Towards NLO for Drell-Yan production at small transverse momentum in the CGC

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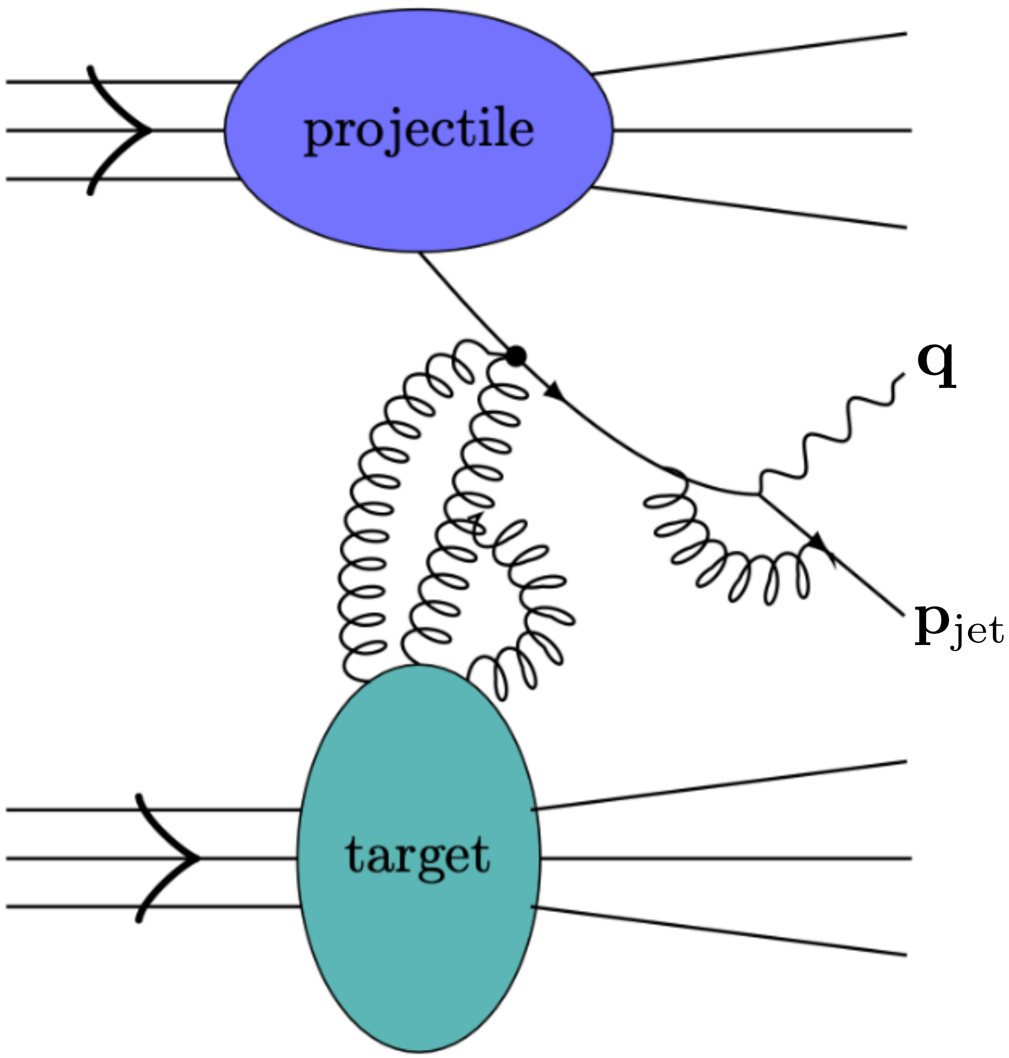
Forward Drell-Yan + jet production in the CGC



$$s \gg M^2 \sim \mathbf{q}^2 \sim \mathbf{p}_{\rm jet}^2 \gg Q_s^2$$
$$q^2 = -M^2$$

Color Glass Condensate (CGC) features saturation scale Q_s^2

Allows for the nonlinear evolution of high-energy logarithms $\ln(s/M^2)$

Incorporates all ‘kinematic’ twists
$$\mathbf{q}^2/M^2 \sim \mathbf{p}_{\rm jet}^2/M^2$$
and ‘genuine’ twists Q_s^2/M^2

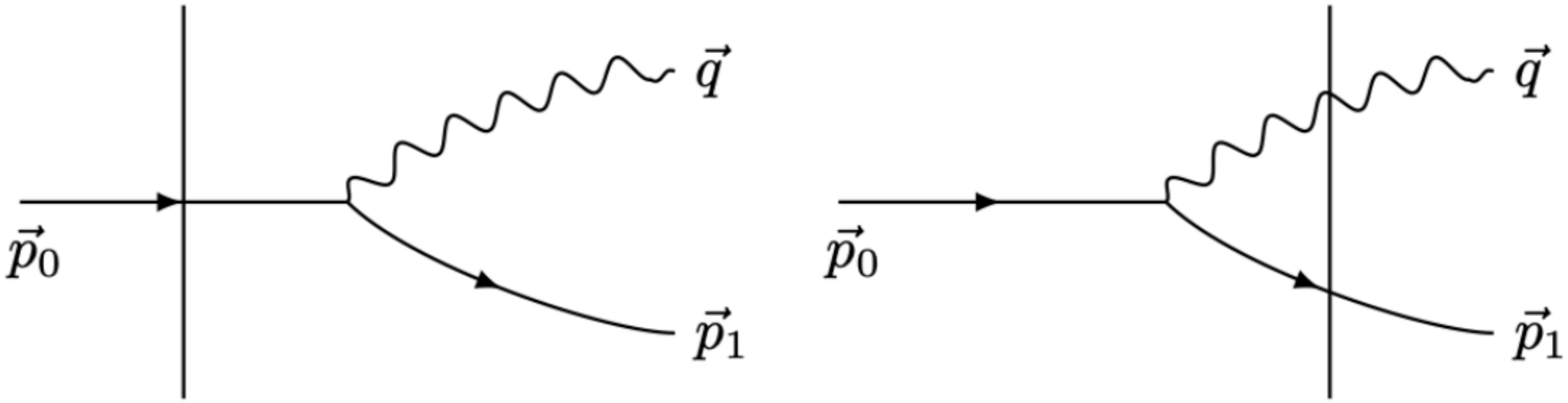
Forward production justifies:

(i) hybrid collinear-CGC approach
$$x_p \sim 10^{-1} - 10^{-2}$$
$$x_A \sim 10^{-4} - 10^{-6}$$
projectile PDF vs target CGC

(ii) eikonal approx. $M^2/s \ll 1$

Gelis, Iancu, Jalilian-Marian, Venugopalan (2010)
Dumitru, Hayashigaki, Jalilian-Marian (2006);
Altinoluk, Kovner (2011)
Altinoluk, Boussarie & Kotko (2019)

Light-Cone Perturbation Theory (LCPT)



LO1 LO2

Consider perturbative time-evolution of Fock states from infinity to interaction point and vice versa

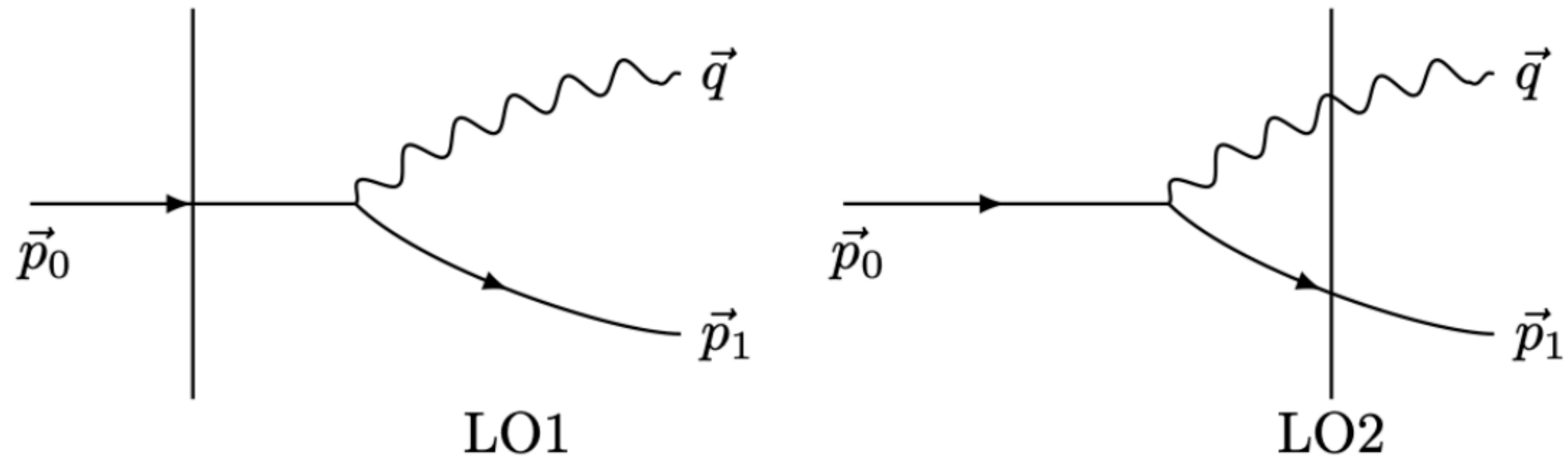
$$
\begin{aligned}
\big|\mathbf{q}(\vec{p}_0)\big\rangle_i &= \hat{\mathcal{U}}(0,-\infty)\big|\mathbf{q}(\vec{p}_0)\big\rangle \\
&= \big|\mathbf{q}(\vec{p}_0)\big\rangle + \int \mathrm{PS}(\vec{\ell},\vec{k})\, \frac{\big\langle \mathbf{q}(\vec{\ell})\boldsymbol{\gamma}^*(\vec{k})\big|\hat{V}\big|\mathbf{q}(\vec{p}_0)\big\rangle}{p_0^- - \ell^- - k^- + i0^+} \big|\mathbf{q}(\vec{\ell})\boldsymbol{\gamma}^*(\vec{k})\big\rangle
\end{aligned}
$$

Interaction modelled as eikonal scattering off classical potential

$$
\begin{aligned}
{}_f\big\langle \mathbf{q}(\vec{p}_1)\boldsymbol{\gamma}^*(\vec{q})\big|\hat{F}-1\big|\mathbf{q}(\vec{p}_0)\big\rangle_i &= \big\langle \mathbf{q}(\vec{p}_1)\boldsymbol{\gamma}^*(\vec{q})\big|\hat{F}-1\big|\mathbf{q}(\vec{p}_0)\big\rangle \\
&+ \int \mathrm{PS}(\vec{\ell})\, \frac{\big\langle \mathbf{q}(\vec{p}_1)\boldsymbol{\gamma}^*(\vec{q})\big|\hat{V}\big|\mathbf{q}(\vec{\ell})\big\rangle}{p_1^- + q^- - \ell^- + i0^+} \big\langle \mathbf{q}(\vec{\ell})\big|\hat{F}-1\big|\mathbf{q}(\vec{p}_0)\big\rangle \\
&+ \int \mathrm{PS}(\vec{\ell},\vec{k})\, \frac{\big\langle \mathbf{q}(\vec{\ell})\boldsymbol{\gamma}^*(\vec{k})\big|\hat{V}\big|\mathbf{q}(\vec{p}_0)\big\rangle}{p_0^- - \ell^- - k^- + i0^+} \big\langle \mathbf{q}(\vec{p}_1)\boldsymbol{\gamma}^*(\vec{q})\big|\hat{F}-1\big|\mathbf{q}(\vec{\ell})\boldsymbol{\gamma}^*(\vec{k})\big\rangle
\end{aligned}
$$

Bjorken, Kogut & Soper (1971)
Beuf (2016)

Leading order



Introduce convenient combinations of transverse momenta

$$\mathbf{P}_\perp \equiv \frac{q^+\mathbf{p}_1 - p_1^+\mathbf{q}}{p_0^+} \qquad \text{and} \qquad \mathbf{k}_\perp \equiv \mathbf{p}_1 + \mathbf{q}$$

and definitions: $z \equiv q^+/p_0^+ \;, \quad \bar{z} \equiv p_1^+/p_0^+ \quad \text{and} \quad q^2 \equiv M^2$

Partonic cross section in transverse case:

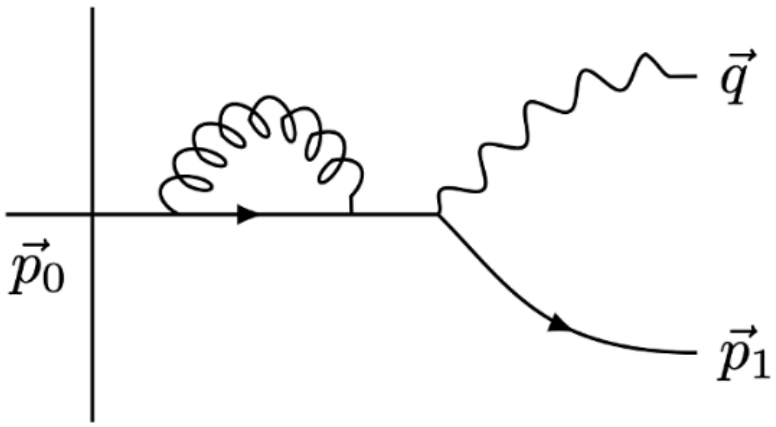
$$\frac{\mathrm{d}\hat{\sigma}^{\mathrm{T}}_{\mathrm{LO}}}{\mathrm{d}z\mathrm{d}\bar{z}\mathrm{d}^2\mathbf{P}_\perp\mathrm{d}^2\mathbf{k}_\perp} = \frac{g_{\mathrm{em}}^2 N_c}{(2\pi)^5}\delta(1-z-\bar{z})\frac{1+(1-z)^2}{z}\left(\frac{\mathbf{P}_\perp}{\mathbf{P}_\perp^2+\bar{z}M^2}+\frac{\mathbf{q}}{\mathbf{q}^2+\bar{z}M^2}\right)^2 \times \int_{\mathbf{x},\mathbf{x}'} e^{-i\mathbf{k}_\perp\cdot(\mathbf{x}-\mathbf{x}')}\left(s_{\mathbf{x}\mathbf{x}'}+1\right)$$

Color dipole:

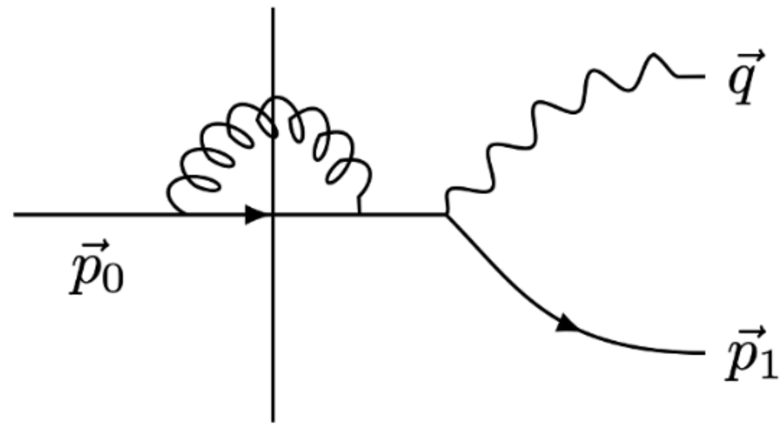
$$s_{\mathbf{x}\mathbf{x}'} = \frac{1}{N_c}\mathrm{Tr}\left(U_{\mathbf{x}}U^\dagger_{\mathbf{x}'}\right)$$

Gelis & Jalilian-Marian (2002)
Stasto, Xiao & Zaslavsky (2012)

UV divergences (1)

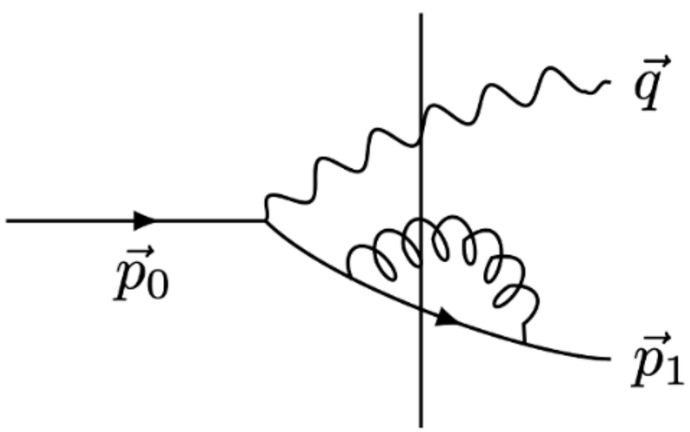


SE1

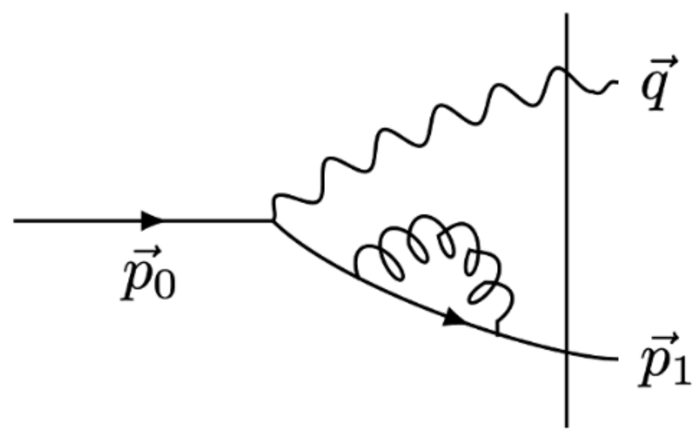


SE2

UV pole cancels between SE1 and SE2



SE3

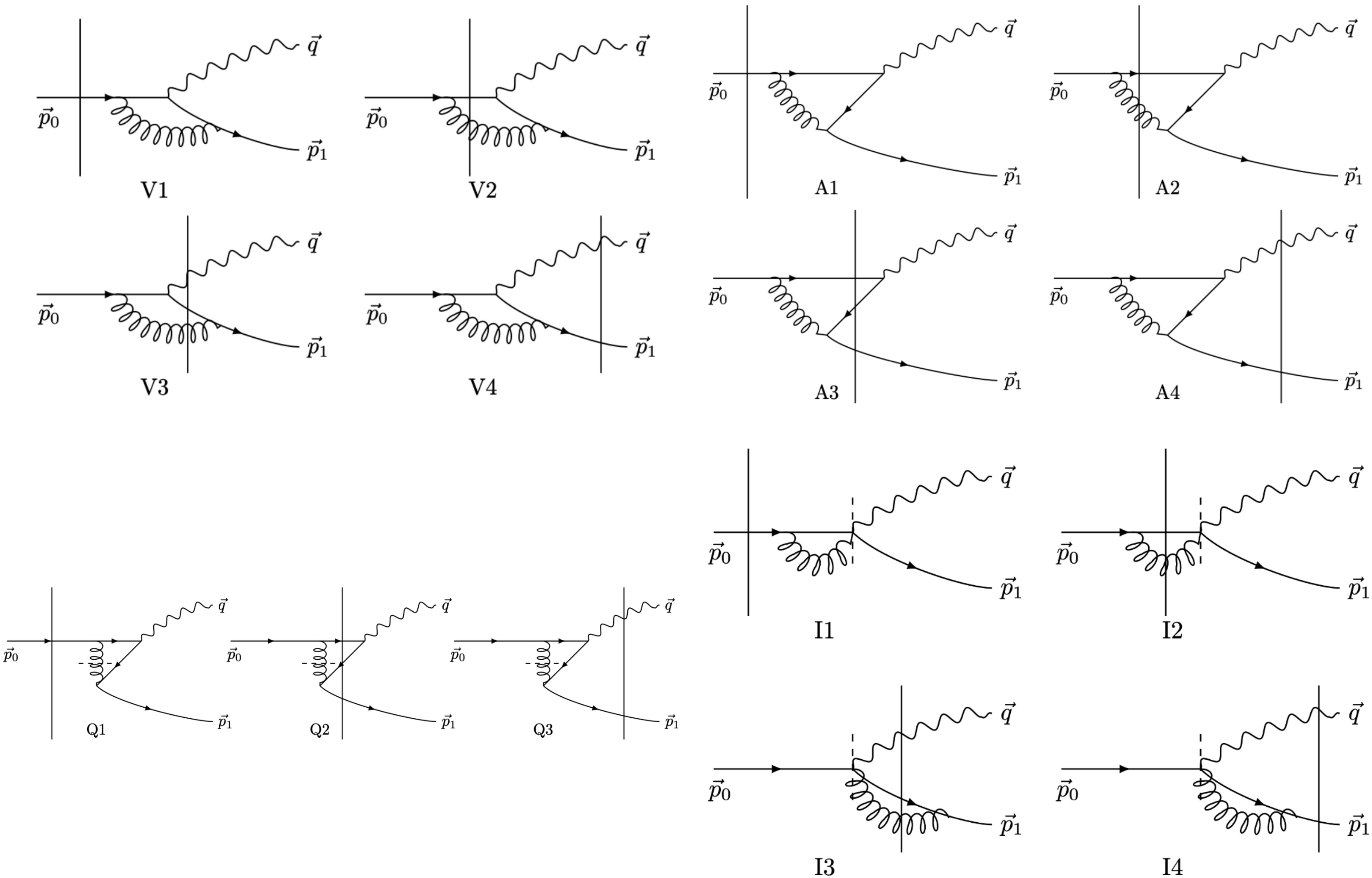


SE4

UV pole cancels between SE3 and SE4

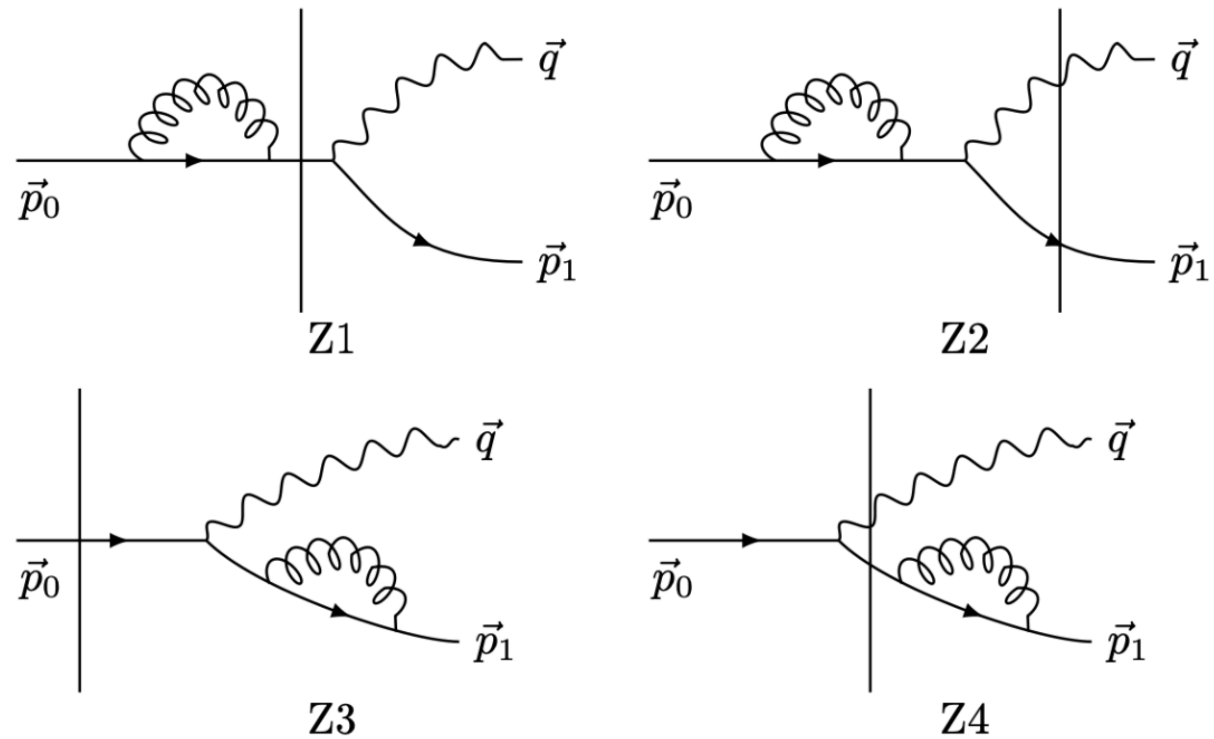
PT (2023)

UV divergences (2)



PT (2023)

UV divergences (3)



Loop corrections on external lines are zero in dimensional regularization and in massless QCD.

$$\mathcal{Z} = 1 - \frac{\alpha_s C_F}{\pi}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{coll}}}\right)\left(-\frac{3}{2} + \ln\frac{p_0^+}{k_{\mathrm{min}}^+} + \ln\frac{p_1^+}{k_{\mathrm{min}}^+}\right)$$

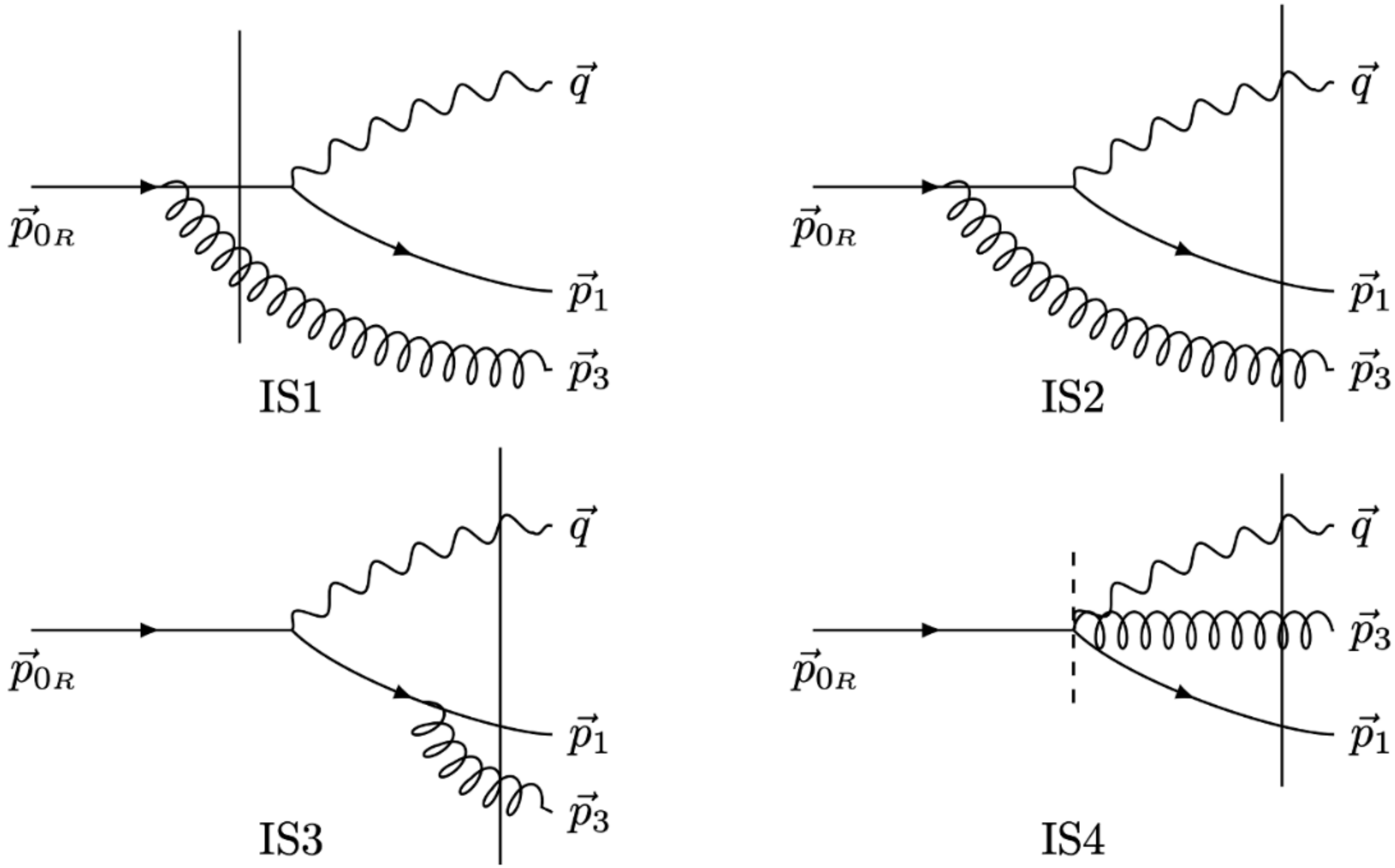
UV/collinear pole in dimreg

rapidity cutoff

All UV poles in the calculation cancel w/o need for running coupling.

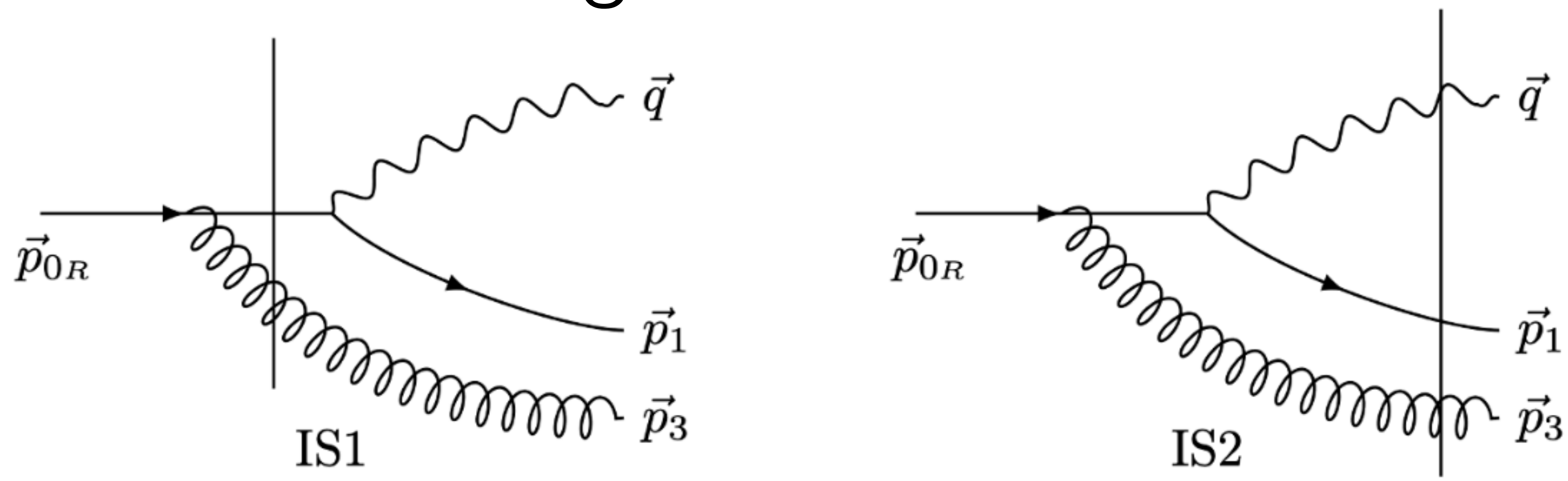
PT (2023)

Real radiative corrections to initial state



PT (2023)

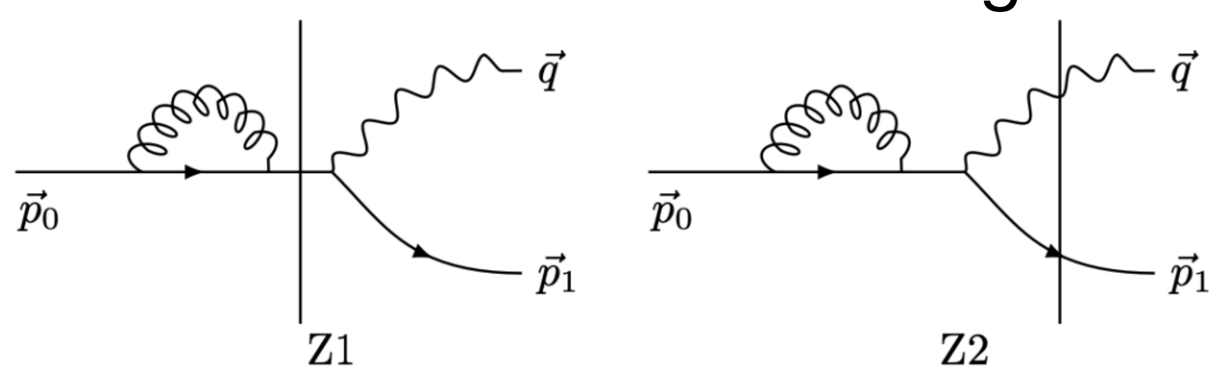
Collinear divergences in the initial state



Contain collinear divergences of the form:

$$\int_{\boldsymbol{\ell}} \frac{1}{\boldsymbol{\ell}^2} e^{-i\boldsymbol{\ell}\cdot(\mathbf{x}-\mathbf{x}')} = -\frac{1}{4\pi}\left(\frac{1}{\epsilon_{\rm coll}} + \gamma_E + \ln\left(\mu^2\pi(\mathbf{x}-\mathbf{x}')^2\right)\right) + \mathcal{O}(\epsilon_{\rm coll})$$

combine with the ones from the scaleless integrals

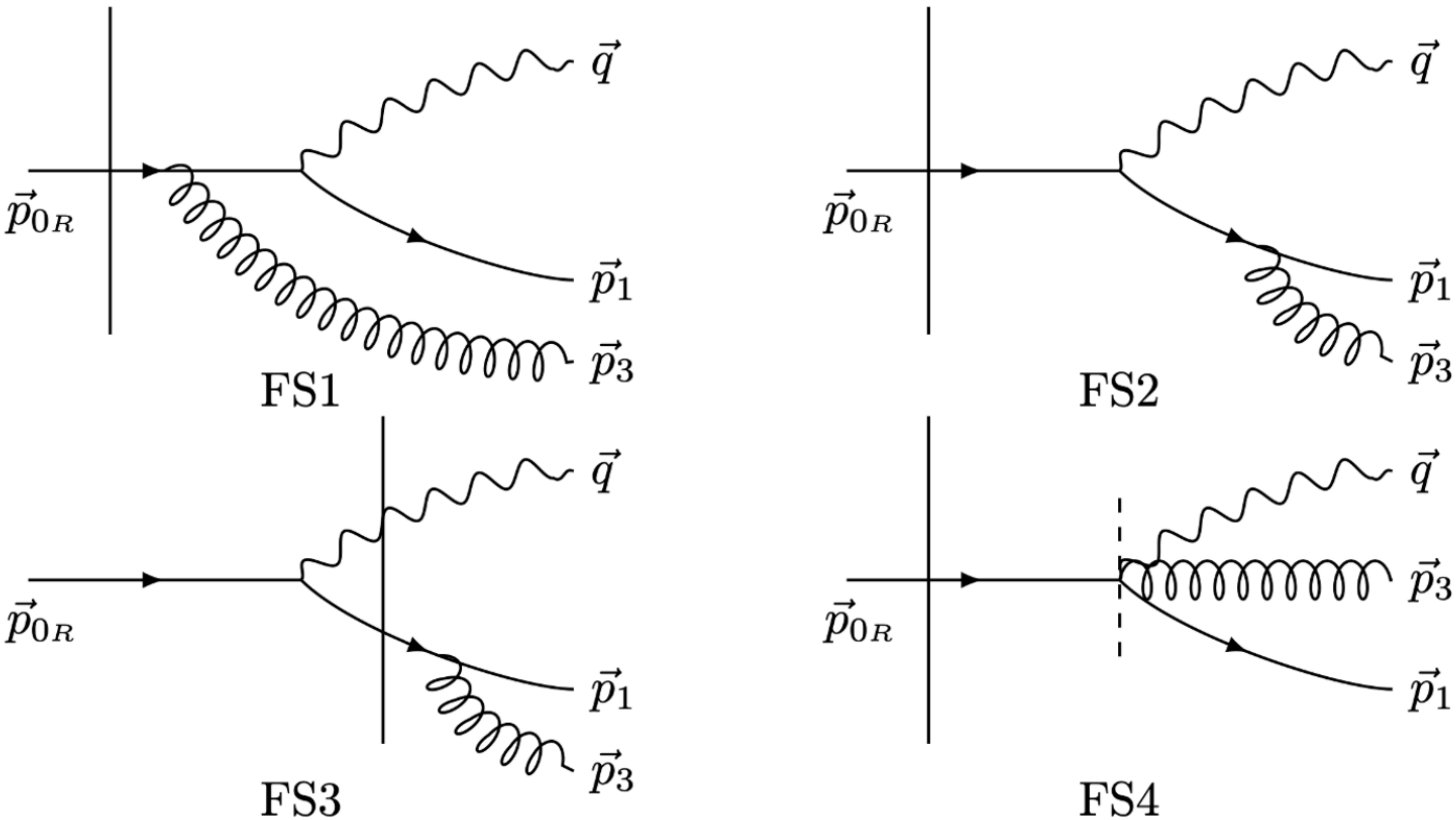


and DGLAP evolution of projectile PDF

$$x_p f_q^{(1)}(x_p,\mu^2) = x_p f_q^{(0)}(x_p)$$
$$-\left(\frac{1}{\epsilon_{\rm coll}} - \gamma_E + \ln 4\pi\right)\frac{\alpha_s}{2\pi}\int_{x_p}^1 \frac{\mathrm{d}\xi}{\xi} P_{qq}^{(0)}(\xi)\, x_p f_q^{(0)}\left(\frac{x_p}{\xi}\right) + \mathcal{O}(\alpha_s^2)$$

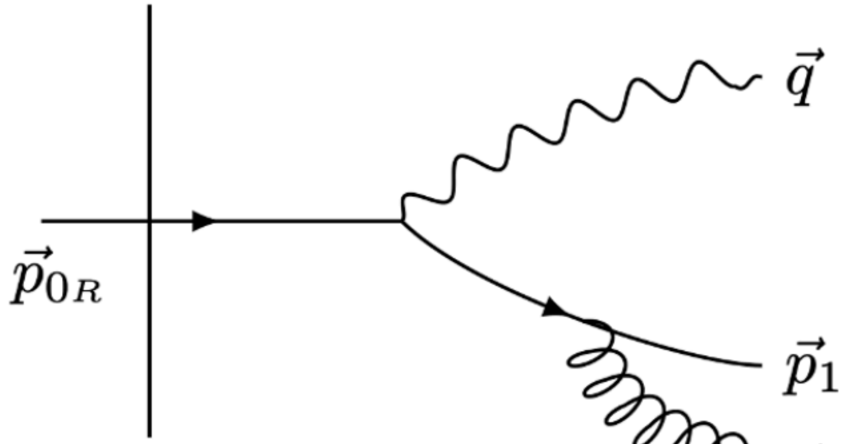
PT (2023)

Real radiative corrections to final state

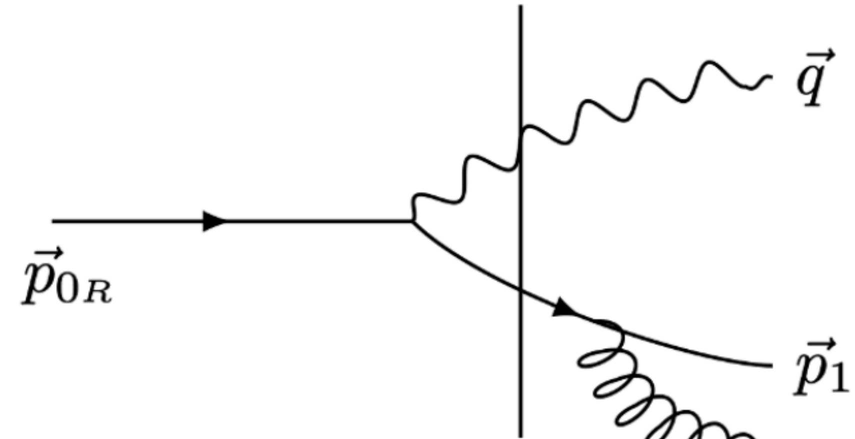


PT (2023)

Collinear divergences in the final state



FS2

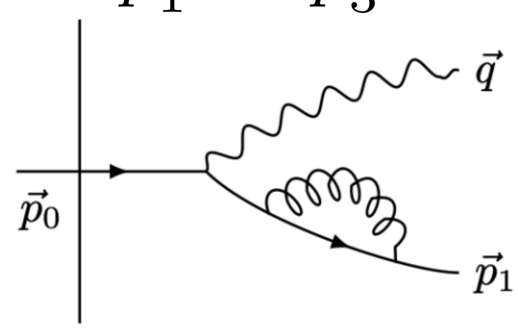


FS3

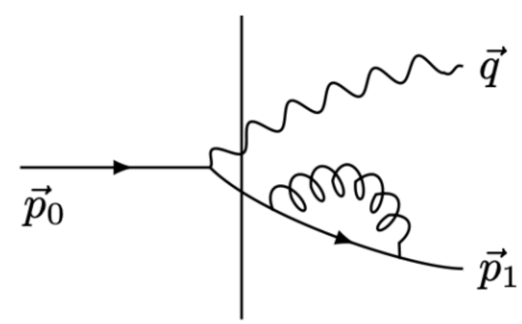
Simple jet algorithm with jet radius *R*: $\frac{p_1^+ + p_3^+}{|\mathbf{p}_1 + \mathbf{p}_3|}\left|\frac{\mathbf{p}_1}{p_1^+} - \frac{\mathbf{p}_3}{p_3^+}\right| < R$

Collinear pole when: $\left|\frac{\mathbf{p}_1}{p_1^+} - \frac{\mathbf{p}_3}{p_3^+}\right| \to 0$

cancel with:



Z3



Z4

'Inside-jet' configuration with $\vec{p}_j \equiv \vec{p}_1 + \vec{p}_3$:

$$\mathrm{d}\sigma_{\mathrm{in}} + \mathrm{d}\sigma_{\mathcal{Z}_{\mathrm{FS}}} = \mathrm{d}\sigma_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[\ln\left(\frac{4\pi e^{-\gamma_E}\mu_R^2}{\mathbf{p}_j^2 R^2}\right)\left(\frac{3}{4} - \ln\frac{p_j^+}{k_{\mathrm{min}}^+}\right) + \frac{13}{4} - \frac{\pi^2}{3} - \ln^2\frac{p_j^+}{k_{\mathrm{min}}^+}\right]$$

Unphysical double log cancels with 'outside-jet' contribution.

PT, Altinoluk, Beuf & Marquet (2022)
PT (2023)

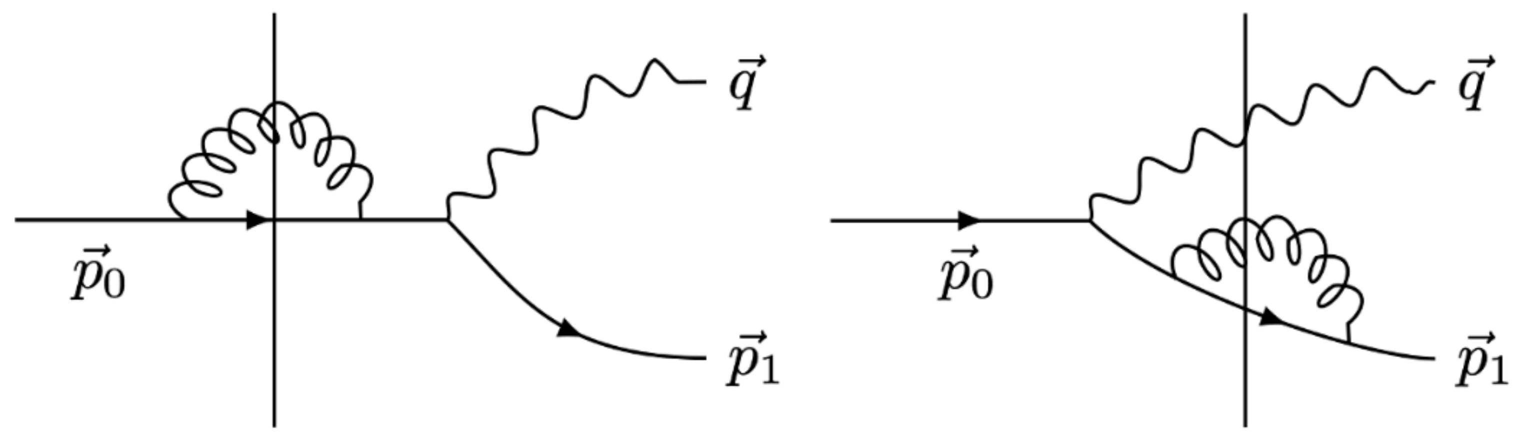
High-energy resummation

The aim is to prove that:

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int_{k^+_{\min}}^{k^+_f} \frac{\mathrm{d}p_3^+}{p_3^+} \hat{H}_{\mathrm{JIMWLK}} \mathrm{d}\sigma_{\mathrm{LO}} + \int_0^{+\infty} \frac{\mathrm{d}p_3^+}{p_3^+} \Big(\mathrm{d}\tilde{\sigma}_{\mathrm{NLO}} - \theta(k_f^+ - p_3^+) \hat{H}_{\mathrm{JIMWLK}} \mathrm{d}\sigma_{\mathrm{LO}} \Big)$$

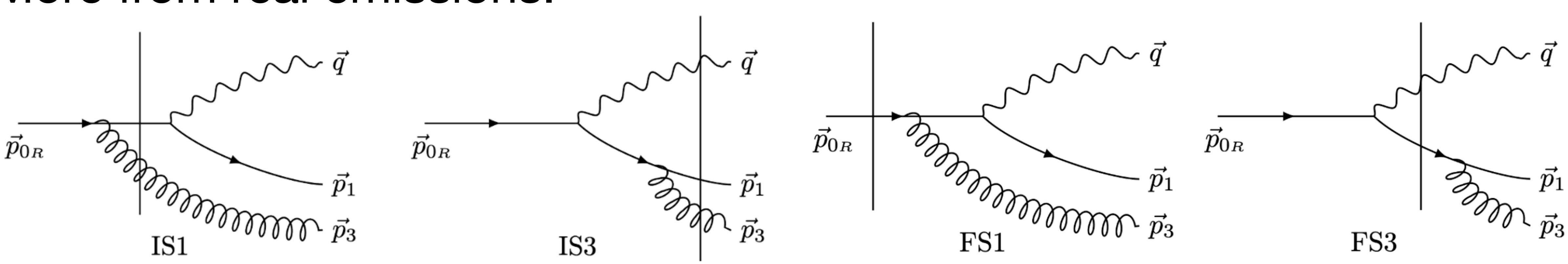
where: $\hat{H}_{\mathrm{JIMWLK}} \langle s_{\mathbf{x}\mathbf{x}'} + 1 \rangle = -\frac{\alpha_s N_c}{2\pi^2} \int_{\mathbf{z}} \frac{(\mathbf{x}-\mathbf{x}')^2}{(\mathbf{x}-\mathbf{z})^2 (\mathbf{x}'-\mathbf{z})^2} \langle s_{\mathbf{x}'\mathbf{x}} - s_{\mathbf{z}\mathbf{x}} s_{\mathbf{x}'\mathbf{z}} \rangle$

Only nonvanishing contributions from virtual diagrams stem from:



SE2 SE3

More from real emissions:



IS1 IS3 FS1 FS3

PT (2023)

Final result

$$\mathrm{d}\sigma^{pA\to\gamma^*+\mathrm{jet}+X}_{\mathrm{LO+NLO}} = \mathrm{d}\sigma_{\mathrm{LO+DGLAP+JIMWLK}} + \mathrm{d}\sigma_{\mathrm{jet}} + \mathrm{d}\sigma_{\mathrm{IS}} + \mathrm{d}\sigma_{\mathrm{virtual}} + \mathrm{d}\sigma_{\mathrm{real}}$$

'Factorized' in the sense of the CGC:

$$\mathrm{d}\sigma^{\mathrm{DY+jet}}_{\mathrm{CGC}} = f_q(x_p,\mu^2) \otimes \langle \mathrm{d}\hat{\sigma}^{\mathrm{DY+jet}}_{\mathrm{CGC}} \rangle_{x_A} + \mathcal{O}(\alpha_s^2)$$

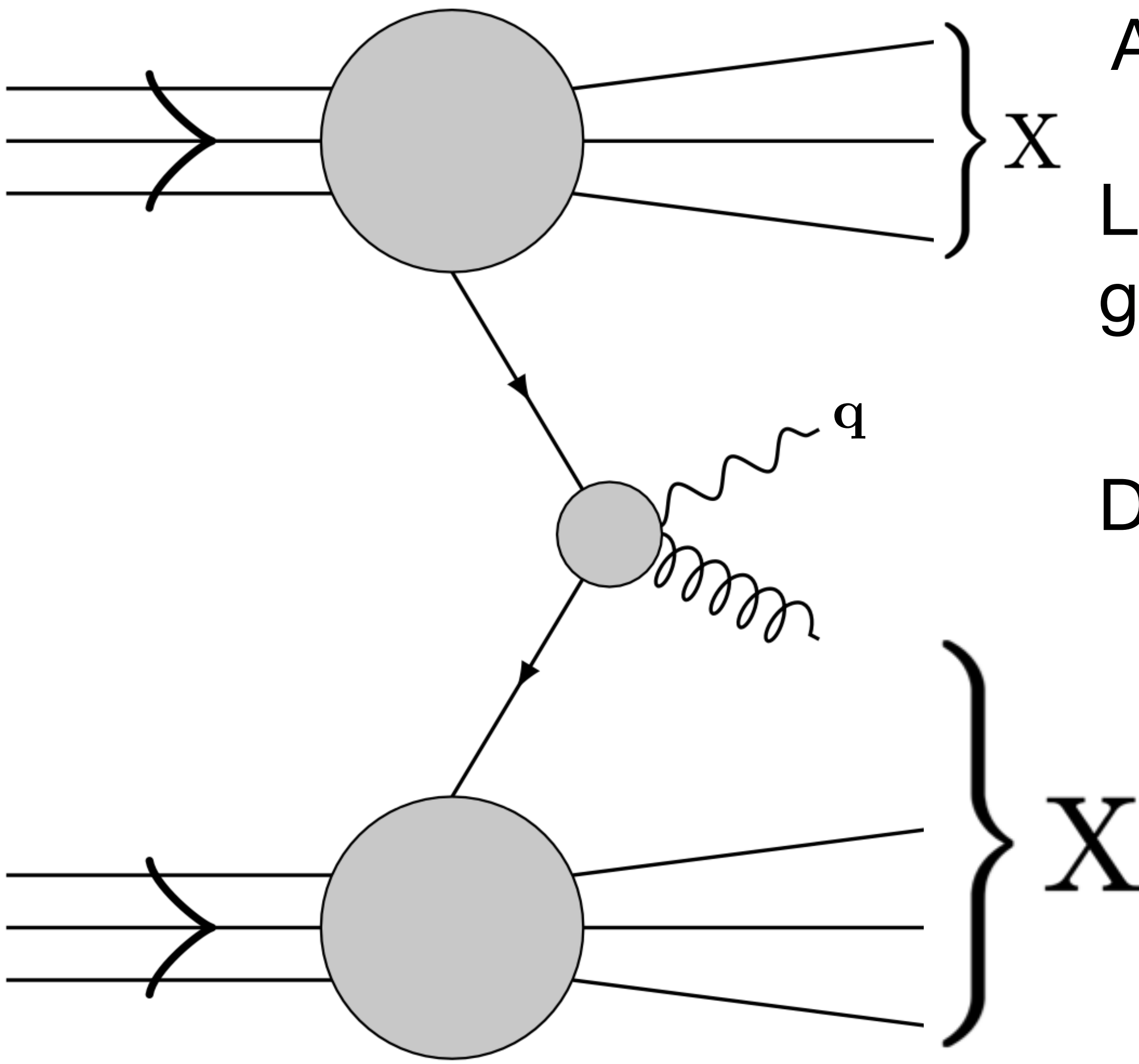
… in contrast to the collinear factorization theorem:

$$\mathrm{d}\sigma^{\mathrm{DY+jet}}_{\mathrm{coll.}} = f_q(x_p,\mu^2) \otimes f_{\bar{q}}(x_A,\mu^2) \otimes \mathrm{d}\hat{\sigma}^{\mathrm{DY+jet}}_{\mathrm{coll.}} + \mathcal{O}(1/M^n)$$

which holds at leading twist but all orders in α_s .

PT (2023)

Single inclusive DY in collinear factorization



Assume $s \sim M^2 \sim \mathbf{q}^2 \gg \Lambda^2$

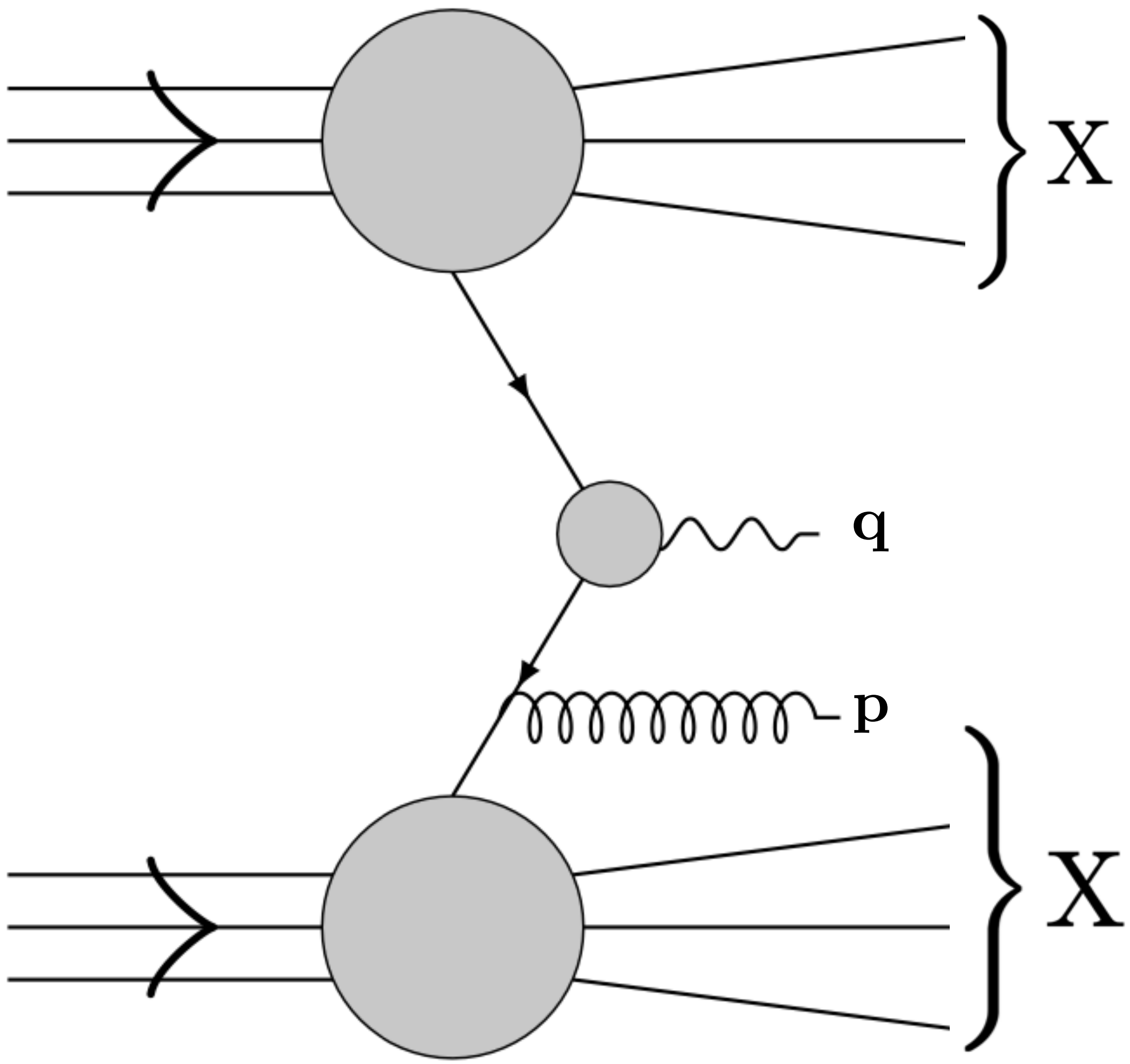
Large transverse momentum generated in hard scattering

DGLAP resummation $\ln M^2/\Lambda^2$

$$\mathrm{d}\sigma^{\mathrm{DY}}_{\mathrm{coll.}} = f_q(x_p, M^2) \otimes f_{\bar{q}}(x_A, M^2) \otimes \mathrm{d}\hat{\sigma}^{\mathrm{DY}}_{\mathrm{coll.}} + \mathcal{O}(1/M^n)$$

Collins, Soper & Sterman (1989)

Single inclusive DY in TMD factorization



$$s \sim M^2 \gg \mathbf{q}^2 \gtrsim \Lambda^2$$

Only collinearly divergent real corrections taken into account

Incomplete cancellation with virtual corrections lead to Sudakov logs

$$\ln \mathbf{q}^2/M^2$$

Real corrections are power suppressed

$$\mathbf{p}^2/M^2$$

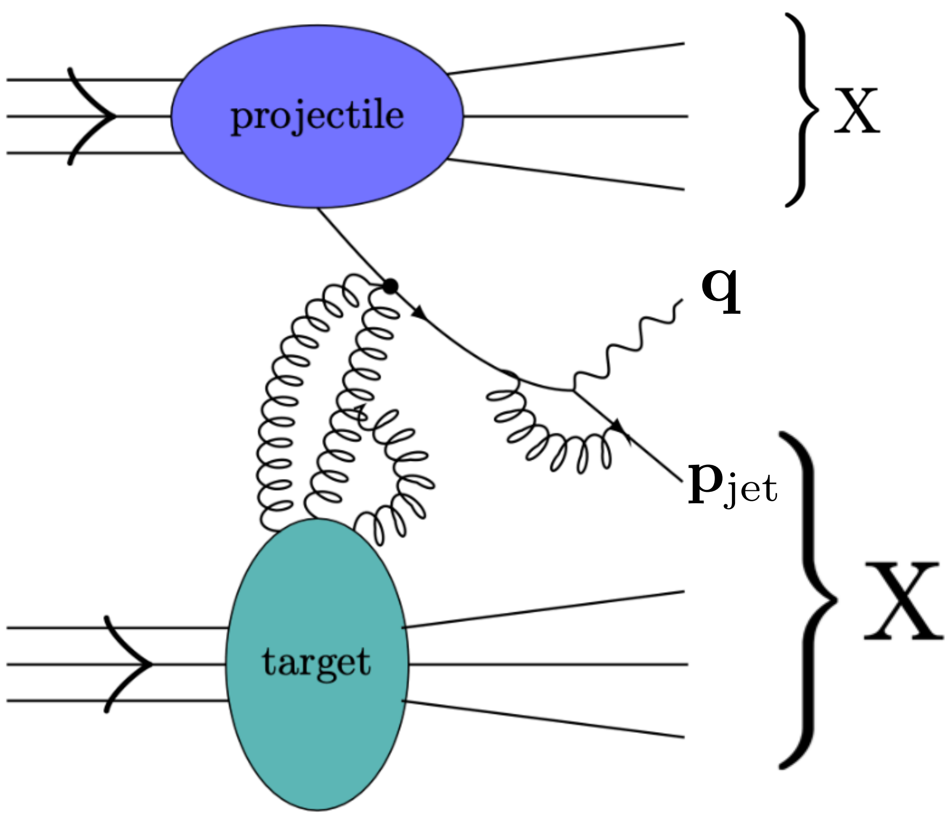
$$\mathrm{d}\sigma^{\mathrm{DY}}_{\mathrm{TMD}} = \delta(\mathbf{q} - \mathbf{p}_p - \mathbf{p}_A) f_q(x_p, \mathbf{p}_p, M^2) \otimes f_{\bar{q}}(x_A, \mathbf{p}_A, M^2) \otimes \mathrm{d}\hat{\sigma}^{\mathrm{DY}}_{\mathrm{TMD}} + \mathcal{O}(1/M^n)$$

Collins, Soper & Sterman (1984)

Towards single inclusive DY in the TMD regime

What about the TMD regime, i.e., $M^2 \gg \mathbf{q}^2 \gtrsim Q_s^2$?

Invariant mass of lepton pair prevents collinear divergence when integrating jet out



Power $\mathbf{p}_{\text{jet}}^2/M^2$ suppressed contribution is leading at low *x*!

Either interpret gluon distribution as low-*x* evolution of (anti)quark PDF

Marquet, Xiao & Yuan (2009)

Either expand gluon density in genuine twist

Altinoluk, Boussarie & Kotko (2019)

Expand hybrid collinear-CGC framework to TMD-CGC

Altinoluk, Armesto, Kovner & Lublinsky (2023)

$$\mathrm{d}\sigma^{\mathrm{DY}}_{\mathrm{CGC,TMD}} = \delta(\mathbf{q}-\mathbf{p}_p-\mathbf{p}_A) f_q(x_p,\mathbf{p}_p,M^2)\otimes f_g(x_A,\mathbf{p}_A,M^2)\otimes \mathrm{d}\hat{\sigma}^{\mathrm{DY}}_{\mathrm{CGC,TMD}}(\mathbf{p}_p,\mathbf{p}_A) + \mathcal{O}(Q_s/M)^n$$

Towards single inclusive DY in the TMD regime

What about the TMD regime, i.e., $M^2 \gg \mathbf{q}^2 \gtrsim Q_s^2$?

Simultaneous resummation of low-*x* $\ln s/M^2$
and Sudakov $\ln \mathbf{q}^2/M^2$ logarithms.

Longstanding problem, studied using many different approaches, including recently:

Rapidity-only: Balitsky (2021-2023)
HEF: Deak, Hautmann, Jung, Kutak, van Hameren, Sapeta, Hentschinski (2016-2021)
BFKL: Nefedov (2021)
PB: Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek (2022)
CGC: Mueller, Xiao, Yuan (2011); Balitsky, Tarasov (2015); Hatta, Xiao, Yuan, Zhou (2017-2021); Stasto, Wei, Xiao, Yuan (2018); PT, Altinoluk, Beuf, Marquet (2022); Caucal, Salazar, Schenke, Venugopalan (2022-2023)
Background: Mukherjee, Skokov, Tarasov, Tiwari (2023)

Crucial role of kinematic improvement of high-energy resummation!