Rethinking running coupling in JIMWLK/BK

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Introduction

- Small-x evolution: BFKL $\to$ BK $\to$ JIMWLK
- JIMWLK allows to evolve arbitrary combination of many Wilson lines without large N_c approximation
- NLO JIMWLK equation was derived $\approx$ 10 years ago

  Kovner, Lublinsky & Mulian (2013), Balitsky & Chirilli (2007), Grabovsky (2013); ML & Mulian (2016)
- Large transverse logs in NLO JIMWLK/BK: improvements are necessary

  Altinoluk, Armesto, Beuf Hatta, Iancu, Lublinsky, Müller, Stasto, Triantafyllopoulos, Xiao, ...
- Simulation of NLO JIMWLK? LO JIMWLK: Langevin formulation
- The principal part: large logs multiplied by QCD β-function

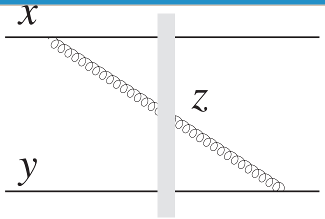
  I will not talk today about large logs due to k^- ordering
- Resummation of these logs led to rcBK with generalization to rcJIMWLK

  Balitsky, Kovchegov & Weigert, ...
- There is no rcJIMWLK implementation that would explicitly reproduce any specific rc prescription consistent with NLO JIMWLK

Positivity semi-definiteness of JIMWLK Hamiltonian

- Positive semi-definitness plays important role in Langevin form of JIMWLK
- For illustration: LO JIMWLK and derivation of its Langevin form

LO JIMWLK Hamiltonian



- JIMWLK Hamiltonian at LO:

$$\frac{d\mathcal{O}}{dY} = -H_{\rm LO}\mathcal{O}, \qquad H_{\rm LO} = \int_z \int_x \int_y \mathcal{K}(X,Y) q^a(x,z) q^a(y,z)$$

$$X = x - z, q^a(x,z) = [U(x) - U(z)]^{ab} J_R^b(x)$$

- Property of LO JIMWLK kernel

$$\mathcal{K}(X,Y) = \frac{\alpha}{2\pi^2} K_i(X) K_i(Y), \quad H_{LO} = \frac{\alpha_s}{2} \int_z Q_i^a(z) Q_i^a(z)$$

  Single gluon emmision operator $Q_i^a(z) = \frac{1}{\pi} \int_x K_i(X)\, q^a(x,z)$.
  The Weizsäcker–Williams (WW) field $K_i(x) = \frac{x_i}{x^2}$.
  $U(x)(V(x))$ is adjoint (fundamental) Wilson line along the light cone.

- The operators $J_R^b(x)$ form a SU(N) algebra

$$J_R^a(z) V(x) = \delta^{(2)}(x-z) V(x) t^a, \qquad J_R^a(z) V^\dagger(x) = -\delta^{(2)}(x-z) t^a V^\dagger(x)$$

$$J_R^a(z) U(x) = \delta^{(2)}(x-z) U(x) T^a, \qquad J_R^a(z) U^\dagger(x) = -\delta^{(2)}(x-z) T^a U^\dagger(x)$$

Langevin form of JIMWLK I

- The rapidity variable η acts as a fictitious time, the evolution operator from η_0 to η_1

$$\mathcal{U}(\eta_0, \eta_1) = \mathcal{P} e^{-\frac{\alpha_s}{2} \int_{\eta_0}^{\eta_1} d\eta \int_z Q_i^a(z) Q_i^a(z)}$$

- Introduce auxiliary noise field

$$\mathcal{U}(\eta_0, \eta_1) = \int D\xi \, \mathcal{P}_\eta \, \exp\{\int_{\eta_0}^{\eta_1} d\eta \int_z \left[-i\sqrt{\alpha_s} Q_i^a(z) \xi_i^a(\eta, z) - \frac{1}{2} \xi^{\,2}(\eta, z) \right] \}$$
$$= \int D\xi \, \mathcal{U}_\xi(\eta_0, \eta_1) \, e^{-\int_{\eta_0}^{\eta_1} d\eta \int_z \frac{1}{2} \xi^{\,2}(\eta, z)}$$

$\mathcal{U}_\xi$ is evolution operator for a fixed configuration of $\xi_i^a(\eta, x)$

- Evolution over small Δ ($\xi_n^{a\,i}(x) \to \hat{\xi}_n^{a\,i}(x) = \sqrt{\Delta} \xi_n^{a\,i}(x)$):

$$\mathcal{U}_\xi \approx 1 - i\sqrt{\alpha_s \Delta} \int_z \xi_n^{a,i}(z) Q_i^a(z) - \frac{1}{2} \alpha_s \Delta \int_z \int_{z'} \xi_n^{a,i}(z) \xi_n^{b,j}(z') Q_i^a(z) Q_j^b(z') \, .$$

Langevin form of JIMWLK II

- Then, one step of the evolution of V

$$\mathcal{U}_\xi V(x) = V(x) - i\frac{\sqrt{\alpha_s \Delta}}{\pi}\int_z K_i(x-z)\left[\xi_n^i(z)V(x) - V(x)V^\dagger(z)\xi_n^i(z)V(z)\right]$$
$$-\frac{\alpha_s\Delta}{2\pi^2}\int_z\int_{z'} K_i(x-z)K_j(x-z')\xi_n^{a,i}(z)\xi_n^{b,j}(z')$$
$$\times\Big[t^a t^b V(x) - t^a V(x)V(z')t^b V(z') - t^b V(x)V(z)t^a V(z)$$
$$+ V(x)V^\dagger(z')t^b V(z')V^\dagger(z)t^a V(z)\Big]$$

- This exponentiates into Langevin form of LO JIMWLK

$$V(x) \to \mathcal{U}_\xi V(x) = \exp\left(-i\frac{\sqrt{\alpha_s\Delta}}{\pi}\int_z \mathbf{K}(x-z)\cdot\xi_n(z)\right)$$
$$\times V(x)\exp\left(i\frac{\sqrt{\alpha_s\Delta}}{\pi}\int_z \mathbf{K}(x-z)\cdot\left(V^\dagger(z)\xi_n(z)V(z)\right)\right).$$

Key property for Langevin formulation

- Naively, factorization of the JIMWLK Hamiltonian is required $\mathcal{K}(X,Y) \to \mathcal{K}(X)\mathcal{K}(Y)$

$$H = \int_{x,y,z} \mathcal{K}(X,Y)\, q^a(x,z)\, q^a(y,z), \quad q^a(x,z) = [U(x) - U(z)]^{ab}\, J_R^b(x).$$

- Actually, positive semi-definiteness of the kernel is sufficient
- For a positive semi-definite

$$\frac{1}{2}\int_V \Psi_i(X,V)\Psi_i(Y,V) = \mathcal{K}(X,Y)\,.$$

- This would enable introduction of the noise field

$$\mathcal{U}(\eta, \eta+\Delta) = \int D\zeta e^{\Delta\left(i\int_{x,y,z} \Psi_i(X,Y)\, q^a(x,z)\, \zeta_i^a(y,z) - \frac{1}{2}\int_{x,z} \zeta_i^a(x,z)\zeta_i^a(x,z)\right)}.$$

Following the same steps as at LO $\rightsquigarrow$ Langevin formulation

Non semi-definite kernels

- No Langevin formultion
- Does it mean JIMWLK evolution is not well defined?
- Alternative form of JIMWLK (α is the phase of the Wilson line):

$$\partial_\eta \mathcal{O}[\alpha] = \frac{1}{2}\int d^2x d^2y \frac{\partial}{\partial \alpha^a(x^-, x)} \left(\eta^{ab}(x,y) \frac{\partial}{\partial \alpha^b(y^-, y)} \mathcal{O}[\alpha] \right)$$

with

$$\eta^{ab}(x,y) = \int d^2z \mathcal{K}(X,Y) \left[(U(x) - U(z))(U^\dagger(y) - U^\dagger(z)) \right]^{ab}$$

- For non positive semi-definite kernels, there exists such distribution $W(U)$ that the evolution is "anti-diffusive"

Constraints from positive semi-definiteness

Positive semi-definite kernel

$$\int d^2X\, d^2Y\, \phi(X)\, \mathcal{K}(X,Y)\, \phi(Y) \geq 0 \quad \forall\, \phi(X)$$

- Consider a trial function

$$\phi(X) = A_1\delta(X - X_1) + A_2\delta(X - X_2)\,.$$

- Positive semi-definiteness means

$$A_1^2\mathcal{K}(X_1, X_1) + A_2^2\mathcal{K}(X_2, X_2) + 2A_1A_2\mathcal{K}(X_1, X_2) \geq 0, \quad \forall A_1 \text{ and } A_2$$

and thus

$$\mathcal{K}(X_1, X_1)\mathcal{K}(X_2, X_2) - \mathcal{K}^2(X_1, X_2) \geq 0,$$
$$\mathcal{K}(X_1, X_1) \geq 0$$

- Potentialy more constraints, but those above are already quite restrictive

rcJIMWLK

- There are several rc prescription which reproduce β_0-dependent terms of the NLO JIMWLK
  - Balitsky
  - Kovchegov-Weigert
- Do they have positive semi-definite kernels?

Balitsky prescription

- For JIMWLK (original prescription for BK)

$$\mathcal{K}^B(x,y,z) = \frac{\alpha_s((X-Y)^2)}{2\pi^2}\frac{X\cdot Y}{X^2Y^2} + \frac{\alpha_s(X^2)}{4\pi^2}\frac{1}{X^2}\left(1-\frac{\alpha_s((X-Y)^2)}{\alpha(Y)}\right) + \frac{\alpha_s(Y^2)}{4\pi^2}\frac{1}{Y^2}\left(1-\frac{\alpha_s((X-Y)^2)}{\alpha(X^2)}\right).$$

- Is it positive semi-definite? E.g. consider $Y = cX$ and $c \to 1$

$$\mathcal{K}^B(X,X)\mathcal{K}^B(Y,Y) - \left(\mathcal{K}^B(X,Y)\right)^2 \approx -\frac{(1-c)^2\alpha_s(X^2)^2}{4\pi^4X^4}\left[1-\frac{\beta_0\alpha_s(X^2)}{4\pi}\right]^2 \leq 0$$

- The same conclusion applied to the un-resummed kernel at fixed NLO order

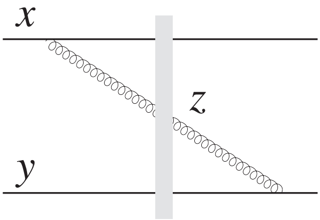
For details see, T. Altinoluk, G. Beuf, M. Lublinsky and V. S., arXiv:2310.10738

Other prescriptions

- Kovchegov-Weigert: More about the difference with Balitsky's rc prescription in the second part of the talk
- KW: The same criterion of positive semi-definiteness is violated
- In arXiv:2310.10738, we tried many different prescriptions. All of them violated positive semi-definiteness
- Open question: rcBK, does not have apparent problems; instability in $(n > 2)$-pointWilson line correlators?
- Do we set rc scale correctly?

LO JIMWLK Hamiltonian

- LO JIMWLK Hamiltonian $\partial \mathcal{O}/\partial Y = -\mathcal{H}^{\text{JIMWLK}}\mathcal{O}$



$$\mathcal{H}^{\text{JIMWLK}}_{\text{LO}} = \int_{x,y,z} K_{\text{LO}} \left[J^a_L(x) J^a_L(y) + J^a_R(x) J^a_R(y) - 2 J^a_L(x) S^{ab}(z) J^b_R(y) \right]$$

$$K_{\text{LO}}(x,y,z) = \frac{\alpha_s}{2\,\pi^2} \frac{(x-z)_i (y-z)_i}{(x-z)^2 (y-z)^2} \equiv \frac{\alpha_s}{2\,\pi^2} \frac{X \cdot Y}{X^2 Y^2}$$

- Eikonal propagation through target

$$S(z) = \mathcal{P} \exp\left(ig \int dz^+ A^-(z_+, z) \right)$$

- Lee derivatives

$$J^a_L(x) S(z) = T^a S(x) \delta^{(2)}(x-z) \qquad J^a_R(x) = S^{\dagger ab}(x) J^b_L(x)$$

NLO JIMWLK Hamiltonian

- The NLO JIMWLK Hamiltonian

$$
\begin{aligned}
H^{\text{NLO}}_{\text{JIMWLK}} &= \int_{\mathbf{x},\mathbf{y},\mathbf{z}} K_{JSJ}(\mathbf{x},\mathbf{y},\mathbf{z}) \left[J^a_L(\mathbf{x})J^a_L(\mathbf{y}) + J^a_R(\mathbf{x})J^a_R(\mathbf{y}) - 2J^a_L(\mathbf{x})U^{ab}(\mathbf{z})J^b_R(\mathbf{y})\right] \\
&+ \int_{\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}'} K_{JSSJ}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}') \left[J^a_L(\mathbf{x})D^{ad}(\mathbf{z},\mathbf{z}')J^d_R(\mathbf{y}) - \frac{N_c}{2}J^a_R(\mathbf{x})J^a_R(\mathbf{y}) - \frac{N_c}{2}J^a_L(\mathbf{x})J^a_L(\mathbf{y})\right] \\
&+ \int_{\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}'} K_{q\bar{q}}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}') \left[2\,J^a_L(\mathbf{x})\,tr[V^\dagger(\mathbf{z})\,t^a\,V(\mathbf{z}')t^b]\,J^b_R(\mathbf{y}) - \frac{1}{2}J^a_R(\mathbf{x})J^a_R(\mathbf{y}) - \frac{1}{2}J^a_L(\mathbf{x})J^a_L(\mathbf{y})\right] \\
&+ \ldots
\end{aligned}
$$

where $D^{ab}(\mathbf{z}_1,\mathbf{z}_2) = \mathrm{Tr}[T^a U(\mathbf{z}_1) T^b U^+(\mathbf{z}_2)]$

NLO JIMWLK Hamiltonian: relevant kernels

$$K_{JSJ}(\mathbf{x},\mathbf{y},\mathbf{z}) \equiv \frac{\alpha_s^2(\mu) X\cdot Y}{4\pi^3 X^2 Y^2}\left[\beta_0\left(\frac{1}{2}\ln[X^2\mu^2]+\frac{1}{2}\ln[Y^2\mu^2]\right)+\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c-\frac{10}{9}N_f\right]$$

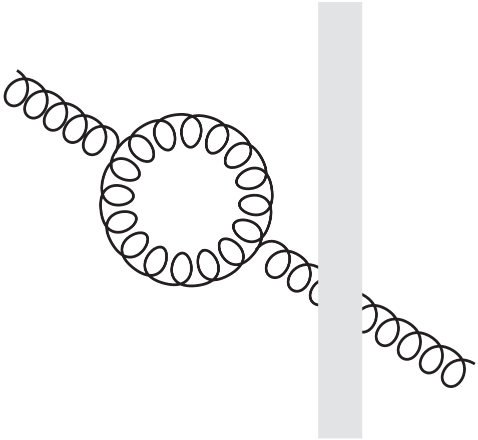
with $\beta_0 = \beta_0^g + \beta_0^q \equiv \frac{11N_c - 2N_f}{3}$.

$$K_{JSSJ}(\mathbf{x},\,\mathbf{y},\,\mathbf{z},\,\mathbf{z}') = \frac{\alpha_s^2(\mu)}{16\pi^4}\left[\frac{4}{Z^4}+\left\{2\frac{X^2(Y')^2+(X')^2Y^2-4(X-Y)^2Z^2}{Z^4(X^2(Y')^2-(X')^2Y^2)}+\frac{(X-Y)^4}{X^2(Y')^2-(X')^2Y^2}\right.\right.$$
$$\left.\times\left(\frac{1}{X^2(Y')^2}+\frac{1}{Y^2(X')^2}\right)+\frac{(X-Y)^2}{Z^2}\left(\frac{1}{X^2(Y')^2}-\frac{1}{Y^2(X')^2}\right)\right\}\ln\left(\frac{X^2(Y')^2}{(X')^2Y^2}\right)$$
$$\left.-\frac{2I(\mathbf{x},\,\mathbf{z},\,\mathbf{z}')}{Z^2}-\frac{2I(\mathbf{y},\,\mathbf{z},\,\mathbf{z}')}{Z^2}\right]+\widetilde{K}(\mathbf{x},\,\mathbf{y},\,\mathbf{z},\,\mathbf{z}'),\quad \text{with } Z_i \equiv \mathbf{z}_i - \mathbf{z}'_i$$

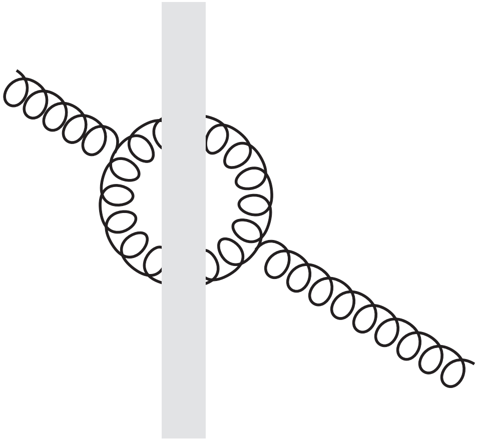
$$\widetilde{K}(\mathbf{x},\,\mathbf{y},\,\mathbf{z},\,\mathbf{z}') = \frac{\alpha_s^2(\mu)}{16\pi^4}\left(\frac{(Y')^2}{(X')^2Z^2Y^2}-\frac{Y^2}{Z^2X^2(Y')^2}+\frac{1}{Z^2(Y')^2}-\frac{1}{Z^2Y^2}+\frac{(X-Y)^2}{X^2Z^2Y^2}\right.$$
$$\left.-\frac{(X-Y)^2}{(X')^2Z^2(Y')^2}+\frac{(X-Y)^2}{(X')^2X^2(Y')^2}-\frac{(X-Y)^2}{X^2(X')^2Y^2}\right)\ln\left(\frac{X^2}{(X')^2}\right)+(\mathbf{x}\leftrightarrow\mathbf{y})$$

NLO JIMWLK Hamiltonian: UV divergent contributions

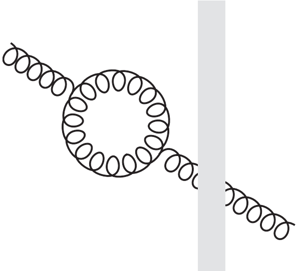
K_{JSJ}



K_{JSSJ}



NLO JIMWLK Hamiltonian: UV divergent contributions I



$$\int_{x,y,z} K_{JSJ} \left[J^a_L(x)J^a_L(y) + J^a_R(x)J^a_R(y) - 2J^a_L(x)U^{ab}(z)J^b_R(y)\right]$$

$$K_{JSJ} = K_{\mathrm{LO}} \frac{\alpha\beta_0}{4\pi} \left(\ln\left(X^2\mu^2\right) + \ln\left(Y^2\mu^2\right)\right) + \ldots$$

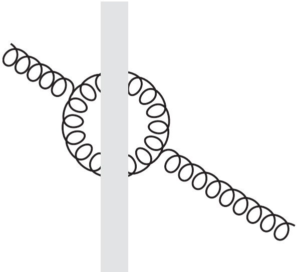
- The structure similar to the leading order
- Proportional to the WW kernel $\frac{X\cdot Y}{X^2Y^2}$
- No reasonable r. c. prescription, as the number of UV logs is twice as many

$$\alpha(X^2) \to \alpha\left(1 + \frac{\alpha\beta_0}{4\pi}\ln X^2\mu^2\right)$$

- Forcing r. c. would lead to $\frac{\alpha(X^2)\alpha(Y^2)}{\alpha}$

Kovchegov & Weigert, Balitsky, Albacete & Kovchegov 2007

NLO JIMWLK Hamiltonian: UV divergent contributions II



$$\int_{x\,y\,z\,z'} K_{JSSJ} f^{abc} f^{def} J_L^a(x) U^{be}(z) U^{cf}(z') J_R^d(y)$$

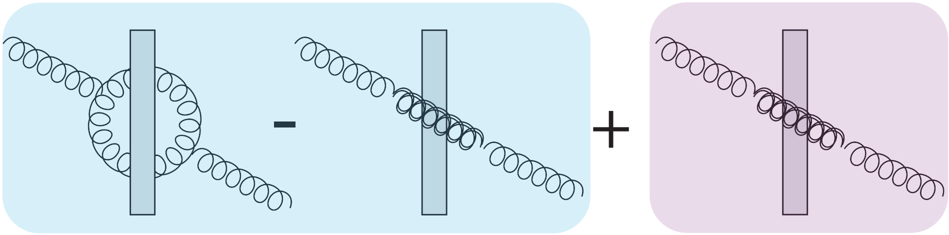
- When $z' \to z$, $f^{abc} f^{def} U^{be}(z) U^{cf}(z') \to N_c U^{ad}(z)$

- In the coincidence limit, integral of JSSJ kernel contains wanted UV singularity

$$N_c \int_{\mathbf{z}'} K_{JSSJ} = \frac{\alpha_s}{2\pi^2} \frac{\alpha_s \beta_0}{4\pi} \left(\frac{1}{X^2} \ln\left(X^2\mu^2\right) + \frac{1}{Y^2} \ln\left(Y^2\mu^2\right) + \frac{(X-Y)^2}{X^2Y^2} \ln\left(\frac{(X-Y)^2}{X^2Y^2\mu^2}\right)\right) + \dots$$

- Strategy is to shift UV divergent "single gluon" scattering part to K_{JSJ}

NLO JIMWLK Hamiltonian: UV divergent contributions III



K_{JSSJ} K_{JSJ}

- ✓ No UV divergence in K_{JSSJ}
- ✓ Allows for r. c. in K_{JSJ}: cancel an extra $\ln \mu^2$
- ✗ UV-finite pieces, including potentially large logarithms, are not uniquely defined. Dependence on the coordinate of the subtraction point
- ◆ All logarithms multiplying β_0 were attributed to r. c.

This led to Balitsky and Kovchegov-Weigert rc prescriptions.

Balitsky and Kovchegov-Weigert UV subtractions

- Balitsky subtraction (position of the quark)

$$K_{JSJ} \to K^{B}_{JSJ} = \frac{\alpha_s^2(\mu)\beta_0}{16\pi^3}\left\{-\frac{(X-Y)^2}{X^2Y^2}\ln(X-Y)^2\mu^2 + \frac{1}{X^2}\ln Y^2\mu^2 + \frac{1}{Y^2}\ln X^2\mu^2\right\}$$

- Kovchegov-Weigert subtraction (position of mother gluon)

$$K_{JSJ} \to K^{KW}_{JSJ} = \frac{\alpha_s^2(\mu)\beta_0}{8\pi^3}\frac{X\cdot Y}{X^2Y^2}\left\{\frac{X^2\ln X^2\mu^2 - Y^2\ln Y^2\mu^2}{X^2-Y^2} - \frac{X^2Y^2}{X\cdot Y}\frac{\ln\frac{X^2}{Y^2}}{X^2-Y^2}\right\}$$

Kovchegov & Weigert, Balitsky, Albacete & Kovchegov 2007

Balitsky and Kovchegov-Weigert rc prescriptions

- rc prescriptions:

$$K^{B}_{JSJ} \to \frac{\alpha_s((X-Y)^2)}{2\pi^2} \frac{X \cdot Y}{X^2 Y^2} + \frac{\alpha_s(X^2)}{4\pi^2} \frac{1}{X^2} \left(1 - \frac{\alpha_s((X-Y)^2)}{\alpha_s(Y^2)}\right) + \frac{\alpha_s(Y^2)}{4\pi^2} \frac{1}{Y^2} \left(1 - \frac{\alpha_s((X-Y)^2)}{\alpha_s(X)}\right),$$

$$K^{KW}_{JSJ} \to \frac{1}{2\pi^2} \frac{\alpha_s(X^2)\alpha_s(Y^2)}{\alpha_s(R^2)} \frac{X \cdot Y}{X^2 Y^2}$$

for the latter

$$R^2 = \sqrt{X^2\, Y^2} \left(\frac{Y^2}{X^2}\right)^{\Theta/2}, \qquad \Theta = \frac{X^2 + Y^2}{X^2 - Y^2} - 2\frac{X^2 Y^2}{X \cdot Y} \frac{1}{X^2 - Y^2}.$$

- Ambiguity due to UV subtraction in K_{JSSJ}.

Dressed gluon state I

- Let's return back to K_{JSJ}; it has twice the factor needed for renormalization of α_s
- K_{JSJ}: production of a *bare* gluon state from the valence charge
- rc in QFT: the matrix element of the interaction Hamiltonian b/w *dressed* states
- Gluon wave function renormalization at arbitrary scale Q in one loop

$$A_\mu^Q(x) = Z^{-1/2}(Q^2) A_\mu(x), \qquad Z^{1/2}(Q^2) = 1 + \frac{\alpha_s}{8\pi}\beta_0 \ln\frac{Q^2}{\mu^2}$$

- This leads to the modification of NLO:

$$K_{JSJ} \to K_{LO} \frac{\alpha_s \beta_0}{4\pi}\left(\ln(X^2\mu^2) + \ln(Y^2\mu^2) - \ln\frac{\mu^2}{Q^2}\right) + \ldots$$

Q^2 is the scale at which the renormalized field is defined

- Correct number of UV logs

Dressed gluon state II

- How to deal with divergence in K_{JSSJ}?

- This divergene also has to cancel, if H_{JIMQLK} is rewritten in term os physical dressed gluon amplitudes

- Simple multiplicative wave function renormalization does not account for scattering of a two gluon component of the dressed gluon state It has to be explicitly considered.

- At NLO the dressed gluon state contains a two-gluon (and $q - \bar{q}$) component due to gluon splitting; to be included in the definition of the dressed gluon scattering. To simplify, I will neglect quarks in this talk.

Dressed gluon states III

- For splitting to two gluons, the S-matrix of the gluon state at order α with the transverse resolution Q:

$$\mathbb{U}_Q^{ab}(z) = U^{ab}(z) + \frac{\alpha_s}{2\pi^2}\int d\xi \underbrace{\frac{1}{\xi_+(1-\xi)_+}\left(\xi^2+(1-\xi)^2+\xi^2(1-\xi)^2\right)}_{\sigma(\xi)}$$

$$\times \int_{\mu^{-1}<Z<Q^{-1}} \frac{1}{Z^2}\left(\underbrace{\mathrm{Tr}[T^a U(z+(1-\xi)Z)T^b U^+(z-\xi Z)]}_{D_{ab}(z+(1-\xi)Z,z-\xi Z)} - N_c U^{ab}(z)\right)$$

Last term: $\frac{\alpha\beta_0}{4\pi}\ln\frac{\mu^2}{Q^2}S^{ab}(z)$

- Expressing LO JIMWLK in terms of $\mathbb{S}_Q$ cancels UV divergence of K_{JSSJ} in NLO
- This expression uses exact DGLAP splitting function; to leading log accuracy this is unnecessary; one may replace $\xi \to \frac{1}{2}$ in D^{ab}.

$$\mathbb{U}_Q^{ab}(\mathbf{z}) = [1 + \frac{\alpha_s\beta_0^g}{4\pi}\ln\frac{\mu^2}{Q^2}]S^{ab}(\mathbf{z}) - \frac{\alpha_s\beta_0^g}{4\pi^2 N_c}\int_{|Z|<Q^{-1}}\frac{d^2Z}{Z^2}D^{ab}(\mathbf{z}+Z/2,\mathbf{z}-Z/2)$$

The linear term is the "virtual" DGLAP log. The quadratic term is due to two gluon component of the dressed gluon – the "real" DGLAP log

Putting everything together

- All occuracnce of U in H_{JIMWK} should be replace with $\mathbb{U}_Q$
- This will eliminate all UV "extra" UV logs
- Subtle point: $\mathbb{U}_Q$ defined for dressed gluons without any possible overlap, as in DGLAP: gluons evolve independently. This cannot be right if sources are in the same region of transverse plane. The probability of gluon splitting has to be modified if the size of the pair is larger that the distance to the closest source.
- The distance to the closest source $\rightsquigarrow$ an IR cutoff: $\bar{Q}^2 = \max\left\{Q^2, \frac{1}{X^2}, \frac{1}{Y^2}\right\}$
- Use $\bar{Q}^2$ instead of Q^2

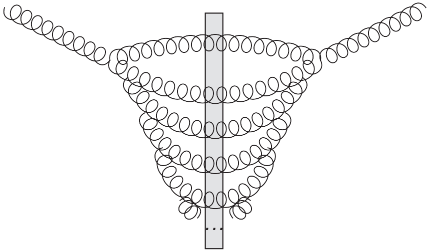
Resolution scale independence and RG

- Promoting to closed equation describing multiple consecutive DGLAP splittings

$$\frac{\partial \mathbb{S}_Q(z)}{\partial \ln Q^2} = -\alpha_s \int_\xi \sigma(\xi)\,(\mathbb{D}_Q - \mathbb{S}_Q(z))$$

- Independence of the introduced scale, Q:

$$\frac{dH}{d\ln Q} = \frac{\partial H}{\partial \ln Q} + \int_u \left[\frac{\delta H}{\delta \mathbb{S}_Q(u)} \frac{\partial \mathbb{S}_Q(u)}{\partial \ln Q}\right] = 0$$



Initial conditions and rc

- Initial conditions: at $Q_{\rm in} = Q_s^P$

$$\mathcal{H}_{\rm in} = \int K_{\rm in} \left[\{ \mathbb{S}_{Q_{\rm in}}(z) - \mathbb{S}_{Q_{\rm in}}(x) \} \{ \mathbb{S}_{Q_{\rm in}}(z) - \mathbb{S}_{Q_{\rm in}}(y) \}^\dagger \right]^{ab} J_L^a(x)\, J_L^b(y)$$

- The kernel at this scale is given by

$$K_{\rm in} = \frac{\alpha_s^\lambda(X^2)\,\alpha_s^\lambda(Y^2)\alpha_s^{1-2\lambda}(XY)}{2\pi^2} \frac{X \cdot Y}{X^2 Y^2} \left[1 + \text{ small NLO corrections}\right]$$

and does not contain large logs, as $Q_s^P |X| \sim 1$

λ is not uniquely fixed by NLO; $\lambda = 1/2$ is our preference; $\lambda = 1$ is "triumvirate" form

c.f. G. Chirilli & Y. Kovchegov, 2013

- Evolve up to $Q_f = Q_s^T$

Dilute/BFKL regime

- Initial JIMWLK kernel is convenient to write in the form:

$$\mathcal{H}_{\rm in} \propto \int_{x,y,z,z_1,z_2} \underbrace{\frac{X\cdot Y}{X^2Y^2}\Big(\underbrace{\delta_{z_1,z_2}}_{\delta(z_1-z_2)} \delta_{z_1,z} + \delta_{x,z_1}\delta_{y,z_2} - \delta_{x,z_1}\delta_{z,z_2} - \delta_{y,z_2}\delta_{z,z_1}\Big)}_{\propto K_{\rm in}} \Big[\mathbb{S}_{Q_0}(z_1)\mathbb{S}^\dagger_{Q_0}(z_2)\Big]^{ab} J_L^a(x)\, J_L^b(y)$$

- DGLAP evolution leads to smearing of δ-functions

$$\mathcal{H}_Q \propto \int_{x,y,z,z_1,z_2} \frac{X\cdot Y}{X^2Y^2}\Big(\underbrace{r_{z_1,z_2}}_{r(z_1-z_2)} r_{z_1,z} + r_{x,z_1} r_{y,z_2} - r_{x,z_1} r_{z,z_2} - r_{y,z_2} r_{z,z_1}\Big) \Big[\mathbb{S}_{Q}(z_1)\mathbb{S}^\dagger_{Q}(z_2)\Big]^{ab} J_L^a(x)\, J_L^b(y)$$

- r function:

$$r(z) = \begin{cases} \delta(z), & \text{for } z > 1/Q_s^P \\ \frac{1}{z^2}\left[\left(\frac{1}{zQ_s^P}\right)^{\frac{\alpha_s\beta_0}{2\pi}} - 1\right], & \text{for } 1/Q_s^P > z > 1/Q_s^T \\ \frac{1}{z^2}\left[\left(\frac{Q_s^T}{Q_s^P}\right)^{\frac{\alpha_s\beta_0}{2\pi}} - 1\right], & \text{for } z < 1/Q_s^T \end{cases}$$

Saturation regime

- Target saturation momentum plays two roles:
  - provides correlation length for Wilson lines
  - provides color neutralization scale: a Wilson line separated from the rest by a distance greater than $1/Q_s$ is vanishingly small

- For evolution in distance range from $1/Q_s^P$ to $1/Q_s^T$, neglect quadratic term in DGLAP evolution $\mathbb{D}_Q - N_c \mathbb{S}_Q(z) \rightarrow - N_c \mathbb{S}_Q(z)$

- The kernel is

$$K_Q = \left[\frac{Q_s^T}{Q_s^P} \right]^{\frac{\alpha_s}{2\pi} b} K_{in}$$

Interpolating equation between two regimes

- Explicit solutions in dilute and saturation regime of DGLAP provided us with Q-dependent kernel for JIMWLK Hamiltonian
- For practical implementation, an interpolating equation is needed

Conclusions

- Conventional rc prescriptions violate positive semi-definiteness of JIMWLK kernel
- Not all large logs of NLO JIMWLK multiplying QCD β-function belong to running coupling
- Subset of the logs comes from DGLAP evolution of the projectile
- We identified both types of logs, and provided the scheme for their resummation:
  - DGLAP logs $\rightsquigarrow$ evolution equation for JIMWLK kernel
  - rc logs $\rightsquigarrow$ simple scale for the QCD running coupling
- This procedure leads to positive semi-definite JIMWLK Hamiltonian