Weak Gravity & Compact Extra Dimensions

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Setup

- **String Theory** is believed to be a **consistent quantum theory of gravity** and other interactions.
- Even if String Theory isn't the theory describing our nature, it can still teach us about the **mathematical structure of consistent theories** of QG.
- One such property is the **weakness of gravity**. (**WGC**)
- Another is the need for **higher-dimensions**, which seems to be a necessity to fit everything in the same framework. String Theory requires 10 (or 11) dimensions.
- Extra dimensions are not an issue as long as the **extra dimensions are "small"**. This is called **Scale-Separation**.

Problem

- Growing suspicions that a **weak gravitational interaction** requires **large extra dimensions.** (Heuristic physical arguments)
- Problem since the extra dimensions can't be "hidden" while maintaining weak gravity.
- **Original Goal**: Show this to be the case with an explicit example. i.e., **Show explicitly** an instance of:

Weak Gravity ⟶ No Scale-Separation

Quantitative Formulation

- **Weakness of gravity** can be **measured by** the value of a function on the particle spectrum of the theory (KK towers)

$$\mathcal{W} \simeq \min_{\text{KK-Spect.}} \left\{ \frac{\text{gravitational coupling}}{\text{gauge coupling}} \times \frac{\text{mass}}{\text{charge}} \right\} \leq 1$$

- **Scale Separation** can be achieved by taking: $\mathcal{M}_{(D)} = \mathcal{M}_{(4)} \times \mathcal{M}_{(D-4)}$

$$\frac{\text{size}(\mathcal{M}_{(D-4)})}{\text{size}(\mathcal{M}_{(4)})} \ll 1$$

- Thus, **need to show:** $\mathcal{W} \leq 1 \implies \frac{\text{size}(\mathcal{M}_{(D-4)})}{\text{size}(\mathcal{M}_{(4)})} \gtrsim 1$

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Field theoretic approach: First signs of trouble.

- **Approach**: Multiple models studied using the usual low-energy **effective field theory (EFT) approach**.
- **Results**: In every case, **negative**. Gravity was maintained as the weakest force while achieving perfect scale-separation. i.e.

$$\mathcal{W} \leq 1 \quad \& \quad \frac{\mathrm{size}(\mathcal{M}_{(D-4)})}{\mathrm{size}(\mathcal{M}_{(4)})} \ll 1$$

- How general are these results? **Difficult to generalize** the field theoretic approach. **Need to reformulate the problem**.

Reformulation

- **New problem: Show** (as general as possible):

Scale-Separation  Weak Gravity

- New approach: **geometrize** what it means for **gravity** to be **weak**.

Geometrization of Weak Gravity

- 1) **Showed** that: $\min_{\text{KK-Spect.}} \Longleftrightarrow \lim_{|\psi\rangle \to (x^\mu,\, p^\mu)}$

- 2) **Constructed** the map between coupling constants, charges and masses with the various components of particle momenta.

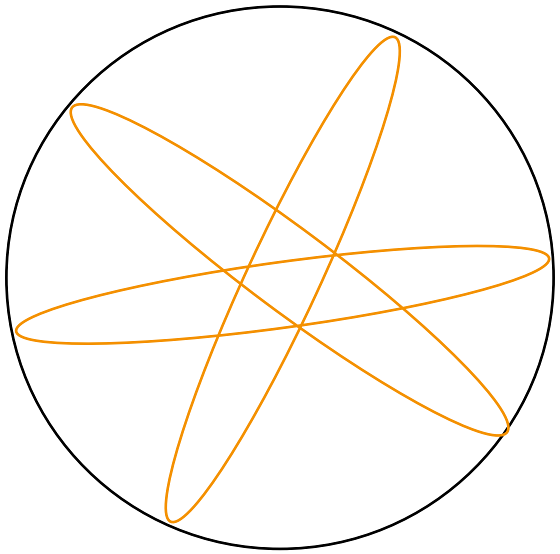
$$(\text{couplings, charges, masses}) = f(p^\mu, \mathcal{M}_{(D-4)})$$

- 3) **Showed** that $\mathcal{W}$ is given by:

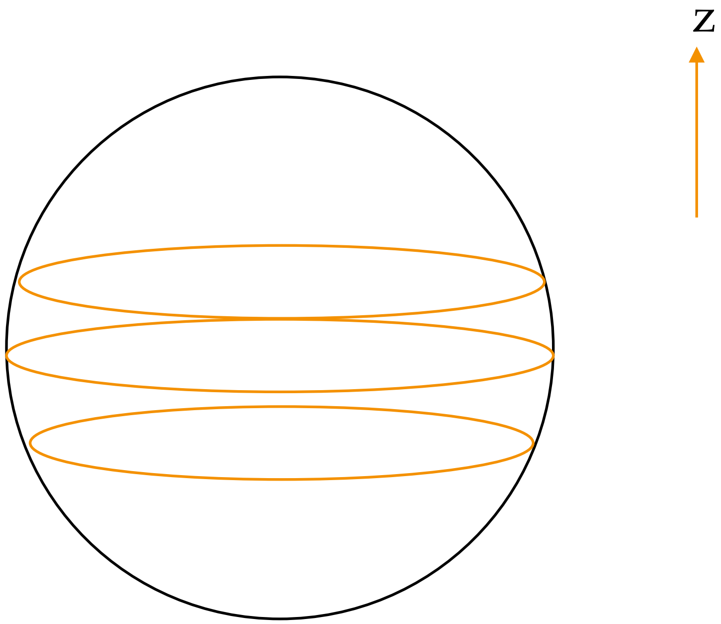
$$\mathcal{W}^{-1} = \text{constant} \times \cos\left(\min_{\mathcal{M}_{(D-4)}} \{\theta = (\text{geo,iso})\}\right)$$

Example: $\mathcal{M}_{(D-4)} = S^2$

Geodesic Curves on S^2



Isometric Curves on S^2



Common: Equator Circle.

Confromality of Weak Gravity

- As $\mathcal{W}$ **depends only on angles** between characteristic curves on the internal space, it is **conformally invariant**.
- Conformal invariance means that changing the size of the internal space does not change the weakness of gravity, i.e.

$$\partial\mathcal{W}/\partial\left[\text{size}\left(\mathcal{M}_{(D-4)}\right)\right] = 0$$

Conclusion: Gravity can be made weak even with small internal spaces, and the weakness of gravity is a geometric statement about the structure of the internal space which can be studied in simple ways using purely geometrical methods.

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Thank you

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