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DIFFERENTIAL EQUATION MODEL OF TUNE RIPPLE EFFECT ON BEAM SPILL RIPPLE IN RFKO SLOW EXTRACTION*

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Abstract

The beam uniformity is an important factor that must be considered in the slow extraction optimization, and the tune ripple caused by the power supply ripple is an important factor that causes the beam uniformity to deteriorate. In this study, based on the beam excitation concept of two regions (extraction region and diffusion region), the differential equation of beam spill under the influence of a mono frequency tune ripple is established. By solving and analysing the differential equation, some new conclusions are found and verified by simulation.

INTRODUCTION

Third order resonance slow extraction [1] is widely used in hadron therapy and space radiation environment simulation devices based on a synchrotron. The extracted beam is referred to as the "spill". The spill uniformity is extremely important in the above applications. Third order resonance extraction is sensitive to the horizontal tune since it is set close to the third order resonance. So, the tune ripple has a significant impact on the temporal structure of the spill. Tune ripple mainly originate from the power supply ripple, especially the main dipole and quadrupole power supply.

In RF-knockout (RFKO) slow extraction [2], the stable phase space region remains constant and a transverse RF field is used to excite the particle emittance growth to leave the stable region. Thus, both the variation of the stable phase space region's area and the variation of the emittance growth rate contribute to the spill ripple.

A model of how the tune ripple transfers to the beam spill ripple in RFKO slow extraction has been proposed in a previous study by HIMAC [3]. Only the variation of the stable phase space region's area was considered and tune ripple influence on the emittance growth was neglected. A novel beam spill ripple model is proposed by W. B. Ye [4] including the effect of tune ripple on the emittance growth. But such two models above can't specify how to optimize RF signal because the excitation effect of the RF signal on the beam is described as only one variable on average. The average excitation effect has already been determined by the demand of extracted beam intensity.

Some experiments have been carried out to optimize RFKO signals to reduce beam spill ripple. In 2002, K. Noda pointed out that there are two tune regions in the extraction process of RFKO slow extraction which are extraction region and diffusion region [5]. By applying a transverse RF field with a mono frequency that matches the tune in the extraction region, the spill ripple is reduced. But the reduction is owed to the decrease of the beamless time. The method is called the separate function method. In 2009, K. Mizushima pointed out that the separate function method can reduce the uncontrollable extracted beam intensity after turning off RFKO [6]. It is explained that the particle density in the extraction region gets smaller with a mono frequency signal on the region. E. C. Cortés García [7], W. B. Ye [8] and so on have successfully reduced beam spill ripple by optimizing RF signal by experiment with qualitative explanations.

So, we attempt to establish a model in which the excitation effect of the RF signal on the beam is not described as only one variable on average but two on the extraction and diffusion region. The excitation effect on the diffusion region is determined by the extracted beam intensity but the effect on the extraction region can be optimized to reduce beam spill ripple.

DIFFERENTIAL EQUATION MODEL

Differential equation without tune ripple influences

Referring to the concept of diffusion region and extraction region, the extraction beam intensity of two regions will be calculated separately. A schematic diagram of two regions is shown in Fig 1.

Set the beam intensity of A_1 entering A_2 is I_1, and the beam intensity exiting A_2 is I_2. I_1 reflects the excitation effect of the RF signal on the diffusion region.

Set the number of particles in A_2 is N_2, and an assumption is made as follow.

Assumption A_2 is a small region in which particles are uniformly distributed.

So, with the above assumption, the number of particles extracted can be considered to be proportional to the number of particles in A_2 and the extraction time. If a short period of time is Δt, an equation can be got as follow.

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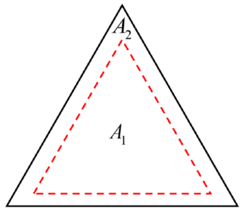

Figure 1: Schematic diagram of two regions. A_1 refers to the diffusion region and A_2 refers to the extraction region.

$$I_2(t+\Delta t)\Delta t = kN_2(t)\Delta t \tag{1}$$

In Eq. (1), k is a coefficient and represents the ratio of the number of extracted particles per unit time to the initial number. And k reflects the excitation effect of the RF signal on the extraction region.

Due to the beam supply of A_1, N_2 varies as follow.

$$N_2(t+\Delta t) - N_2(t) = I_1(t)\Delta t - I_2(t)\Delta t \tag{2}$$

With Eq. (1) and Eq. (2), a differential equation for N_2 can be obtained.

$$\frac{dN_2(t)}{dt} + kN_2(t) = I_1(t) \tag{3}$$

After solving N_2, I_2 can be naturally obtained.

$$I_2(t) = kN_2(t) \tag{4}$$

Differential equation with tune ripple influences

As for the RFKO slow extraction, the tune ripple is transferred to the beam spill ripple by the variation of stable phase space region's area and the variation of the beam emittance growth rate [8].

So, for the area variation, Eq. (2) is adapted to Eq. (5).

$$I_2(t+\Delta t)\Delta t = (k+\rho(t))N_2(t)\Delta t \tag{5}$$

Where $\rho(t)$ represents the ratio of the particles extracted by the area variation per unit time to the initial number.

Define the total area of the stable region as A_0 and the ratio of A_2 to A_0 as γ. A_0 can be calculated from Eq. (6).

$$A_0 = 48\sqrt{3}\pi^2 \frac{q^2}{S^2} \tag{6}$$

Where $q=Q_x-Q_{res}$, Q_x is the horizontal tune and Q_{res} is the resonance tune. S is the virtual normalised sextupole strength. Assuming that the tune ripple is a sine wave, the amplitude is a and the frequency is f. So, q can be written in the following form.

$$q(t) = q_0 + a\sin(2\pi ft) \tag{7}$$

Where a is far less than q_0. So, $\rho(t)$ can be derived as follow considering first-order approximation.

$$\rho(t) = \frac{1}{A_2}\frac{dA_2}{dt} = \frac{1}{\gamma A_0}\frac{dA_0}{dt} \approx \frac{4\pi fa}{\gamma q_0}\cos(2\pi ft) = p\cos(2\pi ft) \tag{8}$$

Where $p = 4\pi fa/\gamma q_0$. In Eq. (8), it is assumed that dS/S is far less than dq/q because the horizontal tune is close to the resonance tune and q is very small.

For the beam emittance growth rate variation, k and $I_l(t)$ are written as follows.

$$k = k_0 + \Delta k\sin(2\pi ft) \tag{9}$$

$$I_1 = I_{10} + I_r\sin(2\pi ft) \tag{10}$$

In Eq. (9) and (10), it is temporarily considered that the amplitude feedback [9] or feedforward [10] is used to control $I_l(t)$ to be approximately a constant I_l for the convenience of analysis. Δk and I_r represent the amplitude of the spill ripple of k and I_l caused by tune ripple.

Thus, with Eq. (5), Eq. (8), Eq. (9) and Eq. (10), the differential equation can be rewritten as follow.

$$\frac{dN_2(t)}{dt} + (k_0 + \Delta k\sin(2\pi ft) + p\cos(2\pi ft))N_2(t) = I_{10} + I_r\sin(2\pi ft) \tag{11}$$

DIFFERENTIAL EQUATION SOLUTION

Eq. (11) is a first order non homogeneous nonlinear differential equation. Although it can be solved through some ways, its result will be hard to understand and be used for physical analysis. A solution convenient for physical analysis was obtained in a relatively simple way with the sacrifice of precision.

Assume that the expression of N_2 is as the follow.

$$N_2(t) = N_{20} + A\sin(\omega t+\phi_1) + B\sin(2\omega t+\phi_2) + \ldots \tag{12}$$

Where $\omega = 2\pi ft$. Substitute Eq. (12) into Eq. (11) and separate the equations for different frequencies. Considering only the ω, 2ω terms and the constant term, three equations are obtained.

$$k_0N_{20} = I_{10} \tag{13}$$

$$A\omega\cos(2\pi ft+\phi_1) + k_0A\sin(2\pi ft+\phi_1) + (\Delta k\sin(2\pi ft) + p\cos(2\pi ft))N_{20} = I_r\sin(\omega t) \tag{14}$$

$$2B\omega\cos(2\omega t+\phi_2) + k_0B\sin(2\omega t+\phi_2) + \sqrt{\Delta k^2+p^2}\sin(\omega t+\phi_0)A\sin(\omega t+\phi_1) = 0 \tag{15}$$

From Eq. (13), we can get N_{20}.

$$N_{20} = \frac{I_{10}}{k_0} \tag{16}$$

A and ϕ_1 are obtained from Eq. (14) as Eq. (17) and (18). From Eq. (15), B is not constant and is quite small than A.

$$A = \frac{\sqrt{(I_r-\Delta kN_{20})^2 + p^2N_{20}^2}}{\sqrt{\omega^2+k_0^2}} \tag{17}$$

$$\sin(\phi_1) = -\frac{pk_0N_{20} + \omega(I_r-\Delta kN_{20})}{\sqrt{k_0^2+\omega^2}\sqrt{(I_r-\Delta kN_{20})^2+p^2N_{20}^2}}$$
$$\cos(\phi_1) = \frac{-p\omega N_{20} + k_0(I_r-\Delta kN_{20})}{\sqrt{k_0^2+\omega^2}\sqrt{(I_r-\Delta kN_{20})^2+p^2N_{20}^2}} \tag{18}$$

With Eq. (13), Eq. (16), Eq. (17) and Eq. (18), I_2 can be obtained as Eq. (19), (20), (21) and (22).

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$$\begin{aligned} I_2(t) &= (k_0 + \Delta k \sin(2\pi f t) + p\cos(2\pi f t))(N_{20} + \\ &\quad A\sin(2\pi f t + \phi_1)) \\ &= I_{10} + I_{2,r,dc} + I_{2,r,\omega}\sin(\omega t + \varphi_1) + \\ &\quad I_{2,r,2\omega}\sin(2\omega t + \varphi_2) + \ldots \end{aligned} \tag{19}$$

$$\frac{I_{2,r,dc}}{I_{10}} = \frac{\frac{I_r}{I_{10}}k_0^2\Delta k - k_0(\Delta k^2 + p^2) - \frac{I_r}{I_{10}}k_0\omega p}{2k_0(\omega^2 + k_0^2)} \tag{20}$$

$$\frac{I_{2,r,\omega}}{I_{10}} = \sqrt{\frac{(\frac{I_r k_0^2}{I_{10}} - p\omega)^2 + \Delta k^2\omega^2}{k_0^2(\omega^2 + k_0^2)}} \tag{21}$$

$$\frac{I_{2,r,2\omega}}{I_{10}} = \frac{\sqrt{p^2 + \Delta k^2}}{2k_0}\frac{\sqrt{(I_r\frac{k_0}{I_{10}} - \Delta k)^2 + p^2}}{\sqrt{\omega^2 + k_0^2}} \tag{22}$$

Since the parameters are hard to be calculated directly from the definitions, only qualitative conclusions can be obtained at present.

From Eq. (19), a mono frequency tune ripple will modulate multiple frequency beam spill ripple.

From Eq. (21) and (22), the relative spill ripple amplitude does not vary monotonically with k. But it is usually agreed that the bigger the better for k due to the cognition that the smaller N_{20} is, the smaller relative spill ripple is [6].

SIMULATION VERIFICATION

The two conclusions above can be verified just by a special case with a simulation by Syntrack [11, 12]. The lattice used in this simulation comes from the Xi'an Proton Application Facility (XiPAF) [13] synchrotron.

Tune ripple is generated by the focus quadrupole strength ripple in the simulation. Amplitude and frequency of the focus quadrupole strength ripple are 200 ppm (parts per million) and 100 Hz.

The revolution frequency is defined as f_{rev}. The RF signal is composed of a mono frequency signal (0.674 f_{rev}) corresponding to the extraction region and a dual frequency modulation (dual FM) signal corresponding to the diffusion region. The dual FM signal's center frequency and the bandwidth are 0.6789 f_{rev} and 0.001 f_{rev}.

k represents the ratio of the number of extracted particles per unit time to the initial number in the extraction region. Thus, the bigger the amplitude ratio of the mono frequency to the dual FM is, the bigger the k is with dual FM signal's amplitude fixed.

Horizontal machine tune is set to 1.6784. 60 Mev proton beam is extracted within 3000000 turns under different ratios of the mono frequency signal's amplitude to the dual FM signal's amplitude.

The extraction spill's time structures and spectrum diagrams under the influence of the 100Hz tune ripple are shown in Fig. 2. The square of the relative beam spill ripple amplitudes before 600 ms are shown in Fig. 3.

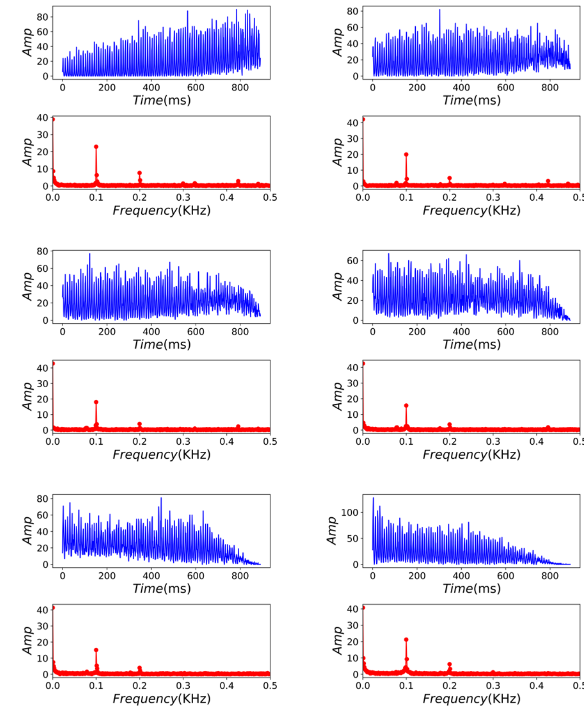

Figure 2: Extraction spill's time structures and spectrum diagrams with 1 kHz sampling. (From left to right and down, the ratio is 0, 0.25, 0.5, 1, 1.5 and 2)

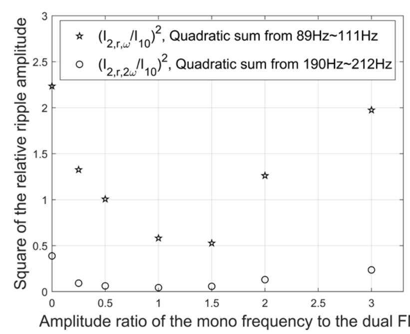

Figure 3: Square of the relative beam spill ripple amplitudes under different amplitude ratios of the mono frequency to the dual FM.

From Fig. 2, it is verified that a mono frequency tune ripple will modulate multiple frequency beam spill ripple. From Fig. 3, it does exist a situation where k is not the bigger the better and an optimal k exists. Therefore, a future work is to make this model's parameters computable so that the optimal k can be given theoretically.

CONCLUSION

Excitation effect of the RF signal on the beam is described as two variables corresponding to the extraction and diffusion region respectively. Then a new differential equation model of the tune ripple effect on beam spill ripple is established and solved. Two preliminary conclusions are obtained and checked by simulation which validates the reliability of this model. At last, this model has the potential to provide the optimal RF signal theoretically.

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REFERENCE

[1] L. Badano *et al*., "Proton-Ion Medical Machine Study", CERN, Geneva, Switzerland, Rep. CERN-PS-99-010-DI, 1999.

[2] K. Noda. *et al*., "Slow beam extraction by a transverse rf field with AM and FM", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 374, p. 269-277, 1996. doi:10.1016/j.nima.2021.16623

[3] K. Mizushima *et al*., "Making beam spill less sensitive to power supply ripple in resonant slow extraction", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 638, p. 19-23, 2011. doi:10.1016/j.nima.2011.02.056

[4] W. B. Ye *et al*., "Analysis and simulation of the tune ripple effect on beam spill ripple in RF-KO slow extraction", Nucl. Sci. Tech., vol. 33, p. 60, 2022. doi:10.1007/s41365-022-01038-6

[5] K. Noda. *et al*., "Advanced RF-KO slow-extraction method for the reduction of spill ripple", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 492, p. 253-263, 2002. doi:10.1016/S0168-9002(02)01319-0

[6] K. Mizushima *et al*., "Reduction of uncontrollable spilled beam in RF-knockout slow extraction", *Nucl. Instrum. Methods Phys. Res., Sect. A,* vol. 606, no. 3, p. 325-329, 2009. doi:10.1016/j.nima.2009.05.154

[7] E. C. Cortés García, "Investigation of RF-signals for the slow extraction at HIT's medical synchrotron", Master Thesis, Technische Universität, Darmstadt, 2022.

[8] W. B. Ye, "Physical and Experimental Research on Slow Extraction at Medical Proton Synchrotron", Master Thesis, Tsinghua University, Beijing, 2022.

[9] S. Sato *et al*., "Dynamic intensity control system with RF-knockout slow-extraction in the HIMAC synchrotron", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 574, p. 226-231, 2007. doi:10.1016/j.nima.2007.01.174

[10] T. Furukawa *et al*., "Global spill control in RF-knockout slow-extraction", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 522, p. 196-204, 2004. doi:10.1016/j.nima.2003.11.395

[11] Y. Yang, "Research on the physics and experiment of low-energy slow extraction for proton synchrotron", Master Thesis, Tsinghua University, Beijing, 2022.

[12] H. J. Yao *et al*., "A High-performance code for beam dynamics simulation of synchrotrons", in *Proc. 10th Int. Particle Accelerator Conf. (IPAC'19)*, Melbourne, Australia, May 2019, pp. 3170-3173. doi:10.18429/JACoW-IPAC2019-WEPTS033

[13] Z. M. Wang *et al*., "Construction and beam commissioning of a compact proton synchrotron for space radiation environment simulation", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 1027, p. 166283, 2022. doi:10.1016/j.nima.2021.166283

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