22nd Conference on Flavour and CP Violation (FPCP 2024)

Theory of rare hadronic decay

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Bangkok, 27-31/05/24

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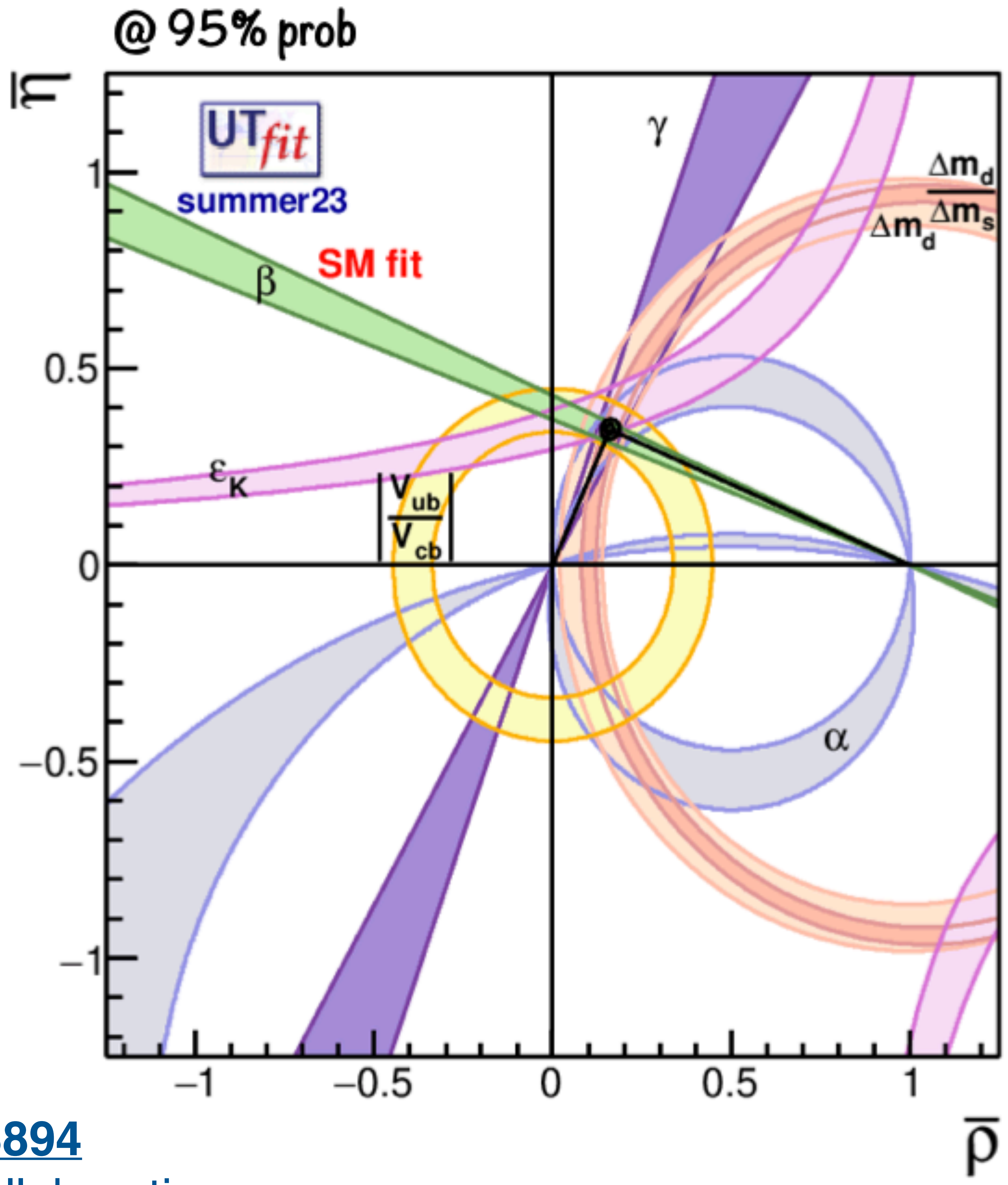
CSIC Consejo Superior de Investigaciones Científicas

The Status of Flavour Physics

Flavour Physics allows for a fantastic playground to test the Standard Model and probe for New Physics effects. The unitarity of the CKM matrix is a fundamental consistency check



$$\bar{\rho} = 0.160 \pm 0.009 \sim 6\%$$
$$\bar{\eta} = 0.346 \pm 0.009 \sim 3\%$$

$$\lambda = 0.2251 \pm 0.0008$$
$$A = 0.827 \pm 0.010$$

Wolfenstein parameters determined with ever-increasing precision, but (un)fortunately all measurements are in perfect agreement!

2212.03894
UTfit Collaboration

The Flavour NP reach

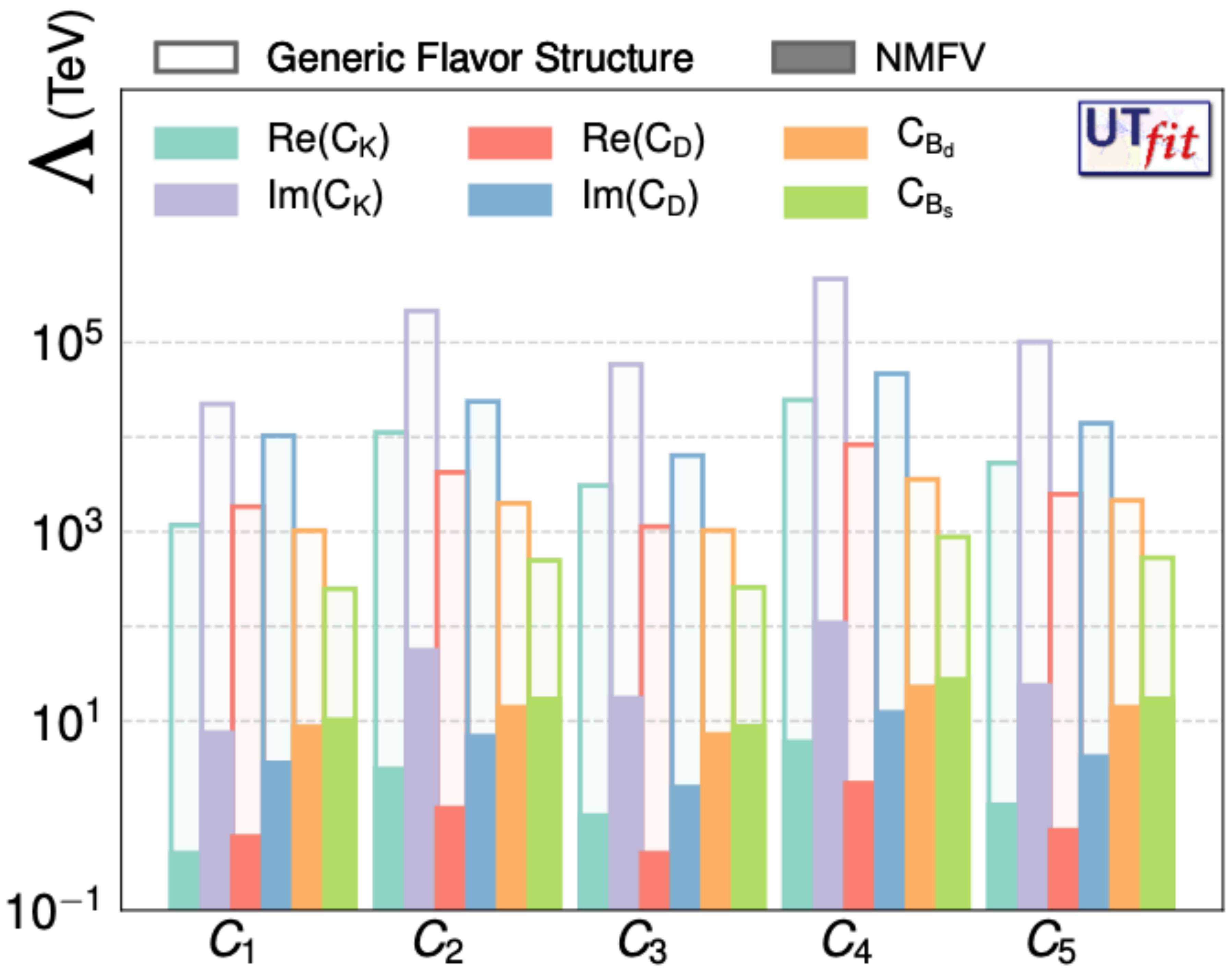
To describe heavy NP effects, it is customary to employ effective Hamiltonians, where the UV degrees of freedom are integrated out and which allow model-independent analyses

couplings parametrizing low-scale footprints of heavy degrees of freedom

$$H^{\text{eff}}(x) = \sum \frac{c_{\mathcal{O}}}{\Lambda^{dim_{\mathcal{O}}-4}} \mathcal{O}(x)$$

high scale of the heavy degrees of freedom, setting as a cutoff of the eff. theory

series of local operator built as monomials in low-energies fields and derivatives



Rare Hadron Decays

- Among the several accidental symmetries of the Standard Model, a particularly interesting one is the absence of tree-level Flavour Changing Neutral Currents (FCNC)

- These hadronic decays occur at loop-level, and are both GIM- and CKM-suppressed: very rare, hence fundamental probe of heavy NP effects

- Indeed, since no NP has been (so far) directly observed at colliders, is fundamental to have input from indirect searches where BSM appears through virtual, intermediate states

I will focus here on rare decays of the B meson, but fundamental information can be extracted from rare D and K decays as well!

Overview

- $B \to \tau\nu$
- $B \to \mu\mu$
- $B \to K^{(*)}\ell\ell, B_s \to \phi\ell\ell$
- $b \to s\gamma$
- $B \to K^{(*)}\nu\nu$

$B \to \tau\nu$: the SM status

- Helicity suppressed, tree-level decay

- Main uncertainties come from CKM elements (UTA) and decay constants (Lattice)

$$\mathcal{B}(B_q^+ \to \tau^+ \nu_\tau)^{\rm SM} = \tau_{B_q^+} \frac{G_F^2 |V_{qb}|^2 f_{B_q^+}^2 m_{B_q^+} m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_{B_q^+}^2}\right)^2 , \quad q = u, c$$

$|V_{cb}|^{\rm UTA} = 42.22(51) \times 10^{-3}, f_{B_c} = 427(6)$ MeV

$|V_{ub}|^{\rm UTA} = 3.70(11) \times 10^{-3}, f_{B^+} = 190.0(1.3)$ MeV

$$\Rightarrow \begin{array}{lcl} \mathcal{B}(B_c^+ \to \tau^+ \nu_\tau)^{\rm SM} & = & 2.29(9) \times 10^{-2} \\ \mathcal{B}(B^+ \to \tau^+ \nu_\tau)^{\rm SM} & = & 0.87(5) \times 10^{-4} \end{array}$$

2212.03894
UTfit Collaboration

2111.09849
FLAG

According to present Lattice estimates, decay constants errors could be halved in the next decade!

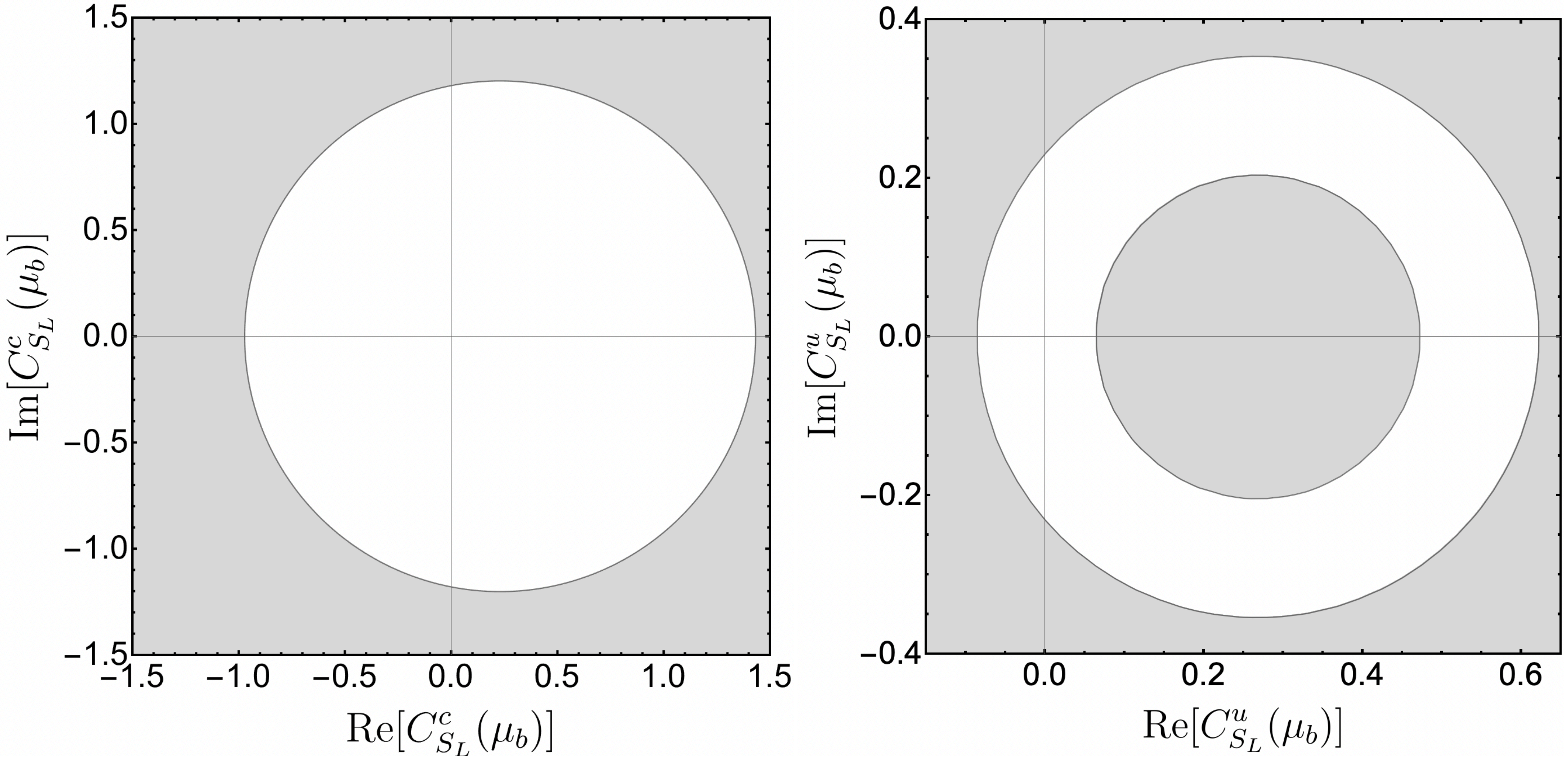
$B \to \tau\nu$: NP implications

Extremely sensitive to scalar BSM extensions (2HDM, LQ), which lift helicity suppression

$$\mathcal{B}(B_q^+ \to \tau^+\nu_\tau) = \mathcal{B}(B_q^+ \to \tau^+\nu_\tau)^{\rm SM} \times \left| 1 - (C_{V_R}^q - C_{V_L}^q) + (C_{S_R}^q - C_{S_L}^q)\,\frac{m_{B_q}^2}{m_\tau(m_b + m_q)} \right|^2$$

$$O_{V_{L(R)}} = (\bar{q}_{L(R)}\gamma_\mu b_{L(R)})(\bar{\tau}_L\gamma_\mu\nu_L)$$

$$O_{S_{L(R)}} = (\bar{q}_{R(L)} b_{L(R)})(\bar{\tau}_R\nu_L)$$



2305.02998
Zuo, MF, Helsen, Hill, Iguro, Klute

Constraints on C_{S_R} obtained by Re → -Re

The $\Delta B = 1$ FCNC Effective Hamiltonian

$$H_{eff}^{\Delta B=1} = H_{eff}^{had} + H_{eff}^{sl+\gamma}$$

$$H_{eff}^{had} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \Big[C_1 P_1^p + C_2 P_2^p + \sum_{i=3,\ldots,6} C_i P_i + C_{8g} Q_{8g} \Big]$$

$$H_{eff}^{sl+\gamma} = \frac{4G_F}{\sqrt{2}} \lambda_t \Big[C_7^{(\prime)} Q_{7\gamma}^{(\prime)} + C_9^{(\prime)} Q_{9V}^{(\prime)} + C_{10}^{(\prime)} Q_{10A}^{(\prime)} + C_S^{(\prime)} Q_S^{(\prime)} + C_P^{(\prime)} Q_P^{(\prime)} \Big]$$

$$\begin{aligned}
P_1^p &= (\bar{s}_L \gamma_\mu T^a p_L)(\bar{p}_L \gamma^\mu T^a b_L) \\
P_2^p &= (\bar{s}_L \gamma_\mu p_L)(\bar{p}_L \gamma^\mu b_L) \\
P_3 &= (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_q (\bar{q} \gamma^\mu q) \\
P_4 &= (\bar{s}_L \gamma_\mu T^a b_L) \textstyle\sum_q (\bar{q} \gamma^\mu T^a q) \\
P_5 &= (\bar{s}_L \gamma_{\mu 1} \gamma_{\mu 2} \gamma_{\mu 3} b_L) \textstyle\sum_q (\bar{q} \gamma^{\mu 1} \gamma^{\mu 2} \gamma^{\mu 3} q) \\
P_6 &= (\bar{s}_L \gamma_{\mu 1} \gamma_{\mu 2} \gamma_{\mu 3} T^a b_L) \textstyle\sum_q (\bar{q} \gamma^{\mu 1} \gamma^{\mu 2} \gamma^{\mu 3} T^a q)
\end{aligned}$$

$$\begin{aligned}
Q_{7\gamma} &= \frac{e}{16\pi^2} \hat{m}_b \bar{s} \sigma_{\mu\nu} P_R F^{\mu\nu} b \\
Q_{8g} &= \frac{\gamma_s}{16\pi^2} \hat{m}_b \bar{s} \sigma_{\mu\nu} P_R G^{\mu\nu} b \\
Q_{9V} &= \frac{\alpha_{em}}{4\pi} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell) \\
Q_{10A} &= \frac{\alpha_{em}}{4\pi} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \gamma^5 \ell) \\
Q_S &= \frac{\alpha_{em}}{4\pi} \frac{\hat{m}_b}{m_W} (\bar{s} P_R b)(\bar{\ell} \ell) \\
Q_P &= \frac{\alpha_{em}}{4\pi} \frac{\hat{m}_b}{m_W} (\bar{s} P_R b)(\bar{\ell} \gamma^5 \ell)
\end{aligned}$$

Matrix elements of quark currents from $Q_{7,9,10,S,P}$ factorize:

$$\mathcal{A} \sim \langle \ell^+ \ell^- | J_{\text{lep}} | 0 \rangle \langle V(P) | J_{had} | B \rangle$$

Not possible for the hadronic Hamiltonian!

$$\tilde{h}_\lambda(q^2) \sim \epsilon_{\lambda,\mu} \int d^4x \, e^{iqx} \langle V(P) | T\{ J_{had}^{\mu,e.m.}(x) \mathcal{H}_{had}^{eff}(0) \} | B \rangle$$

$B \to \mu\mu$: the SM status

- Helicity suppressed, loop-level decay dominated by short-distance effects (C_{10})
- Main uncertainties come from CKM elements (UTA) and decay constants (Lattice)

$$\mathcal{B}(B_q^0 \to \mu^+\mu^-)^{\rm SM} = \tau_{B_q^0} \frac{G_F^4 |V_{tb}^* V_{tq}|^2 f_{B_q}^2 m_W^4 m_{B_q^0} m_\mu^2}{2\pi^5} \sqrt{1 - \frac{4m_\mu^2}{m_{B_q^0}^2}} |C_{10}^{\rm q,SM}|^2, \quad q = d, s$$

$|V_{td}|^{\rm UTA} = 8.59(11) \times 10^{-3}$, $f_{B_d} = 190.5(1.3)$ MeV

$|V_{ts}|^{\rm UTA} = 41.28(46) \times 10^{-3}$, $f_{B_s} = 230.1(1.2)$ MeV

$$\Rightarrow \begin{aligned} \mathcal{B}(B_d \to \mu^+\mu^-)^{\rm SM} &= 9.48(36) \times 10^{-11} \\ \mathcal{B}(B_s \to \mu^+\mu^-)^{\rm SM} &= 3.47(14) \times 10^{-9} \end{aligned}$$

2212.03894
UTfit Collaboration

2111.09849
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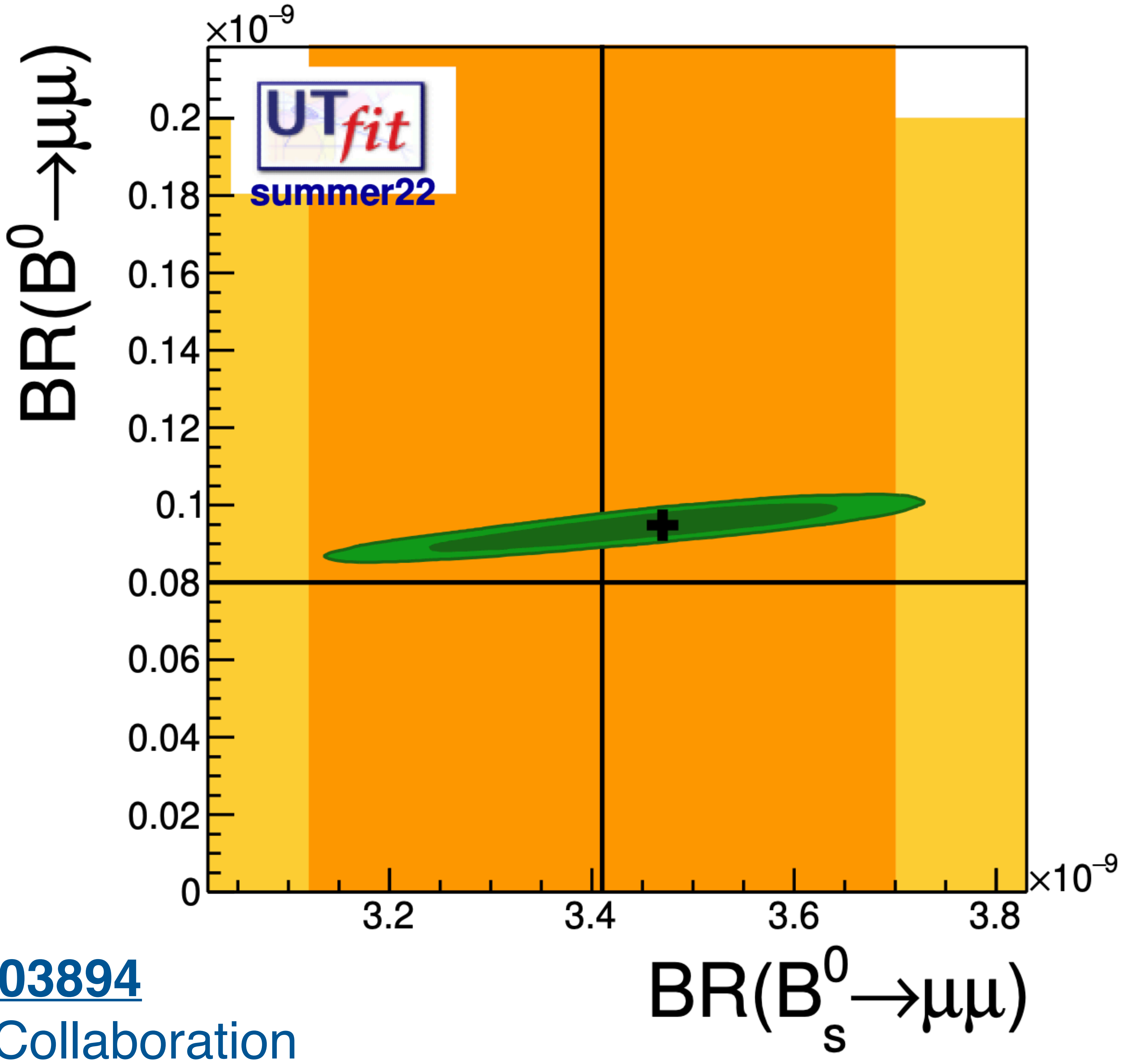
According to present Lattice estimates, decay constants errors could be halved in the next decade!

$B \to \mu\mu$: NP implications

Sensitive to BSM effect on axial and (pseudo)scalar operators, which again lift helicity suppression

$$\mathcal{B}(B_q \to \mu^+\mu^-) = \mathcal{B}^{\rm SM} \times \left(\left| \frac{C_{10}^{q,\rm NP} - C_{10}'^{q,\rm NP}}{C_{10}^{q,\rm SM}} + \frac{m_{B_q}^2}{2m_\mu m_b} \frac{C_P^{q,\rm NP} - C_P'^{q,\rm NP}}{C_{10}^{q,\rm SM}} \right|^2 + \left| \sqrt{1 - \frac{4m_\mu^2}{m_{B_q}^2}} \frac{m_{B_q}^2}{2m_\mu m_b} \frac{C_S^{q,\rm NP} - C_S'^{q,\rm NP}}{C_{10}^{q,\rm SM}} \right|^2 \right)$$



Current results are (now) in perfect agreement with SM prediction, NP strongly constrained

$\Downarrow$

Fundamental player in global fit to $b \to s\ell\ell$ transitions (in a few slides)

2212.03894
UTfit Collaboration

$B \to K^{(*)}\ell\ell, B_s \to \phi\ell\ell$: the SM status

- Loop-level decays dominated by short-distance effects ($C_{9,10}$), important long-distance
- Uncertainties coming from the form factors and from the non-local hadronic parameters

The amplitudes for the K^* and ϕ channel, in the helicity basis, are proportional to

$$
\begin{aligned}
H_\lambda^V(q^2) &\propto (C_9 - C_9')\tilde{V}_\lambda(q^2) + \frac{2m_b m_B}{q^2}(C_7 - C_7')\tilde{T}_\lambda(q^2) - 16\pi^2\frac{m_B^2}{q^2}\tilde{h}_\lambda(q^2) \\
H_\lambda^A(q^2) &\propto (C_{10} - C_{10}')\tilde{V}_\lambda(q^2) \\
H^S(q^2) &\propto \frac{m_b}{m_W}(C_S - C_S')\tilde{S}(q^2) \\
H^P(q^2) &\propto \frac{m_b}{m_W}(C_P - C_P')\tilde{S}(q^2) + \frac{2m_\ell m_B}{q^2}(C_{10} - C_{10}')\left(1 + \frac{m_s}{m_b}\right)\tilde{S}(q^2)
\end{aligned}
$$

$(\lambda = 0, \pm)$

(The K channel has an analogous, simpler description with only $\lambda = 0$)

The form factors

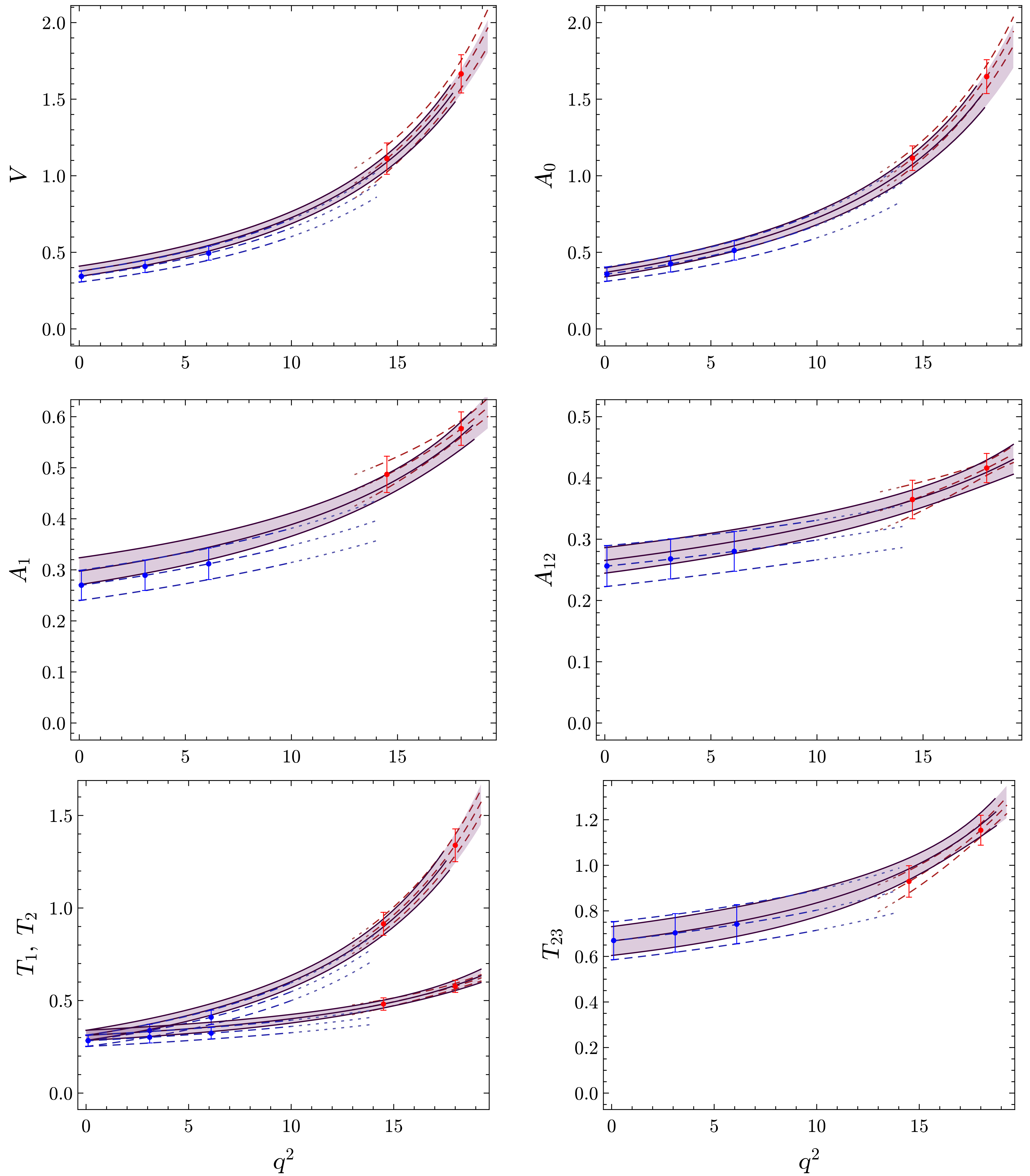
Low recoil region (Lattice, Pos Lattice2014 (2015) 372)

vs.

Large recoil region (LCSR, JHEP 08 (2016) 098)

⇓

Full form factors, together with the *correlation matrix*, have become a reliable option



The non-local hadronic parameter

At first order in α_{em} we can get a contribution from current-current quark operators & QCD penguins

Loop suppressed amplitude, can be enhanced by non-perturbative QCD effects!

In particular, charm current-current insertion not further parametrically suppressed.

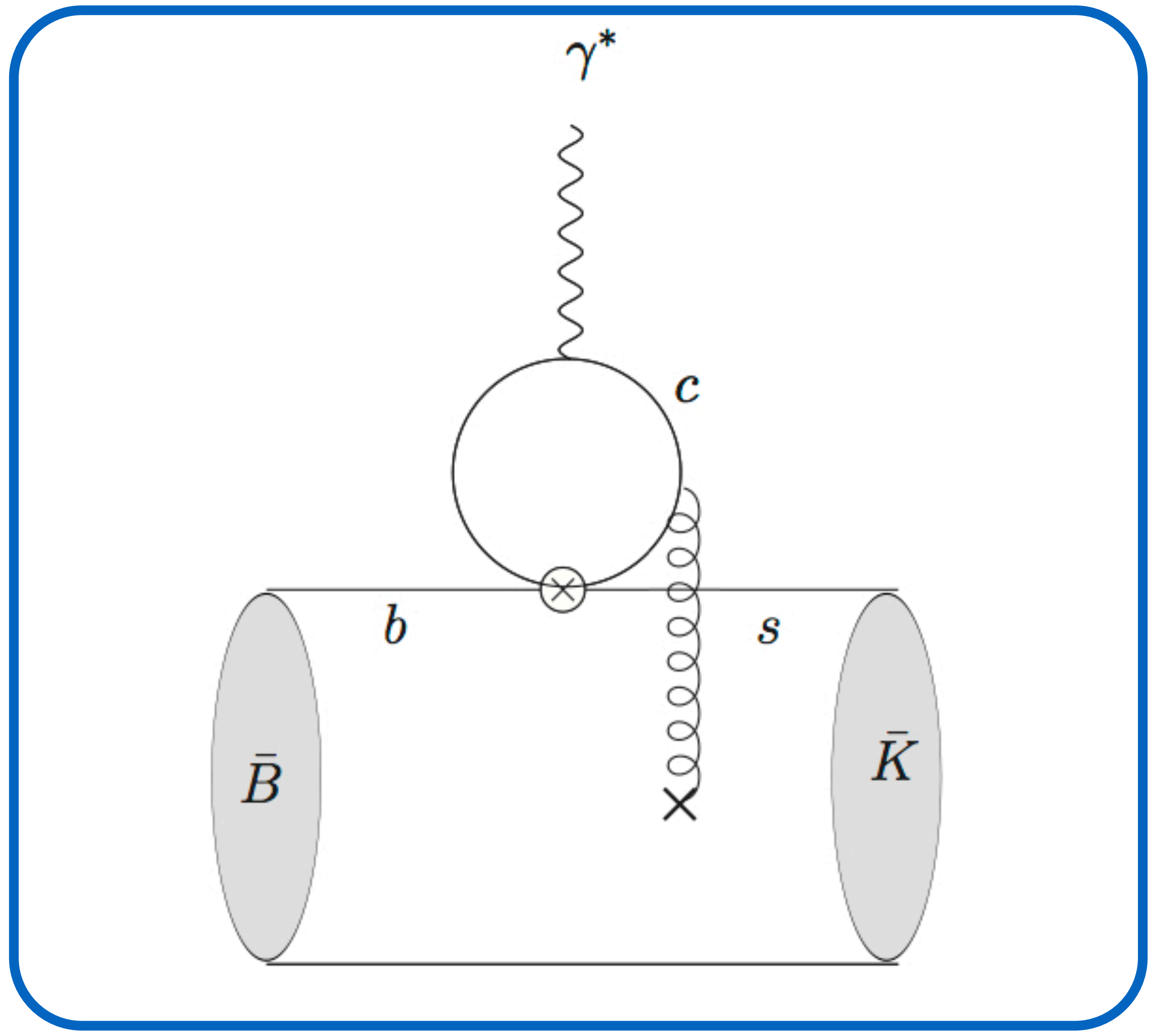
Soft gluon emission from cc-loop estimated for P = K and V = K* with LCSR + dispersion relation. **Sizable effect in K***

$\Rightarrow$ Correlator expanded on the light-cone: **LCSR estimate based on negative/small q²**

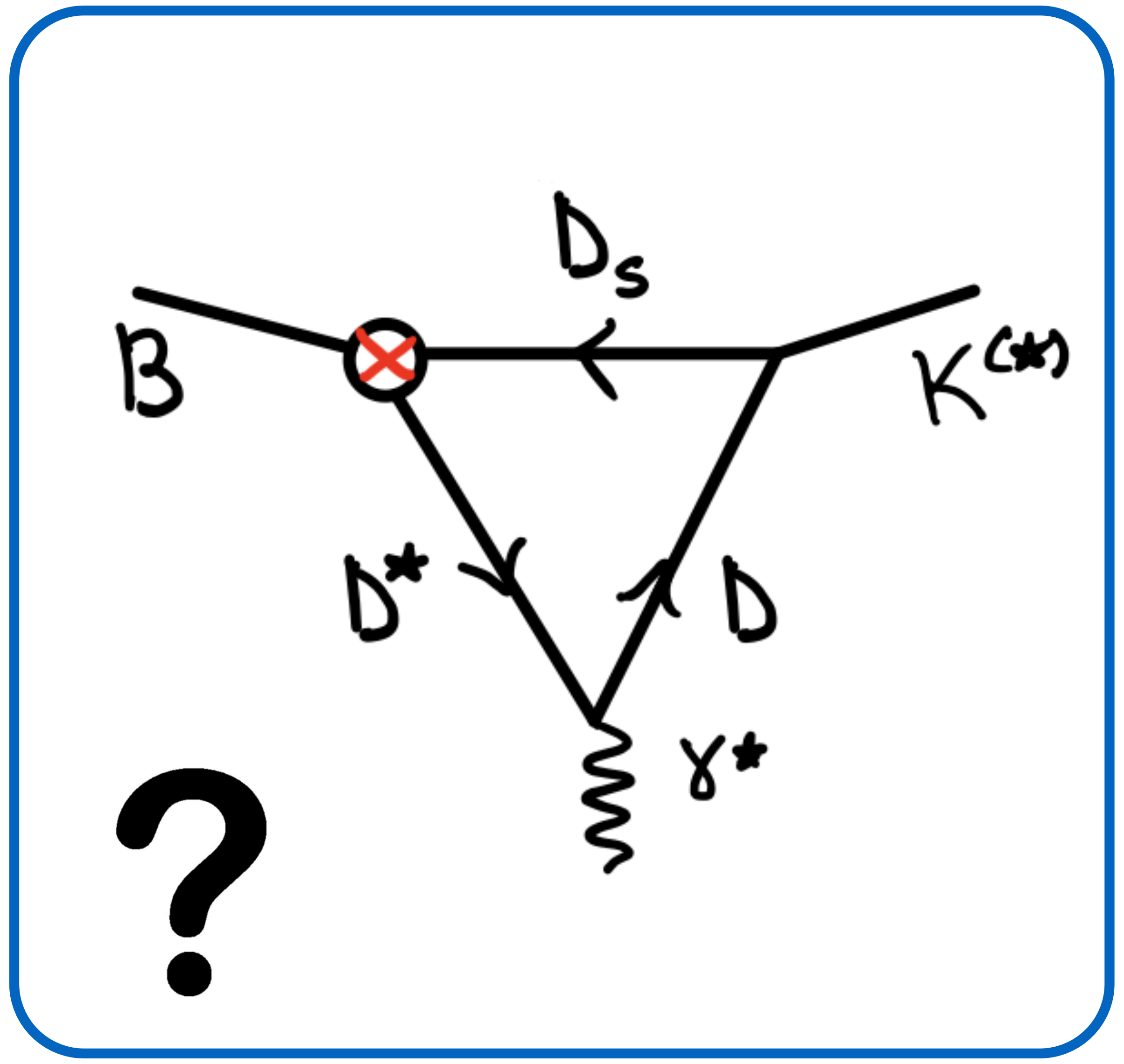
$\Rightarrow$ **Dispersion relation** in order to **extrapolate/ interpolate** LCSR result up to $c\bar{c}$ threshold

$\Rightarrow$ **Single soft gluon approximation:** strictly valid only for $q^2 << m_c^2$

$\Rightarrow$ **Potential effects** coming from D_s-$\bar{D}$ rescattering presently **not included**



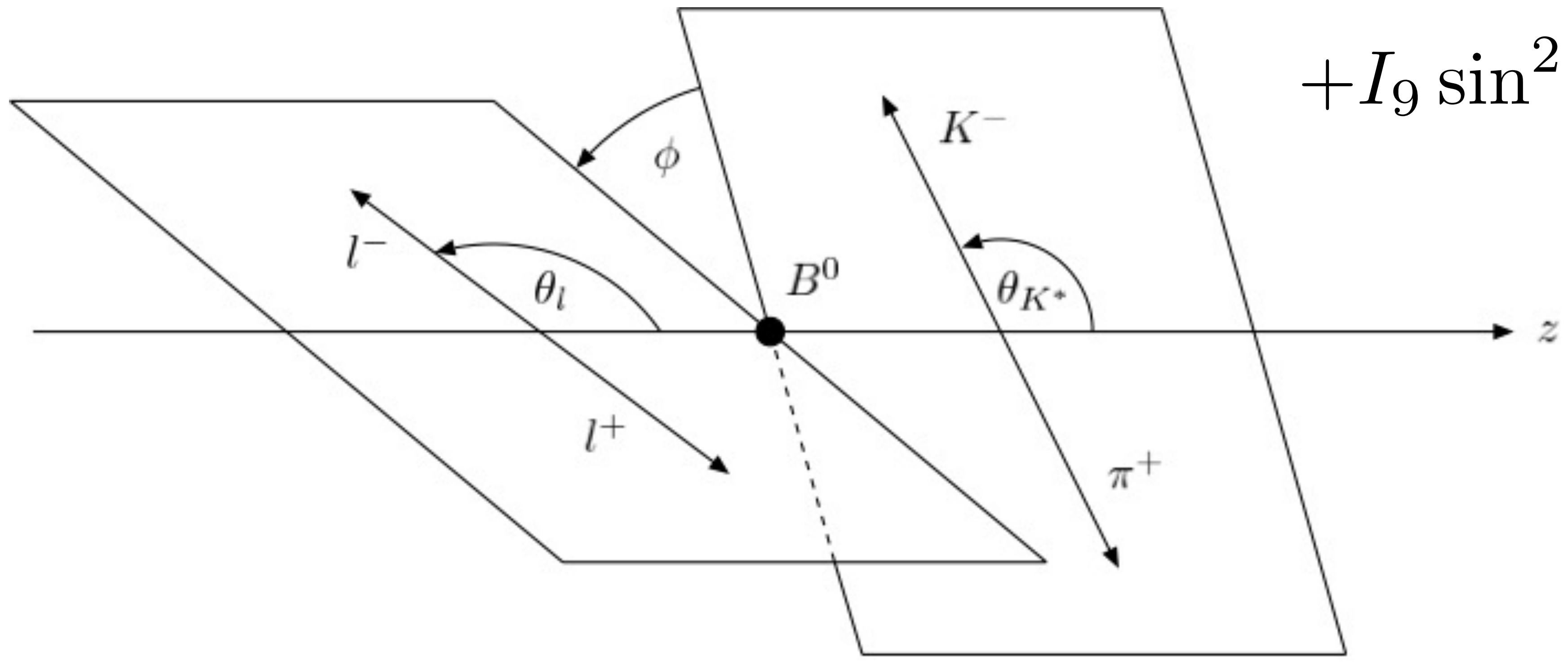
1006.4945
Khodjamirian, Mannel, Pivarov, Wang



2212.10516
Ciuchini, MF, Franco, Paul, Silvestrini, Valli

The Observables

$$\frac{d^{(4)}\Gamma}{dq^2 d(\cos\theta_\ell) d(\cos\theta_K) d\phi} = \frac{9}{32\pi}\Big(I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \cos^2\theta_K)\cos 2\theta_\ell$$
$$+ I_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi$$
$$+ I_5 \sin 2\theta_K \sin\theta_\ell \cos\phi + (I_6^s \sin^2\theta_K + I_6^c \cos^2\theta_K)\cos\theta_\ell$$
$$+ I_7 \sin 2\theta_K \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi$$
$$+ I_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi\Big).$$



$$\Sigma_i = \frac{I_i + \bar{I}_i}{2}$$

CP-Averaged

$$\Gamma' = \frac{1}{2}\frac{d\Gamma + d\bar{\Gamma}}{dq^2} = \Sigma_{1c} + 4\Sigma_{2s}$$

$$A_{FB} = -\frac{3\Sigma_{6s}}{4\Gamma'}$$

$$S_{3,4,5,7,8,9} = \frac{\Sigma_{3,4,5,7,8,9}}{\Gamma'}$$

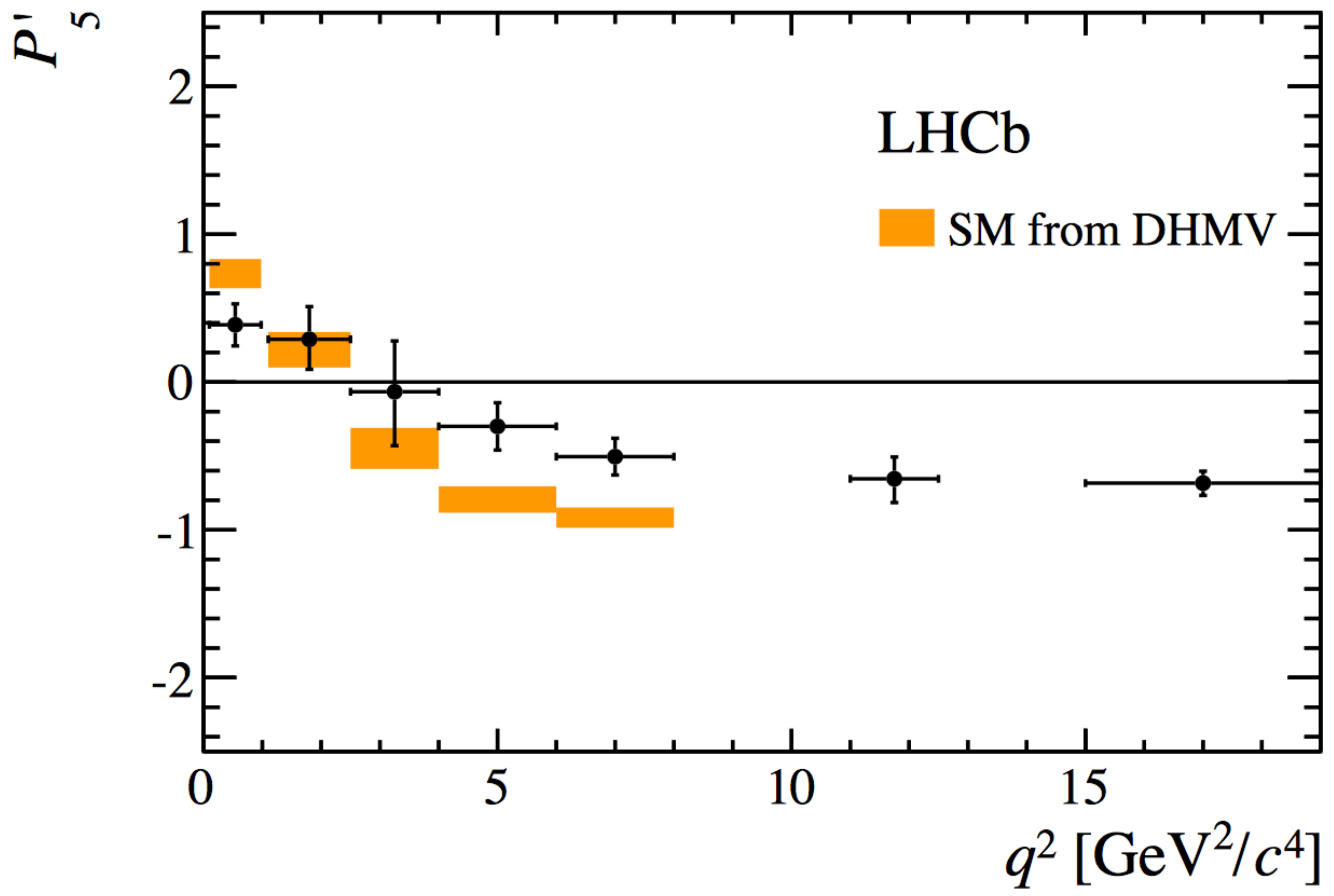
$$F_L = \frac{\Sigma_{1c}}{\Gamma'}$$

$B \to K^{(*)}\ell\ell, B_s \to \phi\ell\ell$: the SM status

A series of consistent deviations has been observed in the last 10 years in decays involving the muon channels

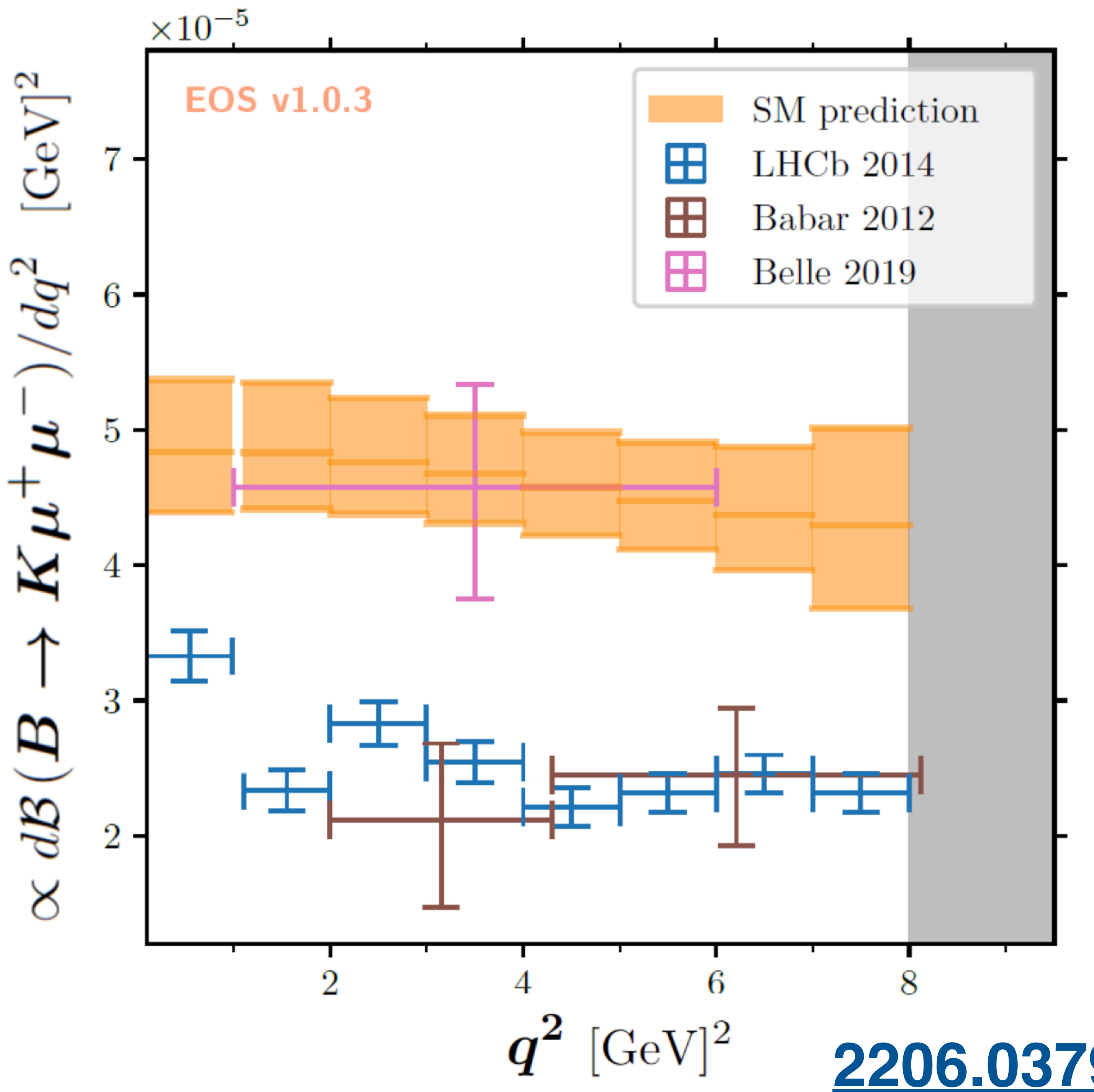
However, many of these observables are potentially plagued by un-accounted hadronic corrections…

1512.04442
LHCb

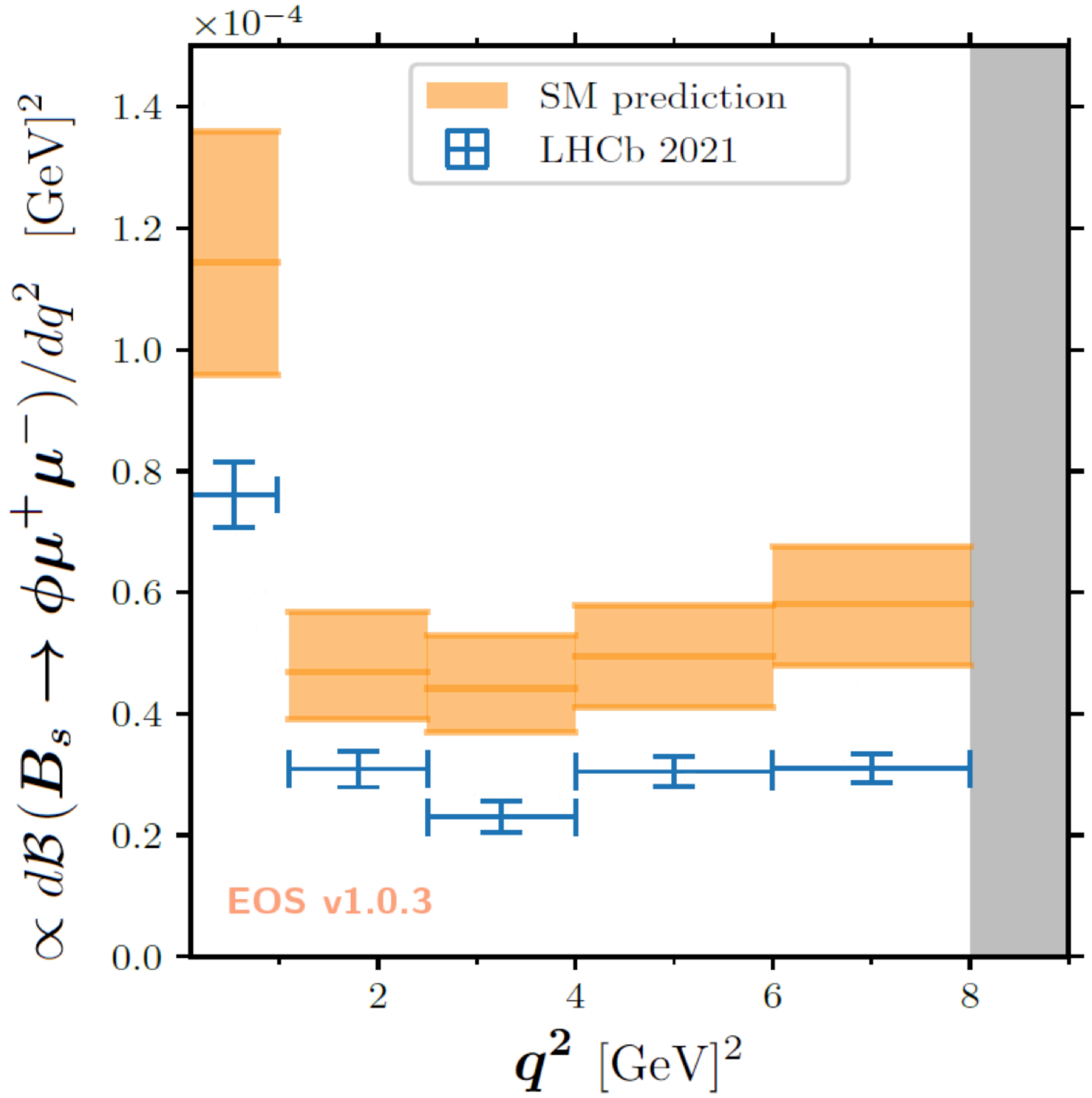


1407.8526
Descotes-Genon, Hofer, Matias, Virto

1403.8044, 1204.3933, 1908.01848
LHCb, BaBar, Belle



2105.14007
LHCb

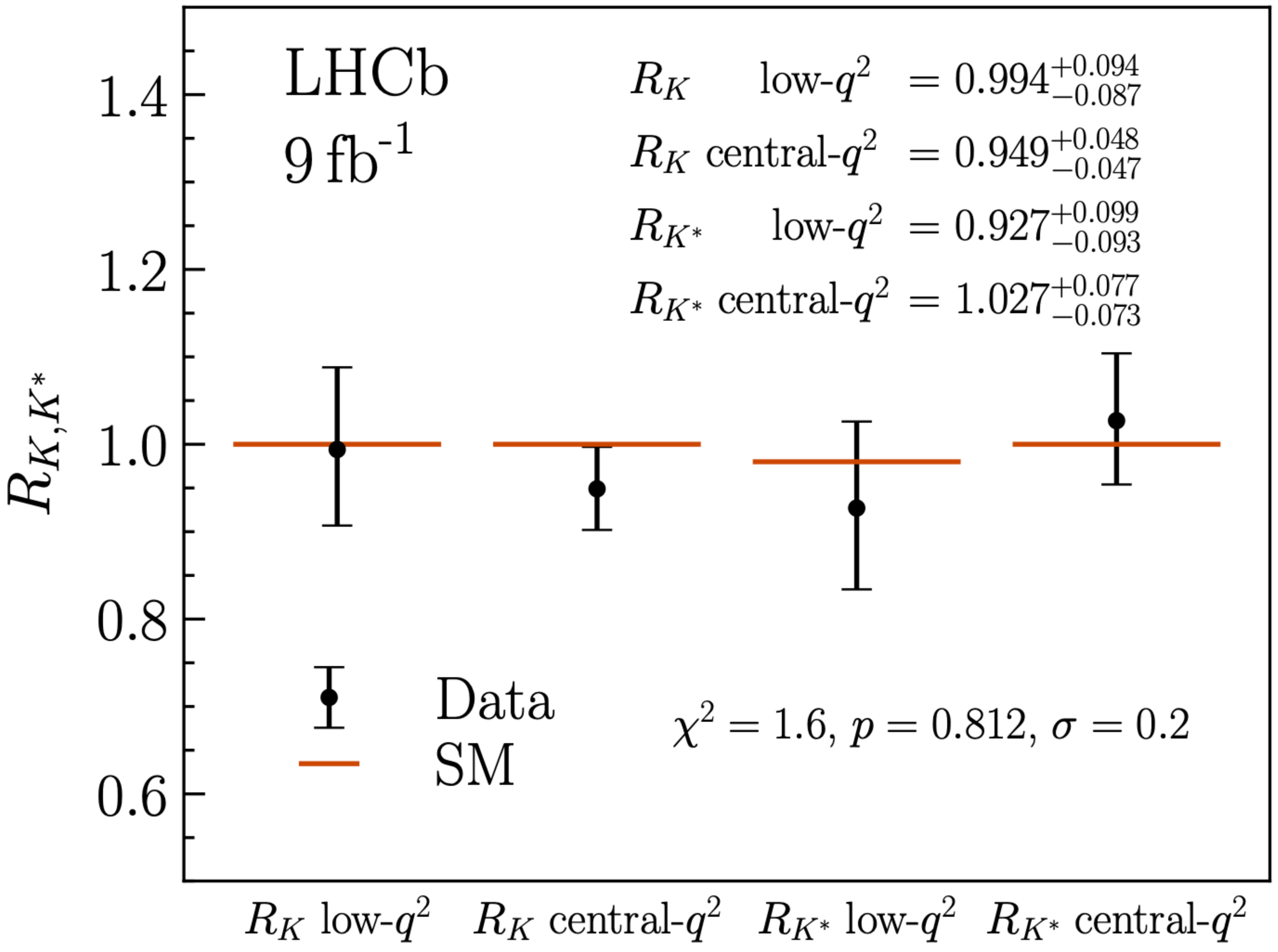


2206.03797
Gubernari, Reboud, van Dyk, Virto

SM predictions above are indeed based on specific (aggressive?) estimates for the hadronic parameters

$B \to K^{(*)}\ell\ell, B_s \to \phi\ell\ell$: NP implications

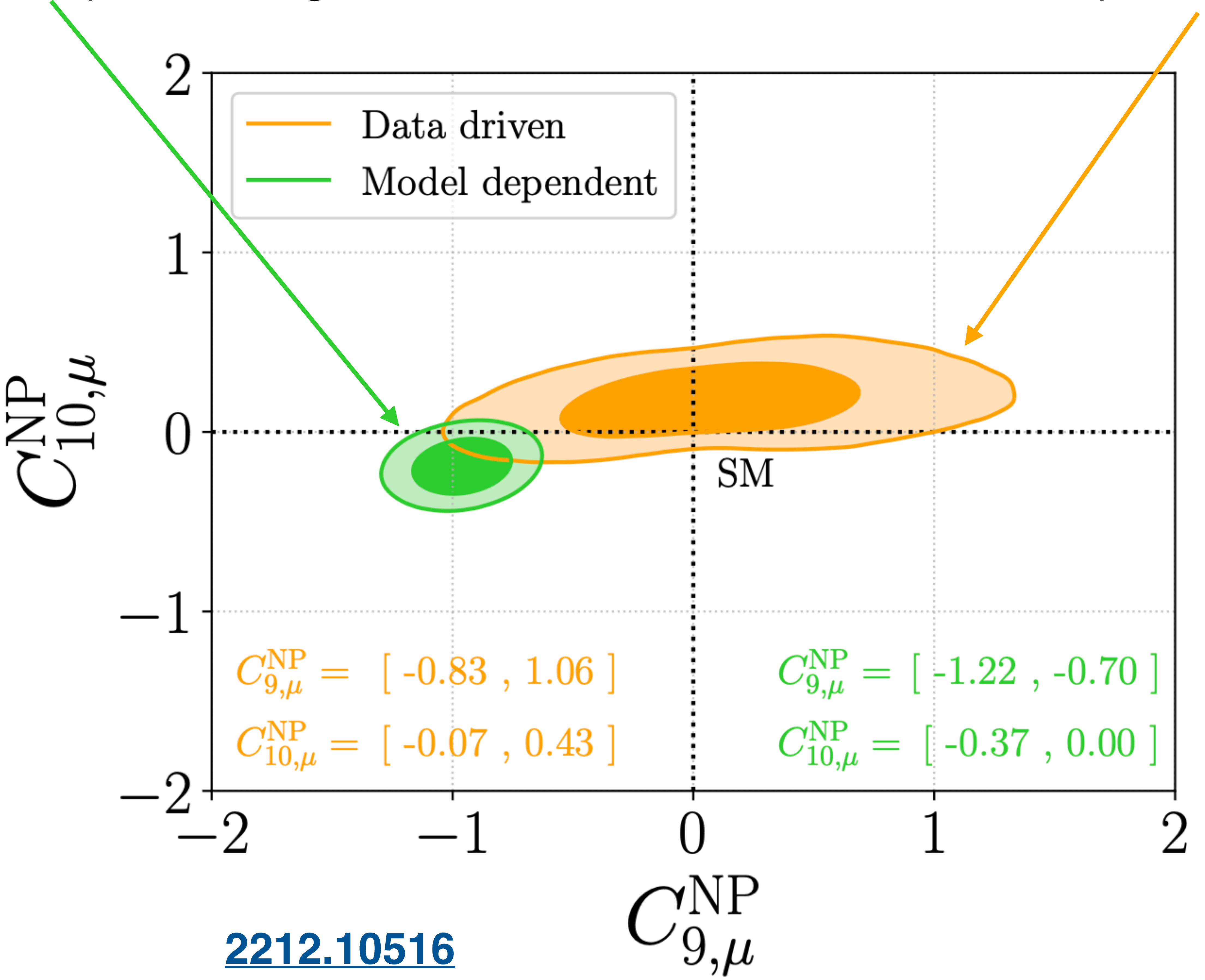
Originally, the set of anomalies could be consistently accounted by a shift in the muon channel. However…



2212.09153
LHCb

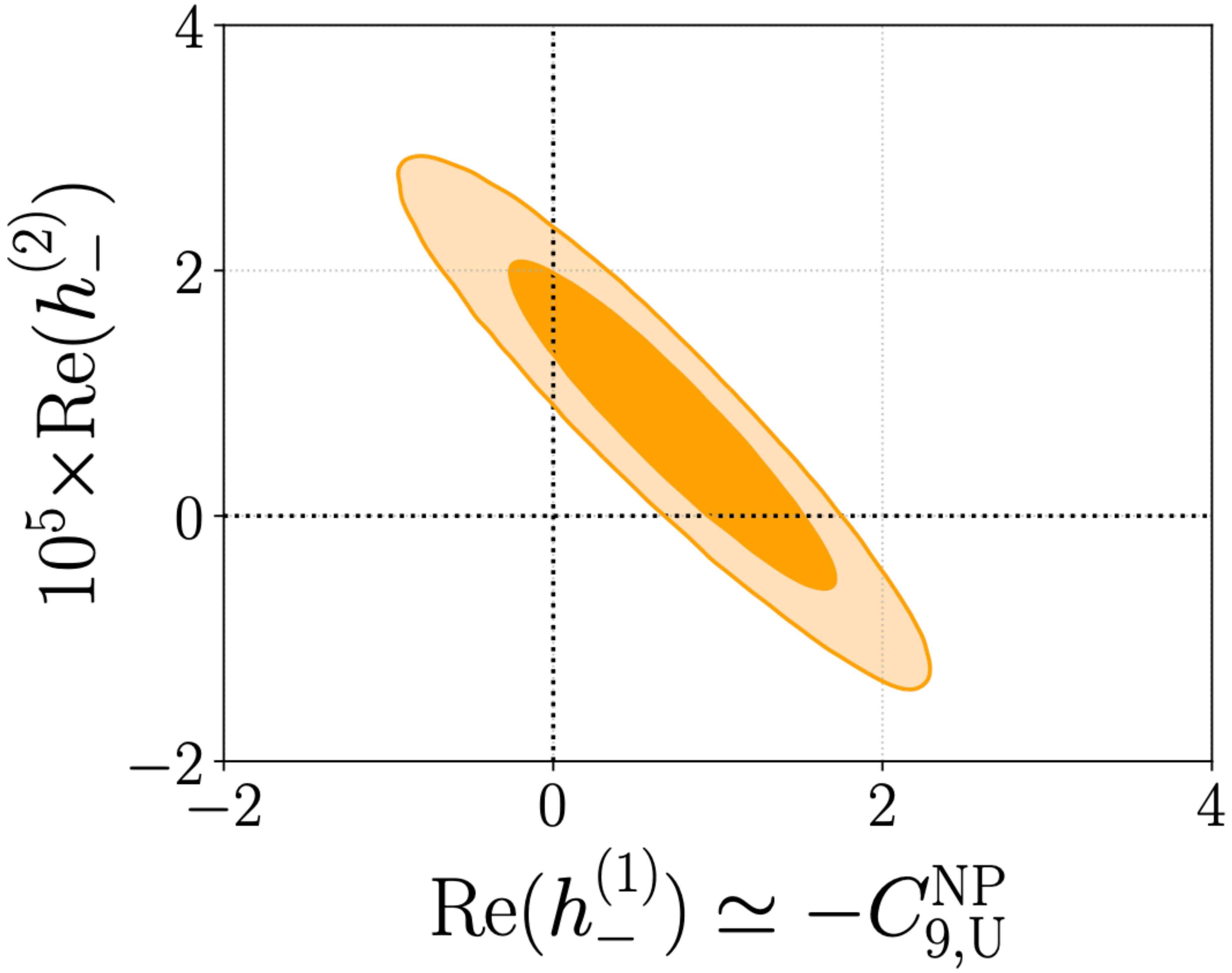
<u>not</u> well reproducing data

well reproducing data



2212.10516
Ciuchini, MF, Franco, Paul, Silvestrini, Valli

$B \to K^{(*)}\ell\ell, B_s \to \phi\ell\ell$: NP implications



We are left with the possibility to address the discrepancies with Lepton Flavour Universal NP effects, which are however indistinguishable from hadronic effects…!

$$\tilde{h}_\lambda(q^2) \sim \epsilon_{\lambda,\mu} \int d^4x\, e^{iqx} \langle V(P)| T\{J_{had}^{\mu,e.m.}(x) \mathcal{H}_{had}^{eff}(0)\}|B\rangle$$

$$H_V^- \propto \left\{ \left(C_9^{\mathrm{SM}} + h_-^{(1)}\right) \widetilde{V}_{L-} + \frac{m_B^2}{q^2}\left[\frac{2m_b}{m_B}\left(C_7^{\mathrm{SM}} + h_-^{(0)}\right)\widetilde{T}_{L-} - 16\pi^2 h_-^{(2)} q^4\right]\right\}$$

$$H_V^+ \propto \left\{ \left(C_9^{\mathrm{SM}} + h_-^{(1)}\right) \widetilde{V}_{L+} + \frac{m_B^2}{q^2}\left[\frac{2m_b}{m_B}\left(C_7^{\mathrm{SM}} + h_-^{(0)}\right)\widetilde{T}_{L+} - 16\pi^2 \left(h_+^{(0)} + h_+^{(1)} q^2 + h_+^{(2)} q^4\right)\right]\right\}$$

$$H_V^0 \propto \left\{ \left(C_9^{\mathrm{SM}} + h_-^{(1)}\right) \widetilde{V}_{L0} + \frac{m_B^2}{q^2}\left[\frac{2m_b}{m_B}\left(C_7^{\mathrm{SM}} + h_-^{(0)}\right)\widetilde{T}_{L0} - 16\pi^2 \sqrt{q^2}\left(h_0^{(0)} + h_0^{(1)} q^2\right)\right]\right\}$$

2212.10516
Ciuchini, MF, Franco, Paul, Silvestrini, Valli

$b \to s\gamma$: the SM status

- Loop-level decay dominated by short-distance effects (C_7)
- Inclusive: main uncertainties come from CKM elements (UTA) and non-perturbative contributions

$$\mathrm{BR}(B \to X_s\gamma)_{E_\gamma > E_0} = \mathrm{BR}(B \to X_c \ell \nu) \left| \frac{\lambda_t}{V_{cb}} \right|^2 \frac{6\alpha_{\mathrm{em}}}{\pi C} \left[|C_7^{\mathrm{eff}}|^2 + |C_7'|^2 + \delta_{\mathrm{nonp.}} \right]$$

2002.01548
Misiak, Rehman, Steinhauser

1908.02812
Gunawardana, Paz

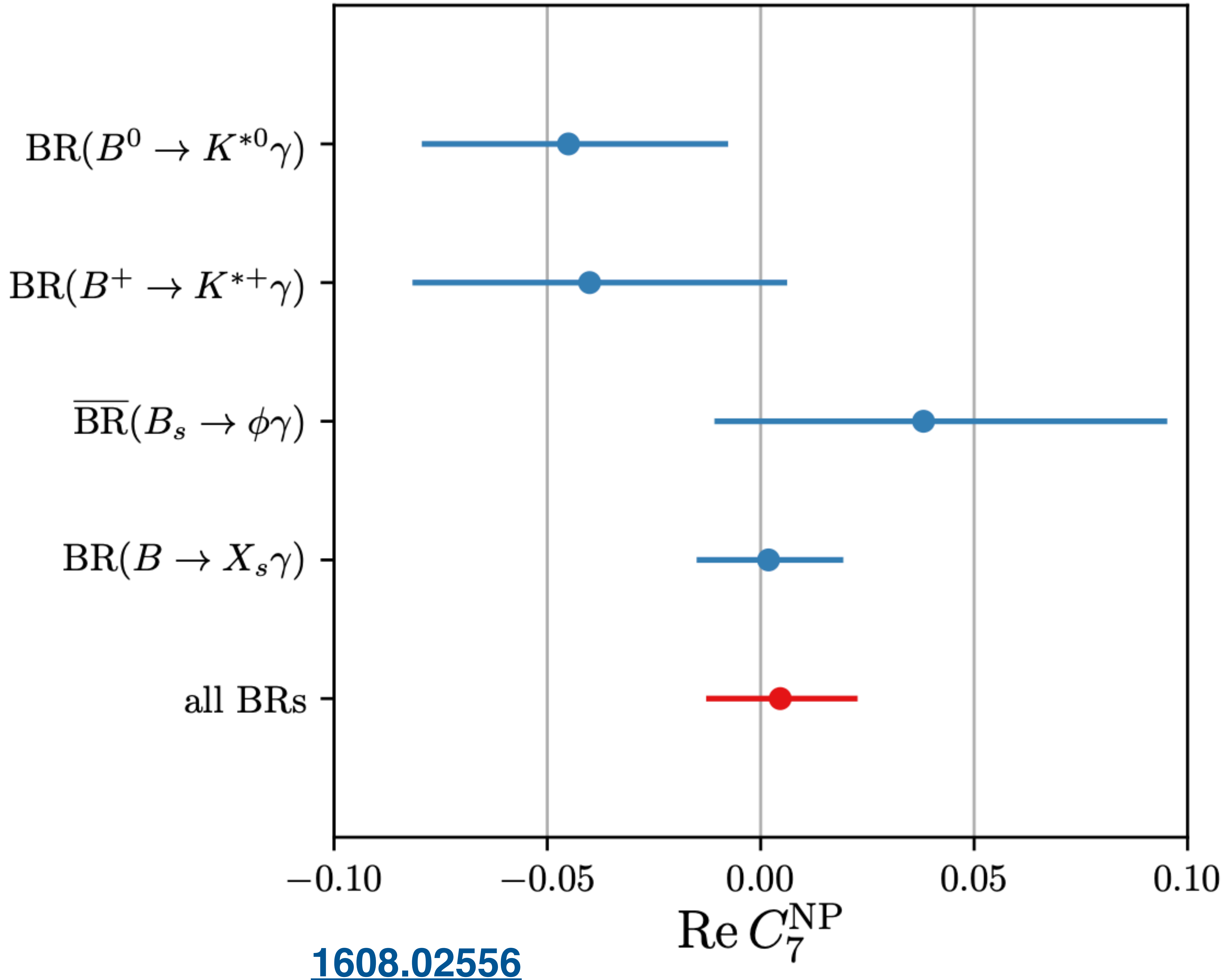
- Exclusive: main uncertainties come from CKM elements (UTA) and form factor (Lattice + LCSR)

$$\mathrm{BR}(B_q \to V\gamma) = \tau_{B_q} \frac{G_F^2 \alpha_{\mathrm{em}} m_{B_q}^3 m_b^2}{32\pi^3} \left(1 - \frac{m_V^2}{m_B^2}\right)^3 |\lambda^t|^2 \left(|\mathcal{C}_7|^2 + |\mathcal{C}_7'|^2 \right) T_1(0)$$

$$A_{\mathrm{CP}}(B_q(t) \to V\gamma) = \frac{\Gamma(\bar{B}_q(t) \to \bar{V}\gamma) - \Gamma(B_q(t) \to V\gamma)}{\Gamma(\bar{B}_q(t) \to \bar{V}\gamma) + \Gamma(B_q(t) \to V\gamma)}$$

$b \to s\gamma$: NP implications

Very strong constraints on possible BSM contribution to the radiative operator, particularly from inclusive decay

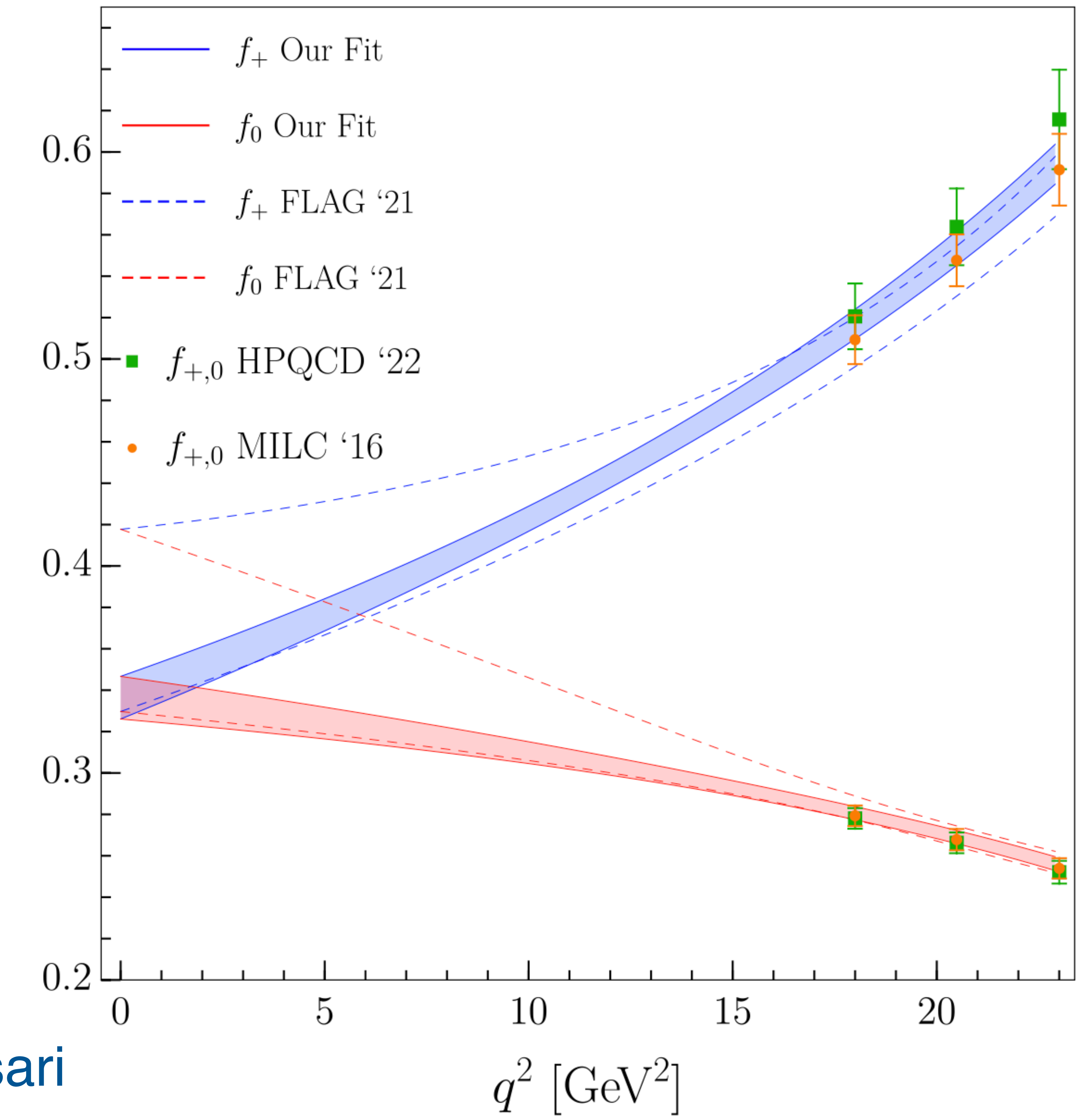


1608.02556
Paul, Straub

$B \to K^{(*)}\nu\nu$: the SM status

- Loop-level decay dominated by short-distance effects (C_L), negligible long-distance
- Main uncertainties as the ones from $B_s \to \mu\mu$, plus additional ones from Form Factors (Lattice)

$$\langle \bar{K}(k)|\bar{s}\gamma^\mu b|\bar{B}(p)\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_K^2}{q^2}q^\mu\right] f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q^\mu f_0(q^2)$$



2301.06990
Bečirević, Piazza, Sumensari

$B \to K^{(*)}\nu\nu$: the SM status

$$\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}q^2}(B \to K\nu\bar{\nu}) = \mathcal{N}_K(q^2)\,|C_L^{\mathrm{SM}}|^2\,|\lambda_t|^2\,\left[f_+(q^2)\right]^2$$

$$\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}q^2}(B \to K^*\nu\bar{\nu}) = \mathcal{N}_{K^*}(q^2)|C_L^{\mathrm{SM}}|^2|\lambda_t|^2\mathcal{F}(q^2)$$

$$\mathcal{O}_L^{\nu_i\nu_j} = \frac{e^2}{(4\pi)^2}(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_i\gamma^\mu(1-\gamma_5)\nu_j)$$

$$\mathcal{O}_R^{\nu_i\nu_j} = \frac{e^2}{(4\pi)^2}(\bar{s}_R\gamma_\mu b_R)(\bar{\nu}_i\gamma^\mu(1-\gamma_5)\nu_j)$$

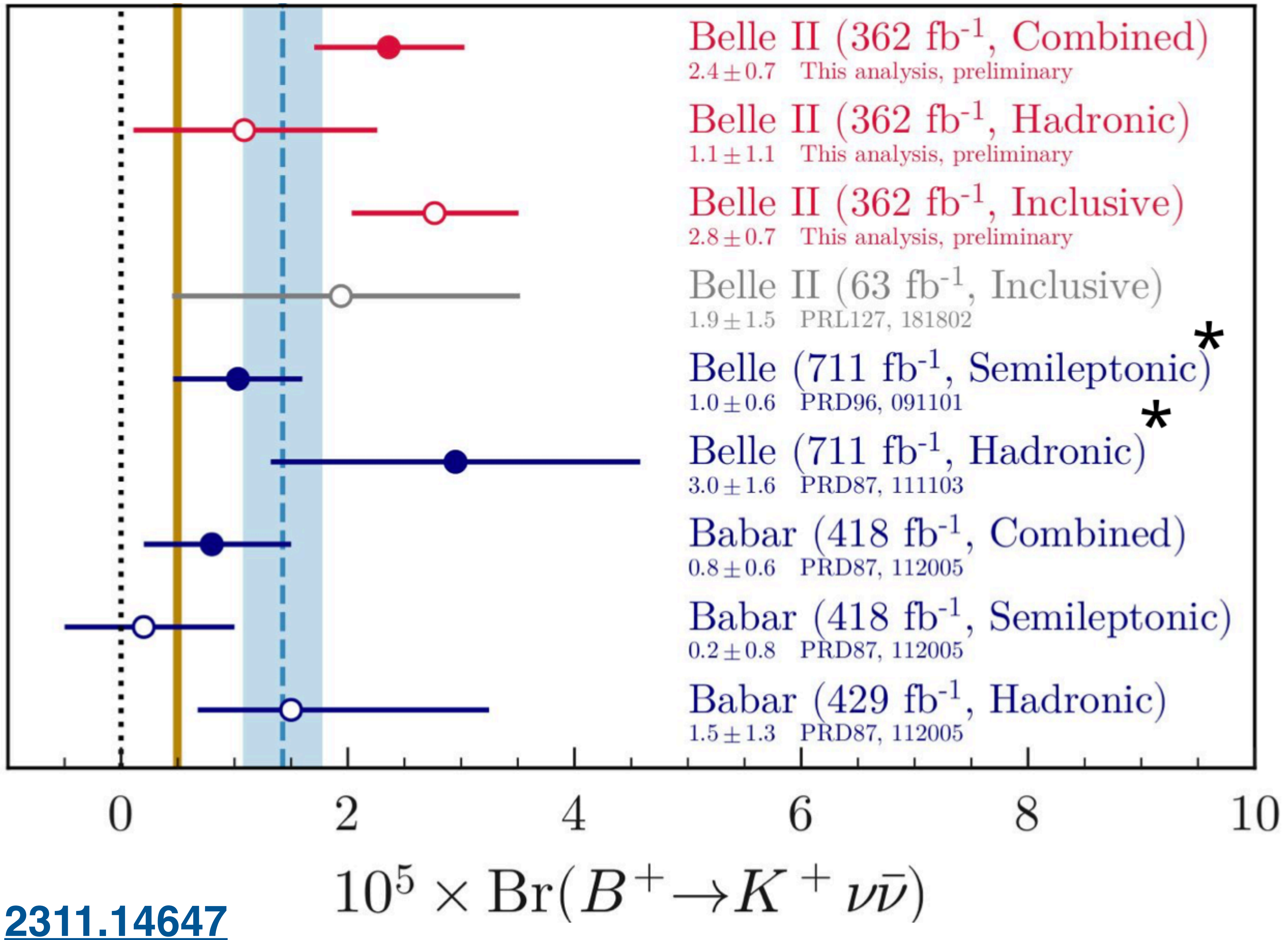
$\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) \times 10^6$	$\sigma_{\mathcal{B}_{K^+}}/\mathcal{B}_{K^+}$	$\mathcal{B}(B^0 \to K_S\nu\bar{\nu}) \times 10^6$	$\sigma_{\mathcal{B}_{K_S}}/\mathcal{B}_{K_S}$
$(5.06 \pm 0.14 \pm 0.28)$	0.06	$(2.05 \pm 0.07 \pm 0.12)$	0.07

$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu}) \times 10^6$	$\sigma_{\mathcal{B}_{K^{*+}}}/\mathcal{B}_{K^{*+}}$	$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu}) \times 10^6$	$\sigma_{\mathcal{B}_{K^{*0}}}/\mathcal{B}_{K^{*0}}$
$(10.86 \pm 1.30 \pm 0.59)$	0.12	$(9.05 \pm 1.25 \pm 0.55)$	0.15

2301.06990
Bečirević, Piazza, Sumensari

$B \to K^{(*)}\nu\nu$: NP implications

Sensitive to BSM effect on both left-handed and right-handed operator



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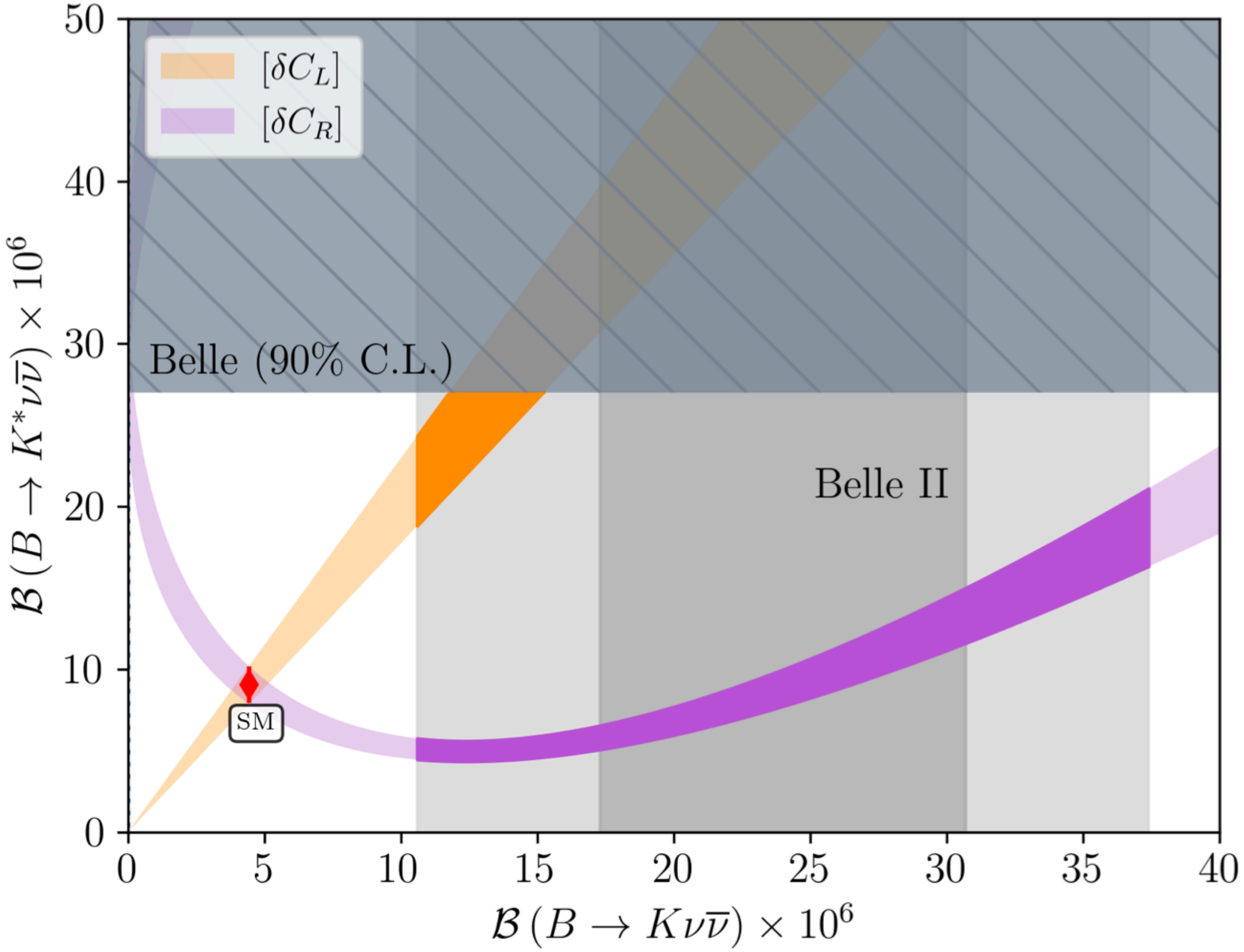

2311.14647
Belle II

2309.02246
Allwicher, Bečirević, Piazza, Rosauro-Alcaraz, Sumensari

Possible interpretation also in terms of weakly interacting light NP (axions)

Conclusions

- Rare decays are a fundamental probe for the search of NP effects. Main theory uncertainties coming from CKM elements, decay constants and form factors

- After re-analysis of LFUV ratios by LHCb, evidence of LFV NP is gone. Remaining hints of LFU NP driven by the muon sector, to be considered with care due to charming penguins

- New discrepancy recently observed in $B \to K\nu\nu$, still much work to do to understand its potential origin and connection with other sectors (light NP?)