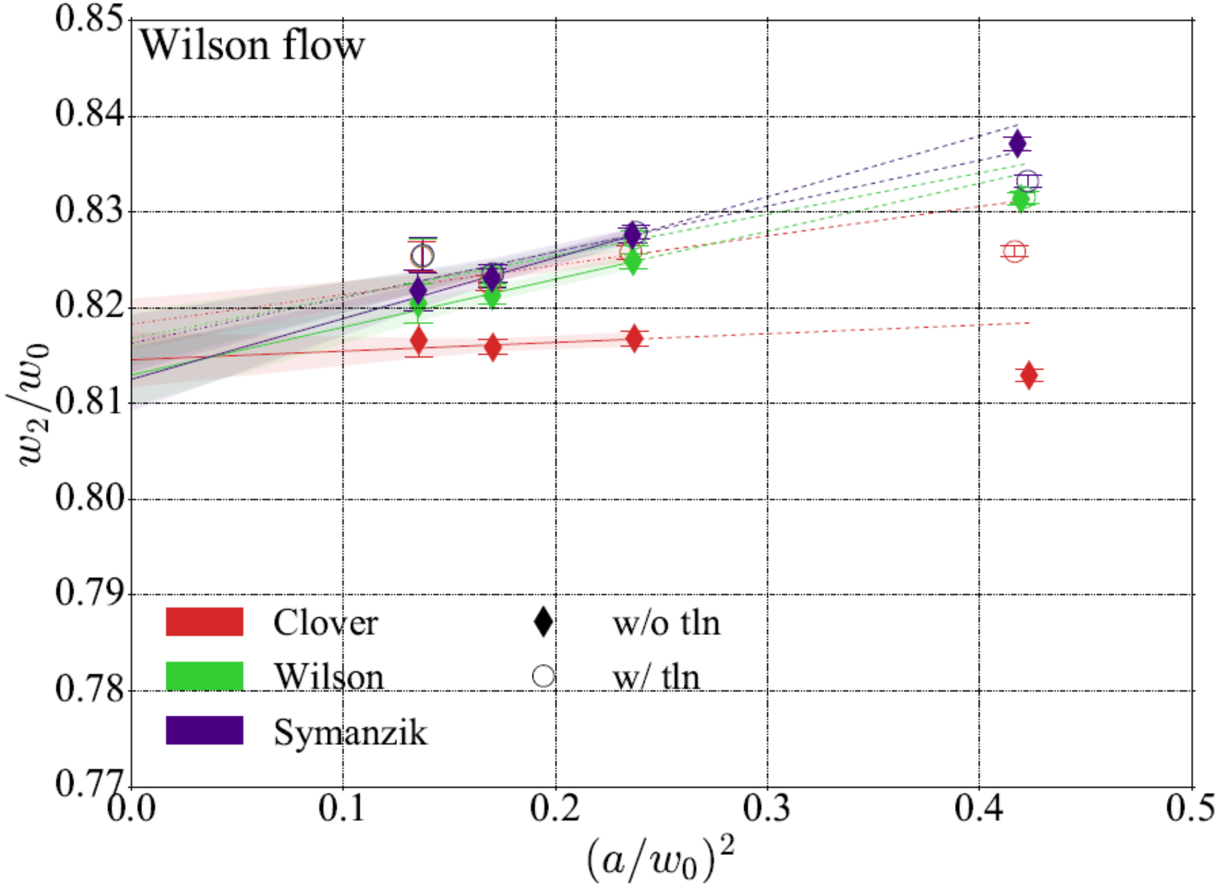
Analysis


Wilson flow
Zeuthen flow
$\sqrt{t_0/t_2}$
a^2/t_0
0.85
0.84
0.83
0.82
0.81
0.80
0.79
0.78
0.77
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Clover
Wilson
Symanzik
w/o tln
w/ tln

Conclusion and Outlook

- tln reduces the spread between operators but cannot remove cutoff effects from the gauge action.
- Successfully computed tln coefficients, which can be used in future studies!
- Parts to improve on:
  - Crude chiral extrapolation.
  - Insufficient data at different bare gauge couplings.
- Next steps: Repeating this exercise using RBC-UKQCD's three physical point ensembles (C0, M0, F0) with Möbius DWF kernel.

Thank you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w_2/w_0
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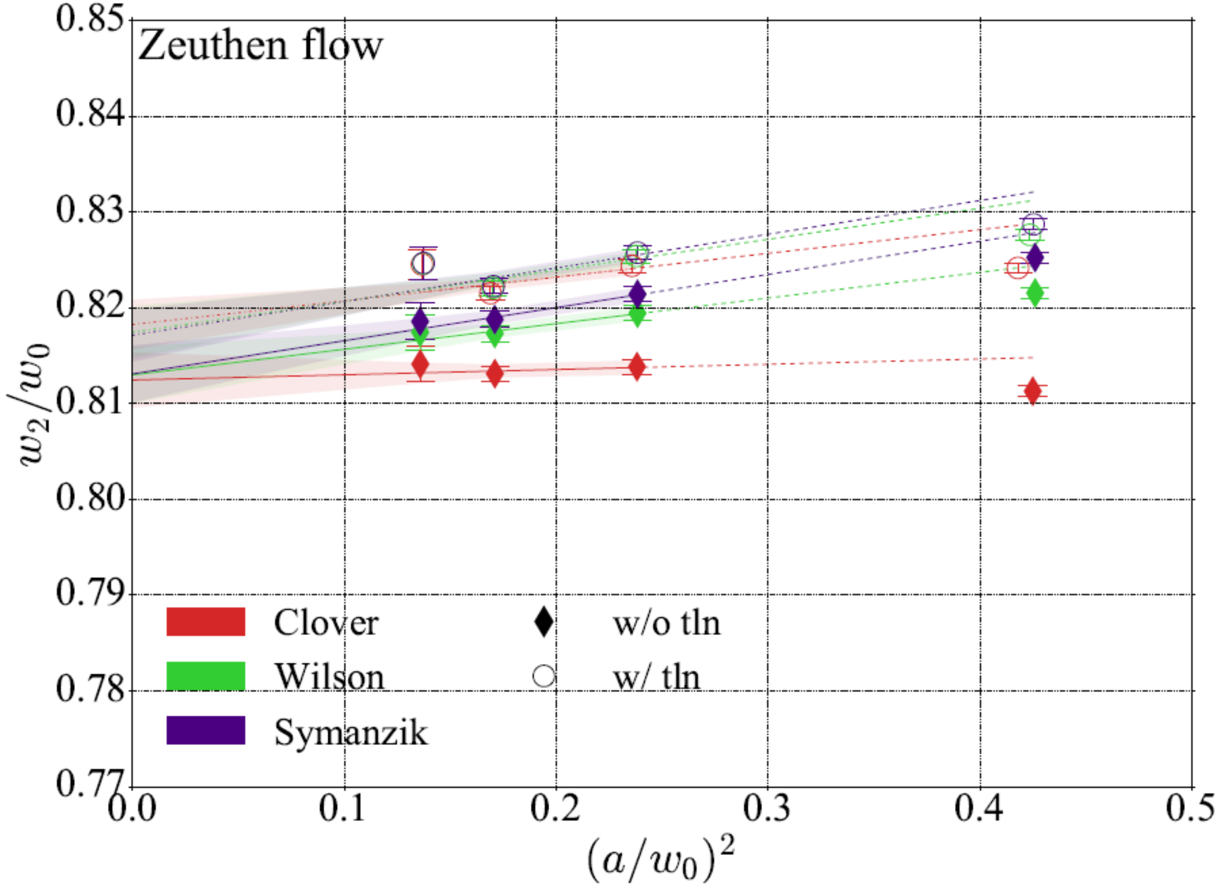
0.4
0.5
$(a/w_0)^2$
Clover
Wilson
Symanzik
w/o tln
w/ tln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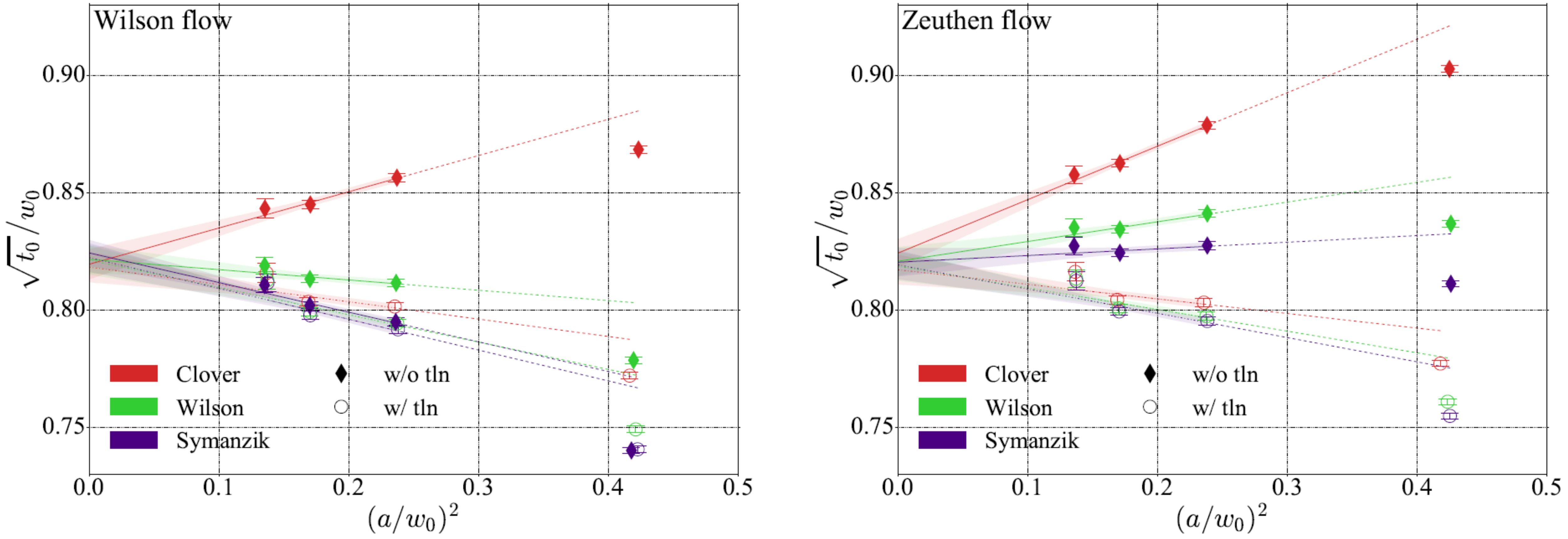
Wilson flow
Zeuthen flow
0.90
0.85
0.80
0.75
$\sqrt{t_0}/w_0$
0.0
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0.3
0.4
0.5
$(a/w_0)^2$
Clover
Wilson
Symanzik
w/o tln
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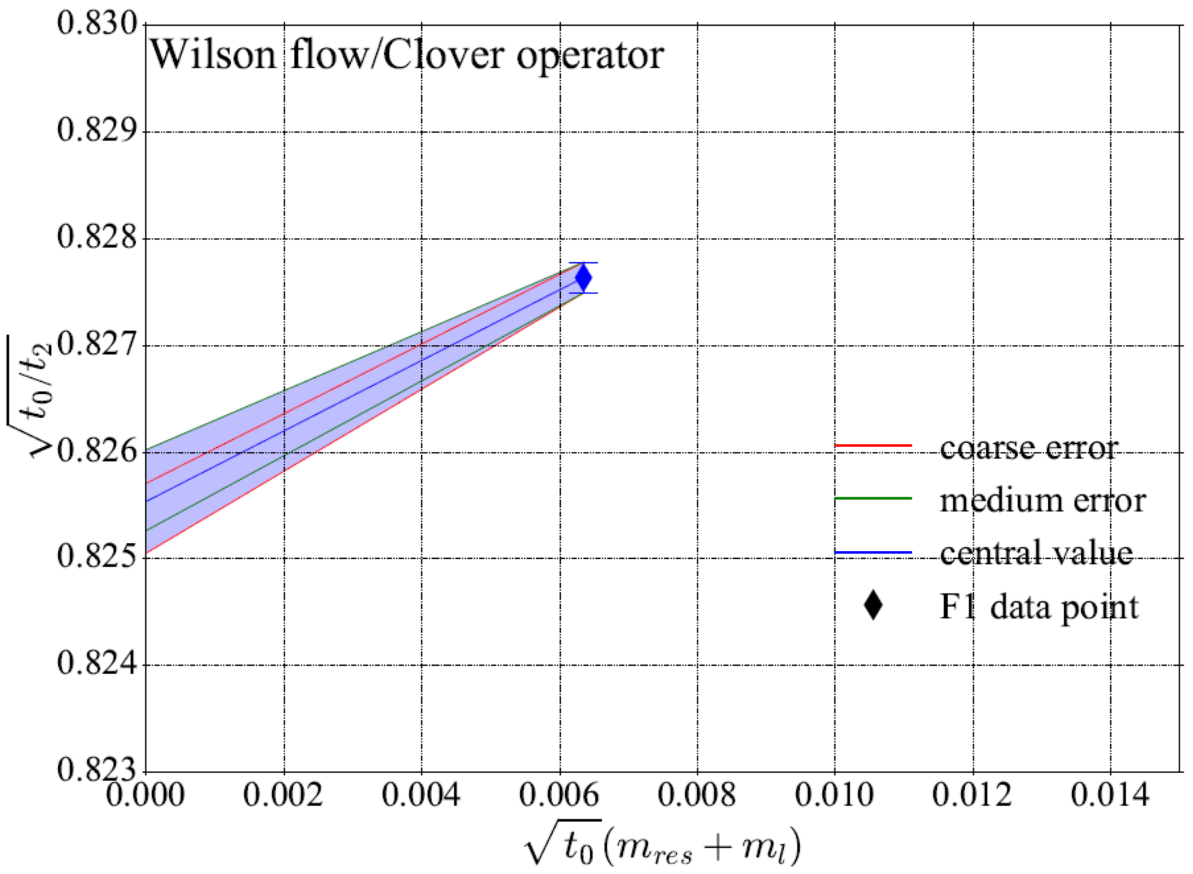
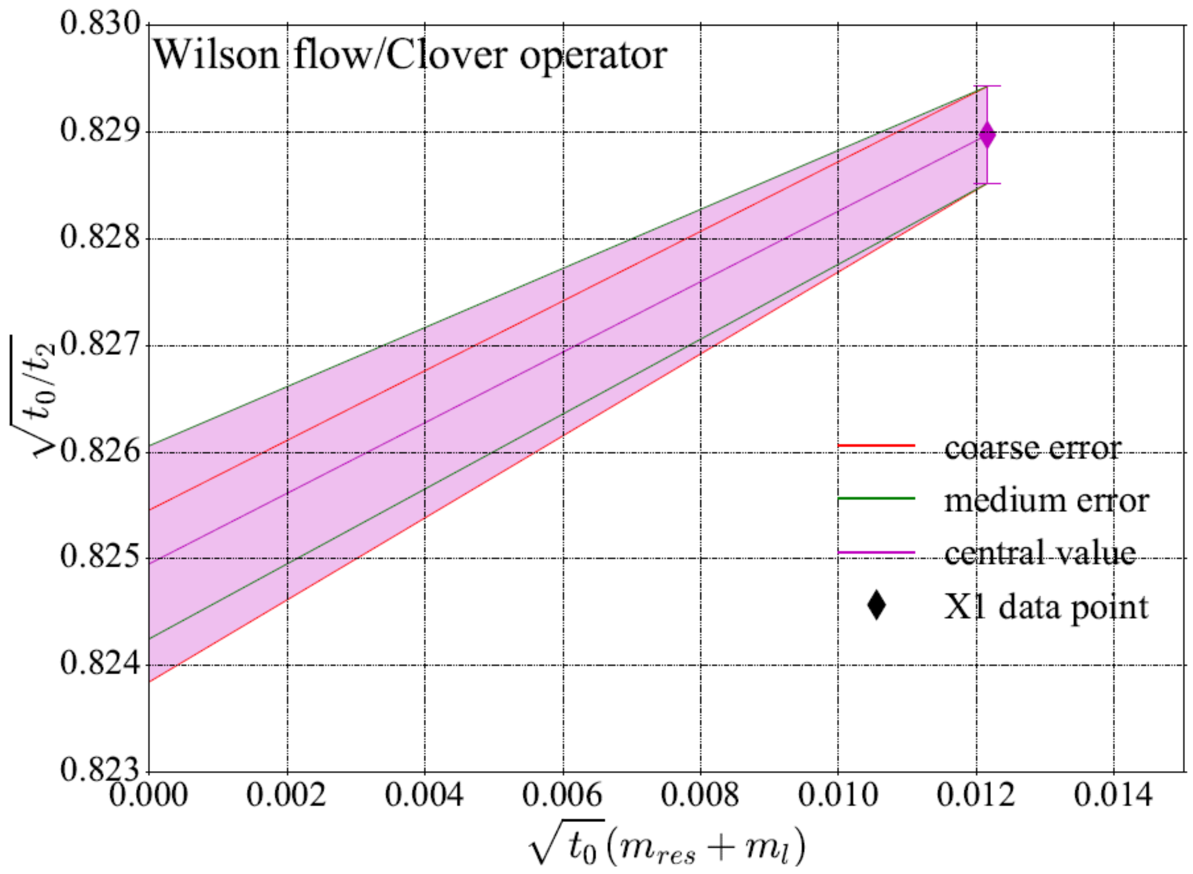

Wilson flow/Clover operator
$\sqrt{t_0/t_2}$
$\sqrt{t_0}(m_{res}+m_l)$
coarse error
medium error
central value
F1 data point
0.830
0.829
0.828
0.827
0.826
0.825
0.824
0.823
0.000
0.002
0.004
0.006
0.008
0.010
0.012
0.014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0.004
0.006
0.008
0.010
0.012
0.014

lattice volume	N_{Sites}	N_{flows}	runtime [h]
$16^3 \times 32$	2,805	200	0.12
$16^3 \times 64$	5,445	200	0.23
$24^3 \times 64$	15,015	450	1.65
$32^3 \times 64$	31,977	800	6.24
$48^3 \times 96$	143,325	1,800	62.9
$64^3 \times 128$	425,425	1,600	160.9

	Wilson flow			Zeuthen flow		
lattice volume	runtime [h]	# nodes	node hrs	runtime [h]	# nodes	node hrs
$24^3 \times 64$	0.33	1	0.33	1.12	1	1.12
$32^3 \times 64$	1.85	1	1.85	6.25	1	6.25
$48^3 \times 96$	5.70	4	22.8	8.19	8	65.52
$64^3 \times 128$	8.36	8	66.88	-	-	-