DIFFERENTIAL EQUATION MODEL OF TUNE RIPPLE EFFECT ON BEAM SPILL RIPPLE IN RFKO SLOW EXTRACTION

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Background

- The beam spill uniformity is an important factor that must be considered in the third order slow extraction optimization.
- The tune ripple caused by the power supply ripple is an important factor that causes the beam uniformity to deteriorate.
- Existing models of how the tune ripple transfers to the beam spill ripple in RFKO slow extraction can't specify how to optimize RF signal because the excitation effect of the RF signal on the beam is described as only one variable on average.
- We attempt to establish a model in which the excitation effect of the RF signal on the beam is not described as only one variable on average but two on the extraction and diffusion region.

Two Regions

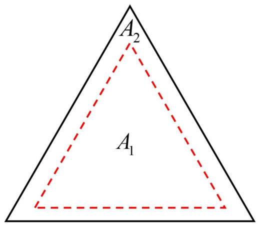

Figure 1: Schematic diagram of two regions. A_1 refers to the diffusion region and A_2 refers to the extraction region.

- Referring to the concept of diffusion region and extraction region, the extraction beam intensity of two regions will be calculated separately.
- Set the beam intensity of A_1 entering A_2 is I_1, and the beam intensity exiting A_2 is I_2. I_1 reflects the excitation effect of the RF signal on the diffusion region.
- Set the number of particles in A_2 is N_2.

Assumption

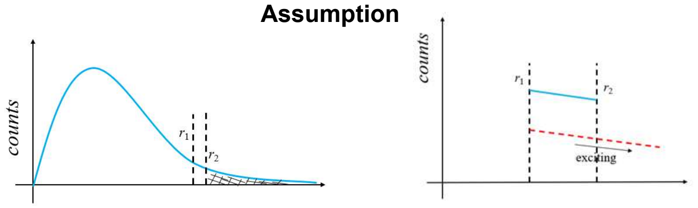

Figure 2: Radial beam distribution with two region's boundaries.

Assumption A_2 is a small region in which particles are uniformly distributed

Differential equation without tune ripple influences

$$I_2(t+\Delta t)\Delta t = kN_2(t)\Delta t \qquad N_2(t+\Delta t) - N_2(t) = I_1(t)\Delta t - I_2(t)\Delta t$$

$$\frac{dN_2(t)}{dt} + kN_2(t) = I_1(t)$$

$$I_2(t) = kN_2(t)$$

k : a coefficient and represents the ratio of the number of extracted particles per unit time to the initial number

Influence of the variation of stable phase space region's area

$$I_2(t+\Delta t)\Delta t = (k+\rho(t))N_2(t)\Delta t$$

$\rho(t)$: the ratio of the particles extracted by the area variation per unit time to the initial number

$$A_0 = 48\sqrt{3}\pi^2\frac{q^2}{S^2}$$

A_0 : total area of the stable region

$$q(t) = q_0 + a\sin(2\pi ft)$$

$$\rho(t) = \frac{1}{A_2}\frac{dA_2}{dt} = \frac{1}{\gamma A_0}\frac{dA_0}{dt} \approx \frac{4\pi fa}{\gamma q_0}\cos(2\pi ft)$$

$$= p\cos(2\pi ft)$$

Influence of the variation of the beam emittance growth rate

$$k = k_0 + \Delta k\sin(2\pi ft) \qquad I_1 = I_{10} + I_r\sin(2\pi ft)$$

$$\frac{dN_2(t)}{dt} + kN_2(t) = I_1(t) \longrightarrow \frac{dN_2(t)}{dt} + (k_0 + \Delta k\sin(2\pi ft) + p\cos(2\pi ft))N_2(t) = I_{10} + I_r\sin(2\pi ft)$$

DIFFERENTIAL EQUATION SOLUTION

$$\frac{dN_2(t)}{dt} + (k_0 + \Delta k\sin(2\pi ft) + p\cos(2\pi ft))N_2(t) = I_{10} + I_r\sin(2\pi ft)$$

Assume that the expression of N_2 can be written as the follow

$$N_2(t) = N_{20} + A\sin(2\pi ft + \phi_1) + B\sin(4\pi ft + \phi_2) + \ldots$$

$$\Downarrow$$

$$A\omega\cos(2\pi ft + \phi_1) + k_0A\sin(2\pi ft + \phi_1) + (\Delta k\sin(2\pi ft) + p\cos(2\pi ft))N_{20} = I_r\sin(\omega t)$$

$$2B\omega\cos(2\omega t + \phi_2) + k_0B\sin(2\omega t + \phi_2) + \sqrt{\Delta k^2 + p^2}\sin(\omega t + \phi_0)A\sin(\omega t + \phi_1) = 0$$

$$\Downarrow$$

$$N_{20} = \frac{I_{10}}{k_0} \qquad A = \frac{\sqrt{(I_r - \Delta kN_{20})^2 + p^2N_{20}^2}}{\sqrt{\omega^2 + k_0^2}}$$

$$\sin(\phi_1) = -\frac{pk_0N_{20} + \omega(I_r - \Delta kN_{20})}{\sqrt{k_0^2 + \omega^2}\sqrt{(I_r - \Delta kN_{20})^2 + p^2N_{20}^2}}$$

$$\cos(\phi_1) = \frac{-p\omega N_{20} + k_0(I_r - \Delta kN_{20})}{\sqrt{k_0^2 + \omega^2}\sqrt{(I_r - \Delta kN_{20})^2 + p^2N_{20}^2}}$$

$$\Downarrow$$

$$\begin{aligned} I_2(t) &= (k_0 + \Delta k\sin(2\pi ft) + p\cos(2\pi ft))(N_{20} + A\sin(2\pi ft + \phi_1)) \\ &= I_{10} + I_{2,r,dc} + I_{2,r,\omega}\sin(\omega t + \varphi_1) + I_{2,r,2\omega}\sin(2\omega t + \varphi_2) + \ldots \end{aligned}$$

$$\frac{I_{2,r,dc}}{I_{10}} = \frac{\frac{I_r}{I_{10}}k_0^2\Delta k - k_0(\Delta k^2 + p^2) - \frac{I_r}{I_{10}}k_0\omega p}{2k_0(\omega^2 + k_0^2)}$$

$$\frac{I_{2,r,\omega}}{I_{10}} = \sqrt{\frac{(\frac{I_rk_0^2}{I_{10}} - p\omega)^2 + \Delta k^2\omega^2}{k_0^2(\omega^2 + k_0^2)}} \qquad \frac{I_{2,r,2\omega}}{I_{10}} = \frac{\sqrt{p^2 + \Delta k^2}}{2k_0}\frac{\sqrt{(I_r\frac{k_0}{I_{10}} - \Delta k)^2 + p^2}}{\sqrt{\omega^2 + k_0^2}}$$

- A mono frequency tune ripple will modulate multiple frequency beam spill ripple.
- The relative spill ripple amplitude does not vary monotonically with k. But it is usually agreed that the bigger the better for k due to the cognition that the smaller N_{20} is, the smaller relative spill ripple is.

Simulation verifycation

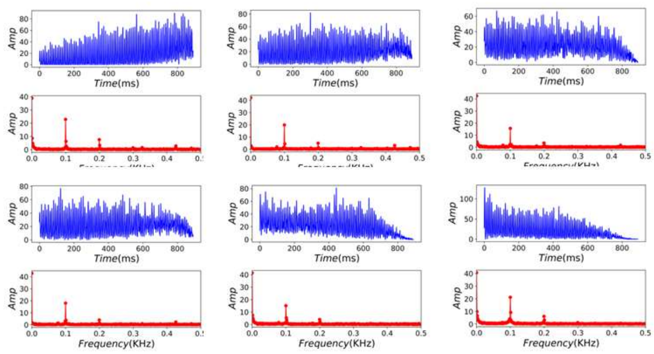

Figure 3: Extraction spill's time structures and spectrum diagrams under the influence of the 100Hz tune ripple with 1 kHz sampling. (From left to right and down, the amplitude ratio of the mono frequency to the dual FM is 0, 0.25, 0.5, 1, 1.5 and 2)

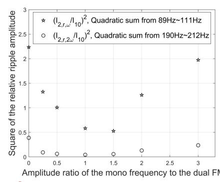

Figure 4: Square of the relative beam spill ripple amplitudes under different amplitude ratios of the mono frequency to the dual FM

Conclusions

- Excitation effect of the RF signal on the beam is described as two variables corresponding to the extraction and diffusion region respectively. Then a new differential equation model of the tune ripple effect on beam spill ripple is established and solved.
- Two preliminary conclusions are obtained and checked by simulation which validates the reliability of this model. At last, this model has the potential to provide the optimal RF signal theoretically.