

# Power corrections to back-to-back DIS dijets: Next-to-Eikonal versus twist 3

Guillaume Beuf

National Centre for Nuclear Research (NCBJ), Warsaw, Poland

with Tolga Altinoluk, Alina Czajka and Cyrille Marquet, (*to appear*).

Workshop on overlap between QCD resummations  
Centre Paul Langevin, Aussois, France, January 14-17, 2024

# TMD vs CGC approaches

For a process with a hard  $\mathbf{P}$  and a not so hard  $\mathbf{k}$  transverse momenta:

- TMD factorization: leading power (twist 2) in the limit  $|\mathbf{k}| \ll |\mathbf{P}| \sim \sqrt{s}$
- CGC result: leading power (eikonal) in the limit  $|\mathbf{k}| \sim |\mathbf{P}| \ll \sqrt{s}$

Consistency of both approaches shown in the double limit  $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$  at leading power ([Dominguez, Marquet, Xiao, Yuan, 2011](#))

Power corrections in  $|\mathbf{k}|/|\mathbf{P}|$  in the regime  $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$  studied from the CGC approach ([Altinoluk, Boussarie, Kotko, 2019](#))

⇒ **What about power corrections in  $\mathbf{P}^2/s$  or  $|\mathbf{P}||\mathbf{k}|/s$  beyond the eikonal limit?**

# Eikonal approximation in the CGC

High-energy dense-dilute scattering in the CGC : Semiclassical and Eikonal approx.

Dense target represented by a **strong semiclassical gluon field**  $\mathcal{A}^\mu(x) \propto 1/g$   
 $\Rightarrow$  Perturbative expansion in  $g$  needs improvement by all order resummation of  $(g \mathcal{A}^\mu(x))^n$

Eikonal approx. : limit of **infinite boost** of  $\mathcal{A}^\mu(x)$  along  $x^-$ :

- $\mathcal{A}^\mu(x)$  **independent on  $x^-$  (static limit)** due to Lorentz time dilation  
 $\Rightarrow$  No  $p^+$  transfer from the target
- Lorentz contraction of  $\mathcal{A}^\mu(x)$  (**shockwave limit**)  
 $\Rightarrow$  Partons from the projectile interact instantly in  $x^+$  with the target, without transverse motion **within** the target
- Under a boost of parameter  $\gamma_t$  along the "–" direction,  $\mathcal{A}^-$  is enhanced and  $\mathcal{A}^+$  is suppressed:  
 $\mathcal{A}^- = O(\gamma_t) \gg \mathcal{A}_\perp = O(1) \gg \mathcal{A}^+ = O(1/\gamma_t)$

Background field in the eikonal limit:  $\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \approx \delta^{\mu-} \mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

$\Rightarrow$  Only  $(g \mathcal{A}^-(x^+, \mathbf{x}))^n$  needs all orders resummation  $\Rightarrow$  Wilson line along  $x^+$

# Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) power corrections to the standard CGC formalism:

- Of order  $1/\gamma_t$  at the level of the boosted background field
- Of order  $1/s$  at the level of a cross section

→ They arise from relaxing either of the 3 approximations:

- 1  $x^-$  dependence of  $\mathcal{A}^\mu(x)$  beyond infinite Lorentz dilation

→ Treated as gradient expansion around a common  $x^-$  value:

$$\frac{\partial_- \mathcal{A}^-(x)}{\mathcal{A}^-(x)} = O(1/\gamma_t)$$

⇒ Possibility of (small)  $p^+$  exchange with the target

- 2 Target with finite width

⇒ transverse motion of the projectile partons within the target

- 3 Interactions with  $\mathcal{A}_\perp$  field taken into account, not only  $\mathcal{A}^-$

Note: Background quark field of the target also relevant at NEik.

⇒ Separate contribution not included in this talk (See [Altinoluk, Armesto, GB, 2023](#)).

# More about NEik corrections beyond the static approx

Effect of relative  $z^-$  dependence of  $\mathcal{A}^-$  insertions along **one** propagator:

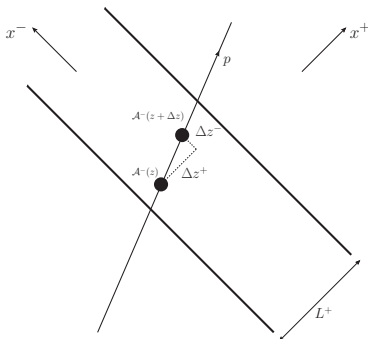
$$\mathcal{A}^-(z^- + \Delta z^-) - \mathcal{A}^-(z^-) \simeq \Delta z^- \partial_- \mathcal{A}^-(z^-)$$

- Slow  $z^-$  dependence from time dilation:

$$\partial_- \mathcal{A}^- \propto \frac{1}{\gamma_t} \mathcal{A}^-$$

- Small  $\Delta z^-$  displacement of the trajectory within the target width  $L^+$ :

$$\Delta z^- \sim \frac{p^-}{p^+} \Delta z^+ \leq \frac{p^-}{p^+} L^+ = O\left(\frac{1}{\gamma_t}\right)$$



Double power suppression, beyond static approx and beyond shockwave approx:

⇒ NNEik effect within a single propagator!

# More about NEik corrections beyond the static approx

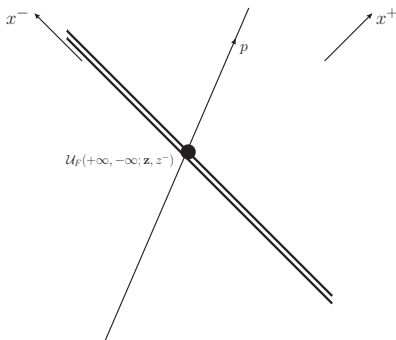
Effect of relative  $z^-$  dependence of  $\mathcal{A}^-$  insertions along **one** propagator is NNEik.

However, dependence on average  $z^-$  is suppressed only once.

⇒ Use Wilson lines with overall  $z^-$  dependence

$$\partial_- \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-) \propto \frac{1}{\gamma t} \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-)$$

→ Accounts for NEik effects beyond static approx



# More about NEik corrections beyond the static approx

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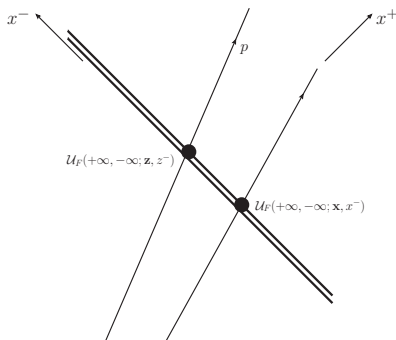
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→ Accounts for NEik effects beyond static approx

In particular: NEik corrections induced by the difference in  $z^-$  between different Wilson lines.



## Full NEik quark propagator through a gluon background field

Propagator from  $y$  before the target to  $x$  after the target:

$$\begin{aligned}
 S_F(x, y) = & \int \frac{dq^+ d^2 \mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2 \mathbf{k}}{(2\pi)^3} \theta(q^+) \theta(k^+) e^{-ix \cdot \bar{q}} e^{iy \cdot \bar{k}} \frac{(\not{k} + m)}{2q^+} \gamma^+ \\
 & \times \int d^2 \mathbf{z} e^{-iz \cdot (\mathbf{q} - \mathbf{k})} \left\{ \int dz^- e^{iz^-(q^+ - k^+)} \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-) \right. \\
 & - 2\pi \delta(q^+ - k^+) \frac{(\mathbf{q}^j + \mathbf{k}^j)}{2(q^+ + k^+)} \int dz^+ \left[ \mathcal{U}_F(+\infty, z^+; \mathbf{z}, 0) \overleftarrow{\mathcal{D}}_{\mathbf{z}^j} \mathcal{U}_F(z^+, -\infty; \mathbf{z}, 0) \right] \\
 & - i \frac{2\pi \delta(q^+ - k^+)}{(q^+ + k^+)} \int dz^+ \left[ \mathcal{U}_F(+\infty, z^+; \mathbf{z}, 0) \overleftarrow{\mathcal{D}}_{\mathbf{z}^j} \overrightarrow{\mathcal{D}}_{\mathbf{z}^j} \mathcal{U}_F(z^+, -\infty; \mathbf{z}, 0) \right] \\
 & \left. + \frac{2\pi \delta(q^+ - k^+)}{(q^+ + k^+)} \frac{[\gamma^i, \gamma^j]}{4} \int dz^+ \mathcal{U}_F(+\infty, z^+; \mathbf{z}, 0) g t \cdot \mathcal{F}_{ij}(z^+, \mathbf{z}, 0) \mathcal{U}_F(z^+, -\infty; \mathbf{z}, 0) \right\} \frac{(\not{k} + m)}{2k^+} \\
 & + \text{NNEik}
 \end{aligned}$$

Altinoluk, G.B, Czajka, Tymowska (2021); Altinoluk, G.B (2022)

$$\mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!} \mathcal{P}_+ \left[ -ig \int_{y^+}^{x^+} dz^+ t \cdot \mathcal{A}^-(z) \right]^N$$

- Generalized Eikonal contribution: also includes the NEik non-static corrections: overall  $z^-$  dependence of the Wilson line.



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 & \times \int d^2 \mathbf{z} e^{-iz \cdot (\mathbf{q} - \mathbf{k})} \left\{ \int dz^- e^{iz^- (q^+ - k^+)} \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-) \right. \\
 & - 2\pi \delta(q^+ - k^+) \frac{(\mathbf{q}^j + \mathbf{k}^j)}{2(q^+ + k^+)} \int dz^+ [\mathcal{U}_F(+\infty, z^+; \mathbf{z}, 0) \overleftrightarrow{\mathcal{D}}_{z^j} \mathcal{U}_F(z^+, -\infty; \mathbf{z}, 0)] \\
 & - i \frac{2\pi \delta(q^+ - k^+)}{(q^+ + k^+)} \int dz^+ [\mathcal{U}_F(+\infty, z^+; \mathbf{z}, 0) \overleftrightarrow{\mathcal{D}}_{z^j} \overrightarrow{\mathcal{D}}_{z^j} \mathcal{U}_F(z^+, -\infty; \mathbf{z}, 0)] \\
 & \left. + \frac{2\pi \delta(q^+ - k^+)}{(q^+ + k^+)} \frac{[\gamma^i, \gamma^j]}{4} \int dz^+ \mathcal{U}_F(+\infty, z^+; \mathbf{z}, 0) g t \cdot \mathcal{F}_{ij}(z^+, \mathbf{z}, 0) \mathcal{U}_F(z^+, -\infty; \mathbf{z}, 0) \right\} \frac{(\not{k} + m)}{2k^+} \\
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 \end{aligned}$$

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$$\mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!} \mathcal{P}_+ \left[ -ig \int_{y^+}^{x^+} dz^+ t \cdot \mathcal{A}^-(z) \right]^N$$

- NEik contributions beyond the shockwave approx or due to  $\mathcal{A}_\perp$ .

Last term: quark helicity coupling with longitudinal chromoelectric field of the target  $\mathcal{F}_{ij}$ .

## Full NEik quark propagator through a gluon background field

Compact notations for the decorated Wilson lines:

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{z}) = \int dz^+ \mathcal{U}_F(+\infty, z^+; \mathbf{z}) \overleftarrow{\mathcal{D}}_{\mathbf{z}^j} \mathcal{U}_F(z^+, -\infty; \mathbf{z})$$

$$\mathcal{U}_F^{(2)}(\mathbf{z}) = \int dz^+ \mathcal{U}_F(+\infty, z^+; \mathbf{z}) \overleftarrow{\mathcal{D}}_{\mathbf{z}^j} \overrightarrow{\mathcal{D}}_{\mathbf{z}^j} \mathcal{U}_F(z^+, -\infty; \mathbf{z})$$

$$\mathcal{U}_{F;ij}^{(3)}(\mathbf{z}) = \int dz^+ \mathcal{U}_F(+\infty, z^+; \mathbf{z}) g t \cdot \mathcal{F}_{ij}(z^+, \mathbf{z}) \mathcal{U}_F(z^+, -\infty; \mathbf{z})$$

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$$\times \left\{ \int dz^- e^{iz^- (q^+ - k^+)} \mathcal{U}_F(\mathbf{z}, z^-) + 2\pi \delta(q^+ - k^+) \left[ -\frac{(\mathbf{q}^j + \mathbf{k}^j)}{2(q^+ + k^+)} \mathcal{U}_{F;j}^{(1)}(\mathbf{z}) \right. \right.$$

$$\left. \left. - \frac{i}{(q^+ + k^+)} \mathcal{U}_F^{(2)}(\mathbf{z}) + \frac{[\gamma^i, \gamma^j]}{4(q^+ + k^+)} \mathcal{U}_{F;ij}^{(3)}(\mathbf{z}) \right] \right\} \frac{(\not{k} + m)}{2k^+} + \text{NNEik}$$

## Full NEik quark propagator through a gluon background field

Compact notations for the decorated Wilson lines:

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$$\mathcal{U}_{F;j}^{(3)}(\mathbf{z}) = \int dz^+ \mathcal{U}_F(+\infty, z^+; \mathbf{z}) g t \cdot \mathcal{F}_{ij}(z^+, \mathbf{z}) \mathcal{U}_F(z^+, -\infty; \mathbf{z})$$

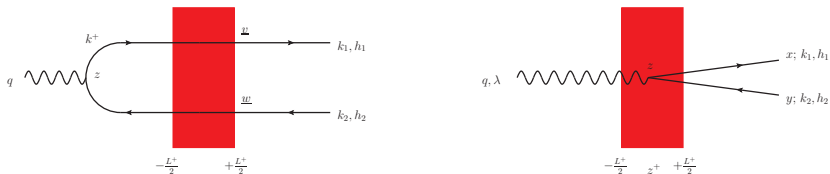
Alternative expressions for the decorated Wilson lines:

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{z}) = -2 \int dz^+ z^+ \mathcal{U}_F(+\infty, z^+; \mathbf{z}) [-igt \cdot \mathcal{F}_j^-(z^+, \mathbf{z})] \mathcal{U}_F(z'^+, -\infty; \mathbf{z})$$

$$\begin{aligned} \mathcal{U}_{F;j}^{(2)}(\mathbf{z}) &= \int dz^+ \int dz'^+ (z^+ - z'^+) \theta(z^+ - z'^+) \mathcal{U}_F(+\infty, z^+, \mathbf{z}) [-igt \cdot \mathcal{F}_j^-(z^+, \mathbf{z})] \\ &\quad \times \mathcal{U}_F(z^+, z'^+, \mathbf{z}) [-igt \cdot \mathcal{F}_j^-(z'^+, \mathbf{z})] \mathcal{U}_F(z'^+, -\infty; \mathbf{z}) \end{aligned}$$

Thanks to the relation:

$$\begin{aligned} &\partial_\mu \mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-) + igt \cdot \mathcal{A}_\mu(x^+, \mathbf{z}, z^-) \mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-) - ig \mathcal{U}_F(x^+, y^+; \mathbf{z}, z^-) t \cdot \mathcal{A}_\mu(y^+, \mathbf{z}, z^-) \\ &= -ig \int_{y^+}^{x^+} dz^+ \mathcal{U}_F(x^+, v^+; \mathbf{z}, z^-) t \cdot \mathcal{F}_\mu^-(z) \mathcal{U}_F(v^+, y^+; \mathbf{z}, z^-) \quad \text{for } \mu \neq + \end{aligned}$$

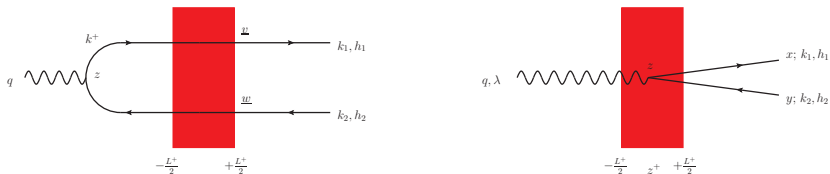
DIS dijet at NEik accuracy: S-matrix for  $\gamma_L^*$ 

DIS dijet cross calculated at NEik accuracy, at LO in  $\alpha_s$  in the CGC.  
 (Altinoluk, G.B., Czajka, Tymowska, (2023))

- Only longitudinal photon contribution will be discussed for simplicity
- Second diagram vanishes in  $\gamma_L^*$  case, but matters in  $\gamma_T^*$  case.

S-matrix at NEik accuracy:  $S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*} = S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*}^{\text{Gen. Eik}} + S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*}^{\text{dyn. target}} + S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*}^{\text{dec. on } q} + S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*}^{\text{dec. on } \bar{q}}$

$$S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*}^{\text{Gen. Eik}} = -2Q \frac{eef}{2\pi} \bar{u}(1) \gamma^+ v(2) \frac{(q^+ + k_1^+ - k_2^+)(q^+ + k_2^+ - k_1^+)}{4(q^+)^2} \int_{\mathbf{v}, \mathbf{w}} e^{-i\mathbf{v} \cdot \mathbf{k}_1} e^{-i\mathbf{w} \cdot \mathbf{k}_2} \\ \times K_0(\hat{Q} |\mathbf{v} - \mathbf{w}|) \int db^- e^{ib^- (k_1^+ + k_2^+ - q^+)} \left[ \mathcal{U}_F(\mathbf{v}, b^-) \mathcal{U}_F^\dagger(\mathbf{w}, b^-) - 1 \right] \\ \hat{Q}^2 = m^2 + \frac{(q^+ + k_1^+ - k_2^+)(q^+ - k_1^+ + k_2^+)}{4(q^+)^2} Q^2.$$

DIS dijet at NEik accuracy: S-matrix for  $\gamma_L^*$ 

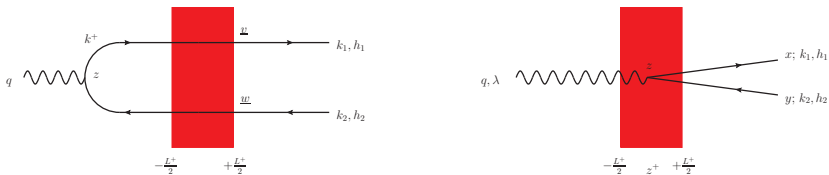
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$$S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*}^{\text{dyn. target}} = 2\pi \delta(k_1^+ + k_2^+ - q^+) iQ \frac{eef}{2\pi} \bar{u}(1) \gamma^+ v(2) \frac{(k_1^+ - k_2^+)}{(q^+)^2} \int d^2 \mathbf{v} e^{-i\mathbf{v} \cdot \mathbf{k}_1} \int d^2 \mathbf{w} e^{-i\mathbf{w} \cdot \mathbf{k}_2} \\ \times \left[ K_0(\bar{Q} |\mathbf{v} - \mathbf{w}|) - \frac{(\bar{Q}^2 - m^2)}{2\bar{Q}} |\mathbf{v} - \mathbf{w}| K_1(\bar{Q} |\mathbf{v} - \mathbf{w}|) \right] \left[ \mathcal{U}_F(\mathbf{v}, b^-) \overleftrightarrow{\partial}_{b^-} \mathcal{U}_F^\dagger(\mathbf{w}, b^-) \right] \Big|_{b^- = 0}$$

$$\bar{Q}^2 = m^2 + Q^2 \frac{k_1^+ k_2^+}{(q^+)^2}$$

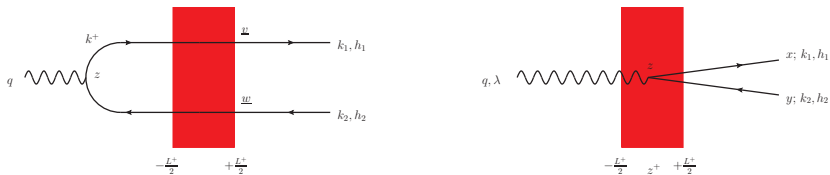
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$$S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*}^{\text{dec. on } q} = 2\pi\delta(k_1^+ + k_2^+ - q^+) \frac{eef}{2\pi} (-1)Q \frac{k_2^+}{(q^+)^2} \int d^2\mathbf{v} e^{-i\mathbf{v}\cdot\mathbf{k}_1} \int d^2\mathbf{w} e^{-i\mathbf{w}\cdot\mathbf{k}_2} K_0(\bar{Q}|\mathbf{v}-\mathbf{w}|) \\ \times \bar{u}(1)\gamma^+ \left[ \frac{[\gamma^i, \gamma^j]}{4} \mathcal{U}_{F;ij}^{(3)}(\mathbf{v}) - i\mathcal{U}_F^{(2)}(\mathbf{v}) + \mathcal{U}_{F;j}^{(1)}(\mathbf{v}) \left( \frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2} + \frac{i}{2} \partial_{\mathbf{w}^j} \right) \right] \mathcal{U}_F^\dagger(\mathbf{w}) v(2)$$

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$$S_{q_1 \bar{q}_2 \leftarrow \gamma_L^*}^{\text{dec. on } \bar{q}} = 2\pi \delta(k_1^+ + k_2^+ - q^+) \frac{eef}{2\pi} (-1) Q \frac{k_1^+}{(q^+)^2} \int d^2 \mathbf{v} e^{-i\mathbf{v} \cdot \mathbf{k}_1} \int d^2 \mathbf{w} e^{-i\mathbf{w} \cdot \mathbf{k}_2} K_0(\bar{Q} |\mathbf{v} - \mathbf{w}|) \\ \times \bar{u}(1) \gamma^+ \left[ \mathcal{U}_F(\mathbf{v}) \left( \frac{[\gamma^i, \gamma^j]}{4} \mathcal{U}_{F;ij}^{(3)\dagger}(\mathbf{w}) - i \mathcal{U}_F^{(2)\dagger}(\mathbf{w}) + \left( \frac{i}{2} \delta_{\mathbf{v}j} - \frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2} \right) \mathcal{U}_{F;j}^{(1)\dagger}(\mathbf{w}) \right) \right] v(2)$$

# Change of variables and back-to-back limit

Back-to-back limit of dijets are conveniently expressed in terms of:

(dijet momentum imbalance)  $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$       and      (relative momentum)  $\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

$z_1 = k_1^+ / (k_1^+ + k_2^+)$  and  $z_2 = k_2^+ / (k_1^+ + k_2^+) = 1 - z_1$  such that

$$\mathbf{k}_1 = \mathbf{P} + z_1\mathbf{k}$$

$$\mathbf{k}_2 = -\mathbf{P} + z_2\mathbf{k}$$

back-to-back correlation limit:  $|\mathbf{k}| \ll |\mathbf{P}|$

In coordinate space:

(conjugate to  $\mathbf{k}$ )  $\mathbf{b} = (z_1\mathbf{v} + z_2\mathbf{w})$       and      (conjugate to  $\mathbf{P}$ )  $\mathbf{r} = \mathbf{v} - \mathbf{w}$

such that

$$\mathbf{v} = \mathbf{b} + z_2\mathbf{r}$$

$$\mathbf{w} = \mathbf{b} - z_1\mathbf{r}$$

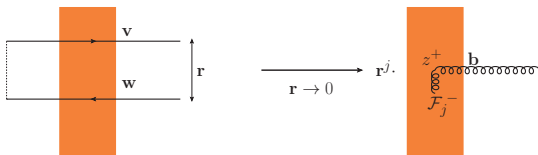
back-to-back correlation limit:  $|\mathbf{r}| \ll |\mathbf{b}|$



# Small $r$ expansion for the eikonal contribution (1)

Open dipole from the Generalized Eikonal term for  $\mathbf{r} = \mathbf{v} - \mathbf{w} \rightarrow 0$ :

$$\begin{aligned}
 & \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ \mathcal{U}_F(\mathbf{b} + z_2 \mathbf{r}, b^-) \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}, b^-) - 1 \right] \\
 &= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ z_2 \mathbf{r}^j (\partial_j \mathcal{U}_F(\mathbf{b}, b^-)) \mathcal{U}_F^\dagger(\mathbf{b}, b^-) - z_1 \mathbf{r}^j \mathcal{U}_F(\mathbf{b}, b^-) (\partial_j \mathcal{U}_F^\dagger(\mathbf{b}, b^-)) + O(r^2) \right] \\
 &= \mathbf{r}^j t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A(+\infty, z^+; \mathbf{b}, b^-)_{ab} (-ig) \mathcal{F}_j^b(z^+, \mathbf{b}, b^-) + O(r^2)
 \end{aligned}$$

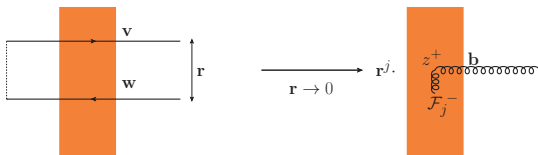


Note: 0th order in the  $\mathbf{r}$  expansion trivial  $\rightarrow$  first order in  $\mathbf{r}$  is the leading power

# Small $r$ expansion for the eikonal contribution (1)

Open dipole from the Generalized Eikonal term for  $\mathbf{r} = \mathbf{v} - \mathbf{w} \rightarrow 0$ :

$$\begin{aligned}
 & \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ \mathcal{U}_F(\mathbf{b} + z_2 \mathbf{r}, b^-) \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}, b^-) - 1 \right] \\
 &= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ z_2 \mathbf{r}^j (\partial_j \mathcal{U}_F(\mathbf{b}, b^-)) \mathcal{U}_F^\dagger(\mathbf{b}, b^-) - z_1 \mathbf{r}^j \mathcal{U}_F(\mathbf{b}, b^-) (\partial_j \mathcal{U}_F^\dagger(\mathbf{b}, b^-)) + O(\mathbf{r}^2) \right] \\
 &= \mathbf{r}^j t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A(+\infty, z^+; \mathbf{b}, b^-)_{ab} (-ig) \mathcal{F}_j^{b-}(z^+, \mathbf{b}, b^-) + O(\mathbf{r}^2)
 \end{aligned}$$



Note: 0th order in the  $\mathbf{r}$  expansion trivial  $\rightarrow$  first order in  $\mathbf{r}$  is the leading power

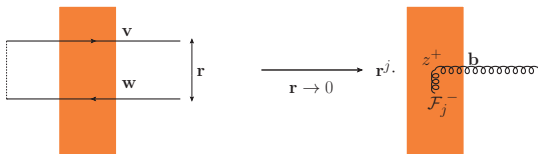
However: the aim is to study the interplay between subleading power corrections

$\Rightarrow$  Terms of order  $\mathbf{r}^2$  needed as well!

# Small $r$ expansion for the eikonal contribution (2)

Open dipole from the Generalized Eikonal term for  $\mathbf{r} = \mathbf{v} - \mathbf{w} \rightarrow 0$ :

$$\begin{aligned}
 & \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ \mathcal{U}_F(\mathbf{b} + z_2 \mathbf{r}, b^-) \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}, b^-) - 1 \right] \\
 = & \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ -i \left( 1 + \frac{i(z_2 - z_1)}{2} \mathbf{r}\cdot\mathbf{k} \right) \mathbf{r}^j t^{a'} \int dv^+ \mathcal{U}_A(+\infty, v^+; \mathbf{b}, b^-)_{a'a} g\mathcal{F}_{j-a}^-(v^+, \mathbf{b}, b^-) \right. \\
 & - \frac{1}{2} \mathbf{r}^i \mathbf{r}^j t^{a'} t^{b'} \int dv^+ \int dw^+ \mathcal{U}_A(+\infty, v^+; \mathbf{b}, b^-)_{a'a} g\mathcal{F}_{i-a}^-(v^+, \mathbf{b}, b^-) \\
 & \left. \times \mathcal{U}_A(+\infty, w^+; \mathbf{b}, b^-)_{b'b} g\mathcal{F}_{j-b}^-(w^+, \mathbf{b}, b^-) + \mathcal{O}(|\mathbf{r}|^3) \right]
 \end{aligned}$$



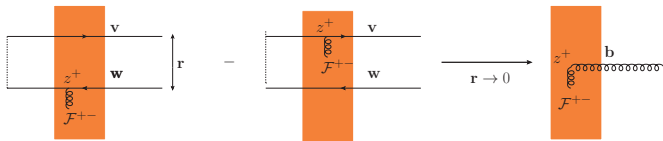
$\Rightarrow$  Order  $|\mathbf{r}|^2$  correction: contributions with either one or two field strength  $\mathcal{F}_\perp^-$

# Small $r$ limit for the non-static NEik correction

For the open decorated dipole due to the dynamics of the target:

$$\begin{aligned}
 & \left[ \mathcal{U}_F(\mathbf{b} + z_2 \mathbf{r}, b^-) \overleftrightarrow{\partial}_{b^-} \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}, b^-) \right] \Big|_{b^- = 0} \\
 &= \int_{z^+} \left\{ \mathcal{U}_F(\mathbf{b}) \mathcal{U}_F^\dagger(z^+, -\infty; \mathbf{b}) i g t \cdot \mathcal{F}^{+-}(z^+, \mathbf{b}) \mathcal{U}_F^\dagger(+\infty, z^+; \mathbf{b}) \right. \\
 & \quad \left. - \mathcal{U}_F(+\infty, z^+; \mathbf{b}) (-i g) t \cdot \mathcal{F}^{+-}(z^+, \mathbf{b}) \mathcal{U}_F(z^+, -\infty; \mathbf{b}) \mathcal{U}_F^\dagger(\mathbf{b}) + O(|\mathbf{r}|) \right\} \\
 &= 2i t a' \int_{z^+} \mathcal{U}_A(+\infty, z^+; \mathbf{b})_{a'a} g \mathcal{F}_a^{+-}(z^+, \mathbf{b}) + O(|\mathbf{r}|)
 \end{aligned}$$

Involves the longitudinal chromoelectric field  $\mathcal{F}^{+-}$  instead of the transverse field  $\mathcal{F}_j^-$

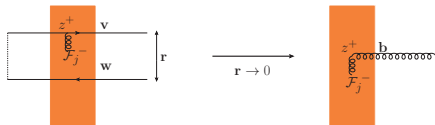


Note: Similar result for the NEik corrections with  $\mathcal{U}_{F;ij}^{(3)}$ , but with  $\mathcal{F}_{ij}$  instead of  $\mathcal{F}^{+-}$

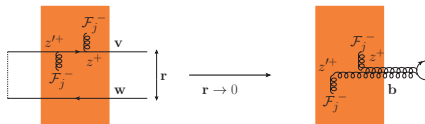
# Small $r$ limit for the NEik corrections in $\mathcal{U}_{F;j}^{(1)}$ and $\mathcal{U}_F^{(2)}$

Terms with  $\mathcal{U}_{F;j}^{(1)}$  and  $\mathcal{U}_F^{(2)}$  decorating the quark line (remembering that  $|\mathbf{r}| \sim 1/|\mathbf{P}|$ ):

$$\begin{aligned}
 & \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[ - \left( \mathbf{P}^j + \frac{(z_1 - z_2)}{2} \mathbf{k}^j \right) \mathcal{U}_{F;j}^{(1)}(\mathbf{b} + z_2 \mathbf{r}) \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}) \right. \\
 & \left. + \frac{i}{2} \mathcal{U}_{F;j}^{(1)}(\mathbf{b} + z_2 \mathbf{r}) \partial_j \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}) - i \mathcal{U}_F^{(2)}(\mathbf{b} + z_2 \mathbf{r}) \mathcal{U}_F^\dagger(\mathbf{b} - z_1 \mathbf{r}) \right] \\
 &= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left\{ \left[ - \mathbf{P}^j + \frac{(z_2 - z_1)}{2} \mathbf{k}^j - iz_2 \mathbf{P}^j (\mathbf{r}\cdot\mathbf{k}) \right] \mathcal{U}_{F;j}^{(1)}(\mathbf{b}) \mathcal{U}_F^\dagger(\mathbf{b}) \right. \\
 & \left. + \left[ \frac{i}{2} \delta^{ij} + \mathbf{P}^j \mathbf{r}^i \right] \mathcal{U}_{F;j}^{(1)}(\mathbf{b}) \partial_i \mathcal{U}_F^\dagger(\mathbf{b}) - i \mathcal{U}_F^{(2)}(\mathbf{b}) \mathcal{U}_F^\dagger(\mathbf{b}) + \mathcal{O}(|\mathbf{r}|) \right\}
 \end{aligned}$$



$$\mathcal{U}_{F;j}^{(1)}(\mathbf{b}) \mathcal{U}_F^\dagger(\mathbf{b}) = 2it^{a'} \int_{z^+} z^+ \mathcal{U}_A(+\infty, z^+; \mathbf{b})_{a'a} g\mathcal{F}_j^{a-}(z^+, \mathbf{b})$$

Small  $r$  limit for the NEik corrections in  $\mathcal{U}_{F;j}^{(1)}$  and  $\mathcal{U}_F^{(2)}$ 

$$\mathcal{U}_F^{(2)}(\mathbf{b})\mathcal{U}_F^\dagger(\mathbf{b}) = -t^{a'}t^{b'} \int_{z^+, z'^+} (z^+ - z'^+) \theta(z^+ - z'^+) \mathcal{U}_A(+\infty, z^+; \mathbf{b})_{a'a} g\mathcal{F}_j^{a-}(z^+, \mathbf{b}) \\ \times \mathcal{U}_A(+\infty, z'^+; \mathbf{b})_{b'b} g\mathcal{F}_j^{b-}(z'^+, \mathbf{b})$$

$$\mathcal{U}_{F;j}^{(1)}(\mathbf{b}) \partial_i \mathcal{U}_F^\dagger(\mathbf{b}) = -2t^{a'}t^{b'} \int dz^+ \int dz'^+ z^+ \mathcal{U}_A(+\infty, z^+; \mathbf{b})_{a'a} g\mathcal{F}_j^{a-}(z^+, \mathbf{b}) \\ \times \mathcal{U}_A(+\infty, z'^+; \mathbf{b})_{b'b} g\mathcal{F}_i^{b-}(z'^+, \mathbf{b})$$

Like in the GEik term: contributions with either 1 or 2  $\mathcal{F}_\perp^-$ ,  
but now with an extra factor  $z^+$  or  $(z^+ - z'^+)$ : NEik suppression with the target width.

Similar results for decorations on the antiquark line instead.


# Back-to-back cross section: (Generalized) Eikonal piece

Squaring the single  $\mathcal{F}_\perp^-$  part of the Generalized Eikonal contribution in the back-to-back limit:

$$\begin{aligned} \frac{d\sigma_{\gamma_L^* \rightarrow q_1 \bar{q}_2}}{d\mathbf{P} \cdot \mathbf{S}} \Bigg|_{\text{Gen.Eik}}^{\mathcal{F}_\perp^- \mathcal{F}_\perp^-} &= g^2 (e e_f)^2 Q^2 (q^+ + k_1^+ - k_2^+)^2 (q^+ - k_1^+ + k_2^+)^2 \frac{k_1^+ k_2^+}{4(q^+)^6} \left[ \frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \hat{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \hat{Q}^2]^4} \right. \\ &\quad \left. + 16(z_2 - z_1) \frac{(\mathbf{k} \cdot \mathbf{P}) \mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \hat{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right) \right] (2q^+) \int d(\Delta b^-) e^{i\Delta b^- (k_1^+ + k_2^+ - q^+)} \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \int_{z^+, z'^+} \\ &\quad \times \left\langle \mathcal{F}_i^a(z'^+, \mathbf{b}', -\frac{\Delta b^-}{2}) \left[ \mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}', -\frac{\Delta b^-}{2}) \mathcal{U}_A(+\infty, z^+; \mathbf{b}, \frac{\Delta b^-}{2}) \right]_{ab} \mathcal{F}_j^b(z^+, \mathbf{b}, \frac{\Delta b^-}{2}) \right\rangle \end{aligned}$$

Strict Eikonal result found by neglecting  $\Delta b^-$  in the fields:

$$\begin{aligned} \frac{d\sigma_{\gamma_L^* \rightarrow q_1 \bar{q}_2}}{d\mathbf{P} \cdot \mathbf{S}} \Bigg|_{\text{Strict.Eik}}^{\mathcal{F}_\perp^- \mathcal{F}_\perp^-} &= (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) (e e_f)^2 g^2 4z_1^3 z_2^3 Q^2 \\ &\times \left[ \frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \hat{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \hat{Q}^2]^4} + 16(z_2 - z_1) \frac{(\mathbf{k} \cdot \mathbf{P}) \mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \hat{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right) \right] \\ &\times \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \int_{z^+, z'^+} \left\langle \mathcal{F}_i^a(z'^+, \mathbf{b}') \left[ \mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b}) \right]_{ab} \mathcal{F}_j^b(z^+, \mathbf{b}) \right\rangle \end{aligned}$$

- **Twist-2 gluon TMDs in the target** (both unpolarized and linearly polarized), with momentum fraction  $x = 0$  and transverse momentum  $\mathbf{k}$ , with a *future staple* gauge link.
- **Kinematical twist 3 corrections**, suppressed by an extra  $|\mathbf{k}|/|\mathbf{P}|$  in the back-to-back dijet limit  $|\mathbf{k}| \ll |\mathbf{P}|$
- Not shown here: **Genuine twist 3 corrections**, involving a correlator of the type  $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$
- Difference between Gen. Eik and strict Eik. : involves correlator  $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \mathcal{F}_\perp^+ \rangle \Rightarrow$  twist 4 and NEik correction! 

# Back-to-back cross section: twist 3 TMDs from NEik

From the interference between the non-static NEik correction and the strict Eikonal amplitudes:

$$\frac{d\sigma_{\gamma_L^* \rightarrow q_1 \bar{q}_2}}{dP.S.} \Bigg|_{NEik}^{\mathcal{F}_\perp^- \mathcal{F}^{+-}} = (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) 8Q^2 e^2 e_f^2 g^2 \frac{z_1^2 z_2^2 (z_2 - z_1)}{q^+} \frac{\mathbf{P}^i (\mathbf{P}^2 + m^2)}{(\mathbf{P}^2 + \bar{Q}^2)^4} \\ \times 2\text{Re} \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \int_{z^+, z'^+} \left\langle \mathcal{F}_i^a(z'^+, \mathbf{b}') \left[ \mathcal{U}_A^\dagger(\infty, z'^+; \mathbf{b}') \mathcal{U}_A(\infty, z^+; \mathbf{b}) \right]_{ab} \mathcal{F}_b^{+-}(z^+, \mathbf{b}) \right\rangle$$

⇒ NEik. correction stemming from the dynamics of the target is a **twist-3 gluon TMD**, (Mulders, Rodrigues (2001)) with momentum fraction  $x = 0$ .

From the interference between the NEik correction with  $\mathcal{U}_{F;ij}^{(3)}$  and the strict Eikonal amplitude:

- Vanishing result in the  $\gamma_L^*$  case due to Dirac algebra.
- An extra contribution to the cross section in the  $\gamma_T^*$  case:

$$\frac{d\sigma_{\gamma_T^* \rightarrow q_1 \bar{q}_2}}{dP.S.} \Bigg|_{NEik}^{\mathcal{F}_\perp^- \mathcal{F}_{ij}} \propto 2\text{Re} \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \int_{z^+, z'^+} \left\langle \mathcal{F}_i^a(z'^+, \mathbf{b}') \left[ \mathcal{U}_A^\dagger(\infty, z'^+; \mathbf{b}') \mathcal{U}_A(\infty, z^+; \mathbf{b}) \right]_{ab} \mathcal{F}_{ij}^b(z^+, \mathbf{b}) \right\rangle$$

⇒ The other **twist-3 gluon TMD** as found in Mulders, Rodrigues (2001), with momentum fraction  $x = 0$ .



# Back-to-back cross section: $x$ dependence from NEik

Including all contributions of the form  $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$ , of order Eik or NEik, and twist 2 or twist 3:

$$\begin{aligned} \frac{d\sigma_{\gamma_L^* \rightarrow q_1 \bar{q}_2}}{d\text{P.S.}} \Big|_{\mathcal{F}_\perp^- \mathcal{F}_\perp^-} &= (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) (ee_f)^2 g^2 4z_1^3 z_2^3 Q^2 \\ &\times \left[ \frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \bar{Q}^2]^4} + 16(z_2 - z_1) \frac{(\mathbf{k} \cdot \mathbf{P}) \mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right) \right] \\ &\times \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \int_{z^+, z'^+} \left[ 1 + i(z^+ - z'^+) \frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} + NNEik \right] \langle \mathcal{F}_i^a(z^+, \mathbf{b}') [\mathcal{U}_A^{\dagger}(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b})]_{ab} \mathcal{F}_j^b(z^+, \mathbf{b}) \rangle \end{aligned}$$

⇒ NEik corrections and kinematic twist 3 corrections to the  $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$  contribution factorize from each other!

The “-” momentum extracted from the target can be defined from the conservation relation (where the  $\mathbf{k}^2$  term is a twist 4 correction):

$$xP_{tar}^- \equiv \bar{k}_1^- + \bar{k}_2^- - q^- = \frac{\mathbf{k}_1^2 + m^2}{2k_1^+} + \frac{\mathbf{k}_2^2 + m^2}{2k_2^+} + \frac{Q^2}{2q^+} = \frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} + \frac{\mathbf{k}^2}{2q^+}$$

The NEik correction can be summed into a phase! ⇒ dependence of the twist 2 gluon TMDs on  $x$

$$\begin{aligned} \frac{d\sigma_{\gamma_L^* \rightarrow q_1 \bar{q}_2}}{d\text{P.S.}} \Big|_{\mathcal{F}_\perp^- \mathcal{F}_\perp^-} &= (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) (ee_f)^2 g^2 4z_1^3 z_2^3 Q^2 \\ &\times \left[ \frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)^4} - 2(z_2 - z_1) \frac{(\mathbf{P}^i \mathbf{k}^j + \mathbf{k}^i \mathbf{P}^j)}{[\mathbf{P}^2 + \bar{Q}^2]^4} + 16(z_2 - z_1) \frac{(\mathbf{k} \cdot \mathbf{P}) \mathbf{P}^i \mathbf{P}^j}{[\mathbf{P}^2 + \bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right) \right] \\ &\times \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \int_{z^+, z'^+} e^{i(z^+ - z'^+) x P_{tar}^-} \langle \mathcal{F}_i^a(z^+, \mathbf{b}') [\mathcal{U}_A^{\dagger}(+\infty, z'^+; \mathbf{b}') \mathcal{U}_A(+\infty, z^+; \mathbf{b})]_{ab} \mathcal{F}_j^b(z^+, \mathbf{b}) \rangle + NNEik \end{aligned}$$

# Back-to-back cross section: 3-body correlators

Including all contributions of the form  $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$ , of order Eik or NEik, at twist 3:

$$\begin{aligned} \frac{d\sigma_{\gamma_L^- \rightarrow q_1 \bar{q}_2}}{dP.S.} \Big|_{\mathcal{F}_\perp^- \mathcal{F}_\perp^- \mathcal{F}_\perp^-} &= (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) (e e_f)^2 16 Q^2 z_1^3 z_2^3 \frac{\mathbf{P}^l}{(\mathbf{P}^2 + \bar{Q}^2)^4} 2\text{Re} \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \\ &\times \int_{v^+, w^+, v'^+} \text{tr}_F(t^{a'} t^{b'} t^{c'}) \langle \mathcal{U}_A(+\infty, v'^+, \mathbf{b}')_{c'e} g \mathcal{F}_i^c{}^- (v'^+, \mathbf{b}') \mathcal{U}_A(+\infty, v^+, \mathbf{b})_{a'a} g \mathcal{F}_i^a{}^- (v^+, \mathbf{b}) \mathcal{U}_A(+\infty, w^+, \mathbf{b})_{b'b} g \mathcal{F}_j^b{}^- (w^+, \mathbf{b}) \rangle \\ &\times \left\{ \delta^{ij} \left[ 1 + i \frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} (\min(v^+, w^+) - v'^+) \right] - \frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)} \left[ 1 + i \frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} (z_2 v^+ + z_1 w^+ - v'^+) \right] \right\} \end{aligned}$$

⇒ **Diagonal NEik corrections** for each tensor structure in the hard factor for  $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$ .

The “-” momentum extracted from the target can be defined from the conservation relation (where the  $\mathbf{k}^2$  term is a twist 4 correction):

$$xP_{tar}^- \equiv \tilde{k}_1^- + \tilde{k}_2^- - q^- = \frac{\mathbf{k}_1^2 + m^2}{2k_1^+} + \frac{\mathbf{k}_2^2 + m^2}{2k_2^+} + \frac{Q^2}{2q^+} = \frac{(\mathbf{P}^2 + \bar{Q}^2)}{2q^+ z_1 z_2} + \frac{\mathbf{k}^2}{2q^+}$$

NEik corrections compatible with a summation into phases for each term

⇒ Non trivial  $x$  dependence of the 3-body twist 3 gluon correlators?

$$\begin{aligned} \frac{d\sigma_{\gamma_L^- \rightarrow q_1 \bar{q}_2}}{dP.S.} \Big|_{\mathcal{F}_\perp^- \mathcal{F}_\perp^- \mathcal{F}_\perp^-} &= (2q^+) 2\pi \delta(k_1^+ + k_2^+ - q^+) (e e_f)^2 16 Q^2 z_1^3 z_2^3 \frac{\mathbf{P}^l}{(\mathbf{P}^2 + \bar{Q}^2)^4} 2\text{Re} \int_{\mathbf{b}, \mathbf{b}'} e^{-i\mathbf{k} \cdot (\mathbf{b} - \mathbf{b}')} \\ &\times \int_{v^+, w^+, v'^+} \text{tr}_F(t^{a'} t^{b'} t^{c'}) \langle \mathcal{U}_A(+\infty, v'^+, \mathbf{b}')_{c'e} g \mathcal{F}_i^c{}^- (v'^+, \mathbf{b}') \mathcal{U}_A(+\infty, v^+, \mathbf{b})_{a'a} g \mathcal{F}_i^a{}^- (v^+, \mathbf{b}) \mathcal{U}_A(+\infty, w^+, \mathbf{b})_{b'b} g \mathcal{F}_j^b{}^- (w^+, \mathbf{b}) \rangle \\ &\times \left\{ \delta^{ij} e^{ixP_{tar}^-(\min(v^+, w^+) - v'^+)} - \frac{4\mathbf{P}^i \mathbf{P}^j}{(\mathbf{P}^2 + \bar{Q}^2)} e^{iz_2 x P_{tar}^-(v^+ - v'^+)} e^{iz_1 x P_{tar}^-(w^+ - v'^+)} \right\} \end{aligned}$$

# Summary

To further understand the interplay between CGC and TMD, we studied the NEik DIS dijet cross-section in the back-to-back jets limit, including twist 3 power corrections. Various types of contributions are obtained:

- $\langle \mathcal{F}_i^- \mathcal{F}_j^- \rangle$ : twist 2 gluon TMDs
  - Factorization of kinematic twist 3 and of NEik corrections
  - NEik correction reproduce the expansion of the phase defining the  $x$  dependence of the TMDs
- 3-body twist 3 correlators  $\langle \mathcal{F}_i^- \mathcal{F}_j^- \mathcal{F}_l^- \rangle$ : beyond TMDs!
  - Already appear in Eikonal contributions
  - NEik corrections also resum into  $x$  dependent phase
  - But different phase for each tensor structure of the hard factor!
- Twist 3 gluon TMDs:  $\langle \mathcal{F}_i^- \mathcal{F}^{+-} \rangle$  and (for  $\gamma_T^*$ )  $\langle \mathcal{F}_l^- \mathcal{F}_{ij} \rangle$ , as further NEik corrections.

Remark: Odd (resp. Even) Twist terms typically proportional to Odd (resp. Even) powers of  $\mathbf{P} \cdot \mathbf{k}$ , for unpolarized target.

⇒ Odd (resp. Even) azimuthal harmonics in terms of  $\mathbf{P}$  and  $\mathbf{k}$

# Recap: Corrections beyond the static approximation

- Relative  $z^-$  dependence along the same propagator : NNEik, higher twist ?
- Relative  $z^-$  dependence between Wilson lines in the amplitude: NEik, twist 3
- Relative  $z^-$  dependence between amplitude and cc. amplitude: NEik, twist 4
- Overall  $z^-$  dependence at cross section level: Gone when performing target average

All these non-static corrections can be written as insertions of the longitudinal chromoelectric field  $\mathcal{F}^{+-}$  of the target.