High-energy amplitudes for forward Higgs production in the infinite-top-mass limit

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in collaboration with

V.S. Fadin, M. Nefedov and A. Papa (paper in preparation)

also based on

JHEP 2022, 92 (2022)

Workshop on overlap between QCD resummations, Aussois, 13 January 2024

Outline

Introduction

BFKL approach

- Reggeization
- BFKL in the LLA
- BFKL in the NLLA

NLO impact factors: Higgs case

- Virtual corrections
- Non-Gribov terms
- Strategy of rapidity regions
- $\mathcal{A}_{gg \to Hg}$ amplitude

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Introduction

- Record energies in the center-of-mass reachable by modern colliders
- **Semi-hard** collision process → stringent *scale hierarchy*

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}, \qquad Q^2 \text{ a hard scale,}$$

↗

Regge kinematic region

$$\alpha_s(Q^2)\,\ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order } \textbf{resummation} \text{ needed}$$

- **Linear regime** of high-energy QCD

The **BFKL** (Balitsky, Fadin, Kuraev, Lipatov) approach

i. Leading-Logarithmic-Approximation (**LLA**): $(\alpha_s \ln s)^n$

ii. Next-to-Leading-Logarithmic-Approximation (**NLLA**): $\alpha_s(\alpha_s \ln s)^n$

iii. Progress on **next-to-NLLA**

[V. Del Duca, R. Marzucca, and B. Verbeek (2022)]

[G. Falcioni, E. Gardi, N. Maher, C. Milloy, L. Vernazza (2022)]

[F. Caola, A. Chakraborty, G. Gambuti, A. von Manteuffel, and L. Tancredi (2022)]

[E. P. Byrne, V. Del Duca, L. J. Dixon, E. Gardi, and J. M. Smillie (2022)]

[V. S. Fadin, M. Fucilla, A. Papa (2023)] [V. S. Fadin (2023)]

Higgs plus jet as a paradigm

- Inclusive **Higgs plus jet** production in proton-proton collision

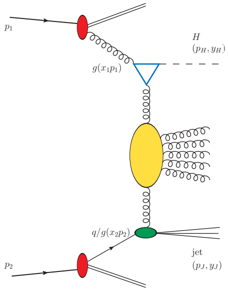
  [V. Del Duca, C. R. Schmidt (1994)]

  i. Full NLL Green function + Partial NLO impact factors (full m_t-dep.)

  [F. G. Celiberto, D. Yu. Ivanov, M. M. A. Mohammed, A. Papa (2021)]

  ii. Same process in HEJ framework (full m_t,m_b-dep.)

  [J. Andersen et al. (2022)]



$$\frac{d\sigma_{\rm pp}}{dy_H\, dy_J\, d|\vec{p}_H|\, d|\vec{p}_J|\, d\phi_1\, d\phi_2} = \frac{1}{(2\pi)^2}$$

$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \left(\mathcal{V}_H^{(g)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \otimes f_g(x_1)\right)$$

$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_H x_J s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \left(\sum_r \mathcal{V}_J^{(p)}(\vec{q}_2, s_0, x_2, \vec{p}_J) \otimes f_r(x_2)\right)$$

- Hadronic cross section expanded in **azimuthal coefficients**

$$\frac{d\sigma_{\rm pp}}{dy_H\, dy_J\, d|\vec{p}_H|\, d|\vec{p}_J|\, d\phi_1\, d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + 2\sum_{n=1}^{\infty} \cos(n\varphi)\,\mathcal{C}_n\right] \qquad \varphi = \phi_1 - \phi_2 - \pi$$

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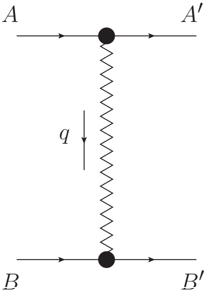
The Reggeized gluon in pQCD

- Elastic scattering process $A + B \longrightarrow A' + B'$

  i. ***Gluon quantum numbers*** in the t-channel

  ii. ***Regge limit*** $\longrightarrow s \simeq -u \to \infty$, $t = q^2$ fixed (i.e not growing with s)

  iii Valid in **LLA** ($\alpha_s^n \ln^n s$ resummed) and **NLLA** ($\alpha_s^{n+1} \ln^n s$ resummed)



$$(\mathcal{A})_{AB}^{A'B'} = \Gamma_{A'A}^{c} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^{c}$$

$$j(t) = 1 + \omega(t), \qquad j(0) = 1$$

$j(t)$-***Reggeized gluon trajectory***

$$\Gamma_{A'A}^{c} = g \langle A' | T^c | A \rangle \, \Gamma_{A'A}$$

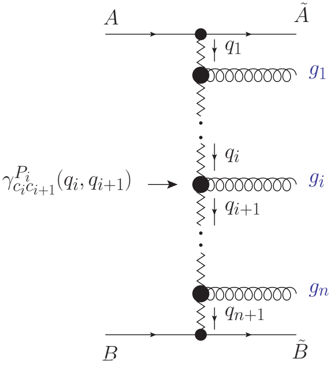
T^c- fundamental(quarks) or adjoint(gluons)

- LLA [L. N. Lipatov (1976)]

$$\Gamma_{A'A}^{(0)} = \delta_{\lambda_{A'}\lambda_A}, \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2} = -g^2 \frac{N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}^{\,2})^{\epsilon}$$

BFKL in LLA

- Inelastic scattering process $A + B \longrightarrow \tilde{A} + \tilde{B} + n$ in the LLA



i. ***Leading-logarithm resummation***

$\downarrow$

Multi-Regge kinematics (MRK)

ii. Exchange of fermions suppressed in LLA

iii. Vertical gluons become Reggeized due to loop radiative corrections

iv. $\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \rightarrow$ ***Lipatov vertex***

- ***Multi-Regge form of inelastic amplitudes***

$$\Re \mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\Gamma^{c_1}_{\tilde{A}A} \left(\prod_{i=1}^{n} \gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \left(\frac{s_i}{s_0} \right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_0} \right)^{\omega(t_{n+1})} \Gamma^{c_{n+1}}_{\tilde{B}B}$$

Multi-Regge kinematics

- ***Sudakov decomposition***

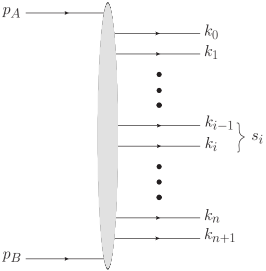
$$k_i = z_i p_A + \lambda_i p_B + k_{i\perp} \qquad p_A^2 = p_B^2 = 0$$

- ***Multi-Regge kinematics (MRK)***

$$z_0 \gg z_1 \gg \dots \gg z_n \gg z_{n+1}$$

$$\lambda_{n+1} \gg \lambda_n \gg \dots \gg \lambda_1 \gg \lambda_0$$

$$k_{0\perp} \sim k_{1\perp} \sim \dots \sim k_{n\perp} \sim k_{n+1\perp}$$



- Cutkosky rules

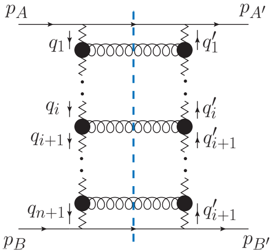
$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{1}{2} \sum\!\!\!\!\!\!\int_n d\Phi_{\tilde{A}\tilde{B}+n} \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} \left(\mathcal{A}_{A'B'}^{\tilde{A}\tilde{B}+n} \right)^*$$

- Integration over phase space

Each integration over s_i (or z_i)

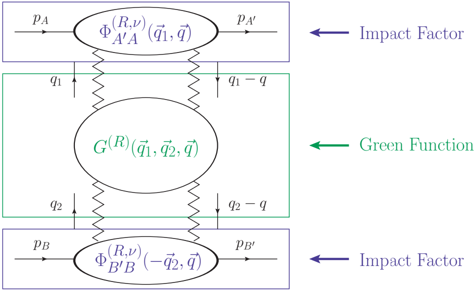
$\downarrow$

One energy logarithm



BFKL resummation

- Diffusion $A + B \longrightarrow A' + B'$ in the ***Regge kinematical region***
- BFKL factorization for $\Im\mathcal{A}_{AB}^{A'B'} \to$ convolution of a ***Green function*** (process independent) with the ***Impact factors*** of the colliding particles (process dependent)



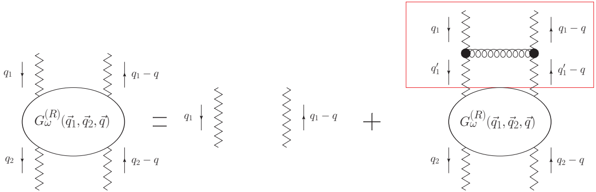
$$\Im\mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^{\,2}} \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q}\,)^{\,2}}$$

$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1,\vec{q},s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0}\right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1,\vec{q}_2;\vec{q}\,)\right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2,\vec{q},s_0)$$

- $\mathcal{R} = 1^{+}(\text{singlet}), 8^{-}(\text{octect}), \ldots$

- $G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$-Mellin transform of the Green function for the Reggeon-Reggeon scattering

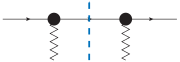
$$\omega G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_1'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime} - \vec{q}\,)^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}\,) G_\omega^{(R)}(\vec{q}_1^{\,\prime}, \vec{q}_2; \vec{q}\,)$$



- ***BFKL equation*** ($\vec{q}^{\,2} = 0$ and singlet color state representation)
  [I. I. Balitsky, V. S. Fadin, E. A. Kuraev, L. N. Lipatov (1975-1978)]

- $\Phi_{P'P}^{(R,\nu)}$- LO impact factor in the t-channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc' | \hat{\mathcal{P}} | \nu \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma^c_{\{f\}P} (\Gamma^{c'}_{\{f\}P'})^*$$



BFKL at NLLA in a nutshell

- Simple factorized form of inelastic amplitudes

$$\downarrow$$

[V. S. Fadin, L. N. Lipatov (1989)]

Straightforward program of computations

- Resummation of subleading logarithms means a ***new kinematics***

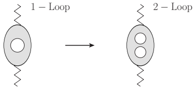
  i. *Multi-Regge kinematics (MRK)*

  ii. *Quasi multi-Regge kinematics (QMRK)*

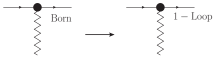
- ***Multi-Regge kinematics***

  Previous quantity must be calculated at higher loops (one α_s more)

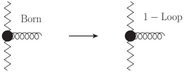
i. $\omega^{(1)}(t) \longrightarrow \omega^{(2)}(t)$



ii. $\Gamma^{c(0)}_{P'P} \longrightarrow \Gamma^{c(1)}_{P'P}$



iii. $\gamma^{G_i(0)}_{c_i c_{i+1}} \longrightarrow \gamma^{G_i(1)}_{c_i c_{i+1}}$

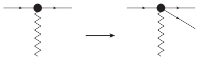


BFKL at NLLA in a nutshell

- ***Quasi Multi-Regge kinematics***

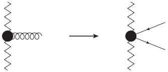
  A pair of particles (but only one!) may have longitudinal Sudakov variables of the same order (one logarithm less)

  i. $\Gamma^{c(0)}_{P'P} \longrightarrow \Gamma^{c(0)}_{\{f\}P}$



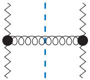
  ii. $\gamma^{G(0)}_{c_i c_{i+1}} \longrightarrow \gamma^{GG(0)}_{c_i c_{i+1}}$

  iii. $\gamma^{G(0)}_{c_i c_{i+1}} \longrightarrow \gamma^{QQ(0)}_{c_i c_{i+1}}$

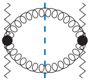


- 3 ***new contributions to the real kernel***

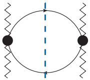
$$\mathcal{K}_r(\vec{q}_1,\vec{q}_2) = \mathcal{K}^{(1)}_{RRG}(\vec{q}_1,\vec{q}_2) + \mathcal{K}^{(0)}_{RRGG}(\vec{q}_1,\vec{q}_2) + \mathcal{K}^{(0)}_{RRQ\bar{Q}}(\vec{q}_1,\vec{q}_2)\,.$$



1−Loop



Born



Born

BFKL at NLLA in a nutshell

- Separating MRK and QMRK → Introduction of s_Λ parameter
- ***QMRK*** $(s_{ij} < s_\Lambda)$

  In the ***two-gluon contribution to the kernel*** the invariant mass should be constrained

$$\mathcal{K}_r(\vec{q}_1,\vec{q}_2) = \frac{\langle c_1 c_1' | \hat{\mathcal{P}}_0 | c_2 c_2' \rangle}{2} \sum_{\{f\}} \int \frac{ds_{RR}}{(2\pi)^D} d\rho_f \; \gamma^{\{f\}}_{c_1 c_2}(q_1,q_2) \left(\gamma^{\{f\}}_{c_1' c_2'}(q_1,q_2) \right)^* \theta(s_\Lambda - s_{RR})$$

- ***MRK*** $(s_{ij} > s_\Lambda)$

  The lower bound of integration over invariant masses is s_Λ

$$-\frac{1}{2} \int d^{D-2}q' \; \vec{q}_1^{\,2} \vec{q}_2^{\,2} \mathcal{K}_r^{(0)}(\vec{q}_1,\vec{q}\,')\mathcal{K}_r^{(0)}(\vec{q}\,',\vec{q}_2) \ln\left(\frac{{s_\Lambda}^2}{(\vec{q}\,'-\vec{q}_1)^2(\vec{q}\,'-\vec{q}_2)^2} \right)$$

- Similarly, for the ***impact factors***

$$\Phi_{AA}(\vec{q}_1; s_0) = \left(\frac{s_0}{\vec{q}_1^{\,2}} \right)^{\omega(-\vec{q}_1^{\,2})} \sum_{\{f\}} \int \theta(s_\Lambda - s_{AR}) \frac{ds_{AR}}{2\pi} \; d\rho_f \; \Gamma^c_{\{f\}A} \left(\Gamma^{c'}_{\{f\}A} \right)^* \langle cc' | \hat{\mathcal{P}}_0 | 0 \rangle$$

$$-\frac{1}{2} \int d^{D-2}q_2 \; \frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \; \Phi^{(0)}_{AA}(\vec{q}_2) \; \mathcal{K}_r^{(0)}(\vec{q}_2,\vec{q}_1) \; \ln\left(\frac{{s_\Lambda}^2}{s_0(\vec{q}_2-\vec{q}_1)^2} \right)$$

- Dependence on s_Λ disappears in the combination

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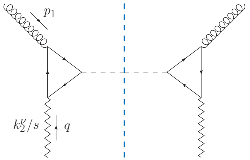
LO Higgs impact factor

- Gluon-Reggeon $\longrightarrow$ Higgs (through the top quark loop)
- Off-shell t-channel gluon with effective k_2^ν/s polarization



- **LO impact factor**

[V. Del Duca, C. R. Schmidt (1994)]

$$\frac{d\Phi_{PP}^{\{H\}(0)}}{dx_H d^2\vec{p}_H} = \frac{\alpha_s^2}{v^2}\frac{\vec{q}^{\,2}|\mathcal{F}(m_t, m_H, \vec{q}^{\,2})|^2}{128\pi^2\sqrt{2(N^2-1)}} f_g(x_H)\delta^{(2)}(\vec{p}_H - \vec{q})$$

$\downarrow$ *Infinite top-mass limit*

$$\frac{d\Phi_{PP}^{\{H\}(0)}}{dx_H d^2\vec{p}_H} = \frac{g_H^2\vec{q}^{\,2} f_g(x_H)\delta^{(2)}(\vec{q} - \vec{p}_H)}{8\sqrt{N^2-1}}$$

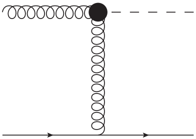
- The study can be upgraded to **Next-to-Leading Order (NLO)**, in the limit $m_t \to \infty$, by using the effective lagrangian

$$\mathcal{L}_{\mathbf{ggH}} = -\frac{\mathbf{1}}{\mathbf{4}}\mathbf{g_H F^a_{\mu\nu} F^{\mu\nu,a} H} \qquad g_H = \frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$$

[M. Nefedov (2019)] , [M. Hentschinski, K. Kutak, A. van Hameren (2020)]

[F. G. Celiberto, M. Fucilla, D. Yu. Ivanov, M. M. A. Mohammed, A. Papa (2022)]

NLO Higgs impact factor: Virtual corrections



- 1-loop ggH effective vertex

$$\Gamma^{ac(1)}_{\{H\}g}(q) = \Gamma^{ac(0)}_{\{H\}g}(q)\,[1+\delta_{\rm NLO}]$$

- General strategy: Comparison of a suitable amplitude (in the high-energy limit) with the ***Regge form***

$$\mathcal{A}^{(8,-)}_{gq\to Hq} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)}+\left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{c}_{qq} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right)+\ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq}+\Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(1)}_{qq}+\Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

NLO Higgs impact factor: Virtual corrections

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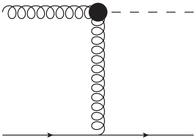


- General strategy: Comparison of a suitable amplitude (in the high-energy limit) with the ***Regge form***

$$\mathcal{A}^{(8,-)}_{gq\to Hq} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{c}_{qq} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right)+\ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(1)}_{qq} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

- **Virtual corrections** to the impact factor

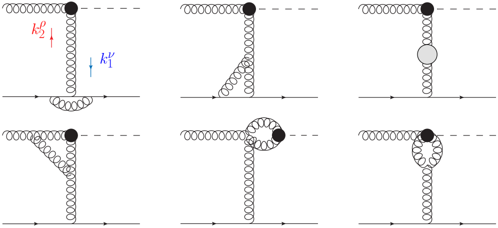
$$\frac{d\Phi^{\{H\}(1)}_{gg}}{dz_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{gg}}{dz_H d^2\vec{p}_H}\,\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{C_A}{\epsilon^2}+\frac{11C_A-2n_f}{6\epsilon}\right.$$

$$\left.-\frac{C_A}{\epsilon}\ln\left(\frac{\vec{q}^{\,2}}{s_0}\right)-\frac{5n_f}{9}+C_A\left(2\,\Re\left({\rm Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)\right)+\frac{\pi^2}{3}+\frac{67}{18}\right)+11\right]$$

- Agreement with [M. Nefedov (2019)]

NLO Higgs impact factor: Virtual corrections

- Single gluon in the t-channel diagrams



Gribov's prescription: $g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\,\frac{k_1^\nu}{s}\,\frac{k_2^\rho}{s}$

NLO Higgs impact factor: Virtual corrections

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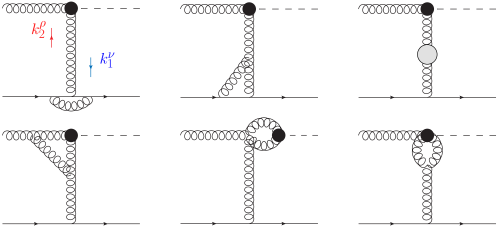
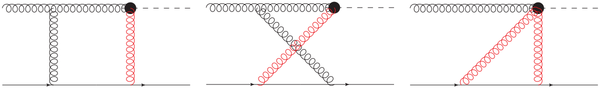


Gribov's prescription: $g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\frac{k_1^\nu}{s}\frac{k_2^\rho}{s}$

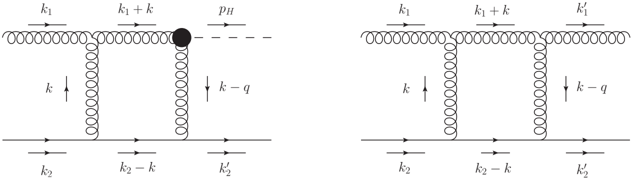
- Two gluons in the t-channel diagrams



Dimension-5 operator in $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{ggH}$ $\to$ **Gribov's trick modification**

$$g^{\rho\nu} = g_{\perp\perp}^{\rho\nu} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\frac{k_1^\nu}{s}\frac{k_2^\rho}{s} + g_{\perp\perp}^{\rho\nu}$$

- Comparison with QCD



- Non-Gribov term in $\mathcal{A}_{gq\to Hq}$

$$-s\bar{u}(k'_2)\gamma^{\xi}u(k_2)\varepsilon^{\nu}_{\perp}(k_1)g^{\perp\perp}_{\sigma\xi}H_{\nu}{}^{\sigma}(-k_1 - k, k - q)$$

$$H^{\nu\sigma}(p_1, p_2) = g^{\nu\sigma}(p_1 \cdot p_2) - p_1^{\nu}p_2^{\sigma}$$

- Non-Gribov term in $\mathcal{A}_{gq\to gq}$

$$-s\varepsilon^{\nu}_{\perp}(k_1)\left(\varepsilon^{*}_{\perp,\beta}(k'_1) - \frac{\varepsilon^{*}_{\perp}(k'_1)\cdot k'_{1,\perp}}{k_{1'}\cdot k_2}k_{2,\beta}\right)A_{\nu}{}^{\sigma\beta}(k-q, k_1+k)\bar{u}(k'_2)\gamma_{\perp,\sigma}u(k_2)$$

$$A^{\nu\sigma\beta}(k-q, -k_1-k) = g^{\sigma\beta}(q-k_1-2k)^{\nu} + g^{\nu\sigma}(k-2q-k_1)^{\beta} + g^{\nu\beta}(2k_1+k+q)$$

What is their impact on the Regge form of the amplitude?

- Born helicity structure: $\mathcal{H}_{\rm Born} \equiv (\varepsilon_\perp(k_1)\cdot q_\perp)\,\bar{u}(k_2-q)\frac{\hat{k}_1}{s}u(k_2)$
- Using the Sudakov decomposition for q

$$\mathcal{H}_{\rm Born} = (\varepsilon_\perp(k_1)\cdot q_\perp)\,\bar{u}(k_2-q)\frac{\hat{q}_\perp}{|q_\perp|^2}u(k_2)$$

- Defining the basis: $n_q^\mu = \frac{q_\perp^\mu}{|q_\perp|}\,, \qquad n_{\tilde{q}}^\mu = \epsilon^{\mu\nu+-}\frac{q_\perp^\nu}{|q_\perp|}$
- Born structure and a non-Gribov term

$$\mathcal{H}_{\rm Born} = \left(\varepsilon_\perp(k_1)\cdot n_q\right)\bar{u}(k_2-q)\hat{n}_q u(k_2)$$

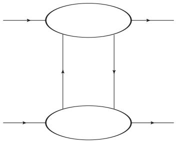
$$\bar{u}(k_2-q)\hat{\varepsilon}_\perp(k_1)u(k_2) = -\bar{u}(k_2-q)\gamma_\mu u(k_2)\left(n_q^\mu n_q^\nu + n_{\tilde{q}}^\mu n_{\tilde{q}}^\nu\right)\varepsilon_{\perp,\nu} = -\mathcal{H}_{\rm Born} - \mathcal{H}_{\rm anomalous}$$

- Taking the interference between $\mathcal{H}_{\rm Born}$ and $\mathcal{H}_{\rm anomalous}$-part and summing over fermions spin gives 0
- The anomalous helicity structure cancels completely at amplitude level
- Nonetheless, these non-Gribov terms give a total contribution

$$\delta^{\rm n.G.}_{gq\to Hq} = g^2(-2C_A)B_0(q^2) = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{2C_A}{\epsilon} - 4C_A\right]$$

Gribov part: Strategy of rapidity regions

- Strategy of rapidity regions for the calculation of two-gluon in the t-channel diagrams [V.S. Fadin, A. D. Martin (1999)] [V.S. Fadin, R. Fiore (2001)]



- Feynman gauge and Gribov's trick

$$g^{\mu\nu} = g^{\mu\nu}_{\perp\perp} + 2\frac{k_2^\mu k_1^\nu + k_2^\mu k_1^\nu}{s} \longrightarrow \frac{2k_2^\mu k_1^\nu}{s}$$

- Loop momenta decomposed à la Sudakov: $k = \beta k_1 + \alpha k_2 + k_\perp$

$$\begin{cases} \textbf{Central region} & |\alpha| \lesssim \alpha_0 \ , \ |\beta| \lesssim \beta_0 \ , \\ \textbf{Region A} & |\alpha| \lesssim \alpha_0 \ , \ |\beta| > \beta_0 \ , \\ \textbf{Region B} & |\alpha| > \alpha_0 \ , \ |\beta| \lesssim \beta_0 \ , \\ \textbf{Region C} & |\alpha| > \alpha_0 \ , \ |\beta| > \beta_0 \ , \end{cases}$$

$$\alpha_0 \ll 1 \ , \qquad \beta_0 \ll 1 \ , \qquad s\alpha_0\beta_0 \gg |t|$$

- Factorization of vertices in different rapidity regions requires that in the region $|\alpha| \ll 1$ ($|\beta| \ll 1$) we can factor out the vertex $\Gamma^{(0)}_{B'B}$ ($\Gamma^{(0)}_{A'A}$) from the diagrams.

Gribov part: Strategy of rapidity regions

- **Region C** is suppressed by a factor $|t|/\alpha_0\beta_0 s \ll 1$
- In the **Central region**

$$\mathcal{A}^{(8,-)}_{\text{box,Central}} = -g^2 C_A s^2 \Gamma^{(0)}_{A'A}\, I^{\text{central}}\, \Gamma^{(0)}_{B'B} = \Gamma^{(0)}_{A'A} \frac{2s}{t} \Gamma^{(0)}_{B'B}\, \omega^{(1)}(t) \left[\frac{1}{2}\ln\left(\frac{-s}{-t}\right) + \frac{\phi(\alpha_0)}{2} + \frac{\phi(\beta_0)}{2}\right]$$

$$\phi(z) = \ln z + \frac{1}{2}\left(-\frac{1}{\epsilon} - \psi(1) + \psi(1+\epsilon) - 2\psi(1-\epsilon) + 2\psi(1-2\epsilon)\right)$$

- Box and cross diagrams in the central region

$$\mathcal{A}^{(8,-)}_{\text{Central}} = \Gamma^{(0)}_{A'A} \frac{2s}{t} \Gamma^{(0)}_{B'B}\, \omega^{(1)}(t) \left[\frac{1}{2}\ln\left(\frac{-s}{t}\right) + \frac{1}{2}\ln\left(\frac{-s}{-t}\right) + \phi(\alpha_0) + \phi(\beta_0)\right]$$

- Correction to the upper and lower effective vertex

$$\Gamma^{(\text{Central})}_{A'A} = \Gamma^{(0)}_{A'A}\, \omega^{(1)}(t)\phi(\beta_0) \qquad \Gamma^{(\text{Central})}_{B'B} = \Gamma^{(0)}_{B'B}\, \omega^{(1)}(t)\phi(\alpha_0)$$

- Correction from the region **Region A**

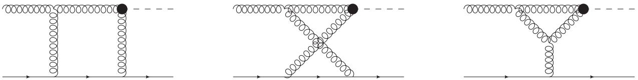
$$\Gamma^{(\text{A})}_{A'A} = \Gamma^{(0)}_{A'A} \delta^{(\text{A})}_{\text{NLO}} = \Gamma^{(0)}_{A'A} \left[-\omega(t)\ln\beta_0 + \tilde{\delta}^{(\text{A})}_{\text{NLO}}\right]$$

- Correction from the region **Region B**

$$\Gamma^{(\text{B})}_{B'B} = \Gamma^{(0)}_{B'B}\, \delta^{(\text{B})}_{\text{NLO}} = \Gamma^{(0)}_{B'B} \left[-\omega(t)\ln\alpha_0 + \tilde{\delta}^{(\text{B})}_{\text{NLO}}\right]$$

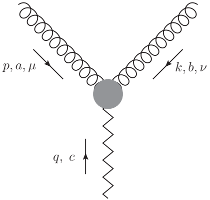
Higgs case (region A)

- In the Higgs case there is a huge simplifications between three diagrams



- **Region A** of box, cross and **triangular diagram**

$$\left[g^{\mu\nu}\frac{k_2(k-p)}{s}+\frac{k_2^\mu}{s}(2p+k)^\nu-\frac{k_2^\nu}{s}(2k+p)^\mu+\frac{2q^2}{s}\frac{k_2^\mu k_2^\nu}{k_2(p-k)}\right]\theta\left(\left|\frac{k_2(p-k)}{s}\right|-\beta_0\right)$$



(a) $g f^{acb} Y^{\mu\nu}(p,k,q)$

Higgs case (central region)

- The full result is very compact

$$\delta_{\rm NLO}^{(\rm Tri+A)} = -\omega^{(1)}(t)\ln\beta_0 + \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}$$
$$\times\left\{\frac{7}{3}\frac{C_A}{\epsilon} + \frac{85}{18}C_A + \frac{1}{6}C_A\ln\left(-\frac{m_H^2}{\vec{q}^{\,2}}\right) + 2C_A\left(\frac{\pi^2}{6} + {\rm Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)\right)\right\} + \mathcal{O}(\epsilon)$$

- The lower $\Gamma_{q'q}^{c(0)}$ vertex properly factorizes in the region $|\alpha| \ll 1$

- **Central region** of the box diagram

$$\mathcal{A}_{\rm Box,Central} = \Gamma_{q'q}^{c(0)}\left(\frac{2s}{t}\right) g_H\epsilon_\mu(k_1)\delta^{ac}\left(-\frac{g^2 C_A st}{2}\right)\frac{s}{2}\int_{-\alpha_0}^{\alpha_0}d\alpha\int_{-\beta_0}^{\beta_0}d\beta$$
$$\times\int\frac{d^{D-2}k_\perp}{(2\pi)^D i}\frac{q_\perp^\mu - k_\perp^\mu}{(\alpha\beta s + k_\perp^2 + i0)(\alpha\beta s + (q-k)_\perp^2 + i0)(-\beta s + i0)(\alpha s + i0)}$$

- The upper $\Gamma_{gH}^{ac(0)}$ vertex does not factorize in the region $|\beta| \ll 1$
- Nonetheless, in this region, we can use the symmetry of denominators under the exchange $k_\perp \to q_\perp - k_\perp$, to replace the numerator by $\frac{1}{2}q_\perp$ and obtain

$$\mathcal{A}_{\rm box,Central}^{(8,-)} = -g^2 C_A s^2 \Gamma_{q'q}^{c(0)}\, I^{\rm central}\,\Gamma_{gH}^{ac(0)}$$

- The result in this region agrees with expectations

Higgs case (region B)

- Contribution from the **Region B**

$$\mathcal{A}_{\rm B} = \Gamma^{c(0)}_{qq'} \left(\frac{2s}{t}\right) \frac{\epsilon_\mu(k_1)\delta^{ac} g_H}{2} g^2 C_A t \int_{\alpha_0}^{1} \frac{d\alpha}{\alpha} (1-\alpha)^2 \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{(q-k)^\mu_\perp}{k_\perp^2 (k_\perp - (1-\alpha) q_\perp)^2}$$

- The expectation is the correction to the quark vertex coming from this region [V.S. Fadin, R. Fiore (2001)]

- In comparison to [V.S. Fadin, R. Fiore (2001)] , an additional rapidity-divergent free term appears

$$\frac{\mathcal{A}_B}{\Gamma^{ac(0)}_{gH} \left(\frac{2s}{t}\right) \Gamma^{c(0)}_{qq'}} = \frac{g^2 C_A t}{2} \int_{\alpha_0}^{1} \frac{d\alpha}{\alpha} (1-\alpha)^2 (1+\alpha) \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}} \frac{1}{k_\perp^2 (k_\perp - (1-\alpha) q_\perp)^2}$$

- *"Anomalous" contribution* from the region B to the Higgs vertex

$$\delta^{(\rm B)}_{\rm NLO} = \frac{\bar{\alpha}_s}{4\pi} \left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon} \left[\frac{2C_A}{\epsilon} + 4C_A\right]$$

Higgs case (region B)

- Contribution from the **Region B**

$$\mathcal{A}_{\rm B} = \Gamma^{c(0)}_{qq'} \left(\frac{2s}{t}\right) \frac{\epsilon_\mu(k_1)\delta^{ac} g_H}{2} g^2 C_A t \int_{\alpha_0}^{1} \frac{d\alpha}{\alpha}(1-\alpha)^2 \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{(q-k)^\mu_\perp}{k_\perp^2 (k_\perp - (1-\alpha)q_\perp)^2}$$

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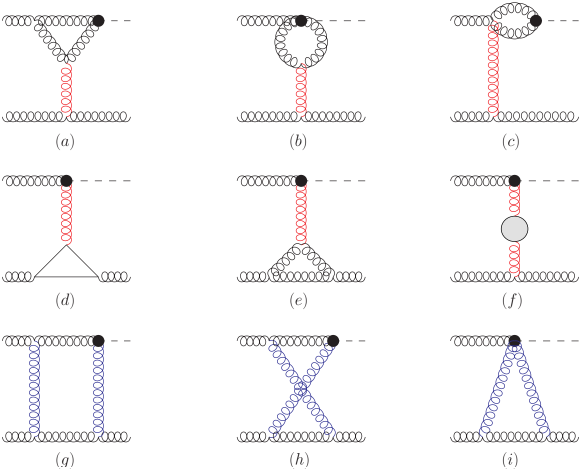
- Let's recall the *non-Gribov contribution*

$$\delta^{\rm n.G.}_{gq\to Hq} = g^2(-2C_A)B_0(q^2) = \frac{\bar{\alpha}_s}{4\pi} \left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon} \left[-\frac{2C_A}{\epsilon} - 4C_A\right]$$

$\mathcal{A}_{gg\to Hg}$ *amplitude*

- Diffusion of a gluon off a gluon to produce a Higgs plus a gluon



$\mathcal{A}_{gg\to Hg}$ amplitude

- Comparing with the Regge form

$$\mathcal{A}^{(8,-)}_{gg\to Hg} = \Gamma^{ac}_{\{H\}g} \frac{s}{t} \left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)} \right] \Gamma^{bcd(0)}_{gg} \approx \Gamma^{ac(0)}_{\{H\}g} \frac{2s}{t} \Gamma^{bcd(0)}_{gg}$$

$$+\Gamma^{ac(0)}_{\{H\}g} \frac{s}{t} \omega^{(1)}(t) \left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right) \right] \Gamma^{bcd(0)}_{gg} + \Gamma^{ac(0)}_{\{H\}g} \frac{2s}{t} \Gamma^{bcd(1)}_{gg} + \Gamma^{ac(1)}_{\{H\}g} \frac{2s}{t} \Gamma^{bcd(0)}_{gg}$$

- The extraction of the effective vertex from $\mathcal{A}_{gg\to Hg}$ leads to the same result

$$\delta_{\mathrm{NLO}} \simeq \frac{\bar{\alpha}_s}{4\pi} \left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon} \left\{ -\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon} - \frac{5n_f}{9} + C_A \left(2\mathrm{Li}_2\left(1 + \frac{m_H^2}{\vec{q}^{\,2}}\right) + \frac{\pi^2}{3} + \frac{67}{18} \right) \right\}$$

$\mathcal{A}_{gg\to Hg}$ *amplitude*

- Comparing with the Regge form

$$\mathcal{A}^{(8,-)}_{gg\to Hg} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{bcd(0)}_{gg} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{bcd(0)}_{gg} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(1)}_{gg} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

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- There are again non-Gribov contributions

$$\delta^{\mathrm{n.G.}}_{gg\to Hg} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[\frac{1}{\epsilon^2} - \frac{5}{\epsilon} - 9 - \zeta(2)\right] + \mathcal{O}(\epsilon)$$

$\mathcal{A}_{gg\to Hg}$ *amplitude*

- Comparing with the Regge form

$$\mathcal{A}^{(8,-)}_{gg\to Hg} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{bcd(0)}_{gg} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{bcd(0)}_{gg} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(1)}_{gg} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

- The extraction of the effective vertex from $\mathcal{A}_{gg\to Hg}$ leads to the same result

$$\delta_{\rm NLO} \simeq \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon} - \frac{5n_f}{9} + C_A\left(2\text{Li}_2\left(1 + \frac{m_H^2}{\vec{q}^{\,2}}\right) + \frac{\pi^2}{3} + \frac{67}{18}\right)\right\}$$

- There are again non-Gribov contributions

$$\delta^{\rm n.G.}_{gg\to Hg} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[\frac{1}{\epsilon^2} - \frac{5}{\epsilon} - 9 - \zeta(2)\right] + \mathcal{O}(\epsilon)$$

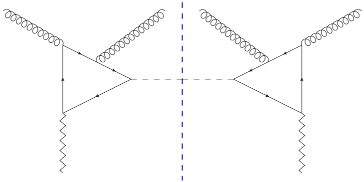
- “Anomalous” contribution from the region B

$$\delta^{\rm (B)}_{\rm NLO} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[-\frac{1}{\epsilon^2} + \frac{5}{\epsilon} + 9 + \zeta(2)\right] + \mathcal{O}(\epsilon)$$

Restoring full top-mass dependece

- Real corrections with full top-mass dependence

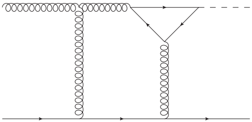
[F. G. Celiberto, L. Delle Rose, M. Fucilla, G. Gatto, A. Papa (in preparation)]



- By expanding the result in $1/m_t$ we recover the result obtained via the effective Lagrangian

  [F. G. Celiberto, M. Fucilla, D. Yu. Ivanov, M. Mohammed, A. Papa (2022)]

- Virtual corrections involve two-loop amplitudes with several scales



Conclusions

Summary

- We investigated the high-energy behaviour of the one-loop $\mathcal{A}_{gq\to Hq}$ and $\mathcal{A}_{gg\to Hg}$ amplitudes in the infinite top-mass limit
- *"Non-Gribov"* contributions endanger the Regge form of the one-loop amplitudes
- Structures not in accordance with the Regge form in different diagrams cancel each other out
- A strategy of regions reveals an anomalous contribution in the rapidity region of the quark/gluon which cancels with the part of the non-Gribov terms that can be cast in the Regge form

Outlook

- Next-to-leading order Higgs impact factor with full top-mass dependence
- Full NLO/NLL Higgs plus jet production

[F. G. Celiberto, L. Delle Rose, M. Fucilla, G. Gatto, A. Papa (in preparation)]

Thanks for your attention!

Backup