Single inclusive particle production in pA collisions at forward rapidities at NLO

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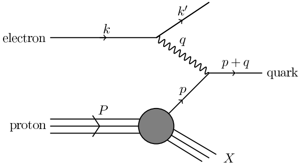
DIS in QCD :



Three Lorentz invariant quantities :

1. $q^2 = -Q^2 \equiv$ virtuality of the incoming photon
2. $x = \frac{Q^2}{2P \cdot Q} \equiv$ longitudinal momentum fraction carried by the parton
3. $s \simeq 2P \cdot Q \equiv$ energy of the colliding $\gamma - p$ system

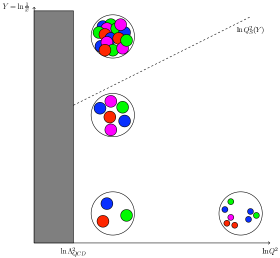
increasing the energy ($s = Q^2/x$) of the system:

Bjorken limit fixed x, $Q^2 \to \infty$

- density of partons decreases/DGLAP

Regge-Gribov limit fixed Q^2, $x \to 0$

- density of partons increases/Saturation!



- $Q_s \equiv$ saturation scale
- In the saturation regime, scattering processes are described by an effective theory: Color Glass Condensate:
  - fast partons : $k^+ > \Lambda^+$ $\to$ color sources: $J^\mu(x) = \delta^{\mu +}\rho(x^-, x_\perp)$
  - slow partons: : $k^+ < \Lambda^+$ $\to$ color fields $A^\mu(x)$
  - interaction: $\int d^4x J^\mu(x) A_\mu(x)$

Motivation to go from LO to NLO in the CGC

Leading Order in α_s CGC calculations:

↙ ↘

(pro) : CGC-based theoretical calculations are in qualitative agreement with the experimental data from all types of collisions

(con): LO CGC lacks precision in order to determine unambiguously whether saturation is exhibited by the experimental data.

increasing precision of theory predictions in order to perform precise quantitative studies:

⋆ relaxing the kinematical approximations performed at LO.

⋆ going from LO to NLO in α_s:

There has been a lot activity to provide expressions of observables at NLO.

↙ ↘

eA collisions

- dipole factorization
- structure functions/ dijets

pA collisions

- hybrid factorization
- single inclusive hadron/jet

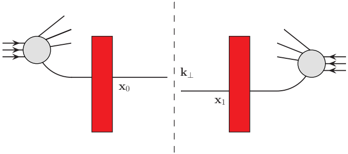
⋆ Many developments on the NLO corrections to the rapidity evolution equations.

Forward hadron production in pA collisions

[Dumitru, Hayashigaki, Jalilian-Marian - hep-ph/0506308]

Accepted calculation framework for forward production in pA collisions: Hybrid factorization

- The wave function of the projectile proton is treated in the spirit of collinear factorization (an assembly of partons with zero intrinsic transverse momenta) - DGLAP gives perturbative corrections.
- Target is treated as distribution of strong color fields which during the scattering event transfer transverse momentum to the propagating partonic configuration. (CGC like treatment)



$$\frac{d\sigma^{q\to H}}{d^2k\, d\eta} = \int_{x_F}^{1} \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) \int e^{ik(x_0-x_1)} \langle s(x_0,x_1)\rangle$$

dipole operator: $s(x_0,x_1) = \frac{1}{N_c}\mathrm{tr}\Big[U(x_0)U^\dagger(x_1)\Big]$

high transverse momentum in the produced hadron is acquired from the interaction with the target.

Forward hadron production

Does LO "Hybrid" formula take into account all contributions at high $k_\perp$?

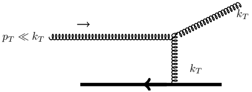
[TA, Kovner - arXiv:1102.5327]

For $k_\perp \gg Q_s$:

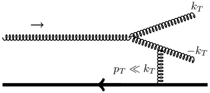
$$\frac{d\sigma}{d^2k d\eta} \propto \left[\frac{d\sigma}{d^2k d\eta}\right]_{el.} + \left[\frac{d\sigma}{d^2k d\eta}\right]_{inel.}$$

Real contributions at NLO.

"Elastic Scattering" (LO)



"Inelastic Scattering" (NLO)

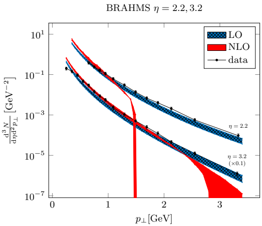


[Chirilli, Xiao, Yuan - arXiv:1112.1061 / arXiv:1203.6139] $\rightarrow$ Full NLO computation.

Collinear divergences: absorbed into DGLAP evolution of PDFs and FFs.

Rapidity divergences: absorbed into evolution of the target.

[Stasto, Xiao, Zaslavsky - arXiv:1307.4057] $\rightarrow$ Numerical studies of full NLO result.



cross sections turn out to be negative at large transverse momentum!

Several solutions proposed to fix the problem:

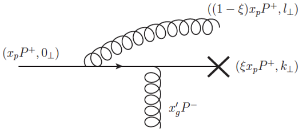
- kinematical constraints
- different choice of rapidity scales
- threshold/ Sudakov resummations

[TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1411.2869]

• choice of frame:
(i) target moves fast and carries almost all the energy.
(ii) projectile moves fast enough: accommodates partons with momentum fraction x_p but does not develop a large low-x tail.
(iii) target is evolved to s from initial s_0 via BK.

• Ioffe time restriction: only pairs whose coherence time is greater than the propagation time through the target can be resolved.

$$\text{coherent scattering} \rightarrow \frac{(1-\xi)\xi x_p}{l_\perp^2} > \frac{1}{s_0}$$



The restriction establishes itself through a modified definition of WW field:

$$A^i_{\xi,x_p}(y-z) \equiv -i\int_{l^2<\xi(1-\xi)x_p s_0} \frac{d^2l}{(2\pi)^2}\frac{l^i}{l^2}e^{-il\cdot(y-z)}$$
$$= -\frac{1}{2\pi}\frac{(y-z)^i}{(y-z)^2}\left[1-\mathrm{J}_0\left(|y-z|\sqrt{\xi(1-\xi)x_p s_0}\right)\right]$$

Neglecting Ioffe time restriction: Modified WW field → standard WW field.

New BK-like terms arise due to Ioffe time restriction.

Revisiting NLO hybrid formula - kinematical constraints

[Watanabe, Xiao, Yuan, Zaslavsky - arXiv:1505.05183] → exact kinematical constraint.
The relevant contribution:

$$\frac{\alpha_s N_c}{2\pi^2}\int_0^1 \frac{\mathrm{d}\xi}{1-\xi}\int \frac{\mathrm{d}^2x_\perp \mathrm{d}^2y_\perp \mathrm{d}^2b_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp-y_\perp)}\Big[-S(x_\perp,y_\perp)+S(x_\perp,b_\perp)S(b_\perp,y_\perp)\Big]$$
$$\times\left\{\frac{\big[1-J_0(u_\perp\Delta)\big]^2}{u_\perp^2}+\frac{\big[1-J_0(u'_\perp\Delta)\big]^2}{u'^2_\perp}-\frac{2u_\perp\cdot u'_\perp}{u_\perp^2 u'^2_\perp}\big[1-J_0(u_\perp\Delta)\big]\big[1-J_0(u'_\perp\Delta)\big]\right\}$$

with $u_\perp = x_\perp - b_\perp$ and $u'_\perp = y_\perp - b_\perp$.
One can approximate:

$$\int_0^1 \frac{\mathrm{d}\xi}{1-\xi}\left[1-J_0\Big(u_\perp\sqrt{x_p s(1-\xi)}\Big)\right]^2 \simeq \ln\frac{x_p s u_\perp^2}{c_0^2} = \ln\frac{1}{x_g}+\ln\frac{k_\perp^2 u_\perp^2}{c_0^2}$$

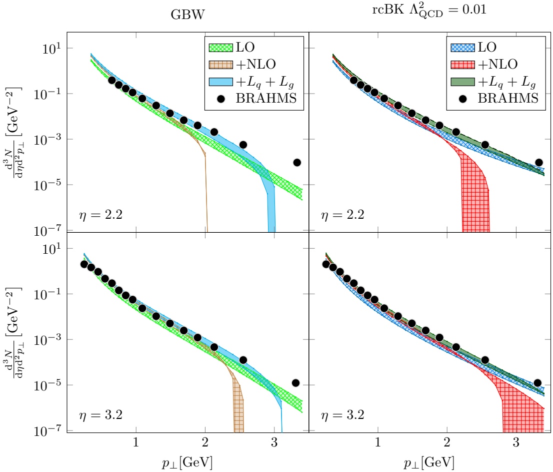
$$\int_0^1 \frac{\mathrm{d}\xi}{1-\xi}\left[1-J_0\Big(u_\perp\sqrt{x_p s(1-\xi)}\Big)\right]\left[1-J_0\Big(u'_\perp\sqrt{x_p s(1-\xi)}\Big)\right]$$
$$\simeq \ln\frac{x_p s u_\perp u'_\perp}{c_0^2} = \ln\frac{1}{x_g}+\ln\frac{k_\perp^2 u_\perp u'_\perp}{c_0^2}$$

New terms ($L_q + L_g$) arise after the implementation of the exact kinematical constraint.

The new terms in both works are consistent and equivalent.

Revisiting NLO hybrid formula - kinematical constraints

[Watanabe, Xiao, Yuan, Zaslavsky - arXiv:1505.05183]



BRAHMS data with $\sqrt{s_{NN}} = 200$ GeV.

The negativity problem is shifted to higher transverse momentum but not cured!

Revisiting NLO hybrid formula

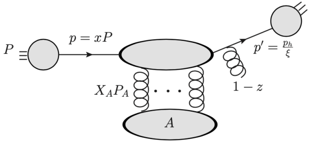
[Liu, Ma, Chao - arXiv:1909.02370]

- a new method to regularize rapidity divergence in the region $\xi \to 1$.

$$(1-\xi)^{-1+\eta} = \frac{\delta(1-\xi)}{\eta} + \frac{1}{(1-\eta)_+} + O(\eta)$$

[Kang, Liu - arXiv:1910.10166], [Liu, Kang, Liu - arXiv:2004.11990]

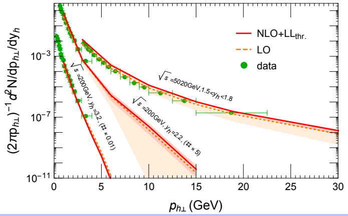
Soft-Collinear Effective Theory (SCET), threshold resummation



$$\sigma^{(n)} \propto \sum_{k=0}^{n-1} \left(\frac{\ln^k(1-z)}{1-z} \right)_+$$

$1 - z$ is the energy fraction carried by the soft radiation.

In the forward region $z \to 1$ very quickly $\Rightarrow$ logs need to be resummed.



charged hadron production $p + Pb$ at LHC and hadron productions at $d + Au$ at RHIC.

Revisiting NLO hybrid formula

[Xiao, Yuan - arXiv:1806.0352], [Shi, Wang, Wei, Xiao - arXiv:2112.06975]

- extra logs from the kinematical constraint written in coordinate space

$$\boxed{\ln\frac{k_\perp^2}{\mu_r^2} \quad , \quad \ln\frac{\mu^2}{\mu_r^2} \quad , \quad \ln^2\frac{k_\perp^2}{\mu_r^2}}$$

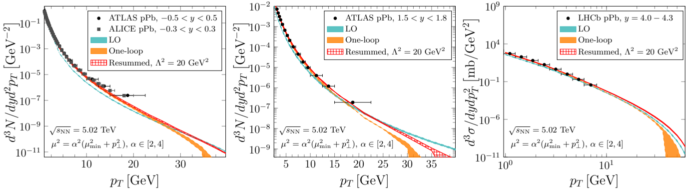
with $\mu_r = 2e^{-\gamma_E}/r_\perp$. In the threshold region ($k_\perp$ or $p_\perp \gg \mu_r$) logs become large and needs to be resummed.

- rewritten in momentum space

$$\boxed{\ln\frac{k_\perp^2}{\Lambda^2} + I_1(\Lambda) \quad , \quad \ln\frac{\mu^2}{\Lambda^2} + I_1(\Lambda) \quad , \quad \ln^2\frac{k_\perp^2}{\Lambda^2} + I_2(\Lambda)}$$

Λ is an auxilary scale in momentum space , $\Lambda \gg \Lambda_{QCD}$

- soft gluon emission $\to \ln\frac{k_\perp^2}{\Lambda^2}$ and $\ln^2\frac{k_\perp^2}{\Lambda^2}$ resummed into Sudakov factor
- collinear logs $\to \ln\frac{\mu^2}{\Lambda^2} \to$ threshold resummation (DGLAP of PDFs and FFs)



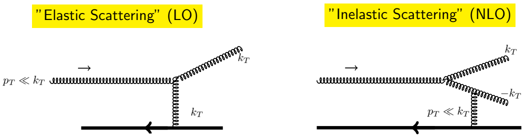
A new approach to forward pA scatterings

Common assumption in all these works: *large logs can be resummed within the collinear factorization.*

[TA, Armesto, Kovner, Lublinsky - arXiv: 2307.14922]

TMD-factorized framework is a natural choice to resum all large logs.

in [arXiv:1102.5327], the mechanisms that give rise to high transverse momentum hadrons:



– It is more natural to think the inelastic contribution in the TMD framework:
produced high k_T quark coming directly from quark TMD PDF.

⋆ another potential source to producing high transverse momentum hadron:

low k_T parton scatters softly, but subsequently fragments into a high transverse momentum hadron.

-Hadron arising from TMD FF.

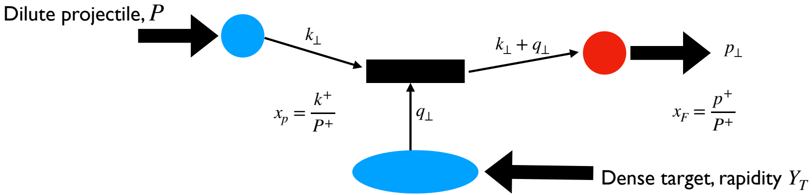
⋆ soft logs – we follow [arXiv:1411.2869]: most of the energy is carried by the target. Projectile wave function does not contain many soft gluons and no large soft logs appear explicitly in the calculation. All such logs implicitly resummed in the dipole scattering amplitude on a highly evolved target.

The setup of the problem

[TA, Armesto, Kovner, Lublinsky - arXiv:2307.14922]

TMD-factorized parton model expression:

$$\frac{d\sigma^{LO+NLO}}{d^2p_\perp d\eta} \propto \int \frac{d\zeta}{\zeta^2} \int_{k_\perp q_\perp} \mathcal{T}(x_F/\zeta, k_\perp; \mu_T^2) P(k_\perp, q_\perp) \mathcal{F}(\zeta, p_\perp, (k_\perp + q_\perp); \mu_F^2) + \text{Gen. NLO}$$



$\mathcal{T}(x_F/\zeta = x_p, k_\perp; \mu_T^2) \rightarrow$ initial TMD PDF

$\mathcal{F}(\zeta, p_\perp, (k_\perp + q_\perp)) \rightarrow$ TMD FF

$P(k_\perp, q_\perp) \rightarrow$ differential probability to produce a parton with momentum $(k_\perp + q_\perp)$ from a parton with momentum $k_\perp$

The setup of the problem

The factorization scales:

$$\mu_T^2 = \max\left\{k_\perp^2, q_\perp^2, Q_s^2, \left(\frac{p}{\zeta}\right)^2\right\} \approx \max\left\{(k_\perp + q_\perp)^2, Q_s^2, \left(\frac{p}{\zeta}\right)^2\right\}$$
$$\mu_F^2 = [(q_\perp + k_\perp) - p_\perp/\zeta]^2 \approx \max\left\{(q_\perp + k_\perp)^2, (p_\perp/\zeta)^2\right\}.$$

for the initial TMD pdf:

- if $k_\perp$ is the largest scale, then TMD is taken at this resolution scale , since the scale has to be at least $k_\perp^2$ in order to resolve the parton.
- if $q_\perp > k_\perp$, then $q_\perp$ defines the factorization scale since it is the highest scale.
- if $q_\perp, k_\perp < Q_s$, then it means that the partner of the incoming quark is scattered with Q_s and it this scale that resolves quark from the rest of the wave function.
- if $q_\perp, k_\perp \ll p_\perp/\zeta$, then the momentum $p_\perp/\zeta$ is acquired during the fragmentation and it sets the resolution scale.

for the fragmentation: fragmentation process proceeds in two steps

first: quark with momentum $(p^+, k_\perp + q_\perp)$ fragments perturbatively into a quark with momentum $(p^+, p_\perp/\zeta)$.
second: the quark fragments nonperturbatively collineraly into a hadron with momentum $(p^+\zeta, p_\perp)$

TMD distributions

Quark TMD PDF is defined as

$$x\mathcal{T}_q(x,k^2;k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1} d\xi \frac{1+(1-\xi)^2}{\xi}\,\frac{x}{1-\xi}\, f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$
$$+\frac{g^2}{(2\pi)^3}\frac{1}{2}\int_{\xi_0}^{1} d\xi \left[\xi^2+(1-\xi)^2\right]\frac{x}{1-\xi}\, f^g_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$

similarly, the quark TMD FF is

$$\mathcal{F}^q_H(\zeta,k^2;k^2,\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1} d\xi\,\frac{1+(1-\xi)^2}{\xi}\,\frac{1}{1-\xi}\, D^q_{H,k^2}\left(\frac{\zeta}{1-\xi}\right)\frac{1}{k^2}$$
$$+\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1} d\xi\,\frac{1+\xi^2}{1-\xi}\,\frac{1}{1-\xi}\, D^g_{H,k^2}\left(\frac{\zeta}{1-\xi}\right)\frac{1}{k^2},$$

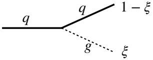
For simplicity, let us only consider the quarks and assume no gluons (the inclusion of the gluons are straight forward albeit tedious)

$$\boxed{x\mathcal{T}_q(x,k^2,k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1} d\xi \frac{1+(1-\xi)^2}{\xi}\,\frac{x}{1-\xi} f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}}$$

TMD distributions

- TMD PDFs are generated from the collinear ones (large k)

$$\boxed{x\mathcal{T}_q(x,k^2,k^2;\xi_0)=\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^{1}d\xi\,\frac{1+(1-\xi)^2}{\xi}\,\frac{x}{1-\xi}f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}}$$



– The soft divergence of the gluon emission is regulated by the cut off ξ_0 (longitudinal resolution scale).

– Partons with high longitudinal momentum are produced from partons with lower longitudinal momentum by DGLAP splitting.

– Transverse resolution scale in these splittings is equal to the transverse momentum of the parton ($\mu^2=k^2$).

The transverse resolution scale (or factorization scale) dependence of the TMD PDF is given by DGLAP like equation

$$x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)=\theta(\mu^2-k^2)\Bigg[x\mathcal{T}_q(x,k^2;k^2;\xi_0)-\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^{1}d\xi\,\frac{1+(1-\xi)^2}{\xi}\,x\,\mathcal{T}_q\left(x,k^2;l^2;\xi_0\right)\Bigg]$$

Increasing the transverse resolution $\Rightarrow$ number of q at a fixed transverse momentum decreases due to DGLAP splittings into qg pair with higher long. momentum given by the resolution scale.

TMD distributions

With these definitions, collinear quark PDF and quark TMD PDF are related via

$$x f^q_{\mu^2}(x) = \int_0^{\mu^2} \pi dk^2 \, x\mathcal{T}_q(x, k^2; \mu^2; \xi_0).$$

And it satisfies the DGLAP evolution equations...

$$\begin{aligned}
\frac{dx f^q_{\mu^2}(x)}{d\mu^2} &= \pi x\mathcal{T}_q(x, \mu^2; \mu^2; \xi_0) + \int_0^{\mu^2} \pi dk^2 \, \frac{d}{d\mu^2} x\mathcal{T}_q(x, k^2; \mu^2; \xi_0) \\
&= \pi \, \frac{g^2}{(2\pi)^3} \frac{N_c}{2} \int_{\xi_0}^1 d\xi \frac{1+(1-\xi)^2}{\xi} \, \frac{x}{1-\xi} \, f^q_{\mu^2}\left(\frac{x}{1-\xi}\right) \frac{1}{\mu^2} \\
&- \pi \, \frac{g^2}{(2\pi)^3} \frac{N_c}{2} \int_{\xi_0}^1 d\xi \frac{1+(1-\xi)^2}{\xi} \, x \frac{1}{\mu^2} \int_0^{\mu^2} \pi dk^2 \, \mathcal{T}_q\left(x, k^2; \mu^2; \xi_0\right) \\
&= \pi \, \frac{g^2}{(2\pi)^3} \frac{N_c}{2} \int_{\xi_0}^1 d\xi \left[\frac{1+(1-\xi)^2}{\xi}\right]_+ \frac{x}{1-\xi} \, f^q_{\mu^2}\left(\frac{x}{1-\xi}\right) \frac{1}{\mu^2},
\end{aligned}$$

Forward pA - quark channel

– start from the expressions obtained in LCPT (with Ioffe time restriction) in [arXiv:1411.2869] (no collinear subtraction and no + prescription)

– projectile contains quarks with transverse momentum smaller than μ_0, target sits at some rapidity with no need of further evolution.

– assumptions: large N_c, factorization of the dipoles, and translationally invariant dipoles.

After Including the fragmentation and FT to momentum space:

$$\frac{d\sigma^{q\to q\to H}}{d^2pd\eta} = \frac{d\sigma_0^{q\to q\to H}}{d^2pd\eta} + \frac{d\sigma_{1,\mathrm{r}}^{q\to q\to H}}{d^2pd\eta} + \frac{d\sigma_{1\mathrm{v.}}^{q\to q\to H}}{d^2pd\eta}$$

LO term

$$\boxed{\frac{d\sigma_0^{q\to q\to H}}{d^2pd\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) s(p/\zeta)}$$

Forward pA - quark channel

NLO real corrections:

$$\frac{d\sigma}{d^2p\,d\eta}\bigg|^{\mathrm{q}\to\mathrm{q}}_{\mathrm{NLO,r}} = \frac{g^2}{(2\pi)^3}S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\,D^q_{\mu_0^2}(\zeta)\int_{k^2,q^2>\mu_0^2}\int_{\xi_0}d\xi\,\frac{x_F}{\zeta(1-\xi)}f^q_{\mu_0^2}\left(\frac{x_F}{\zeta(1-\xi)}\right)\frac{N_c}{2}\left[\frac{1+(1-\xi)^2}{\xi}\right]s(k)s(q)$$
$$\times\left\{\frac{1}{2}\frac{(q-k)^2}{(p/\zeta-k)^2(p/\zeta-q)^2}+\frac{1}{2}\frac{(1-\xi)^2(q-k)^2}{[p/\zeta-(1-\xi)k]^2[p/\zeta-(1-\xi)q]^2}\right\}+(\mathrm{Gen.\,NLO})_1$$

- first term contributes to the quark TMD PDF
- second term contributes to the quark TMD FF

- the leftover genuine NLO correction (no large logs) is given by

$$(\mathrm{Gen.\ NLO})_1 = \frac{g^2}{(2\pi)^3}\,S_\perp\int\frac{d\zeta}{\zeta^2}D^q_{\mu_0^2}(\zeta)\int_{k,q}\int d\xi\,\frac{x_F}{\zeta(1-\xi)}\,f^q_{\mu_0^2}\left(\frac{x_F}{\zeta(1-\xi)}\right)\frac{N_c}{2}\left[\frac{1+(1-\xi)^2}{\xi}\right]s(k)s(q)$$
$$\times\left[\frac{(p/\zeta-k)^i}{(p/\zeta-k)^2}-\frac{(p/\zeta-(1-\xi)k)^i}{(p/\zeta-(1-\xi)k)^2}\right]\left[\frac{(p/\zeta-q)^i}{(p/\zeta-q)^2}-\frac{(p/\zeta-(1-\xi)q)^i}{(p/\zeta-(1-\xi)q)^2}\right].$$

NLO real terms

The first term can be cast into

$$\frac{1}{2}\int_{k,q} s(k)s(q)\frac{(q-k)^2}{(p/\zeta-k)^2(p/\zeta-q)^2} = \int_{k,q}\frac{1}{k^2}s(-k+p/\zeta)\Big[1-\frac{k\cdot q}{q^2}\Big]\, s(-q+p/\zeta)$$

Second term (after rescaling $\zeta(1-\xi)\to\zeta'$) can be acts into the same form.
Using the definition of TMD PDF (analogously TMD FF)

$$x\mathcal{T}_q(x,k^2,k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}\frac{x}{1-\xi}f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$

The real contribution reads

$$\frac{d\sigma^{q\to q\to H}_{1,\mathrm{r}}}{d^2p d\eta} = S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\int_{k^2>\mu_0^2}\frac{x_F}{\zeta}\Big\{D^q_{\mu_0^2}(\zeta)\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;k^2,\xi_0=\frac{k^2\zeta}{x_F s_0}\right)+f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\mathcal{F}^q\left(\zeta,k^2;k^2,\xi_0=\frac{k^2\zeta}{x_F s_0}\right)\Big\}$$
$$\times\int_q s(-k+p/\zeta)\Big[1-\frac{k\cdot q}{q^2}\Big]s(-q+p/\zeta)+(\mathrm{Gen.\,NLO})$$

– incoming quark with mom. k, scatters with mom exchange $-k+p/\zeta$, outgoing quark with mom. p/ζ collinearly fragments into a hadron with mom. p.

– (shift $k\to -q+p/\zeta$ and $q\to -k+p/\zeta$) incoming quark with vanishing mom., scatters with mom. transfer q, first perturbatively fragments into a quark with mom p/ζ, which then fragments into a hadron with momentum p.

NLO virtual contributions

Starting from the expressions in [arXiv:1411.2869], adopting the same assumptions:

$$\frac{d\sigma^{q\to q\to H}_{1,v}}{d^2p d\eta} = -2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta)\int_{k^2>\mu_0^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi\, \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\frac{1+(1-\xi)^2}{\xi}$$
$$\times\int_q s\left(\frac{p}{\zeta}\right) s(q)\left\{\left[\frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right]+\left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}-\frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2}\right]\frac{k}{k^2}\right\}$$

- incoming q $\to$ qg pair, pair scatters, recombines into q.
- NLO corr. to LO elastic q scattering.

- qg loop that appears either before or after the scattering.
- "proper" virtual diagram

Does not contain any large logs (a Gen. NLO correction)

in the first term one can perform the angular integration over the angle of vector k:

$$\int_{\mu_0^2} d^2k\left[\frac{\frac{p}{\zeta}-q-k}{(\frac{p}{\zeta}-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right] = \int_{\mu_0^2}^{(q-\frac{1}{\zeta}p)^2}\frac{d^2k}{k^2}$$

NLO virtual contributions

The virtual NLO contribution can be split into two intervals

$$-2\frac{g^2}{(2\pi)^3}S_\perp \frac{N_c}{2}\int_{x_F}^{1}\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\int_q\left[\int_{\mu_0^2}^{\mu^2}+\int_{\mu^2}^{(q-\frac{1}{\zeta}p)^2}\right]\frac{d^2k}{k^2}\int_{k^2\zeta/(x_F s_0)}^{1}d\xi\,\frac{x_F}{\zeta}\,f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\,\frac{1+(1-\xi)^2}{\xi}s\left(\frac{p}{\zeta}\right)\,s(q)$$

- the first term combines with LO to evolve the resolution scale of the TMD to μ^2.
- contribution from the pairs of the transverse size close to the resolution scale.

(no large logs & Gen. NLO correction)

LO + NLO virtual:

$$S_\perp\int_{x_F}^{1}\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\,\frac{x_F}{\zeta}f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)s\left(\frac{p}{\zeta}\right)$$
$$-2\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^{1}\frac{d\zeta}{\zeta^2}\,D^q_{H,\mu_0^2}(\zeta)\int_q\int_{\mu_0^2}^{\mu^2}\frac{d^2k}{k^2}\int_{k^2\zeta/(x_F s_0)}^{1}d\xi\,\frac{x_F}{\zeta}\,f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\,\frac{1+(1-\xi)^2}{\xi}s\left(\frac{p}{\zeta}\right)$$
$$=S_\perp\int_{x_F}^{1}\frac{d\zeta}{\zeta^2}\int_0^{\mu_0^2}d^2k\left[D^q_{H,\mu_0^2}(\zeta)\,\frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right)+\mathcal{F}^q_H\left(\zeta,k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right)\,\frac{x_F}{\zeta}f^q_{\mu_0^2}(\frac{x_F}{\zeta})\right]s\left(\frac{p}{\zeta}\right)$$

NLO virtual contributions

- evolve the factorization scale in the collinear PDFs and FFs up to μ^2

$$D^q_{H,\mu_0^2}(\zeta) \to \int_0^{\mu_0^2} d^2l \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right); \quad f^q_{\mu_0^2}(\frac{x_F}{\zeta}) \to \int_0^{\mu_0^2} d^2k \mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)$$

this introduces the term at $O(\alpha_s^2)$ therefore legitimate in our $O(\alpha_s)$ calculation.

- alter the scattering amplitude

$$s(\frac{p}{\zeta}) \to \int_q s\left(-(k+l) + \frac{p}{\zeta}\right)\left[1 - \frac{(k+l)\cdot q}{q^2}\right] s\left(-q + \frac{p}{\zeta}\right)$$

($|k+l|^2 \lesssim \mu_0^2 \ll p^2/\zeta^2$ & $q^2 \sim \max(Q_s^2, p^2/\zeta^2)$ & $\int_q s(q) = 1$) $\Rightarrow$ this modification only adds subleading power corrections of the order μ_0^2/Q_s^2

LO + NLO virtual:

$$S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int_q \int_0^{\mu_0^2} d^2l \int_0^{\mu_0^2} d^2k$$
$$\times \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \frac{x_F}{\zeta} \mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) s\left(-(k+l) + \frac{p}{\zeta}\right)\left[1 - \frac{(k+l)\cdot q}{q^2}\right] s\left(-q + \frac{p}{\zeta}\right)$$

LO+NLO virtual+NLO real: Final TMD factorized expression

$$S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int_q \int d^2l \int d^2k \, \mathcal{F}^q_H\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \frac{x_F}{\zeta} \mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)$$
$$\times s\left(-(k+l) + \frac{p}{\zeta}\right)\left[1 - \frac{(k+l)\cdot q}{q^2}\right] s\left(-q + \frac{p}{\zeta}\right) + (\text{Gen. NLO})$$

Summary

• The progress continues in order to provide full NLO results which will provide the necessary precision for quantitive studies to determine whether saturation is exhibited by experimental data.

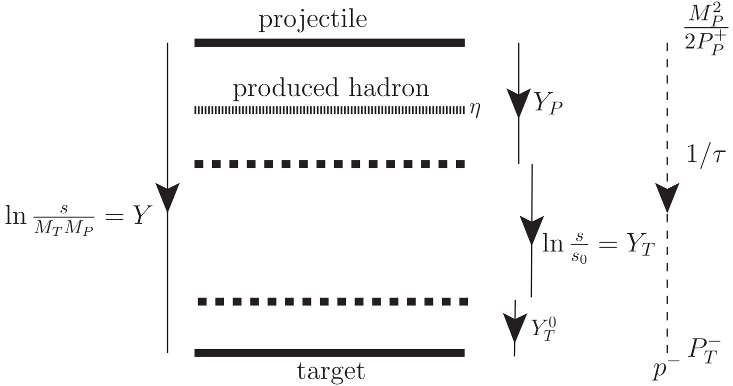
• We have discussed the issues and the suggested solutions for pA collisions at forward rapidities for NLO calculations.

• A new approach to forward pA scatterings is discussed.
Still have a lot of work to do:

- NLO calc. without TMD FFs, single jet production in forward pA collisions.

- NLO calc. without TMD PDFs, single inclusive hadron production in DIS.

Back up - Rapidity balance



Back up - Exact kinematical constraint

$$L_q(k_\perp) = \frac{\alpha_s N_c}{2\pi^2} \int \frac{\mathrm{d}^2x_\perp \mathrm{d}^2y_\perp \mathrm{d}^2b_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} \left[S(x_\perp - b_\perp)S(y_\perp - b_\perp) - S(x_\perp - y_\perp)\right]$$
$$\times \left[\frac{1}{u_\perp^2}\ln\frac{k_\perp^2 u_\perp^2}{c_0^2} + \frac{1}{u_\perp'^2}\ln\frac{k_\perp^2 u_\perp'^2}{c_0^2} - \frac{2u_\perp\cdot u_\perp'}{u_\perp^2 u_\perp'^2}\ln\frac{k_\perp^2|u_\perp||u_\perp'|}{c_0^2}\right]$$

$$\frac{1}{u_\perp^2}\ln\frac{k_\perp^2 u_\perp^2}{c_0^2} = \frac{1}{8\pi}\int \mathrm{d}^2l_\perp e^{il_\perp\cdot u_\perp}\left(\ln\frac{k_\perp^2}{l_\perp^2}\right)^2$$
$$\frac{\vec{u}_\perp}{u_\perp^2}\ln\frac{k_\perp^2 u_\perp^2}{c_0^2} = \frac{1}{2\pi}\int \mathrm{d}^2l_\perp e^{il_\perp\cdot u_\perp}\frac{i\vec{l}_\perp}{l_\perp^2}\ln\frac{k_\perp^2}{l_\perp^2}$$

Back up - All channels

$$\frac{d\sigma^{q\to q\to H,r}}{d^2p d\eta} = \frac{g^2}{(2\pi)^3} C_F S_\perp \int \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \int_{k,q} \int d\xi \frac{x_F}{\zeta\xi} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta\xi}\right) \left[\frac{1+\xi^2}{1-\xi}\right] s(k)s(q)$$
$$\times \Bigg\{ \underbrace{\frac{1}{2}\frac{(q-k)^2}{(p/\zeta-k)^2(p/\zeta-q)^2}}_{\#1\text{ - quark TMD pdf 1}} + \underbrace{\frac{1}{2}\frac{\xi^2(q-k)^2}{(p/\zeta-\xi k)^2(p/\zeta-\xi q)^2}}_{\#2\text{ - quark TMD ff 1}} \Bigg\} + (\text{Gen. NLO})_1 \tag{C1}$$

$$\frac{d\sigma^{g\to q\to H,r}}{d^2p d\eta} = \frac{g^2}{(2\pi)^3}\frac{1}{2} S_\perp \int \frac{d\zeta}{\zeta^2} D^q_{\mu_0^2}(\zeta) \int_{k,q} \int d\xi \frac{x_F}{\zeta(1-\xi)} f^g_{\mu_0^2}\left(\frac{x_F}{\zeta(1-\xi)}\right) \left[\xi^2+(1-\xi)^2\right] s(k)s(q)$$
$$\times \Bigg\{ \underbrace{\frac{1}{2}\frac{(k-q)^2}{(p/\zeta-k)^2(p/\zeta-q)^2}}_{\#3\text{ - quark TMD pdf 2}} + \underbrace{\int_t s(t)\frac{1}{2}\frac{(1-\xi)^2[(q-k)-(q-t)]^2}{\left[p/\zeta-(1-\xi)(q-k)\right]^2\left[p/\zeta-(1-\xi)(q-t)\right]^2}}_{\#4\text{ - gluon TMD ff 1}} \Bigg\} + (\text{Gen. NLO})_2 \tag{C2}$$

$$\frac{d\sigma^{g\to g\to H,r}}{d^2p d\eta} = \frac{g^2}{(2\pi)^3} 2N_c S_\perp \int \frac{d\zeta}{\zeta^2} D^g_{\mu_0^2}(\zeta) \int_{k,q,t} \int d\xi \frac{x_F}{\zeta(1-\xi)} f^g_{\mu_0^2}\left(\frac{x_F}{\zeta(1-\xi)}\right) \left[\frac{1-\xi}{\xi}+\frac{\xi}{1-\xi}+\xi(1-\xi)\right] s(k)s(q)s(t)$$
$$\times \Bigg\{ \underbrace{\frac{1}{2}\frac{[(q-k)-(q-t)]^2}{\left[p/\zeta-(q-k)\right]^2\left[p/\zeta-(q-t)\right]^2}}_{\#5\text{ - gluon TMD pdf 1}} + \underbrace{\frac{1}{2}\frac{(1-\xi)^2[(q-k)-(q-t)]^2}{\left[p/\zeta-(1-\xi)(q-k)\right]^2\left[p/\zeta-(1-\xi)(q-t)\right]^2}}_{\#6\text{ - gluon TMD ff 2}} \Bigg\} + (\text{Gen. NLO})_3 \tag{C3}$$

$$\frac{d\sigma^{q\to g\to H,r}}{d^2p d\eta} = \frac{g^2}{(2\pi)^3} C_F S_\perp \int \frac{d\zeta}{\zeta^2} D^g_{\mu_0^2}(\zeta) \int_{k,q} \int d\xi \frac{x_F}{\zeta\xi} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta\xi}\right) \left[\frac{1+(1-\xi)^2}{\xi}\right] s(k)s(q)$$
$$\times \Bigg\{ \underbrace{\int_t s(t)\frac{1}{2}\frac{[(q-k)-(q-t)]^2}{\left[p/\zeta-(q-k)\right]^2\left[p/\zeta-(q-t)\right]^2}}_{\#7\text{ - gluon TMD pdf 2}} + \underbrace{\frac{1}{2}\frac{\xi^2(q-k)^2}{(p/\zeta-\xi k)^2(p/\zeta-\xi q)^2}}_{\#8\text{ - quark TMD ff 2}} \Bigg\} + (\text{Gen. NLO})_4 \tag{C4}$$

Back up - quark channel

Defining the quark TMD PDF as

$$\mathcal{T}_q(x,k^2,\mu^2,\xi_0) \equiv \frac{1}{2}\frac{g^2}{(2\pi)^3}\int_{\xi_0} d\xi \left\{ \frac{1}{\xi} f^q_{\mu^2}\left(\frac{x}{\xi}\right) C_F \left[\frac{1+\xi^2}{1-\xi}\right] + \frac{1}{1-\xi} f^g_{\mu^2}\left(\frac{x}{1-\xi}\right)\frac{1}{2}\left[\xi^2+(1-\xi)^2\right]\right\}\frac{1}{k^2}$$

then $(\#1+\#3)$ can be written as

$$(\#1+\#3) = S_\perp \int \frac{d\zeta}{\zeta^2}\frac{x_F}{\zeta}\int_k D^q_{\mu_0^2}(\zeta)\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2,k^2,\xi_0\right) s\left(-k+p/\zeta\right)\int_q \left[1-\frac{k\cdot q}{q^2}\right] s\left(-q+p/\zeta\right)$$

After defining the quark TMD FF as

$$\mathcal{F}_q(x,k^2,\mu^2,\xi_0) \equiv \frac{1}{2}\frac{g^2}{(2\pi)^3}\int_{\xi_0} d\xi \left\{\frac{1}{\xi} D^q_{\mu^2}\left(\frac{x}{\xi}\right) C_F\left[\frac{1+\xi^2}{1-\xi}\right] + \frac{1}{\xi} D^g_{\mu^2}\left(\frac{x}{\xi}\right) C_F\left[\frac{1+(1-\xi)^2}{\xi}\right]\right\}\frac{1}{k^2}$$

then $(\#2+\#8)$ can be written as

$$(\#2+\#8) = S_\perp \int \frac{d\zeta}{\zeta^2}\frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\int_k \mathcal{F}_q(\zeta,k^2,k^2,\xi_0) s\left(-k+p/\zeta\right)\int_q \left[1-\frac{k\cdot q}{q^2}\right] s\left(-q+p/\zeta\right)$$

Back up - quark channel

Finally, for the quark production we get

$$(\#1+\#3)+(\#2+\#8)=S_{\perp}\int\frac{d\zeta}{\zeta^2}\,\frac{x_F}{\zeta}\int_k\left\{D^q_{\mu_0^2}(\zeta)\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2,k^2,\xi_0\right)+f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\mathcal{F}_q(\zeta,k^2,k^2,\xi_0)\right\}$$
$$\times\, s\,(-k+p/\zeta)\int_q\left[1-\frac{k\cdot q}{q^2}\right]s\,(-q+p/\zeta)$$