Small x resummation of photon impact factors and the $\gamma^*\gamma^*$ high energy scattering

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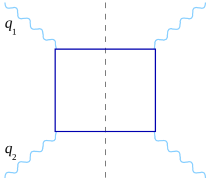
Outline

- BFKL approach to $\gamma^* \gamma^*$ scattering
  - High-energy factorization formula
  - BFKL kernel and gluon Green's function
  - Impact factors
- DGLAP (collinear) description
- Renormalization group improved approach
- Results

$\gamma^* \gamma^*$ at lowest orders

- At LO, interaction is mediated by fermion lines (quark box + cross)
- Spin $j = 1/2$ exchange $\Longrightarrow \sigma_0^{\mathrm{box}} \sim s^{2(j-1)} = 1/s$
- Dominates at low energies



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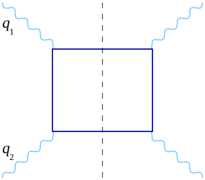
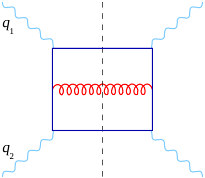


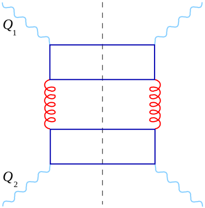
Perturbative corrections to this diagram provide logarithmic corrections

$$\sigma = \sigma_0^{\rm box}[1 + \alpha_{\rm s} \log s + (\alpha_{\rm s} \log s)^2 + \cdots]$$



BFKL approach to $\gamma^* \gamma^*$ scattering

- A constant cross section is obtained by exchanging a gluon, which couples to photons via quark lines.
- Spin $j = 1$ exchange $\Longrightarrow \sigma_0 \sim \alpha_s^2 \, s^{2(j-1)} = \text{const}$



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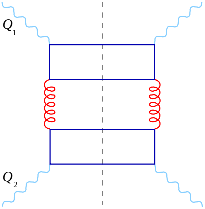
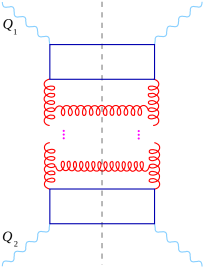


Perturbative corrections to this diagram provide logarithmic terms

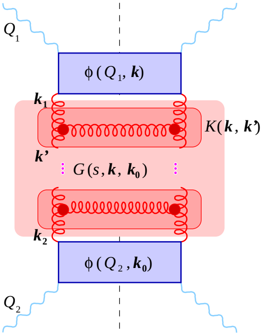
$$\sigma \simeq \alpha_s^2 \sum_{n=0} c_n [\alpha_s log(s)]^n$$

$\alpha_s \log s \sim 1 \Longrightarrow$ all-order resummation



BFKL approach for $\gamma^* \gamma^*$

- The PT series of the leading $\log s$ can be organized as the sum of effective ladder-like diagrams with gluon exchanges in the t-channel and real gluon emissions with strongly ordered rapidities.
- The effective vertex is the celebrated Lipatov's vertex.
- By squaring of the Lipatov's vertex and including LL virtual terms (correction to the gluon trajectory), each rung is described by the so-called BFKL kernel $K(\boldsymbol{k}_1, \boldsymbol{k}')$.



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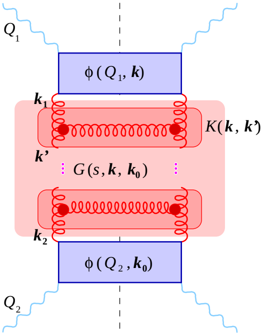
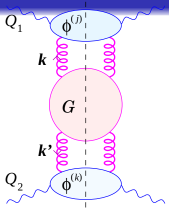


- Due to the extreme Lorentz contraction, the dynamics is essentially transverse
- Integrating over the (ordered) gluon rapidities generates the powers $(\log s)^n$
- the GGF (the blob with 4 gluon legs) obeys the BFKL equation

$$\frac{\partial}{\partial \log s} G(s, \boldsymbol{k}_1, \boldsymbol{k}_2) = \delta^2(\boldsymbol{k}_1 - \boldsymbol{k}_2) + \alpha_s \int \mathrm{d}^2\boldsymbol{k}' \; K(\boldsymbol{k}_1, \boldsymbol{k}') G(s, \boldsymbol{k}', \boldsymbol{k}_2)$$

BFKL approach for $\gamma^* \gamma^*$



High-energy factorization formula
(Mellin variables $s \leftrightarrow \omega$, $Q^2, \boldsymbol{k}^2 \leftrightarrow \gamma$)

$$\sigma^{(j,k)}(s, Q_1, Q_2)$$
$$= \int \mathrm{d}^2\boldsymbol{k}\mathrm{d}^2\boldsymbol{k}_0 \; \phi^{(j)}(Q_1, \boldsymbol{k}) G(s, \boldsymbol{k}, \boldsymbol{k}_0) \phi^{(k)}(Q_2, \boldsymbol{k}_0)$$
$$= \frac{1}{Q_1 Q_2} \int \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \left(\frac{s}{Q_1 Q_2}\right)^{\omega} \int \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}} \left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma} \phi^{(j)}(\gamma) G(\omega, \gamma) \phi^{(k)}(1-\gamma)$$

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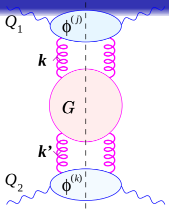

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BFKL equation and solution in LLA

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$$\omega \quad G(\omega,\gamma) \quad = \quad 1 \quad + \quad \alpha_{\mathrm{s}} \quad \chi(\gamma) \quad G(\omega,\gamma) \quad \Longrightarrow \quad G(\omega,\gamma) = \frac{1}{\omega - \alpha_{\mathrm{s}}\chi(\gamma)}$$

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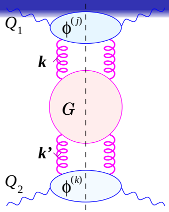

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$$= \frac{1}{Q_1 Q_2} \int \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \left(\frac{s}{Q_1 Q_2}\right)^{\omega} \int \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}} \left(\frac{Q_1^2}{Q_2^2}\right)^{\gamma} \phi^{(j)}(\gamma) G(\omega, \gamma) \phi^{(k)}(1-\gamma)$$

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$\chi = \chi_0 + \alpha_s \chi_1$
$\phi = \alpha_s \phi_0 + \alpha_s^2 \phi_1$ are known at LO and NLO *[BFKL, Camici, Ciafaloni]* *[Catani et al, Balitsky, Chirilli, Ivanov et al]*

BFKL approach for $\gamma^* \gamma^*$

LL kernel in Mellin space ($\gamma \leftrightarrow \boldsymbol{k}^2$)

$$G(\omega, \gamma) = \frac{1}{\omega - \alpha_s \chi(\gamma)}$$

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

Solution to the intercept (growth exponent for $s \gg Q^2$)

$$G(s, Q^2, Q^2) \sim s^{\omega_{\mathbb{P}}} , \qquad \omega_{\mathbb{P}} = \alpha_s \chi(1/2) \simeq 2.7\alpha_s$$

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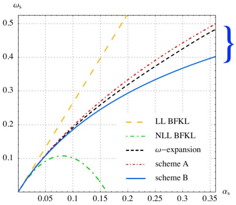

- NLL correction s to BFKL are large and negative causing instability of the BFKL expansion
- Improvement is needed
  [Ciafaloni, DC, Salam, Stasto]
  $\chi_0(\omega, \gamma) = 2\psi(1) - \psi(\gamma + \frac{\omega}{2}) - \psi(1 - \gamma + \frac{\omega}{2})$

Structure of BFKL kernel

$$K(\boldsymbol{k},\boldsymbol{k}') = \frac{1}{\boldsymbol{k}^2}\int \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}} \left(\frac{\boldsymbol{k}^2}{\boldsymbol{k}'^2}\right)^{\gamma} \chi(\gamma)$$

$$\boldsymbol{k} \gg \boldsymbol{k}' \leftrightarrow \gamma \simeq 0 \,, \qquad \frac{1}{\boldsymbol{k}^2}\log^m \frac{\boldsymbol{k}^2}{\boldsymbol{k}'^2} \leftrightarrow \frac{1}{\gamma^{1+m}}$$

$$\boldsymbol{k} \ll \boldsymbol{k}' \leftrightarrow \gamma \to 1$$

$$K_0(\boldsymbol{k},\boldsymbol{k}') = \frac{1}{|\boldsymbol{k}-\boldsymbol{k}'|^2} + \mathrm{virt} \simeq \frac{\Theta(\boldsymbol{k}-\boldsymbol{k}')}{\boldsymbol{k}^2} + \frac{\Theta(\boldsymbol{k}'-\boldsymbol{k})}{\boldsymbol{k}'^2}$$

$$\chi_0(\gamma) \simeq \frac{1}{\gamma} + \frac{1}{1-\gamma}$$

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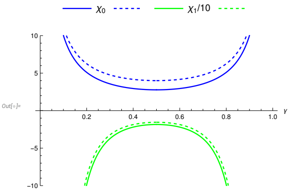
$$\chi_0(\gamma) \simeq \frac{1}{\gamma} + \frac{1}{1-\gamma}$$

$$\chi_1(\gamma) \simeq \frac{-1}{2\gamma^3} + \frac{A_1}{\gamma^2} + (\gamma \to 1 - \gamma)$$



Change of energy scale and ω-shift

The presence of the cubic poles in χ_1 is due to a "wrong" choice of energy scale. Consider the Mellin representation of the GGF

$$G(s, \boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{1}{\boldsymbol{k}_1 \boldsymbol{k}_2} \int \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \left(\frac{s}{s_0}\right)^{\omega} \int \frac{\mathrm{d}\gamma}{2\pi\mathrm{i}} \left(\frac{\boldsymbol{k}_1^2}{\boldsymbol{k}_2^2}\right)^{\gamma} \frac{1}{\omega - \alpha_\mathrm{s}\chi(\gamma)}$$

In collinear limit $\boldsymbol{k}_1 \gg \boldsymbol{k}_2$ the "natural" energy scale $s_0 = \boldsymbol{k}_1^2$, because $s/\boldsymbol{k}_1^2 \simeq 1/x_\mathrm{Bj}$
However, if we adopt the symmetric scale $s_0' = \boldsymbol{k}_1 \boldsymbol{k}_2$

$$\left(\frac{s}{\boldsymbol{k}_1^2}\right)^{\omega} \left(\frac{\boldsymbol{k}_1^2}{\boldsymbol{k}_2^2}\right)^{\gamma} = \left(\frac{s}{\boldsymbol{k}_1 \boldsymbol{k}_2}\right)^{\omega} \left(\frac{\boldsymbol{k}_2}{\boldsymbol{k}_1}\right)^{\omega} \left(\frac{\boldsymbol{k}_1^2}{\boldsymbol{k}_2^2}\right)^{\gamma} = \left(\frac{s}{\boldsymbol{k}_1 \boldsymbol{k}_2}\right)^{\omega} \left(\frac{\boldsymbol{k}_1^2}{\boldsymbol{k}_2^2}\right)^{\gamma - \frac{\omega}{2}}$$

corresponding to an ω-shift $\gamma \to \gamma + \frac{\omega}{2}$ in the integrand and thus in the eig. func. χ.
The position of the ω-pole in the factorization formula is given by

$$\omega = \alpha_\mathrm{s}\chi(\gamma) = \mathcal{O}(\alpha_\mathrm{s})$$

In the collinear limit with $s_0 = \boldsymbol{k}_1^2$ we have at LO $\chi(\gamma) \simeq \frac{1}{\gamma} \Longrightarrow \omega \simeq \alpha_\mathrm{s}\frac{1}{\gamma}$
With symmetric $s_0 = \boldsymbol{k}_1 \boldsymbol{k}_2$ we find

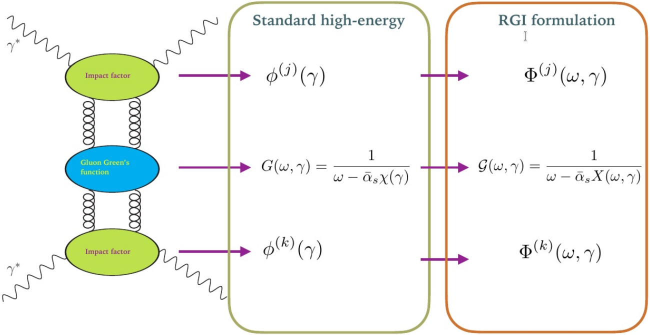
$$\omega \simeq \alpha_\mathrm{s} \frac{1}{\gamma + \frac{\omega}{2}} \simeq \alpha_\mathrm{s} \left(\frac{1}{\gamma} - \frac{\omega}{2\gamma^2} + \mathcal{O}\left(\omega^2\right)\right) \simeq \alpha_\mathrm{s} \left(\frac{1}{\gamma} - \frac{\alpha_\mathrm{s}}{2\gamma^3} + \mathcal{O}\left(\alpha_\mathrm{s}^2\right)\right)$$

Renormalization group improvement

- The functional form of the eigenvalue function (and also of the impact factors) depends on the choice of the energy scale s_0
- In the collinear limit $Q_1 \gg Q_2$, the "physical" γ-poles, i.e., the standard single-logarithmic terms, of the cross section $\alpha_s^m/\gamma^{m+1} \quad \Longleftrightarrow \quad \alpha_s^m \log^m Q_1^2/Q_2^2$ are found only for $s_0 = Q_1^2$
- In the anti-collinear limit $Q_2 \gg Q_1$ the "physical" poles at $\gamma = 1$ are found only for $s_0 = Q_2^2$
- At symmetric scale $s_0 = Q_1 Q_2$ both $\chi(\gamma)$ and impact factors develop spurious poles α_s^m/γ^{2m+1} and $\alpha_s^m/(1-\gamma)^{2m+1}$, i.e., double logarithmic terms
- Such spurious poles can be resummed to all orders by allowing both eigenvalue function and impact factors to be ω-dependent with collinear (anti-collinear) poles are at $\gamma = -\omega/2 \quad (\gamma = 1 + \omega/2)$
- The RGI LO eigenv. function is $X_0(\omega, \gamma) = 2\psi(1) - \psi(\gamma + \frac{\omega}{2}) - \psi(1 - \gamma + \frac{\omega}{2})$

  In momentum space $K(z, k, k')$ has kinematical constraints $\boldsymbol{k}_1 z < \boldsymbol{k}_2 < \boldsymbol{k}_1/z$

Renormalization group improvement



Consistency conditions at $\omega = 0$ $\quad \chi_1(\gamma) = X_1(0,\gamma) + \chi_0(\gamma)\partial_\omega X_0(0,\gamma)$

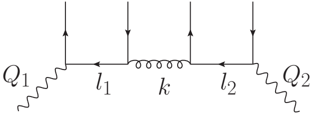
$$\phi_0^{(j)}(\gamma)\phi_1^{(k)}(1-\gamma) + \phi_1^{(j)}(\gamma)\phi_0^{(k)}(1-\gamma) = \Phi_0^{(j)}(0,\gamma)\left[\Phi_1^{(k)}(0,1-\gamma) + \chi_0(1-\gamma)\partial_\omega \Phi_0^{(k)}(0,1-\gamma)\right]$$
$$+ \left[\Phi_1^{(j)}(0,\gamma) + \chi_0(\gamma)\partial_\omega \Phi_0^{(j)}(0,\gamma)\right]\Phi_0^{(k)}(0,1-\gamma)$$
$$+ \Phi_0^{(j)}(0,\gamma)\Phi_0^{(k)}(0,1-\gamma)\partial_\omega X_0(0,\gamma)$$

Renormalization group improvement

- Compute cross section in (anti-)collinear limit, according to standard DGLAP evolution at leading log Q_1^2/Q_2^2, so as to determine the γ-pole structure of the cross section in Mellin space
- Factorize such cross section in RGI impact factors and GGF. This determines their (anti)-collinear poles.
- Expand impact factors in $\omega \sim \alpha_{\mathrm{s}}$ so as to reproduce spurious and physical poles of standard BFKL impact factors
- Complete the RGI impact factor with the "regular" remainder of the BFKL ones
- Use RGI factorization formula to compute the cross section

Transverse impact factor

Consider LO cross section for transverse virtual photons with one gluon exchange



$$\sigma_0^{(TT)}(s, Q_1 \gg Q_2) \simeq \frac{4\pi\alpha^2}{Q_1^2} F_T(x_{\rm Bj}, Q_1^2; Q_2^2) \,, \qquad x_{\rm Bj} \simeq Q_1^2/s$$

$$= \int_{x_{\rm Bj}}^{1} \frac{{\rm d}z_1}{z_1} \frac{{\rm d}z_k}{z_k} \frac{{\rm d}z_2}{z_2} \Theta(z_1 < z_k < z_2) \delta\left(1 - \frac{x_{\rm Bj}}{z_1}\right) P_{qg}\left(\frac{z_1}{z_k}\right) P_{gq}\left(\frac{z_k}{z_2}\right) P_{q\gamma}(z_2)$$

$$\times \int_{Q_2^2}^{Q_1^2} \frac{{\rm d}\boldsymbol{l}_1^2}{\boldsymbol{l}_1^2} \frac{{\rm d}\boldsymbol{k}^2}{\boldsymbol{k}^2} \frac{{\rm d}\boldsymbol{l}_2^2}{\boldsymbol{l}_2^2} \Theta(\boldsymbol{l}_2^2 < \boldsymbol{k}^2 < \boldsymbol{k}_1^2) \alpha\Big(\sum_q e_q^2\Big) \frac{\alpha_{\rm s}(\boldsymbol{l}_1^2)}{2\pi} \frac{\alpha_{\rm s}(\boldsymbol{k}^2)}{2\pi} \frac{\alpha}{2\pi} \Big(\sum_q e_q^2\Big)$$

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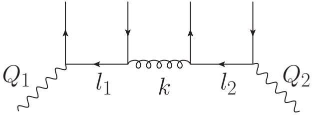


$$\sigma_0^{(TT)}(s, Q_1 \gg Q_2) \simeq \frac{4\pi\alpha^2}{Q_1^2} F_T(x_{\mathrm{Bj}}, Q_1^2; Q_2^2) \,, \qquad x_{\mathrm{Bj}} \simeq Q_1^2/s$$

$$= \int_{x_{\mathrm{Bj}}}^1 \frac{\mathrm{d}z_1}{z_1}\frac{\mathrm{d}z_k}{z_k}\frac{\mathrm{d}z_2}{z_2} \Theta(z_1 < z_k < z_2)\delta\left(1 - \frac{x_{\mathrm{Bj}}}{z_1}\right) P_{qg}\left(\frac{z_1}{z_k}\right) P_{gq}\left(\frac{z_k}{z_2}\right) P_{q\gamma}(z_2)$$

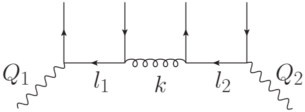
$$\times \int_{Q_2^2}^{Q_1^2} \frac{\mathrm{d}\boldsymbol{l}_1^2}{\boldsymbol{l}_1^2}\frac{\mathrm{d}\boldsymbol{k}^2}{\boldsymbol{k}^2}\frac{\mathrm{d}\boldsymbol{l}_2^2}{\boldsymbol{l}_2^2} \Theta(\boldsymbol{l}_2^2 < \boldsymbol{k}^2 < \boldsymbol{k}_1^2)\alpha(\sum_q e_q^2)\frac{\alpha_s(\boldsymbol{l}_1^2)}{2\pi}\frac{\alpha_s(\boldsymbol{k}^2)}{2\pi}\frac{\alpha}{2\pi}(\sum_q e_q^2)$$

Mellin transform $x_{\mathrm{Bj}} \to \omega$, $Q_1^2/Q_2^2 \to \gamma$ (with fixed coupling)

$$\sigma_0^{(TT)}(\omega, \gamma) \simeq \alpha \left(\sum_{q\in A} e_q^2\right)\frac{1}{\gamma} \cdot \frac{\alpha_s}{2\pi}\frac{P_{qg}(\omega)}{\gamma} \cdot \frac{\alpha_s}{2\pi}\frac{P_{gq}(\omega)}{\gamma} \cdot \frac{\alpha}{2\pi}\left(\sum_{q\in B} e_q^2\right)\frac{P_{q\gamma}(\omega)}{\gamma}$$

Transverse impact factor

Use exact LO DGLAP anomalous dimensions



$$P_{qq}(\omega) = C_F \omega A_{qq}(\omega) \qquad A_{qq}(0) = \frac{5}{4} - \frac{\pi^2}{3}$$

$$P_{gq}(\omega) = \frac{2C_F}{\omega}\left[1 + \omega A_{gq}(\omega)\right] \qquad A_{gq}(0) = -\frac{3}{4}$$

$$P_{qg}(\omega) = \frac{2}{3} T_R \left[1 + \omega A_{qg}(\omega)\right] \qquad A_{qg}(0) = -\frac{13}{12}$$

$$P_{gg}(\omega) = \frac{2C_A}{\omega}\left[1 + \omega A_{gg}(\omega)\right] \qquad A_{gg}(0) = -\frac{11}{6} + \bar{b}$$

$$P_{q\gamma}(\omega) = \frac{N_c}{T_R} P_{qg}(\omega) \qquad \bar{b} = \frac{11}{12} - \frac{T_R N_f}{3C_A}$$

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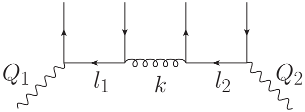

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$$P_{qg}(\omega) = \frac{2}{3} T_R \left[1 + \omega A_{qg}(\omega)\right] \qquad A_{qg}(0) = -\frac{13}{12}$$

$$P_{gg}(\omega) = \frac{2C_A}{\omega}\left[1 + \omega A_{gg}(\omega)\right] \qquad A_{gg}(0) = -\frac{11}{6} + \bar{b}$$

$$P_{q\gamma}(\omega) = \frac{N_c}{T_R} P_{qg}(\omega) \qquad \bar{b} = \frac{11}{12} - \frac{T_R N_f}{3C_A}$$

$$\sigma_0^{(TT)}(\omega,\gamma) = \Phi_0^{(T)} G_0 \Phi_0^{(T)}$$

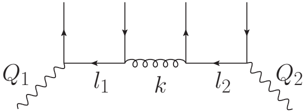
$$\simeq \left[\alpha\alpha_s \left(\sum_q e_q^2\right) 2P_{qg}(\omega)\sqrt{2(N_c^2-1)}\left(\frac{1}{\gamma^2}\right)\right]^2 \times \frac{1}{\omega} \times \left(1 + \omega A_{gq}(\omega)\right)$$

Double poles of BNP impact factors with exact kinematics $\quad G_0 \quad$ additional term

Possible choice of RGI: $\Phi^{(T)}(\omega,\gamma) = \Phi^{(T,\mathrm{BNP})}(\omega,\gamma) \times \left(1 + \frac{\omega}{2} A_{gq}(\omega)\right)$

Transverse impact factor

Use exact LO DGLAP anomalous dimensions



$$P_{qq}(\omega) = C_F \omega A_{qq}(\omega) \qquad A_{qq}(0) = \frac{5}{4} - \frac{\pi^2}{3}$$

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Double poles of BNP impact factors with exact kinematics G_0 additional term

Possible choice of RGI: $\Phi^{(T)}(\omega,\gamma) = \Phi^{(T,\mathrm{BNP})}(\omega,\gamma) \times \left(1 + \frac{\omega}{2} A_{gq}(\omega)\right)$

Transverse impact factor

Having the ω-dependent LO impact factor,
we can predict the spurious (quartic + cubic) poles of the NLO BFKL impact factor

$$\phi_1^{(T)}(\gamma) \simeq \phi_0^{(T)}(\gamma) \left[-\frac{1}{\gamma^2} - \frac{3}{2}\frac{1}{(1-\gamma)^2} \right] \qquad \text{with} \qquad \phi_0 \sim \frac{1}{\gamma^2}$$

Compatibility between BFKL and RGI factorization formula yields

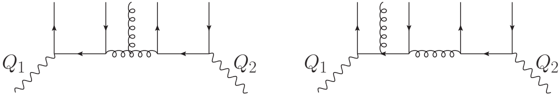
$$\phi_1(\gamma) + \phi_1(1-\gamma) = \Phi_1(0,\gamma) + \Phi_1(0,1-\gamma) + \chi_0(\gamma)[\partial_\omega \Phi_0(0,\gamma) + \partial_\omega \Phi_0(0,1-\gamma)] + \phi_0(\gamma)\partial_\omega X_0(0,\gamma)$$

For the highest $\gamma \to 0$ poles (quartic in this case)

$$\phi_0(\gamma)\left[-\frac{5}{2}\frac{1}{\gamma^2} \right] = \frac{1}{\gamma}\left[\phi_0(\gamma)\frac{-1}{\gamma} \times 2 \right] + \phi_0(\gamma)\frac{-1}{2\gamma^2}$$

Transverse impact factor at NLO

Collinear analysis at $\mathcal{O}\left(\alpha_s^3\right)$ provides the remaining physical cubic poles



$$\sigma_1^{(TT)}(\omega,\gamma) = \sigma_0^{(TT)}(\omega,\gamma) \times \left[\frac{\alpha_s}{2\pi} \frac{P_{gg}}{\gamma} + 2 \frac{\alpha_s}{2\pi} \frac{P_{qq}}{\gamma} - \frac{\alpha_s b_0}{\gamma} + \mathcal{O}\left(\gamma^0\right) \right]$$

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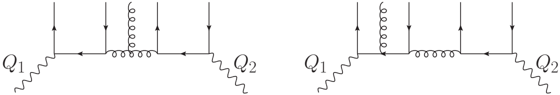

(a) (b) + sym

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(a) Can attribute it to GGF

(b) Can attribute it to the impact factor

- Running coupling term: can attribute it to either or both

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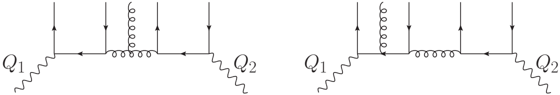

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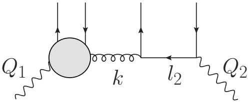
- Running coupling term: can attribute it to either or both
- Determine the pole structure of $\Phi_1^{(T)}(\omega,\gamma)$ (reproducing cubic poles of ϕ_1)
- Add "regular" part of ϕ_1 according to compatibility condition

$$\Phi_1(0,\gamma) = \frac{1}{2}\left[\phi_1(\gamma) + \phi_1(1-\gamma) - \phi_0(\gamma)\partial_\omega X_0(0,\gamma) - \chi_0(\gamma)\left(\partial_\omega \Phi_0(0,\gamma) + \partial_\omega \Phi_0(0,1-\gamma)\right)\right]$$

Longitudinal impact factor

LO cross section for longitudinal (Q_1) against transverse (Q_2) virtual photons with one gluon exchange

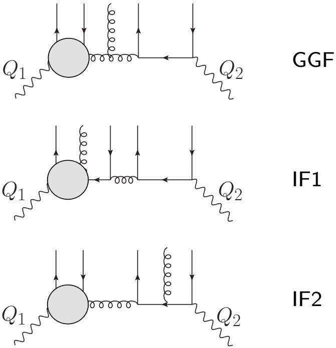


Collinear analysis reproduce simple poles of BNP impact factors with exact kinematics, up to the same additional factor from P_{gq}

$$\Phi^{(L)}(\omega,\gamma) = \Phi^{(L,\mathrm{BNP})}_{(\omega,\gamma)} \times \left(1 + \frac{\omega}{2} A_{gq}(\omega)\right)$$

Note: $\phi_0^{(L)} \sim 1/\gamma$ while $\phi_0^{(T)} \sim 1/\gamma^2$

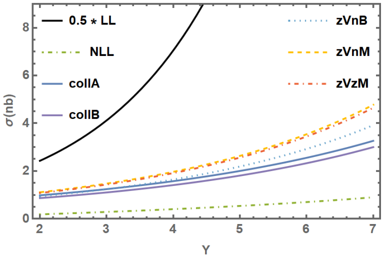
NLO cross section in collinear limit



Second diagram provides a C_F term which is absent in BFKL impact factor (stems from $C_{L,q}$ coeff. function)

Numerical results for $\gamma^* - \gamma^*$ cross section

We compare BFKL results with RGI ones in various schemes for $\mathrm{d}\sigma/\mathrm{d}Y$
$Y = \log(s/Q^2)$, $Q^2 = 17$ GeV^2 , $N_f = 4$ active flavours



- NLL BFKL is much smaller than LL
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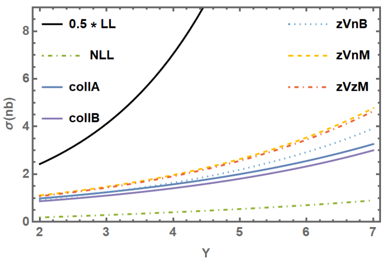

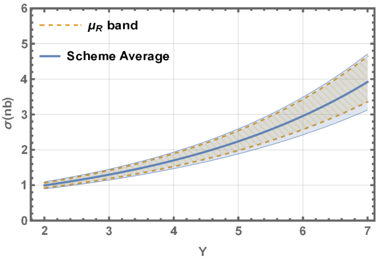

- NLL BFKL is much smaller than LL
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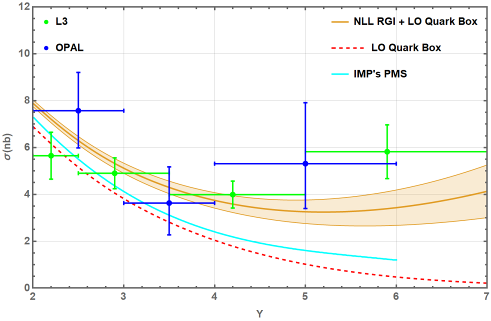
- Spread in resummation scheme and renormalization scale choice are of the same order

Numerical results for $\gamma^* - \gamma^*$ cross section

We sum the "quark-box" and the RGI BFKL contributions
We compare with previous estimates *[Ivanov et al]*
and with LEP data: L3 ($Q^2 = 16$ GeV^2) and OPAL ($Q^2 = 17.9$ GeV^2)



- Quark box dominates at small Y, BFKL with RGI dominates at large Y
- RGI results provide a fair description of data, compatible within uncertainties

Conclusions and outlook

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- Resummation gives result consistent with LEP data, lower than LL and higher than NLL
- NLO DGLAP improvement
- NLO and double log resummation in the quark box could slightly modify the result, though not large energy behavior