SLOW EXTRACTION TECHNIQUES FROM FIXED FIELD ACCELERATORS

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Abstract

Fixed Field Accelerators are a candidate for future hadron cancer therapy facilities as their high repetition rate and large energy acceptance enables novel treatment modalities such as high dose rate FLASH. However, conventional dose delivery mechanisms are still necessary, requiring continuous beam delivery over 1–30s. This work is the first study of slow extraction from a scaling Fixed Field Accelerator, using the LhARA facility for baseline parameters. At a horizontal tune of 10/3, the intrinsic sextupole strength of the nonlinear FFA magnetic field is sufficient to excite the resonance, although extraction is better controlled using an additional excitation sextupole at a tune close to 8/3, with radiofrequency knockout extraction. Including considerations of issues due to nonlinear fields and limitations required to keep the tune energy-independent, slow extraction from Fixed Field Accelerators is successfully demonstrated.

INTRODUCTION

Fixed Field Accelerators (FFAs) are being considered for future hadron therapy treatment facilities, as beam current and novel treatment modalities may both benefit from their high repetition rate and large momentum acceptance. Several particle therapy FFAs have been proposed, including RACCAM, PAMELA, NORMA, and LhARA[2, 8, 5, 3], with successful demonstration of key concepts at KURNS [12, 11]. These FFAs have highly nonlinear magnetic fields, required to keep the tunes constant throughout beam acceleration without ramping the magnets [9]. The high packing factor and periodicity of these lattices present a challenge for beam extraction. Further, decisions taken early in the design process, such as the working point, will influence feasibility of extraction mechanisms. As such, beam delivery must be considered concurrently with the optical design.

This is especially true for slow extraction, used in hadron therapy synchrotrons to deliver a continuous uniform beam to the patient for up to thirty seconds per spill [4]. This is achieved by setting the fractional tune close to ⅓, which is then driven with strong sextupole magnets. Under these conditions, the stable phase space ellipse is transformed into a triangular region with three extraction separatrices. Particles are driven into the resonance via excitation methods, and are kicked out by the Electrostatic Septum (ES).

This study is the first investigation of slow extraction from a scaling FFA using the third-order resonance. Previous work has investigated resonance crossing in FFAs with non-constant tune [13, 10, 6], but extraction in these cases has been achieved with fast kicker magnets [1, 6]. This study uses design specifications based on the LhARA Stage 2 [3] accelerator, to outline geometric and optical adjustments required for resonant slow extraction. Finally, potential difficulties for slow extraction with FFAs are considered.

LhARA Facility

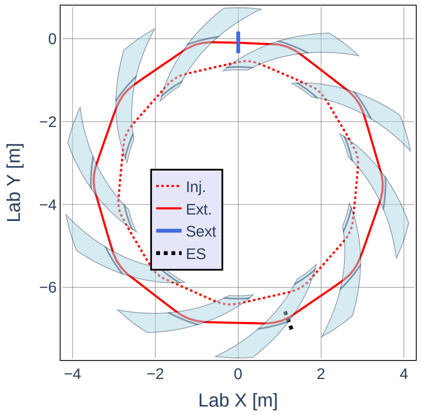

Figure 1: A LhARA-like spiral scaling FFA with parameters of Table 1. The orbits at injection and extraction are shown, as well as possible locations for an extra sextupole and ES.

The Laser-hybrid Accelerator for Radiobiological Applications (LhARA) [3] is a proposed accelerator for the UK Ion Therapy Research Facility for in-vitro and in-vivo irradiation and radiobiological studies. The beam is generated from a laser-driven source with a frequency up to 10 Hz. A spiral FFA has been proposed, to reduce the footprint: key parameters are given in Table 1, and the layout is illustrated in Fig. 1. Although the static magnetic fields make it theoretically possible to simultaneously transport many energies, LhARA will operate with only one circulating beam per cy-

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cle. The FFA is intended to operate as an ‘energy multiplier’, where the extracted beam energy is varied by controlling the injection energy and ramping the maximum magnetic field to match. For this work, it is assumed that the closed orbit trajectory at extraction is energy-independent, and that the beam dynamics of lower energy orbits can be neglected.

Table 1: Parameters of the LhARA FFA [3]

Parameter	**Value**	**Parameter**	**Value**
E [MeV]	15 - 127	Packing Factor	0.34
B_0 [T]	1.4	r_0 [m]	3.47704
Magnet length	0.7537	ε_x [πmm·mrad]	0.41

EXTRACTION CONSIDERATIONS

FFA Multipoles

To keep the tune of an FFA constant, the magnetic field must adhere to the scaling law. For circular FFAs with horizontal excursion [9], in cylindrical coordinates this is given by Equation 1, where B_0 and r_0 are a reference field and radius respectively, the magnet spiral angle ζ provides edge-focusing, and the k-index determines the field gradient. The form factor $\mathcal{F}$ describes magnet fringe and body fields.

$$B = B_0 \left(\frac{r}{r_0}\right)^k \mathcal{F}\left(\theta - \tan(\zeta) \ln \frac{r}{r_0}\right) \tag{1}$$

The magnetic multipole terms are entirely determined by B_0, r_0, and k: In a Taylor expansion of Equation 1, the first term determines the dipole field, the second is the quadrupole gradient, and so on for higher orders. Neglecting edge focusing (which has no first-order impact on the horizontal tune), the multipole strengths and working point can’t be decoupled. This is illustrated in Fig. 2: relative multipole strengths at each resonance are fixed, and varying them (for example, to increase the sextupole strength) is not possible without breaking the scaling law.

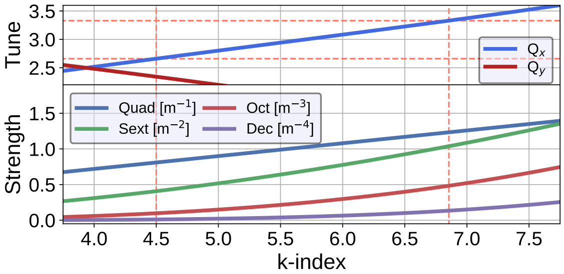

Figure 2: Tunes and normalised integrated multipole strengths at the extraction energy, for ζ=57.0°. Multipoles at the 8⁄3 and 10⁄3 resonances are indicated by dashed lines.

Slow extraction requires a strong sextupole strength S to drive the third-order resonance. In synchrotrons, excitation sextupoles are usually added in a section with zero dispersion to avoid introducing additional chromaticity. However, it is not possible to include dispersion-free regions in scaling FFAs, as this would violate the scaling law required for constant tunes. In an FFA, if the intrinsic sextupolar term is insufficient to drive the resonance, an additional sextupole is necessary: to keep the tune constant, any extra magnets must be kept at zero field until the beam has been accelerated to extraction energy. Alternatively, a family of sextupoles or other multipoles could be considered, countering the chromatic effects but keeping the resonance-driving terms.

Septum Positioning

The ES provides a kick to direct high-amplitude particles towards the field region of a Magnetic Septum (MS), which has sufficient strength to extract particles from the accelerator. Between the septa, the phase advance should be around 90°. Here the ES is modelled as a single point.

The angles of the extraction separatrices must be aligned with respect to the horizontal septum position, X_{ES}. This angle depends on the phase advance between the excitation sextupole and ES, ideally around 225° [4]. In an FFA with a high packing factor, locations for the ES are limited, and may need to be within a magnet. In addition, the slow extraction spiral step is proportional to $S \cdot X_{\mathrm{ES}}^2$, so these variables must be chosen to control the extracted particle distribution, and to avoid losses on the ES electrodes.

Excitation Methods

The excitation method affects the rate at which particles enter the resonance and are extracted, defining spill rate and quality. Radiofrequency knock-out (RF-KO) can excite these particles into the resonance whilst preserving the constant tune of the FFA. This requires the installation of a transverse kicker that applies a sinusoidal excitation at the frequency of the tune of the beam. For a uniform distribution with time, an exponential increase in the amplitude of this kick must be added, corresponding to the increasing density of the beam.

Alternatively, the working point could be ramped towards the resonance once the beam has reached the extraction energy. This could be achieved by varying the k-index, however this would affect the optics and multipoles, and require a specialised magnet design. Although it may be feasible, this option is not considered here.

SIMULATIONS

Simulations were performed in Zgoubi [7] with the Zgoubidoo Python package. Starting from the LhARA FFA design, the parameters in Table 1 were kept constant, varying k and ζ to investigate particular resonances.

Intrinsically Driven Resonance

Extraction without additional sextupoles would be ideal to preserve the zero-chromaticity of the FFA. A strong resonance was observed at $Q_x = 10/3$. The LhARA design spiral angle of 48.7° was unsuitable for this study, as the FFA was vertically unstable at this horizontal tune: this may be countered by increasing vertical focusing by optimising the fringe fields. Here, k and ζ were set to 6.678 and 59.876° respectively, with a 25 mm wide ES centred at 52.5 mm.

In this case, particles extract without the need for a resonance-driving sextupole: the $10/3$ resonance is particularly strong as the FFA has a superperiodicity of 10, meaning each individual cell has a $1/3$ phase-advance. The $1/3$ phase-advance per cell is limiting, as separatrix orientation is identical in each cell, making septum positioning difficult.

Typically with a third-order resonance, the particles will experience a spiral step in amplitude once every three turns around the lattice. This particular tune produces that step every three magnets, causing particles to leave the machine ten times faster than expected; this forces the spiral step within one revolution to be larger than the reasonable width of the ES. This issue is illustrated in Fig. 3, which shows the maximum horizontal position of each particle, before tracking fails. In a lattice with only one ES (blue), 90 % of all particles jump over the septum, and are lost elsewhere in the ring. If septa could be placed at the same position for all 10 cells (red), more particles could be successfully extracted, however including so many septa is not feasible.

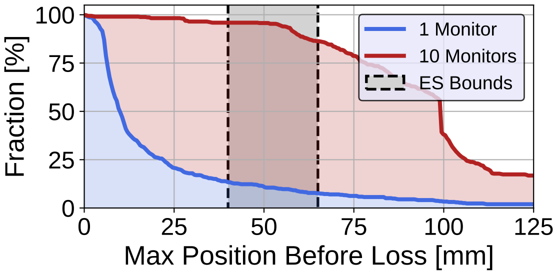

Figure 3: Maximum offset relative to the closed orbit before beam loss during extraction, comparing a single virtual screen to one in each cell.

It may be possible to use this self-exciting resonance in a lattice with a smaller spiral step, but it would require significant modifications to the FFA design. Instead, a new working point was chosen, requiring an excitation sextupole.

Sextupole-Driven Resonance

The resonance at $Q_x = 8/3$ was chosen for extraction with RF-KO, using k-index= 4.9, $\zeta = 57.175°$, and an ES centred at 32.5 mm with X_{ES} =15 mm; although it had been hoped that the LhARA spiral angle of 48.7° could be used, a clean spill could not be achieved due to sextupole amplitude detuning affecting separatrix linearity. Near the $8/3$ resonance, the FFA's intrinsic sextupole strength was insufficient for extraction, requiring an additional resonant sextupole. Both the sextupole and the ES were modelled with zero length, positioned as shown in Fig. 1.

A sinusoidal excitation of amplitude $2.5 \times 10^{-5}\ \sqrt{\mathrm{m}}$ and frequency 0.6590 /turn with a bandwidth of 1.5×10^{-3} /turn was applied to the angular component of the beam. Fig. 4 shows the cumulative phase-space distribution over 30 000 turns for 256 particles, and the tune shift towards the resonance at large amplitudes. The beam is successfully extracted, although further optimisation of the excitation signal is recommended for analysis of extracted spill quality.

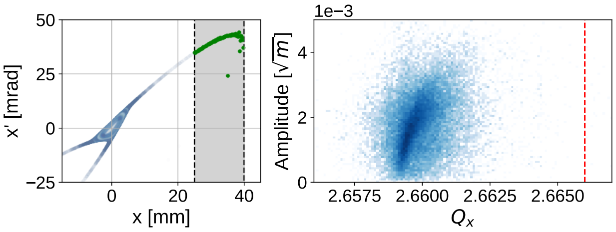

Figure 4: Extraction with RF-KO, in phase space and tune-amplitude. On the left, the ES field region is shown in grey, with the extracted particle distribution in green. On the right, the centre of the resonance is given by the dashed line.

CONCLUDING REMARKS

The approximations required for the theory of slow extraction are derived for synchrotrons, so they may not hold at large amplitudes in FFAs: for example, the spiral step may not be symmetric along the horizontal axis, due to the large field gradient and high curvature giving rise to asymmetric separatrices. In such a case, the spiral step of the particles may be larger in the direction towards the machine centre. An asymmetry was not observed in this study, however it may be evident in lattices suitable for clinical use, as the increased beam rigidity would require a larger k-index.

This study modelled septa as point elements, but their finite length is likely to introduce a significant spread in the extracted beam, due to the large phase advance per cell. In addition, particles traversing from the ES to the MS will go through fringe fields at large amplitudes, distorting the distribution of extracted beam: this will be exacerbated if the horizontal position of the coasting beam's orbit is far from the ES, as may be the case for multi-energy extraction.

The design and placement of the sextupole is challenging, as it must be centred on the extraction orbit, and the magnet yoke must not interrupt the path of lower energy orbits. All inserted components, including septa and sextupoles may need to curve with the magnet spiral in order to follow the closed orbits: this would give rise to cylindrical or angled multipole effects, which slow extraction is sensitive to at large amplitudes. This could be solved by having a racetrack design with longer straight sections, however that would introduce tune variations that must be compensated.

In spite of these difficulties, it is clear that slow extraction from a scaling Fixed Field Accelerator is feasible. The high periodicity may introduce issues, seen here at a tune of $10/3$, therefore in this work, extraction was carried out at $Q_x = 8/3$ with the addition of an extra sextupole and a transverse RF-KO exciter. This work has demonstrated for the first time that slow extraction from an FFA can be achieved, highlighting some challenges that future studies should consider further.

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