Rare decays from lattice QCD: with a focus on $b \to s, d$

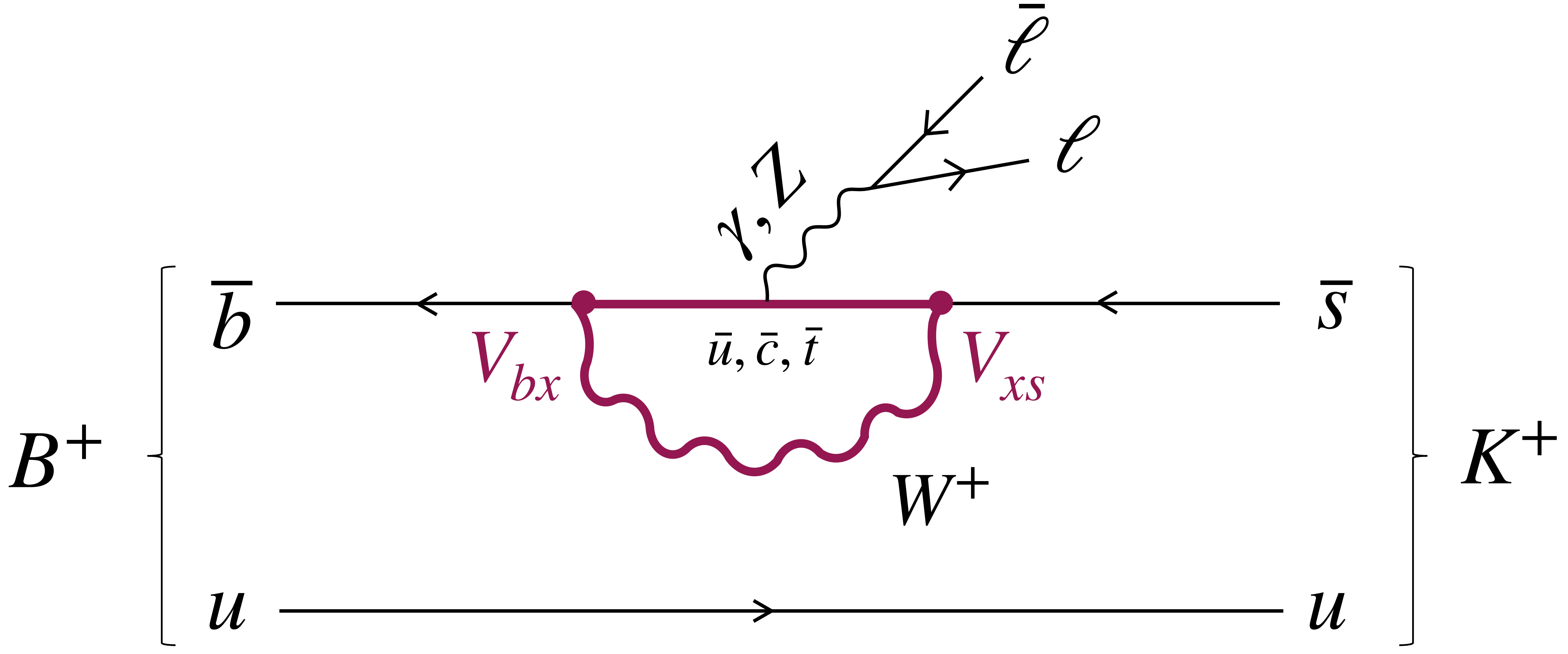
Chris Bouchard, University of Glasgow

FPCP 2024, Chulalongkorn University, Bangkok, Thailand

27 May 2024

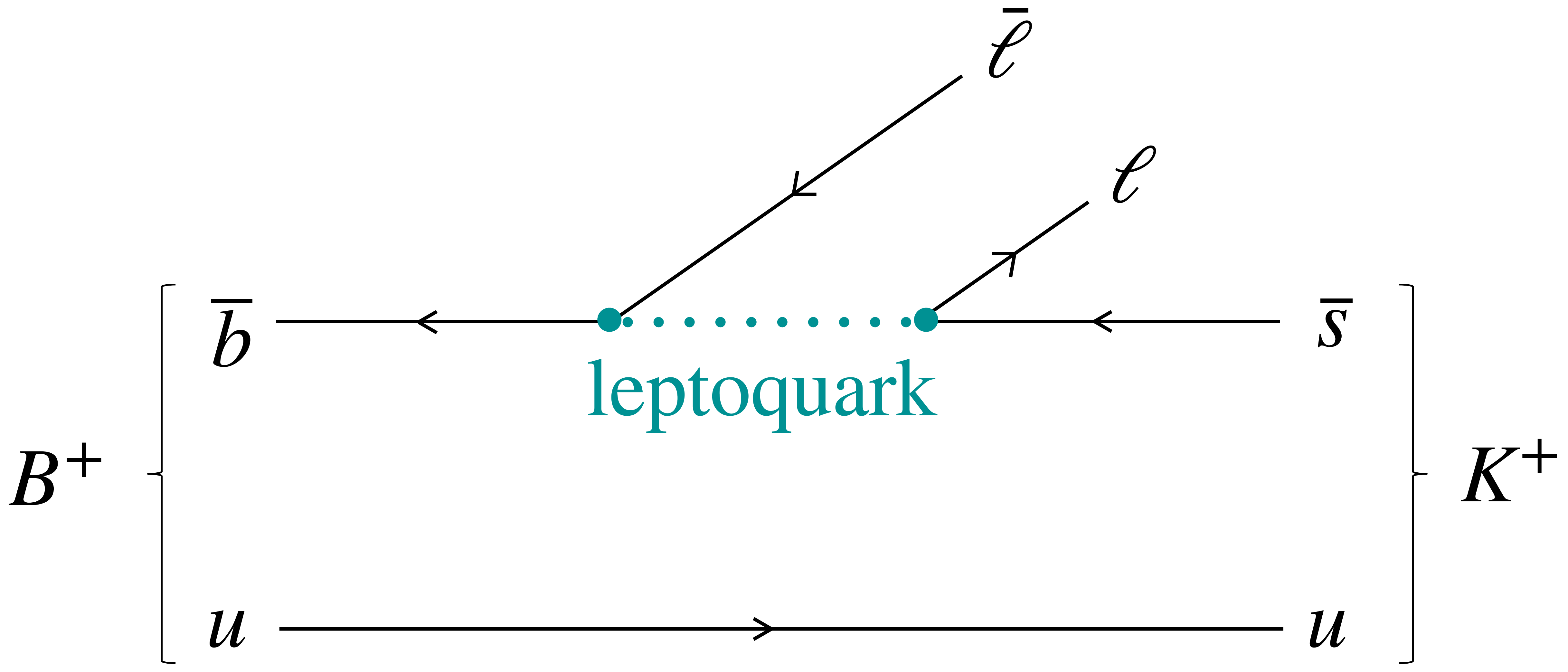
- Motivation for rare decays
- The role of lattice QCD (LQCD) in rare decays
- An improvement in LQCD heavy quark decays
- Status:
  - $B \to K$ and $B \to \pi$
  - $B \to K^*$ and $B_s \to \phi$
  - $\Lambda_b \to \Lambda$ and $\Lambda_b \to \Lambda^*(1520)$
- Summary

Motivation: small SM contribution



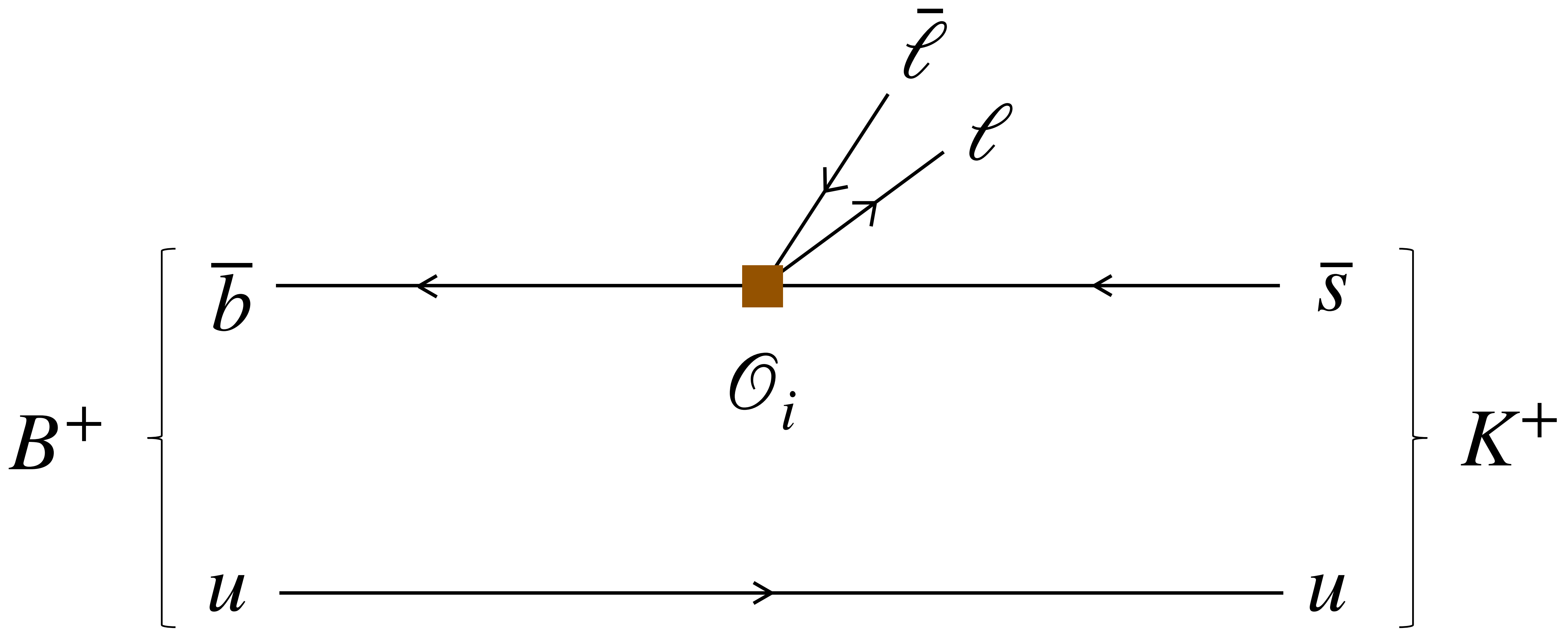
- Loop and CKM suppression makes SM contribution very small.

Motivation: small SM contribution



- Loop and CKM suppression makes SM contribution very small.
- New physics effects (e.g., from a leptoquark) may be large enough to be measured.

Role of LQCD



- Loop and CKM suppression makes SM contribution very small.
- New physics effects (e.g., from a leptoquark) may be large enough to be measured.
- Use a low energy effective theory description for both. See, e.g., Buras, hep-ph/9806471

$$\mathscr{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\, \mathcal{O}_i(\mu)$$

Role of LQCD

- Observables calculated from matrix element, $\langle K\bar{\ell}\ell\,|\,\mathscr{H}_{\text{eff}}^{b\to s}\,|\,B\rangle$.
- This factorizes* giving a hadronic matrix element, like $\langle K\,|\,V^{\mu}\,|\,B\rangle$ for vector current $V^{\mu} = \bar{s}\gamma^{\mu}b$. Also have scalar $S = \bar{s}b$ and tensor $T^{\mu\nu} = \bar{s}\, i\sigma^{\mu\nu}b$ currents.
  - involves QCD at low energy, so nonperturbative
  - has kinematic dependence, typically in terms of $q^2 = (p_B - p_K)^2$
- LQCD allows calculation of hadronic matrix element over (nearly) full kinematic range.
  - fully nonperturbative
  - uncertainties quantifiable and systematically reducible
  - hadronic uncertainties typically the limiting factor for theory

* Approximately - nonfactorizable contributions exist and not yet under full control (see Marco Fedele’s talk).

Role of LQCD

- Form factors parametrize hadronic matrix elements

$$\langle K|S_{\rm latt}|B\rangle = \frac{M_B^2 - M_K^2}{m_b - m_s}\, f_0^{B\to K}(q^2)$$

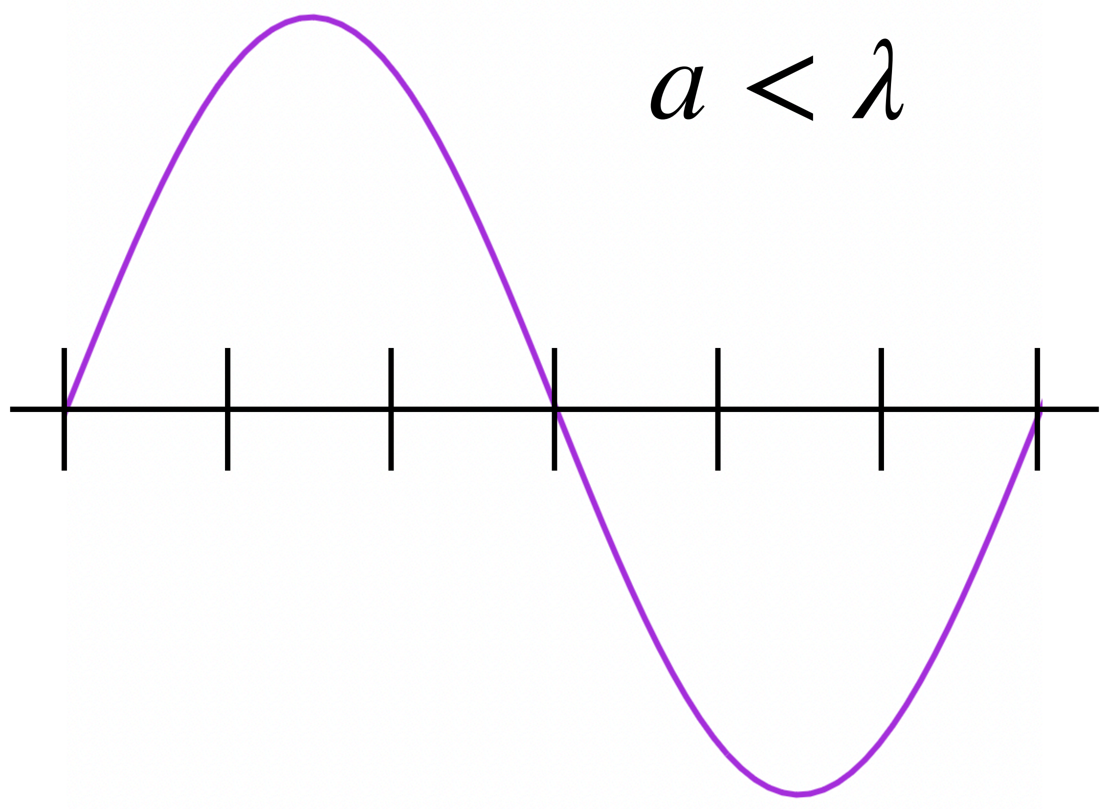
$$Z_V\langle K|V^\mu_{\rm latt}|B\rangle = f_+^{B\to K}(q^2)\left(p_B^\mu + p_K^\mu - \frac{M_B^2 - M_K^2}{q^2}q^\mu\right) + f_0^{B\to K}(q^2)\,\frac{M_B^2 - M_K^2}{q^2}q^\mu$$

$$Z_T(\mu_b)\langle K|T^{\mu\nu}_{\rm latt}|B\rangle = 2\frac{p_B^\mu p_K^\nu - p_B^\nu p_K^\mu}{M_B + M_K}\, f_T^{B\to K}(\mu_b, q^2)$$

- Lattice matrix elements extracted from amplitudes of 2pt and 3pt correlation functions
- If necessary, lattice matrix elements matched to continuum
- Form factors extrapolated to continuum, infinite volume, and physical quark masses
- q^2 dependence determined after (or as part of) physical extrapolation

An improvement for heavy quark decays

- Historically, available lattice spacings have limited ability to "resolve" heavy quarks
- Need a smaller than characteristic size of the quark, $\lambda \sim 1/m$



- For b quark, need $a \lesssim 0.04\ \mathrm{fm}$
- Equivalent to requiring controlled discretization errors, i.e. $am < 1$,

$$O_{\mathrm{cont}} = O_{\mathrm{latt}}\left(1 + (am)^n + \ldots\right)$$

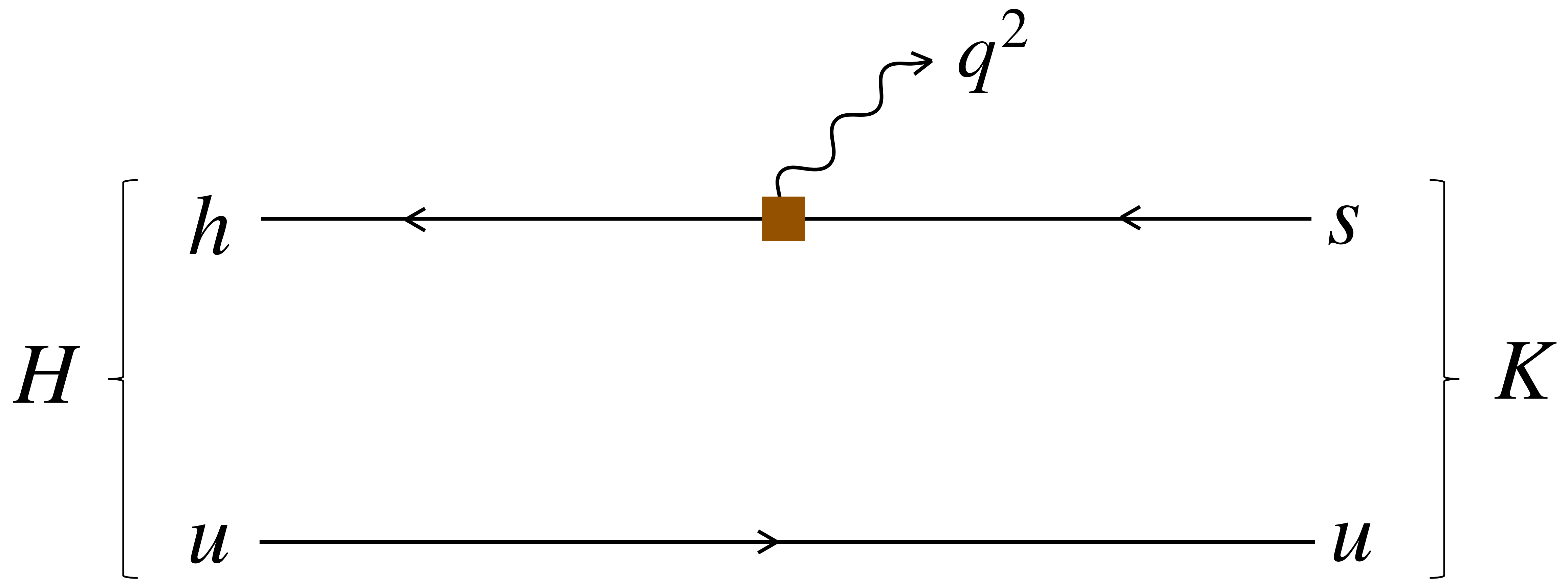
An improvement for heavy quark decays

- Available lattice spacings continue to get smaller. Now have

$$a \sim 0.03 \text{ fm.}$$ FNAL/MILC, PRD 98, 074512 (2018)

  - computationally expensive to create and use, especially if light quarks involved
  - limited statistics on (and community use of) these ensembles
  - not yet at point where we can just use a small a to do the calculation and get sufficient errors to fully leverage experiment ⇒ must address $am_b > 1$
- Historically have used an effective theory for b quark - NRQCD or HQET
  - matching effective theory to QCD introduces systematic error very hard to reduce
  - was becoming dominant error in HPQCD calculations

Heavy HISQ for $B \to K$: a solution



- Highly Improved Staggered Quarks (HISQ) are fully relativistic, no EFT matching

  Follana et al., (HPQCD and UKQCD), PRD 75, 054502 (2007)

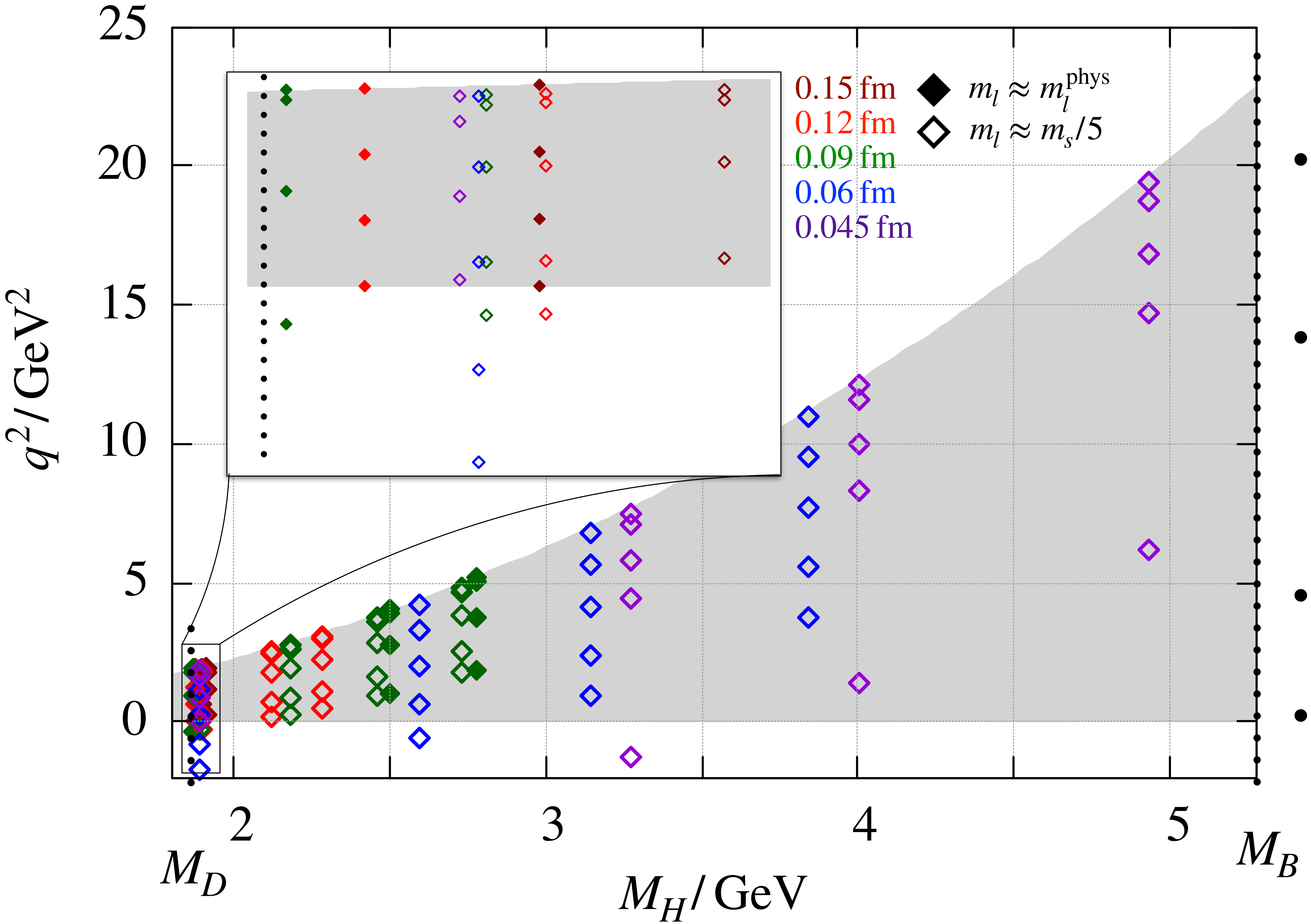
- Heavy HISQ approach uses HISQ for heavy quark h

  McNeile, Davies, Follana, Hornbostel, Lepage (HPQCD), PRD 85, 031503 (2012)

- Simulate a range of m_h such that $m_c \leq m_h \lesssim m_b$ and interpolate/extrapolate

- With $M_D \leq M_H \leq M_B$, obtain results for both B and D decays from one calculation

Heavy HISQ for $B \to K$: kinematic coverage



- HPQCD 23 calculation

  HPQCD, PRD 107, 014510 (2023)

- MILC HISQ $n_f = 2 + 1 + 1$ ensembles

  FNAL/MILC, PRD 82, 074501 (2010)
  FNAL/MILC, PRD 87, 054505 (2012)

- for large range of M_H, cover q^2

- near M_B on finest lattice ◇

Heavy HISQ for $B \to K$: simplified matching

- Lattice matrix elements matched to continuum to give form factors (negligible uncertainty)

$$\langle K|S_{\rm latt}|H\rangle = \frac{M_H^2 - M_K^2}{m_h - m_s} f_0^{H\to K}(q^2)$$

$$Z_V \langle K|V^\mu_{\rm latt}|H\rangle = f_+^{H\to K}(q^2)\left(p_H^\mu + p_K^\mu - \frac{M_H^2 - M_K^2}{q^2}q^\mu\right) + f_0^{H\to K}(q^2)\frac{M_H^2 - M_K^2}{q^2}q^\mu$$

$$Z_T(\overline{\rm MS}, M_H)\langle K|T^{jo}_{\rm latt}|H\rangle = \frac{2iM_H p_K^j}{M_H + M_K} f_T^{H\to K}(\overline{\rm MS}, M_H; q^2)$$

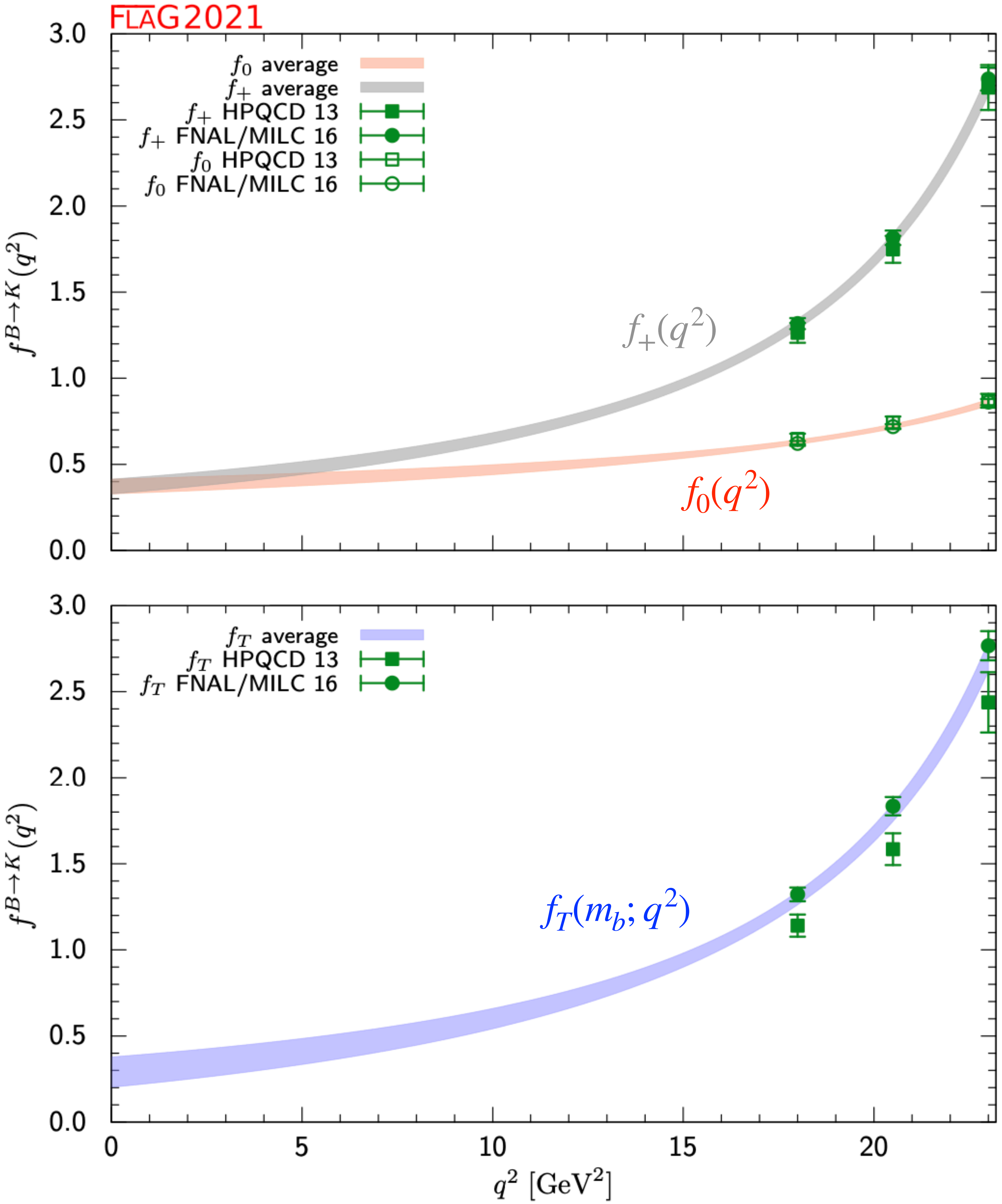
- Z_V calculated via PCVC relation, $Z_V = \left.\frac{(m_h - m_s)\langle K|S|H\rangle}{(M_H - M_K)\langle K|V^0|H\rangle}\right|_{\vec{p}_K=0}$

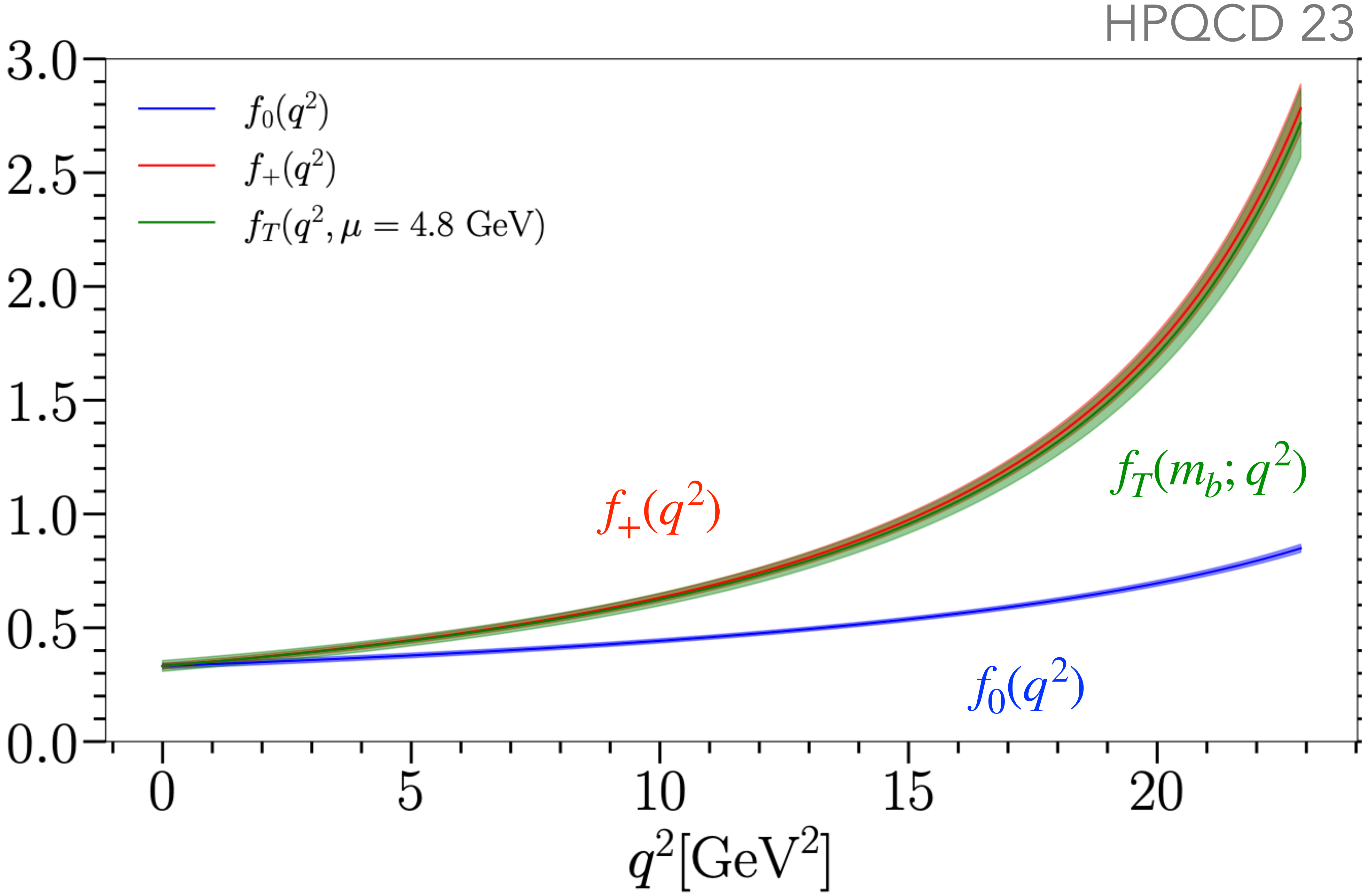
Na, Davies, Follana, Lepage, (HPQCD) PRD 82, 114506 (2010)

- Z_T calculated nonperturbatively (RI-SMOM scheme)

Hatton, Davies, Lepage, Lytle, (HPQCD) PRD 102, 094509 (2020)

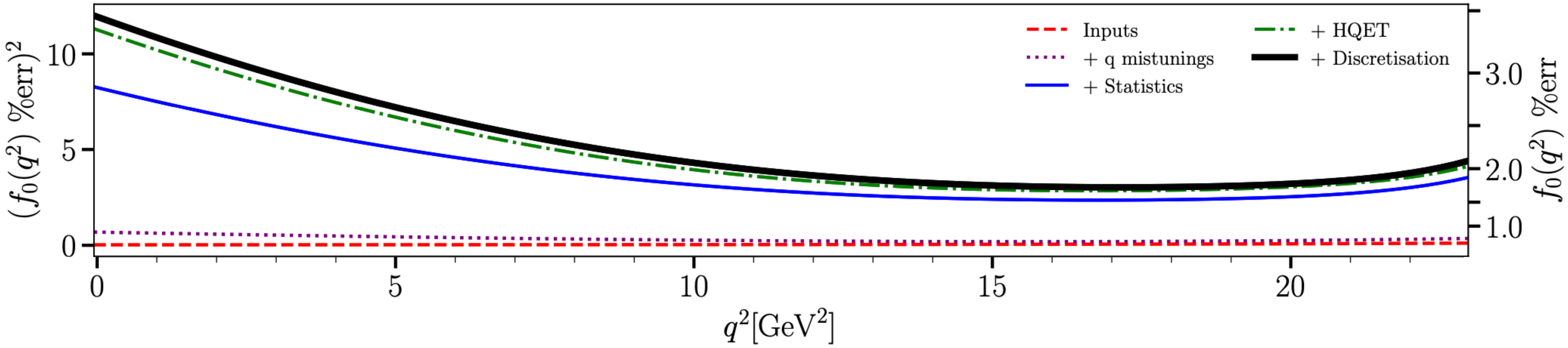
$B \to K$: current status





- HPQCD 23 first $B \to K$ with fully relativistic b quark
- heavy HISQ removes matching EFT to QCD
- better coverage of kinematic range
- at least 3x more precise at $q^2 = 0$

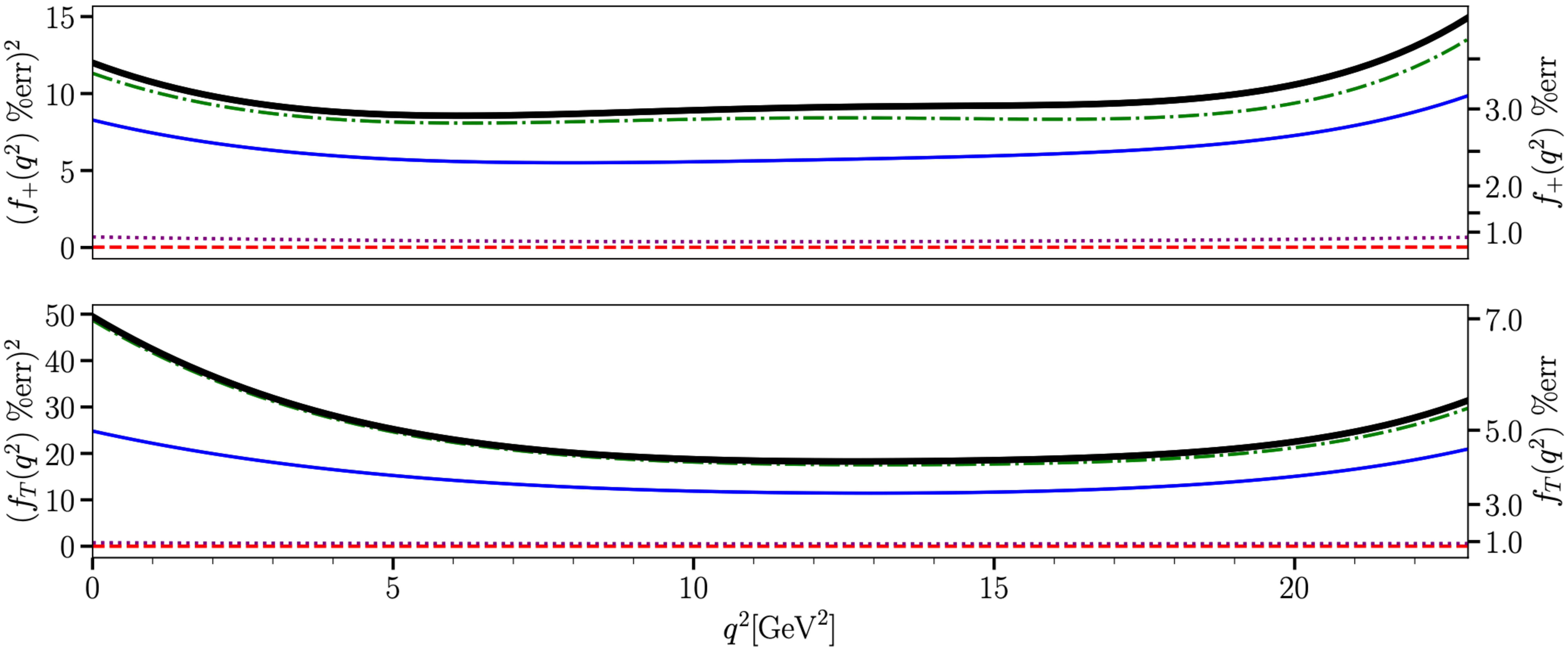
$B \to K$: error budget vs q^2



error budget (stacked variances)

- "Inputs" from errors of input quantities, e.g. meson masses
- "q mistunings" mistuned simulation light quark masses and chiral extrapolation
- "Statistics" from finite ensemble size
- "HQET" from extrapolation $m_h \to m_b$
- "Discretisation" from uncertainty in extrapolation $a \to 0$

$B \to K$: error budget vs q^2



- improved precision, especially at low q^2, where it is needed
- **statistics dominated**, so further improvement straightforward

Phenomenology: $B \to K\ell\bar{\ell}$

- LHS - differential decay rate Γ is measured

$$\frac{d\Gamma(B \to K\ell\bar{\ell})}{dq^2} = 2a_\ell + \frac{2}{3}c_\ell$$

- RHS - prediction depends on $F_{P,A,V}$; functions of form factors and Wilson coefficients

$$a_\ell = \mathscr{C}\left[q^2|F_P|^2 + \frac{\lambda(q, M_B, M_K)}{4}(|F_A|^2 + |F_V|^2) + 4m_\ell^2 M_B^2 |F_A|^2 + 2m_\ell(M_B^2 - M_K^2 + q^2)\mathrm{Re}(F_P F_A^*)\right]$$

$$c_\ell = -\mathscr{C}\frac{\lambda(q, M_B, M_K)\beta_\ell^2}{4}(|F_A|^2 + |F_V|^2)$$

Phenomenology: $B \to K\ell\bar{\ell}$

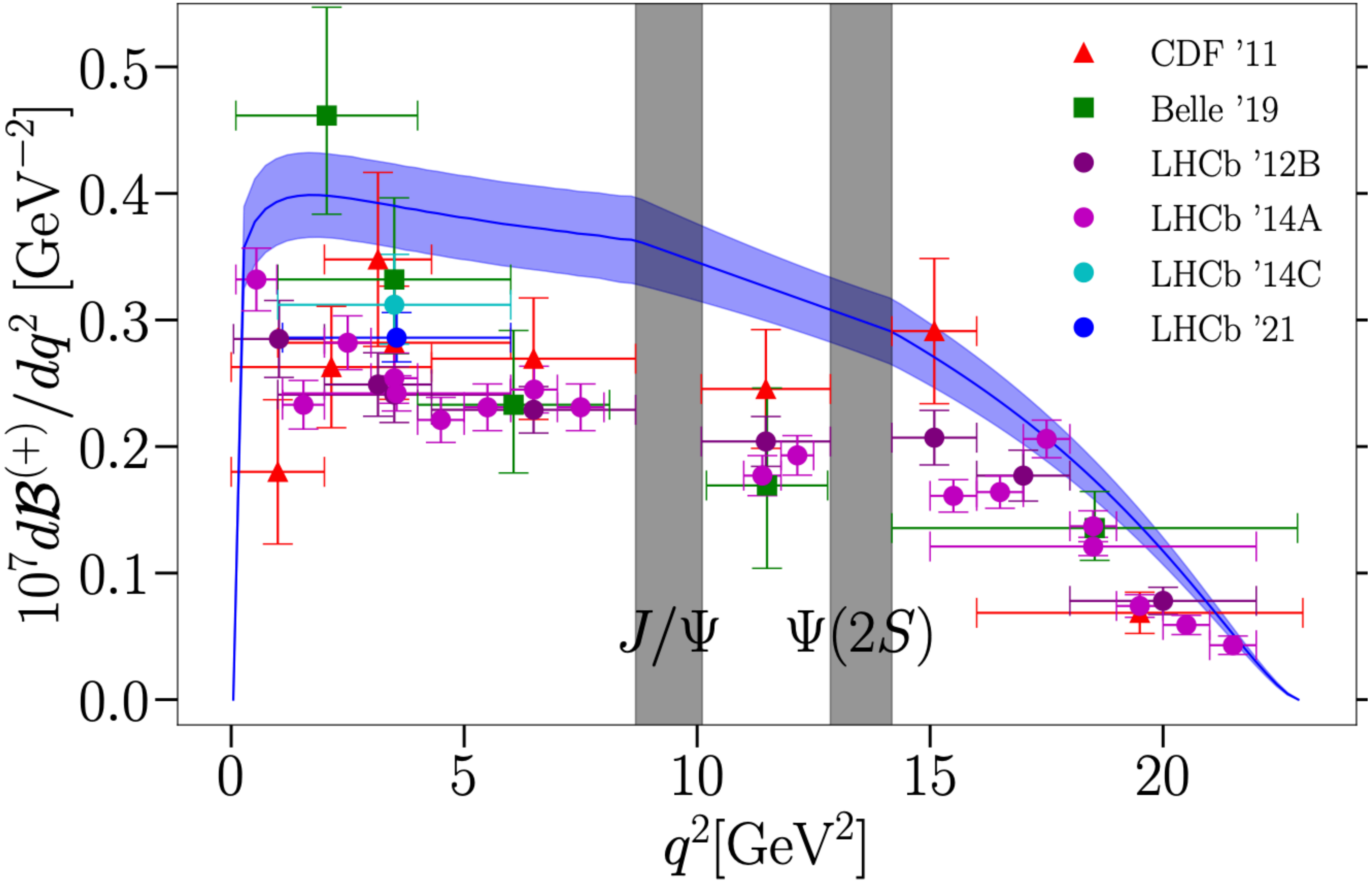
$$F_P = -m_\ell C_{10}\left[f_+^{B\to K} - \frac{M_B^2 - M_K^2}{q^2}(f_0 - f_+^{B\to K})\right]$$

$$F_A = C_{10} f_+^{B\to K}$$

$$F_V = C_9^{\mathrm{eff},1} f_+^{B\to K} + \frac{2m_b^{\overline{\mathrm{MS}}}(\mu_b)}{M_B + M_K} C_7^{\mathrm{eff},1} f_T^{B\to K}(\mu_b)$$

- $C_9^{\mathrm{eff},1}$ includes $\mathcal{O}(\alpha_s)$ perturbative QCD and estimates of nonfactoriazable corrections
  Gubernari et al, 2206.03797
- $C_7^{\mathrm{eff},1}$ includes $\mathcal{O}(\alpha_s)$ corrections FNAL/MILC, PRD 93, 034005 (2016)
  - these corrections give $< 1\sigma$ shift, slightly reducing tension with expt
- QED effect from final state radiation: 2% (5%) in $d\mathcal{B}/dq^2$ for $\mu(e)$; 1% in ratio R_K
- other small uncertainties included (scale dependence of Wilson coefficients, $m_u \neq m_d$, …)

Phenomenology: $B \to K\ell\bar{\ell}$ vs experiment



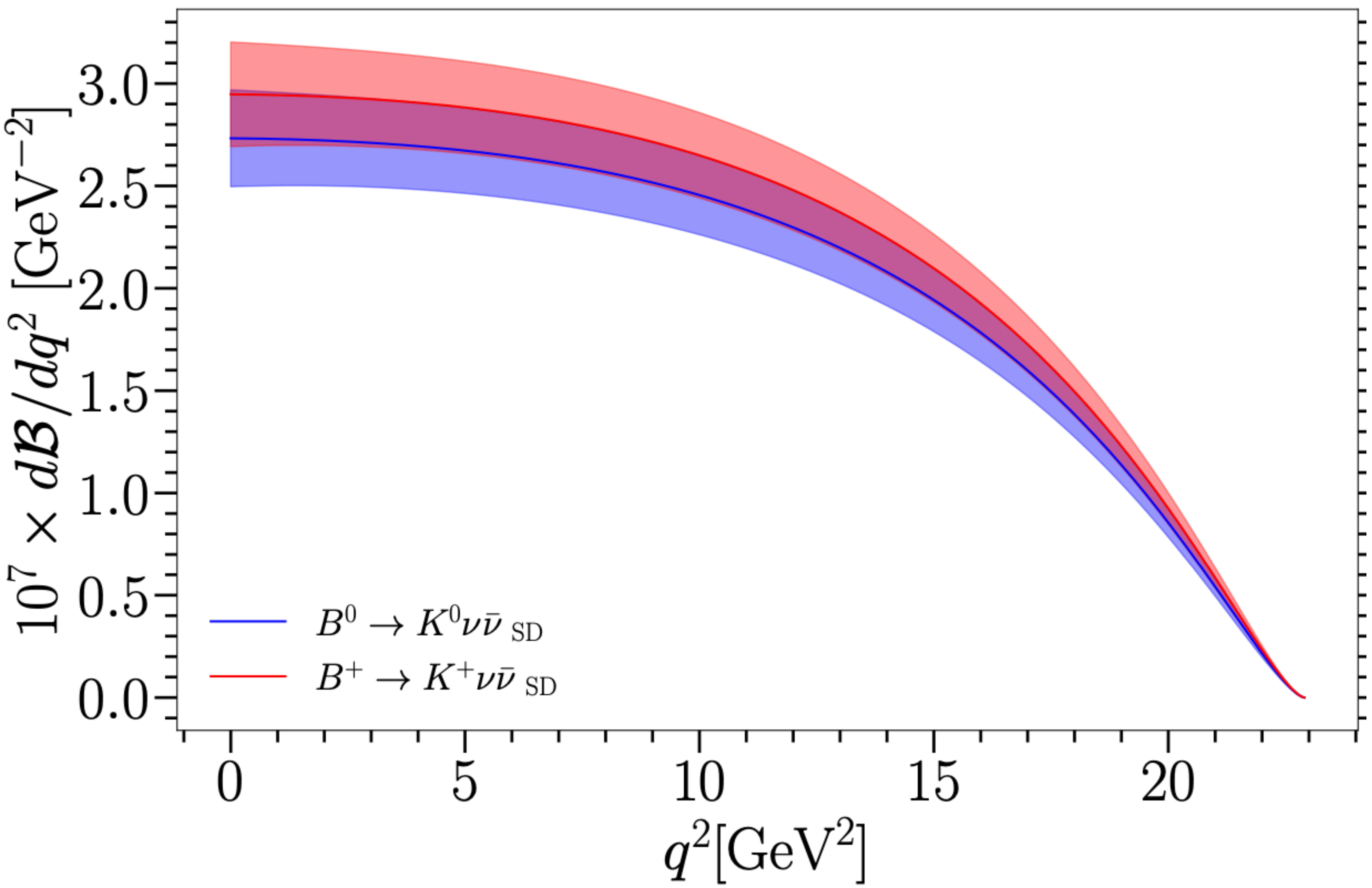
- $1.1 \le q^2/\mathrm{GeV}^2 \le 6$: below $c\bar{c}$ resonances; improved precision

- New result from CMS is consistent with LHCb '14A ● and more precise at low q^2

CMS, submitted, 2401.07090 (2024)

- Tension with LHCb'14A and CMS for $1.1 \le q^2/\mathrm{GeV}^2 \le 6$ is ~4σ

Phenomenology: $B \to K\nu\bar{\nu}$

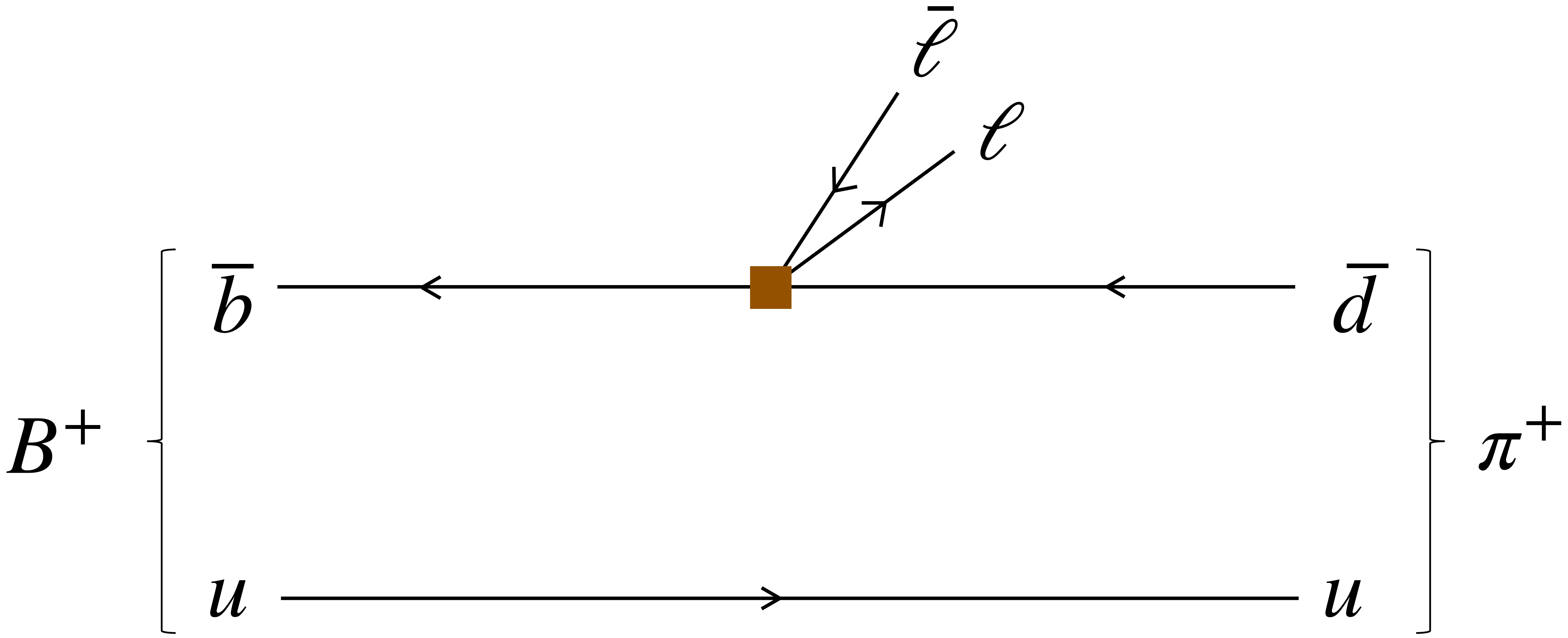


Decay	$\mathcal{B} \times 10^6$	Reference	
$B^0 \to K_S^0\nu\bar{\nu}$	< 13 (90% CL) Exp.	[32]	Belle '17
$B^0 \to K^0\nu\bar{\nu}$	< 49 (90% CL) Exp.	[34]	BaBar '13
	4.01(49)	[9]	FNAL '16
	$4.1^{+1.3}_{-1.0}$	[37]	Wang, Xiao '12
	4.60(34)	HPQCD '22	
$B^+ \to K^+\nu\bar{\nu}$	< 16 (90% CL) Exp.	[34]	BaBar '13
	< 19 (90% CL) Exp.	[32]	Belle '17
	< 41 (90% CL) Exp.	[33]	Belle II '21
	5.10(80)	[79, 81]	Altmanshoffer et al '09, Kamenik, Smith '09
	$4.4^{+1.4}_{-1.1}$	[37]	Wang, Xiao '12
	3.98(47)	[45]	Buras et al '14
	4.94(52)	[9]	FNAL '16
	4.53(64)	[86]	Buras, Venturini '21
	4.65(62)	[87]	Buras, Venturini '22
	5.58(37)	HPQCD '22	

24(7) Belle II $2.6\,\sigma$

- Cleaner theoretically; no resonances or nonfactorizable contributions
- Improved precision for SM prediction

$B \to \pi$: current status



- Only difference from $B \to K$ is final state quark is down not strange
- LQCD calculation computationally more expensive
- LQCD calculations don't typically distinguish u/d, so same form factors as tree-level FCCC decay needed to get V_{ub}

$B \to \pi$: current status

currently dominated by FNAL/MILC 2015a ($f_{0,+}$), FNAL/MILC 2015b (f_T)

$f_{0,+}^{B\to\pi}$: HPQCD 2006

FNAL/MILC 2015a

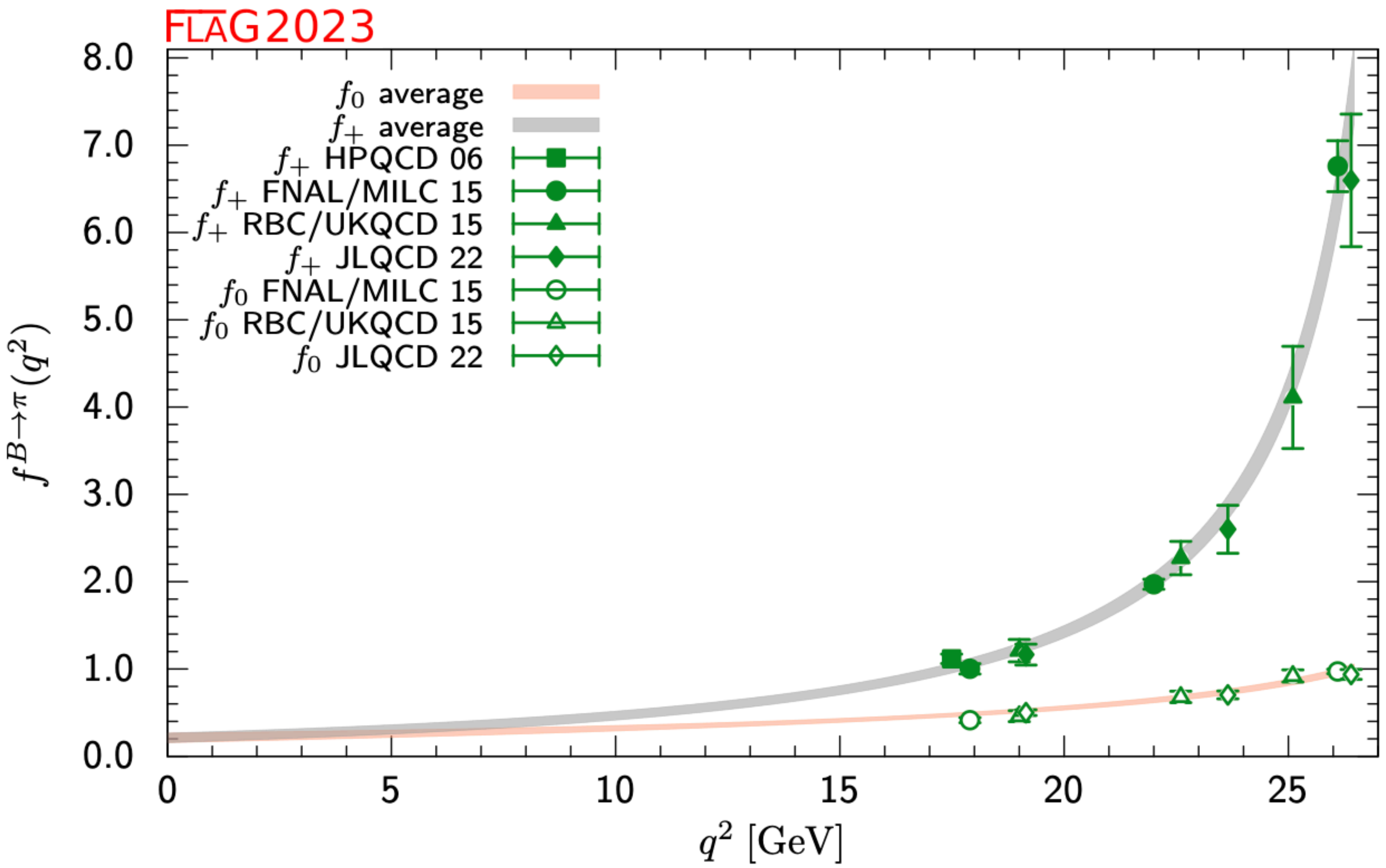
RBC/UKQCD 2015

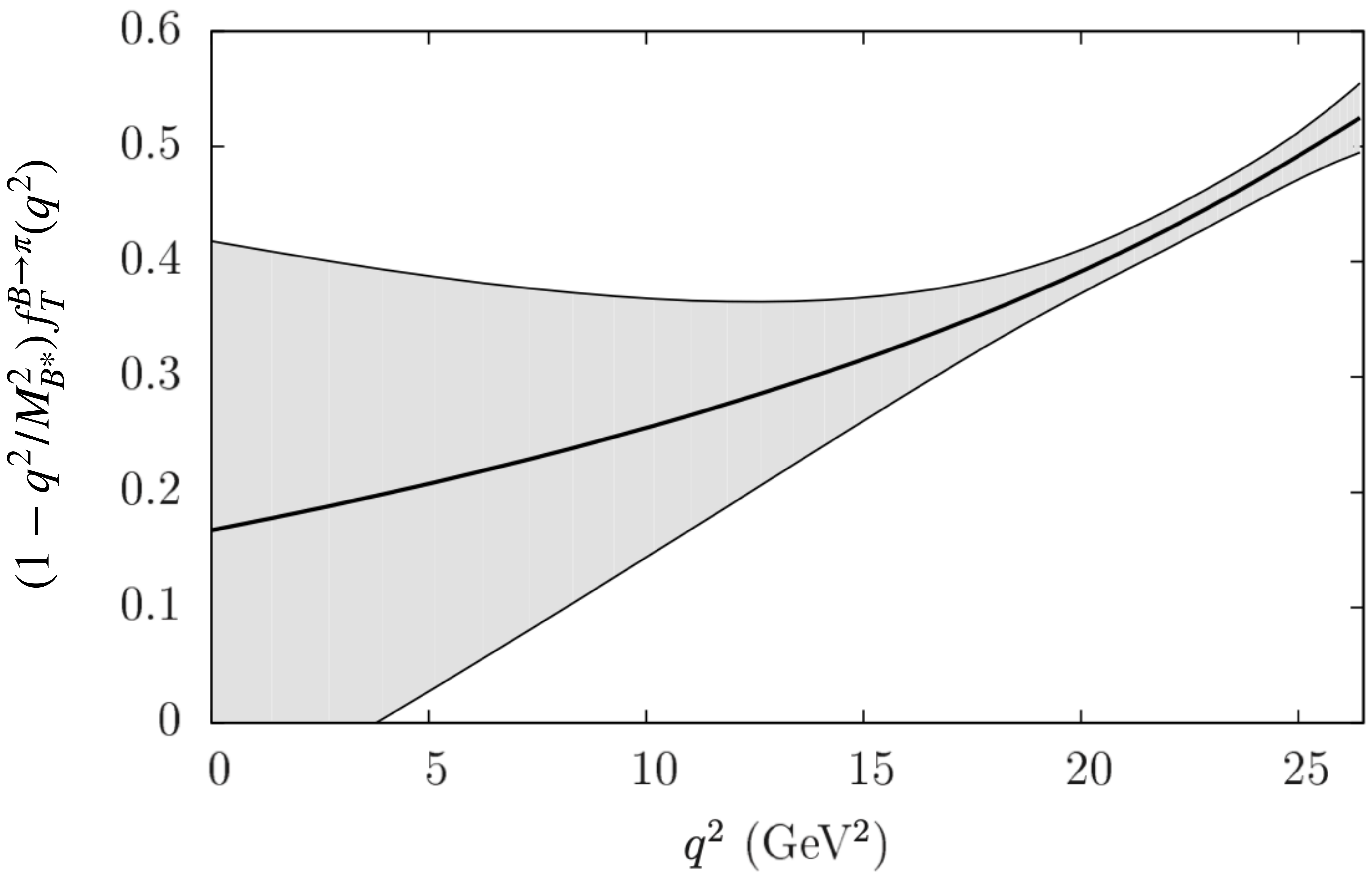
JLQCD 2022

$\left.\begin{matrix}\text{HPQCD 2006}\\ \text{FNAL/MILC 2015a}\\ \text{RBC/UKQCD 2015}\end{matrix}\right\}$ updates underway using relativistic b

$f_T^{B\to\pi}$: FNAL/MILC 2015b

- FNAL/MILC update underway
- calculations by other groups underway





$B \to \pi$

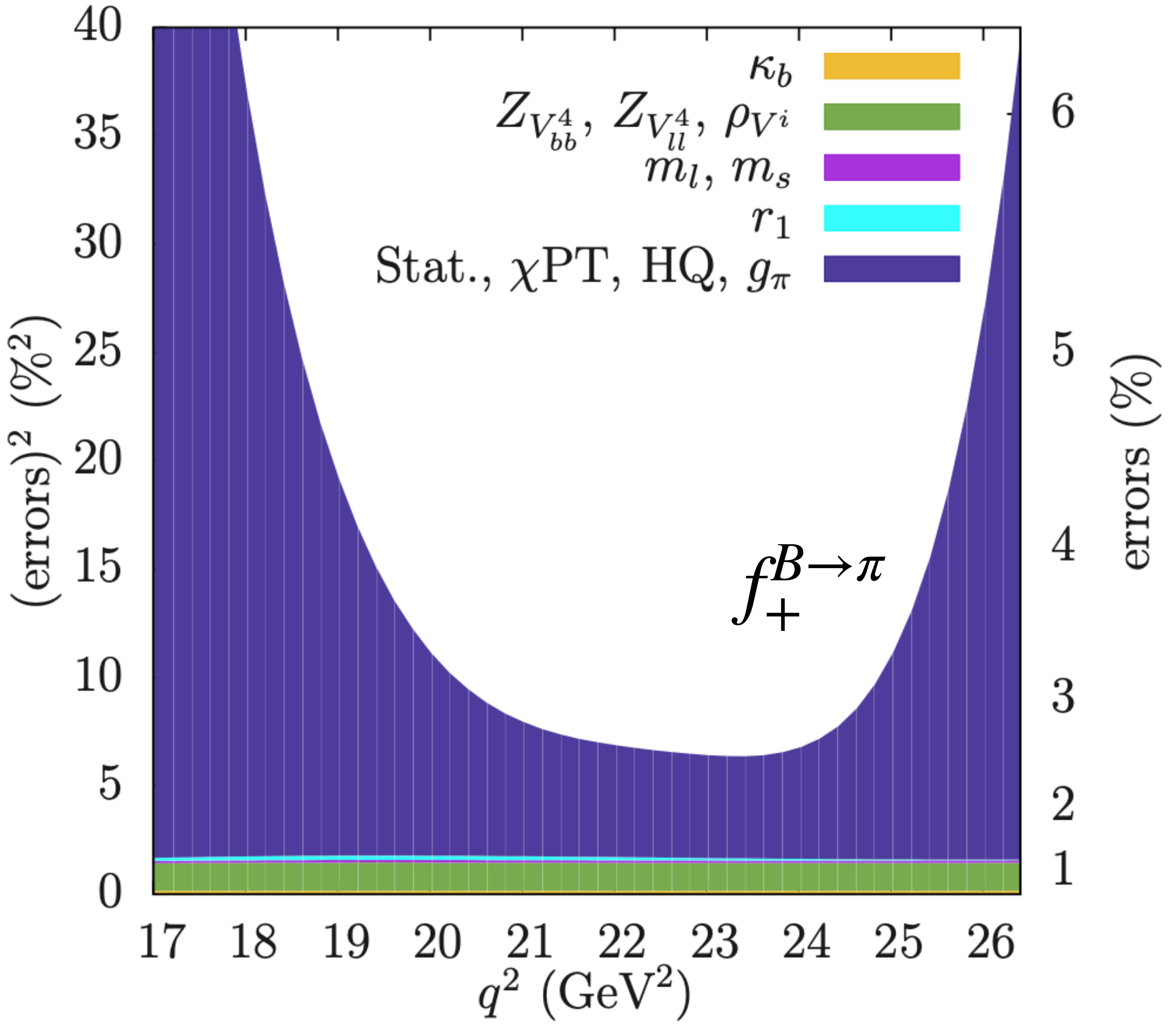
FNAL/MILC 2015a ($f_{0,+}^{B\to\pi}$), FNAL/MILC 2015b ($f_T^{B\to\pi}$)

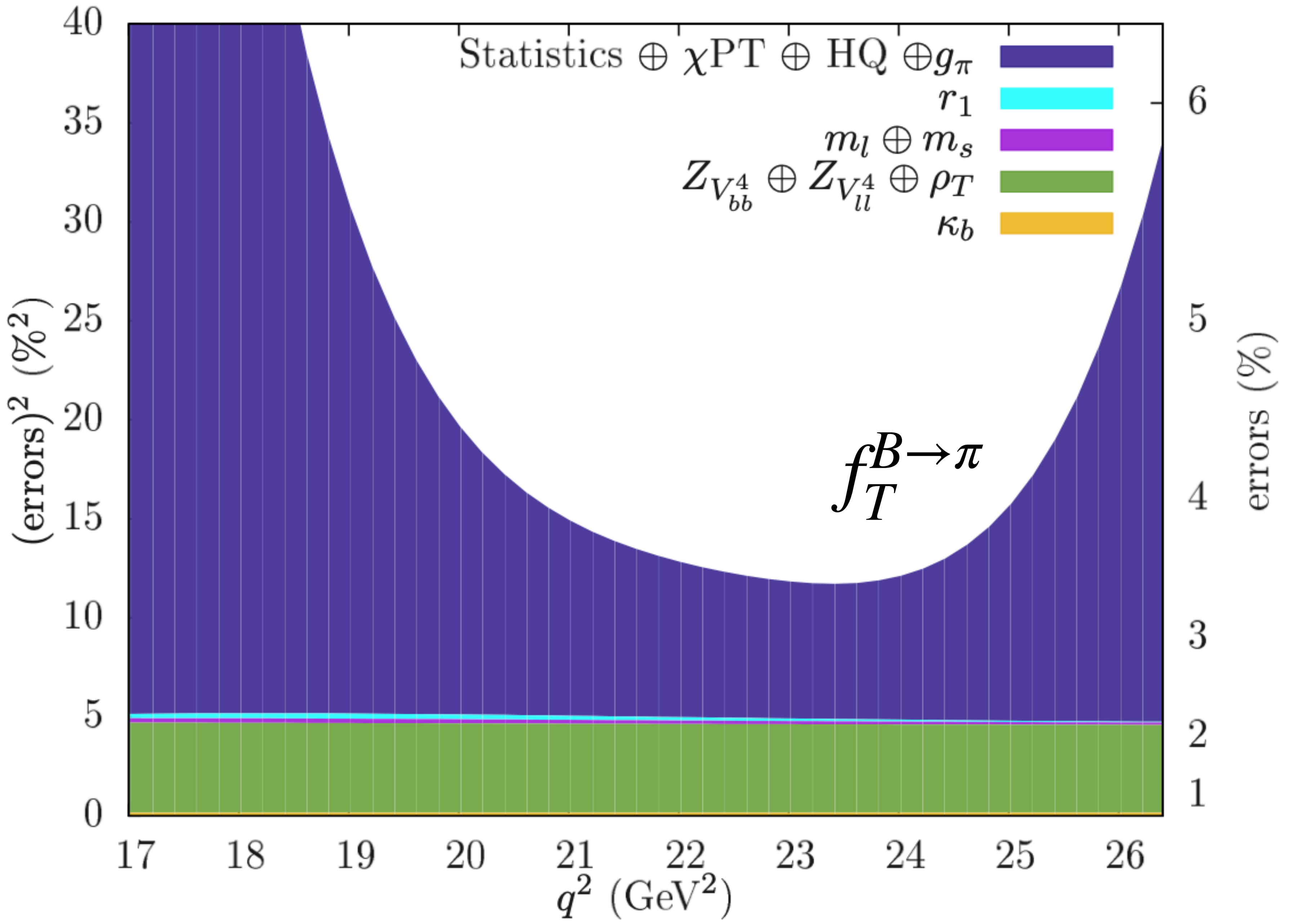
FNAL/MILC 2015a: PRD 92 (2015) 014024
FNAL/MILC 2015b: PRL 115 (2015) 152002

- MILC asqtad $n_f = 2 + 1$ flavor ensembles
- Fermilab b quark (must match to QCD)
- asqtad light valence quarks
- 4 lattice spacings, from 0.12 - 0.045 fm

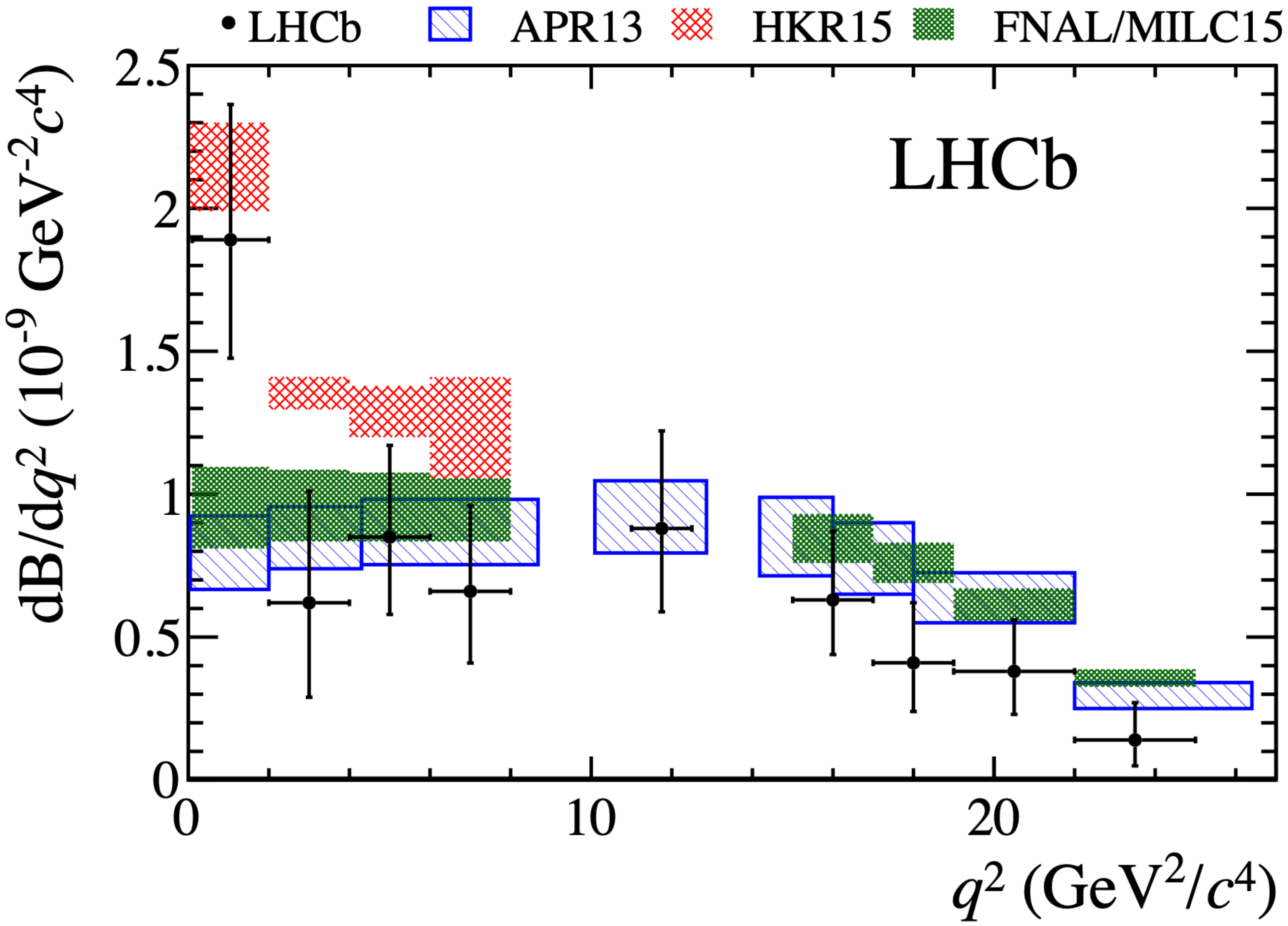
dominant errors: statistics, chiral extrapolation, HQ discretization

- most reduced by MILC's HISQ $n_f = 2 + 1 + 1$ ensembles
- relativistic treatment of b quark will address HQ





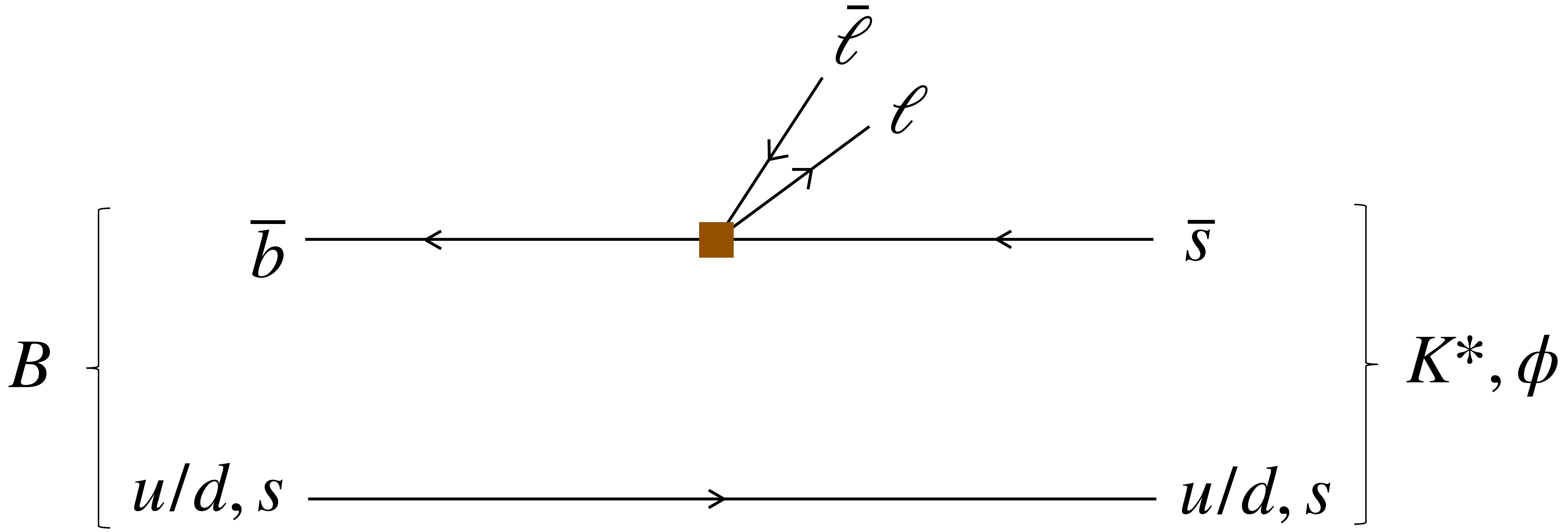
$B \to \pi$



LHCb, Aaij et al., JHEP 10 (2015) 034

- APR13 uses lattice form factors with SU(3) breaking ansatz Ali, Parkhomenko, Rusov, PRD 89, 094021 (2014)
- HKR15 uses light cone sum rules Hambrock, Khodjamirian, Rusov, PRD 92, 074020 (2015)
- $f_{0,+,T}^{B\to\pi}$ should improve similar to $B \to K$

$B \to K^*$ and $B_s \to \phi$: current status

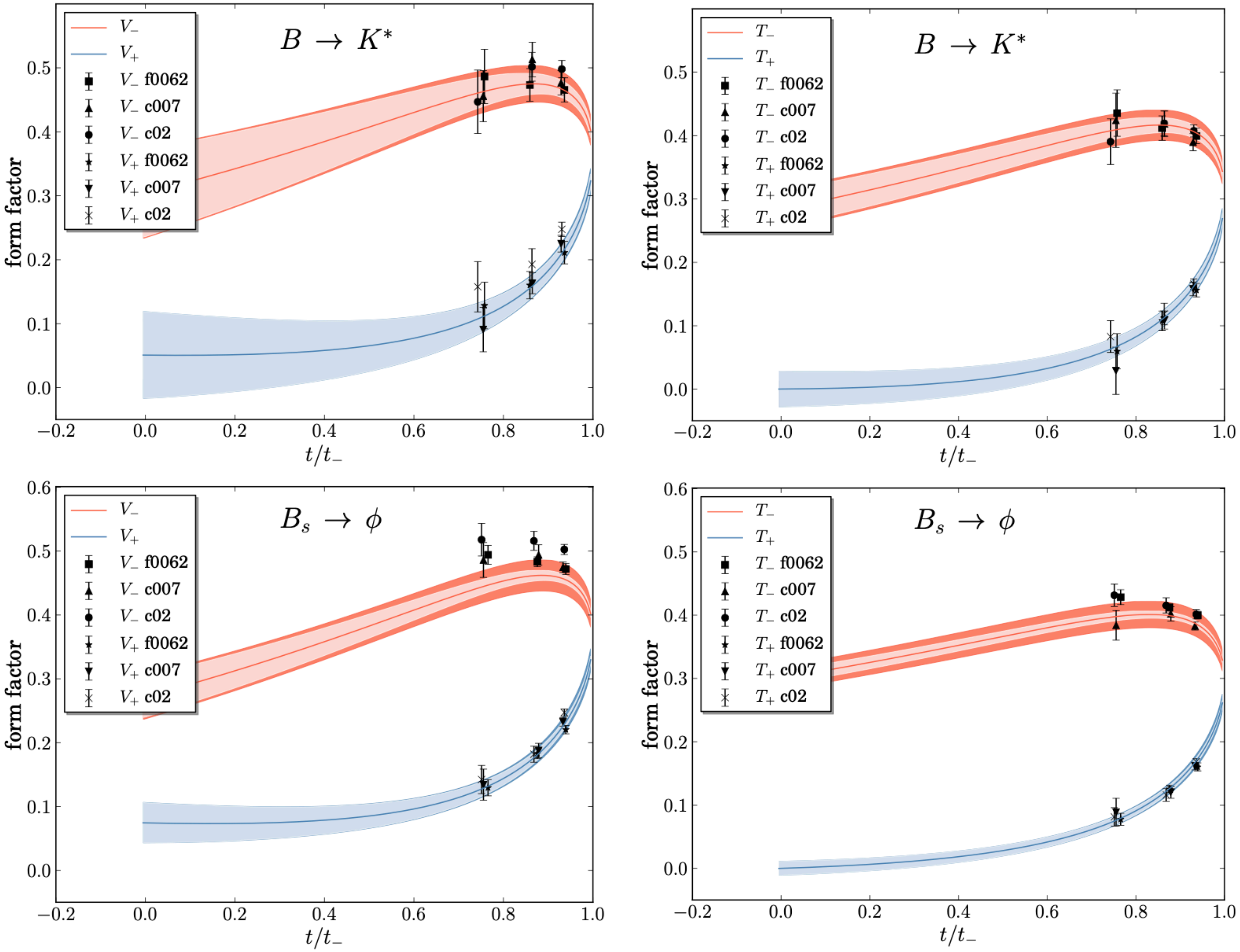


- Pseudoscalar to vector involves seven form factors
- Final states unstable, e.g., $K^* \to K\pi$, where $K\pi$ scattering must be accounted for
  - technology for this has been developed and groups are working on pilot calculations
  - difficulty means they have not received same attention; only one calculation to date
  - RBC/UKQCD, HPQCD, … working but in preliminary stages

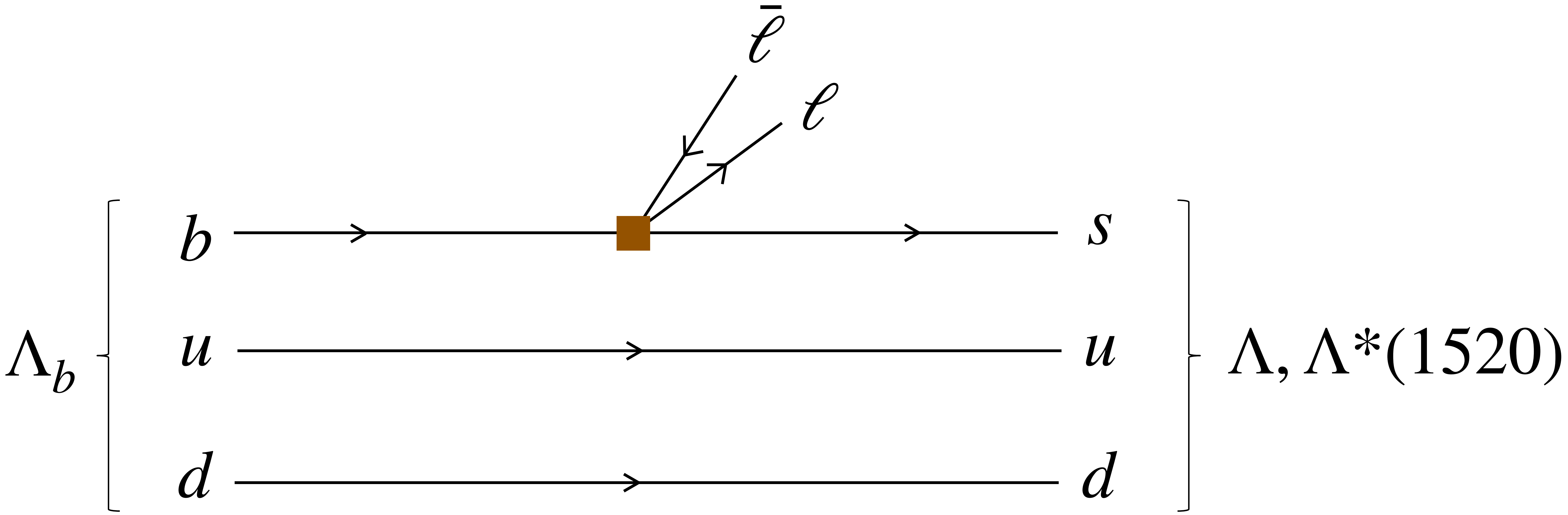
$B \to K^*$ and $B_s \to \phi$: current status

Horgan et al., PRD 89, 094501 (2014)

- Representative plots shown for four of seven form factors
- MILC asqtad $n_f = 2 + 1$ flavor ensembles
- NRQCD b quark (must match to QCD) and ASQTAD (precursor to HISQ) staggered light quarks
- Final states assumed to be stable



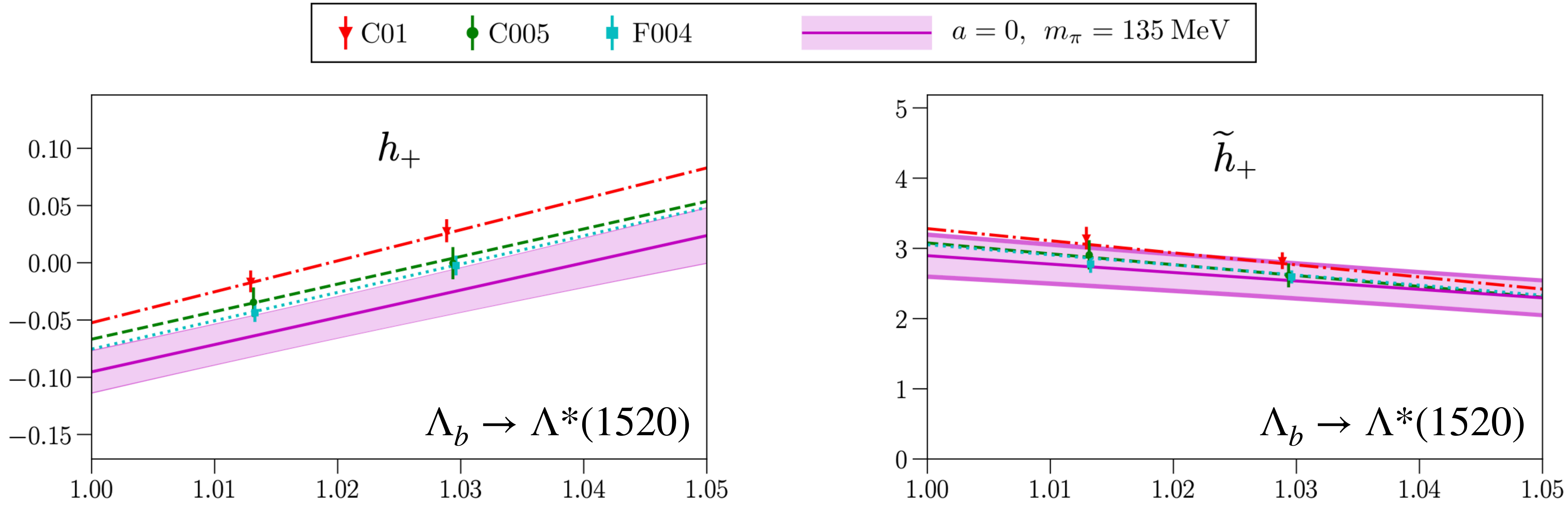
$\Lambda_b \to \Lambda$ and $\Lambda_b \to \Lambda^*(1520)$: current status



- First LQCD calculations
  - $\Lambda_b \to \Lambda$ Detmold and Meinel, PRD 93, 074501 (2016)
  - $\Lambda_b \to \Lambda^*(1520)$ Meinel and Rendon, PRD 103, 074505 (2021)
- Λ is stable; $\Lambda^*(1520)$ unstable but decay neglected
- $\Lambda_b \to \Lambda$ has 10 form factor; $\Lambda_b \to \Lambda^*(1520)$ has14 form factors

$\Lambda_b \to \Lambda$ and $\Lambda_b \to \Lambda^*(1520)$: current status

Meinel and Rendon, PRD 103, 074505 (2021)



- Two representative $\Lambda_b \to \Lambda^*(1520)$ tensor form factors shown
- RBC/UKQCD DWF $n_f = 2 + 1$ flavor ensembles
- Columbia Relativistic Heavy Quark treatment of b quark

Summary

- $B \to K$ form factors are most advanced, with $B \to \pi$ close behind. Benefit from
  - fully relativistic treatment of b quark, e.g., Heavy HISQ
  - fewer form factors and stable final states
- Work ongoing to account for unstable final states in and update $B \to K^*$ and $B_s \to \phi$
- Limited baryonic decay form factors calculated
  - only $\Lambda_b \to \Lambda$ and $\Lambda_b \to \Lambda^*(1520)$
  - only one group/using only one method
- LQCD community has work to do here, but future is exciting

Thank you

and thanks to collaborators:

- Bipasha Chakraborty
- Christine Davies
- Dan Hatton
- Jonna Kopponen
- Peter Lepage
- Will Parrott

Form Factor calculation: ensembles

MILC $n_f = 2+1+1$ HISQ ensembles Bazavov et al., PRD 82, 074501 (2010); Bazavov et al., PRD 87, 054505 (2012)

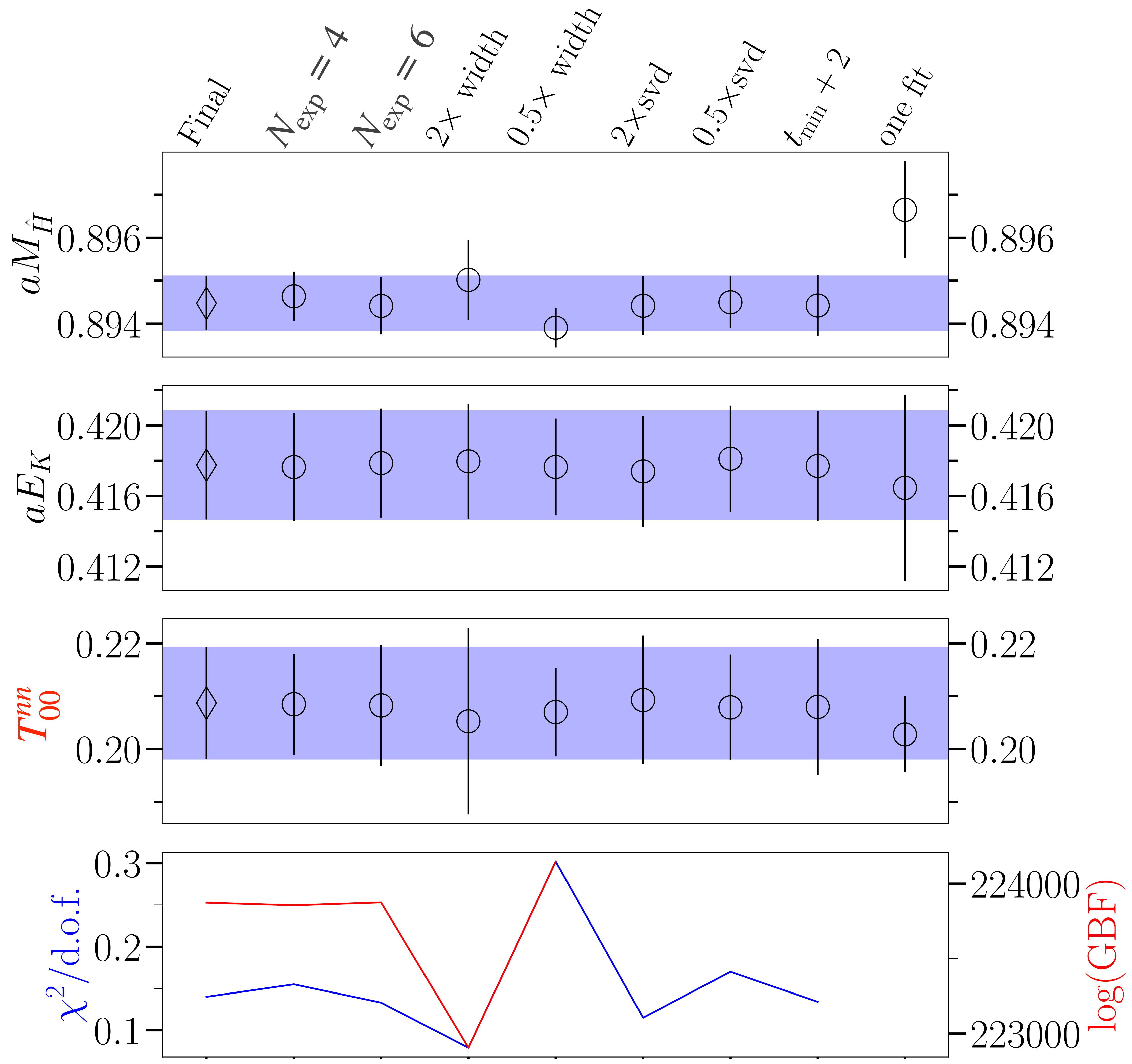
$\approx a/\mathrm{fm}$	$N_s^3 \times N_t$	N_{cfg}	N_{src}	$am_l^{\mathrm{val,\,sea}}$	am_h
0.15	32^3 x 48	998	16	$0.00235 \approx am_l^{\mathrm{phys}}$	0.8605
0.15	16^3 x 48	1020	16	$0.013 \approx am_s/5$	0.888
0.12	48^3 x 64	985	16	$0.00184 \approx am_l^{\mathrm{phys}}$	0.643
0.12	24^3 x 64	1053	16	$0.0102 \approx am_s/5$	0.664, 0.8, 0.9
0.09	64^3 x 96	620	8	$0.0012 \approx am_l^{\mathrm{phys}}$	0.433, 0.683, 0.8
0.09	32^3 x 96	499	16	$0.0074 \approx am_s/5$	0.449, 0.566, 0.683, 0.8
0.06	48^3 x 144	413	8	$0.0048 \approx am_s/5$	0.274, 0.45, 0.6, 0.8
0.045	64^3 x 192	375	4	$0.00316 \approx am_s/5$	0.194, 0.45, 0.6, 0.8

Form Factors: matrix elements from correlators

- matrix element extracted from amplitudes of 3pt correlators (built with MILC code)
- simultaneous fit 2pt and 3pt correlators, e.g.
  Parrott, Bouchard, Davies, Hatton, PRD 103, 094506 (2021)
- fits use Lepage's gvar, lsqfit and corrfitter

$$C_2(t) = \sum_{i=0}^{N_{\exp}} \left[|d_i^n|^2 (e^{-E_i^n t} + e^{-E_i^n (N_t - t)}) - (-1)^t |d_i^o|^2 (e^{-E_i^o t} + e^{-E_i^o (N_t - t)}) \right]$$

$$\begin{aligned} C_3^J(t,T) = \sum_{i,j=0}^{N_{\exp}} \Big[& d_i^{H,n} J_{ij}^{nn} d_j^{K,n} e^{-E_i^{H,n} t} e^{-E_j^{K,n}(T-t)} \\ & + (-1)^{(T-t)} d_i^{H,n} J_{ij}^{no} d_j^{K,o} e^{-E_i^{H,n} t} e^{-E_j^{K,o}(T-t)} \\ & + (-1)^t d_i^{H,o} J_{ij}^{on} d_j^{K,n} e^{-E_i^{H,o} t} e^{-E_j^{K,n}(T-t)} \\ & + (-1)^T d_i^{H,o} J_{ij}^{oo} d_j^{K,o} e^{-E_i^{H,o} t} e^{-E_j^{K,o}(T-t)} \Big] \end{aligned}$$



representative fit stability, from $a \approx 0.045$ fm

Form Factors: modified z-expansion

- form factors at simulated a, m_{quarks}, V and q^2
- extrapolate to $a \to 0$, $m_{\text{quarks}} \to m_{\text{quarks}}^{\text{phys}}$ and $V \to \infty$ using modified z-expansion

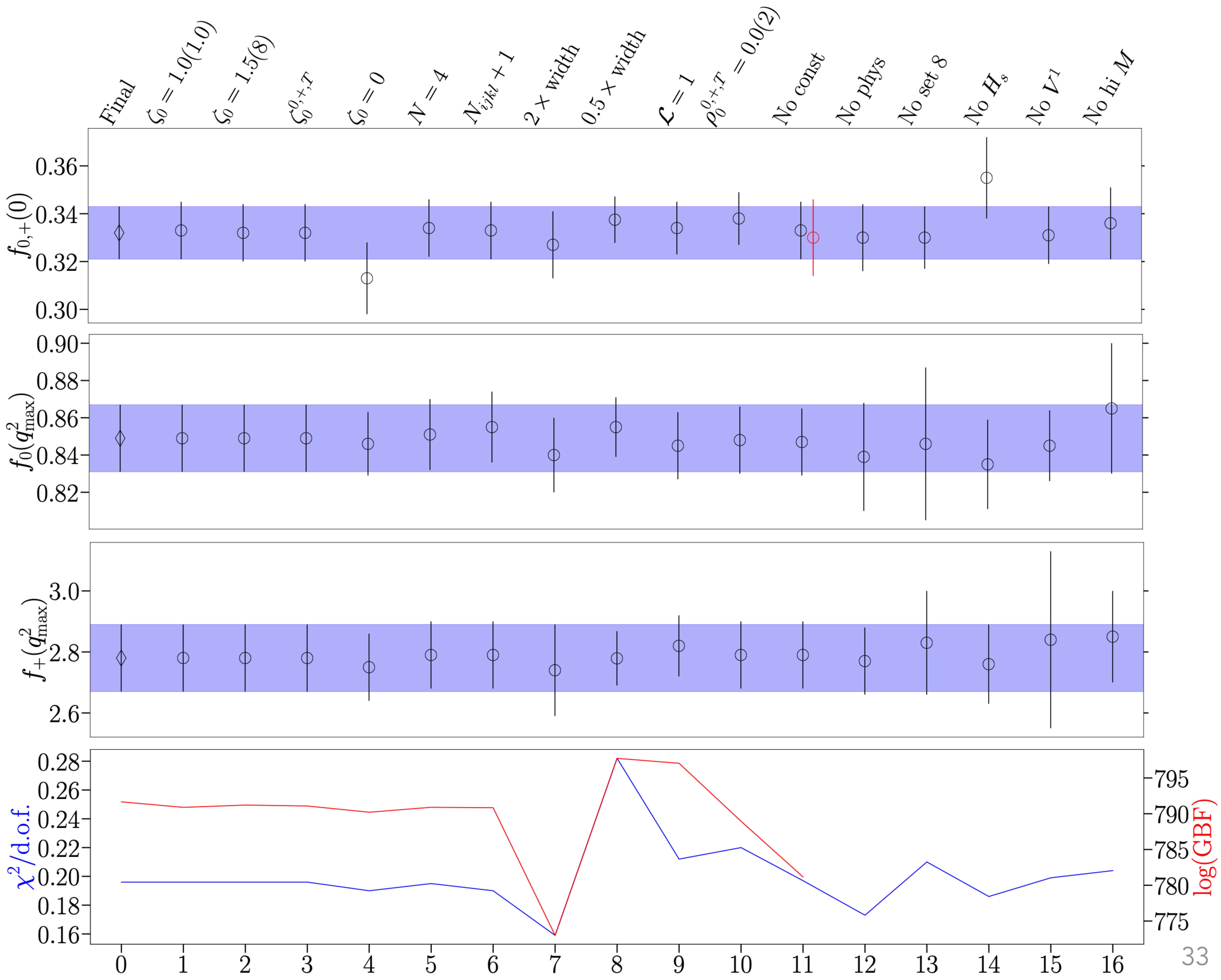
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}} \; ; \quad t_+ = (M_H + M_K)^2 \;, \quad \text{we choose} \quad t_0 = 0$$

$$f_{+,T}(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M_{H_s^*}^2} \sum_{n=0}^{N-1} a_n^{+,T}\left(z^n - \frac{n}{N}(-1)^{n-N} z^N\right) \;, \quad f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M_{H_s^0}^2} \sum_{n=0}^{N-1} a_n^0 z^n$$

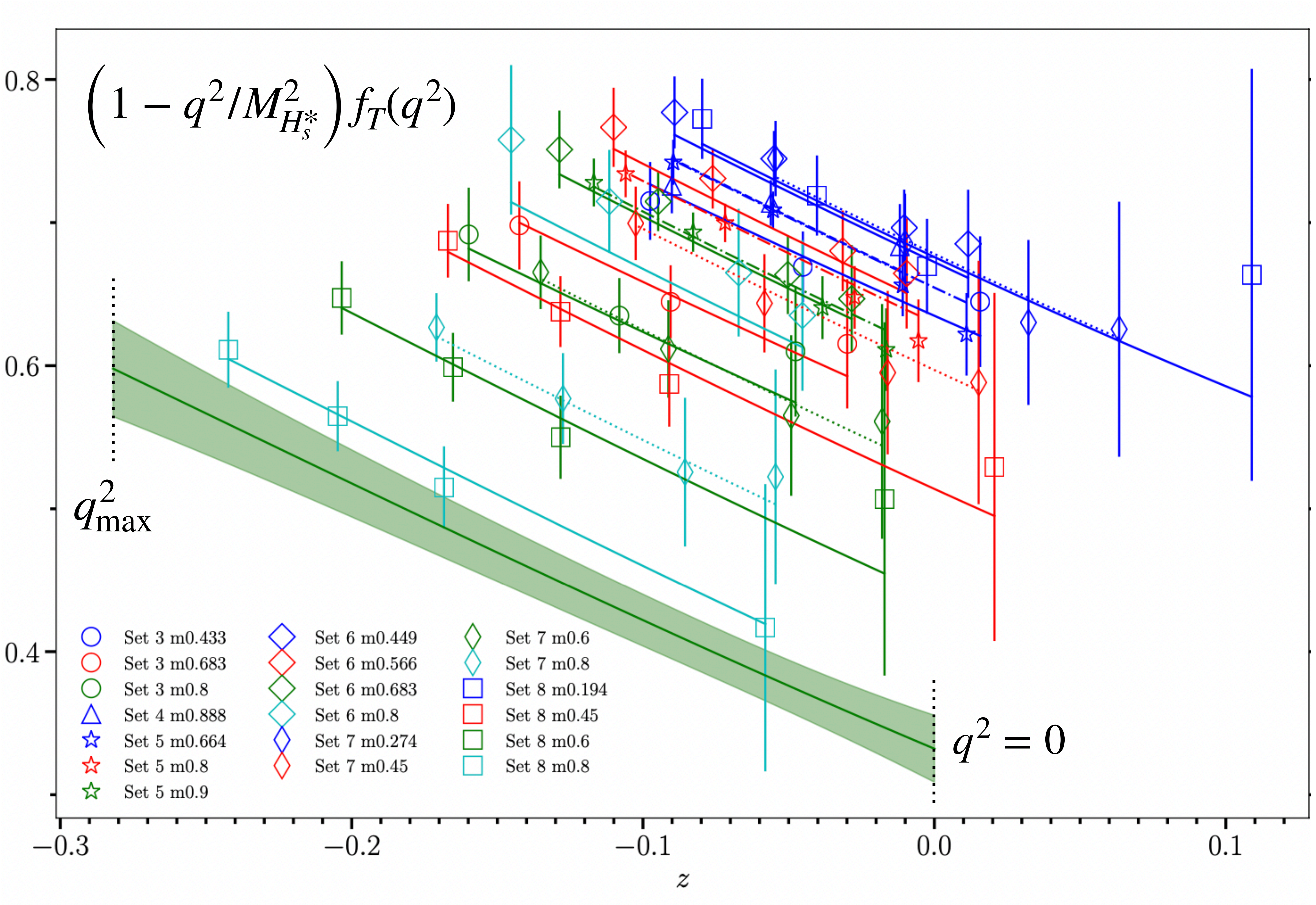
- $\mathscr{L}(V)$ are hard pion ChPT logs including (small) FV corrections Bijnens, Jemos, NPB 846, 145-166 (2011)
- a_n contains mistuning, heavy quark expansion, discretization, and analytic chiral terms

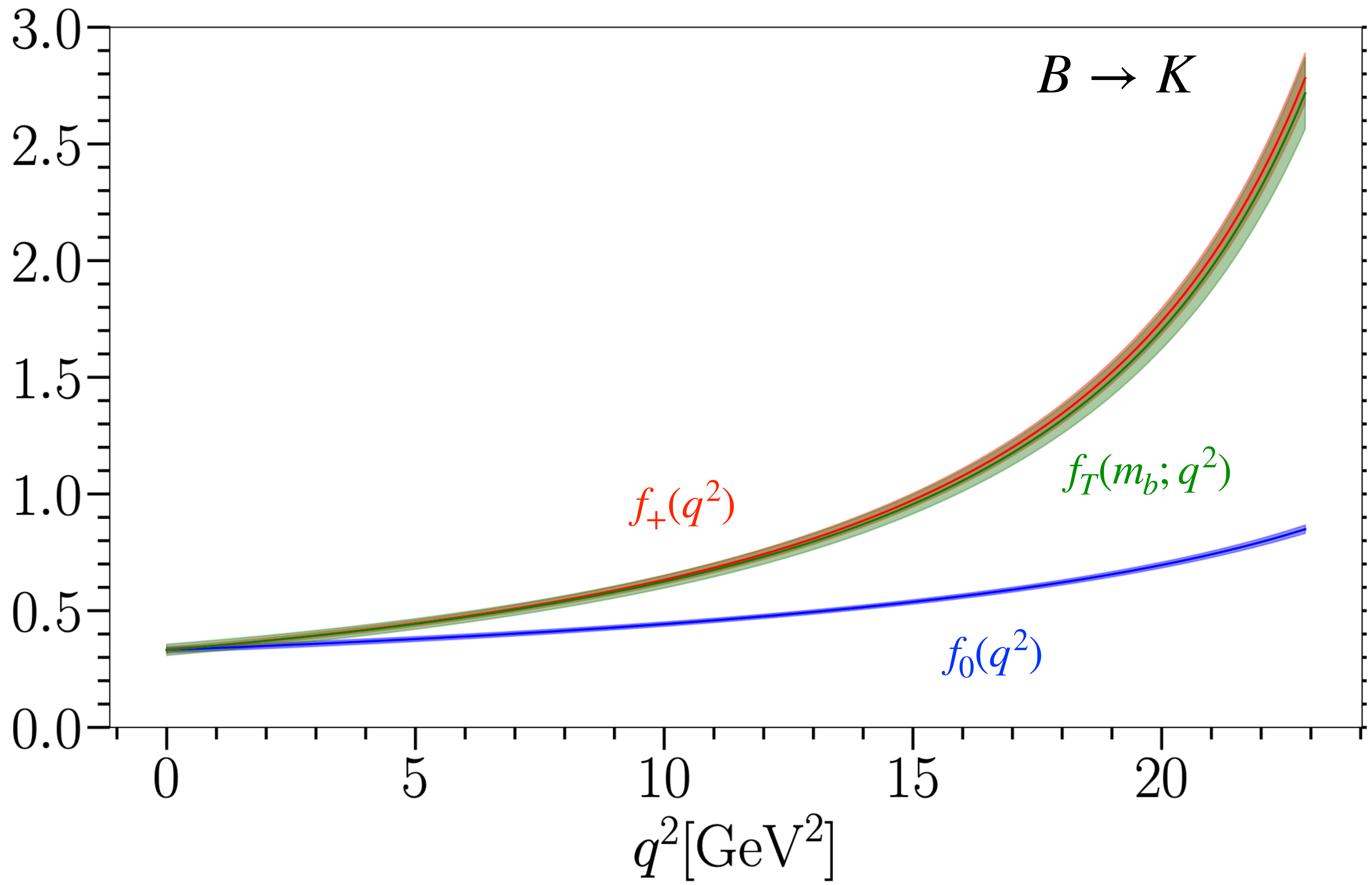
$$a_n^f = \left(1 + \mathcal{N}_n^f\right)\left(\frac{M_D}{M_H}\right)^{\zeta_n}\left(1 + \rho_n^f \log\left(\frac{M_H}{M_D}\right)\right) \sum_{i,j,k,l=0}^{N_{ijkl}-1} d_{ijkln}^f \left(\frac{\Lambda}{M_H}\right)^i \left(\frac{am_h}{\pi}\right)^{2j} \left(\frac{a\Lambda}{\pi}\right)^{2k} \left(\frac{m_\pi^2 - (m_\pi^{\text{phys}})^2}{(4\pi f_\pi)^2}\right)^l$$

Form Factors: modified z-expansion stability for $B \to K\ell\bar{\ell}$

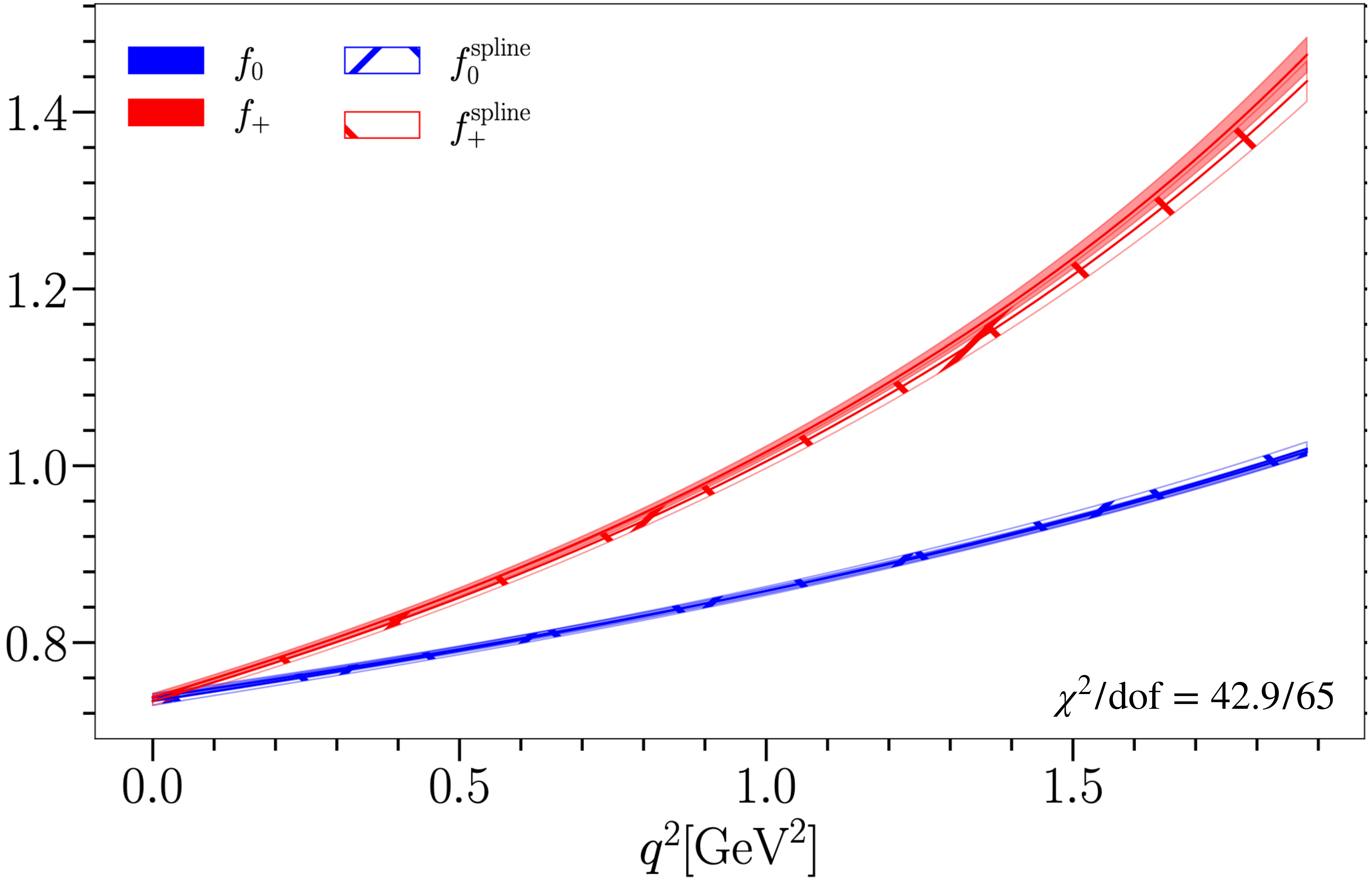


Form Factors: $B \to K\ell\bar{\ell}$ extrapolation results





Form Factors: $B \to K\ell\bar{\ell}$ test of modified z-expansion



- for $D \to K$, try cubic spline instead of modified z-expansion

$$f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M^2_{H_s^0}} \sum_{n=0}^{N-1} a_n^0 z^n$$

$$\downarrow$$

$$f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M^2_{H_s^0}} \left[\sum_{j=0}^{N} g_j(q^2) \left(\frac{am_c}{\pi} \right)^{2j} + \mathscr{N} \right]$$

- $g_j(q^2)$ are Steffen spline functions
- 4 knots {-3.25, -1.5, 0.25, 2.0} GeV^2

Form Factors: $B \to K\ell\bar{\ell}$ variation with m_h

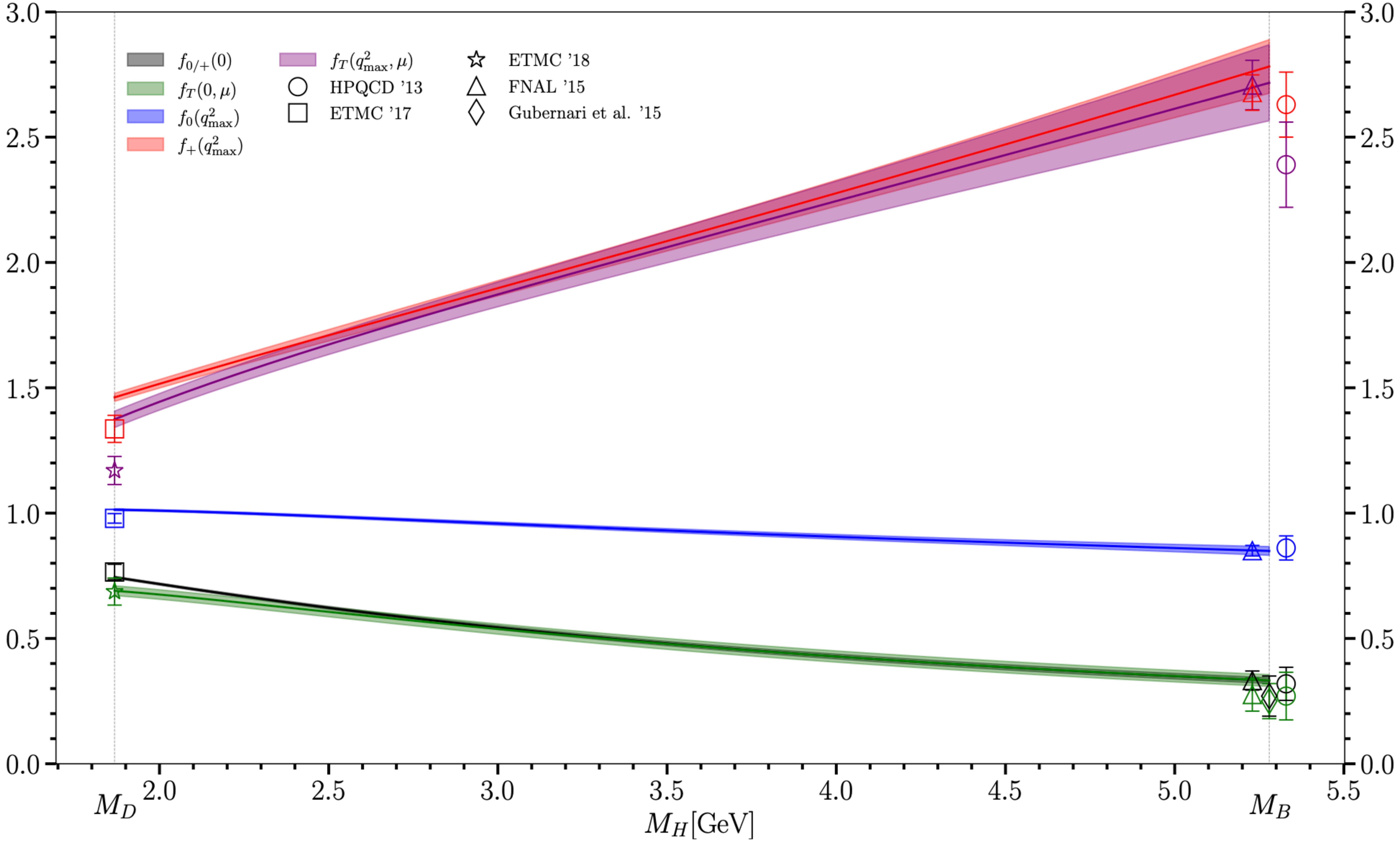
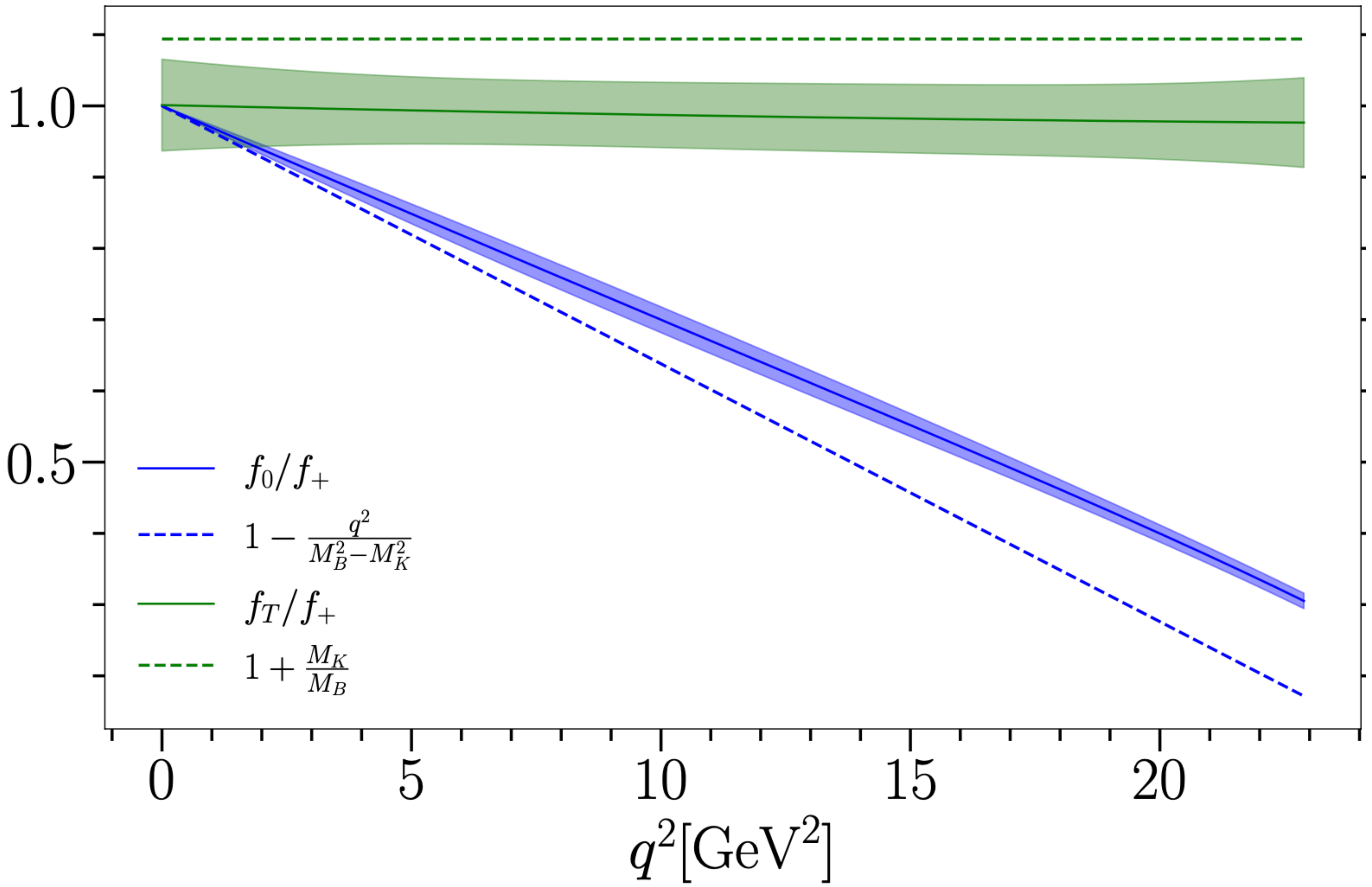


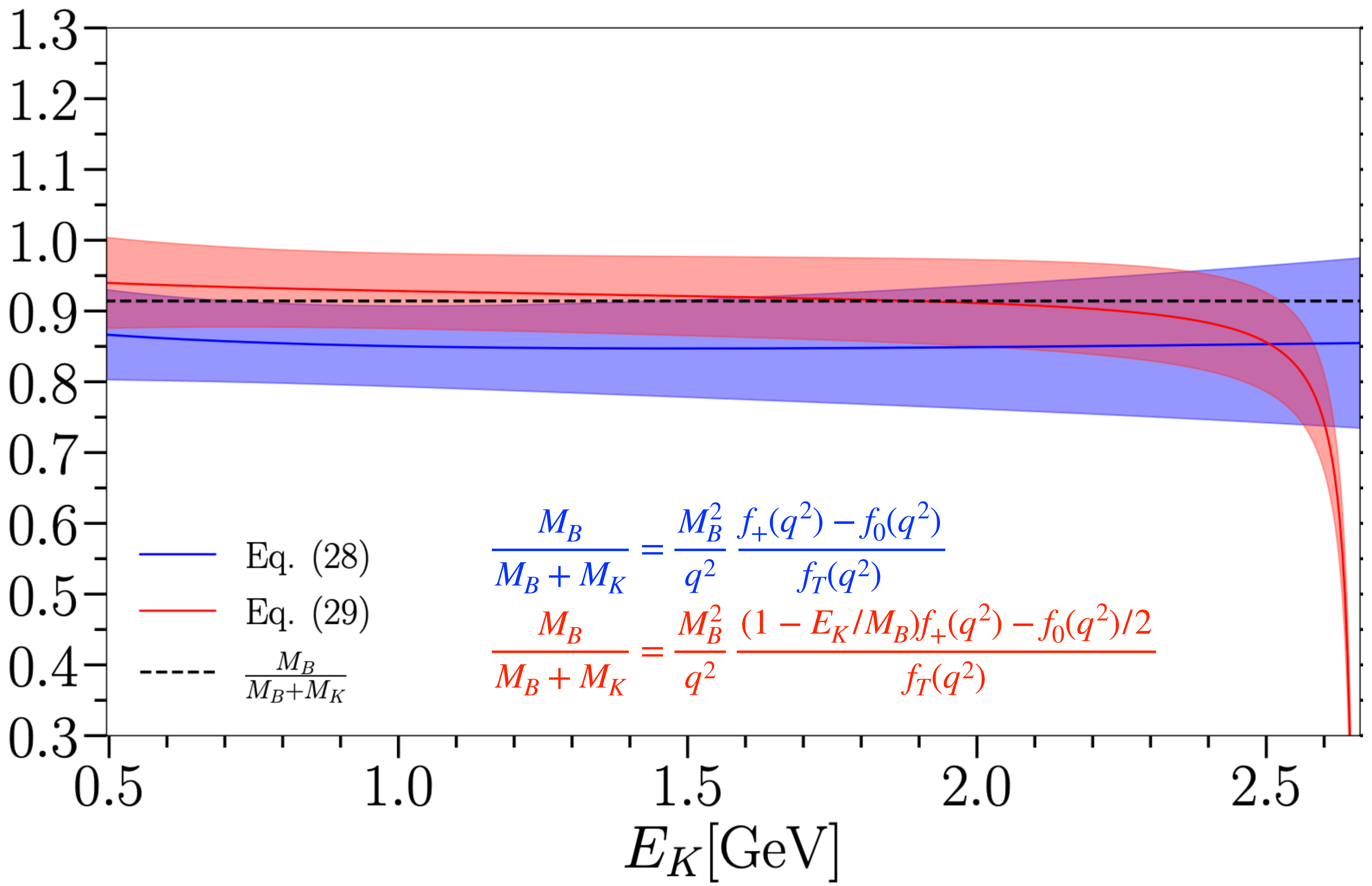
FIG. 10. The form factors at $q^2_{\max}$ and $q^2 = 0$ evaluated across the range of physical heavy masses from the D to the B. Other lattice studies [25, 28, 68, 69] of both $D \to K$ and $B \to K$ are shown for comparison. We also include some $B \to K$ results at $q^2 = 0$ from Gubernari et al. [70], a calculation using light cone sum rules. We do not include HPQCD's $D \to K$ results that share data with our calculation here [36]; see text for a discussion of that comparison. At the B end, data points are offset from M_B for clarity. Note that we have run Z_T to scale μ in this plot, where μ is defined linearly between 2 GeV and $m_b = 4.8$ GeV, according to Equation (26). The full running to 2 GeV from m_b results in a factor of 1.0773(17), applied to $f_T^{D\to K}$.

Form Factors: $B \to K\ell\bar{\ell}$ testing EFT expectation



Large Energy Effective Theory expectations

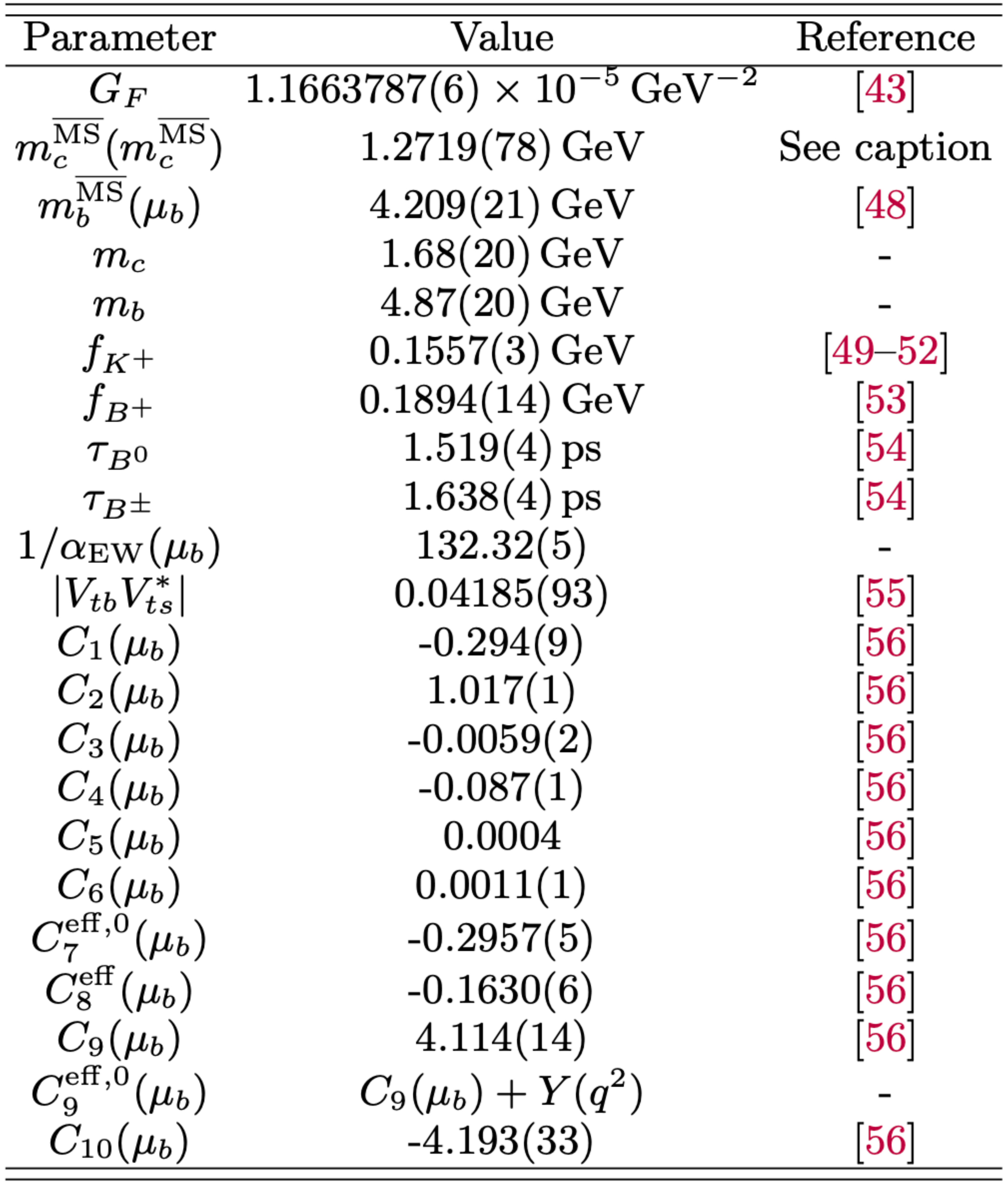
Charles, Le Yaouanc, Oliver, Pene, Raynal, PRD 60, 014001 (1999)



HQET expectations

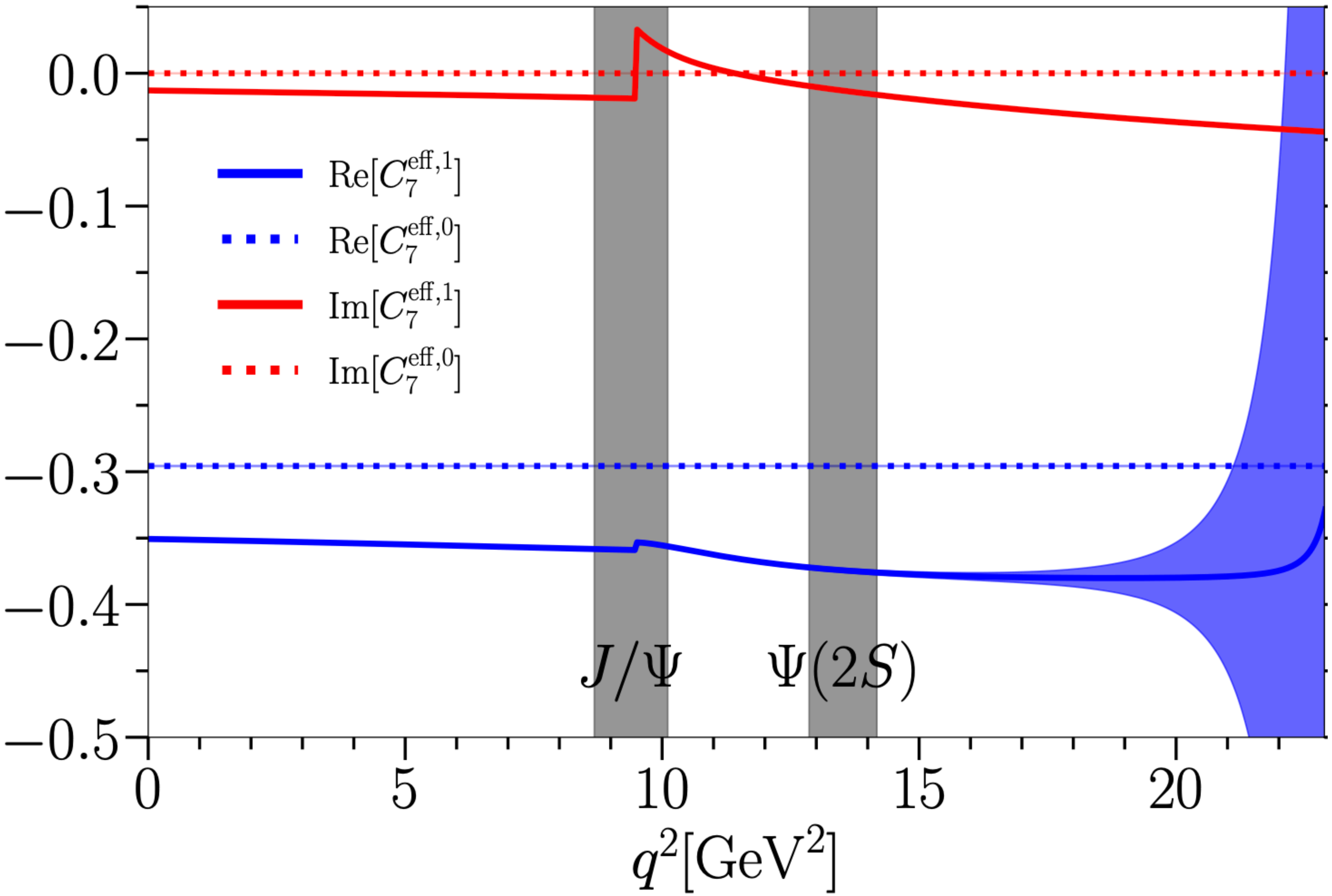
Hill, PRD 73, 014012 (2006)

Phenomenology: $B \to K\ell\bar{\ell}$ inputs



Parameter	Value	Reference
G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[43]
$m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}})$	1.2719(78) GeV	See caption
$m_b^{\overline{\text{MS}}}(\mu_b)$	4.209(21) GeV	[48]
m_c	1.68(20) GeV	-
m_b	4.87(20) GeV	-
f_{K^+}	0.1557(3) GeV	[49–52]
f_{B^+}	0.1894(14) GeV	[53]
τ_{B^0}	1.519(4) ps	[54]
$\tau_{B^\pm}$	1.638(4) ps	[54]
$1/\alpha_{\text{EW}}(\mu_b)$	132.32(5)	-
$\|V_{tb}V_{ts}^*\|$	0.04185(93)	[55]
$C_1(\mu_b)$	-0.294(9)	[56]
$C_2(\mu_b)$	1.017(1)	[56]
$C_3(\mu_b)$	-0.0059(2)	[56]
$C_4(\mu_b)$	-0.087(1)	[56]
$C_5(\mu_b)$	0.0004	[56]
$C_6(\mu_b)$	0.0011(1)	[56]
$C_7^{\text{eff},0}(\mu_b)$	-0.2957(5)	[56]
$C_8^{\text{eff}}(\mu_b)$	-0.1630(6)	[56]
$C_9(\mu_b)$	4.114(14)	[56]
$C_9^{\text{eff},0}(\mu_b)$	$C_9(\mu_b) + Y(q^2)$	-
$C_{10}(\mu_b)$	-4.193(33)	[56]

Phenomenology: $B \to K\ell\bar{\ell}$ corrections

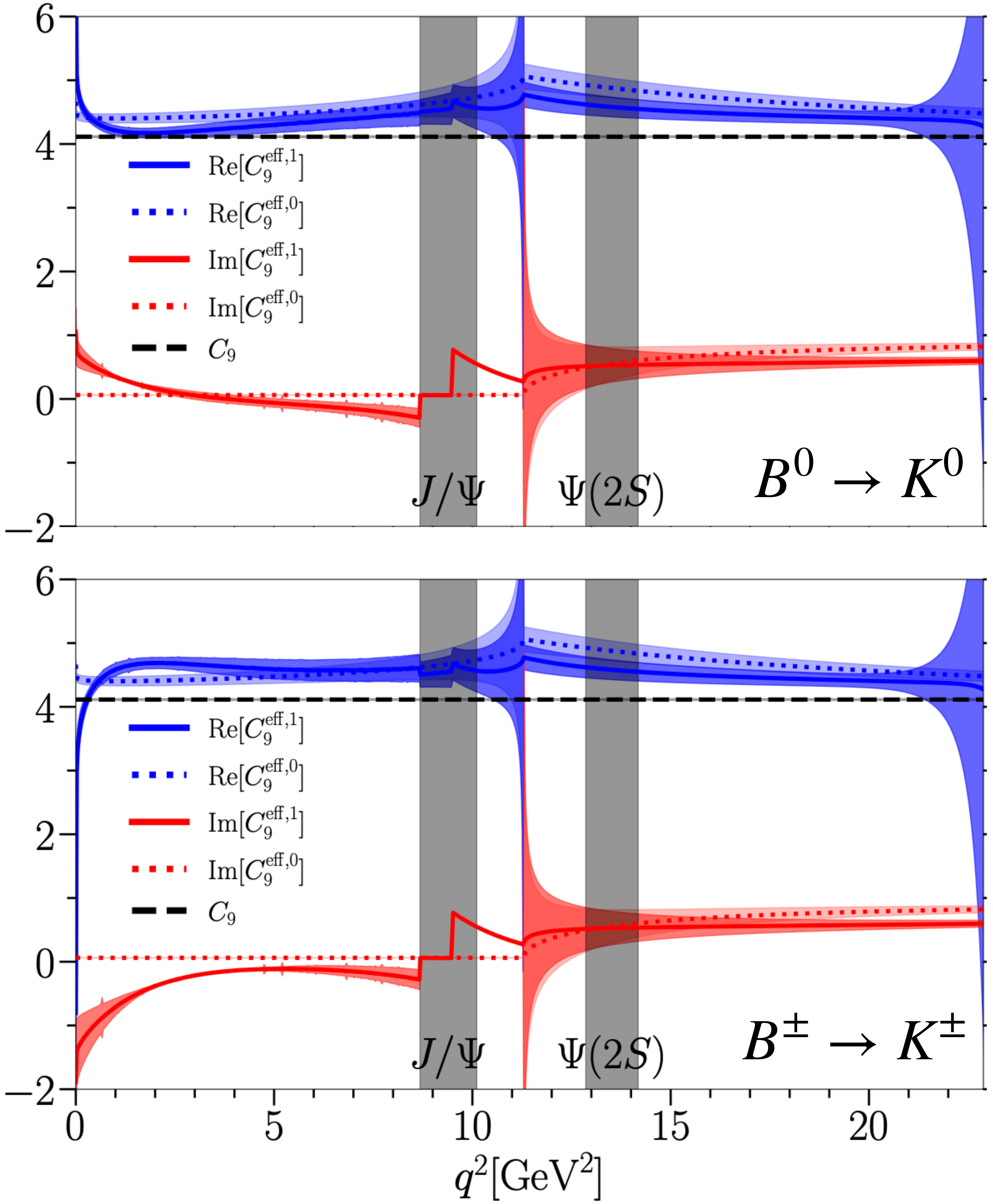


⋯⋯ uncorrected C_7^{eff}

═ corrected C_7^{eff}

Corrections to C_7^{eff} are $\mathcal{O}(\alpha_s)$ perturbative QCD effects for all q^2

Phenomenology: $B \to K\ell\bar{\ell}$ corrections



····· uncorrected C_9^{eff}

━━ corrected C_9^{eff}

corrections to C_9^{eff} include:

- $\mathcal{O}(\alpha_s)$ perturbative QCD effects for all q^2
- non-factorizable corrections at low q^2

Beneke, Feldmann, Seidel, NPB 612, 25-58 (2001)

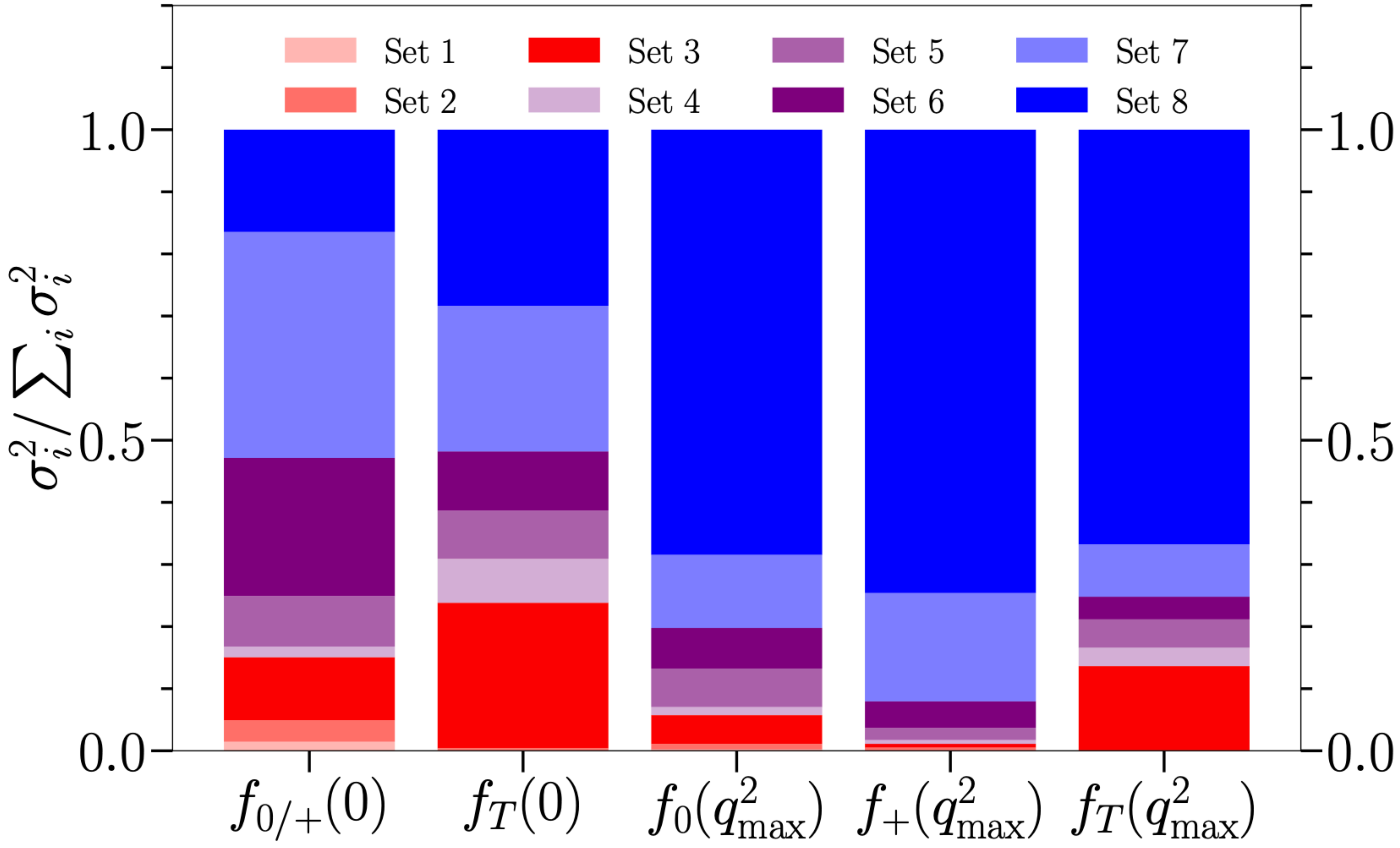
- would be interesting to compare non-factorizable corrections to results of data driven determination

talk by Andrea Mauri (LHCb), Wed@11:30

$B \to K$: extrapolation results

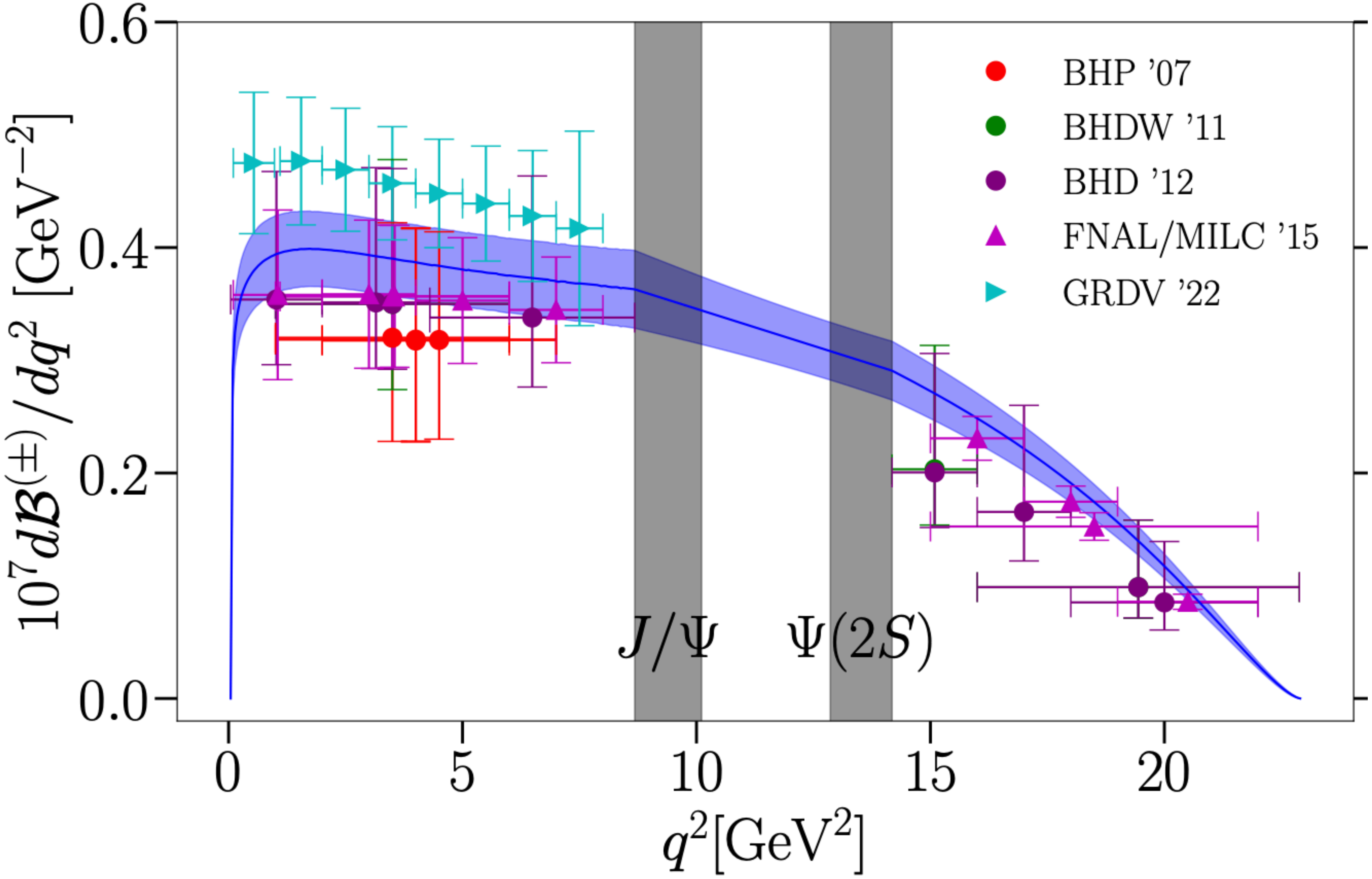
- bands show continuum, infinite volume, physical quark mass ($m_h = m_b$) form factors
- large f_+ errors at large q^2, when using V^0
  - using spatial component V^k fixes this *

$B \to K$: error budget by ensemble



- blue are lattices with finest lattice spacing, needed to reach m_b
- red are lattices with physical light quark mass

Phenomenology: $B \to K\ell\bar{\ell}$ vs other theory



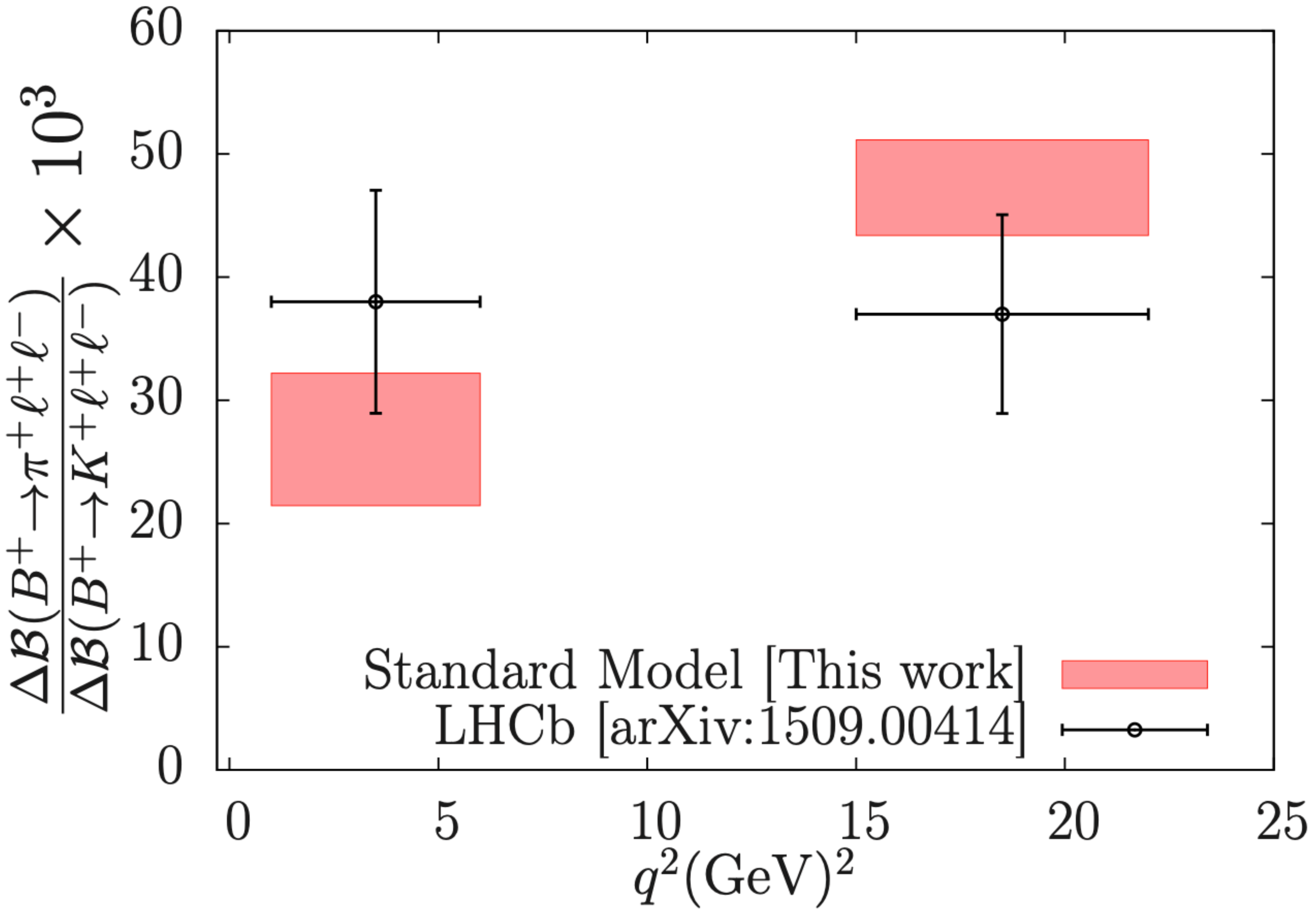
Bobeth, Hiller, and Piranishvili

Bobeth, Hiller, van Dyk, and Wacker

Bobeth, Hiller, and van Dyk

Gubernari, Reboud, van Dyk, and Virto

Phenomenology: $B \to \pi$ with $B \to K$



$B \to \pi$

(FNAL/MILC) Bailey et al., PRD 92 (2015) 014024
(FNAL/MILC) Bailey et al., PRL 115 (2015) 152002

$B \to K$

(FNAL/MILC) Bailey et al., PRD 93 (2016) 2, 025026

- FNAL/MILC combined phenomenological analysis on $B \to \pi, K$

(FNAL/MILC) Du et al., PRD 93 (2016) 3, 034005

- capitalises on correlations in lattice calculations
- both calculations (have been/are being) improved with heavy-HISQ