NLO corrections to particle production in DIS at small x

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Workshop on overlap between QCD resummations

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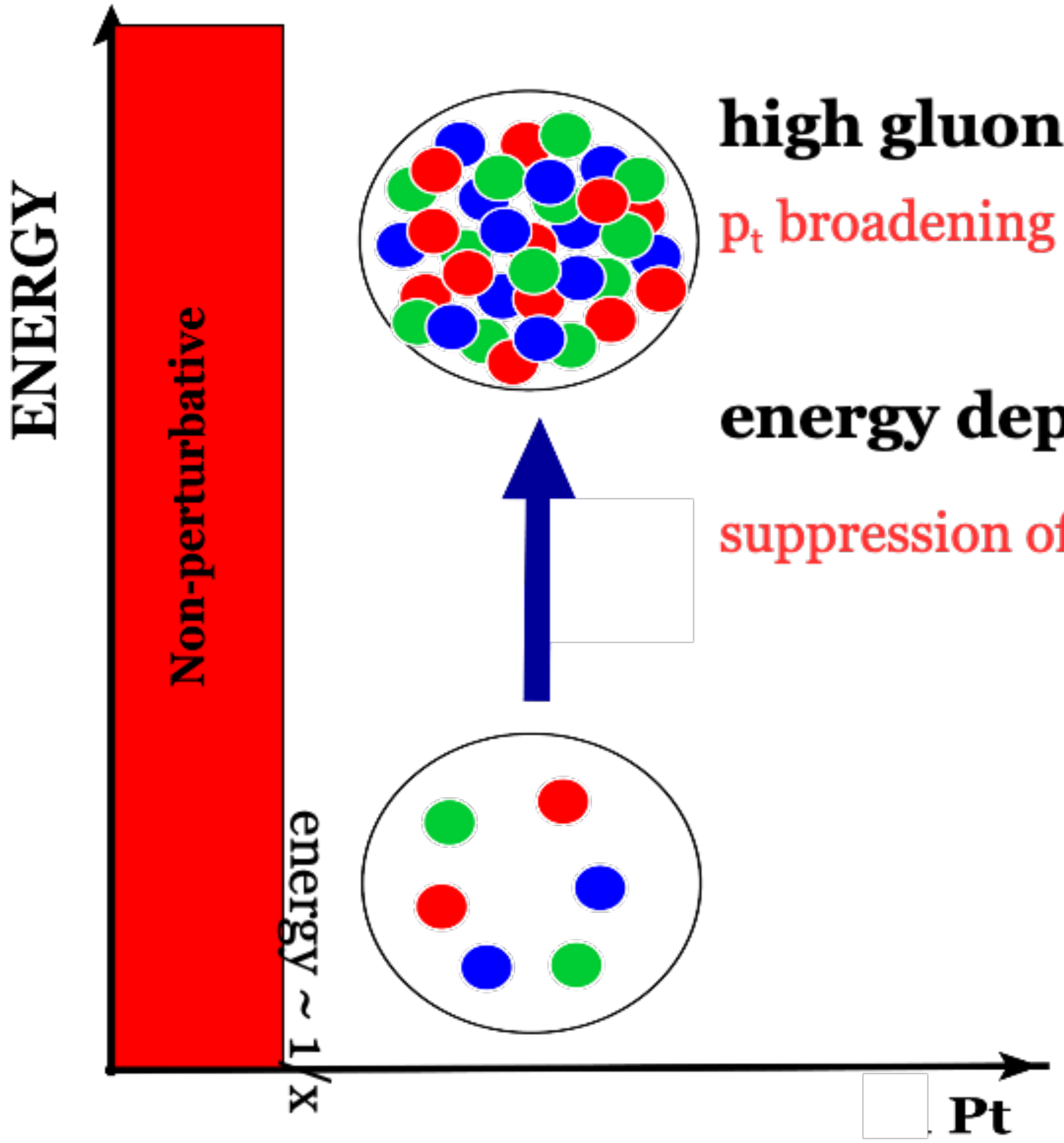
QCD at high energy: gluon saturation



high gluon density: Eikonal multiple scattering

p_t broadening

energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} (\frac{1}{x})^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

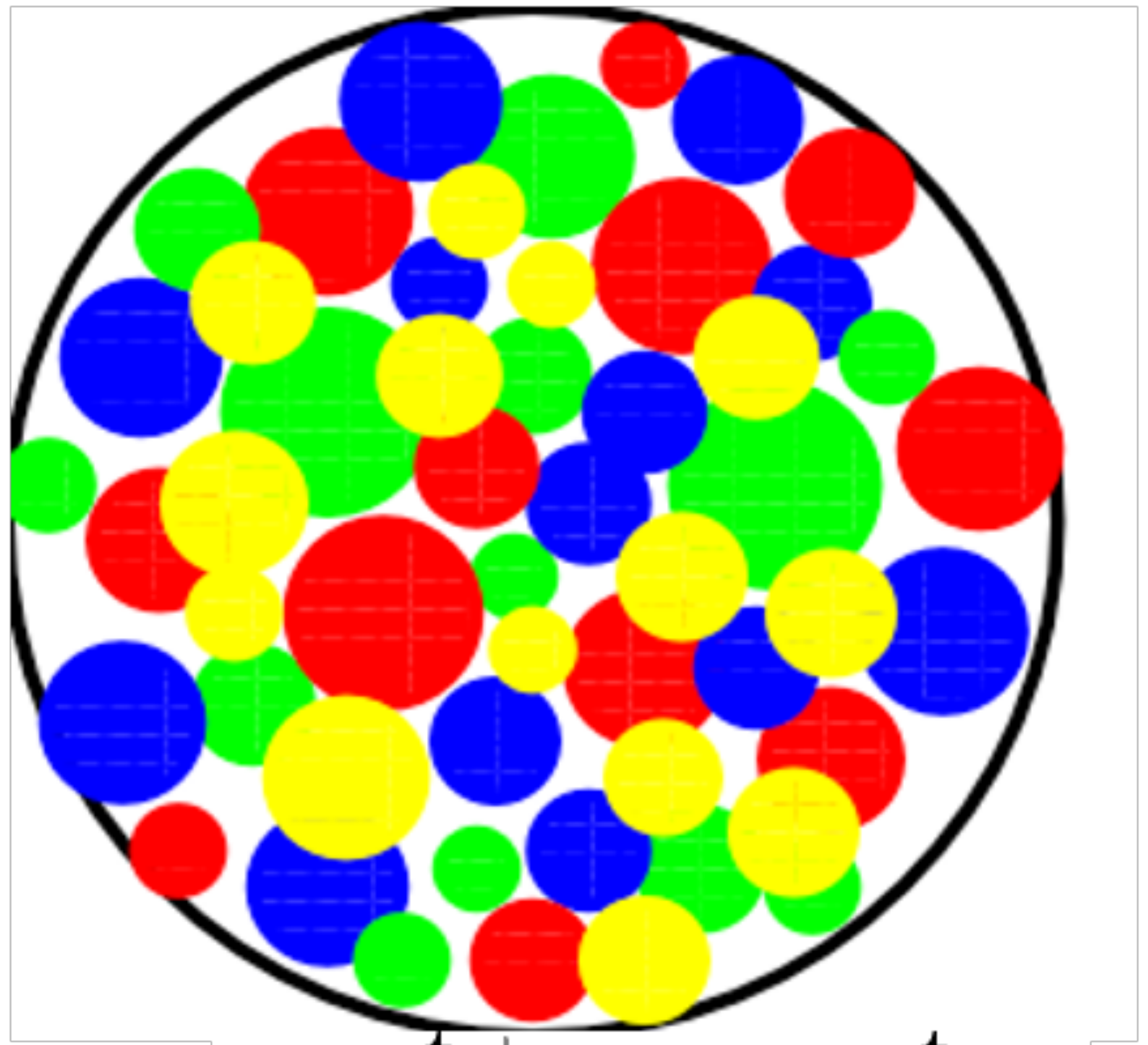
for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

Shadowing/Nuclear modification factor
Azimuthal angular correlations (dihadrons/dijets,...)
Long range rapidity correlations (ridge,...)
Connections to TMDs,...

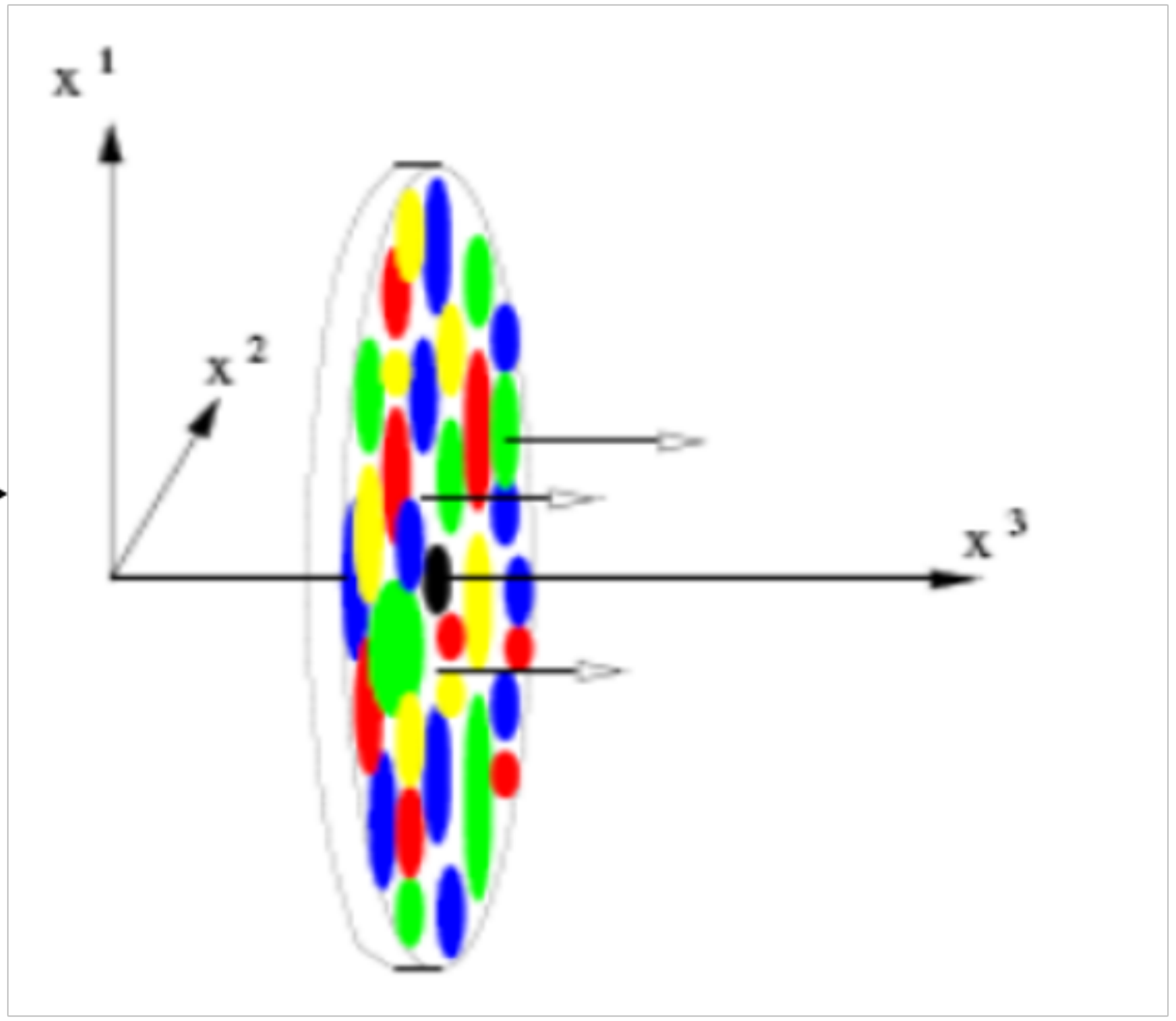
$$\mathbf{x \leq 0.01}$$

A very large nucleus at high energy: MV model

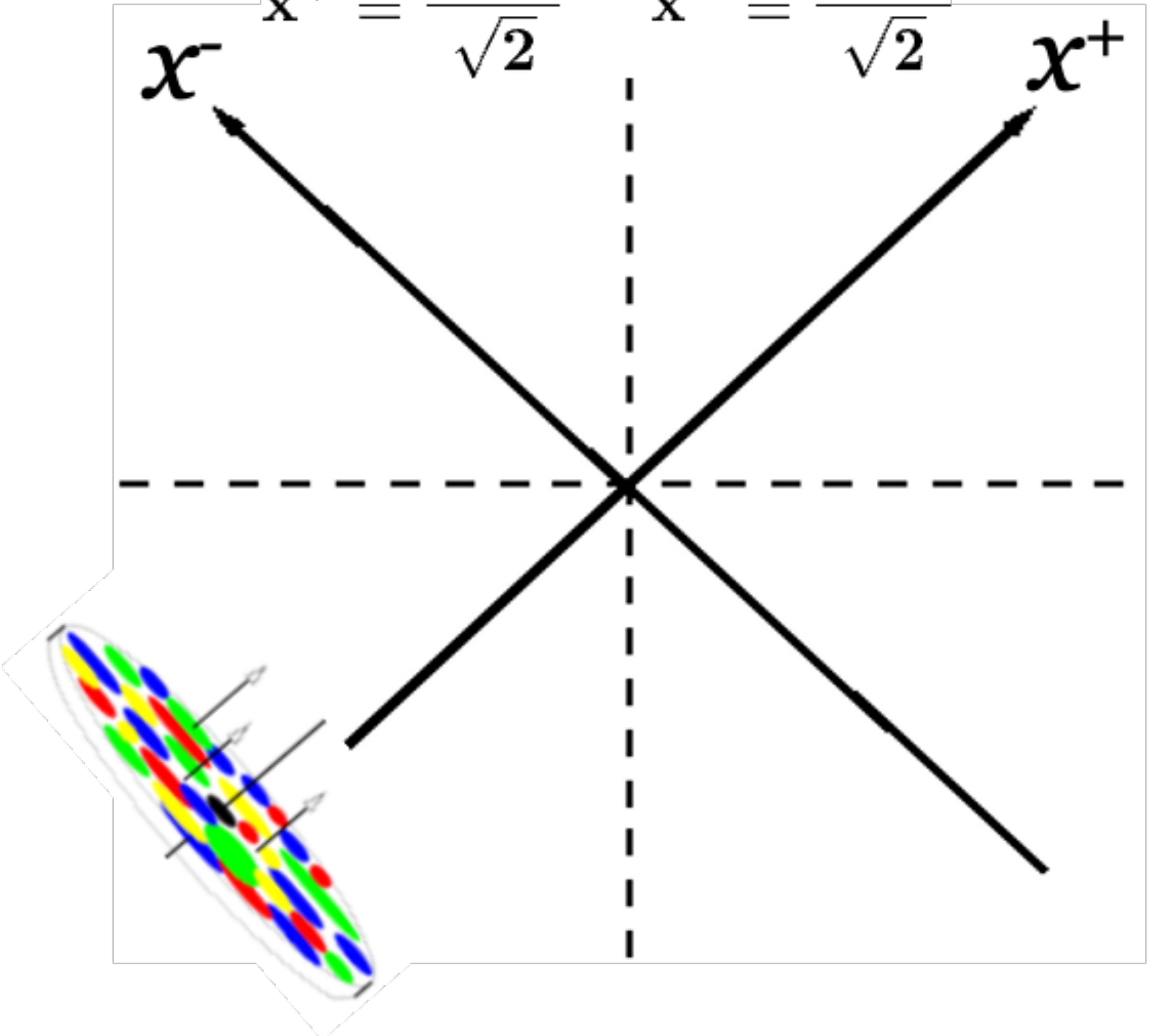


boost

$$R \rightarrow \frac{R}{\gamma}$$
$$\gamma \sim 100 \quad \text{RHIC}$$
$$\gamma \sim 2500 \quad \text{LHC}$$



$$\mathbf{x^+ \equiv \frac{t+z}{\sqrt{2}}} \qquad \mathbf{x^- \equiv \frac{t-z}{\sqrt{2}}}$$



sheet of color charge moving along x^+ and sitting at $x^- = 0$

$$\mathbf{J^\mu_a(x) \equiv \delta^{\mu +}\, \delta(x^-)\, \rho_a(x_t)}$$

color current *color charge*

$$\mathbf{A^a_i(x^-, x_t) = \theta(x^-)\, \alpha^a_i(x_t)}$$

with $\partial_i\, \alpha^a_i = g\, \rho^a$

Dense proton/nucleus: multiple eikonal scatterings

sum over all scatterings $$i\mathcal{M} = \sum_n i\,\mathcal{M}_n$$

$$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\, [V(x_t) - 1]\; u(p)$$

with

$$V(x_t) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a\right\}$$

Wilson lines: effective degrees of freedom that contain all the target information

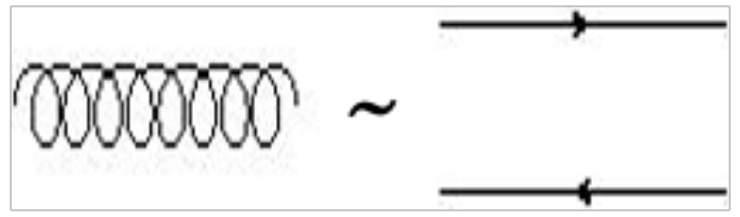
$$\frac{d\,\sigma^{q\,T\to q\,X}}{d^2p_t\,dy} \sim |i\mathcal{M}|^2 \sim F.T. < Tr\, V(x_t)\, V^\dagger(y_t) >$$

dipole

One-loop corrections: BK-JIMWLK eq.

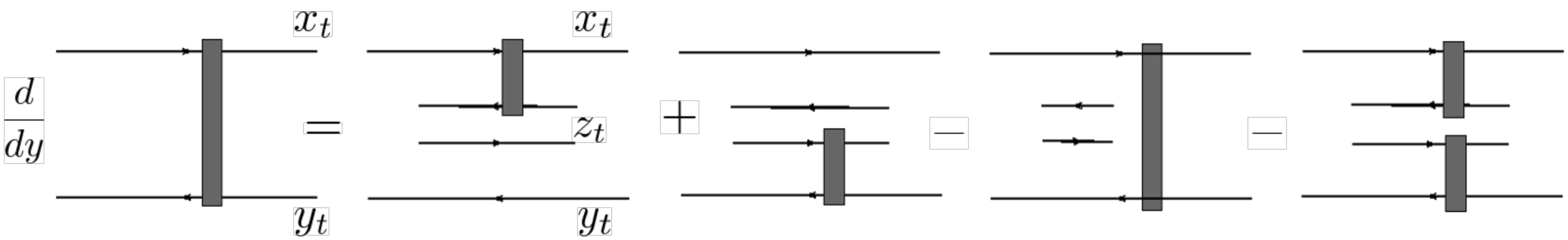
at large N_c

$$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$$



$$\frac{d}{dy}T(x_t,y_t) = \frac{N_c\alpha_s}{2\pi^2}\int d^2z_t\,\frac{(x_t-y_t)^2}{(x_t-z_t)^2(y_t-z_t)^2}\left[T(x_t,z_t)+T(z_t,y_t)-T(x_t,y_t)-T(x_t,z_t)T(z_t,y_t)\right]$$

$$T \equiv 1 - S$$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^\gamma \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2p_t\,dy}}{A^{1/3}\,\frac{d\sigma^{pp}}{d^2p_t\,dy}}$$

nuclear shadowing

suppression of p_t spectra

disappearance of back to back peaks

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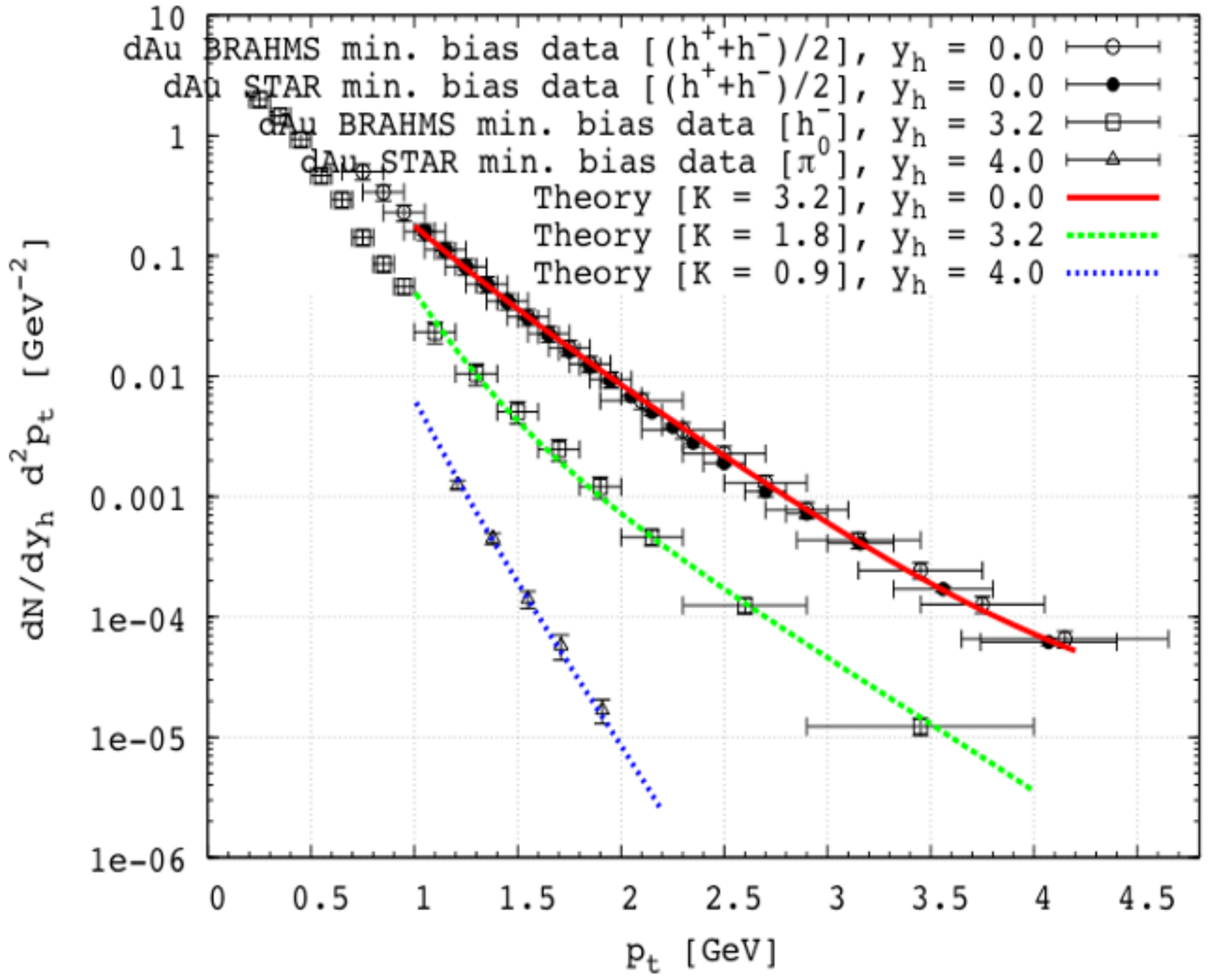
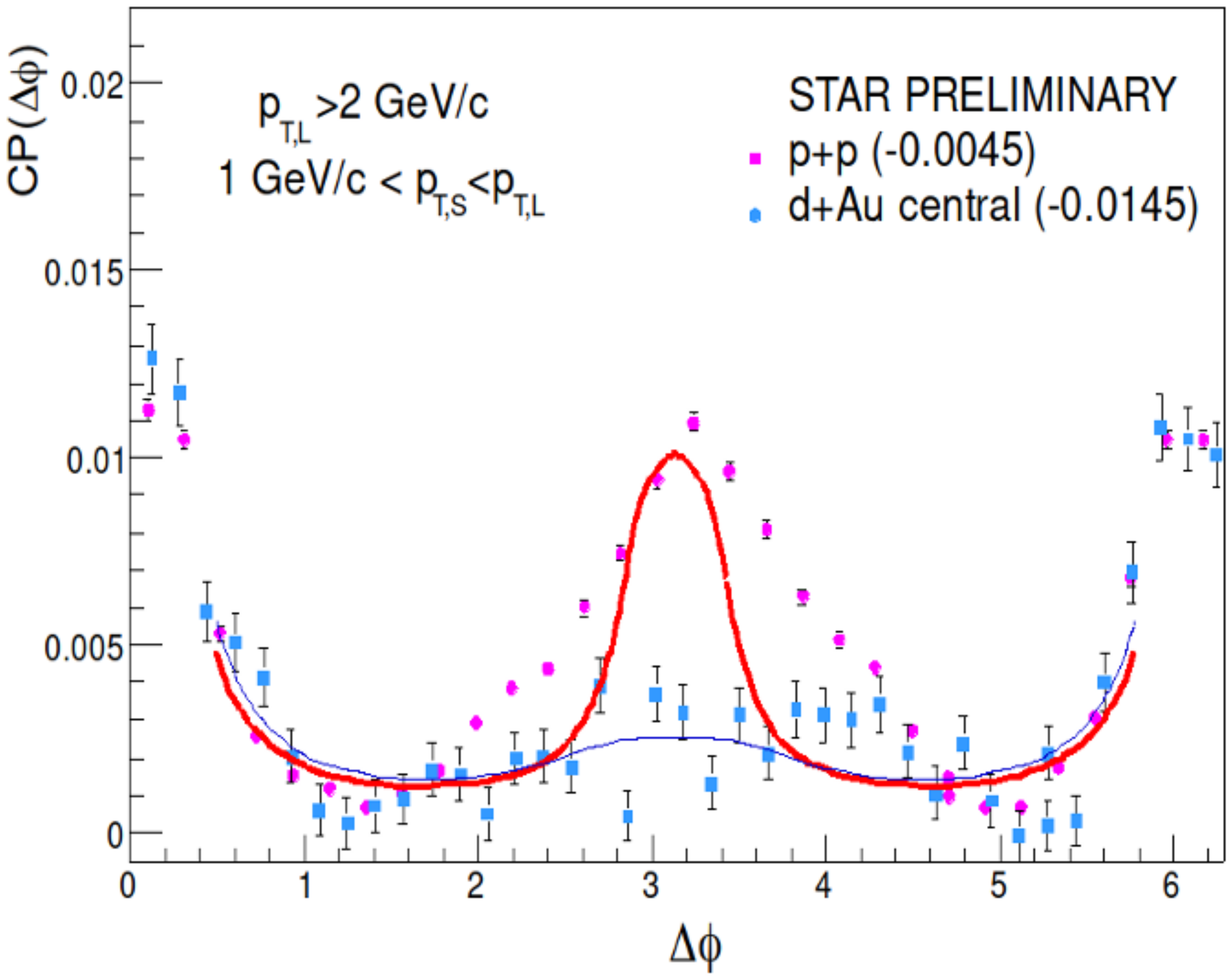
CGC at RHIC

Single and double inclusive hadron production in dA collisions



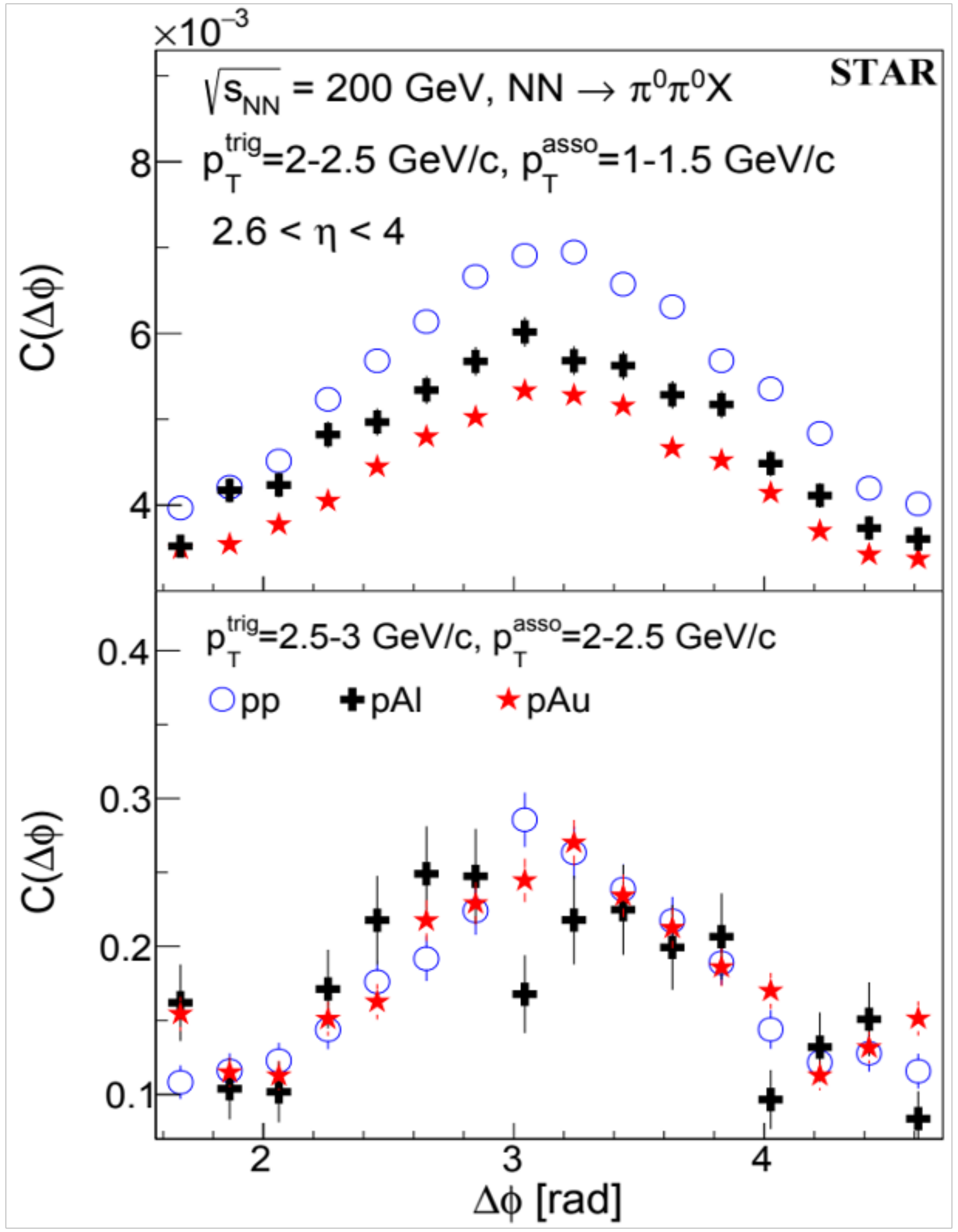
Dumitru,Hayashigaki, JJM, NPA770 (2006) 57

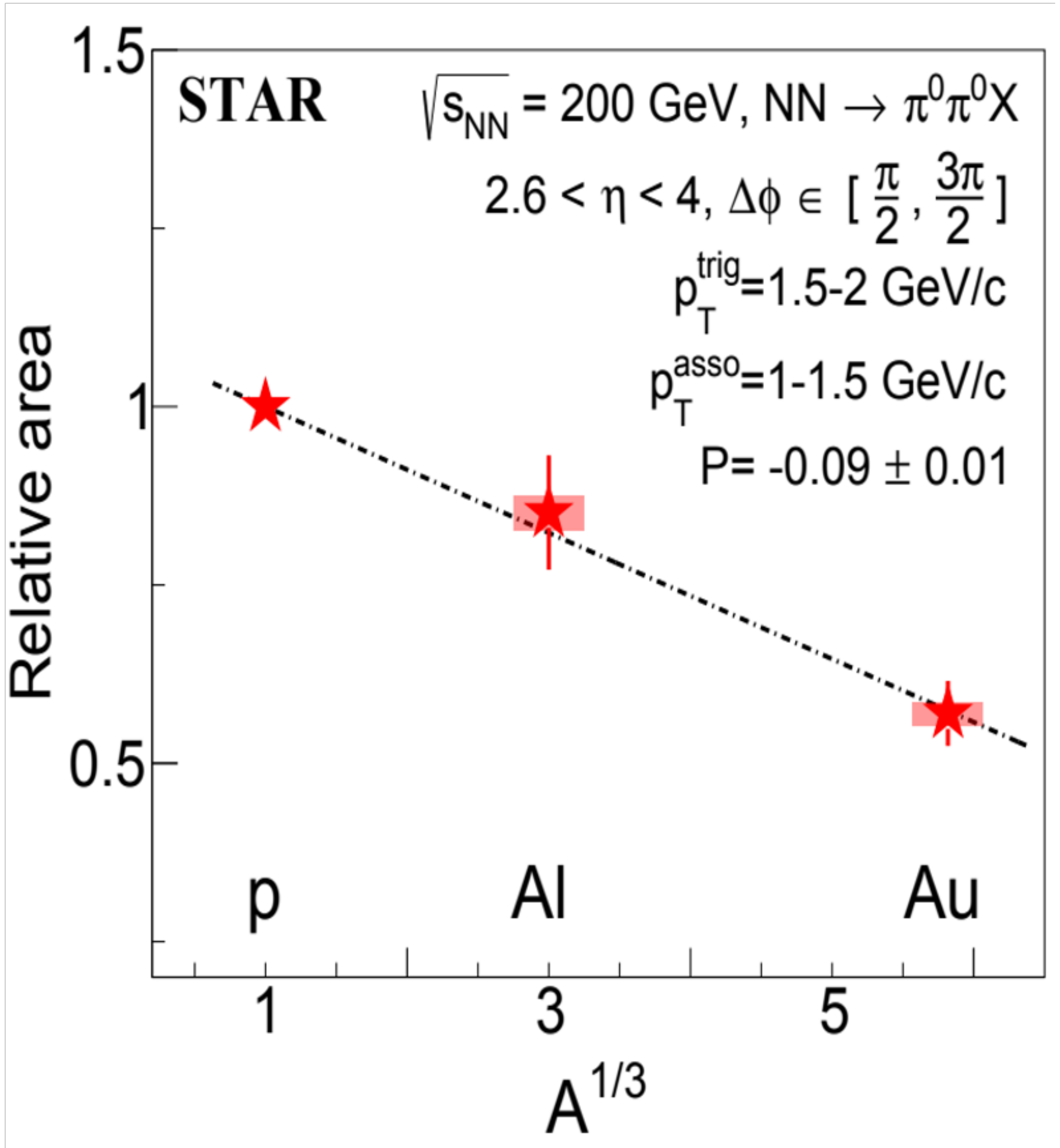


Albacete, Marquet, PRL105 (2010) 162301

Back to back hadron production in pA collisions: forward rapidity

STAR collaboration(2021)
arXiv:2111.10396





Toward precision CGC: inclusive DIS

NLO BK/JIMWLK evolution equations

Kovner, Lublinsky, Mulian (2013)

Balitsky, Chirilli (2007)

NLO corrections to structure functions

Beuf, Lappi, Paatelainen (2022)

Beuf (2017)

NLO corrections to SIDIS

Bergabo, JJM (2023, 2024)

Caucal, Ferrand, Salazar (2024)

NLO corrections to dihadron/dijets (+)

Bergabo, JJM (2022, 2023)

Iancu, Mulian (2023)

Caucal, Salazar, Schenke, Stebel, Venugopalan (2023), Caucal, Salazar, Schenke, Venugopalan (2022)

Taels, Altinoluk, Beuf, Marquet (2022), Taels (2023)

Caucal, Salazar, Venugopalan (2021)

Ayala, Hentschinski, JJM, Tejeda-Yeomans (2016,2017),…………

Toward precision CGC: exclusive/diffractive DIS

NLO corrections to diffractive structure functions

Beuf, Hanninen, Lappi, Mulian, Mantyssari (2022)

………………………

NLO corrections to diffractive dihadron/dijets (+)

Boussarie, Grabovsky, Szymanowski, Wallon (2016)

Iancu, Mueller, Triantafyllopoulos (2021, 2022)

Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)

………………………

NLO corrections to exclusive light/heavy vector meson production (+)

Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)

Mantyssari, Penttala (2021, 2022)

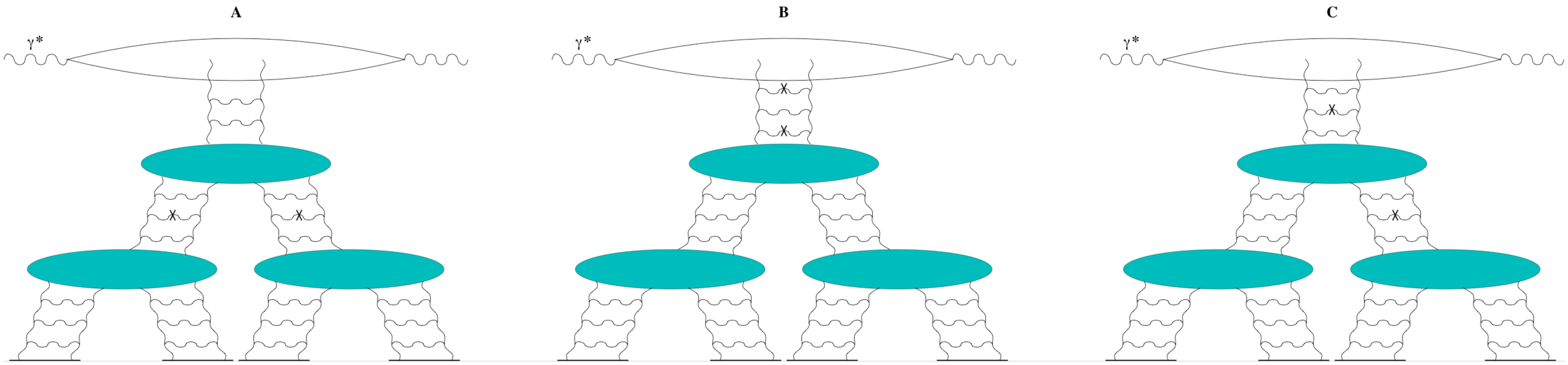
………………………

Inclusive dihadron production in DIS at small x:

<u>central vs forward rapidity</u>

Inclusive dihadron production in midrapidity: LO

JJM, Yu. Kovchegov
PRD70 (2004) 114017



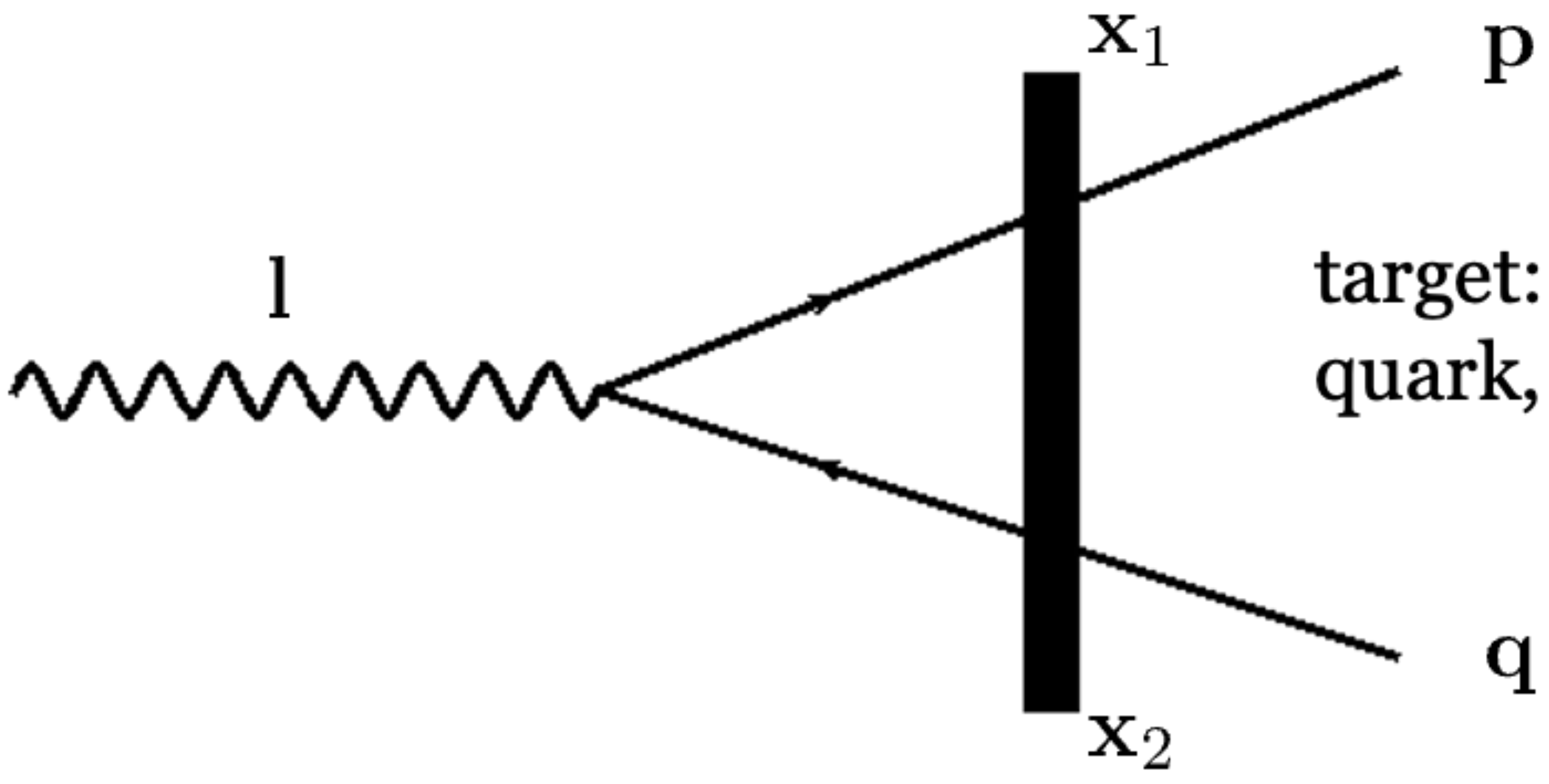
need a very large rapidity window

target is treated as a classical color field $\mathbf{A}_a^\mu = \delta^{\mu -}\, n^\mu\, S_a(x^+, \mathbf{x})$

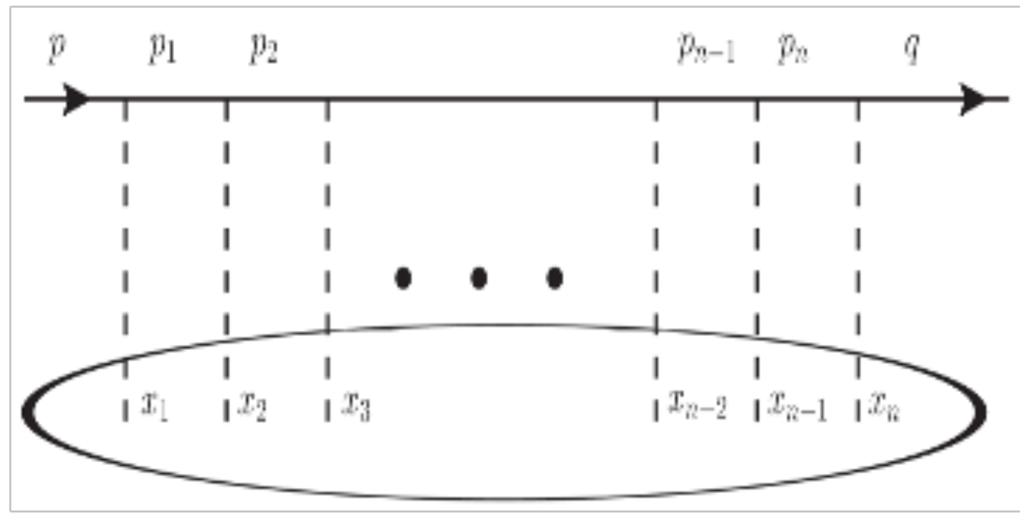
scatterings of gluons on the target encoded in Wilson lines $\mathbf{U}\ (\mathbf{x}_1), U^\dagger(\mathbf{x}_2)$

leading log evolution included

Inclusive dihadron production in <u>forward rapidity</u>: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A \to q\bar{q} X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7} \delta(1 - z_1 - z_2)$$

$$\int d^8x_\perp e^{ip\cdot(x_1' - x_1)} e^{iq\cdot(x_2' - x_2)} \left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{ 4 z_1 z_2 K_0(|x_{12}|Q_1) K_0(|x_{1'2'}|Q_1) +$$

$$(z_1^2 + z_2^2) \frac{x_{12} \cdot x_{1'2'}}{|x_{12}||x_{1'2'}|} K_1(|x_{12}|Q_1) K_1(|x_{1'2'}|Q_1) \Big\}$$

with

dipole $\mathbf{S}_{12} \equiv \frac{1}{N_c} Tr\, V(x_1)\, V^\dagger(x_2)$

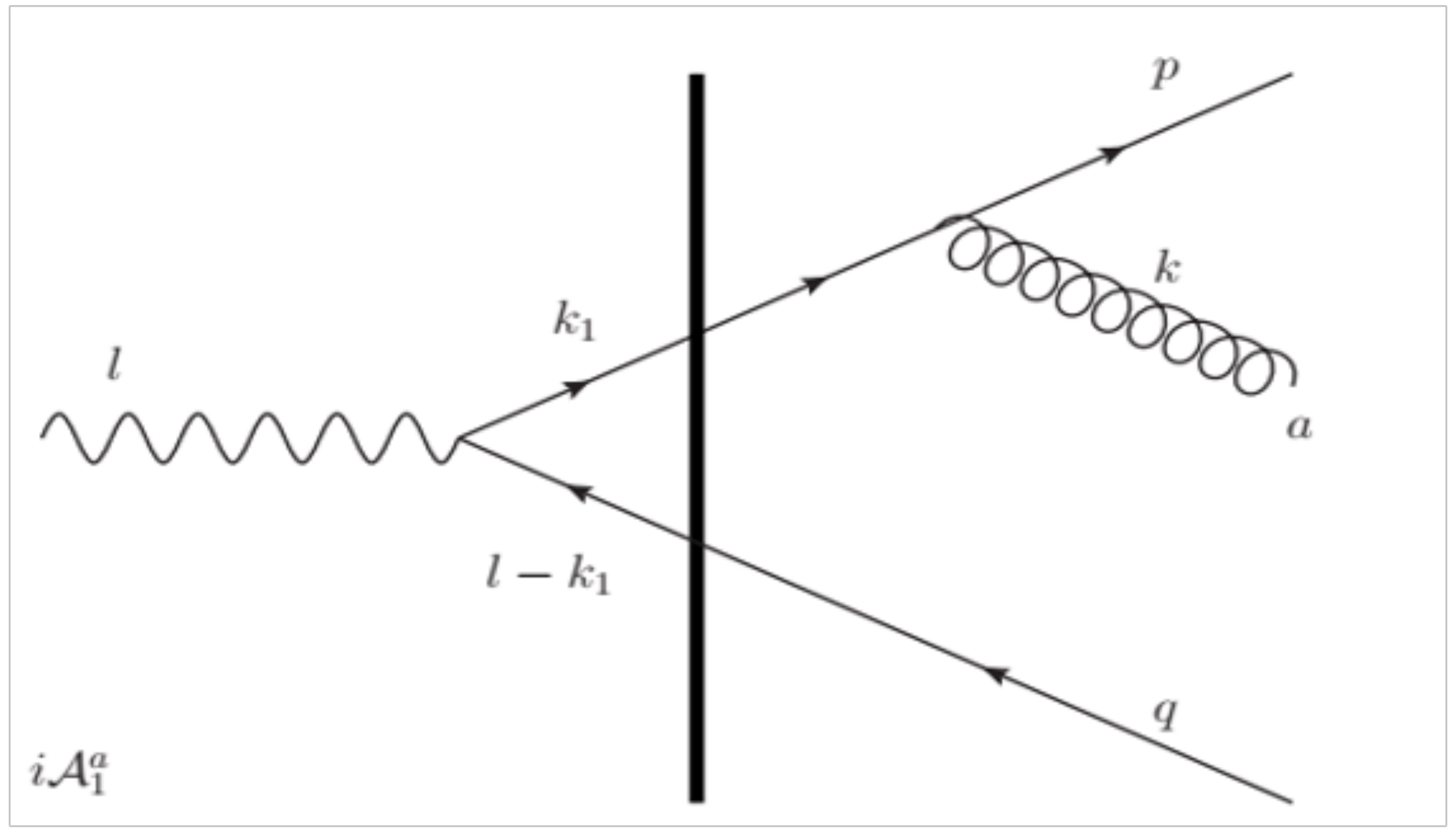
$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

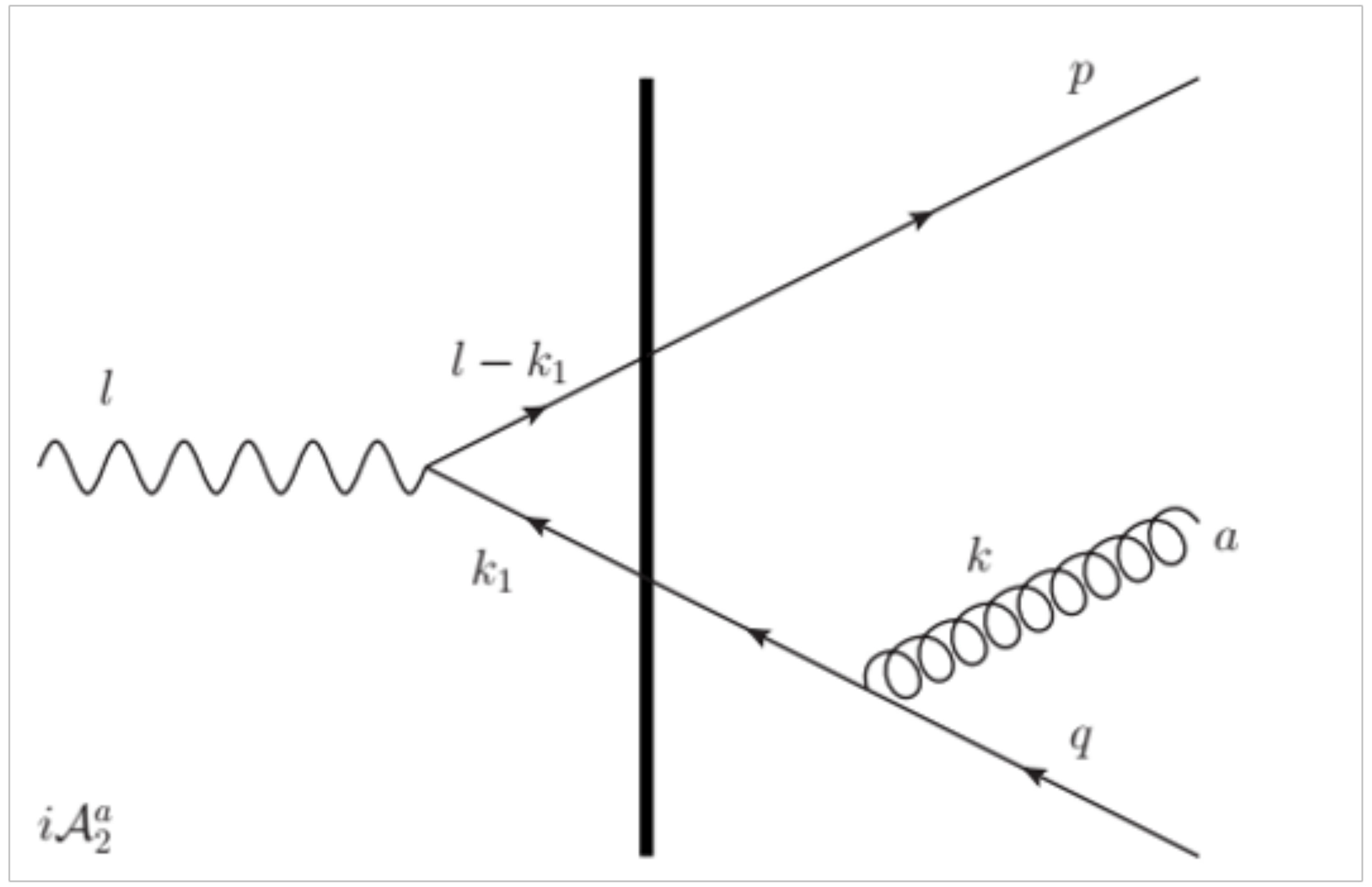
quadrupole

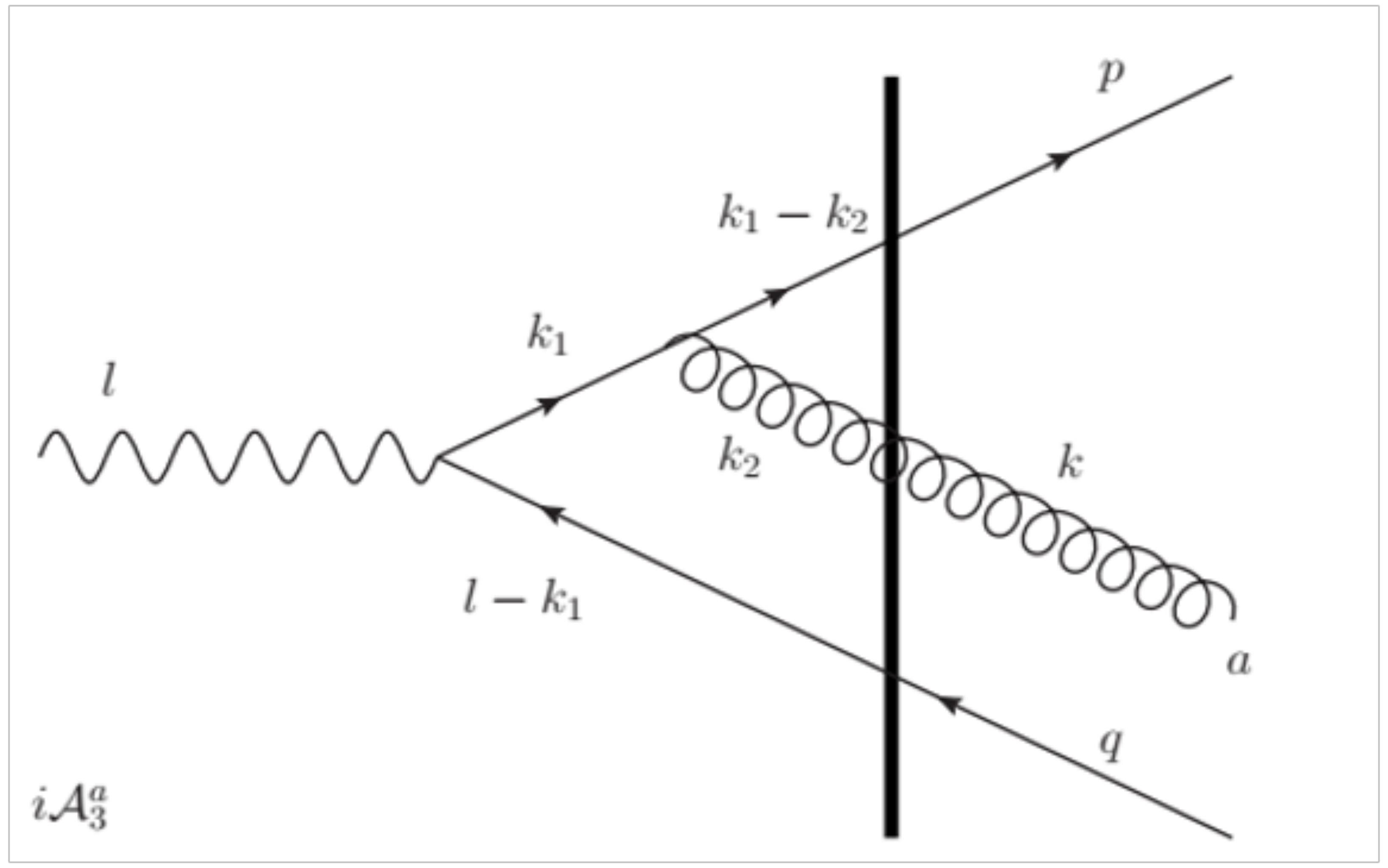
$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'})$$

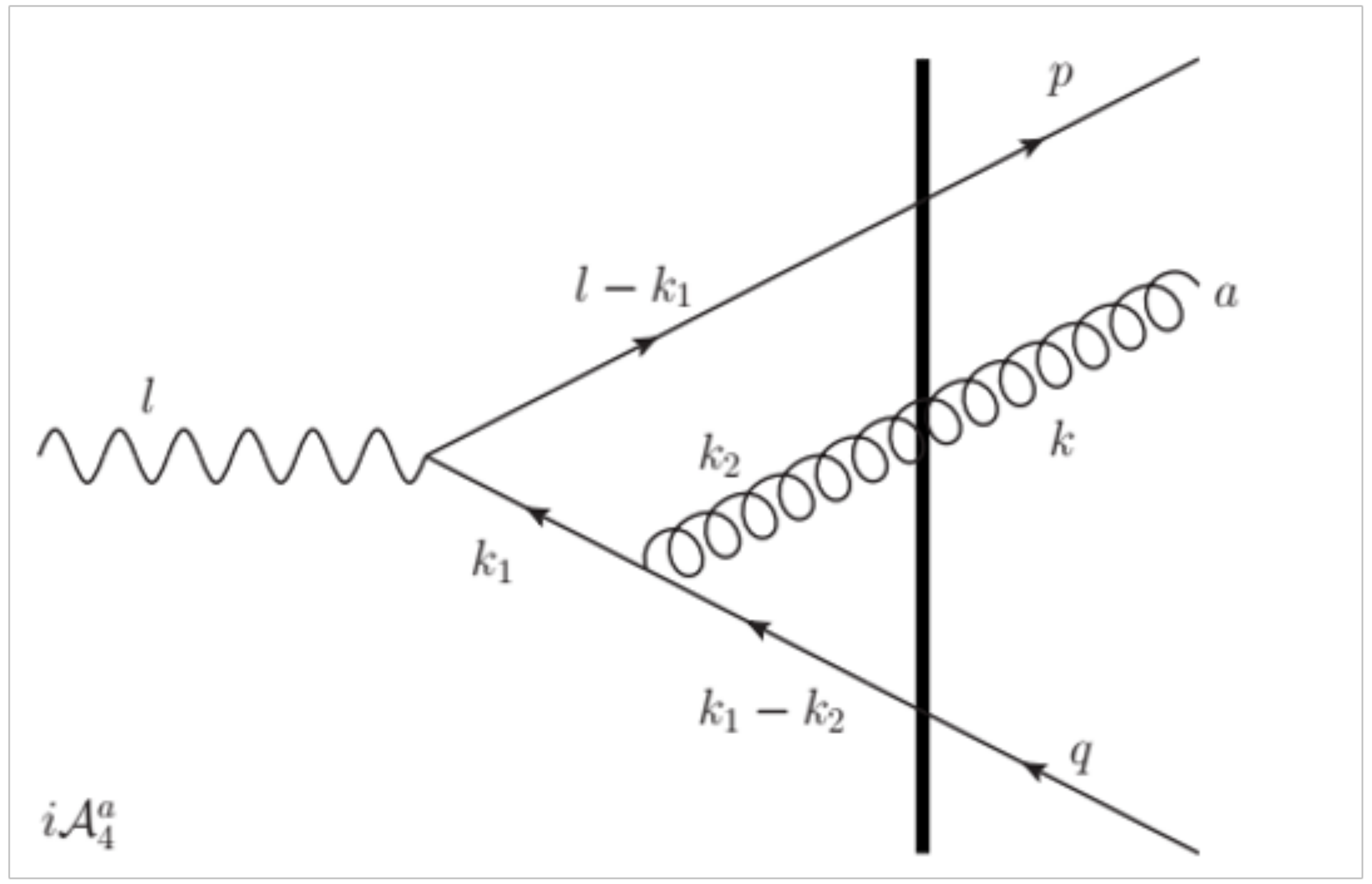
Only dipoles and quadrupoles contribute: DMXY, PRD 83 (2011) 105005

One loop corrections - real diagrams



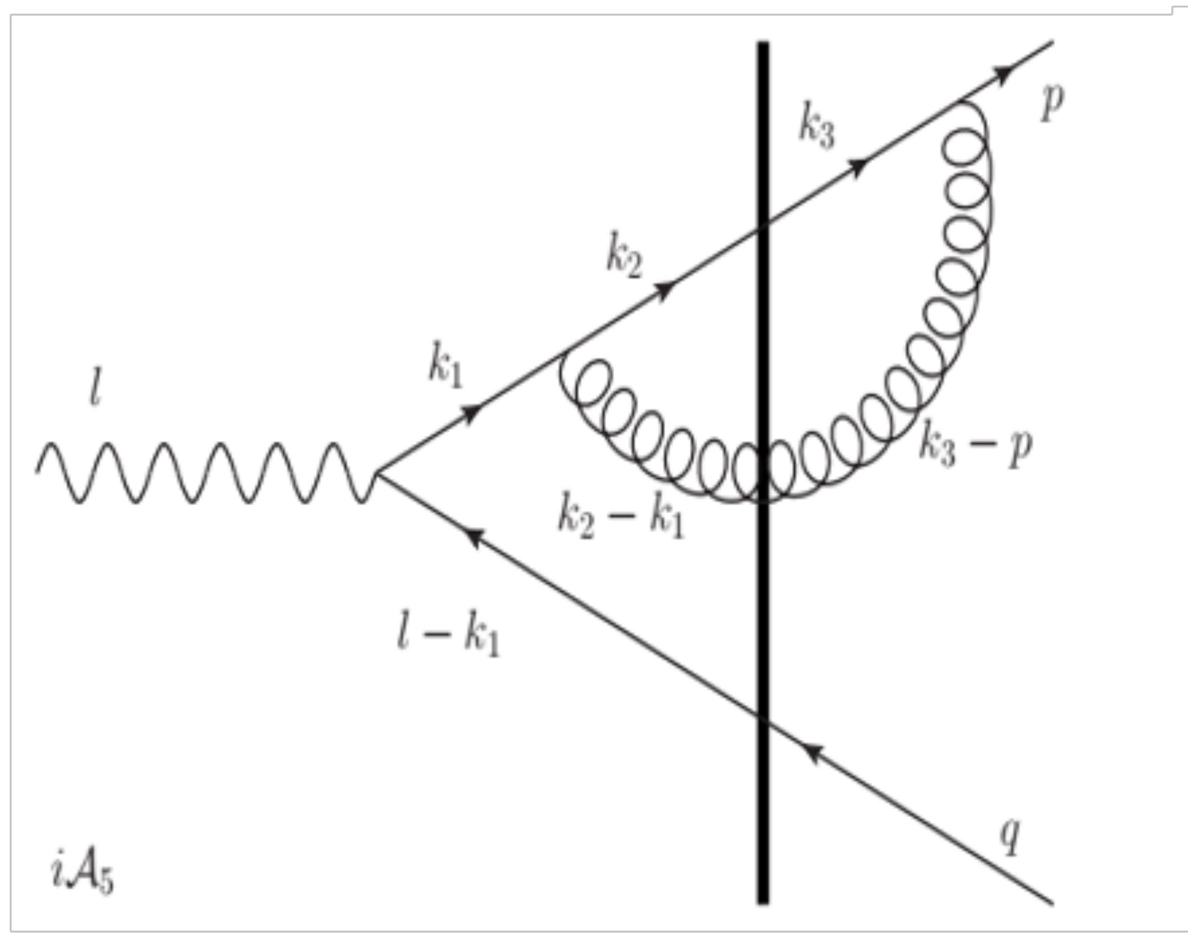


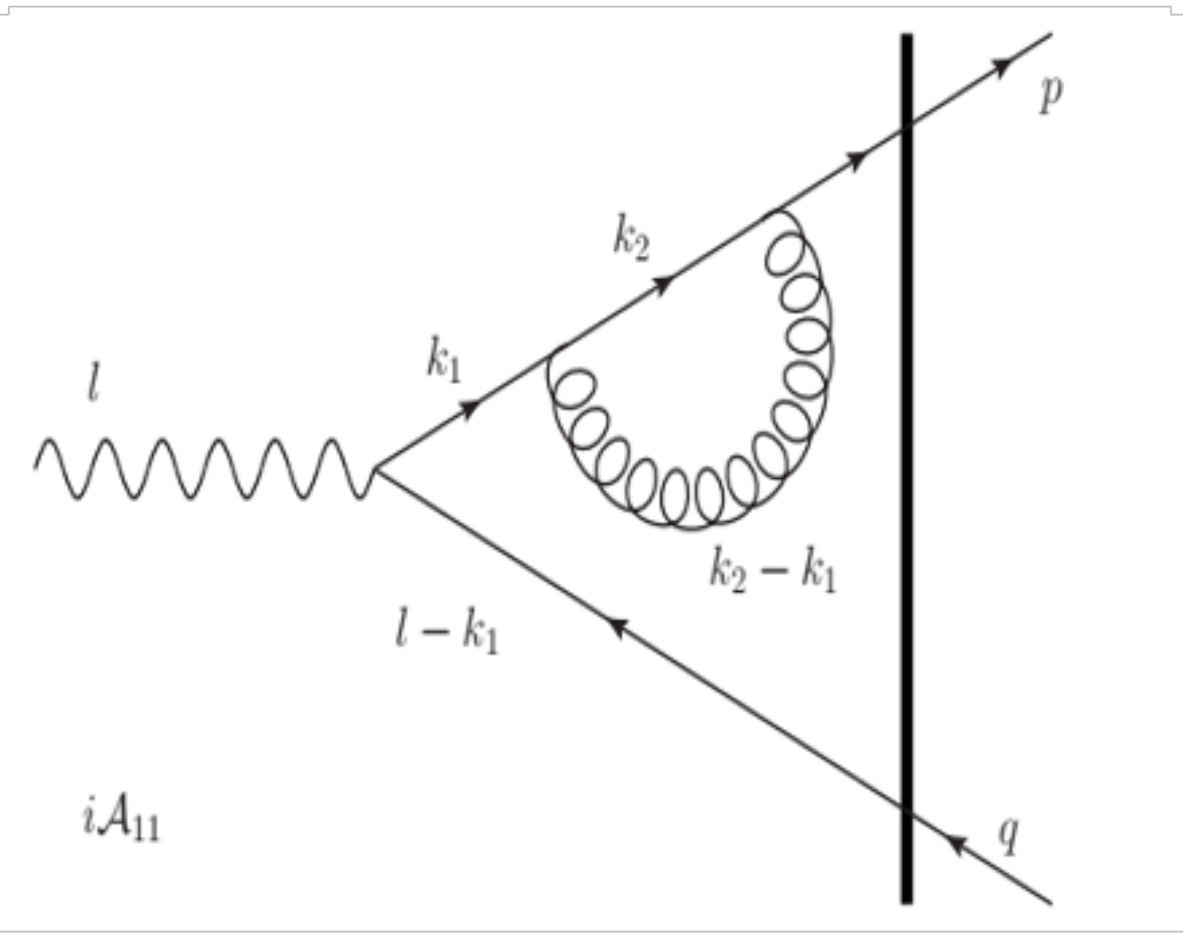


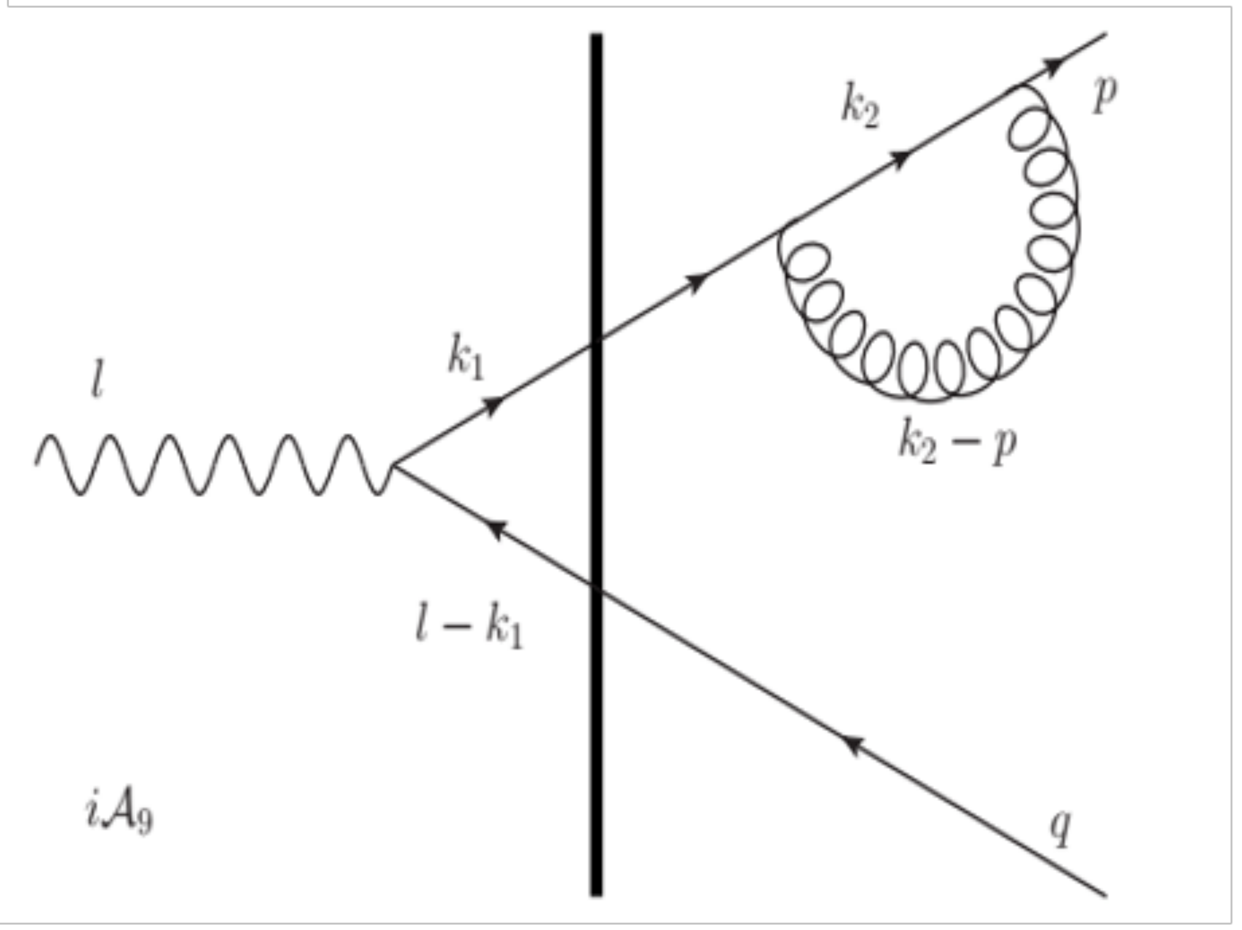


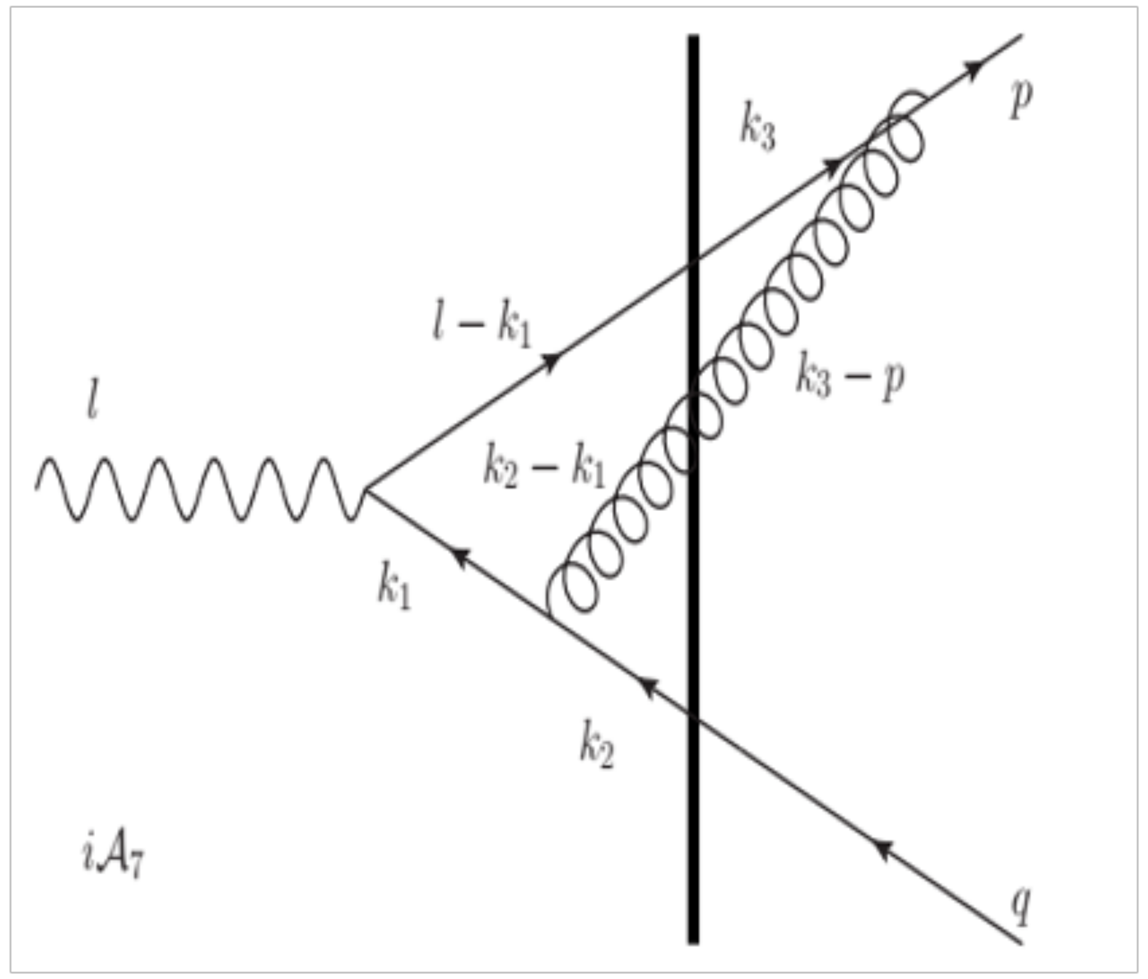
3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

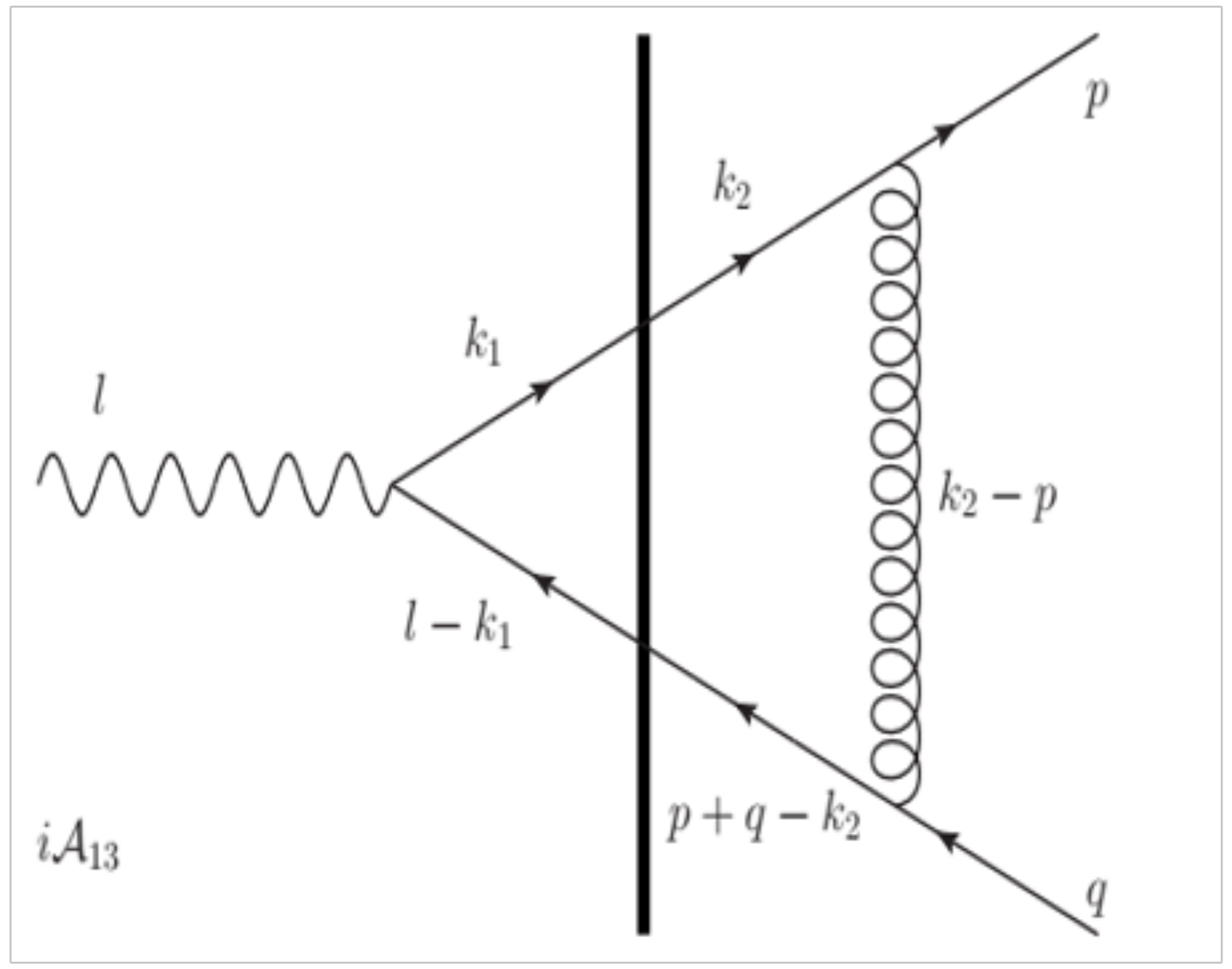
One loop corrections – virtual diagrams

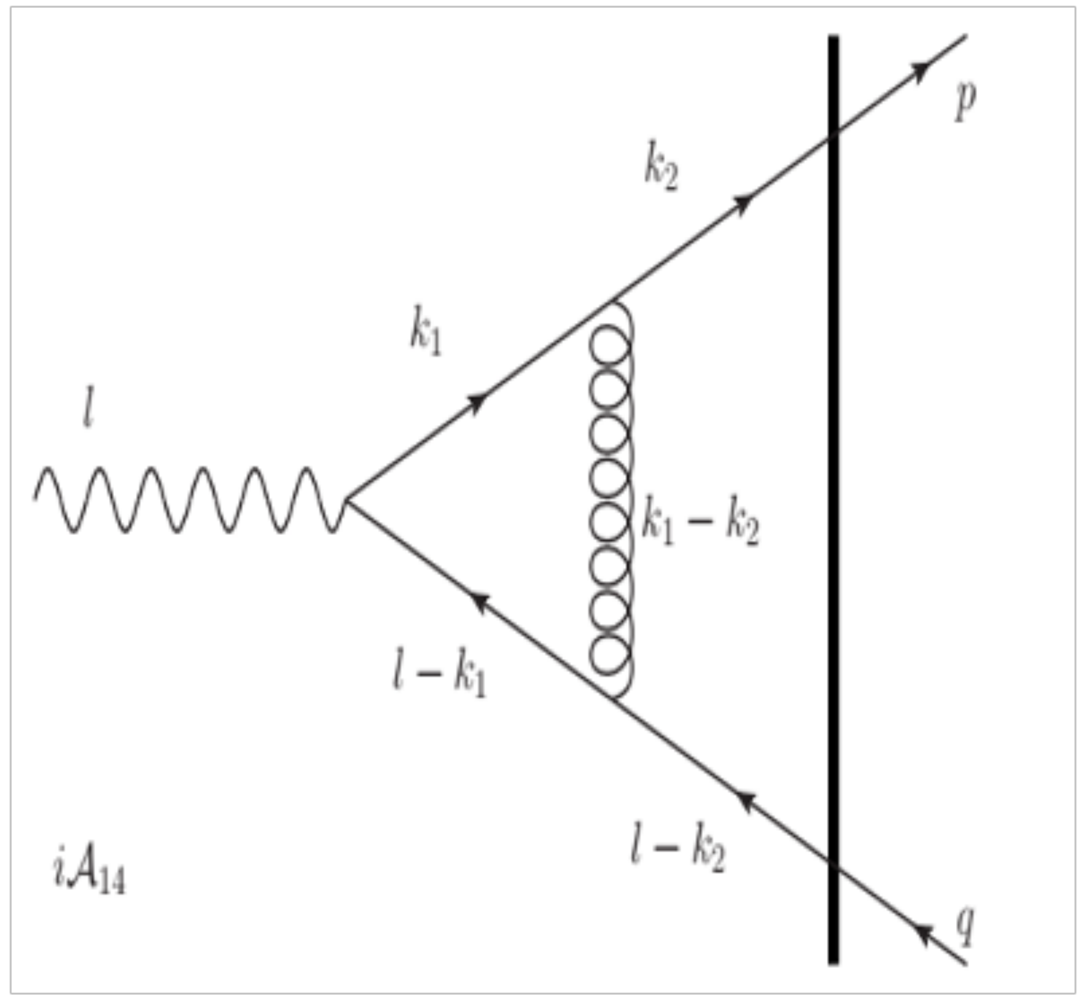












<u>F. Bergabo</u> and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

Spinor helicity formalism: helicity amplitudes

Numerator	$\lambda_\gamma; \lambda_q, \lambda_g$	$N_i^{\lambda_\gamma;\lambda_q,\lambda_g}$
N_1	$L; +, +$	$-Q(z_1 z_2)^{3/2}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}$
	$L; +, -$	$-Q(z_2)^{3/2}\sqrt{z_1}(1-z_2)^2\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}$
	$+; +, +$	$-(z_1)^{3/2}\sqrt{z_2}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}(\mathbf{k}_1\cdot\epsilon)$
	$+; +, -$	$-\sqrt{z_1 z_2}(1-z_2)^2\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon^*]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}(\mathbf{k}_1\cdot\epsilon)$
	$+; -, +$	$(z_2)^{3/2}\sqrt{z_1}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}(\mathbf{k}_1\cdot\epsilon)$
	$+, -, -$	$(z_1 z_2)^{3/2}\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon^*]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}(\mathbf{k}_1\cdot\epsilon)$
N_2	$L; +, +$	$Q(z_1)^{3/2}\sqrt{z_2}(1-z_1)^2\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}$
	$L; +, -$	$Q(z_1 z_2)^{3/2}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^*]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}$
	$+; +, +$	$-(z_1)^{3/2}\sqrt{z_2}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}(\mathbf{k}_1\cdot\epsilon)$
	$+; +, -$	$-(z_1 z_2)^{3/2}\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^*]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}(\mathbf{k}_1\cdot\epsilon)$
	$+; -, +$	$(z_2)^{3/2}\sqrt{z_1}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}(\mathbf{k}_1\cdot\epsilon)$
	$+, -, -$	$\sqrt{z_1 z_2}(1-z_1)^2\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^*]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}(\mathbf{k}_1\cdot\epsilon)$
N_3	$L; +, +$	$Q(z_1 z_2)^{3/2}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)$
	$L; +, -$	$Q(z_2)^{3/2}\sqrt{z_1}(1-z_2)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{1-z_2}\right)$
	$+; +, +$	$(z_1)^{3/2}\sqrt{z_2}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+; +, -$	$\sqrt{z_1 z_2}(1-z_2)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+; -, +$	$-(z_2)^{3/2}\sqrt{z_1}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+, -, -$	$-(z_1 z_2)^{3/2}\left[\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{(1-z_2)}\right)\mathbf{k}_1\cdot\epsilon+\frac{\mathbf{k}_1^2+z_2(1-z_2)Q^2}{2z_2(1-z_2)}\right]$
N_4	$L; +, +$	$-Q(z_1)^{3/2}\sqrt{z_2}(1-z_1)^2\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)$
	$L; +, -$	$-Q(z_1 z_2)^{3/2}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{1-z_1}\right)$
	$+; +, +$	$(z_1)^{3/2}\sqrt{z_2}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$
	$+; +, -$	$(z_1 z_2)^{3/2}\left[\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{(1-z_1)}\right)\mathbf{k}_1\cdot\epsilon+\frac{\mathbf{k}_1^2+z_1(1-z_1)Q^2}{2z_1(1-z_1)}\right]$
	$+; -, +$	$-(z_2)^{3/2}\sqrt{z_1}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$
	$+, -, -$	$-\sqrt{z_1 z_2}(1-z_1)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$

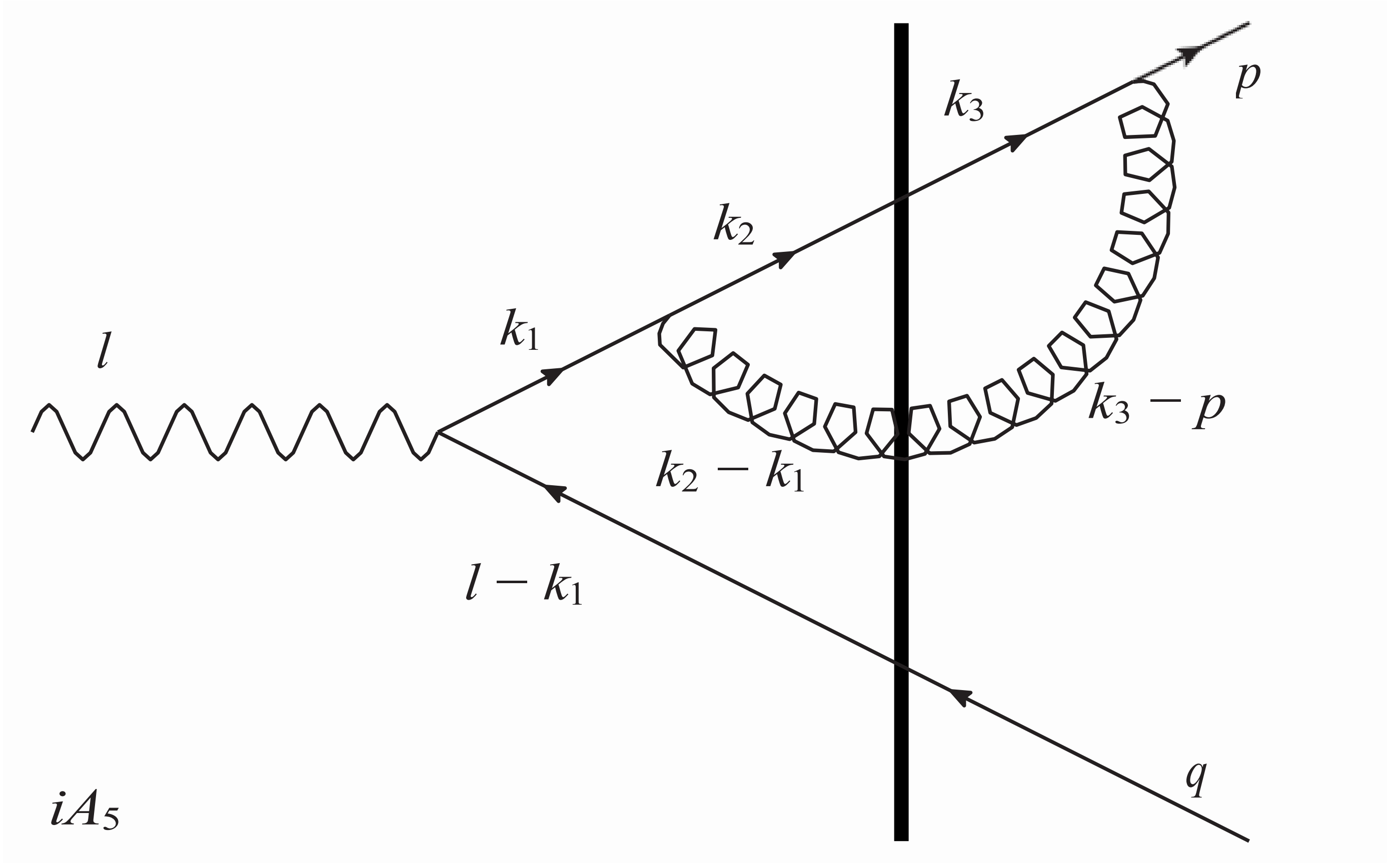
divergences

- ***Ultraviolet:***

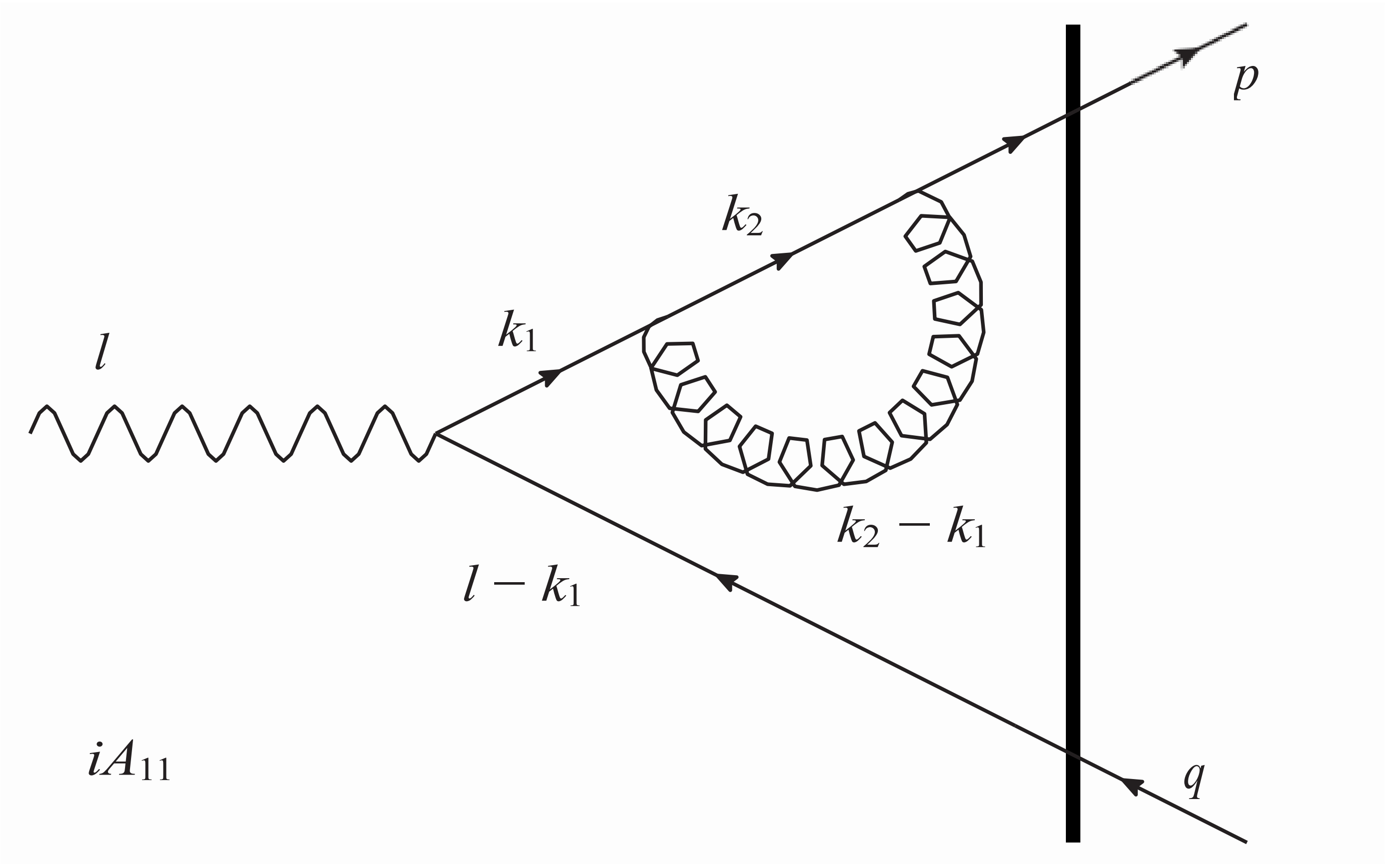
Real corrections are UV finite

UV divergences cancel among virtual corrections $\quad \mathbf{k} \to \infty \quad$ **or** $\quad \mathbf{x_3} \to \mathbf{x_i}$

$$\left(d\sigma_5 + d\sigma_{11}\right)_{UV} = 0$$
$$\left(d\sigma_6 + d\sigma_{12}\right)_{UV} = 0$$
$$\left(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\right)_{UV} = 0$$



$+$



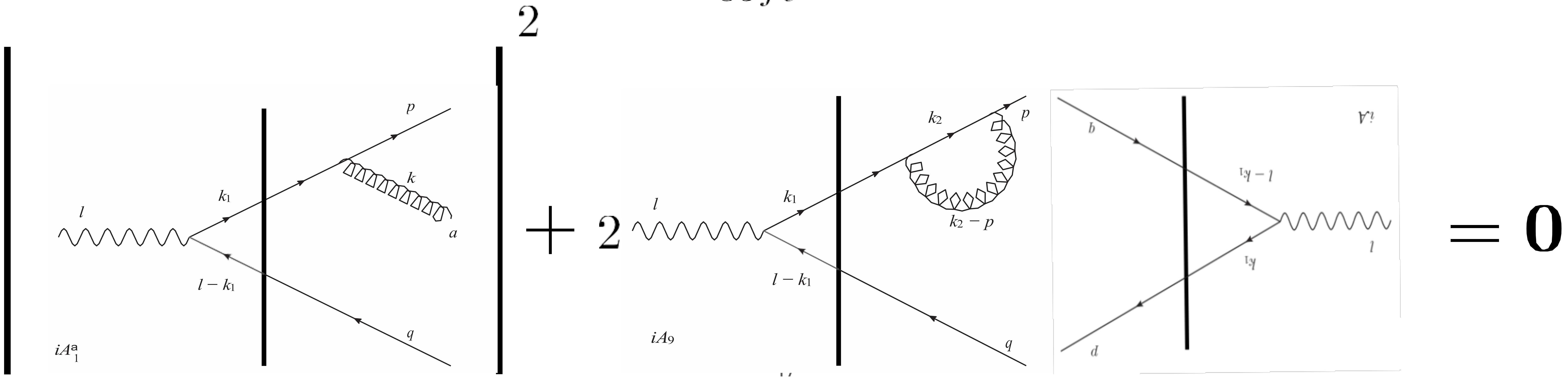
$= 0$

divergences

- ***Soft:*** $\mathbf{k}^{\mu} \to \mathbf{0} \;\; (\mathbf{x_3} \to \infty \;\; \textbf{AND} \;\; \mathbf{z} \to \mathbf{0})$

Soft divergences cancel between real and virtual corrections

$$(d\sigma_{1-1} + d\sigma_9)_{soft} = 0,$$
$$\left(d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)}\right)_{soft} = 0$$
$$(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4})_{soft} = 0$$
$$(d\sigma_{1-3} + d\sigma_{1-4})_{soft} = 0$$
$$(d\sigma_{2-3} + d\sigma_{2-4})_{soft} = 0$$
$$(d\sigma_5 + d\sigma_7)_{soft} = 0$$
$$\left(d\sigma_{11} + d\sigma_{14}^{(1)}\right)_{soft} = 0$$



divergences

- ***Rapidity:*** $\mathbf{z \to 0}$ ***, but finite*** $\boldsymbol{k_t}$

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma^L_{\rm NLO}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}}\, \delta(1 - z_1 - z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$

$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Big\{ \left(\tilde\Delta_{12} + \tilde\Delta_{22'} - \tilde\Delta_{12'}\right) S_{132'1'} S_{23} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{22'} - \tilde\Delta_{21'}\right) S_{1'321} S_{2'3}$$

$$+ \left(\tilde\Delta_{12} + \tilde\Delta_{11'} - \tilde\Delta_{21'}\right) S_{322'1'} S_{13} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{11'} - \tilde\Delta_{12'}\right) S_{32'21} S_{1'3}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} + \tilde\Delta_{12} + \tilde\Delta_{1'2'}\right) S_{122'1'} - \left(\tilde\Delta_{12} + \tilde\Delta_{1'2'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{12} S_{1'2'}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{11'} S_{22'} - 2\tilde\Delta_{12} \left(S_{13} S_{23} - S_{12}\right) - 2\tilde\Delta_{1'2'} \left(S_{1'3} S_{2'3} - S_{1'2'}\right) \Big\}$$

JIMWLK evolution of quadrupoles

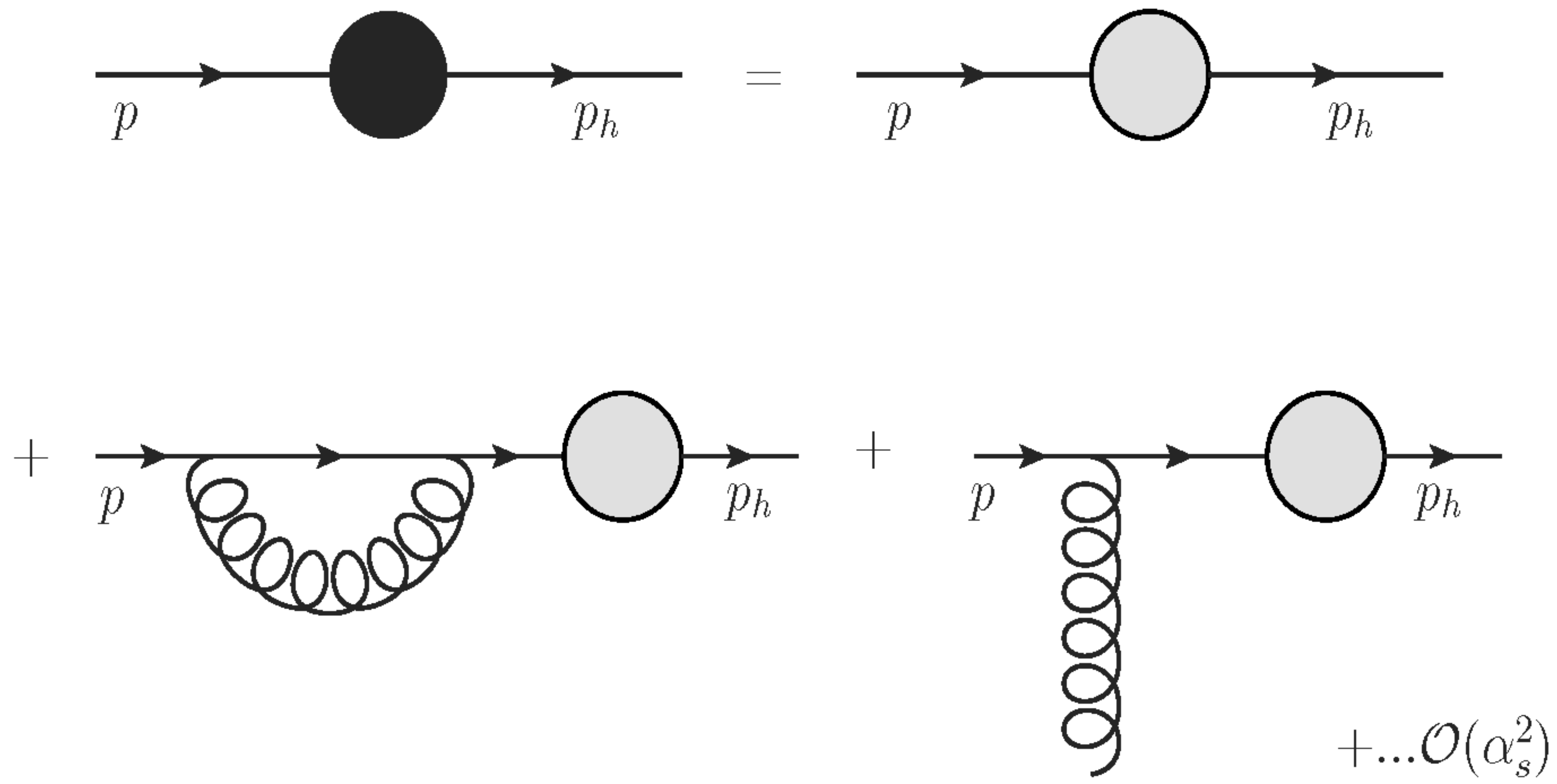
JIMWLK evolution of dipoles

$$\tilde{\boldsymbol{\Delta}}_{\mathbf{12}} \equiv \frac{(\mathbf{x_1} - \mathbf{x_2})^2}{(\mathbf{x_1} - \mathbf{x_3})^2 (\mathbf{x_2} - \mathbf{x_3})^2}$$

divergences

- ***Collinear:*** $$\frac{1}{(p+k)^2} = \frac{1}{|\vec{p}||\vec{k}|(1-cos\,\theta)} \longrightarrow \infty \quad as \quad \theta \to 0$$

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



collinear divergences

real corrections

$$\frac{d\sigma_{LO+1-1}^{\gamma^* A\to h_1 h_2 X}}{d^2\mathbf{p}_h\, d^2\mathbf{q}_h\, dy_1\, dy_2} = \int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2 Q^2 N_c (z_1 z_2)^3}{(2\pi)^7 (z_{h_1} z_{h_2})^2} H\left(\mathbf{p},\mathbf{q},z_2\right) D^0_{h_1/q}(z_{h_1}) D^0_{h_2/\bar{q}}(z_{h_2})$$

$$\int \frac{d\xi_1}{\xi_1^3} \delta(1-z_2-z_1/\xi_1) \left[\delta(1-\xi_1) + 2\alpha_s\, P_{qq}(\xi_1) \int d^2\mathbf{k} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}}{(\xi_1\mathbf{k}-(1-\xi_1)\mathbf{p})^2}\right]$$

with $$P_{qq}(\xi_1) = C_F \frac{(1+\xi_1^2)}{(1-\xi_1)}$$

virtual corrections

$$\frac{d\sigma_9^{\gamma^* A\to h_1 h_2 X}}{d^2\mathbf{p}_h\, d^2\mathbf{q}_h\, dy_1\, dy_2} = -\int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2 Q^2 (z_1 z_2)^3 N_c}{(2\pi)^7 (z_{h_1} z_{h_2})^2} H(\mathbf{p},\mathbf{q},z_2) D^0_{h_1/q}(z_{h_1}) D^0_{h_2/\bar{q}}(z_{h_2})$$

$$\times\, \alpha_s \int_0^1 d\xi\, P_{qq}(\xi) \int d^2\mathbf{k} \frac{1}{(\mathbf{k}-(1-\xi)\mathbf{p})^2} \delta(1-z_1-z_2)$$

these are combined into DGLAP evolution of fragmentation functions

$$D_{h_1/q}(z_{h1},\mu^2) = \int_{z_{h1}}^1 \frac{d\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h1}}{\xi}\right) \left[\delta(1-\xi) + \frac{\alpha_s}{2\pi} P_{qq}(\xi)\, \log\left(\frac{\mu^2}{\Lambda^2}\right)\right]$$

Divergences

- Ultraviolet

  real corrections are UV finite

  UV divergences cancel among virtual diagrams

- Soft

  soft divergences cancel Soft

  soft divergences cancel between real and virtual diagrams real and virtual diagrams

- Collinear

  collinear divergences are absorbed into fragmentation functions

- Rapidity

  Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\sigma^{\gamma^* A \to h_1 h_2 X} = \sigma_{LO} \otimes \mathrm{JIMWLK} + \sigma_{LO} \otimes D_{h/q}(z_h, \mu^2) \otimes D^{(0)}_{h/q}(z_h) + \sigma^{\mathrm{finite}}_{NLO}$$

Back to back limit: deep connections to physics of TMDs, Sudakov effect,....

Dihadron/dijets kinematics at EIC

Larger kinematic phase space than dihadrons

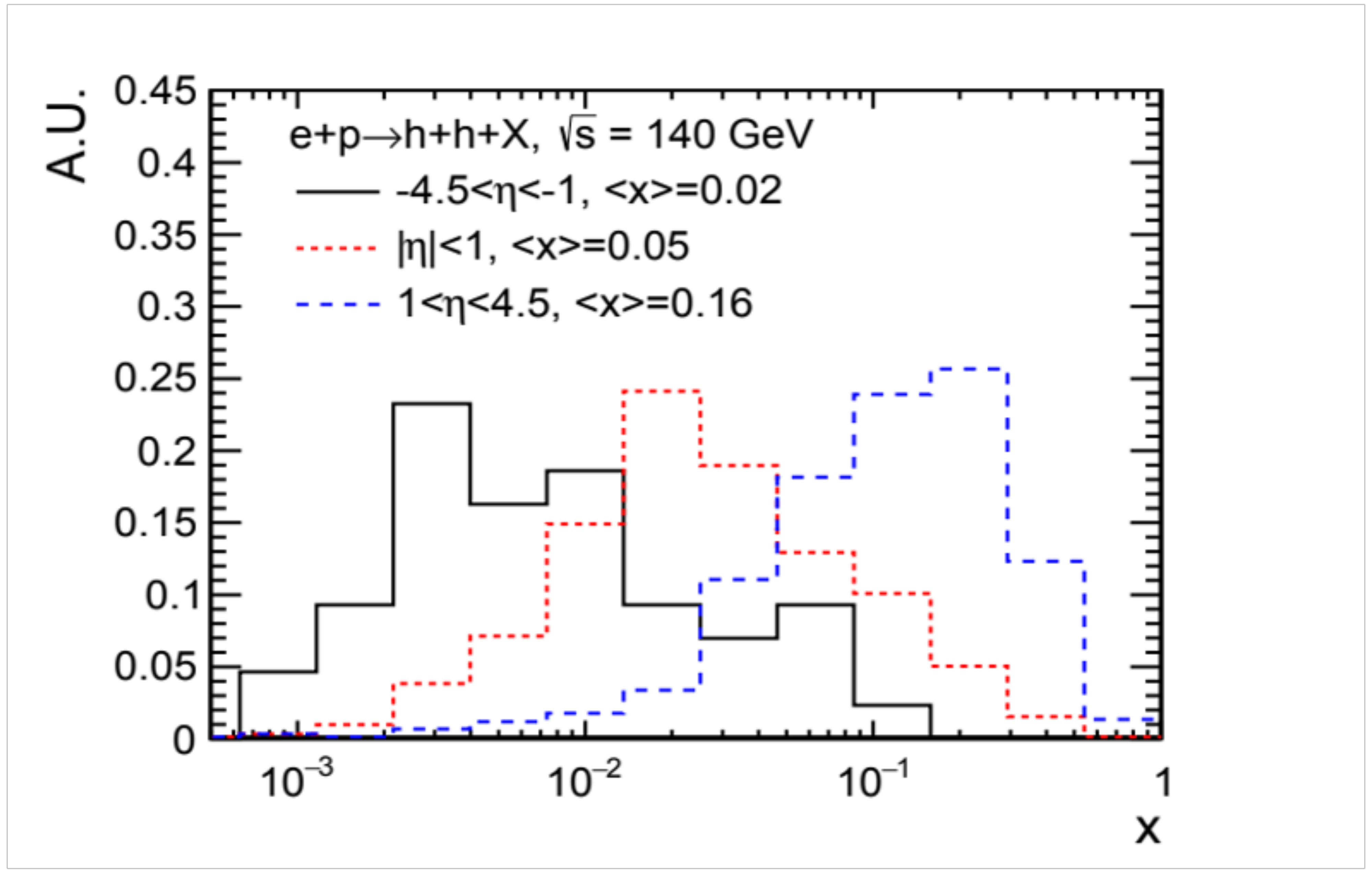
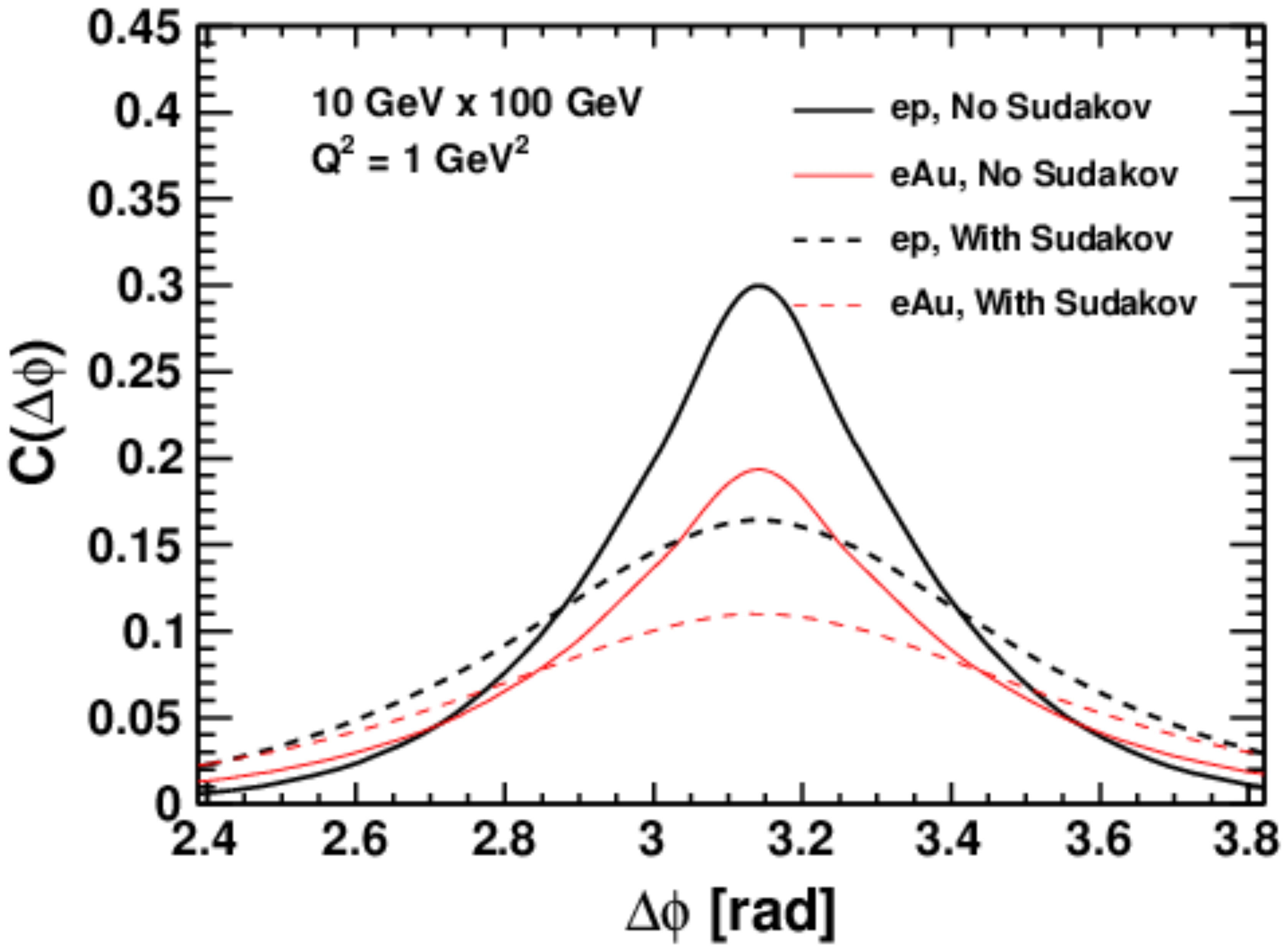


Fig courtesy of Xiaoxuan Chu

Sudakov can be avoided(?)



Zheng, Aschenauer,Lee,Xiao, arXiv:1403.2413

SIDIS at small x: NLO corrections

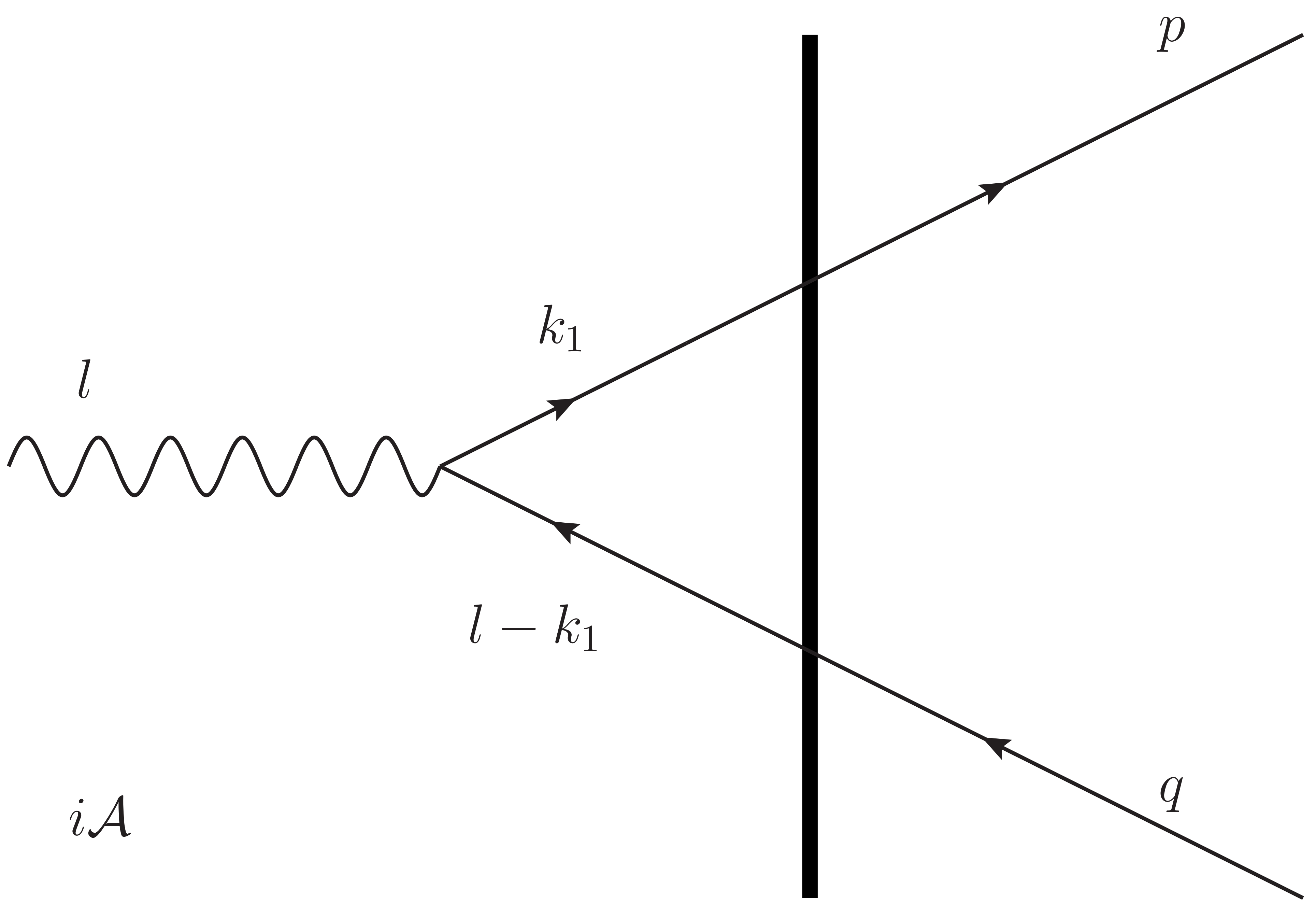
F. Bergabo, JJM, JHEP 01 (2023) 095, and arXiv:2401.06259

Caucal, Ferrand, Salazar, arXiv:2401.01934

Forward rapidity

LO:

integrate over final state antiquark

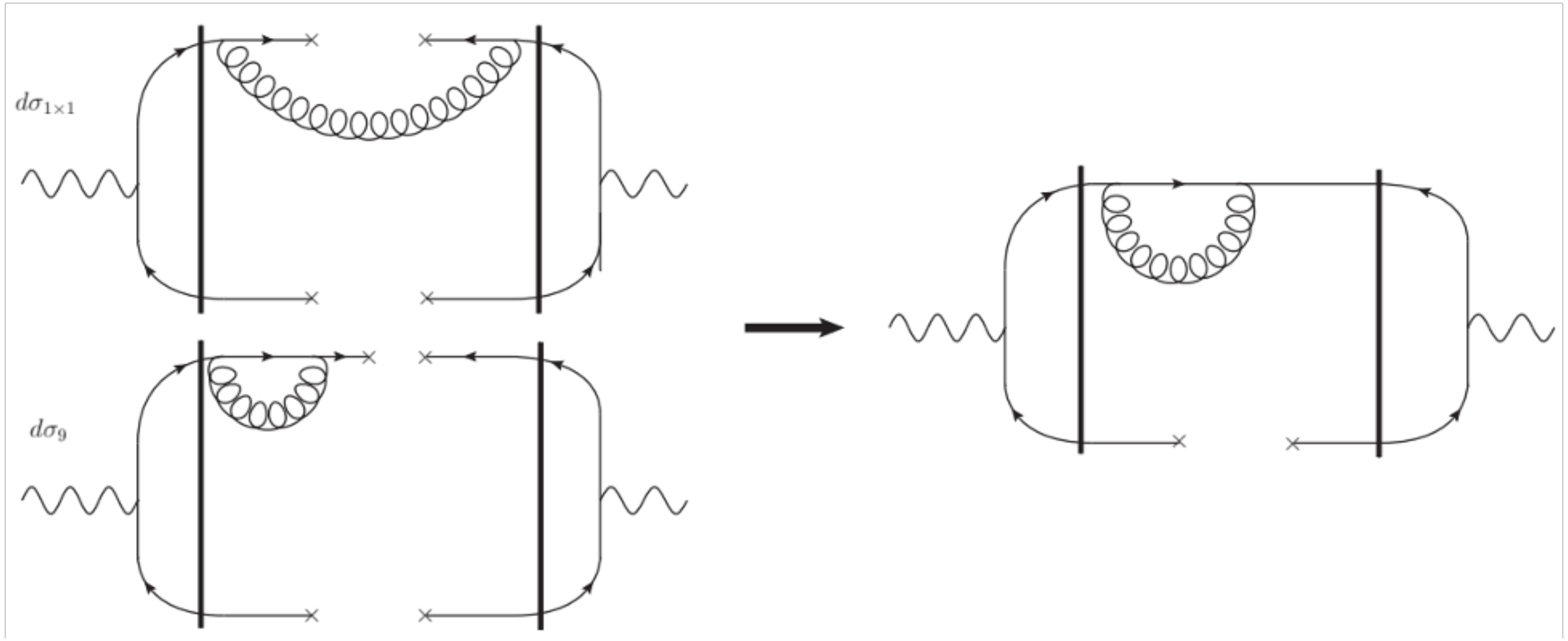


$$\frac{\mathrm{d}\sigma^{\gamma^* p/A \to q(\mathbf{p}, y_1) X}}{\mathrm{d}^2\mathbf{p}\, \mathrm{d}y_1} = \frac{e^2 Q^2 N_c}{(2\pi)^5} \int \mathrm{d}z_2\, \delta(1 - z_1 - z_2)\, (z_1^2\, z_2) \int \mathrm{d}^6\mathbf{x}\, [S_{11'} - S_{12} - S_{1'2} + 1]\, e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$\left\{ 4 z_1 z_2 K_0(|\mathbf{x}_{12}| Q_1) K_0(|\mathbf{x}_{1'2}| Q_1) + (z_1^2 + z_2^2)\, \frac{\mathbf{x}_{12} \cdot \mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\, K_1(|\mathbf{x}_{12}| Q_1) K_1(|\mathbf{x}_{1'2}| Q_1) \right\}$$

Single inclusive hadron production in DIS at small x: NLO

start with NLO corrections to dihadron production and integrate out quark

cancellations among diagrams



Single inclusive hadron production in DIS at small x: NLO

all terms with quadrupoles cancel; only dipoles contribute to the cross section

cancellations of divergences as before

$$\sigma^{\gamma^* A \to hX} = \sigma_{LO} \otimes \mathrm{JIMWLK} + \sigma_{LO} \otimes D_{h/\bar{q}}(z_h, \mu^2) + \sigma_{NLO}^{\mathrm{finite}}$$

phenomenology: need to consider hadronization of any of the 3 partons

consider hadronization of the gluon

Gluon production

integrate out both quark and antiquark

finite N_c corrections included

$$\frac{\mathrm{d}\sigma^L_{1\times 1}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} = 4\frac{e^2Q^2g^2N_c}{(2\pi)^6}\,C_F\int \mathrm{d}z_1\mathrm{d}z_2\,\delta(1-z_1-z_2-z)\,z_2^2(1-z_2)^2\left[z_1^2+(1-z_2)^2\right]$$
$$\int \mathrm{d}^6\,\mathbf{x}e^{i\mathbf{k}\cdot\mathbf{x}_{1'1}}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2}|Q_2)\,\left[S_{11'}-S_{12}-S_{1'2}+1\right]\int\frac{\mathrm{d}^2\mathbf{p}}{(2\pi)^2}\frac{e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}}{\left[z\mathbf{p}-z_1\mathbf{k}\right]^2}$$

$$\frac{\mathrm{d}\sigma^L_{2\times 2}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} = 4\frac{e^2Q^2g^2N_c}{(2\pi)^6}\,C_F\int \mathrm{d}z_1\mathrm{d}z_2\,\delta(1-z_1-z_2-z)\,z_1^2(1-z_1)^2\left[z_2^2+(1-z_1)^2\right]$$
$$\int \mathrm{d}^6\mathbf{x}\,e^{i\mathbf{k}\cdot\mathbf{x}_{22'}}\,K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{12'}|Q_1)\,\left[S_{22'}-S_{12}-S_{12'}+1\right]\int\frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{e^{i\mathbf{q}\cdot\mathbf{x}_{22'}}}{\left[z\mathbf{q}-z_2\mathbf{k}\right]^2}$$

$$\frac{\mathrm{d}\sigma^L_{1\times 2}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} = 4\frac{e^2Q^2g^2N_c}{(2\pi)^6}\int \mathrm{d}z_1\mathrm{d}z_2\,\delta(1-z_1-z_2-z)\,\left[\frac{z_1(1-z_1)z_2(1-z_2)}{z}\right]\left[\frac{z_1(1-z_1)+z_2(1-z_2)}{z}\right]$$
$$\frac{1}{(2\pi)^2}\int \mathrm{d}^8\mathbf{x}\,\frac{\mathbf{x}_{1'1}\cdot\mathbf{x}_{2'2}}{\mathbf{x}^2_{1'1}\mathbf{x}^2_{2'2}}\,K_0(|\mathbf{x}_{12}|Q_2)\,K_0(|\mathbf{x}_{1'2'}|Q_1)\,e^{i\mathbf{k}\cdot\left[\mathbf{x}_{2'1}+\frac{z_1}{z}\mathbf{x}_{1'1}+\frac{z_2}{z}\mathbf{x}_{2'2}\right]}$$
$$\left[\frac{N_c}{2}S_{12}S_{1'2'}+C_F\left[1-S_{12}-S_{2'1'}\right]-\frac{1}{2N_c}S_{122'1'}\right]$$
$$+ \quad \cdots\cdots$$

F. Bergabo, JJM, arXiv:2024.0000

Gluon hadronizing

$$k_h^+ \frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{k}_h\,\mathrm{d}k_h^+} \quad = \quad \frac{8e^2Q^2N_c}{(2\pi)^5} \int \frac{\mathrm{d}z_h}{z_h^2}\, z^3(1-z)^2 \int \mathrm{d}^6\,\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{12'}|Q_2)\ [S_{22'} - S_{12} - S_{12'} + 1]$$
$$e^{i\frac{\mathbf{k}_h}{z_h}\cdot\mathbf{x}_{22'}} \int \frac{\mathrm{d}\xi}{\xi}\, D^{(0)}_{h/g}(\frac{z_h}{\xi})\, C_F\, \frac{\alpha_s}{\pi}\, P_{gq}(\xi)\ \log\frac{\mu^2}{\Lambda^2}$$
$$+ \quad \cdots\cdots$$

with $z \equiv \frac{k_h^+}{z_h l^+}$ and $P_{gq}(\xi) = \frac{1+(1-\xi)}{\xi}$

this can be combined with quark hadronizing contributions

$$\mathrm{d}\sigma^{\gamma^* A\to hX} = \mathrm{d}\sigma_{LO}\otimes\mathrm{JIMWLK} + \mathrm{d}\sigma_{LO}\otimes[D_{h/q}(z_h,\mu^2)+D_{h/g}(z_h,\mu^2)] + \mathrm{d}\sigma^{\mathrm{finite}}_{NLO}$$

so far: SIDIS for longitudinal photons

work on transverse photons is almost complete (see also Caucal, Salazar, arXiv:2401.01934)

Summary I

QCD at high energy

dense hadron/nucleus: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,..., to be probed precisely at EIC

toward precision: NLO, sub-eikonal corrections, ...

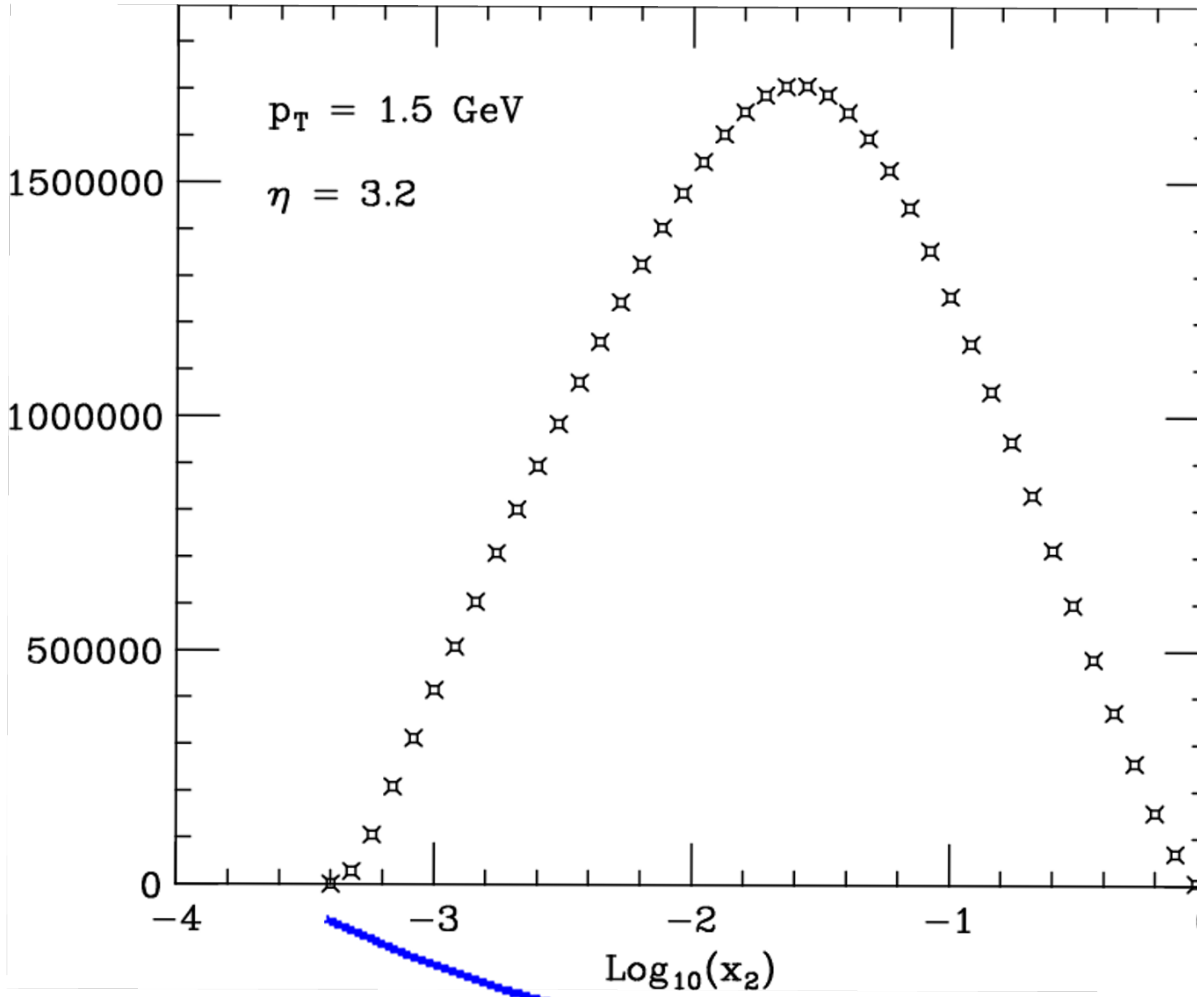
CGC is limited to small x (low p_t)

How good is eikonal approximation?

Single inclusive pion production in pp at RHIC

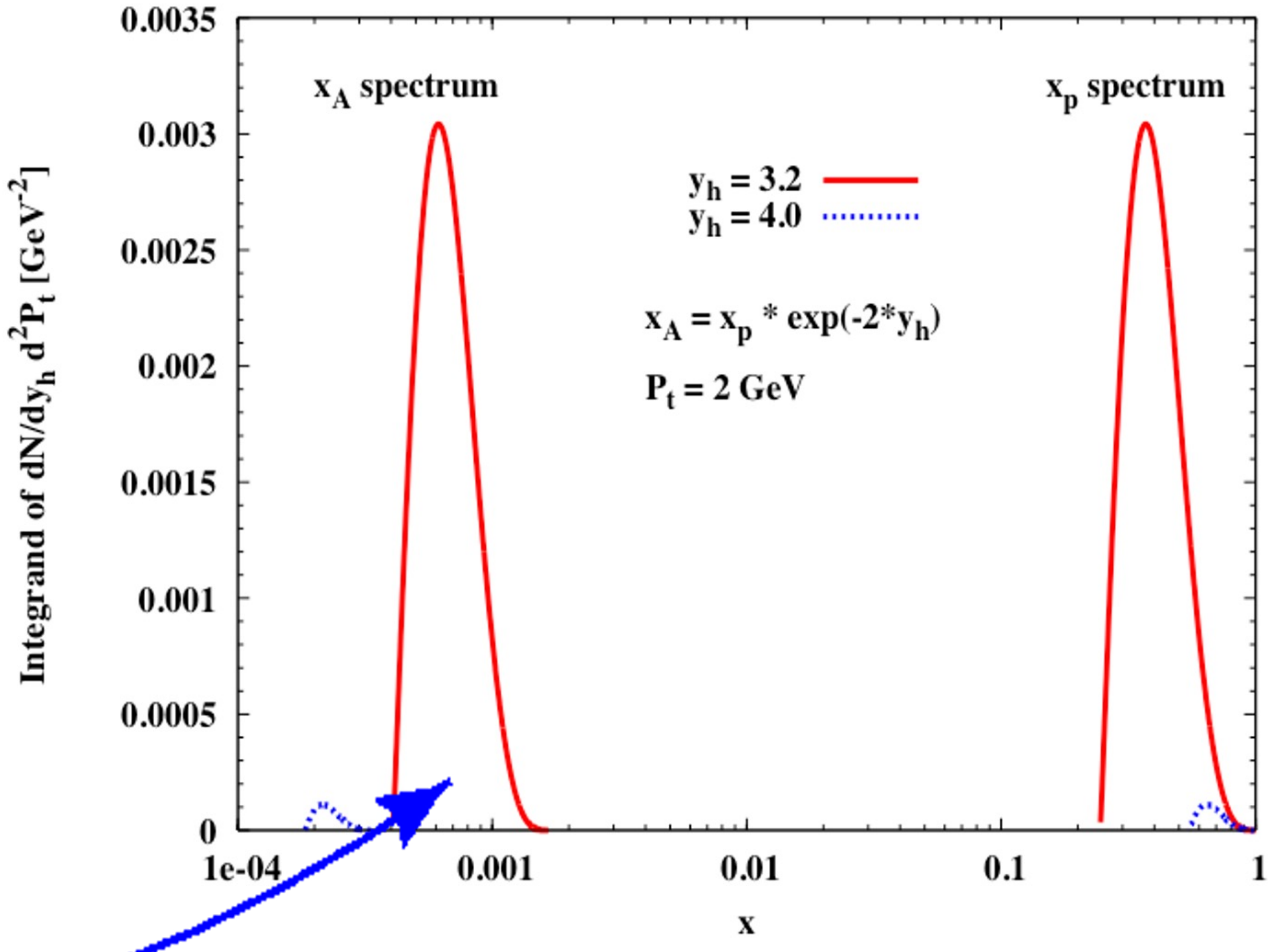
collinear factorization

GSV, PLB603 (2004) 173-183



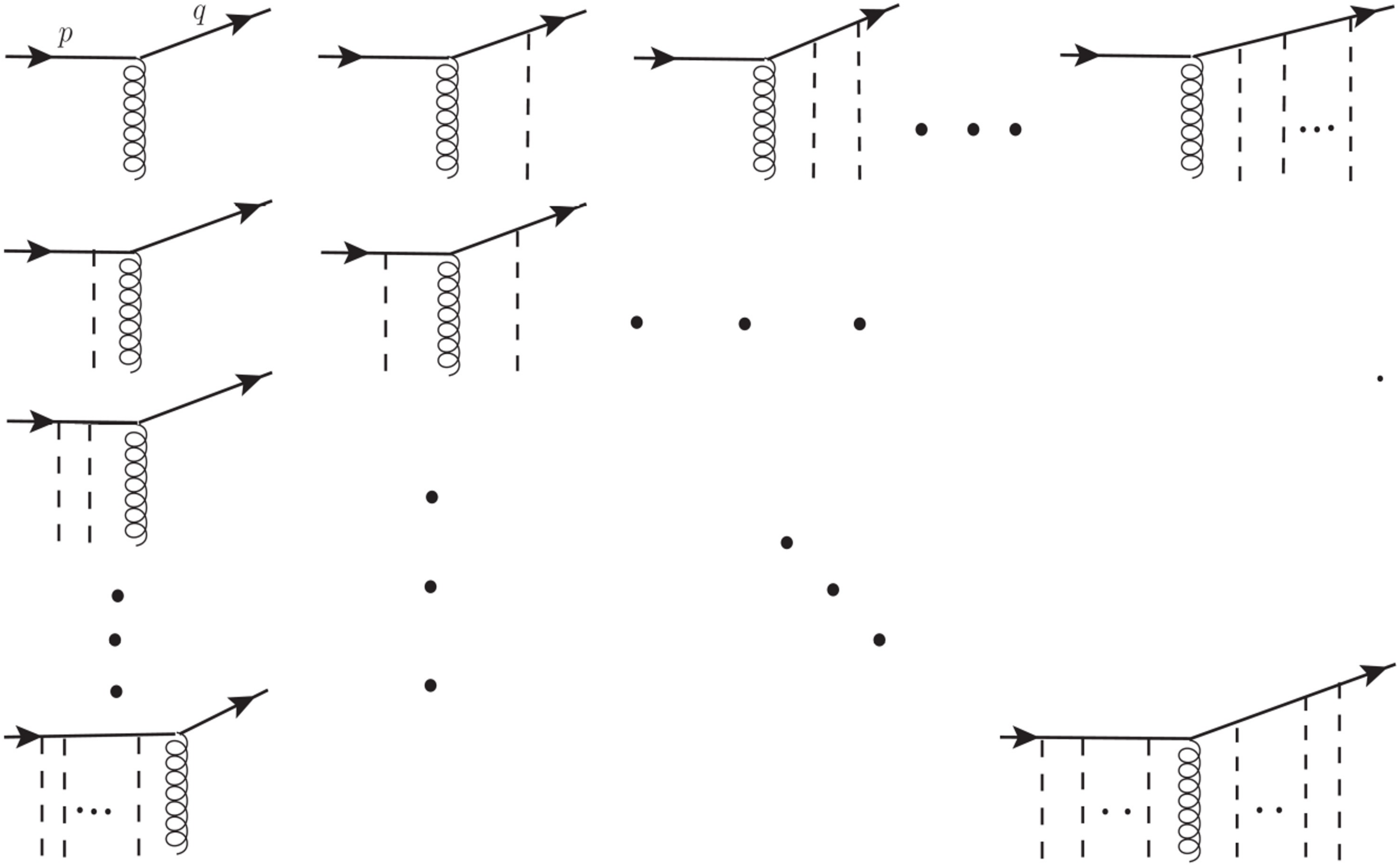
CGC

DHJ, NPA765 (2006) 57-70

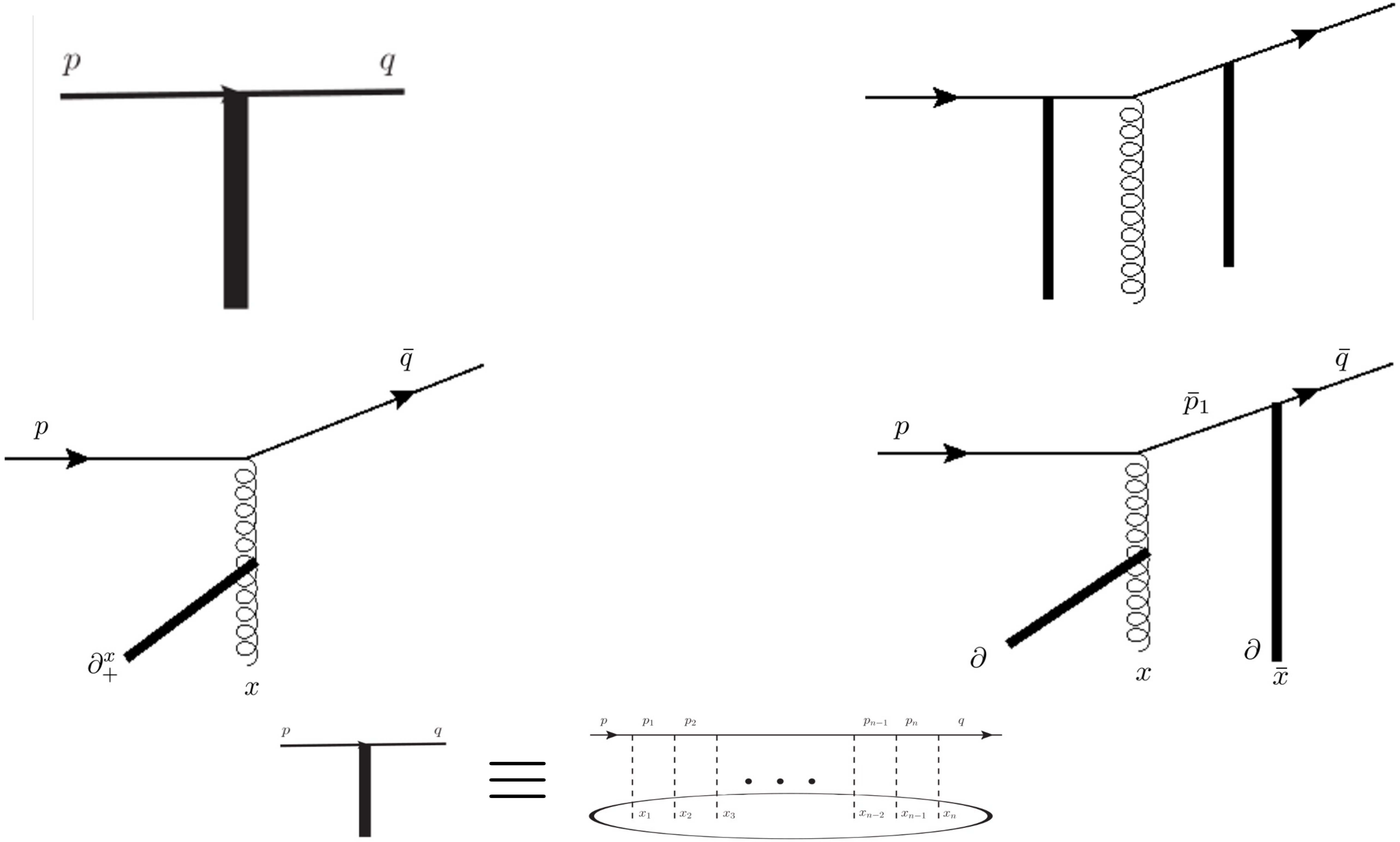


$$\int_{x_{min}}^{1} dx\, xG(x, Q^2) \cdots\cdots \longrightarrow x_{min} G(x_{min}, Q^2) \cdots$$

which kinematics are we in?


p
q

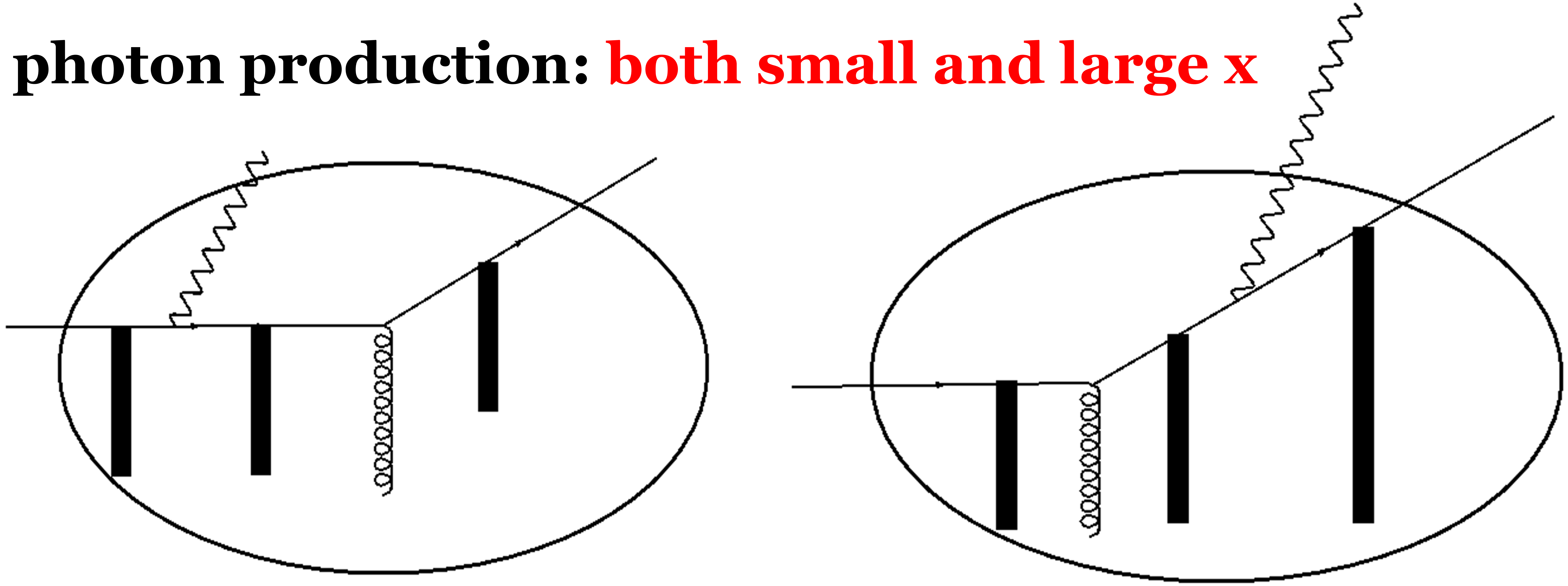
full amplitude: $\mathbf{i\mathcal{M} = i\mathcal{M}_{eik} + i\mathcal{M}_1 + i\mathcal{M}_2 + i\mathcal{M}_3}$



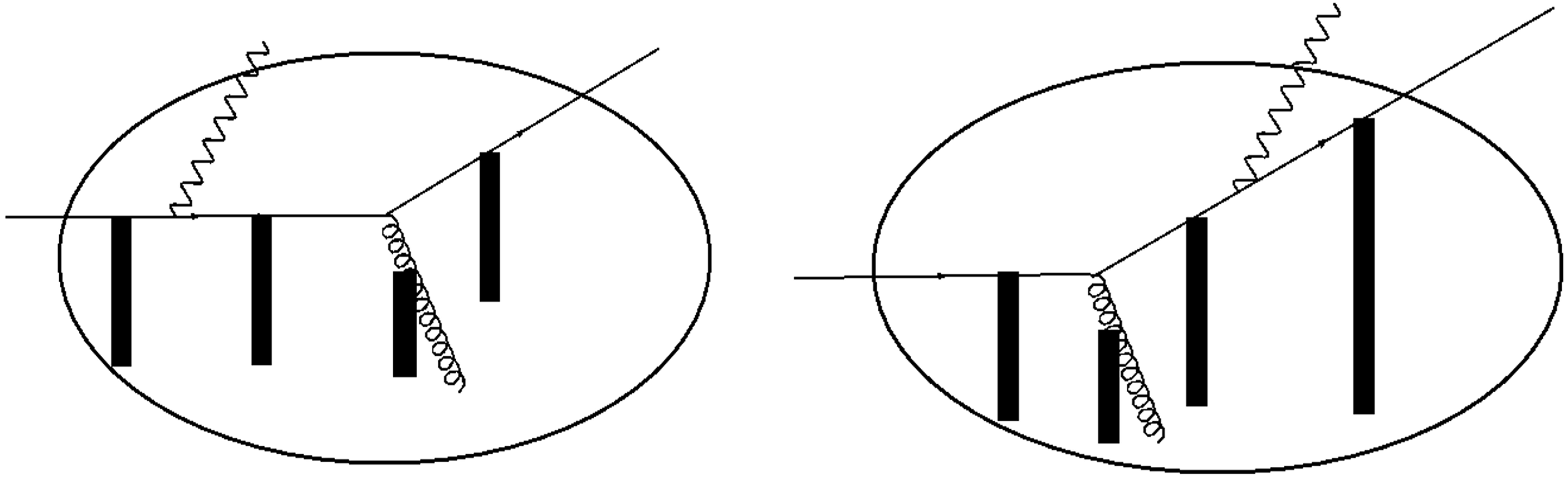
soft (eikonal) limit: $A^\mu(x) \to n^- \, S(x^+, x_t) \qquad n \cdot \bar{q} \to n \cdot p$

$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

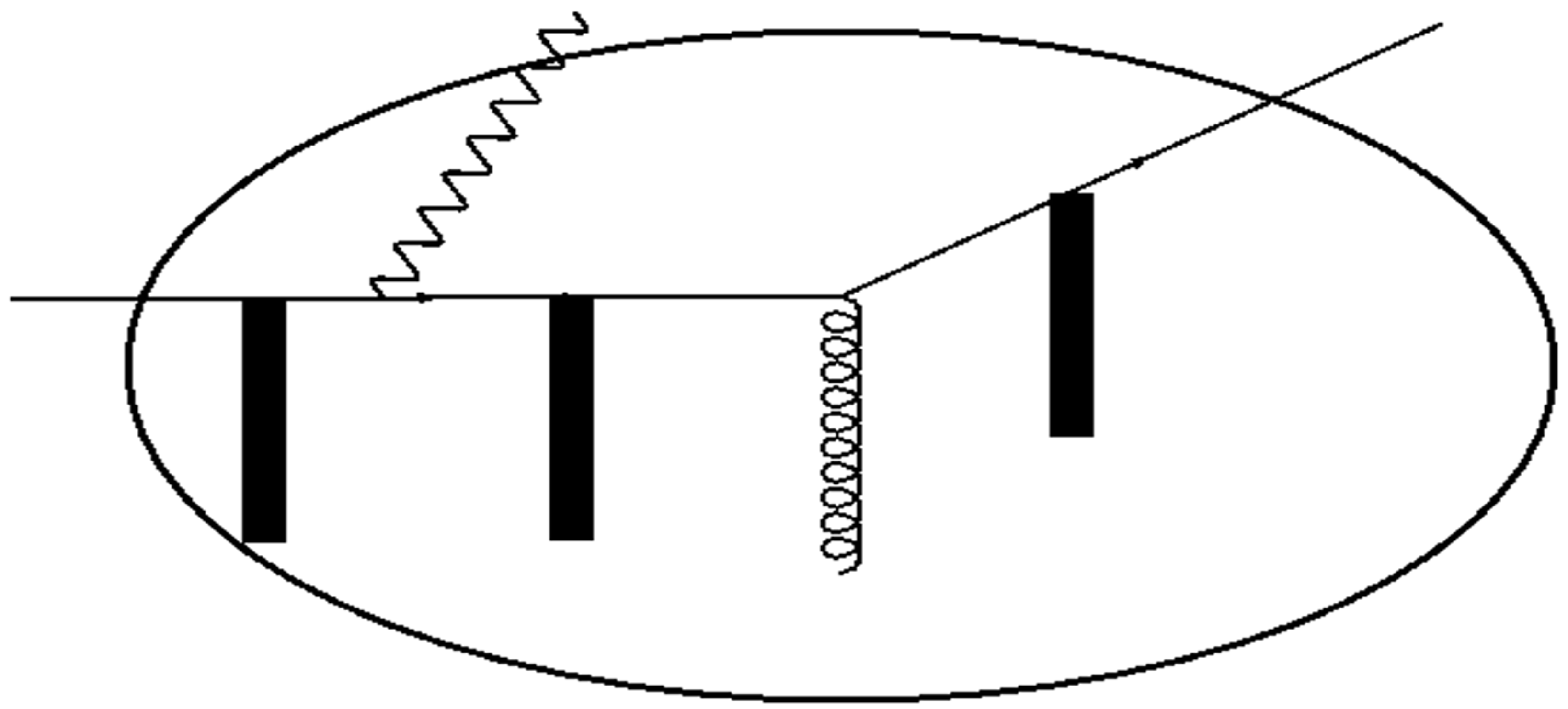
photon production: both small and large x

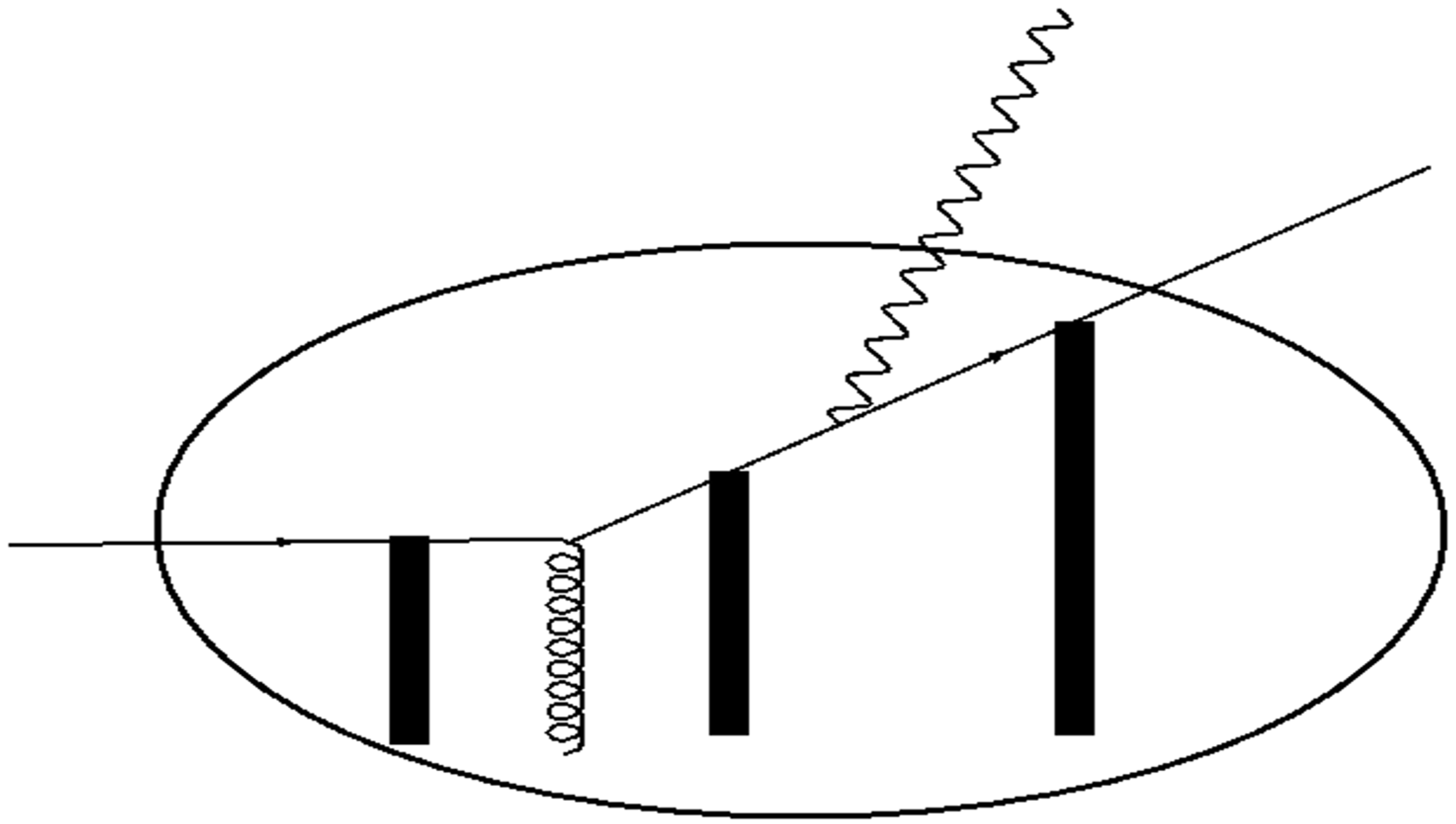


before hard scattering | after hard scattering

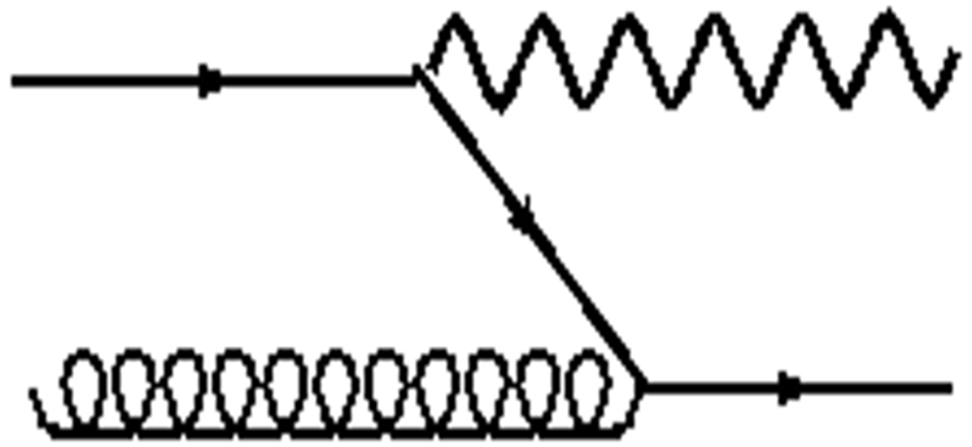


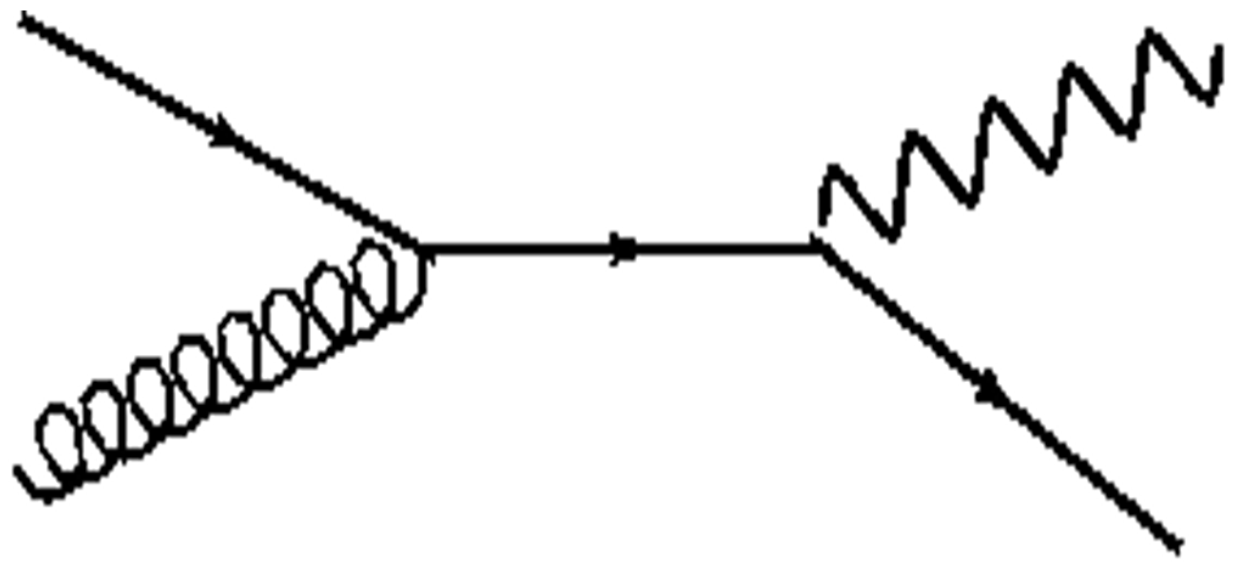
pQCD limit (large x: gluon PDF X partonic cross section):





V = U = 1





Summary II

- **EIC will have a large arm in x**
- **EIC will be able to probe the small x - large x transition region**
- **significant progress in relating various approaches**
- **need a unifying framework!**