Inclusive back-to-back dijet in DIS: Sudakov vs high-energy resummation of the WW TMD

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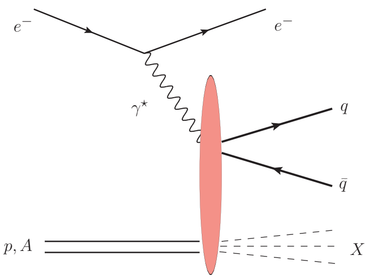
Overlap between QCD resummations - Aussois - January 2024

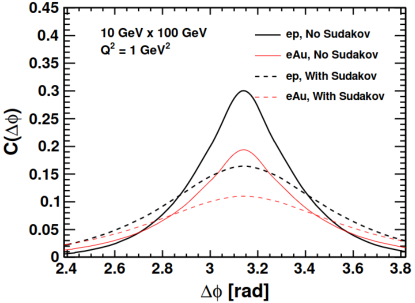
with F. Salazar, B. Schenke, T. Stebel, R. Venugopalan
JHEP 2021 (11), 1-108, JHEP 2022 (11), 1-77, JHEP 2023, (62) and arXiv:2308.00022

Back-to-back di-jets in DIS

⇒ probe of the saturated regime of QCD

⇒ access to the Weizsäcker-Williams gluon TMD in the back-to-back limit.

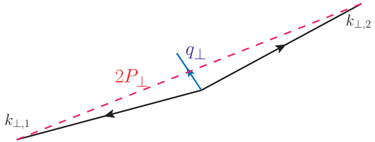




Zheng, Aschenauer, Lee, Xiao, 1403.2413

LO: common language between small-x and TMD communities

- Def: $|\boldsymbol{P}_\perp| = |z_2 \boldsymbol{k}_{\perp,1} - z_1 \boldsymbol{k}_{\perp,2}| \gg |\boldsymbol{q}_\perp| = |\boldsymbol{k}_{\perp,1} + \boldsymbol{k}_{\perp,2}|$



- LO in photon-gluon fusion channel: TMD factorization Dominguez, Marquet, Xiao, Yuan, 1101.0715

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star \to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{LO}} \propto \mathcal{H}^{ij}(\boldsymbol{P}_\perp) G_Y^{ij}(\boldsymbol{q}_\perp) + \mathcal{O}\left(\frac{q_\perp}{P_\perp}\right) + \mathcal{O}\left(\frac{Q_s}{P_\perp}\right)$$

See also del Castillo, Echevarria, Makris, Scimemi, 2008.07531

- $G_Y(\boldsymbol{q}_\perp)$: WW gluon TMD

$$G^{ij}_{Y=\ln(1/x)}(\boldsymbol{q}_\perp) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - iq_\perp \xi_\perp} \left\langle P \left| F^{+i}(\xi^-, \boldsymbol{\xi}_\perp) U_\xi^{[+]\dagger} F^{+j}(0) U_\xi^{[+]} \right| P \right\rangle \quad \text{TMD}$$

$$= \frac{-2}{\alpha_s} \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp}{(2\pi)^4} e^{-iq_\perp \cdot r_{bb'}} \left\langle \mathrm{Tr}\left[\partial^i V^\dagger(\boldsymbol{b}_\perp) V(\boldsymbol{b}'_\perp) \partial^j V^\dagger(\boldsymbol{b}'_\perp) V(\boldsymbol{b}_\perp)\right]\right\rangle_Y \quad \text{CGC}$$

See talk by Alexey this morning.

Conceptual questions relevant for TMD and small-x communities

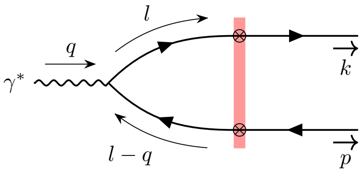
Small x and back-to-back regime

We work in the regime $W^2 \gg Q^2 \sim P_\perp^2 \gg q_\perp^2 \sim Q_s^2$.
Two kinds of large logs: $\ln(W^2/Q^2) \sim \ln(1/x)$ and $\ln(P_\perp/q_\perp)$

- Does TMD factorization hold at NLO in the small x limit?
- Do we recover the same NLO hard factor as in TMD calculations?
  Becher, Schwartz, 0911.0681, del Castillo, Echevarria, Makris, Scimemi, 2111.03703, Zhang, 1709.08970
- Can we isolate Sudakov from small-x logarithms beyond double logarithmic accuracy ?
  At DLA, conjecture from Mueller, Xiao, Yuan, 1308.2993 based on Higgs production in pA: yes!
- What value of $Y = \ln(1/x)$ enters the CGC definition of the WW TMD?
- Can we prove CSS evolution at small x? See Balitsky, Tarasov, arXiv:1505.02151, Mukherjee, Skokov, Tarasov, Tiwari, 2311.16402

Dipole picture and CGC EFT

- We work in the dipole picture of DIS, large q^-.



- Covariant perturbation theory.

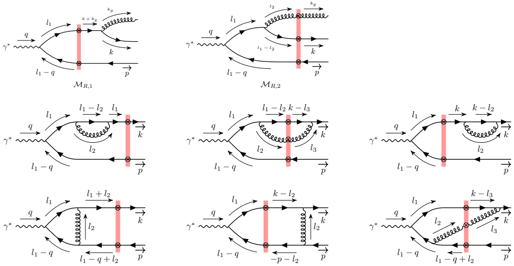
- CGC effective vertex:

$$= (2\pi)\delta(q^- - p^-)\gamma^- \int \mathrm{d}^2\boldsymbol{x}_\perp e^{-i(\boldsymbol{q}_\perp - \boldsymbol{p}_\perp)\boldsymbol{x}_\perp} V_{ij}(\boldsymbol{x}_\perp)$$

$\Rightarrow$ multiple gluon interactions with the target resummed via Wilson lines $V(\boldsymbol{x}_\perp)$

Outline of the NLO calculation

- We have done the full computation for general kinematics in 2108.06347
  Similar calculations & cross-checks in Taels, Altinoluk, Beuf, Marquet, 2204.11650, Iancu, Mulian, 2211.04837, Bergago, Jalilian-Marian, 2301.03117
- In the CGC EFT+ dipole picture of DIS, the diagrams are



- Rapidity divergence $\int_{\Lambda^-}^{k_f^-} \frac{k_g^-}{k_g^-}$ isolated $\Rightarrow$ gives JIMWLK evolution of the LO cross-section.
- Explicit computation of the NLO impact factor.

Definition of the NLO impact factor

- Schematically, we have the following one loop result

$$\frac{\mathrm{d}\sigma_{1\mathrm{L}}^{\lambda}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp} = \ln\left(\frac{z_f}{z_0}\right) H_{\mathrm{LL}} \otimes \mathrm{d}\sigma_{\mathrm{LO}}^{\lambda} + \alpha_s \int_0^1 \frac{\mathrm{d}z_g}{z_g} \int \mathrm{d}^2\boldsymbol{z}_\perp \left[\mathrm{d}\widetilde{\sigma}_{1\mathrm{L}}^{\lambda}(z_g, \boldsymbol{z}_\perp) - \mathrm{d}\widetilde{\sigma}_{1\mathrm{L}}^{\lambda}(0, \boldsymbol{z}_\perp)\Theta(z_f - z_g)\right]$$

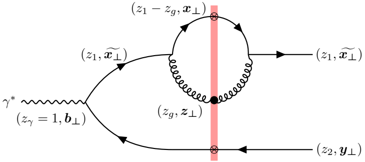
- $z_0 = \Lambda^- / q^-$, rapidity cut-off, $z_f = k_f^- / q^-$ is the projectile rapidity factorization scale.
- $\boldsymbol{z}_\perp$ is the transverse coordinate of the gluon while eikonally interacting with the dense target.
- H_{LL} leading log BK-JIMWLK hamiltonian such that

$$H_{\mathrm{LL}} \otimes S(\boldsymbol{r}_{bb'}) = \bar{\alpha}_s \int \frac{\mathrm{d}^2\boldsymbol{z}_\perp}{2\pi} \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2} \langle S_Y(\boldsymbol{r}_{zb}) S_Y(\boldsymbol{r}_{zb'}) - S_Y(\boldsymbol{r}_{bb'}) \rangle$$

- Goal: extract the leading power in $q_\perp / P_\perp$ term in the NLO impact factor.

Concrete example with the quark dressed self-energy

- Back-to-back limit of virtual graphs are very challenging! Need to find a judicious expansion in coordinate space.



SE1

$$u_\perp = \widetilde{x_\perp} - y_\perp \sim r_\perp = z_\perp - x_\perp \ll b_\perp = z_1 \widetilde{x_\perp} + z_2 y_\perp$$

$$u_\perp \sim r_\perp \sim 1/P_\perp, \; b_\perp \sim 1/q_\perp$$

- In the end, the leading power contribution can be extracted and computed fully analytically within a TMD factorized expression:

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star \to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp}\right|_{\mathrm{SE1}} \propto \mathcal{H}^{ij}_{\mathrm{NLO,se}}(\boldsymbol{P}_\perp) G^{ij}_Y(\boldsymbol{q}_\perp) + \mathcal{O}\left(\frac{q_\perp}{P_\perp}\right) + \mathcal{O}\left(\frac{Q_s}{P_\perp}\right)$$

$$\mathcal{H}^{\lambda=L,ij}_{\mathrm{NLO,se}} = \alpha_s \mathcal{H}^{ij}_{\mathrm{LO}} \int_0^{z_1} \frac{\mathrm{d}z_g}{z_g} \left(1 - \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2}\right) \left[-1 + \ln\left(\frac{1+\chi^2}{\chi^2}\right) - \ln\left(\frac{z_g}{z_2(z_1 - z_g)}\right)\right] - \text{rapidity div.}$$

- Byproduct: a double logarithmic divergence as $z_g \to 0$ arises.

Cancellation of rapidity divergences in the back-to-back limit

- To cure the double log divergence in the rapidity cut-off,

$$\mathrm{d}\sigma^{\lambda,\mathrm{b2b}}_{\mathrm{NLO}} = \alpha_s \int_0^1 \frac{\mathrm{d}z_g}{z_g} \int \mathrm{d}^2\boldsymbol{z}_\perp \Big[\mathrm{d}\widetilde{\sigma}^{\lambda,\mathrm{b2b}}_{\mathrm{1L}}(z_g,\boldsymbol{z}_\perp) - \mathrm{d}\widetilde{\sigma}^{\lambda,\mathrm{b2b}}_{\mathrm{1L}}(0,\boldsymbol{z}_\perp)\Theta(z_f - z_g)\Theta\left(-\ln(z_g) - \ln(\min(\boldsymbol{r}^2_{zb},\boldsymbol{r}^2_{zb'})2k^+_c q^-)\right)\Big]$$

- Include an additional constraint in the small-x evolution which is k_g^- dependent.
- Effectively imposes lifetime ordering $1/k_g^+ \leq 1/q^+$ of gluon emissions.
- We have now

$$\mathcal{H}^{\lambda=L,ij}_{\mathrm{NLO,se}} = \alpha_s \mathcal{H}^{ij}_{\mathrm{LO}} \left\{ \int_0^{z_1} \frac{\mathrm{d}z_g}{z_g}\left[\dots - \ln\left(\frac{z_g}{z_2(z_1-z_g)}\right)\right] - \int_0^{z_f}\frac{\mathrm{d}z_g}{z_g}\text{"kc rapidity div."}\right\}$$

- Cancellation of $z_g \to 0$ singularity demands kinematic constraint **and**

$$\frac{k_c^+}{P^+} = \frac{1}{ec_0^2}\underbrace{\frac{M^2_{q\bar{q}}+Q^2}{W^2+Q^2}}_{\equiv x_g}$$

Universality of small-x resummation: target rapidity evolution

- Our kinematically constrained evolution equation seems process dependent:

$$\frac{\partial S_Y(\boldsymbol{r}_{bb'})}{\partial Y} = \bar{\alpha}_s \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta\left(-Y - \ln(r_<^2 \mu_\perp^2)\right) \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2} \left[S_Y(\boldsymbol{r}_{zb}) S_Y(\boldsymbol{r}_{zb'}) - S_Y(\boldsymbol{r}_{bb'})\right]$$

$(\mu_\perp \sim P_\perp,\ r_<^2 = \min(\boldsymbol{r}_{zb}^2, \boldsymbol{r}_{zb'}^2))$

- It is because it is formulated in terms of the projectile rapidity $Y = \ln(k_f^- / q^-)$.

- Change of variable $\eta = Y + \ln(r_<^2 Q^2) - \ln(x_{\mathrm{Bj}}/x_0)$:

$$\frac{\partial S_\eta(\boldsymbol{r}_{bb'})}{\partial \eta} = \bar{\alpha}_s \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta\left(\eta - \delta_{bb'z}\right) \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2} \left[S_{\eta-\delta_{zb}}(\boldsymbol{r}_{zb}) S_{\eta-\delta_{zb'}}(\boldsymbol{r}_{zb'}) - S_\eta(\boldsymbol{r}_{bb'})\right]$$

- Recover result by Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos, 1902.06637+ NLO matching relation for the coefficient function (for $\eta_f = \ln(P^+/k_f^+)$)

$$\eta_f \equiv Y_f + \ln(\mu_\perp^2 \boldsymbol{r}_{bb'}^2) + \ln(1/x_c)$$

- Choosing $x_f \sim x_g \Rightarrow Y_f \sim \ln(\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2) \Rightarrow z_f \sim q_\perp^2 / P_\perp^2$

Kinematic constraint in high energy resummation vs Sudakov

- Kinematic improvement: impose both k_g^- and k_g^+ ordering (lifetime ordering).

  $\Longrightarrow$ Resum large transverse double logarithms to all orders.

  $\Longrightarrow$ Solve the instability of NLO BFKL or BK-JIMWLK evolution.

  Ciafaloni, Colferai, 9812366, Kwiecinski, Martin, Stasto, 9703445, Salam, 9806482, Vera, 0505128, Beuf, 1401.0313, ...

  See talk by Agustin and Dimitri this morning

- With this modification $H_{\mathrm{LL}} \to H_{\mathrm{LL,coll}}$, one gets the expected Sudakov double logarithm in the NLO impact factor

  Mueller, Xiao, Yuan, 1308.2993, Taels, Altinoluk, Beuf, Marquet, 2204.11650

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} \sim \mathcal{H}_{\mathrm{LO}}(\boldsymbol{P}_\perp) \int \mathrm{d}^2 \boldsymbol{r}_{bb'} \, e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}}$$

$$\times \left[1 - \frac{\alpha_s N_c}{4\pi} \ln^2 \left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2} \right) + \ldots + \alpha_s \ln \left(\frac{x_0}{x_f} \right) H_{\mathrm{LL,coll}} \otimes \right] G_{\mathrm{WW}}(\boldsymbol{r}_{bb'}) + \mathcal{O}(\alpha_s)$$

Kinematic constraint in high energy resummation vs Sudakov

In summary

- Without kinematic constraint in high energy evolution: uncancelled light cone singularities in NLO coefficient functions for back-to-back dijets.

- Without kinematic constraint: Sudakov double logarithms do not match the CSS ones.
  Taels, Altinoluk, Beuf, Marquet, 2204.11650

- Remarkable that the need for the kinematic constraint arises at leading log-x! Due to additional constraint on the final state (back-to-back config).

Final TMD factorized result

$$
\begin{aligned}
\left\langle \mathrm{d}\sigma^{(0),\lambda}_{\mathrm{LO}} + \alpha_s \mathrm{d}\sigma^{(0),\lambda}_{\mathrm{NLO}} \right\rangle_{\eta_f} = \; & \frac{1}{2}\mathcal{H}^{\lambda,ii}_{\mathrm{LO}} \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{G}^0(x_f, \boldsymbol{r}_{bb'}) \\
& \times \left\{ 1 + \frac{\alpha_s}{\pi} \left[-\frac{N_c}{4} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) - s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \beta_0 \ln\left(\frac{\mu_R^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) \right.\right. \\
& \left.\left. + \frac{N_c}{2} f_1^\lambda(Q/M_{q\bar{q}}, z_1, R, x_f/x_g) + \frac{1}{2N_c} f_2^\lambda(Q/M_{q\bar{q}}, z_1, R) \right] \right\} \\
& + \frac{\alpha_s}{2\pi}\mathcal{H}^{\lambda,ii}_{\mathrm{LO}} \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{h}^0(x_f, \boldsymbol{r}_{bb'}) \left\{ \frac{N_c}{2}\left[1 + \ln(R^2)\right] - \frac{1}{2N_c} \ln(z_1 z_2 R^2) \right\}
\end{aligned}
$$

- x_f dependence of TMD given by k-c. k_g^+ ordered non-linear evolution. Saturation corrections $\mathcal{O}(Q_s/q_\perp)$ fully included in this dependence!
- First line should be exponentiated à la CSS to resum large double and single Sudakov logs.
- $s_L = -C_F \ln(z_1 z_2 R^2) + N_c \ln(1 + Q^2/M_{q\bar{q}}^2) \Rightarrow$ agreement with collinear calculations.

  Hatta, Xiao, Yuan, Zhou, 2106.05307
- Last line: dependence on linearly polarized WW, due to real soft gluon radiation.

Analytic results for NLO coefficient functions

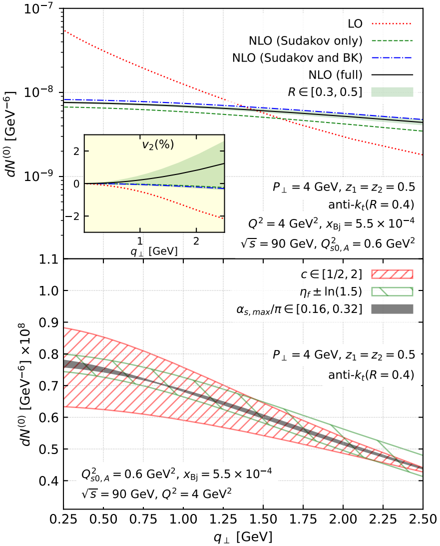
- Gathering all diagrams together:

$$\mathcal{O}(\alpha_s) = \mathcal{H}^{ij} \times G^{ij}(\boldsymbol{q}_\perp) \times \left[\frac{\alpha_s N_c}{2\pi} f_1^{\lambda=L} + \frac{\alpha_s}{2\pi N_c} f_2^{\lambda=L}\right]$$

$$\begin{aligned} f_1^{\lambda=L}(\chi = Q/M_{q\bar{q}}, z_1, R, x_f = x_c) &= 9 - \frac{3\pi^2}{2} - \frac{3}{2}\ln\left(\frac{z_1 z_2 R^2}{\chi^2}\right) - \ln(z_1)\ln(z_2) - \ln(1+\chi^2)\ln\left(\frac{1+\chi^2}{z_1 z_2}\right) \\ &+ \left\{\mathrm{Li}_2\left(\frac{z_2 - z_1\chi^2}{z_2(1+\chi^2)}\right) - \frac{1}{4(z_2 - z_1\chi^2)} \right. \\ &\left. + \frac{(1+\chi^2)(z_2(2z_2 - z_1) + z_1(2z_1 - z_2)\chi^2)}{4(z_2 - z_1\chi^2)^2} \ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right) + (1 \leftrightarrow 2)\right\} \end{aligned}$$

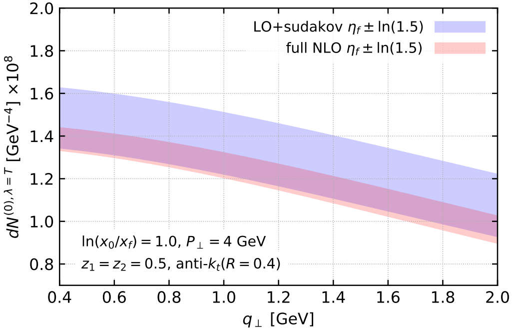
- Similar expressions for subleading $1/N_c$ term f_2 and for transversely polarized virtual photons.
- Very fast numerical implementation.
- Still potentially large logs as $z_{1/2} \to 1 \Rightarrow$ link with threshold resummation?

Numerical NLO results: inclusive back-to-back dijet cross-section



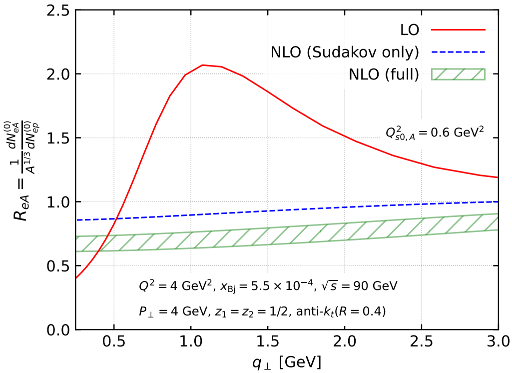
- EIC kinematics
- Dominant log is the Sudakov ones.
- Small x evolution yields an increase of the cross-section tamed by non -linear saturation corrections.

Rapidity factorization scale dependence at EIC kinematics



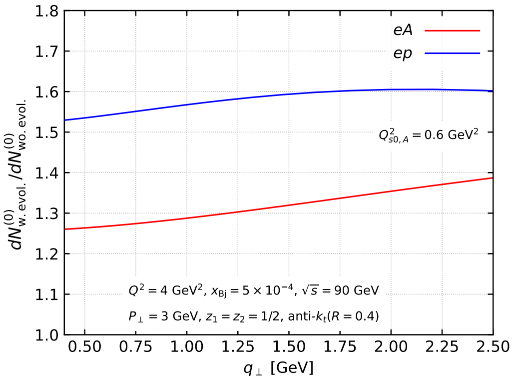
- x_f variation around a central value to gauge the sensitivity to missing N^2LO corrections.
- Scale variations shrink from LO to NLO.
- One expects thinner NLO bands when $\alpha_s \ln(x_0/x_f) = \mathcal{O}(1)$.

Numerical NLO results: nuclear modification factor



- In R_{eA} ratio, "vacuum" physics largely cancels.
- High energy resummation gives a strong suppression.
- These results depends on the initial condition: need to fit the WW TMD at small x.

Non-linear saturation effects in back-to-back dijet



- $q_\perp$ dependence of the x-section ratio with/without high energy resummation.
- In ep: mild $q_\perp$ dependence.
- In eA: slower evolution especially at small $q_\perp$.

Summary

- First proof of WW gluon TMD factorization at NLO at small x: non trivial because of "all twist" $Q_s/q_\perp$ power corrections.
- TMD factorization and isolation of Sudakov logs demand kinematic constraint + target rapidity small x evolution.
- First calculation of Sudakov single log for this process at small x, agreement with collinear calculations.
- We postulate exponentiation of Sudakov logs à la CSS, a rigorous proof will require to go beyond our one-loop computation
- Outlook: look at dihadron production and other TMD factorizable processes at small x. See also Jamal's talk tomorrow and Pieter's talk right after.

Back-up slides

LO cross-section

- Differential cross-section at leading order:

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}^2\boldsymbol{p}_\perp \mathrm{d}\eta_q \mathrm{d}\eta_{\bar{q}}}\right|_{\mathrm{LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{r}_{xx'}} e^{-i\boldsymbol{p}_\perp \boldsymbol{r}_{yy'}} \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}'_{xy})$$

- **Factorization** between perturbative factor describing the $\gamma^\star \to q\bar{q}$ splitting...

$$\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy}, \boldsymbol{r}'_{xy}) = 8 z_q^3 z_{\bar{q}}^3 Q^2 K_0(\bar{Q} r_{xy}) K_0(\bar{Q} r_{xy'})$$

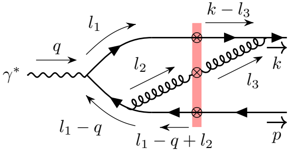
- ... and a color structure describing the interaction of $q\bar{q}$ with the dense target

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{x}'_\perp, \boldsymbol{y}'_\perp) = \left\langle \underbrace{Q(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)}_{quadrupole} - D(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) - \underbrace{D(\boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)}_{dipole} + 1 \right\rangle_Y$$

$$\text{Dipole:} \qquad D(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = \frac{1}{N_c}\langle \mathrm{Tr}(V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp))$$

Structure of NLO amplitudes in the CGC

- Example: the dressed vertex correction for longitudinally polarized $\gamma^\star$.



$$= \frac{ee_f q^-}{\pi}\int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp \mathrm{d}^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp - i\boldsymbol{p}_\perp \boldsymbol{y}_\perp}[t^a V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - t^a t_a]$$

$$\times \frac{\alpha_s}{\pi^2} 2(z_q z_{\bar{q}})^{3/2} Q\delta_{\sigma,-\bar{\sigma}} \int_0^{z_q} \frac{\mathrm{d}z_g}{z_g} e^{-iz_g \boldsymbol{k}_\perp / z_q \cdot \boldsymbol{r}_{zx}} \left(1+\frac{z_g}{z_{\bar{q}}}\right)\left(1-\frac{z_g}{z_q}\right) K_0\left(QX_V\right)$$

$$\times \left\{\left[1-\frac{z_g}{2z_q}-\frac{z_g}{2(z_{\bar{q}}+z_g)}\right]\frac{\boldsymbol{r}_{zx}\cdot \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} + i\sigma\left[\frac{z_g}{2z_q}-\frac{z_g}{2(z_{\bar{q}}+z_g)}\right]\frac{\boldsymbol{r}_{zx}\times \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2}\right\}$$

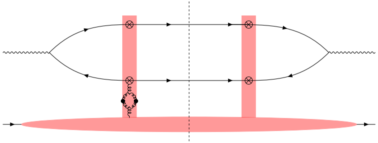
with

$$X_V^2 = z_{\bar{q}}(z_q - z_g \boldsymbol{r}_{xy}^2 + z_g(z_q - z_g)\boldsymbol{r}_{zx}^2 + z_g z_{\bar{q}} \boldsymbol{r}_{zy}^2$$

The single log proportional to β_0

- At NLO, quantum correction to the classical field: $\boldsymbol{A}^i_\perp = \boldsymbol{A}^{i,(0)}_\perp + \underbrace{\boldsymbol{A}^{i,(1)}_\perp}_{\mathcal{O}(\alpha_s)}$

Gelis, Venugopalan, 0601209



- We have (see Ayala, Jalilian-Marian, McLerran, Venugopalan, 9508302)

$$\boldsymbol{A}^{i,(1)}_\perp = \frac{\alpha_s N_c}{\pi}\beta_0[1/\varepsilon + \text{finite}]\boldsymbol{A}^{i,(0)}_\perp \qquad (1)$$

- UV divergence removed by renormalization $\Rightarrow$ renormalization scale dependence of the WW gluon TMD: See also Zhou, 1807.00506

$$\frac{\partial \hat{G}_Y(\boldsymbol{r}_{bb'}, \mu)}{\partial \ln(\mu)} = \alpha_s\beta_0 \times \hat{G}_Y(\boldsymbol{r}_{bb'}, \mu)\,. \qquad (2)$$

Dijet azimuthal anisotropy