Hadronic cross sections measurement for the muon *g-2* calculation

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on behalf of the BABAR Collaboration

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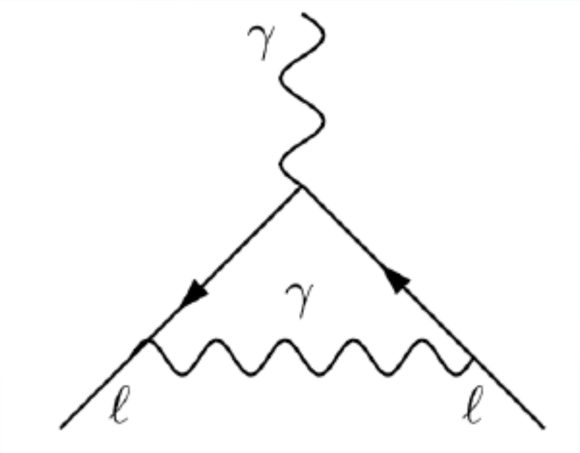
The anomalous magnetic moment of the leptons

$$\vec{\mu} = g\frac{e}{2m}\vec{s}, \qquad a = \frac{g-2}{2}$$

- **(1928) pointlike Dirac particles: g=2, a=0**
- (1948) **anomaly discovered for the electron:**
  - **a_e^{exp}= (1.19 ± 0.05) 10^{-3}** (Kusch-Foley)
- (1948) **explained by $O(\alpha)$ QED corrections**
  - **$a_e^{th} = \alpha/2\pi = 1.16\ 10^{-3}$ (Schwinger)**
- First triumph of QED!



a_l sensitive to quantum fluctuations, not only from QED.

==> Must include all contributions for a precise calculation

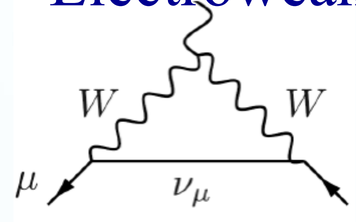
More quantum fluctuations

$$a^{Th} = a^{SM} + a^{NP} \,?$$

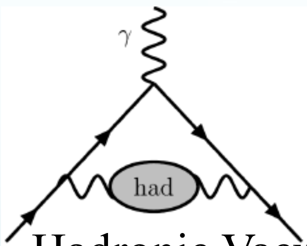
$$a^{SM} = a^{QED} + a^{had} + a^{EW}$$

QED

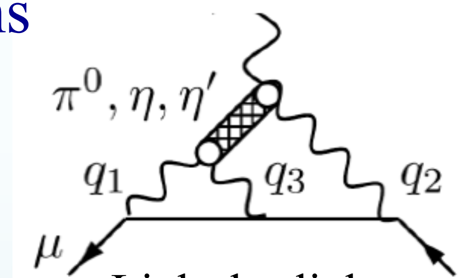
Electroweak



Hadrons

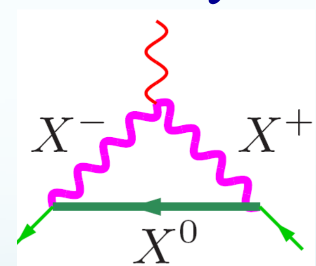


Hadronic Vacuum Polarisation



Light-by-light

New Physics



Main theoretical uncertainties from hadronic contributions

— HLBL: best present approach a combination of data and lattice calculations

— **Leading HVP (HVP-LO): estimated using experimental data on e^+e^- annihilations**

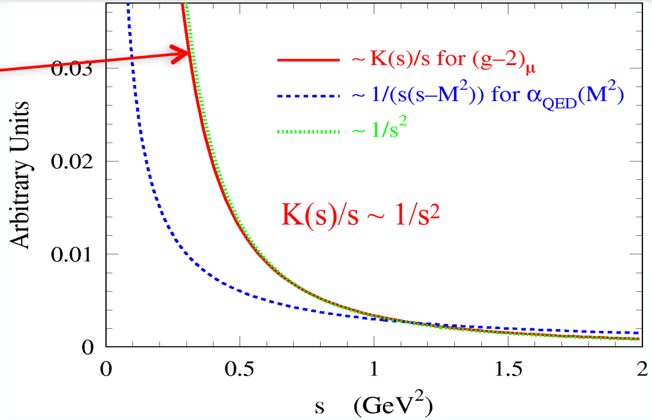
Leading Order HVP calculations

Can be calculated by using dispersion relations

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

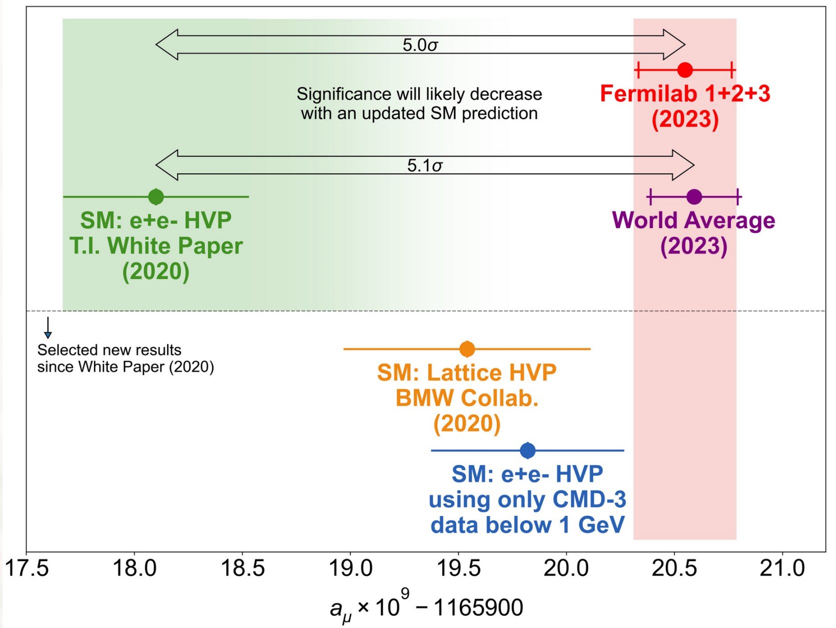
$$R(s) = \frac{\sigma^0 [e^+e^- \to \text{hadrons} (\gamma)]}{\sigma_{pt}}$$

σ^{had} => experimental input to a_μ^{had} calculation



- Main contribution to a_μ from very low energy region
  - $\sigma(e^+e^- \to \pi^+\pi^-)$ provides ~73% of the total
- Uncertainty from multi-hadronic channels significant
- Strategy to calculate the total cross section:
  - Sum up exclusive cross sections up to √s~2 GeV
  - Estimate the remaining (*e.g.* by using isospin relations)
  - Use inclusive cross sections measurements and/or pQCD above √s~2 GeV

Present status



Muon g-2 Theory Initiative White Paper:

T. Toyama *et al.*, Phys.Rep. 887 (2020) 1

$a_\mu^{SM} = (116\,591\,810 \pm 43) \cdot 10^{-11}$

Experiment (BNL + FNAL Run 1-3)

$a_\mu^{exp} = (116\,592\,059 \pm 22) \times 10^{-11}$ (0.19 ppm)

BNL: Phys. Rev. D. 73 (2006) 072003
FNAL (Run1): Phys. RevL ett. 126 (2021) 141801
FNAL (Run2+3): Phys.Rev.Lett. 131 (2023) 161802
BMW20 (lattice): Nature 593 (2021) 51
CMD-3: arXiv:2303.08834 (2023)

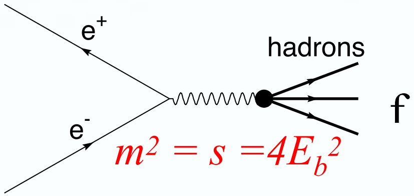
Theory-experiment deviation at 5 sigma! But…
After WP2020 publication:

- **First lattice calculation with small uncertainties (BMW Collaboration)**
- **Preliminary results on $e^+e^- \to \pi^+\pi^-$ by CMD-3 at VEPP2000**
- Both in tension with previous SM results from e^+e^- data

Hadronic cross sections measurement

How to measure σ_{had}

$e^+e^- \rightarrow hadrons$

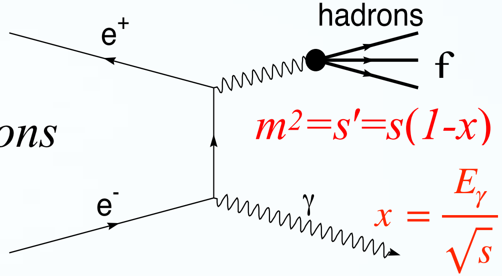


"Conventional" method: Energy Scan

- σ_{had} measured varying the beam energies within the accessible range ==> $\sqrt{s} = 2E_{beam}$
- Well-defined center-of-mass energy
- Low background
- Very good energy resolution
  - precise study of narrow resonances

- Systematics from point-to-point normalization
- Limited energy range covered
- $\varepsilon \rightarrow 0$ approaching processes thresholds (zero-momentum tracks)

$e^+e^- \rightarrow \gamma_{ISR} + hadrons$



"Novel method": Radiative Return

- Developed by KLOE and *BABAR*
- Effective c.m. energy: $\sqrt{s'} = \sqrt{s(1-x)}$
- Continuous and wide spectrum of energies $\sqrt{s'}$ below the nominal $\sqrt{s}$
- Uniform data quality all-over energy range
- Boost of hadronic system: $\varepsilon \neq 0$ at threshold

- Higher (and different) background sources
  - ➢ main backgrounds from different ISR processes and $e^+e^- \rightarrow q\bar{q}$ production
- Limited mass resolution (~ few MeV)

The players

Energy scan

SND and CMD-2 @VEPP-2M:
- $\sqrt{s} < 1.4$ GeV
- $\pi^+\pi^-$ and several multi-hadron channels

SND and CMD-3 @VEPP-2000:
- $0.32 < \sqrt{s} < 2.0$ GeV
- $L_{int} = > 1 fb^{-1}$; ~65 pb^{-1} in the ρ region
- $\pi^+\pi^-$ and many multi-hadron channels

KEDR @VEPP-4M:
- $2 < \sqrt{s} < 4$ GeV
- inclusive cross section

Older experiments:
- DM1, DM2, FENICE, BES, BESII,…

ISR

BABAR @PEP-II:
- $\sqrt{s}$ =10.6 GeV (Υ(4S) peak)
- L_{int} = 470 fb^{-1} ; 232 fb^{-1} used for $e^+e^- \to \pi^+\pi^-$
- Covered energy range: 0.3 - 4.5 GeV
  - >50 final states measured

KLOE @DAΦNE:
- $\sqrt{s}$ =1.02 GeV (ϕ(1020) peak)
- L_{int} = ~2 fb^{-1} ; 240 pb^{-1} used for $e^+e^- \to \pi^+\pi^-$
- Covered energy range: 0.3 - 0.9 GeV

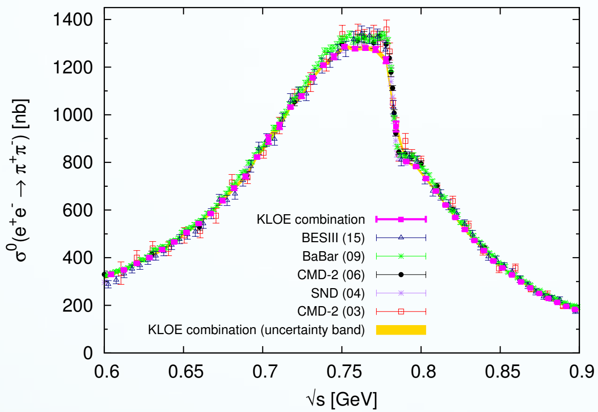
BES III @BEPC-II:
- $\sqrt{s}$ = 2 - 4.7 GeV
- main ISR results from data at ψ(3770)
- L_{int}[ψ(3770)] = ~20 fb^{-1} ; 2.93 fb^{-1} used for $e^+e^- \to \pi^+\pi^-$
- ISR: 0.6 - 0.9 GeV ($\pi^+\pi^-$)
- Scan: 2 - 3.8 GeV (spectroscopy and inclusive cross section)

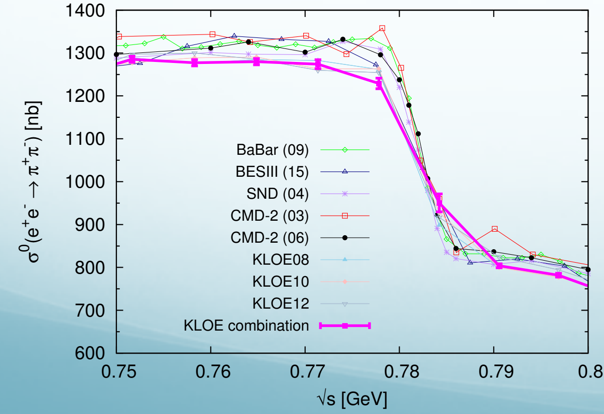
CLEO-c data
- $\sqrt{s}$ = 3.77 & 4.17GeV
- L_{int} = ~1.4 fb^{-1} used for $e^+e^- \to \pi^+\pi^-$

Belle II @SuperKEK-B
- $\sqrt{s}$ =10.6 GeV (Υ(4S) peak)
- ISR program in progress

$e^+e^- \to \pi^+\pi^-$

- Channel dominated by the huge "ρ" peak.
- Cross section must be measured at few per mil level in the ρ region
- Accuracy of all measurements limited by systematic uncertainties
- Several measurements with sub percent error available, but tension observed among some of them



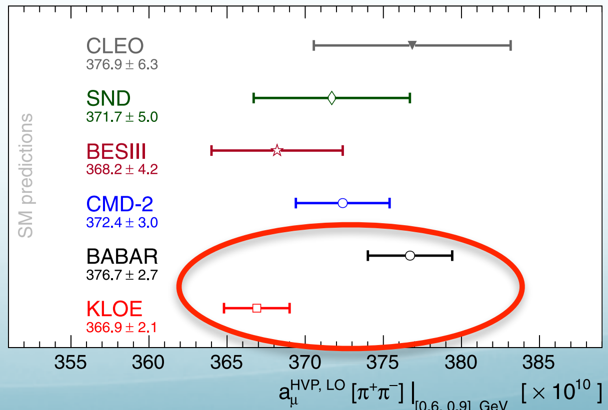


<u>Precision at the ρ peak:</u>

- *BABAR*: 0.5%
- KLOE: 0.7% (combination of 3 analyses)
- BESIII: 0.9%
- CLEO-c: 1.5%
- SND, CMD-2 (VEPP-2M): 1.3% and 0.8%, respectively
- SND, CMD-3 (VEPP-2000): 0.8% and 0.7%, respectively

not in WP 2020

$a_\mu^{\pi\pi}$ in the region $0.6 < \sqrt{s} < 0.9$ GeV

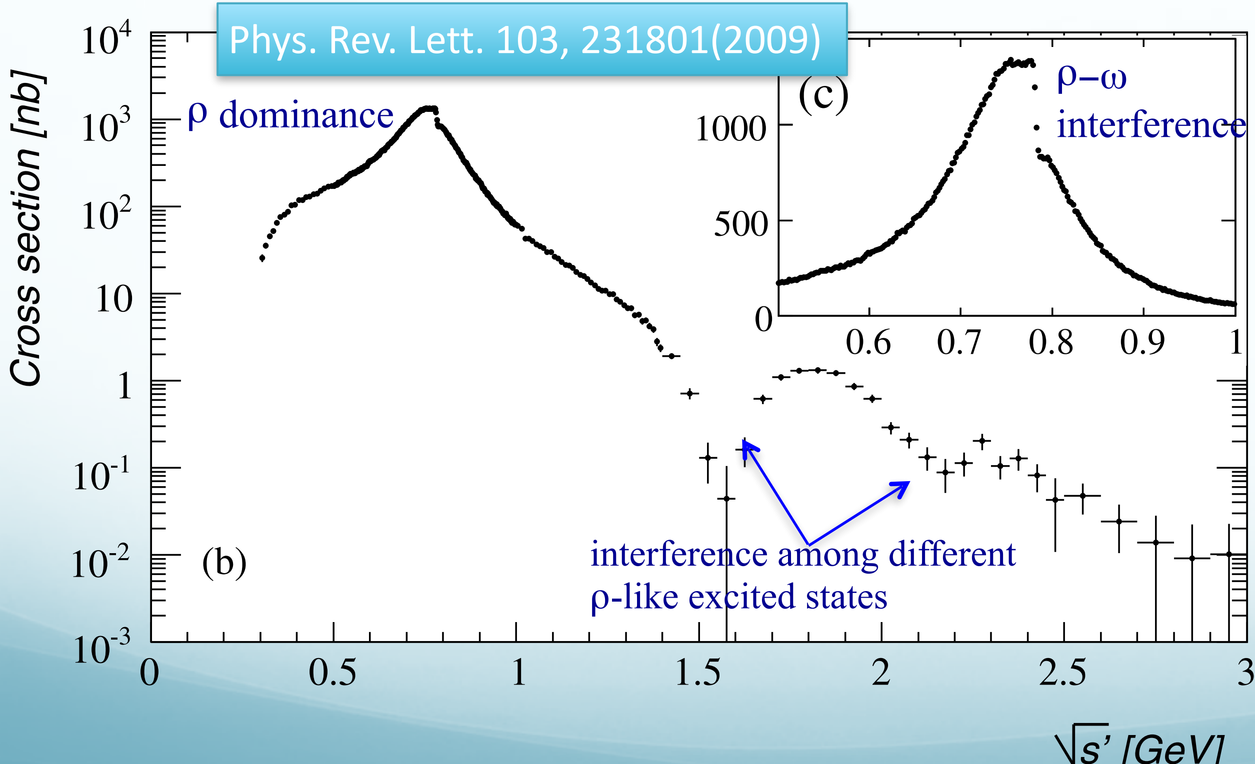


~2.7σ KLOE-BABAR tension

$e^+e^- \to \pi^+\pi^-$: *BABAR*

- Data taken at Υ(4S) peak. Tagged analysis (require $E_{\gamma ISR} > 3$ GeV).
- **Measure the ratio** $\frac{\sigma(e^+e^- \to \pi^+\pi^-(\gamma))}{\sigma(e^+e^- \to \mu^+\mu^-(\gamma))}$ ==> Results rather insensitive to details of MC generator
- Kinematic fit to data allows for one extra photon => measurement at NLO:
- Systematic uncertainty at the ρ peak at 0.5% level, dominated by Particle-ID effects

Phys. Rev. Lett. 103, 231801(2009)



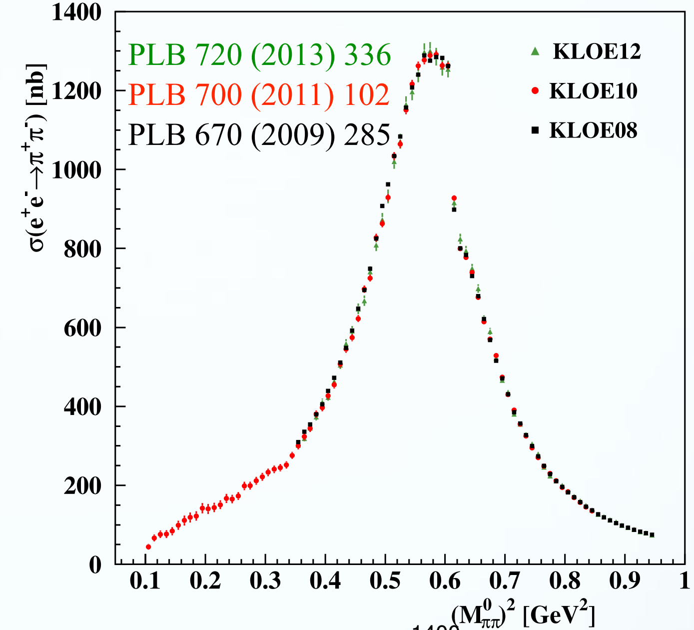
- Measurement based on half data set (232 fb^{-1}).
  - ~500M selected $\pi^+\pi^-$ pairs
- It is the only experiment to cover the cross section from threshold up to ~3 GeV
- New measurement with different analysis strategy in progress.

$e^+e^- \to \pi^+\pi^-$: KLOE

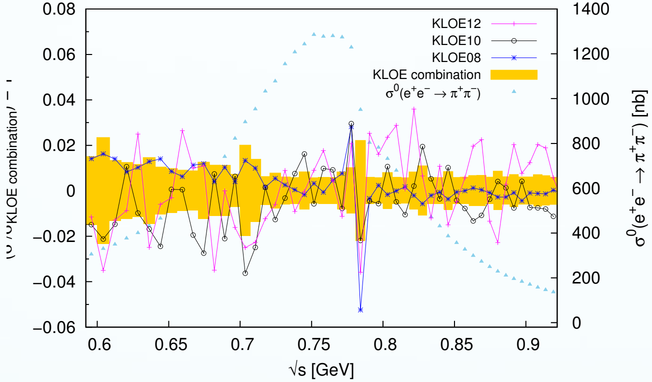
- Three KLOE results (data from 2005 publication discarded):

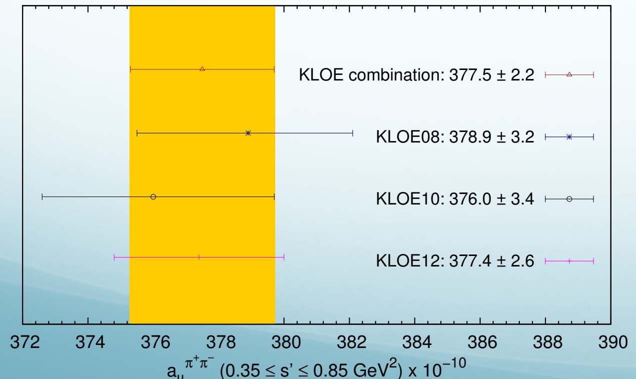
1. KLOE08: small angle ISR, normalisation from MC generator (Phokara)
2. KLOE10: large angle ISR (tagged photon), normalisation from MC
3. KLOE12: small angle ISR, normalisation from $e^+e^- \to \mu^+\mu^-$ pairs

- note that the selected $\pi^+\pi^-$ pairs in KLOE08 and KLOE12 are the same

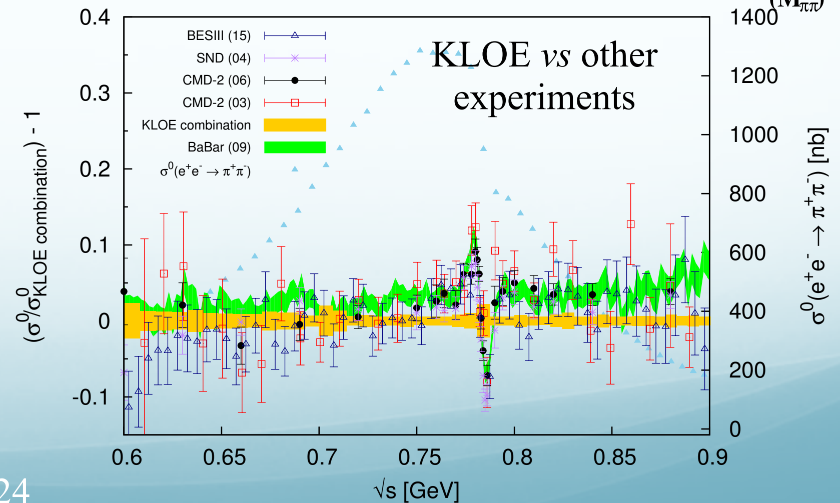


Cross sections comparison within KLOE





KLOE *vs* other experiments



CMD-3: new $e^+e^- \to \pi^+\pi^-$

arXiv:2302.08834 (2023)

Three VEPP-2000 data taking periods used:

- **RHO2013** mainly ρ-peak L = 17.8 pb^{-1}
- **RHO2018** mainly ρ-peak L = 45.4 pb^{-1}
- **LOW2020** √s < 0.6 GeV L = 1 pb^{-1}
- additional data taken after 2021 at √s > 1.2 GeV
- Significant accelerator and detector (new drift chamber) upgrade after RHO2013

$N_{\pi\pi}$ = 34 x 10^6

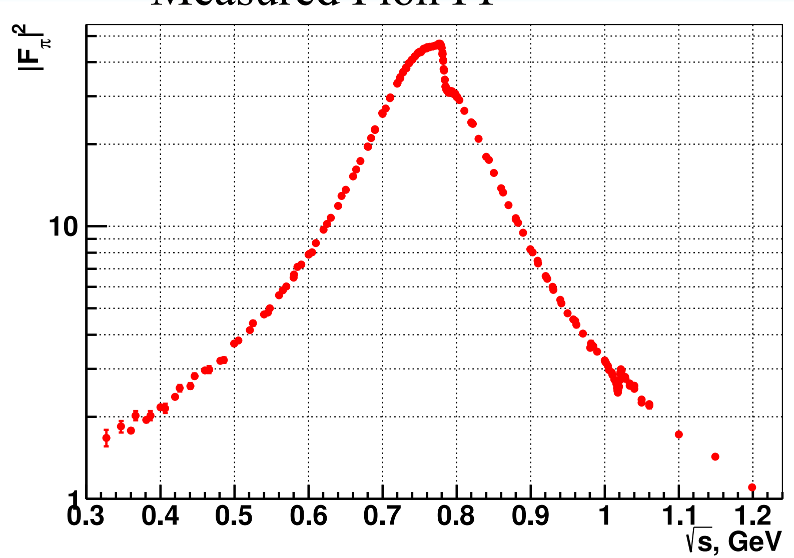
==> x30 CMD-2

Total uncertainties in the ρ-peak region:

- 0.7% (RHO2018) / 0.9% (RHO2013)

	uncertainty
Radiative corrections	0.3%
ee/μμ/ππ separation	0.2%
Fiducial volume	0.5% / 0.8%(2013)
Correlated inefficiency	0.1%
Trigger	0.05%
Beam energy	0.1%
Bremsstrahlung loss	0.05%
Pion specific loss	0.2% nuclear interaction 0.1% pion decay
	0.7% / 0.9%(2013)

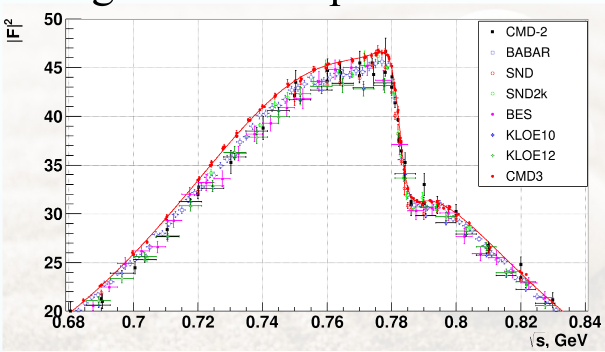
Measured Pion FF



CMD-3 $e^+e^- \to \pi^+\pi^-$: Results

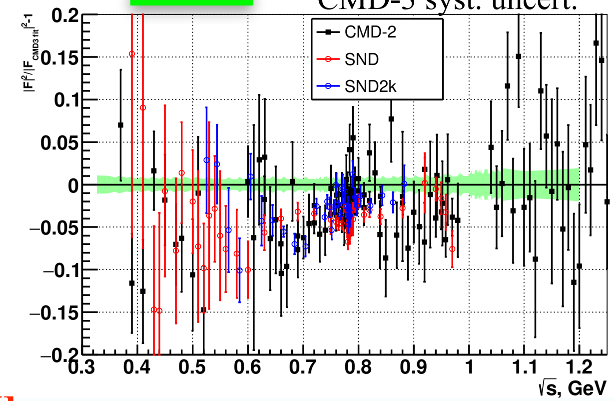
arXiv:2302.08834 (2023)

- $|F_\pi|^2$ (and $a_\mu^{\pi\pi}$) significantly higher than all previous results!



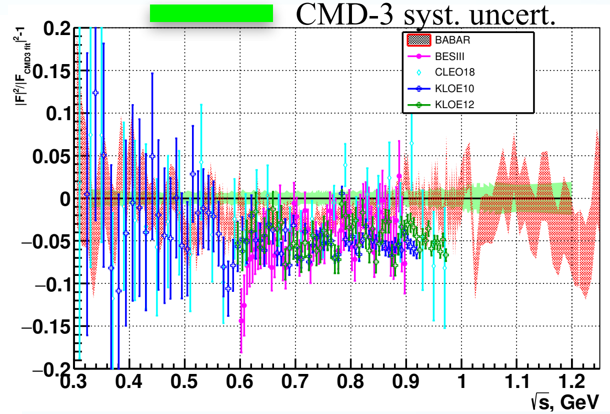
CMD-3 *vs* energy scan measurements

CMD-3 syst. uncert.

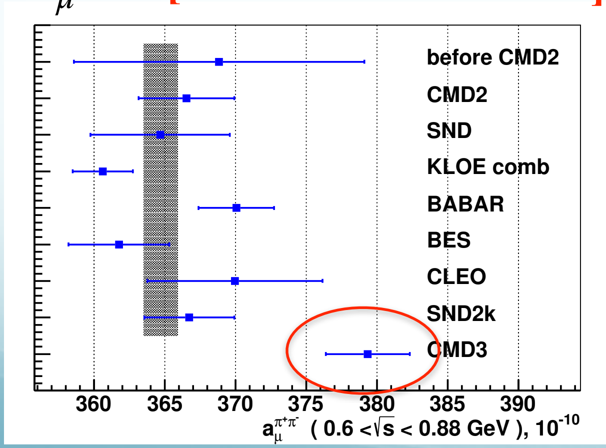


CMD-3 *vs* ISR measurements

CMD-3 syst. uncert.



$a_\mu^{\pi\pi,LO}$ [0.6 < √s < 0.88 GeV]



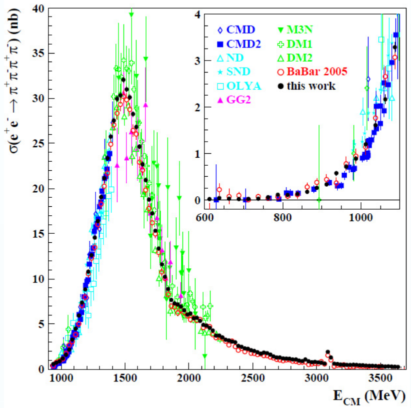
- **CMD-3:** $a_\mu^{\pi\pi,LO} = (379.35 \pm 0.30 \pm 2.95)10^{-10}$

- No clear smoking gun that would explain the difference
- **A new scan in the ρ region is started** with the aim to collect ~1 fb^{-1}
  - huge amount of data for better understanding of systematic effects

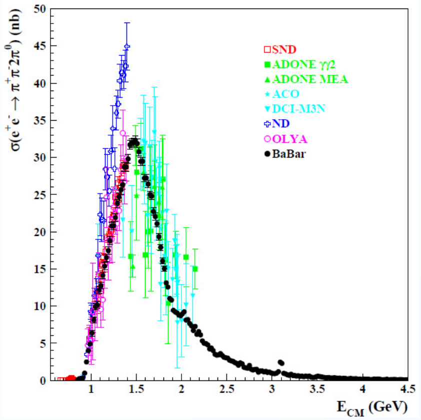
Exclusive cross sections other than $\pi^+\pi^-$

- Main contributions from **3π, 2K and 4π final states**
- ***BABAR***: almost complete set of measurements of final states with 2 to 7 hadrons (π, K and η's)
- VEPP-2M: √s < 1.4 GeV and relatively low statistics
- VEPP-2000: √s < 2 GeV with much higher luminosity. Many new results coming
- BES-III measured a few channels included in the WP2020. More results on the way

$e^+e^- \rightarrow 4\pi$



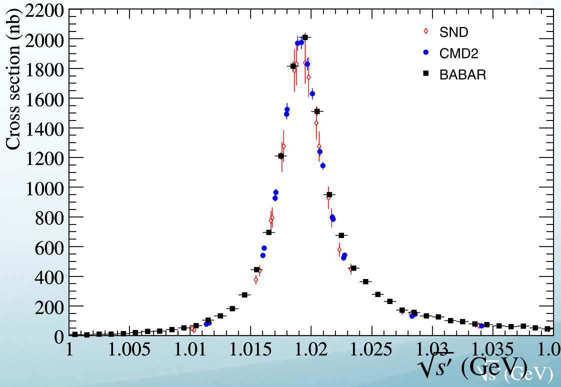
$e^+e^- \rightarrow 2\pi2\pi^0$



Contribution to $a_\mu^{HVP,LO}$ ($\sqrt{s}$ < 1.8 GeV) from DHMZ19 Phys. J. C 80 (2020) 410

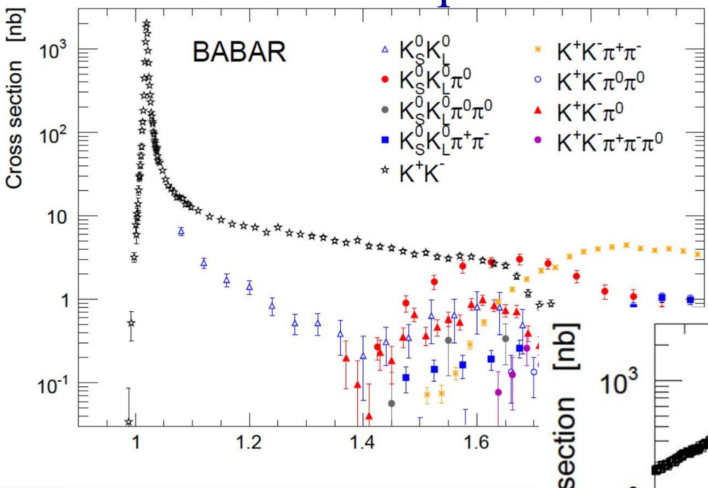
channel X	$a_\mu^X(10^{-10})$
$\pi^+\pi^-$	507.85 ± 3.38
$\pi^+\pi^-\pi^0$	46.21 ± **1.45**
$\pi^+\pi^-\pi^+\pi^-$	13.68 ± 0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03 ± 0.55
K^+K^-	23.08 ± 0.44
K_SK_L	13.02 ± 0.24

$e^+e^- \rightarrow K^+K^-$

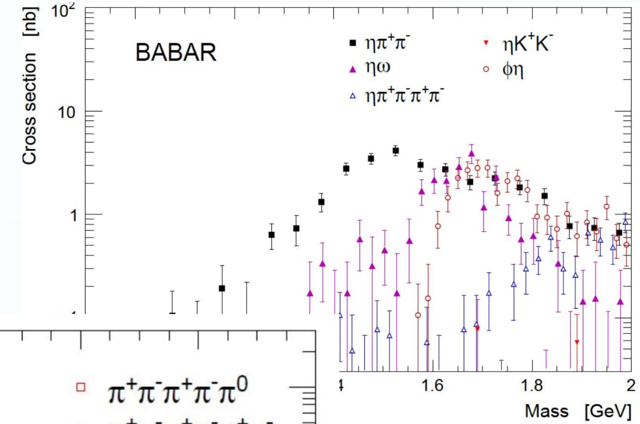


$e^+e^- \to$ hadrons: 'BABAR only' collection

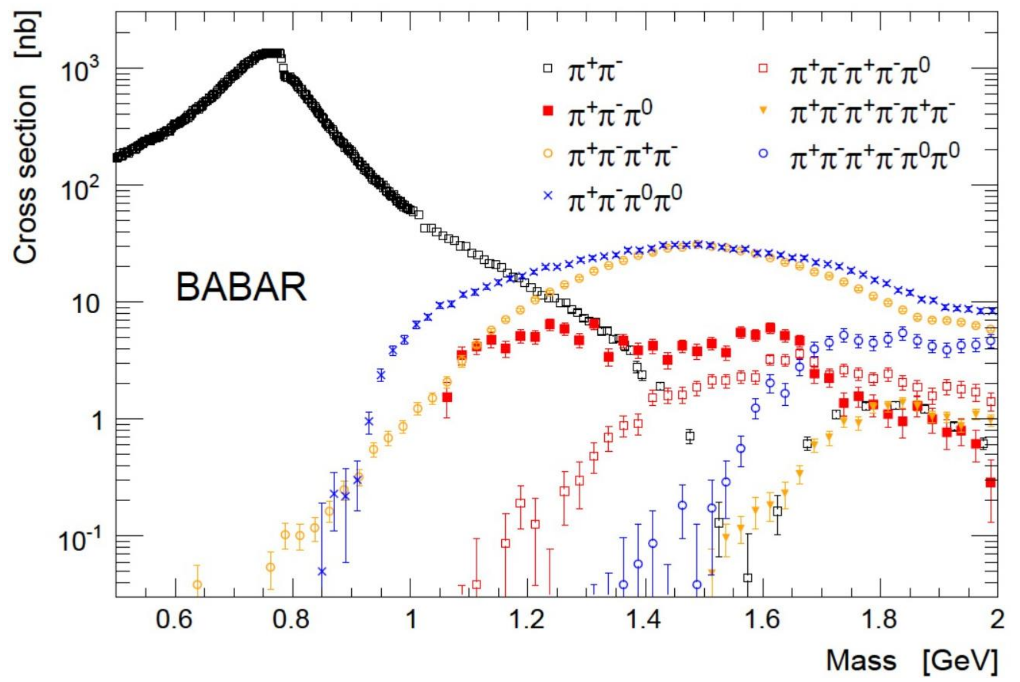
$e^+e^- \to 2K$ + pions



$e^+e^- \to \eta$ + pions



$e^+e^- \to$ pions



Recent $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ results

- Second largest contribution to $a_\mu^{had,LO}$ and its uncertainty
- Dominated at low energies by the ω and φ resonances, and then by ω recurrences
- Old data:
  - Precise SND and CMD-2 @VEPP-2M below 1.1 GeV
  - *BABAR* (80 fb^{-1}) above the φ

Five new analyses, not in the WP2020, recently produced:

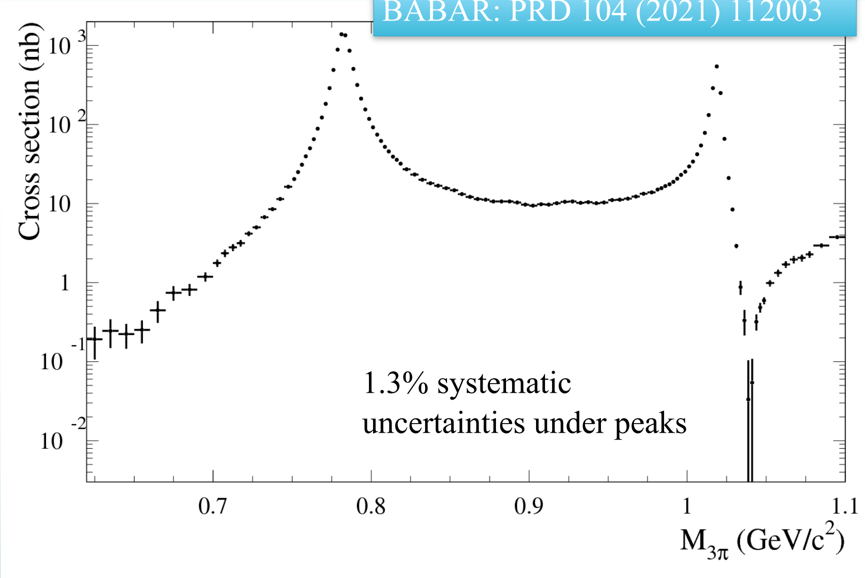
- ***BABAR***: ISR@Υ(4S) $0.62 < m_{3\pi} < 3.5$ GeV [PRD 104 (2021) 112003]
- **BESIII**: ISR@ψ(3770) $0.7 < m_{3\pi} < 3$ GeV [arXiv:1912.11208]
- **SND**: scan $1.2 < \sqrt{s} < 2$ GeV [EPJC80, 993 (2020)]
- **BESIII**: scan $2 < \sqrt{s} < 3.08$ GeV [arXiv2401.14711]
- **CMD-3**: scan $0.66 < \sqrt{s} < 0.97$ GeV [preliminary]

BABAR analysis ~469 fb^{-1}

- tagged ISR: 2.5 MeV bin-width at the resonances
- Syst. uncert. at the ω and ϕ peaks ~1.3%.
- Precision on $a_\mu^{3\pi}$ improved by a factor ~2 (for $m_{3\pi} < 2$ GeV):

$$a_\mu^{3\pi} = (45.86 \pm 0.14 \pm 0.58)\ 10^{-10}$$

BABAR: PRD 104 (2021) 112003



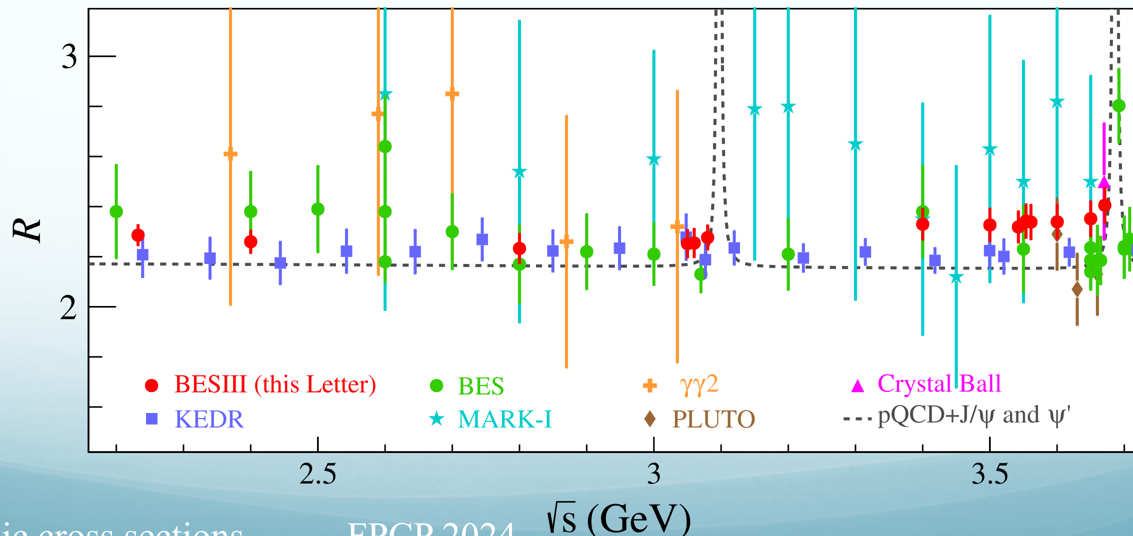
BESIII: Inclusive measurement

Inclusive cross sections above 2 GeV

- Previous results:
- KEDR
  - $1.84 < \sqrt{s} < 3.72$ [PLB 770 (2017) 174] [PLB 788 (2019) 42]
  - tot. uncert. 2.6% -> 4%
  - Agree well with pQCD in the region between 2 and 3.8 GeV

- New results from BESIII (not in the WP)
  - 14 points: $2.23 < \sqrt{s} < 3.67$ GeV
  - Precision slightly better than KEDR
    - dominated by systematic uncertainties
    - 2% -> 3% from 2.2 to 3.7 GeV
  - Measured cross section somewhat above KEDR (by 1.9σ) data and pQCD predictions (by 2.7σ)

BESIII: PRL 128, 062004 (2022)

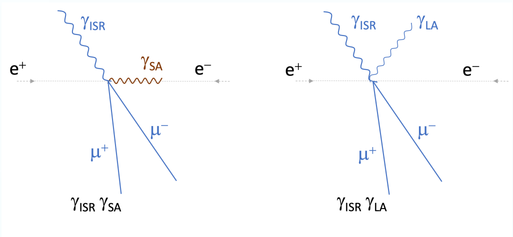


Measurement of additional radiation in ISR processes

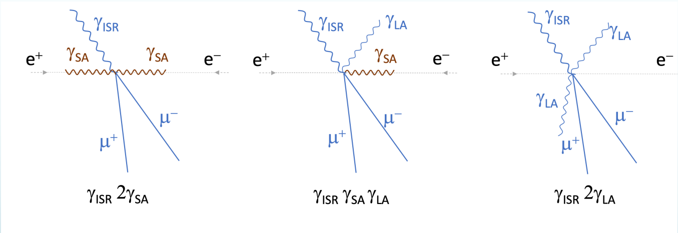
- Study of $e^+e^- \to \mu^+\mu^-\gamma_{ISR}$ and $e^+e^- \to \pi^+\pi^-\gamma_{ISR}$ with 1 (NLO) or 2 (NNLO) additional hard photons
- The full BaBar data sample (~468 fb^{-1}) is used. Data compared with MC generators:
  - **AfkQed** : up to NNLO additional ISR collinear to beams, FSR from PHOTOS
  - **Phokara 9.1**: full matrix elements at NLO (no NNLO)

- All events are subjected to several fits consistent with the event topology
- The fit with the best χ^2 determine the category of that event
- Small-angle (SA) γ's assumed collinear with the beams. Large-angle (LA) γ's detected

NLO event topologies



NNLO event topologies



Two NLO fits:

- $\gamma_{ISR}\gamma_{SA}$ ($E^*_{\gamma_{SA}} > 200$ MeV)
- $\gamma_{ISR}\gamma_{LA}$ ($E_{\gamma_{LA}} > 200$ MeV)

Three categories:

- NLO LA
- NLO SA
- LO: events with no γ's above threshold

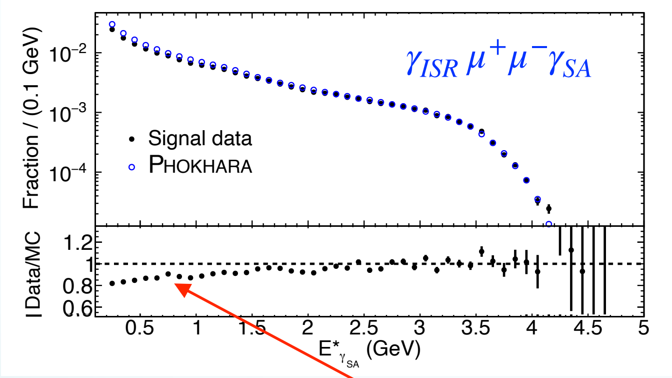
Three NNLO fits and three categories:

- $\gamma_{ISR}\gamma_{SA}\gamma_{SA}$ -> NNLO 2SA
- $\gamma_{ISR}\gamma_{SA}\gamma_{LA}$ -> NNLO SA+LA
- $\gamma_{ISR}\gamma_{LA}\gamma_{LA}$ -> NNLO 2LA

BABAR: Phys. Rev. D108 (2023) 111103

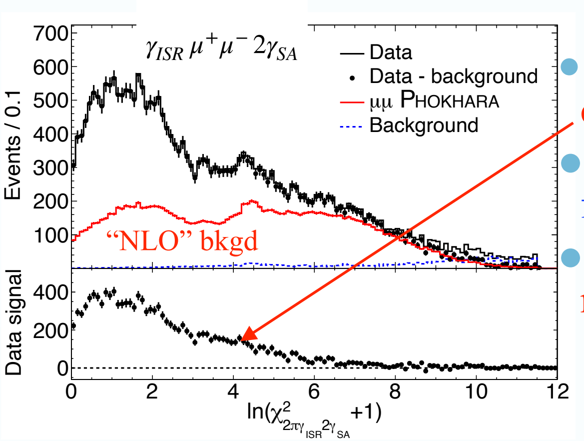
NLO fits results

- Similar results for $\mu\mu$ and $\pi\pi$ samples
  - $\pi\pi$ affected by larger background due to other multihadronic processes, largely suppressed by a BDT-based selection



- Phokara: clear energy-dependent excess w.r.t. data at low additional SA γ energy
- Overall Phokara excess is ~25%
- Good MC/data agreement for extra γ_{LA}

NNLO fits results



- Significant NNLO component observed
- Dominant contribution from 2SA process
- Overall ~3.5% both for muons and pions

Category	$\mu\mu$ $m_{\pi\pi} < 1.4\,\mathrm{GeV}/c^2$	$\pi\pi$ $0.6 < m_{\pi\pi} < 0.9\,\mathrm{GeV}/c^2$
LO	0.7716(4)(14)	0.7839(5)(12)
NLO SA-ISR	0.1469(3)(36)	0.1401(2)(16)
NLO LA-ISR	0.0340(2)(9)	0.0338(2)(9)
NLO ISR	0.1809(4)(35)	0.1739(3)(20)
NLO FSR	0.0137(2)(7)	0.0100(1)(16)
NNLO ISR [a]	0.0309(2)(38)	0.0310(2)(39)
NNLO FSR [b]	0.00275(6)(9)	0.00194(12)(50)
NNLO 2LA [c]	0.00103(3)(1)	0.00066(4)(4)

[a] NNLO ISR = 2SA-ISR or SA-ISR + LA-ISR
[b] NNLO FSR = SA-ISR + LA-FSR
[c] NNLO 2LA = 2LA-ISR, LA-ISR + LA-FSR or 2LA-FSR

- AfkQed reproduces well the additional SA photons energy distributions

Consequences for the $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ measurements

- ***BABAR*** analyses essentially unaffected:
  - loose selection, ππ/μμ ratio, efficiencies from data => no particular dependence from any MC
  - Phokara used for acceptance corrections => Estimated effect (0.03±0.01)% well below the quoted syst. uncertainty of ~0.5%
  - NLO already included in 2009 analysis (one extra photon, SA or LA) allowed
  - New analysis in progress will allow for two extra photons (NNLO)

- Other ISR results (**KLOE**, **BESIII**), relying on Phokara, might be affected by:
  - Missing NNLO and higher order contributions
  - Too large MC SA hard-photon rate
  - Recent DHMZ study with fast simulation to estimate such effects [arXiv:2312.02053]
  - A precise estimate of such effects could/should be performed by the experiment themselves

What's in the future

- The $\pi\pi$ channel is still the major source of uncertainty
  - CMD-3 result increases the tension
  - Progress in the theoretical description of radiative corrections is necessary to reach few per mil precision
- Impact of other final states becoming very important.
  - Goal: measure the main channels at 1-3% total uncertainty
  - Sophisticated amplitude analyses needed in most cases
- KEDR @VEPP-4M, BESIII and BelleII may provide new inclusive measurement at $\sqrt{s} > 2$ GeV

SND & CMD-3

$1 fb^{-1}$ collected above 1 GeV ==> many analysis in progress on multi hadron production

$1 fb^{-1}$ planned in the ρ region ==> Future $\pi\pi$ analyses with much improved control of systematic effects

KLOE

New $\pi\pi$ analysis on full data set in progress => ~x10 data sample

BESIII

20 fb^{-1} total collected data at √s =3770 MeV (~x6 previous analyses).
Expected improved ISR analysis on many final states

BABAR

New $\pi\pi$ analysis on full data set, with different technique, in progress => Effectively ~x7 data sample

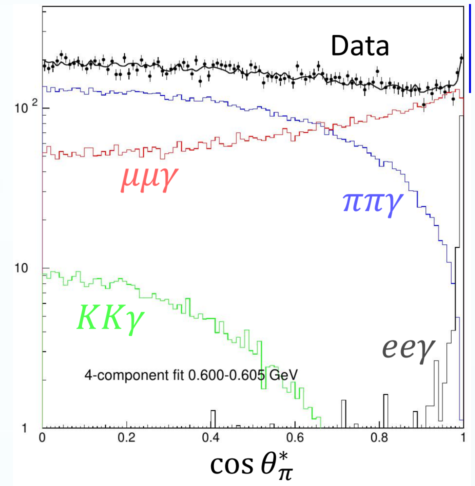
Belle II

ISR program, analogous to BABAR, already started

New *BABAR* $e^+e^- \to \gamma_{ISR}\,\pi^+\pi^-(\gamma\gamma)$

Published analysis (2009)

- Run 1-4 (232 fb^{-1})
- Main systematics from Particle Identification (PID) related effects
- Track momentum selection: p>1 GeV/*c*
- Total uncert. [0.6 — 0.9 GeV] = 0.5%

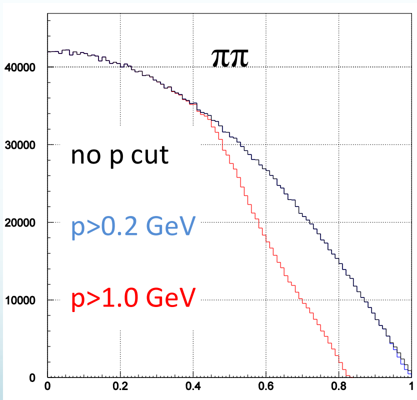


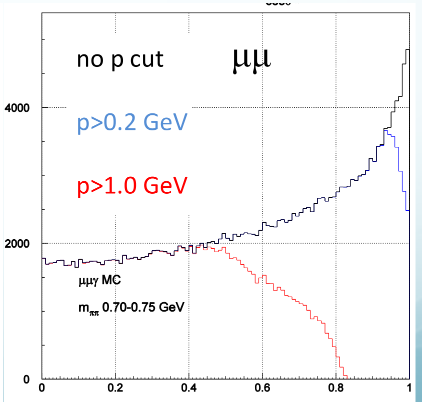
New analysis (in progress)

- Run 1-6 (469 fb^{-1})
- New muon detector in additional data
- No PID selection and no momentum cut (detector limit p_T>0.1 GeV/*c*
- π/μ/K separation from angular distribution

==> effectively x7 statistics increase

- Up to two extra photon admitted in the fit ==> NNLO measurement

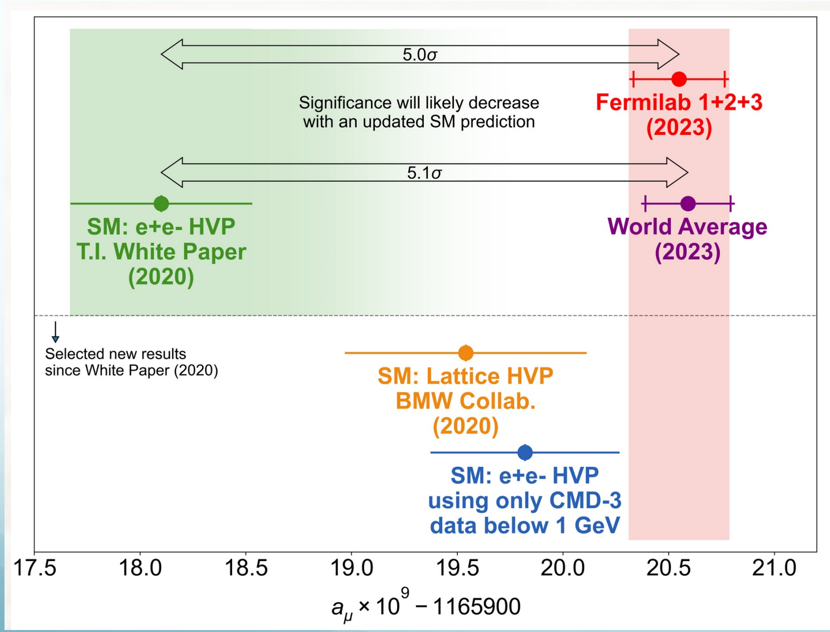




- Challenge: understand data/MC differences for tracking&trigger efficiencies below 1 GeV

Conclusions

- The muon *g*-2 is a long standing and among the most solid and significant discrepancies between experiment and SM theory
- Impressive recent improvements in every direction:
  - high quality e^+e^- data from new experiments/analyses
  - new direct measurement at Fermilab $(g\text{-}2)_\mu$ experiment
  - accuracy of lattice calculations becoming competitive



But the puzzle now is not at all simpler w.r.t. WP!!

Many open questions:

- KLOE vs BABAR vs CMD-3
  Where the differences come from?
- Lattice *vs e+e-* data?
- Will the final FNAL and JPARC results confirm the current central value?

BACKUP slides

Why studying e^+e^- annihilation

- All had begin with ADA in Frascati….

Bernardini,Corazza,Ghigo,Touschek
Il Nuovo Cimento 18, 1293 (1960)

- Many more e^+e^- collider since ADA:
  - c.m. energy up to 200 GeV (LEP-2)
  - Luminosity above 10^{34} cm^{-2}s^{-1} (*B*-factories) and 10^{35} cm^{-2}s^{-1} (SuperKEKB)

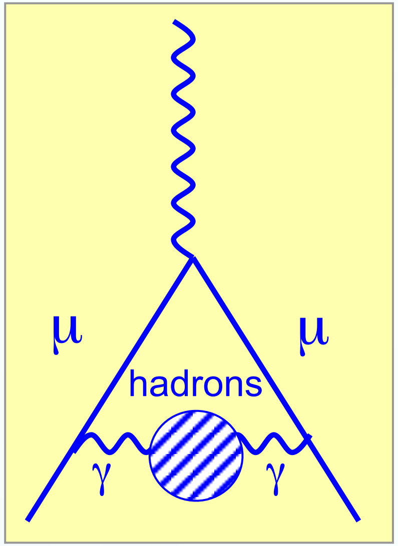
e^+e^- annihilation at low energies is a very effective tool to study the structure of the hadrons

- initial state well defined (momentum and quantum numbers)
- the final state can be fully reconstructed
- in general very simple event topology

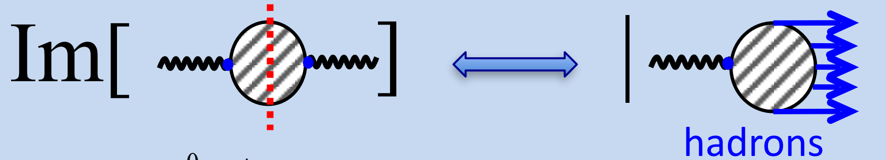
Perturbative QCD approaches do not apply at low energies

- QCD-based models need experimental data as input
- Nature of many observed exotic states not yet understood
- Precise knowledge of **Hadronic Vacuum Polarization (HVP)** needed for the calculation of:
  - $\alpha_{QED}(M_Z^2)$: running fine structure constant at the Z^0 mass
  - **$(g\text{-}2)_\mu$: anomalous magnetic moment of the muon**

HVP calculations



- Quark loops not computable from QCD (low mass scale)
- Can use dispersion relations, with analyticity and optical theorem to relate the vertex corrections to the $e^+e^- \to$ *hadrons* cross section

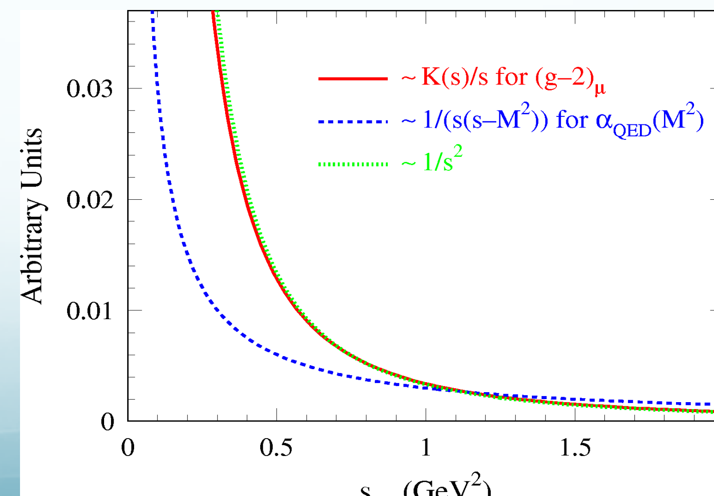


$$12\pi\ \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0[e^+e^- \to \mathrm{hadrons}\,(\gamma)]}{\sigma_{pt}} \equiv R(s)\ ; \quad \sigma^0(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$$

Dispersion integral

$$a_\mu^{HVP,LO} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty} ds \frac{K(s)}{s} R(s)$$

$K(s)/s \sim 1/s^2$ emphasizes the role of the processes at low energies



Muon *g*-2 Theory Initiative and data combination

Data combination for calculation of a_μ^{HVP-LO} is quite a delicate procedure:

- >300 data sets from >50 channels by dozens of experiments

Various approaches used to perform the data combination and integration. Among them:

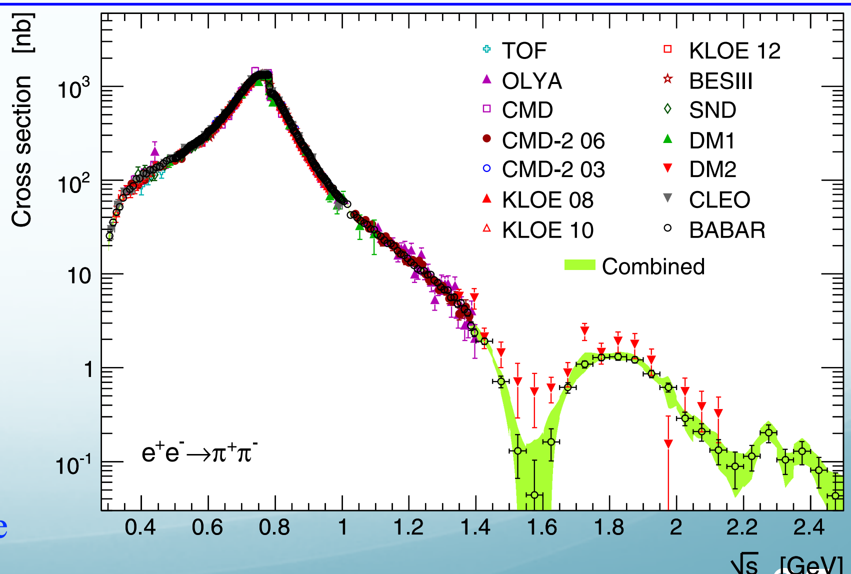
- DHMZ, Eur. Phys. J. C 80 (3) (2020) 241
- KNT, Phys. Rev. D 101 (2020) 014029
- F. Jegerlehner, Springer Tracts Mod. Phys. 274 (2017) 1

Muon g-2 Theory Initiative

Goal: improve the **SM evaluation of a_μ** to a precision comparable to that expected from experiments (**~100 ppb**)

White Paper: T. Toyama *et al.*, Phys.Rep. 887 (2020) 1

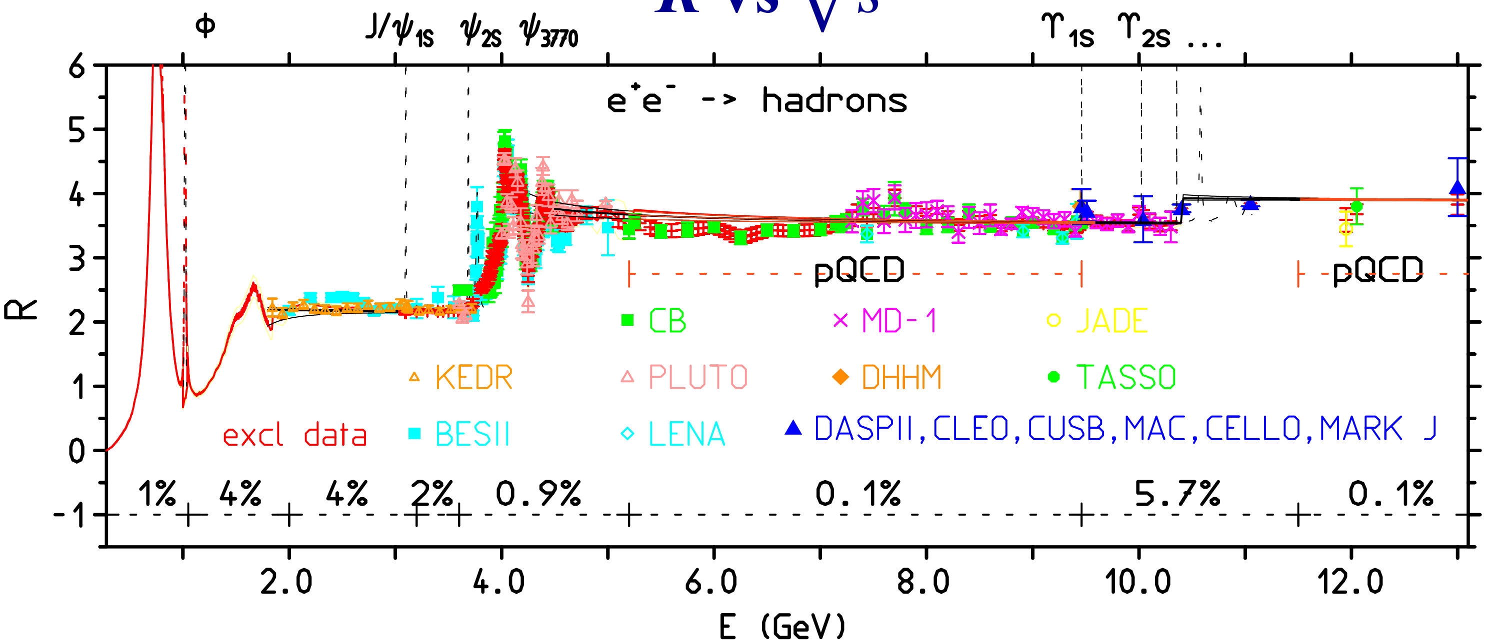
- Provide a comprehensive update about the g-2 question
  - Data-driven SM calculation (using e^+e^- and τ data)
  - SM Lattice calculations
  - Direct measurements



- WP 2020 recommended $a_\mu^{HVP,LO}$ value is based on a combination of DHMZ and KNT evaluations
- They agree within roughly one sigma.
- Results on $a_\mu^{\pi\pi}$ (for √s < 1.8 GeV):
  - KNT: $a_\mu^{HVP,LO}[\pi\pi] = (503.2 \pm 1.9) \cdot 10^{-10}$
  - DHMZ: $a_\mu^{HVP,LO}[\pi\pi] = (507.9 \pm 0.8 \pm 3.2) \cdot 10^{-10}$

BaBar-KLOE difference

R vs $\sqrt{s}$



Recommended WP2020 value from conservative merging of the various evaluations (dominated by the results from DHMZ19 and KNT19):

$$a_\mu^{HVP,LO} = (693.1 \pm 2.8_{exp} \pm 2.8_{sys} \pm 0.7_{DV\text{-}QCD}) \cdot 10^{-10}$$

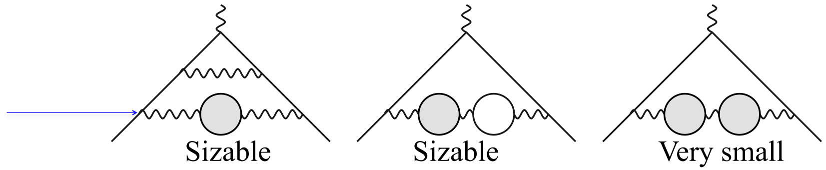
experimental uncertainties — KLOE-BABAR ππ discrepancy — data *vs* QCD differences

The other contributions to a_μ

Higher order HVP:

various calculations, all in agreement

- NLO: $a_\mu^{HVP,NLO} = (-98.3 \pm 0.7) \cdot 10^{-11}$
- NNLO: $a_\mu^{HVP,NNLO} = (12.4 \pm 0.1) \cdot 10^{-11}$



Hadronic Light-By-Light:

estimates with a dispersive relations approach and with lattice calculations compatible. Average value:

- $a_\mu^{HLBL} = (92 \pm 18) \cdot 10^{-11}$

QED contributions:

full perturbative calculation ($\alpha = 1/137.035$)

10th order corrections calculated!

- $a_\mu^{QED} = (116\ 584\ 718.931 \pm 0.104) \cdot 10^{-11}$

Electroweak contributions:

calculated up to two loops and an estimate of leading log contribution beyond 2-loop level

- $a_\mu^{EW} = (153.6 \pm 1.0) \cdot 10^{-11}$

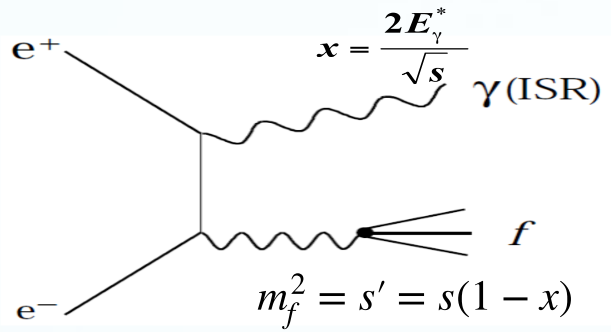
Recommended WP20 theory (SM) value:

$$a_\mu^{SM} = (116\ 591\ 810 \pm 43) \cdot 10^{-11}$$

Overall estimate of $a_\mu^{HVP,LO}$

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

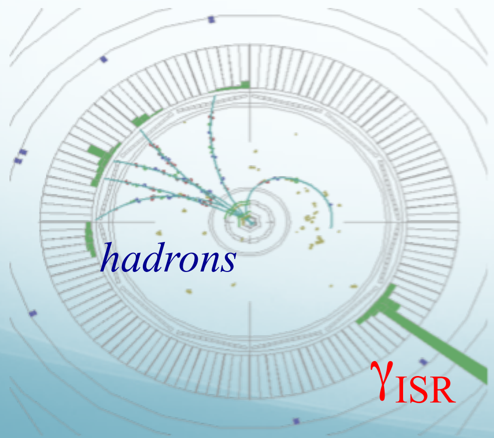
The Initial State Radiation method



$$\frac{d\sigma_{e^+e^-\to f\gamma}(s,m_f)}{dm_f\, d\cos\theta^*_\gamma} = \frac{2m_f}{s} W(s,x,\theta^*_\gamma)\cdot\sigma_{e^+e^-\to f}\left(m_f\right)$$

- The hadronic cross section $e^+e^- \to f$ can be extracted from the ISR cross section $e^+e^- \to \gamma f$.
- The radiator function $W(s,x)$ is calculated in QED with accuracy better than 1% level

BABAR display of a typical ISR event



Common ISR analysis strategy

- Tagged analysis (E_γ^*>3 GeV)
- Back-to-back topology btw ISR γ and the rest of the event
- π/K/*p* discrimination based on dE/dx e Cherenkov angle
- Kinematic fit for 4-momentum conservation
- Fitted χ^2 used for signal selection and background subtraction
- Detector acceptances and selection efficiencies estimated with MC simulation

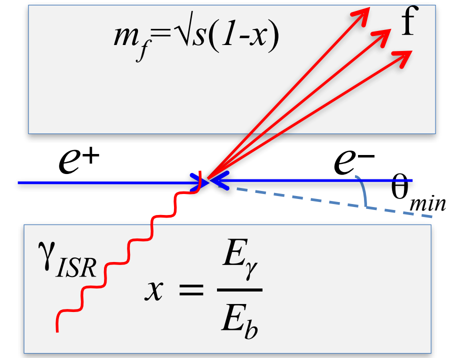
ISR method in a nutshell

Born approximation $$\frac{d\sigma_{e^+e^-\to f\gamma}(s,m_f,\theta^*_\gamma)}{dm\, d\cos\theta^*_\gamma} = \frac{2m}{s} W(s,x,\theta^*_\gamma)\cdot\sigma_{e^+e^-\to f}(m_f)$$

$x = \frac{E_\gamma}{E_b}$ $\quad m^2 = s' = s(1-x)$ $\quad \theta_\gamma^*$: ISR photon polar angle in the e^+e^- c.m.

Radiator function (at lowest order):

$$w_0(s,x,\theta^*) = \frac{\alpha}{\pi x}\left[\frac{(2-2x+x^2)\sin^2\theta^* - \frac{x^2}{2}\sin^4\theta^*}{\left(\sin^2\theta^* + \frac{4m_e^2}{s}\cos^2\theta^*\right)^2} - \frac{4m_e^2}{s}\frac{(1-2x)\sin^2\theta^* - x^2\cos^4\theta^*}{\left(\sin^2\theta^* + \frac{4m_e^2}{s}\cos^2\theta^*\right)}\right]$$



cross section $\sigma_0(e^+e^- \to f)$ $$\sigma_0(m_i) = \frac{\Delta N(m_i)}{\Delta m}\frac{1}{\varepsilon(s,m_i)(1+\delta_{rad})\, d\mathcal{L}(m_i)/dm}$$

reconstruction efficiency — radiative corrections — ISR luminosity

ISR differential luminosity $$\frac{d\mathcal{L}}{dm} = \frac{2m}{s}\frac{\alpha}{\pi\cdot x}\cdot\left((2-2x+x^2)\log\frac{1+C}{1-C} - x^2 C\right)L_{ee}$$

L_{ee}: Luminosity integrated by the collider

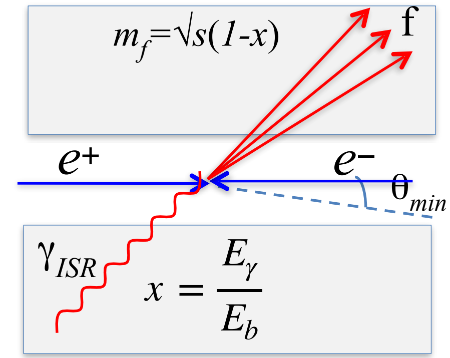
C: $\cos\theta^*_{min}$ Detector angular acceptance

- obtained from integration of the radiator function over θ_γ^*
- **$20^o < \theta_\gamma^* < 160^o$ ==> acceptance for ISR photon ~15% in *BABAR***
- known at <1% level

To tag or not to tag

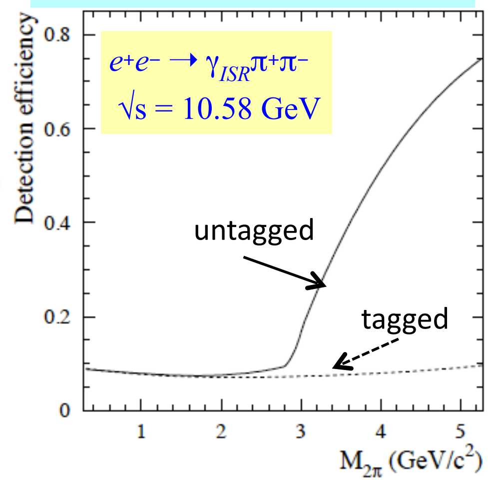
Tagged approach:
☺ fully reconstructed events ➞ great background reduction
☹ ~90% signal loss

Untagged approach:
☺ typically higher efficiency
☹ higher background reduced by requiring the missing mass consistent with zero



- So, what is the more convenient approach?
  ==> It depends on experimental situation
- At $\sqrt{s}$=10.58 GeV and for low m_f, (i.e. large x) the hadronic system has a large boost opposite to the photon direction ==> the efficiency is almost insensitive to tagging

Druzhinin et *al*, arXiv:1105.4975

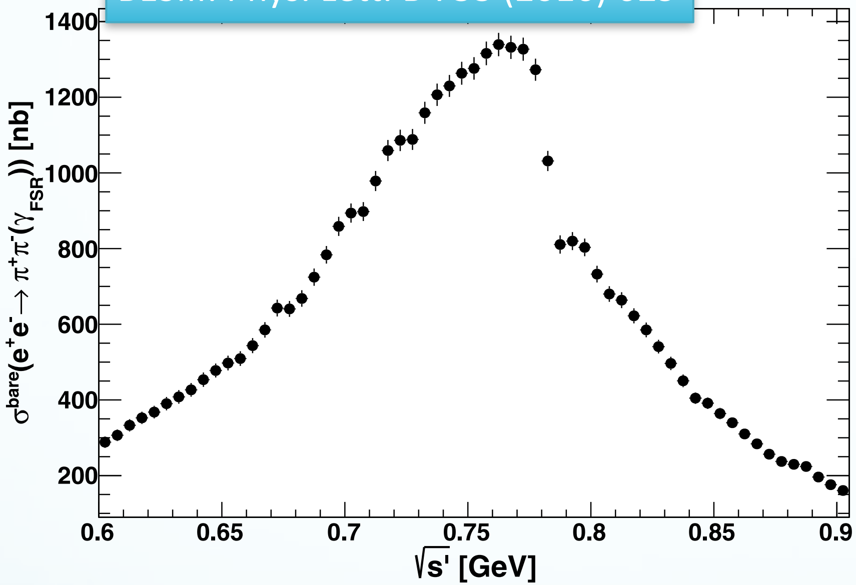


- This is why, at BABAR:
- **Light Quarks final states ⟺ Tagged analyses**
- Heavy Quarks final states ⟺ Untagged analyses

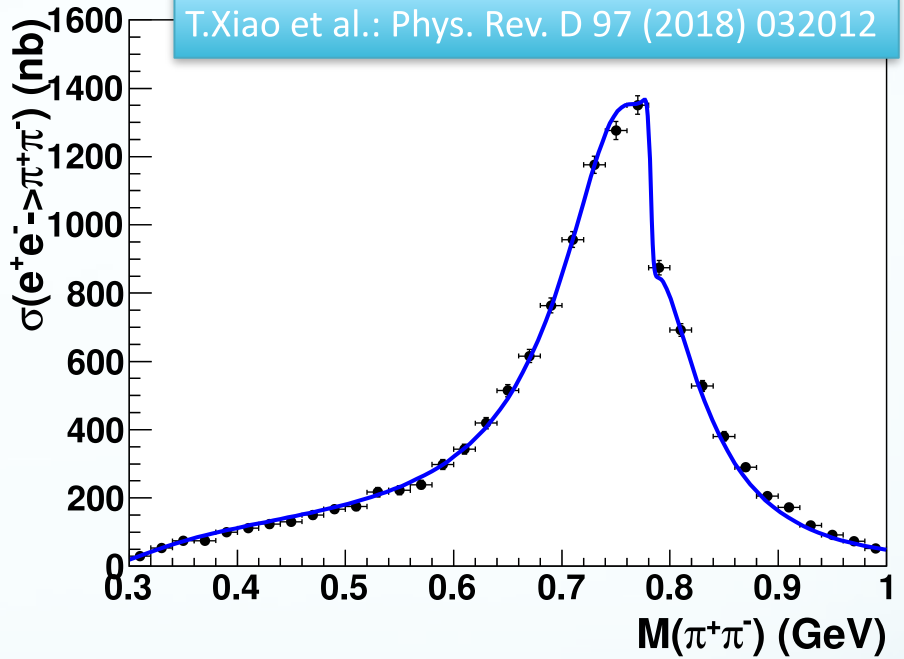
- At $\sqrt{s} \approx 1$ GeV (**KLOE**) **untagged analyses** are more efficient

$e^+e^- \to \pi^+\pi^-$: BESIII and "CLEO-c"

BESIII: Phys. Lett. B 753 (2016) 629



T.Xiao et al.: Phys. Rev. D 97 (2018) 032012



BESIII:

- Tagged ISR. $600 < \sqrt{s} < 900$ MeV.
- Normalisation from MC, cross checked with $e^+e^- \to \mu^+\mu^-$.
- Syst. uncertainties at the ρ-peak: ~0.9%

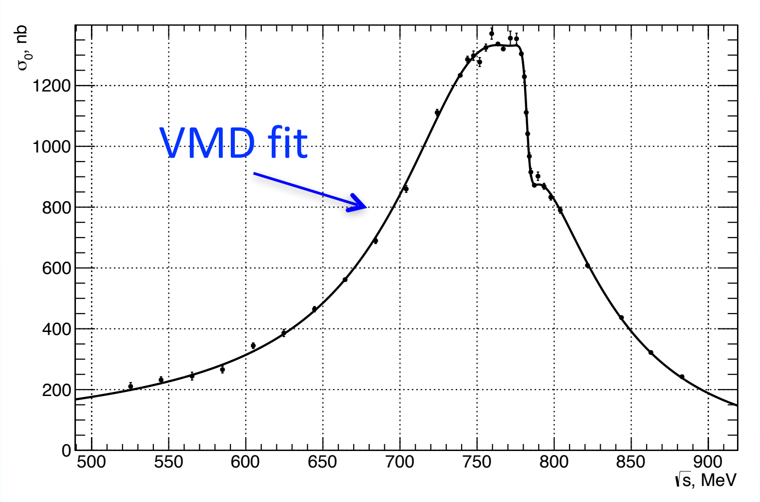
Analysis of **CLEO-c** data:

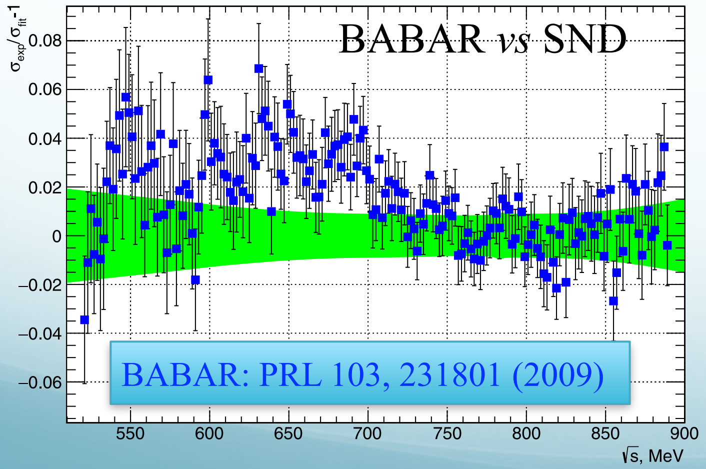
- Tagged ISR $300 < \sqrt{s} < 1000$ MeV.
- Normalisation from MC
- Statistical uncertainty: 0.7%
- Syst. uncertainties on a_μ: 1.5%

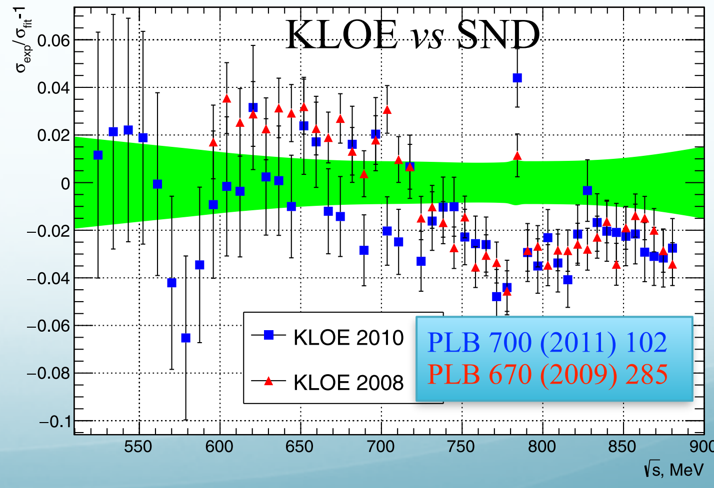
SND: new $e^+e^- \to \pi^+\pi^-$

SND: JHEP 01 (2021) 113

- Energy scan at **VEPP-2000**: 0.32 < √s < 2 GeV
  - peak luminosity 7×10^{31} $cm^{-2}s^{-1}$
  - higher than VEPP-2M (CMD-2)
  - ~70% of data in the ρ-peak region
  - This analysis:
    - 10% of collected data, with 525 < √s < 883 MeV
    - Syst. uncertainties ~0.8%
- $a_\mu^{\pi\pi}[525 - 883\,\mathrm{MeV}] = (409.79 \pm 1.44 \pm 3.87) \times 10^{-10}$

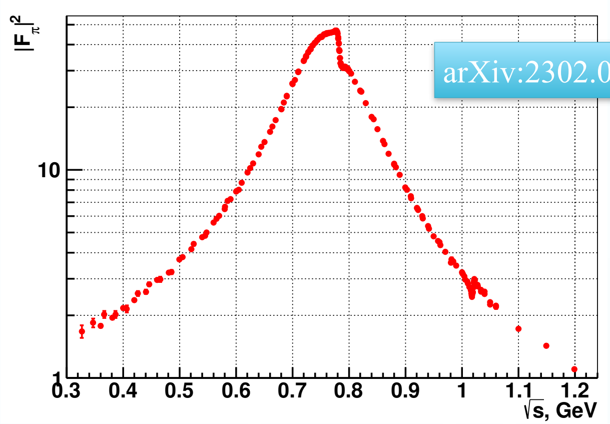




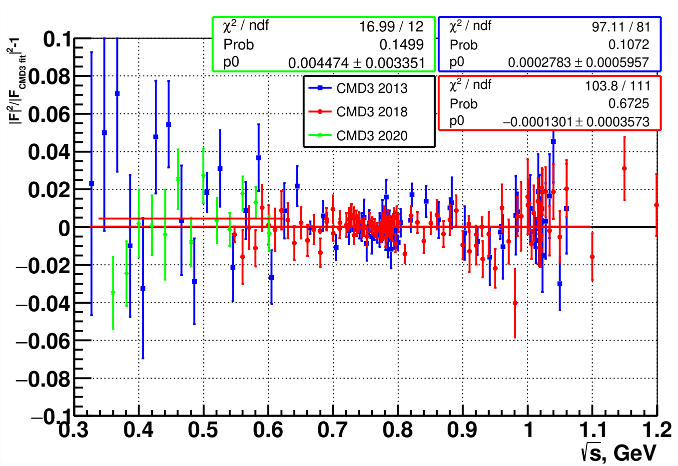


CMD-3 $e^+e^- \to \pi^+\pi^-$: Results

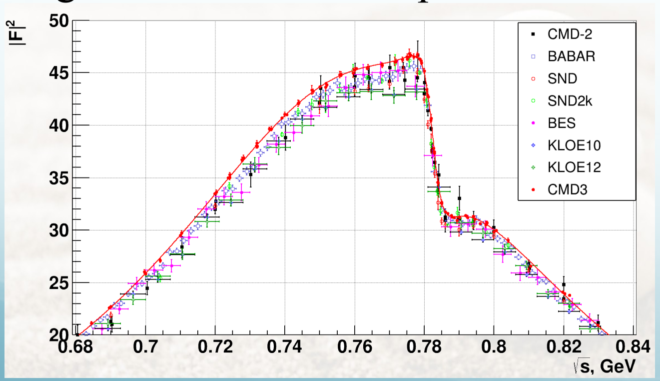
Pion FF measured by CMD-3



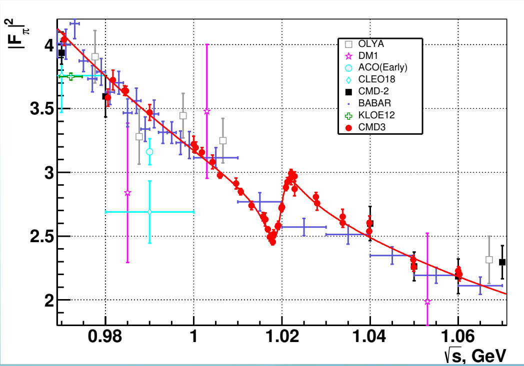
Good internal consistency among data sets



Details of ρ-region: CMD-3 significantly higher than all other experiments



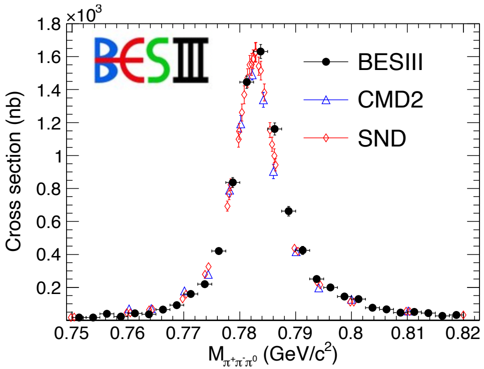
Interference between $\phi(1020)$ and non-resonant $\pi\pi$

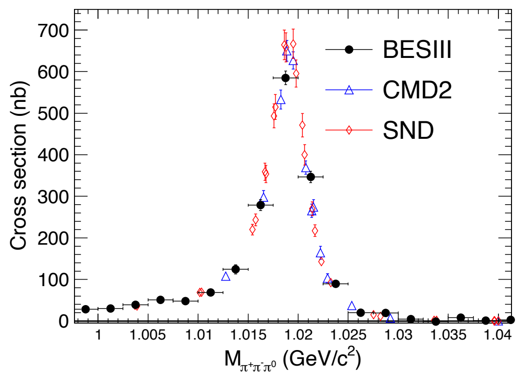


$e^+e^- \rightarrow \pi^+\pi^-\pi^0$

BESIII preliminary results BESIII: arXiv:1912.11208

- tagged ISR in this energy range, Ntot?, binning?
- Good consistency with previous SND and CMD-2 data



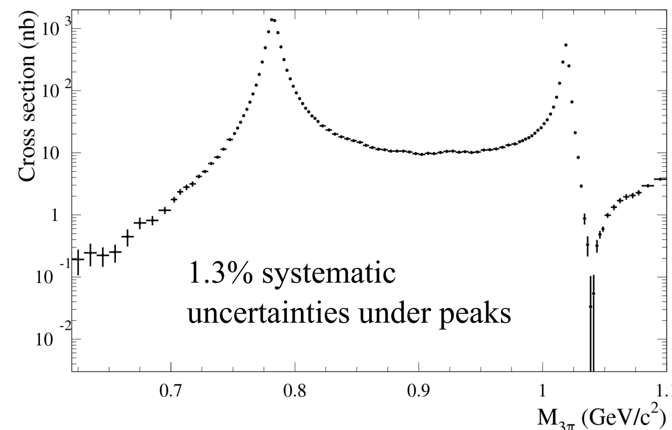


BABAR analysis ~469 fb^{-1} BABAR: PRD 104 (2021) 112003

- tagged ISR. 2.5 MeV bin-with at the resonances
- Syst. uncert. at the ω and ϕ peaks ~1.3%.

Precision on $a_\mu^{3\pi}$ improved by a factor ~2 (for $m_{3\pi}$ < 2 GeV):

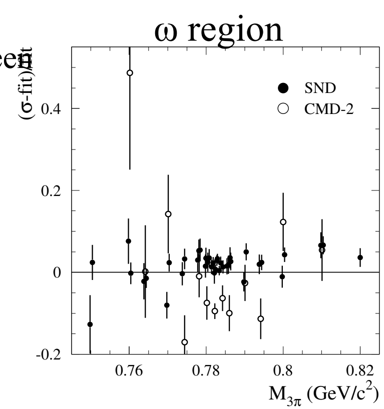
- $a_\mu^{3\pi} = (45.86 \pm 0.14 \pm 0.58)\, 10^{-10}$

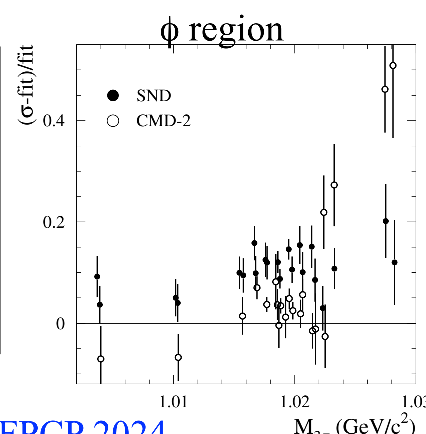


relative difference between measured cross sections and *BABAR* fit.

Some tension seen.

Only statistical uncertainties shown.





SND:
- PRD 63, 072002 (2001)
- PRD 68, 052006 (2003)

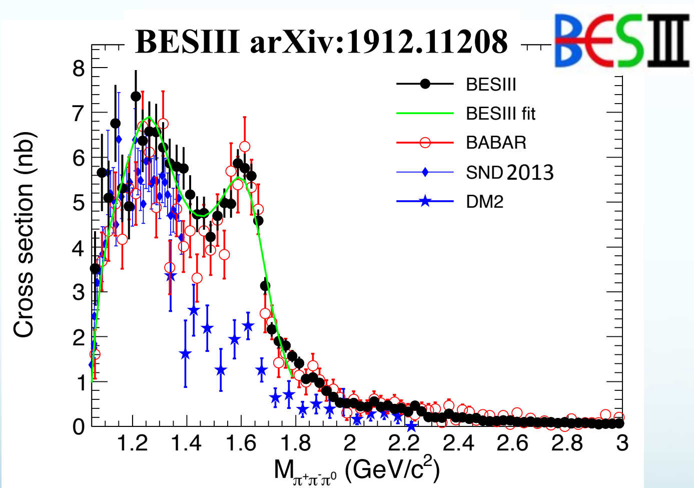
CMD-2:
- PLB 578, 285 (2004)
- PLB 642, 203 (2006)

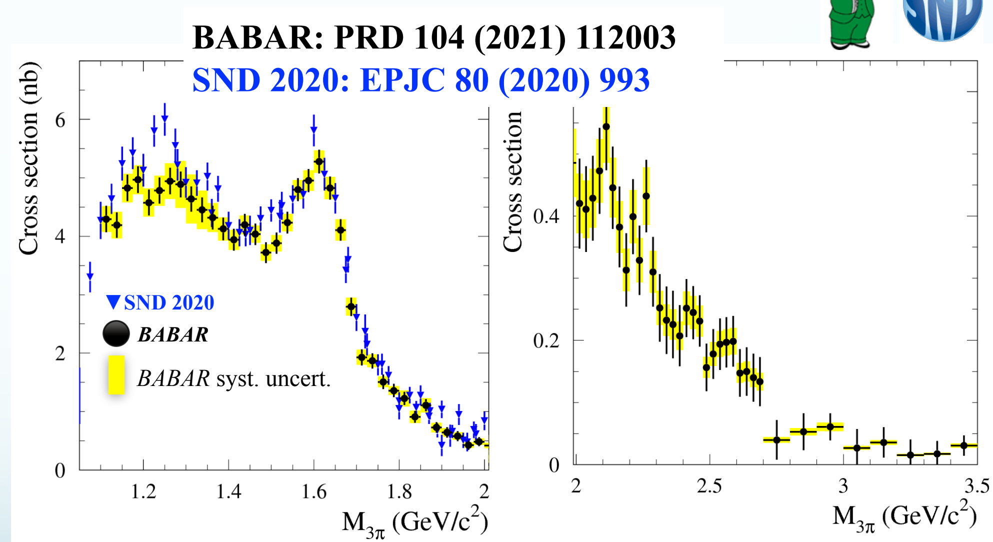
Recent $e^+e^- \to \pi^+\pi^-\pi^0$ results

- Second largest contribution to $a_\mu^{had,LO}$ and its uncertainty
- Dominated at low energies by the ω and φ resonances, and then by ω recurrences
- Old data:
    - Precise SND and CMD-2 @VEPP-2M below 1.1 GeV
    - *BABAR* (80 fb^{-1}) above the φ

Five new analyses, not in the WP, recently produced:

- ***BABAR***: ISR@Υ(4S) $0.62 < m_{3\pi} < 3.5$ GeV [PRD 104 (2021) 112003]
- **BESIII**: ISR@ψ(3770) $0.7 < m_{3\pi} < 3$ GeV [arXiv:1912.11208]
- **SND**: scan $1.2 < \sqrt{s} < 2$ GeV [EPJ C80, 993 (2020)]
- **BESIII**: scan $2 < \sqrt{s} < 3.08$ GeV [arXiv2401.14711]
- **CMD-3**: scan $0.66 < \sqrt{s} < 0.97$ GeV [preliminary]



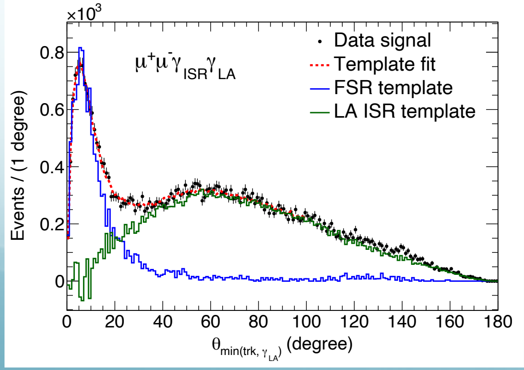


Clear observation of ω(1420) and ω(1650)
All recent data inconsistent with old DM2 data

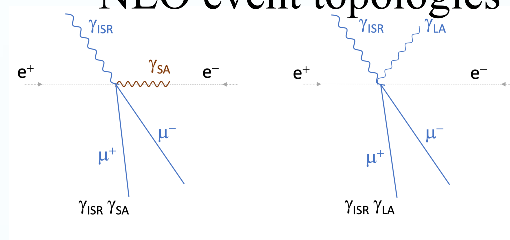
Measurement of additional radiation in ISR processes

- Study of $e^+e^- \to \mu^+\mu^-\gamma_{ISR}$ and $e^+e^- \to \pi^+\pi^-\gamma_{ISR}$ with 1 (NLO) or 2 (NNLO) additional hard photons
- The full BaBar data sample (~468 fb^{-1}) is used. Data compared with MC generators:
  - **AfkQed** : up to NNLO additional ISR collinear to beams, FSR from PHOTOS
  - **Phokara 9.1**: full matrix elements at NLO (no NNLO)

- All events are subjected to several fits consistent with the event topology
- The fit with the best χ^2 determine the category of that event
- Small-angle (SA) γ's assumed collinear with the beams. Large-angle (LA) γ's detected
- FSR/ISR separation from angular distance of γ_{LA} to the closest track



NLO event topologies



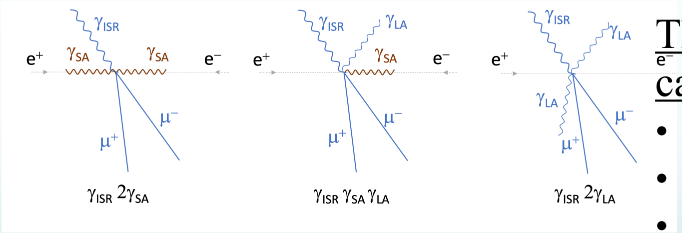
Two NLO fits:

- $\gamma_{ISR}\gamma_{SA}$ ($E^*_{\gamma_{SA}} > 200$ MeV)
- $\gamma_{ISR}\gamma_{LA}$ ($E_{\gamma_{LA}} > 200$ MeV)

Three categories:

- NLO LA
- NLO SA
- LO: events with no γ's above threshold

NNLO event topologies

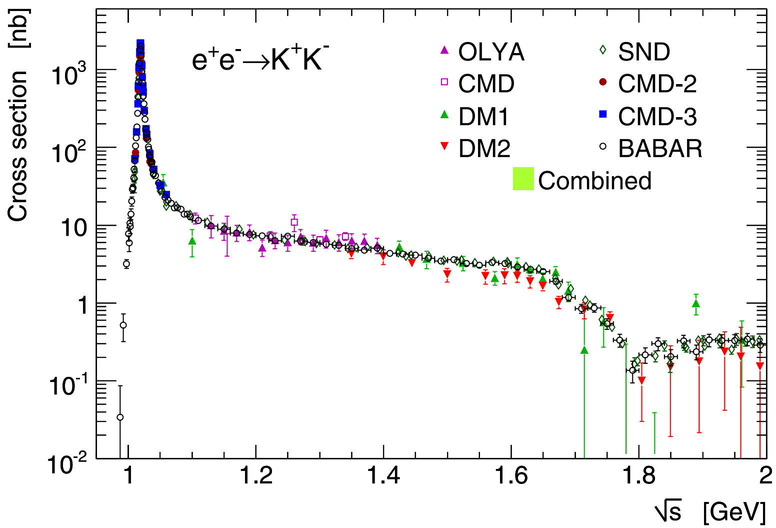


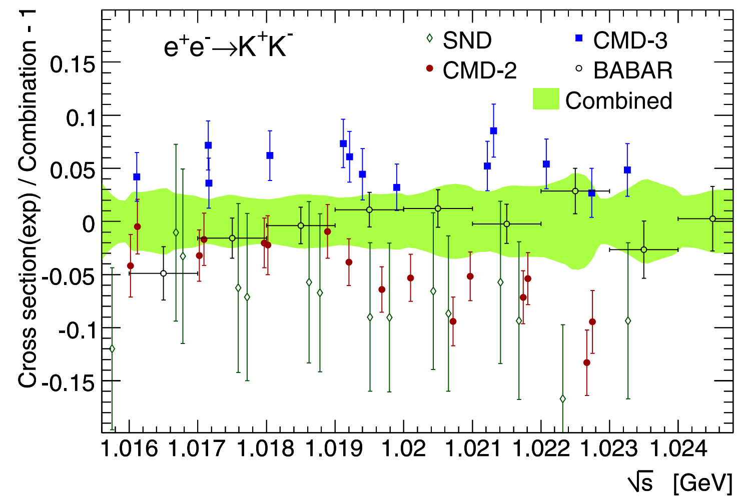
Three NNLO fits and three categories:

- $\gamma_{ISR}\gamma_{SA}\gamma_{SA}$ -> NNLO 2SA
- $\gamma_{ISR}\gamma_{SA}\gamma_{LA}$ -> NNLO SA+LA
- $\gamma_{ISR}\gamma_{LA}\gamma_{LA}$ -> NNLO 2LA

BABAR: Phys. Rev. D108 (2023) 111103

$e^+e^- \rightarrow K^+K^-$



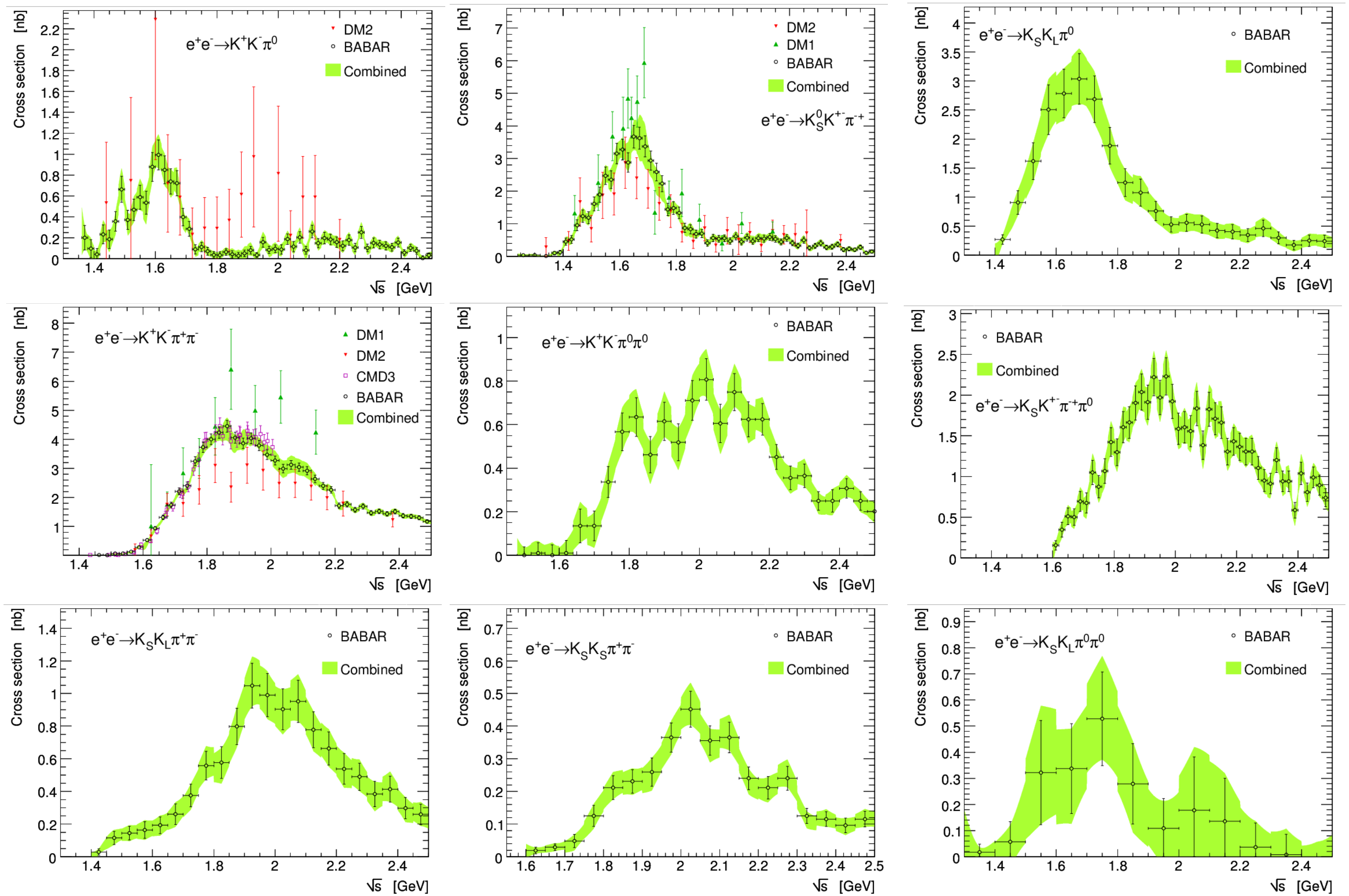


- Cross section dominated by the φ(1020) peak
- Precise measurements from BABAR, CMD-2/3 and SND

- Some tension at the peak with new CMD-3 data significantly above the older Novosibirsk results and BABAR data in the middle
- Note: φ(1020) just above KK threshold. Very low momentum tracks in energy scan experiments

- Green combination from DHMZ

$e^+e^- \rightarrow KK\pi$ and $KK\pi\pi$

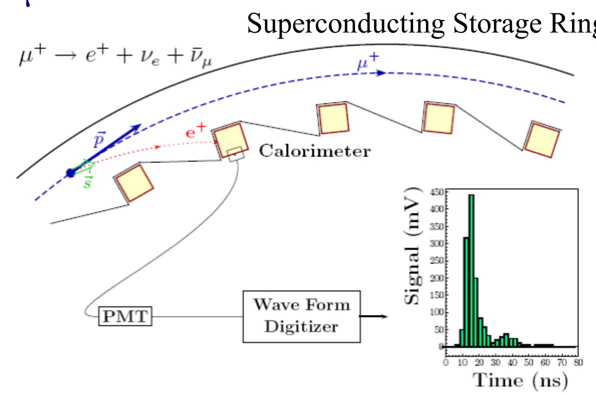


F. Anulli

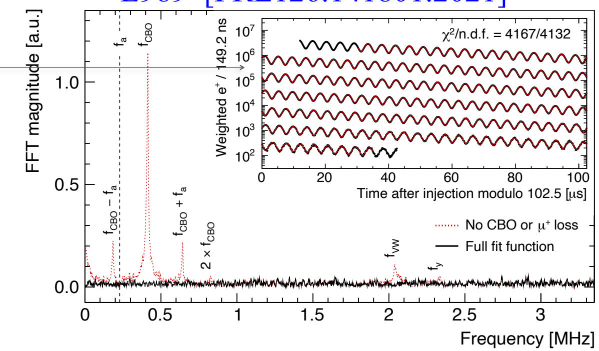
The E821 and E989 direct a_μ measurements

- Intense proton beam on target to produce the pion beam.
  - Pions of 3.1 GeV are selected.
- $\pi^+ \to \mu^+ \nu$ violates P ==> 95% polarisation for forward muons
- μ^+ stored in a cyclotron: constant B field
  - cyclotron frequency ω_c; spin precessing with freq. ω_s
  - $\omega_a = \omega_s - \omega_c = a_\mu \, eB/m_\mu$
- $\mu \to e\nu\bar{\nu}$ violates P ==> e^+ direction (energy in LAB) "remembers" the μ polarization
  - fraction of detected e^+ with $E>E_{threshold}$ modulated with frequency ω_a

$$N_{ideal}(t) = N_0 \exp(-t/\gamma\tau_\mu)\left[1 - A\cos(\omega_a t + \phi)\right],$$



E989 [PRL126.141801.2021]



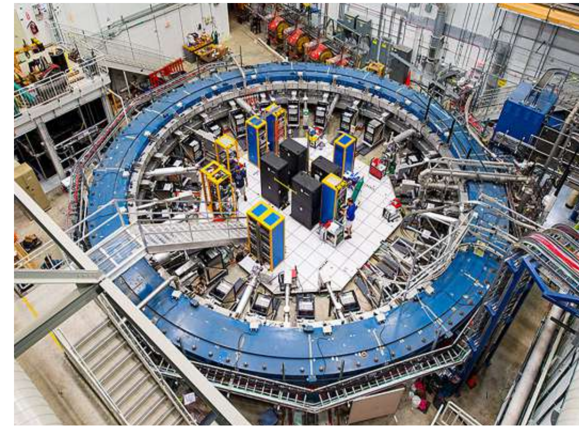
- Precise knowledge of B field critical
  - use of pulsed proton NMR
  - B determined from proton precession frequency and magnetic moment
- a_μ extracted from:

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

  - measure ω_a and ω_p
  - Other quantities from external inputs

Perspectives in direct measurements

1. The “Muon g-2” experiment at Fermilab is taking new data

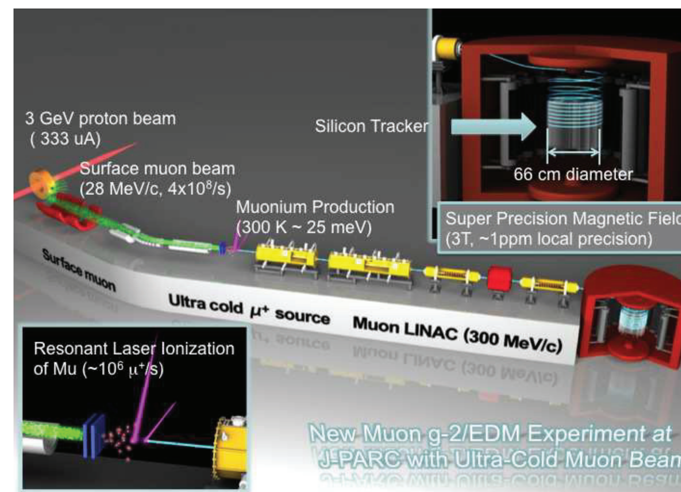


- Final goal: **uncertainty of 0.14 *ppm*** (0.1 *ppm* stat., 0.07 *ppm* syst.)
- (now they reached 0.35 *ppm*)
- very ambitious program, but it looks under reach

2. J-PARC E34 Collaboration

- novel method for muon g-2 and EDM measurements
- ultra-cold muons produced from pion decay at rest
- accelerated by a linac at p = 300 MeV
- beam with negligible transverse momentum
    - no need for strong electromagnetic focusing

  => no need of the 3.1 GeV *magic momentum*
- small storage ring (33 cm radius); cyclotron period only 7.4 ns; 3T dipole magnet
- Predicted statistics 10x FNAL
- Final goal: **uncertainty of 0.1 *ppm***



Alternative approach for a_μ^{HLO}: MUonE experiment

- Standard approach using e^+e^- data ($s > 0$ integral in the time-like region)

$$a_\mu^{\rm HLO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \, \frac{\hat{K}(s) R_{\rm had}(s)}{s^2},$$

- Alternative approach:

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi} \int_0^1 dx \, (1-x) \, \Delta\alpha_{\rm had}[t(x)] \, , \qquad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$t = q^2 < 0$ integral in the space-like region => integrand smooth and free of resonances

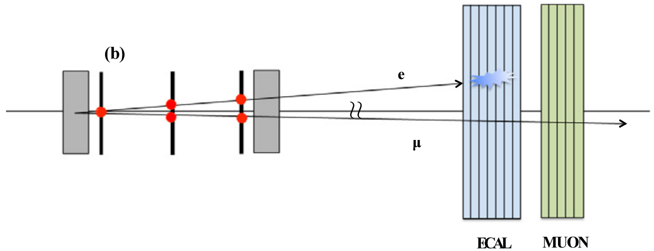
- Measure the running of $\alpha(t)$ from the elastic scattering $\mu^+e^- \to \mu^+e^-$ to extract $\Delta\alpha(t)$

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left| \frac{\alpha(t)}{\alpha(0)} \right|^2 , \qquad \alpha(t) = \frac{\alpha(0)}{1 - \Delta\alpha(t)},$$

- Subtract the leptonic contribution (calculable in perturbation theory) to get: $\Delta\alpha_{had}(t) = \Delta\alpha(t) - \Delta\alpha_{lep}(t)$

MUonE at CERN North-Area

- Fixed target experiment with a 150 GeV muon beam
- t variable related to electron scattering angle (and energy)
- Experimental coverage:
  - $-143 < t = q^2 < 0$ GeV^2
  - ~87% of all $a_\mu^{HVP,LO}$
- Remaining part estimated using time-like data + pQCD, or Lattice+QCD



- 40 Be targets, 1.5cm-thick, equipped with Si trackers
- EM calorimeter and muon tracking downstream to measure all components of the final state

- With three years of running at planned conditions, expect a statical uncertainty of ~0.3%
- Systematic and theoretical uncertainties to be carefully evaluated
- Results should be competitive and fully independent from e^+e^- data