Complete NLO Single-Inclusive π^0 Production in Forward pA Collisions

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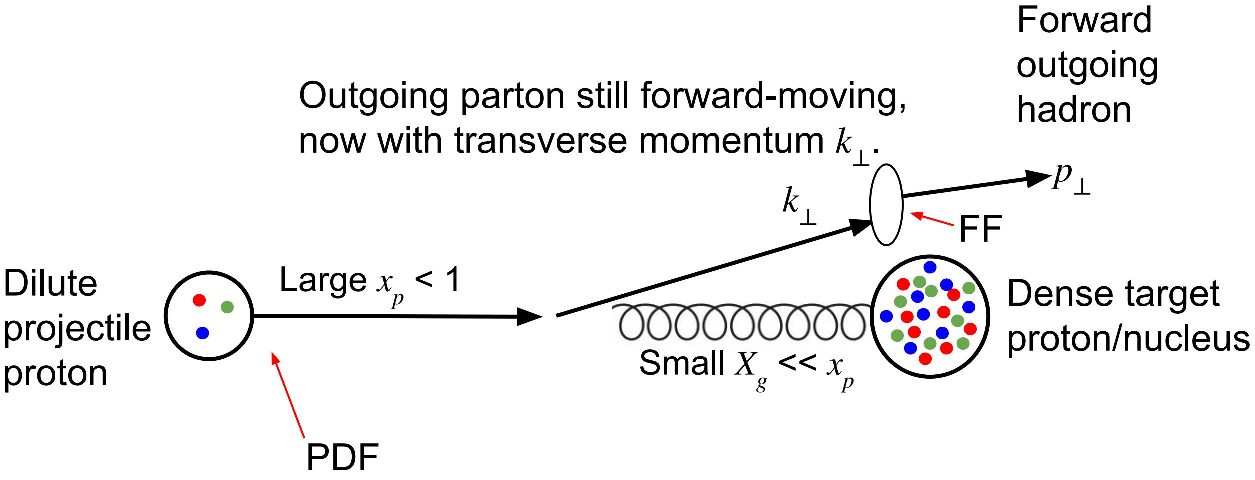
Based on: 2310.06640

Motivation

- Single-inclusive particle production provides a way to probe heavy nuclei at small Bjorken x.



Outgoing parton still forward-moving, now with transverse momentum $k_\perp$.

Forward outgoing hadron

$k_\perp$

$p_\perp$

FF

Dilute projectile proton

Large x_p < 1

PDF

Small X_g << x_p

Dense target proton/nucleus

With sq CM energy s and (large) parton rapidity y,

$$x_p = \frac{k_\perp}{\sqrt{s}} e^y$$

$$X_g = \frac{k_\perp}{\sqrt{s}} e^{-y}$$

Setup

[Chirilli, Xiao, Yuan, 1203.6139]

Hard factor

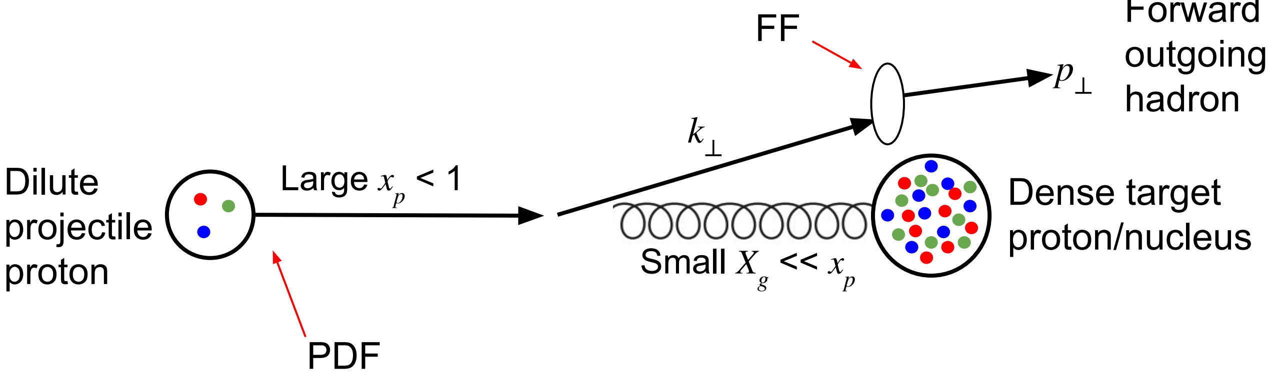
Semi-Hard factor
("The dipole")

PDF

FF

$$\sigma^{p+\mathrm{A}\to h+X} \to \sum_{a,c} f_a \otimes \mathcal{H}_{a\to c} \otimes S_{a,c} \otimes D_{h,c}$$

Three (anti)quark flavors + gluon



FF

$p_\perp$

Forward
outgoing
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$k_\perp$

Dilute
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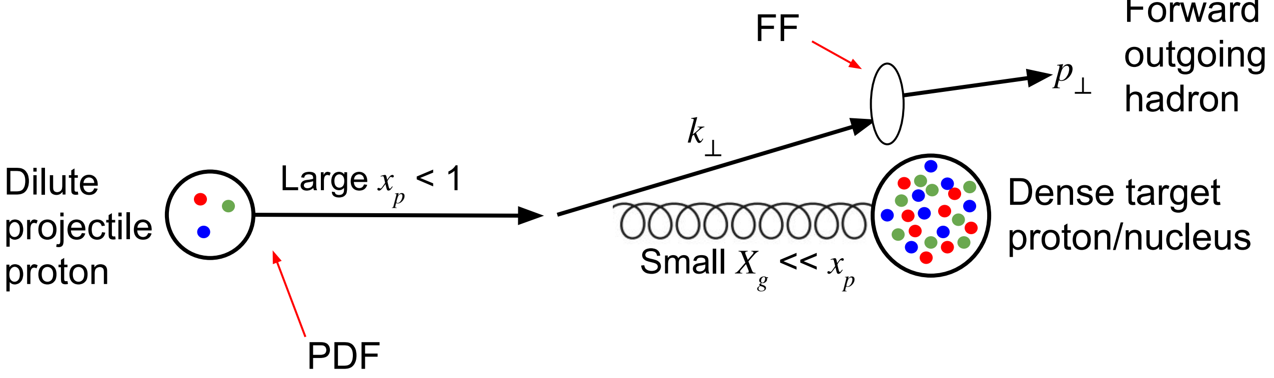

$$x_p = \frac{k_\perp}{\sqrt{s}} e^{y}$$

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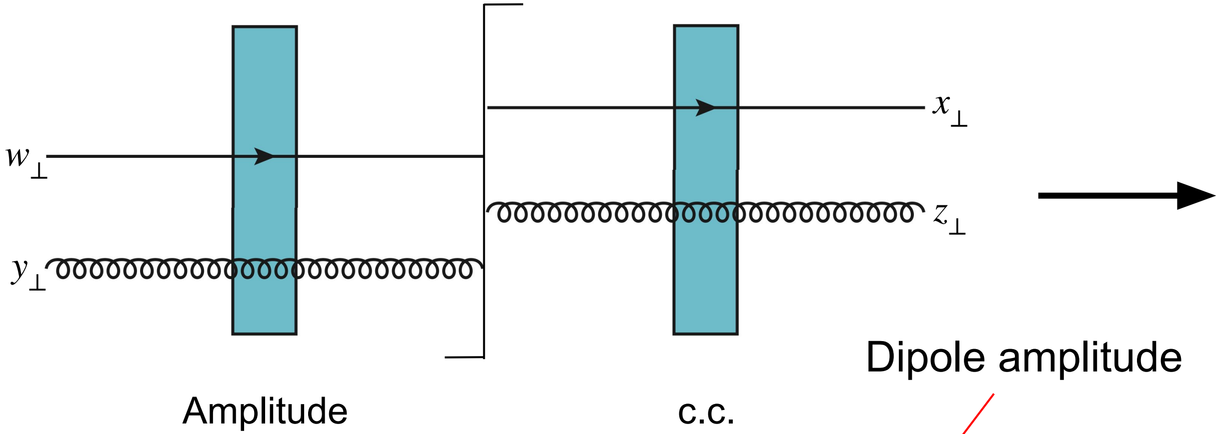
Semi-Hard Factor

- Includes soft-parton (small plus momenta) emissions by the quarks/gluons from the dilute projectile.
- Collectively expressed in terms of Wilson lines, averaged over target state.



$$\left\langle V_{w_\perp} V^\dagger_{x_\perp} \otimes U_{y_\perp} U^\dagger_{z_\perp} \right\rangle$$

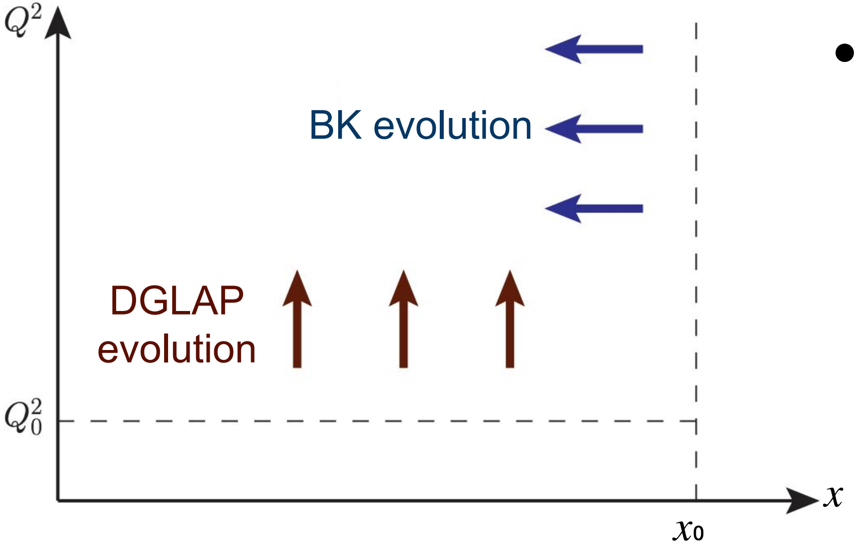
Dipole amplitude

Available rapidity interval, usually ~ $X_g \ll 1$.

- Large-N_c limit → products of $S(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, X) = \frac{1}{N_c} \left\langle \mathrm{tr} \left[V_{\boldsymbol{b}_\perp + \boldsymbol{r}_\perp/2} V^\dagger_{\boldsymbol{b}_\perp - \boldsymbol{r}_\perp/2} \right] \right\rangle_X$

Dipole Amplitude at small X

- For each $b_\perp$, determine $S(r_\perp, b_\perp, X)$ by numerically solving **BK evolution** from initial X_0 = 0.01 (relatively large).



- Renormalization group equation in ln(1/X).
- Includes non-linear effects, i.e. saturation
- At leading order (and large N_c) for each $b_\perp$,

$$\frac{\partial}{\partial \ln(1/X)} S(r_\perp = x_\perp - y_\perp, X)$$

$$= \frac{\alpha_s N_c}{2\pi^2} \int d^2 z \, \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}$$

$$\times \left[S(x_\perp - z_\perp, X)\, S(z_\perp - y_\perp, X) - S(x_\perp - y_\perp, X) \right]$$

  - In this work, we include largest NLO corrections to BK evolution.

NLO Corrections to BK

- LO: single rapidity logarithm per α_s
- NLO contains terms with double-log in anti-collinear (large daughter dipole) region.
  - ➢ Dominate the NLO corrections.
  - ➢ Needs to resum separately
- In this work, we consider 3 different resummation schemes:
  - ➢ Kinematically constraint BK (**KCBK**) [Beuf, 1401.0313]
  - ➢ Local-rapidity resum BK (**ResumBK**) [Iancu et al, 1502.05642]
  - ➢ Target momentum fraction BK (**TBK**) [Ducloué et al, 1902.06637]

  with running coupling.

Initial Condition for BK Evolution

- At $X = X_0$, we take the generalized MV model for **proton target**:

$$S_{pp}^{(0)}(r_\perp, b_\perp, X_0) = \exp\left[-\frac{(r_\perp^2 Q_{s,0}^2)^\gamma}{4} \ln\left(\frac{1}{r_\perp \underset{\nwarrow \text{0.241 GeV}}{\Lambda_{\mathrm{QCD}}}} + e\right)\right]$$

for any $b_\perp$ within the disk of area $\sigma_0/2$.

Parameters

- ➢ $Q_{s,0}$: saturation scale at $X = X_0$
- ➢ γ: “anomalous dimension”, determining the shape of S at small $r_\perp$
- ➢ $\sigma_0/2$: proton’s effective area

See also: [Casuga, Karhunen, Mäntysaari, 2311.10491]

Determined by a fit of **structure functions** to HERA data [Beuf et al, 2007.01645].

Separately for each r.c. and resummation scheme (KCBK, ResumBK, TBK)

Initial Condition for BK Evolution

- For a **nucleus target**, generalize pp via optical Glauber model [Lappi, Mäntysaari, 1309.6963]:

$$S_{pA}^{(0)}(r_\perp, b_\perp, X_0) \;=\; \exp\left[-\frac{\sigma_0}{2}\, AT_A(\boldsymbol{b}_\perp)\, \frac{(r_\perp^2 Q_{s,0}^2)^\gamma}{4} \ln\left(\frac{1}{r_\perp \Lambda_{\rm QCD}} + e\right)\right]$$

  - ➢ $T_A(b_\perp)$: Nuclear transverse thickness function, obtained from Woods-Saxon dist.
- Essentially, modify the initial saturation scale to account for nuclear profile in impact parameter, $b_\perp$.
- Evolve S separately for each discrete value of $b_\perp$.

Setup

[Chirilli, Xiao, Yuan, 1203.6139]

PDF

Hard factor

Semi-Hard factor
("The dipole")

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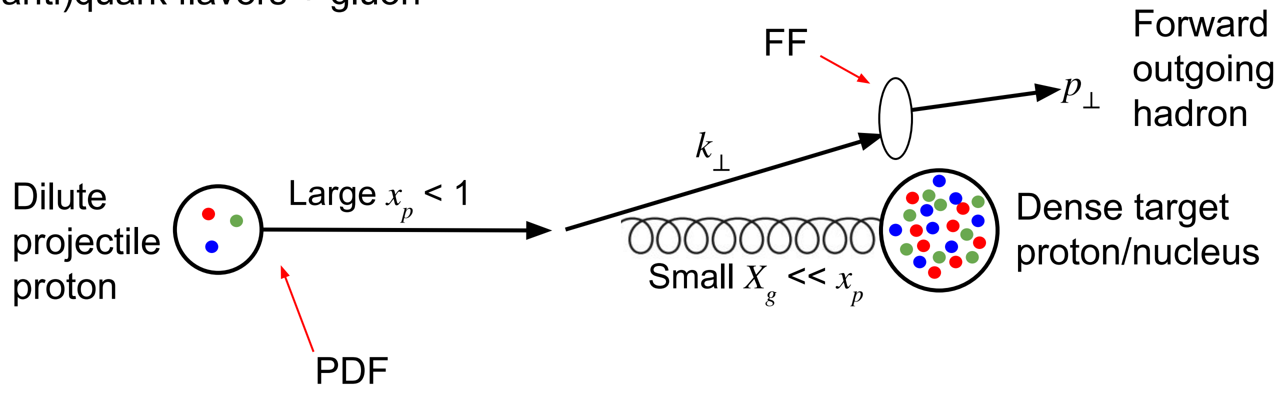
Three (anti)quark flavors + gluon



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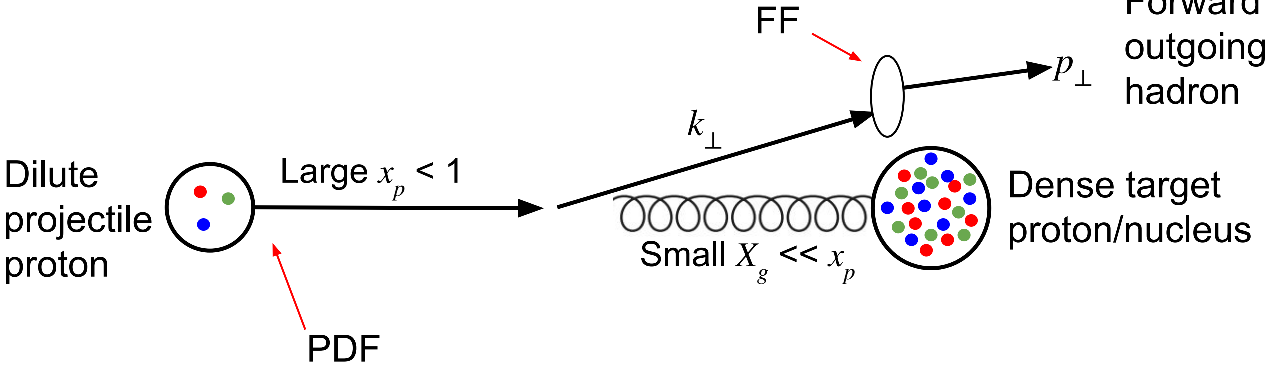


$$x_p = \frac{k_\perp}{\sqrt{s}} e^{y}$$

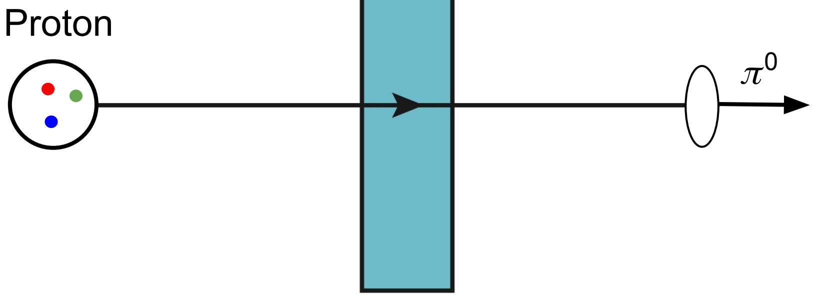
$$X_g = \frac{k_\perp}{\sqrt{s}} e^{-y}$$

Hard Factor – Leading Order

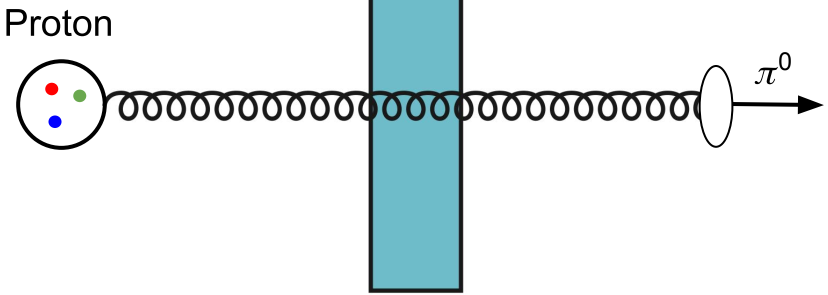
$$X_g = \frac{k_\perp}{\sqrt{s}} e^{-y}$$

- Quark(gluon) from the proton interacts with the target and fragments into π^0.
- At small X_g, interaction time scale is short → **shockwave** picture
- Transverse Fourier transform (over $r_\perp$) of the semi-hard factor (the dipoles).

q channel:



g channel:



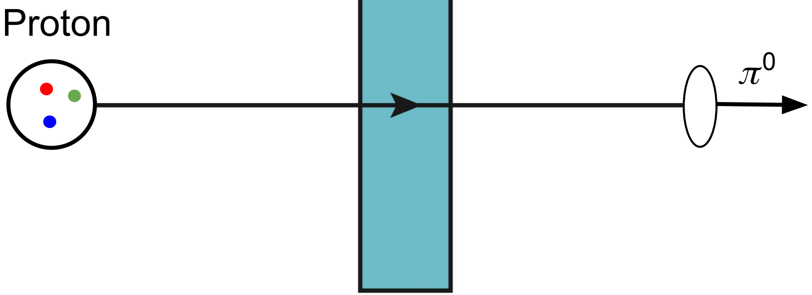
Hard Factor – Leading Order

$$X_g = \frac{k_\perp}{\sqrt{s}}\, e^{-y}$$

$$\left.\frac{\mathrm{d}\sigma^{p+A\to h+X}}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{p}_\perp}\right|_{\mathrm{LO,\ q}} = \frac{1}{4\pi^2}\int_\tau^1 \frac{\mathrm{d}z}{z^2}\, D_{h/q}(z)\, x_p\, q(x_p) \int \mathrm{d}^2\boldsymbol{b}_\perp \int \mathrm{d}^2\boldsymbol{r}_\perp\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} S(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, X_g)$$

$$\left.\frac{\mathrm{d}\sigma^{p+A\to h+X}}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{p}_\perp}\right|_{\mathrm{LO,\ g}} = \frac{1}{4\pi^2}\int_\tau^1 \frac{\mathrm{d}z}{z^2}\, D_{h/g}(z)\, x_p\, g(x_p) \int \mathrm{d}^2\boldsymbol{b}_\perp \int \mathrm{d}^2\boldsymbol{r}_\perp\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} \left[S(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, X_g)\right]^2$$

q channel:



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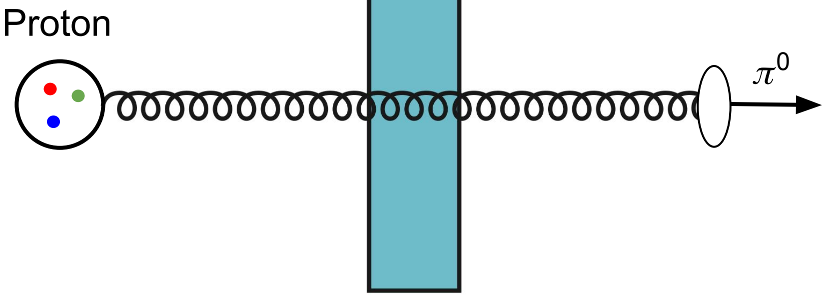
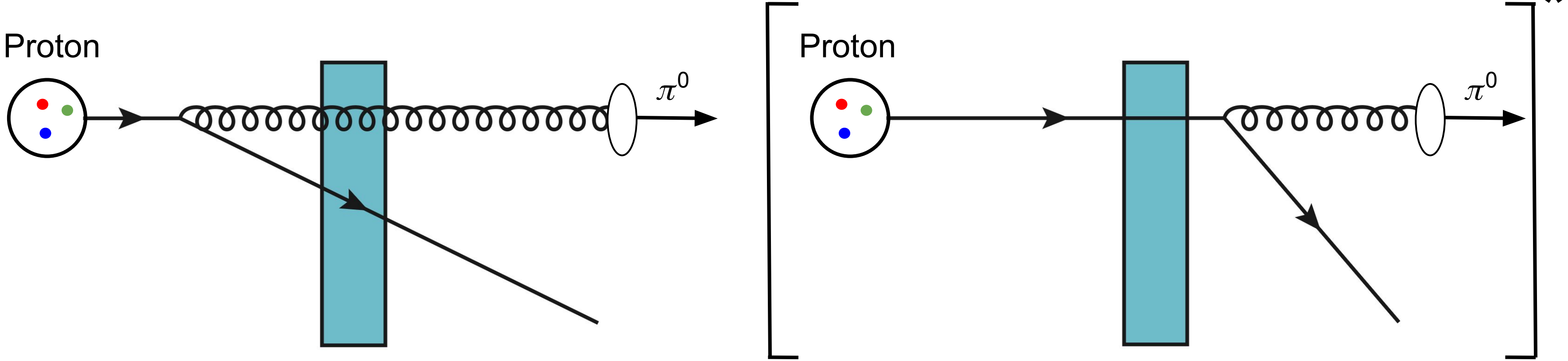


Hard Factor – NLO

- Contains one emission & absorption of **hard** parton (real & virtual)
- With real emission, only one outgoing parton is measured ("single-inclusive"). The momentum of the other outgoing parton is integrated over.

Example qg-channel contribution:



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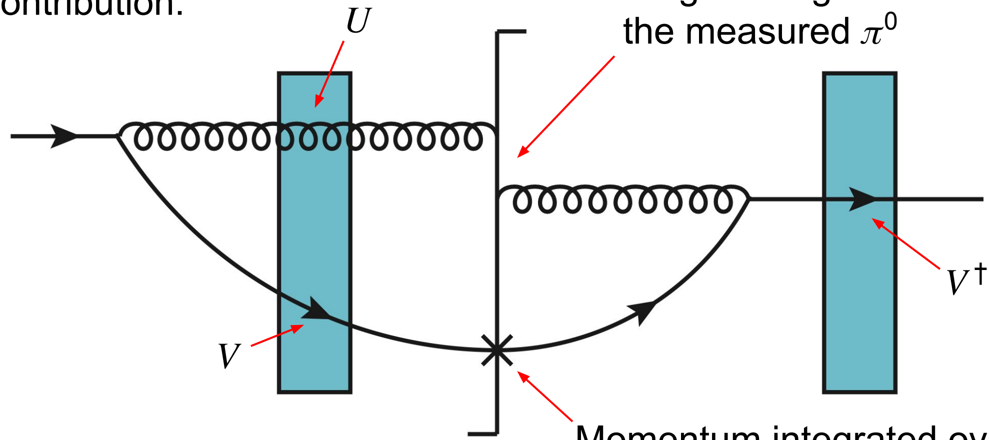


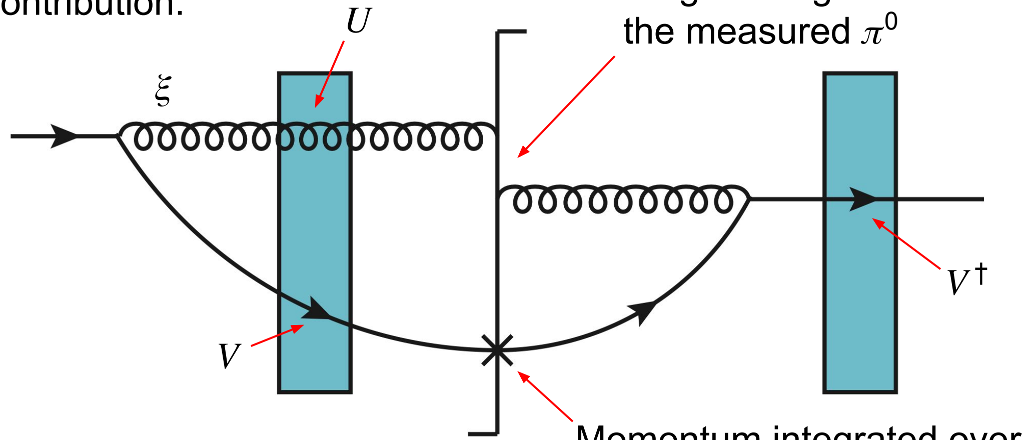
Fragmenting into the measured π^0

Momentum integrated over

Hard Factor – NLO

- The measured parton (below: gluon) has momentum fraction ξ ~ 1.
- The Wilson lines correlation is evaluated at $X(\xi) = \dfrac{X_g}{1-\xi}$ [Ducloué et al, 1712.07480].
  - ➢ Based on available rapidity interval (initially ln(1/X_g), quark emission takes away ln[1/(1-ξ)].)

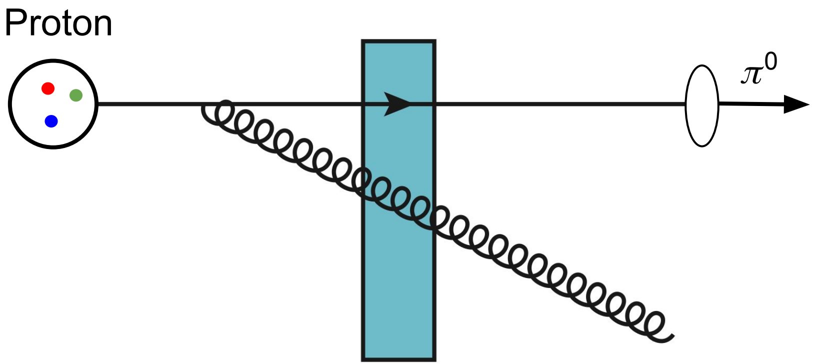
Example qg-channel contribution:

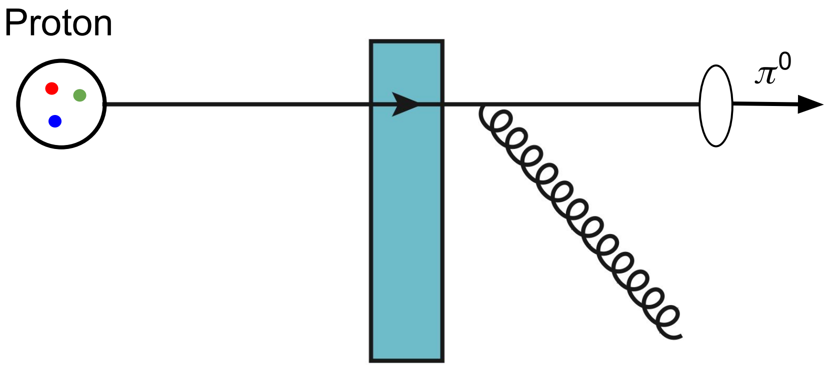


Fragmenting into the measured π^0

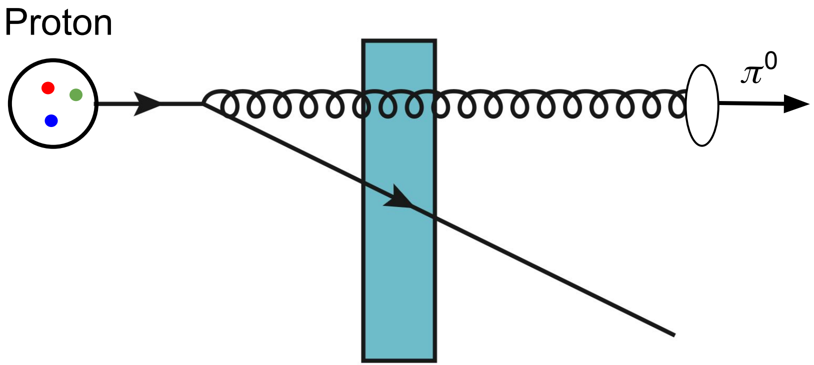
Momentum integrated over

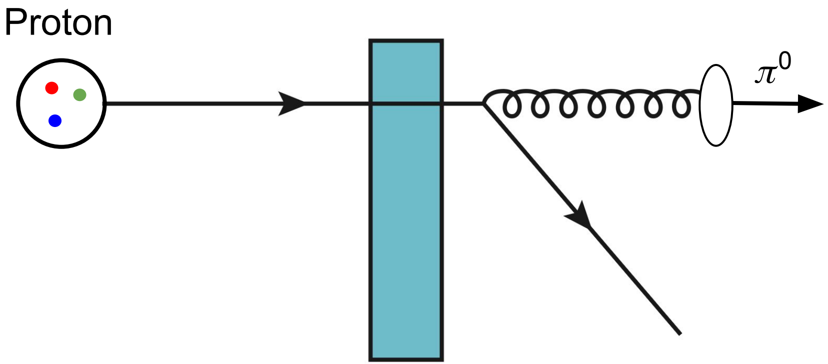
qq channel:



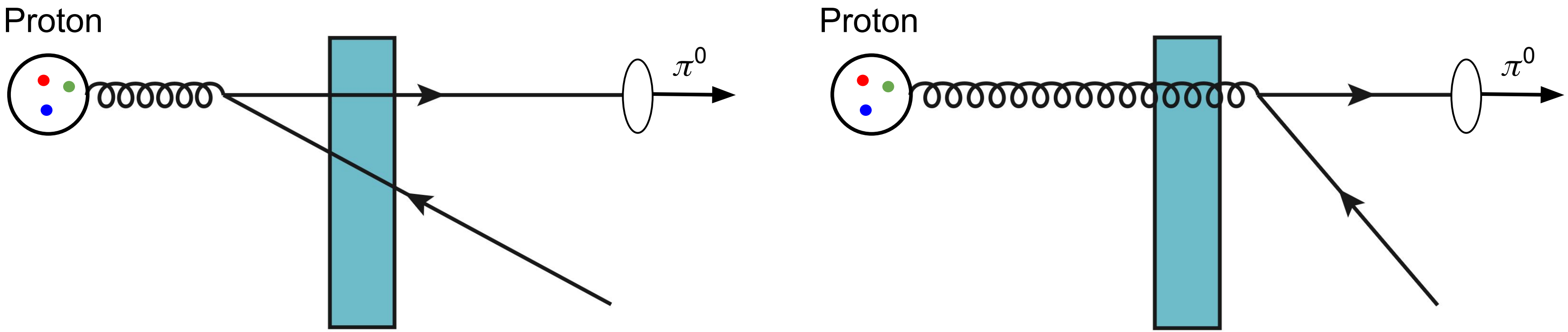


qg channel:

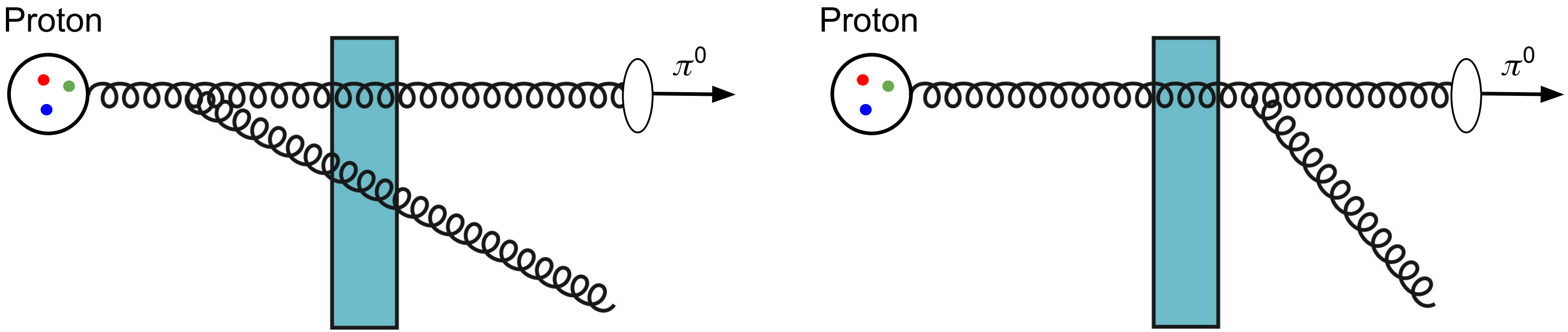




gq channel:



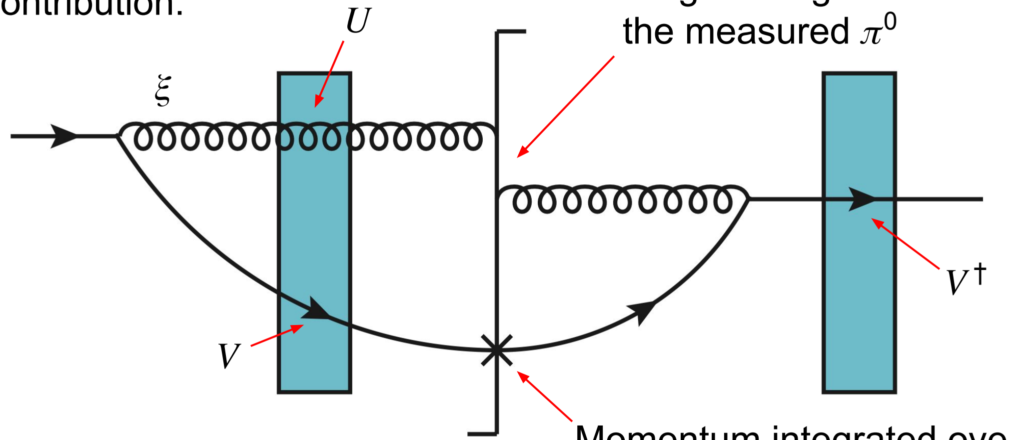
gg channel:



Collinear Divergence

- The emission contains integrals over transverse positions, which lead to collinear divergence.
  - ➢ Cancels with LO DGLAP evolution of PDF and FF [Chirilli, Xiao, Yuan, 1203.6139].

Example qg-channel contribution:



Fragmenting into the measured π^0

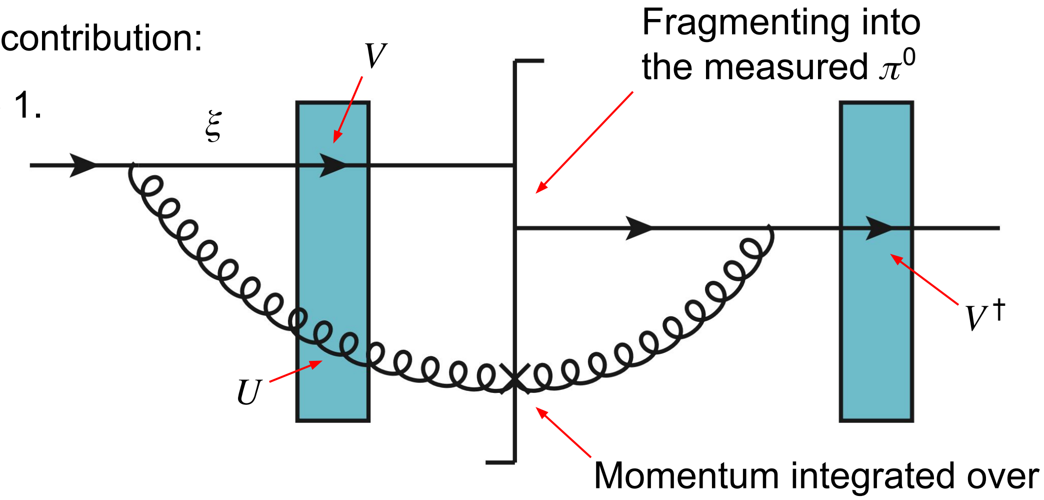
Momentum integrated over

Rapidity Divergence

- In qq and gg channels, the integrals over ξ lead to rapidity divergence.
- This corresponds to soft-parton emission included in BK evolution at LO in the hard factor [Chirilli, Xiao, Yuan, 1203.6139].

Example qq-channel contribution:

Rapidity div from $\xi \to 1$.



Fragmenting into the measured π^0

Momentum integrated over

Rapidity Divergence

- In qq and gg channels, the integrals over ξ lead to rapidity divergence.
- This corresponds to soft-parton emission included in BK evolution at LO in the hard factor [Chirilli, Xiao, Yuan, 1203.6139].
- Two “subtraction schemes”:
  - Subtracted scheme:

Subtracted ξ-integral dominated by $\xi \to 1$ and put it with LO. But making the replacement would lead to negative cross section [Ducloué et al, 1712.07480].

$$\mathcal{H}^{\mathrm{LO}} \otimes S + \mathcal{H}^{\mathrm{NLO}} \otimes S = \mathcal{K}^{\mathrm{LO}}(x_p)\, S(X_g) + \int d\xi \left[\mathcal{K}^{\mathrm{NLO}}(\xi) - \mathcal{K}^{\mathrm{NLO}}(\xi = 1)\right] S(X(\xi))$$

  - Unsubtracted scheme:

$$\mathcal{H}^{\mathrm{LO}} \otimes S + \mathcal{H}^{\mathrm{NLO}} \otimes S = \mathcal{K}^{\mathrm{LO}}(x_p)\, S(X_0) + \int d\xi\, \mathcal{K}^{\mathrm{NLO}}(\xi)\, S(X(\xi))$$

Ingredients

PDF

Hard factor

FF

$$\sigma^{p+\mathrm{A}\to h+X} \to \sum_{a,c} f_a \otimes \mathcal{H}_{a\to c} \otimes S_{a,c} \otimes D_{h,c}$$

Three (anti)quark flavors + gluon

Semi-Hard factor

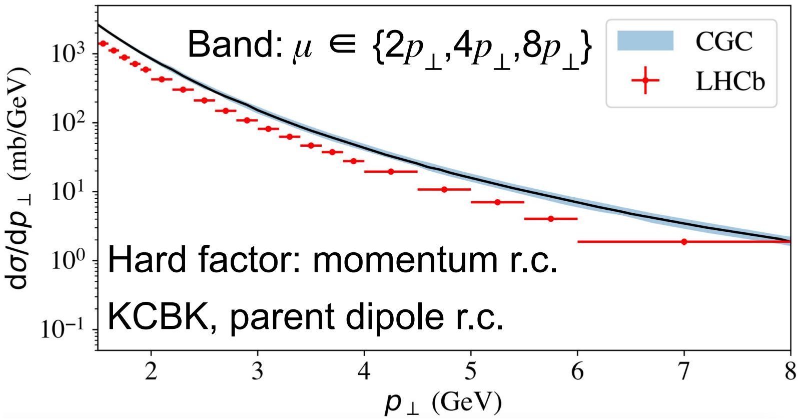
- Hard factor
  - ➢ NLO with momentum-space running coupling
  - ➢ Unsubtracted scheme (calculated all other channels except for the existing qq-channel result from [Ducloué et al, 1712.07480])
- Semi-hard factor (The dipole)
  - ➢ NLO BK evolution (KCBK [Beuf, 1401.0313], ResumBK [Iancu et al, 1502.05642] and TBK [Ducloué et al, 1902.06637]) with running coupling
  - ➢ Generalized MV model (MV$^{\gamma}$ model) for pp dipole at X_0, with parameters fitted to HERA data [Beuf et al, 2007.01645].
  - ➢ For pA, use optical Glauber model to incorporate nuclear profile [Lappi, Mäntysaari, 1309.6963].
- NLO PDF [Martin et al, 0901.0002] and FF [de Florian et al, hep-ph/0703242].

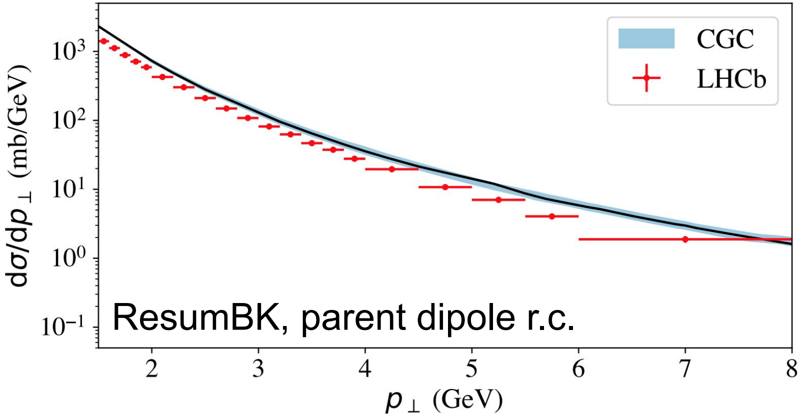
Results

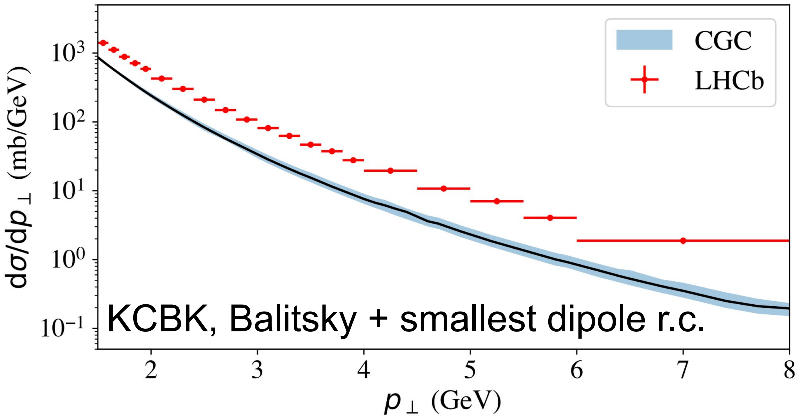
Neutral Pion Spectra (p+Pb)

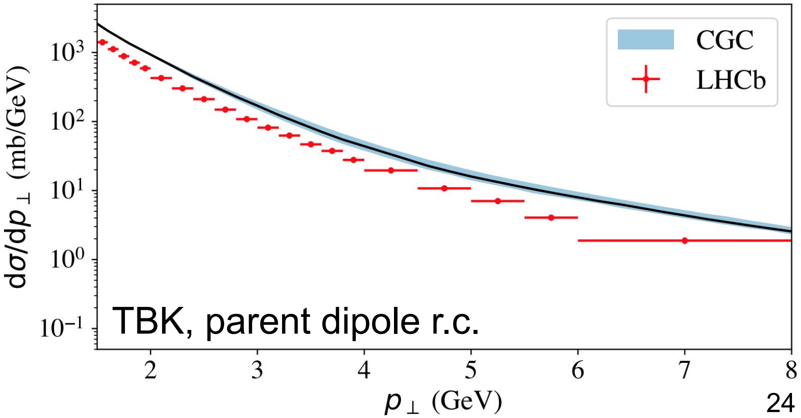
Kinematics: $y = 3$ and $\sqrt{s} = 8.16$ TeV.

LHCb: $y \in [2.5, 3.5]$ [LHCb, 2204.10608].









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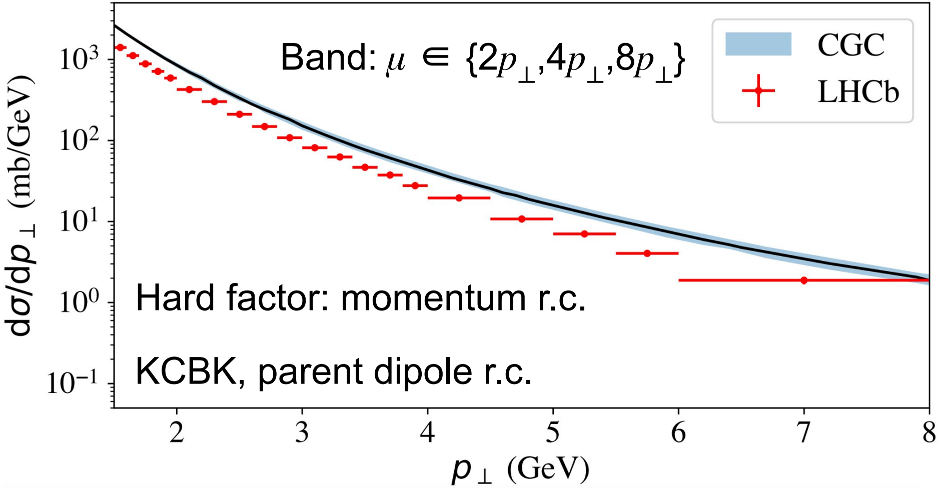
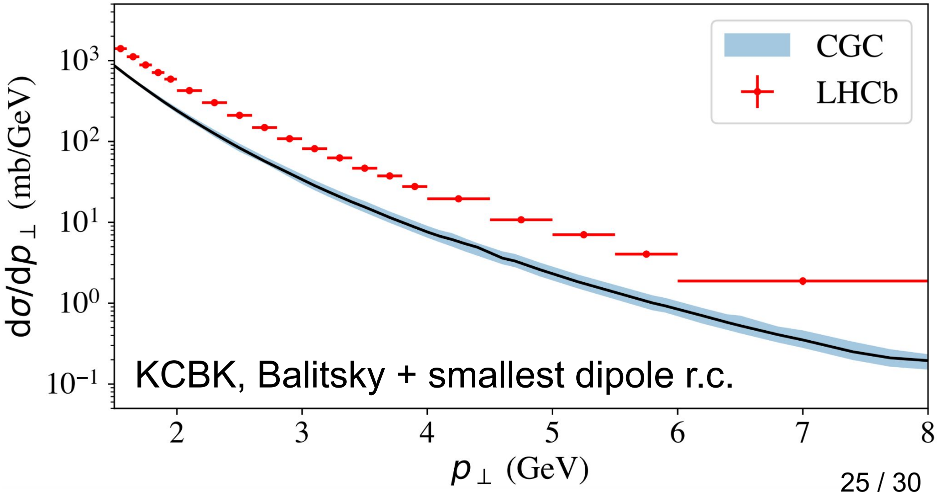
Kinematics: y = 3 and $\sqrt{s}$ = 8.16 TeV.

LHCb: $y \in [2.5, 3.5]$ [LHCb, 2204.10608].

- All cases: normalization mismatch.
- Balitsky + smallest dipole: falls more steeply than LHCb results.
- Each r.c. has different γ, $Q_{s,0}^2$ and σ_0 in the IC, such that DIS structure functions come out identical.
- Forward pA collisions put additional constraints on NLO BK parameters.





Nuclear Modification Factor

Kinematics: y = 3 and $\sqrt{s}$ = 8.16 TeV.

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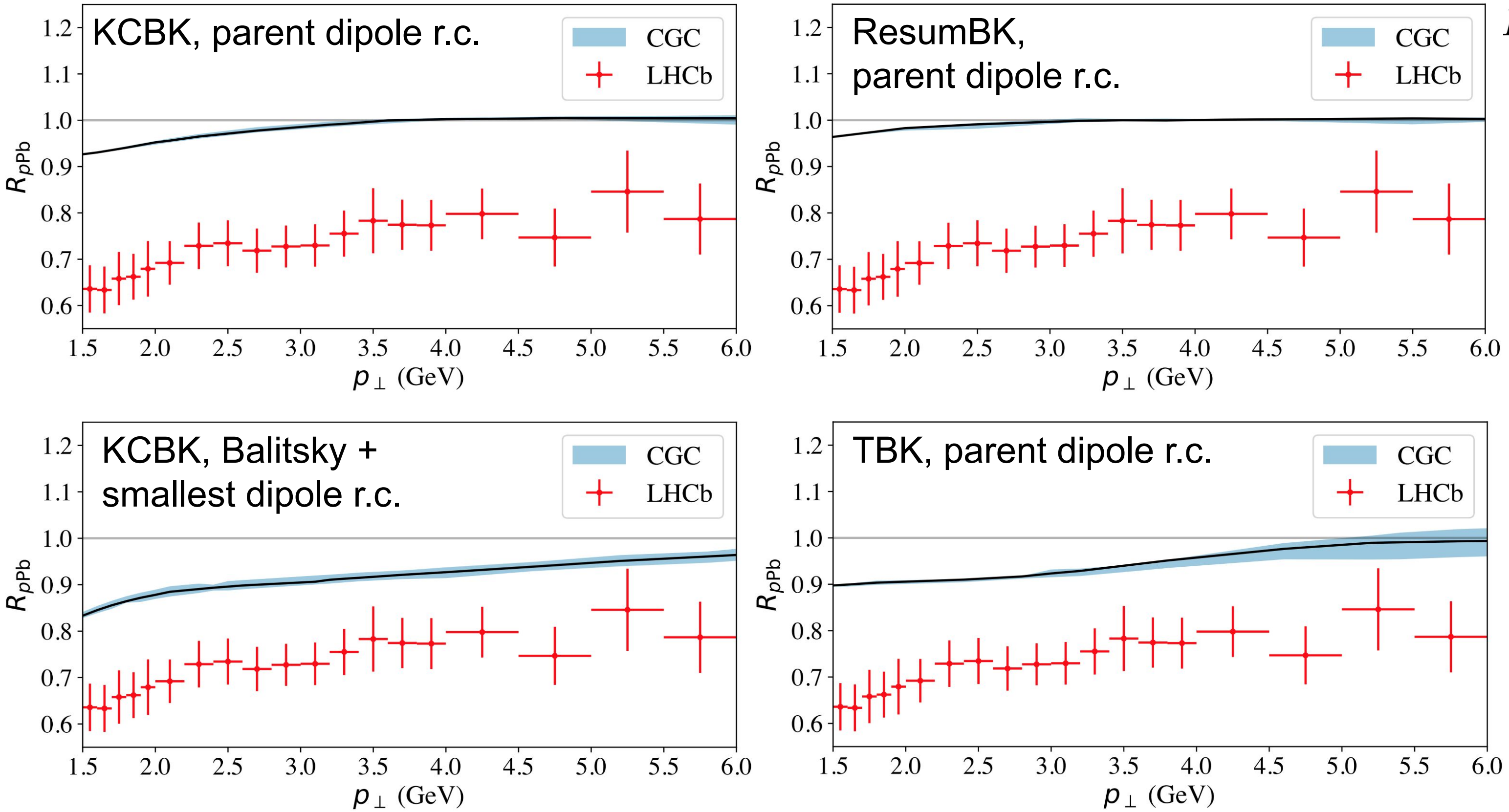
$$R_{p\mathrm{Pb}} = \frac{\mathrm{d}\sigma^{pA\to h+X}}{A\,\mathrm{d}\sigma^{pp\to h+X}}$$

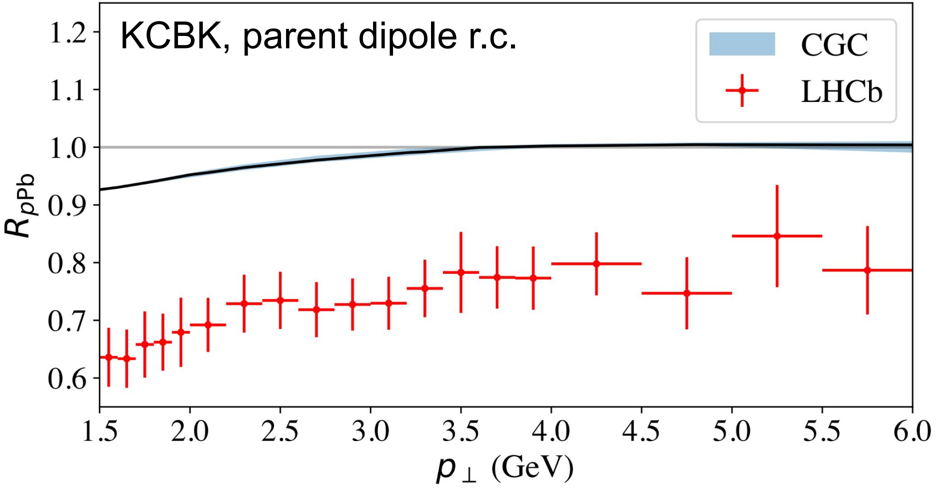


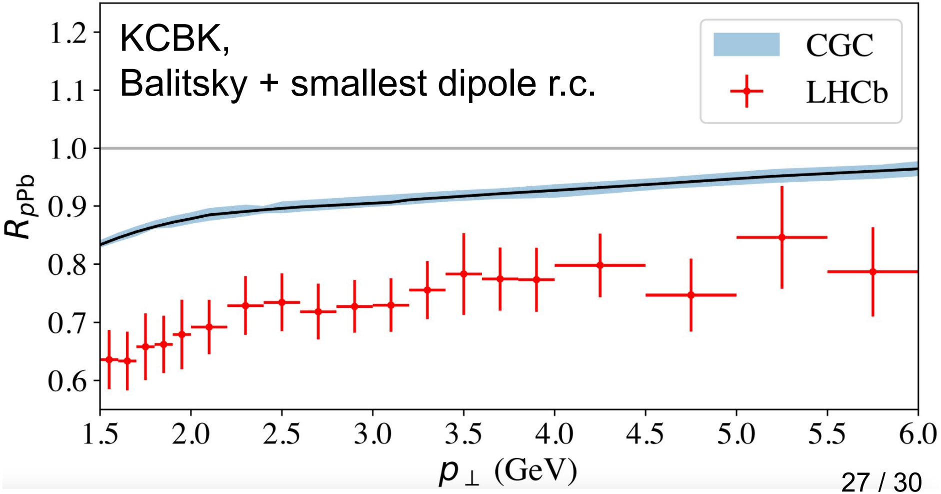
Nuclear Modification Factor

$$R_{p\mathrm{Pb}} = \frac{\mathrm{d}\sigma^{pA\to h+X}}{A\,\mathrm{d}\sigma^{pp\to h+X}}$$

Similarly for both cases,

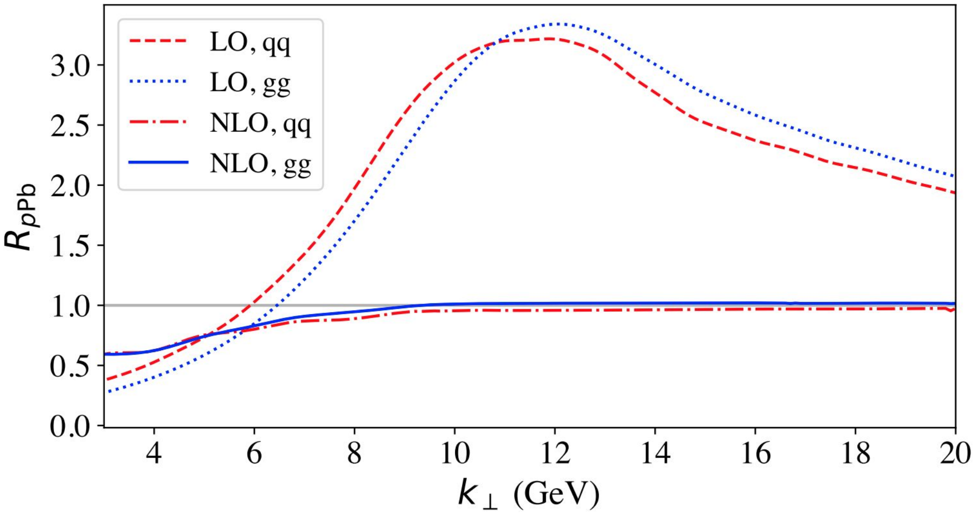
- Weak nuclear suppression at low $p_\perp$.
- $R_{p\mathrm{Pb}} \to 1$ at moderate to high $p_\perp$, overshooting LHCb data.
- Resulted from small σ_0 from the DIS fit, which is mostly sensitive to $\sigma_0 Q^2_{s,0}$.
- Small σ_0 is also preferred by other analyses, e.g. exclusive J/ψ production [Caldwell, Kowalski, 0909.1254].





Cronin Effect at Parton Level

- Large Cronin peak at $k_\perp \approx$ 12 GeV for LO. So, NLO corrections to BK have qualitative effects on R_{pA}.
- Since measurements have no Cronin peak, need to include NLO corrections to all ingredients.
- Weaker low-$k_\perp$ nuclear suppression at NLO due to its 1→2 kinematics.
- NLO case approaches $R_{\mathrm{pA}} \to 1$ at much lower $k_\perp$ compared to $\mathcal{H}^{\mathrm{LO}} \otimes S^{\mathrm{LO}}$ case (not shown), since NLO BK preserves qualitative features of IC.



<u>Kinematics</u>: y = 3 and $\sqrt{s}$ = 8.16 TeV

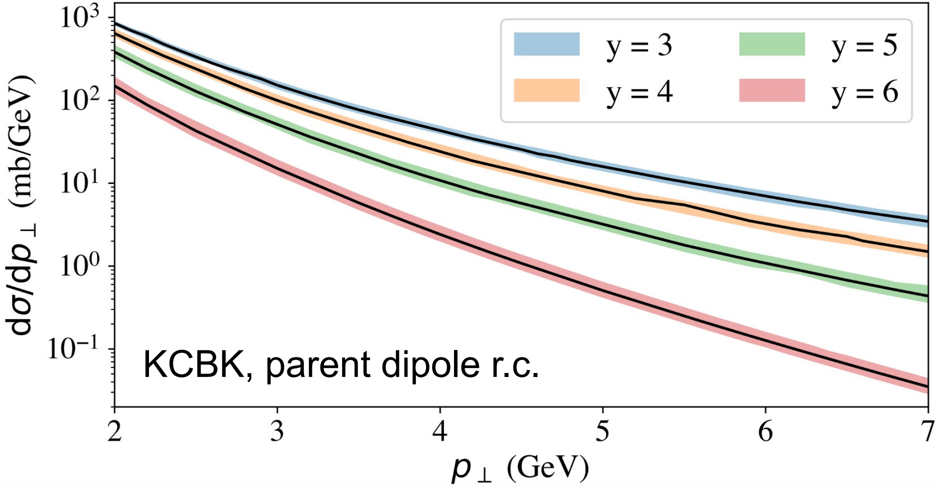
Factorization scale: $\mu = 4k_\perp$

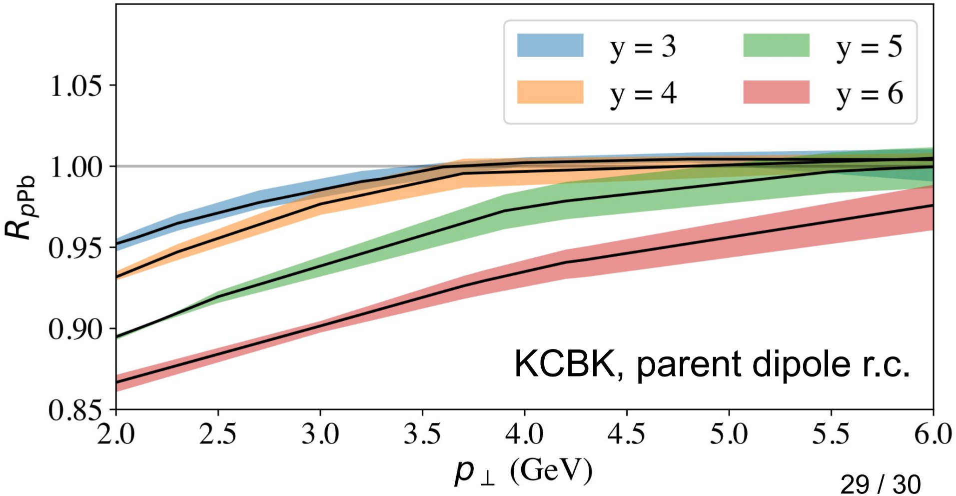
LO: $\mathcal{H}^{\mathrm{LO}} \otimes S^{\mathrm{NLO}}$, NLO: $\mathcal{H}^{\mathrm{NLO}} \otimes S^{\mathrm{NLO}}$

Rapidity Dependence

Kinematics: $\sqrt{s}$ = 8.16 TeV.

- Spectra suppressed as y increases, since PDFs vanish as $x_p \to 1$.
- Stronger low-$p_\perp$ nuclear suppression at larger y because nuclear saturation scale increases.
- Still see $R_{p\mathrm{Pb}} \to 1$ at high $p_\perp$ for all y.
- Qualitatively consistent with the charged hadron data from LHCb [LHCb, 2108.13115]. Here, we get a slightly weaker y dependence.





Conclusion and Outlook

[Mäntysaari, Tawabutr, 2310.06640]

- For the first time, we compute the forward single inclusive hadron production with **NLO hard factor** and **NLO dipole.** The latter employs parameters fitted to HERA structure function data.
- NLO corrections have significant effects on π^0 spectra and $R_{p\mathrm{Pb}}$.
- The spectra qualitatively agree with the LHCb π^0 data, while $R_{p\mathrm{Pb}}$ overestimates LHCb data and approaches 1 at high $p_\perp$.
- **This calls for a comprehensive global analysis of NLO BK evolution, including both DIS and forward pA collision data.**
- The NLO corrections to dipole's BK evolution is important in **removing the Cronin peak** that comes in with NLO corrections to the hard factor.
- Spectra and $R_{p\mathrm{Pb}}$ are suppressed at high rapidities, in qualitative agreement with LHCb charged hadron data.

Parameters for Initial Condition of BK Equation

Resummation + r.c. schemes for BK	$Q_{s,0}^2$ (GeV^2)	γ	$\sigma_0/2$ (mb)
KCBK, parent dipole r.c.	0.0833	0.98	9.74
ResumBK, parent dipole r.c.	0.0964	0.98	7.66
TBK, parent dipole r.c.	0.0917	0.90	6.19
KCBK, Balitsky + smallest dipole r.c.	0.0905	1.21	8.68

All cases: X_0 = 0.01 and momentum-space r.c. in the hard factor