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Theoretical progress in CP violation in D-meson system

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The Acclaimed Dark Horse

Bigi and Sanda, *CP Violation*, Chapter 14

- CP violation in charm decays is considered as the dark horse.

"… charm's SM phenomenology provides us with a dual opportunity, namely to

(1) probe our *quantitative* understanding of QCD's non-perturbative dynamics thus calibrating our theoretical tools for describing B decays;

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Importance of Hadronic D Meson Decays

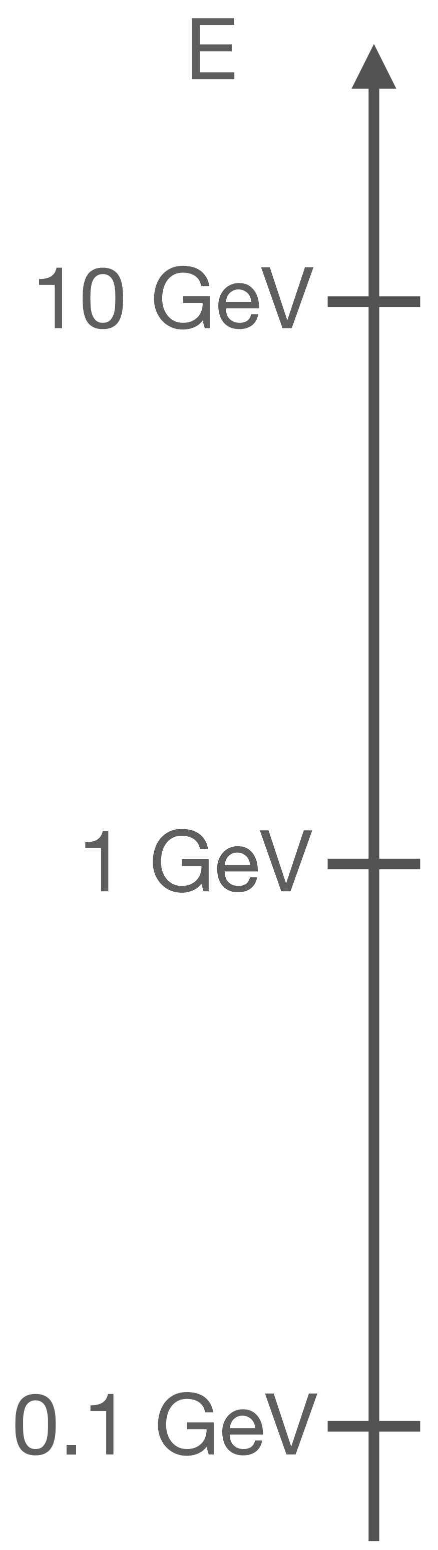
- D mesons decay dominantly (~84%) into hadronic final states, 3/4 of which are two-body modes.
  - ➠ importance of hadronic decay modes in understanding the D meson

Mode	BR
PP	$\sim 10\%$
VP	$\sim 28\%$
VV	$\sim 10\%$
SP	$\sim 4.2\%$
AP	$\sim 10\%$
TP	$\sim 0.3\%$
2-body	$\sim 63\%$
hadronic	$\sim 84\%$
semileptonic	$\sim 16\%$

– most dominant ones

P: pseudoscalar meson
V: vector meson
A: axial vector meson
T: tensor meson

Peculiarities of Charm System



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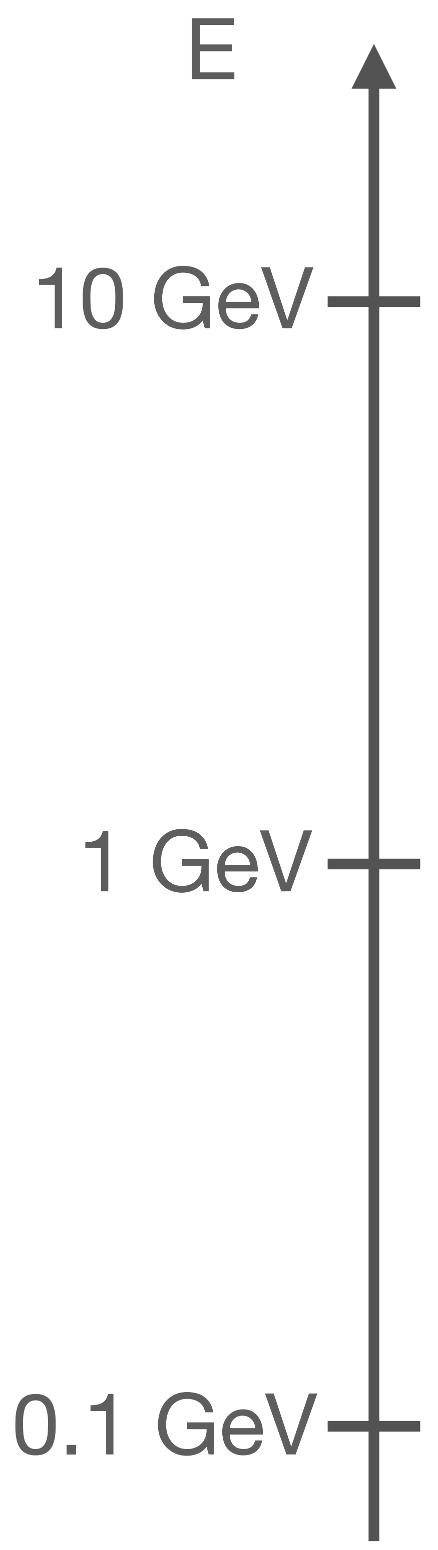
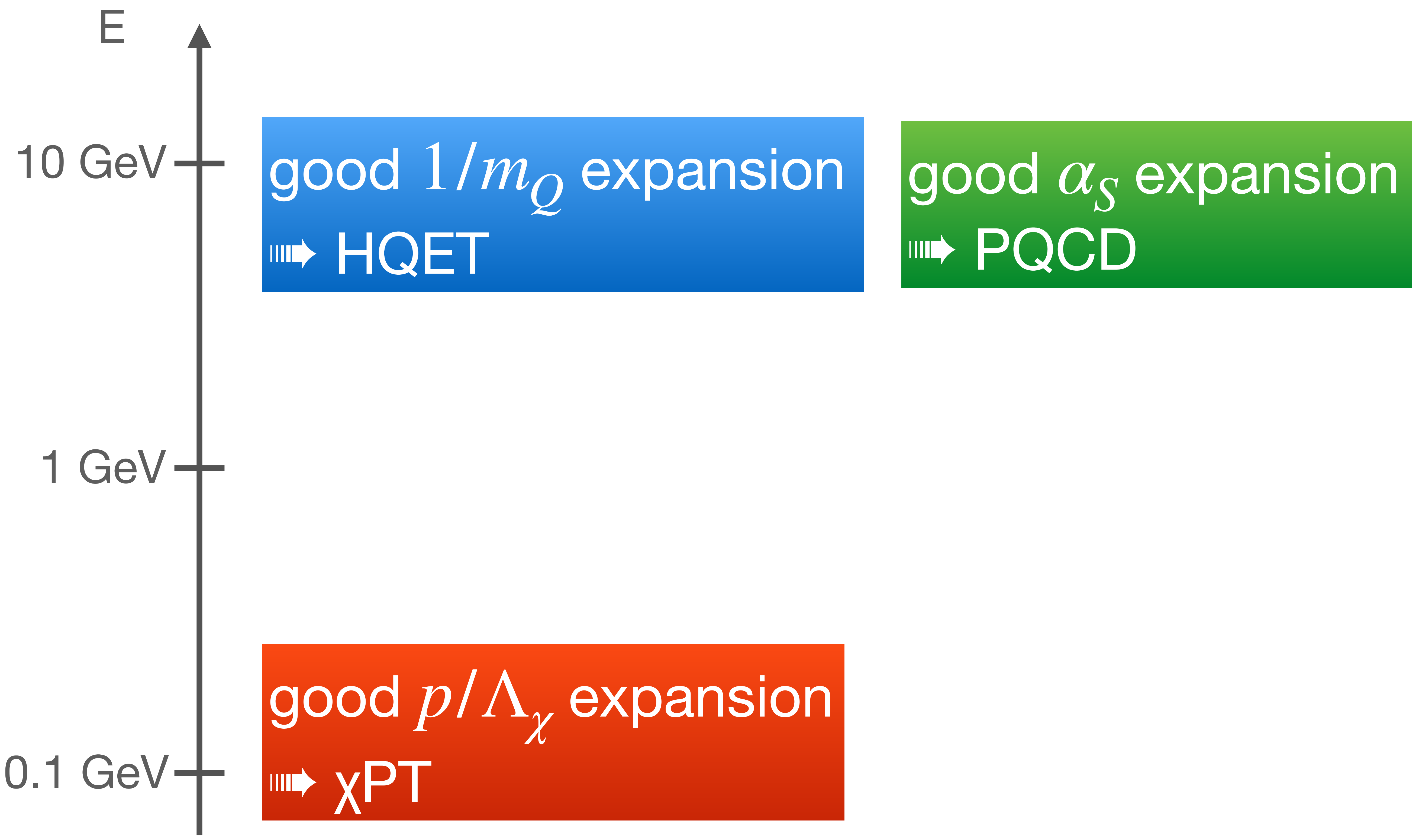


good $1/m_Q$ expansion
➠ HQET

good α_S expansion
➠ PQCD

Peculiarities of Charm System



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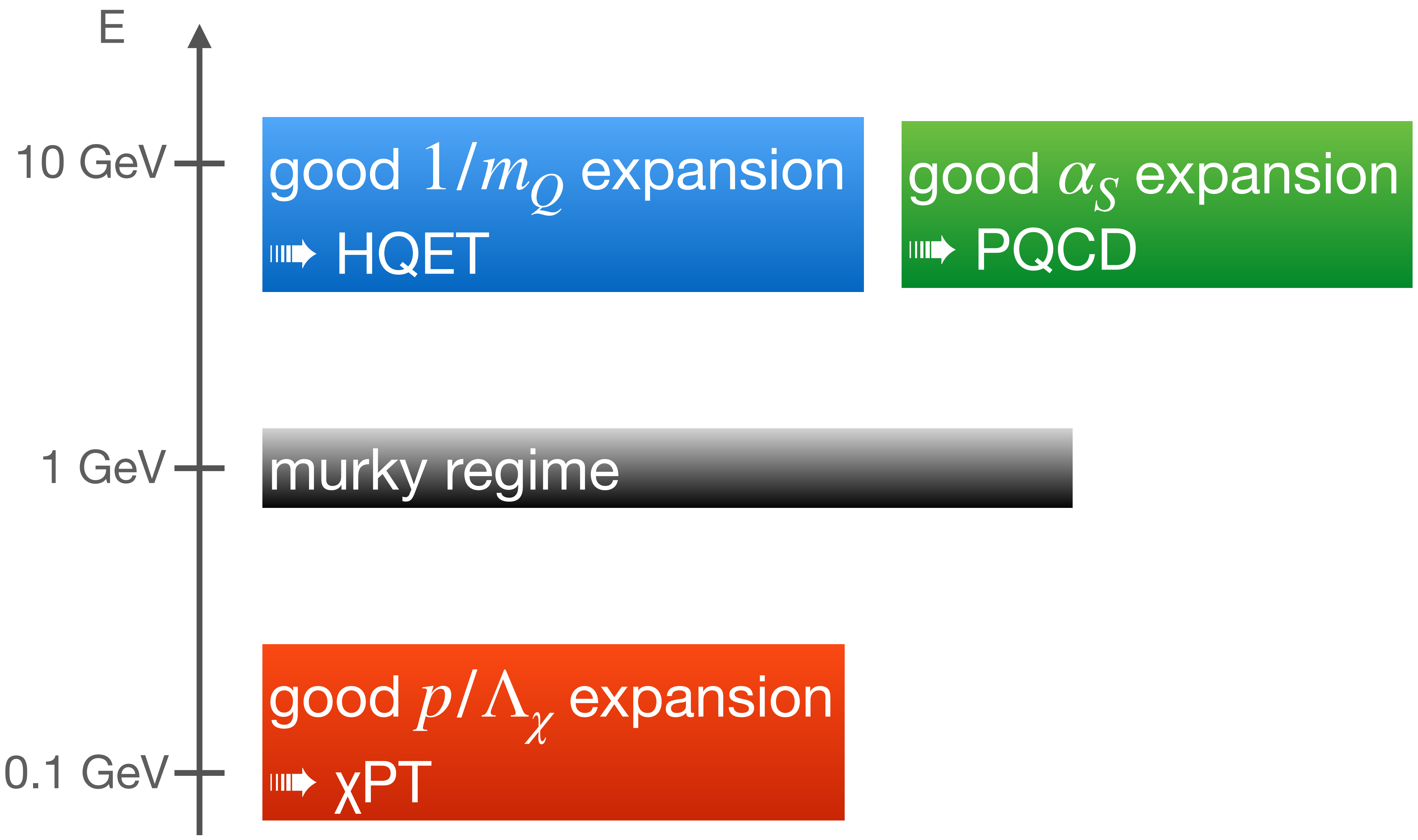

- $\Lambda_{\mathrm{QCD}}/m_c \sim 0.3$
  ➠ bad heavy quark expansion
  ➠ higher power corrections
- $\alpha_S(m_c) \sim 0.3$
  ➠ bad PQCD expansion
  ➠ higher-order perturbations and/or nonperturbative effects
- Many nearby resonances
  ➠ final-state rescattering effects

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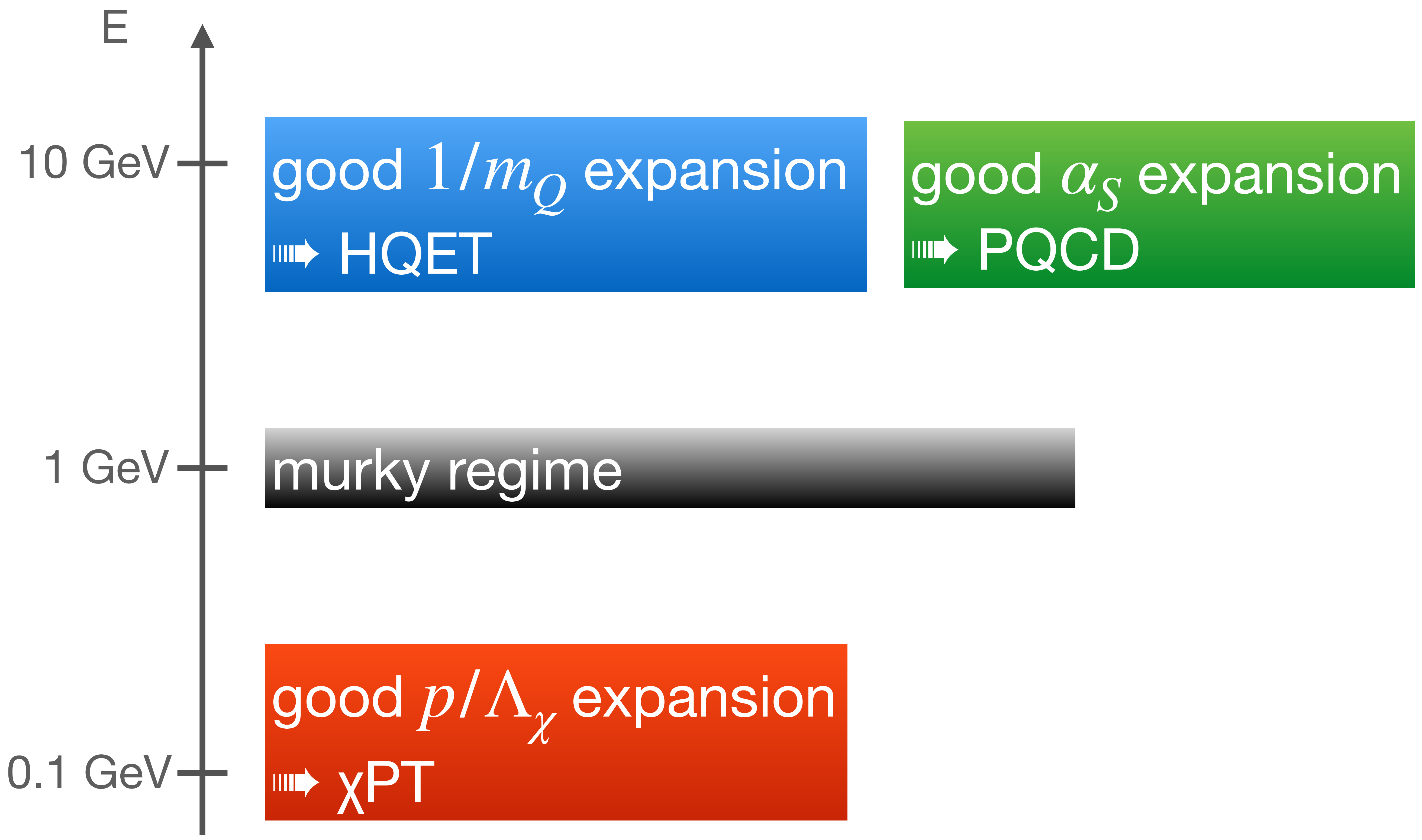

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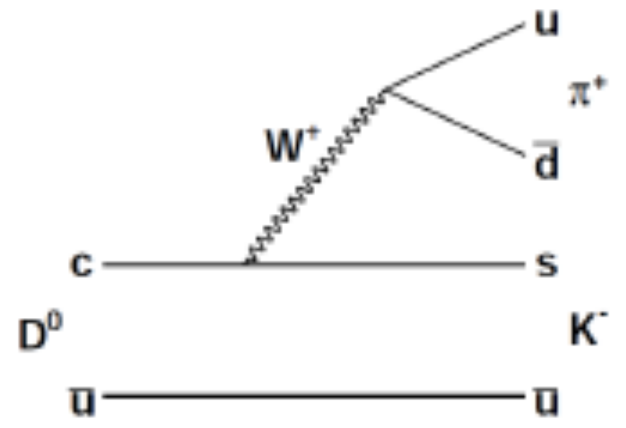
- There is no satisfactory effective theory that allows us to study the charm system reliably, particularly for the hadronic decays.
- Common approaches: symmetry-based and perturbation-based, assisted with lattice inputs and sometimes phenomenological models.

Cabibbo Hierarchy in Hadronic D Decays

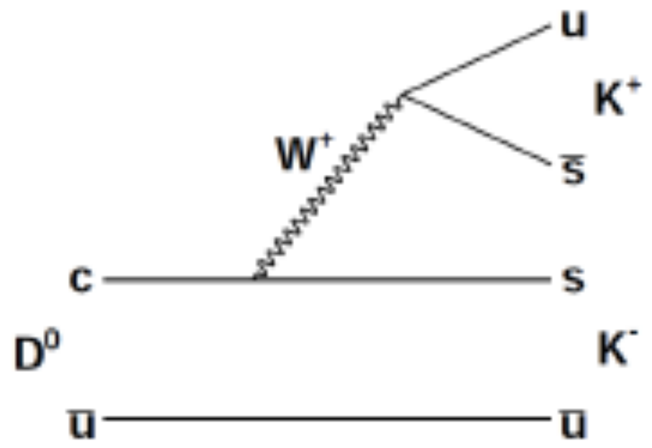
- Cabibbo-favored (CF):

  involving $V_{cs}^* V_{ud} \sim 1 - \lambda^2 \sim 0.95$



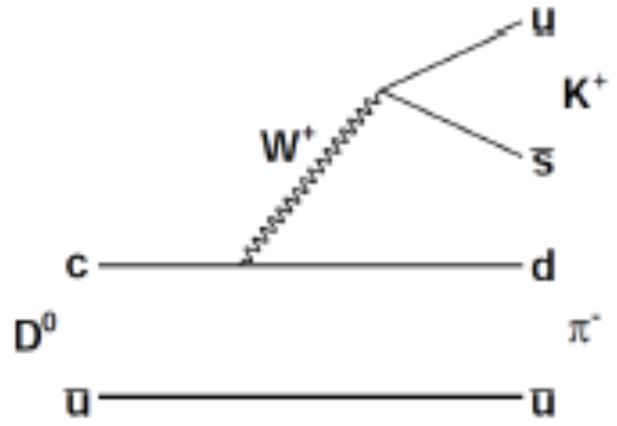
- Singly Cabibbo-suppressed (SCS):

  involving $V_{cd}^* V_{ud}$ or $V_{cs}^* V_{us} \sim \lambda \sim 0.22$



- Doubly Cabibbo-suppressed (DCS):

  involving $V_{cd}^* V_{us} \sim \lambda^2 \sim 0.05$



- SCS decays can involve diagrams with different CKM phases and have CPA's:

$$\mathrm{Amp} = V_{cd}^* V_{ud}(\mathrm{trees} + \mathrm{penguins}) + V_{cs}^* V_{us}(\mathrm{trees} + \mathrm{penguins})$$

(after using unitarity identity)

tiny relative weak phase between the two combinations

GIM Mechanism

- In the SM, the CKM matrix takes the hierarchical form:

Cabibbo 1963; Kobayashi, Maskawa 1973
Wolfenstein 1983

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} \boxed{\begin{matrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \end{matrix}} \\ \begin{matrix} A\lambda^3[(1-\bar\rho) - i\bar\eta] & -A\lambda^2 & 1 \end{matrix} \end{pmatrix}$$

where the unitarity condition for the upper two rows (relevant for charm physics)

$$\lambda_d + \lambda_s + \lambda_b = 0 \qquad (\text{where } \lambda_q \equiv V_{cq}^* V_{uq})$$

renders an extremely squashed unitarity triangle (roughly $1 : 1 : 10^{-3}$ sides).

- The loop function for the $c \to u$ penguin amplitude $\sim \dfrac{1}{(4\pi)^2}\dfrac{m_q^2}{m_W^2}$, which is of order $(10^{-10},\ 10^{-8},\ 10^{-5})$ for the d, s, and b quark, respectively, resulting in a very effective Glashow-Iliopoulos-Maiani (GIM) cancellation.

CP Violation in SCS Decays

- Typically, direct CPA's in SCS decays are given by

relative strong phase

$$a_{\rm CP}^{\rm dir} = \frac{2{\rm Im}(V_{cd}^* V_{ud} V_{cs} V_{us}^*)}{|V_{cd}^* V_{ud}|^2} \left|\frac{A_2}{A_1}\right| \sin\delta = 2 \left|\frac{V_{cb}^* V_{ub}}{V_{cd}^* V_{ud}}\right| \sin\gamma \left|\frac{A_2}{A_1}\right| \sin\delta \sim 10^{-3} \left|\frac{A_2}{A_1}\right| \sin\delta$$

weak phase of V_{ub}^*

where A_1 and A_2 generically denote amplitudes (including tree and penguin types in general) associated with different CKM factors.

- With $|A_2/A_1| \lesssim 1$, the CPA is at most $\mathcal{O}(10^{-3})$.
  ➠ new physics, if measured to be more sizable
  ➠ current data at the borderline…

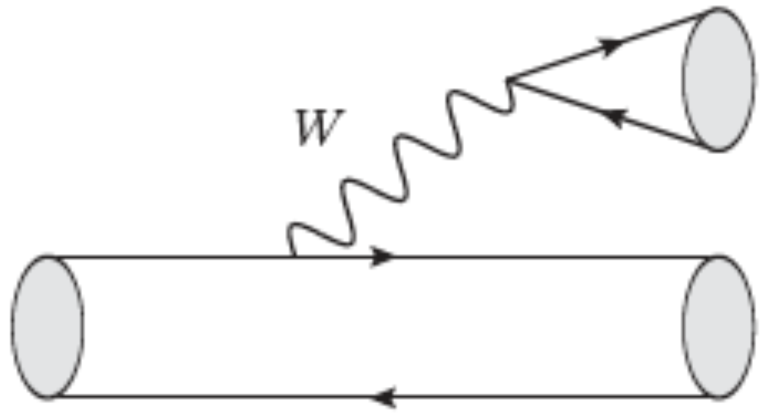
- The estimates of $|A_2/A_1|$ differ between perturbative and symmetry approaches.

Symmetry-based Approach — Topological Diagrams

- Diagrams for 2-body hadronic D meson decays can be classified more intuitively according to flavor topology into the tree- and loop-types, universal in flavor SU(3) limit:

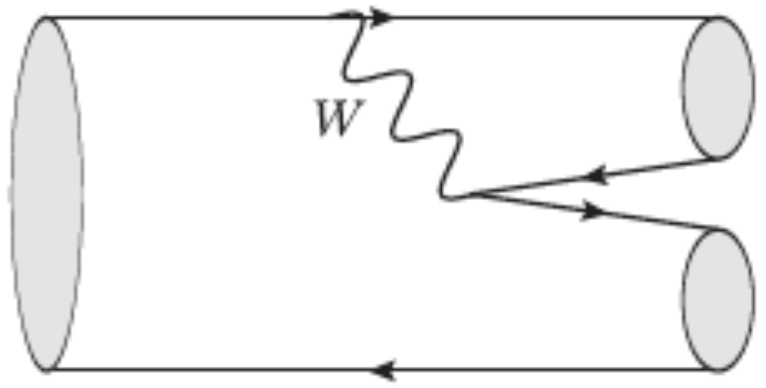
Zeppenfeld 1981
Chau and Cheng 1986, 1987, 1991
Savage and Wise 1989
Grinstein and Lebed 1996
Gronau et. al. 1994, 1995, 1995

Tree-type



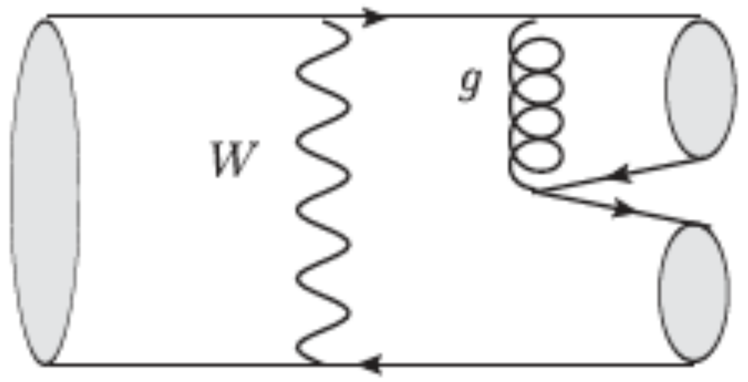
(a) T

color-allowed tree



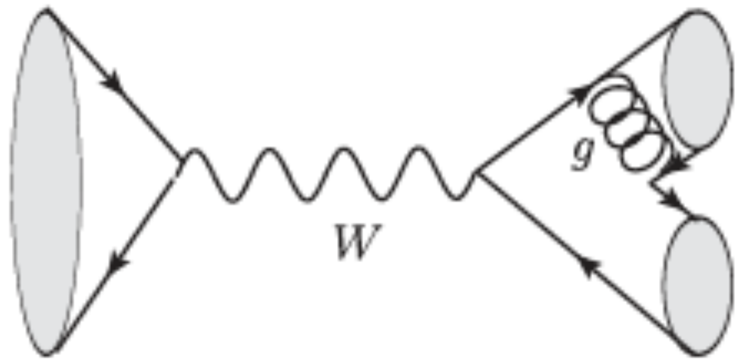
(b) C

color-suppressed tree



(e) E

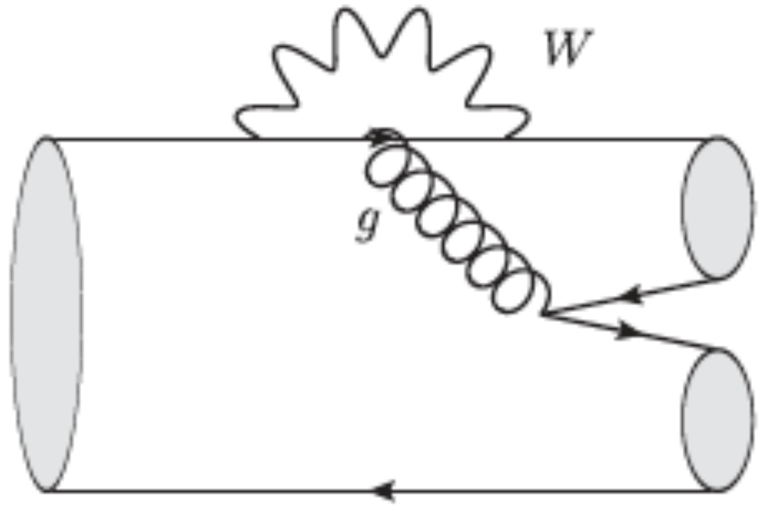
exchange



(f) A

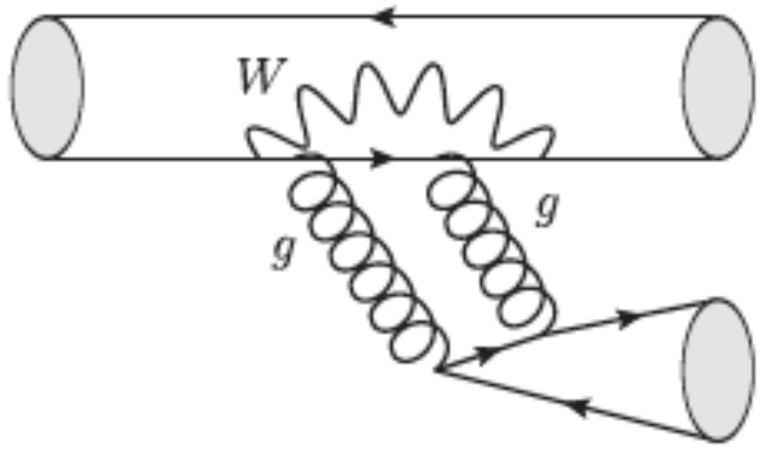
annihilation

Loop-type



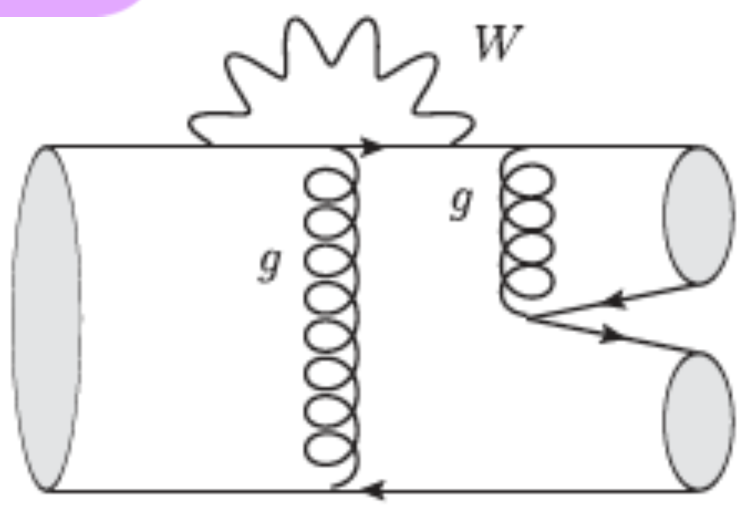
(c) $P,\ P_{\rm EW}^C$

QCD penguin
color-suppressed EW penguin



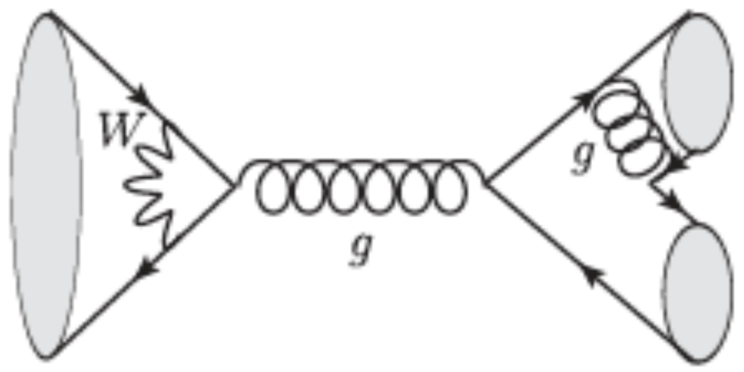
(d) $S,\ P_{\rm EW}$

singlet penguin
EW penguin



(g) $PE,\ PE_{\rm EW}$

penguin exchange
EW penguin exchange



(h) $PA,\ PA_{\rm EW}$

penguin annihilation
EW penguin annihilation

From Partial Width to Decay Amplitude

- Partial decay widths of $D \to PP$ and VP decays are related to their decay amplitudes as follows:

magnitude of 3-momentum
of final-state particle

$$\Gamma(D \to PP) = \frac{p_c}{8\pi m_D^2}|\mathcal{M}|^2$$

P-wave state

$$\Gamma(D \to VP) = \frac{p_c^3}{8\pi m_V^2}|\mathcal{M}|^2$$

due to polarization sum

- Flavor SU(3) breaking due to phase space difference is thus removed.
- In the symmetry limit, flavor SU(3) is assumed at the amplitude level (decay strength and strong phase), where the decay amplitude $\mathscr{M}$ is a linear combination of the topological amplitudes multiplied by the corresponding CKM factors.

Symmetry-based Approach

- As far as BR's are concerned, penguin diagrams are negligible because of the GIM mechanism ($V^*_{cd}V_{ud} = -V^*_{cs}V_{us}$ and $V^*_{cb}V_{ub} \sim A^2\lambda^5$).
  ➠ tree-type diagrams are dominant in determining the BR's

- Perform a χ^2 fit to the BR's of all CF modes, extracting magnitudes and strong phases (up to discrete ambiguities) of all the topological amplitudes.
- Make flavor SU(3) symmetry-breaking corrections (mostly in the magnitude) as demanded by data.
- Include penguin amplitudes, particularly certain diagrams that could be enhanced by final-state rescattering, to induce large direct CPA's.
- Using the extracted information, make predictions of BR's and CPA's for SCS and DCS modes.
  ➠ testable by current/future data

Tree-level Amplitudes and Long-distance Effects

- A fit to the BR's of CF PP modes gives (in units of 10^{-6}):

Cheng and CWC 2019 with slight updates in Cheng and CWC 2024

$$T = 3.113 \pm 0.011$$ — real by convention

such magnitudes and strong phases signify that amplitudes other than T all receive significant nonfactorizable, long-distance FSI's ➠ more on this later

$$C = (2.767 \pm 0.029)e^{-i(151.3\pm 0.3)^\circ}$$

$$E = (1.48 \pm 0.04)e^{i(120.9\pm 0.4)^\circ}$$

large strong phases

$$A = (0.55 \pm 0.03)e^{i(23+7)^\circ}$$

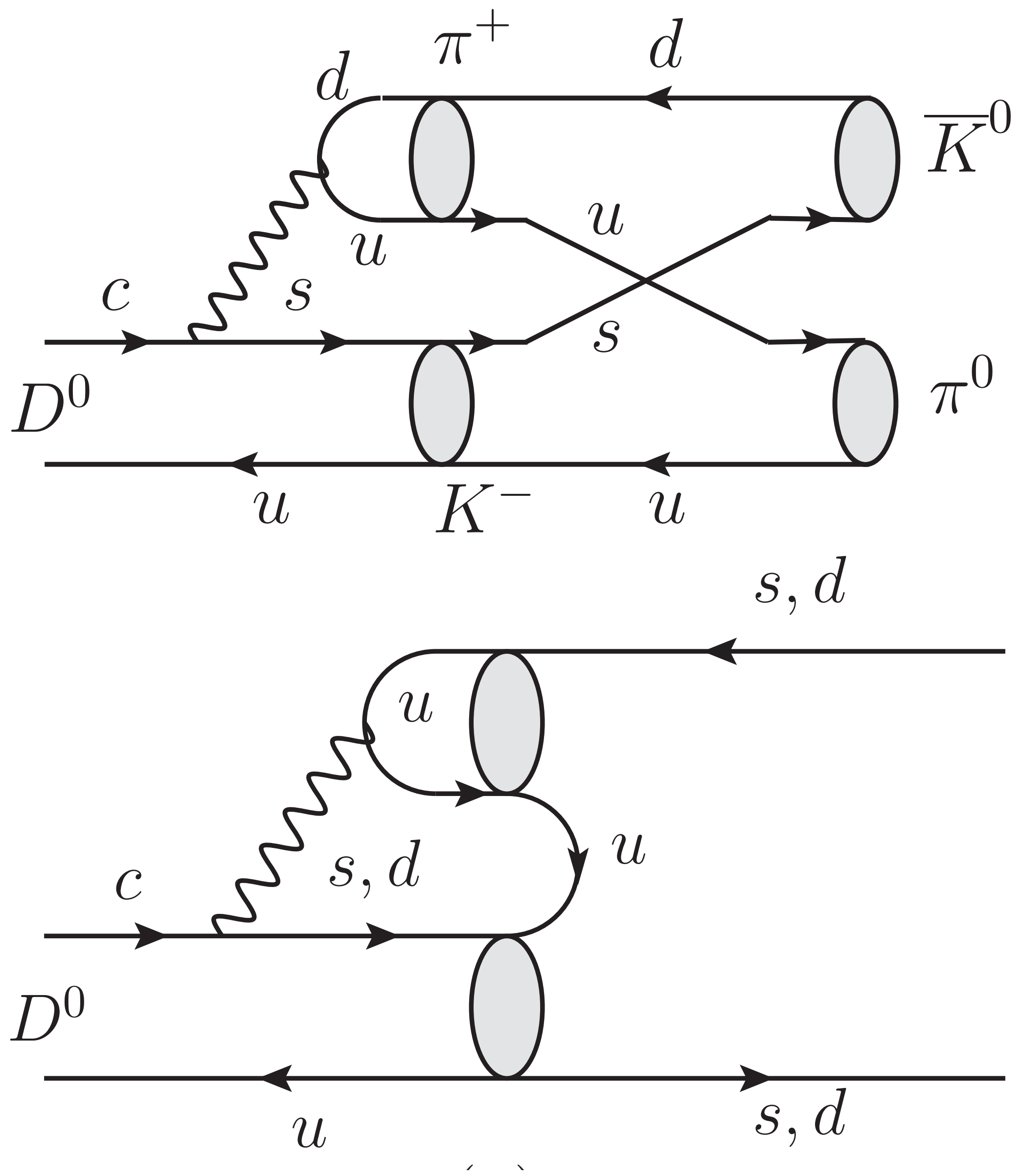
cf. B decays

- For example, the effective Wilson coefficients for the T and C amplitudes are given by:

extracted from data —
$$a_1(\bar{K}\pi) \approx 1.22 \quad \text{and} \quad a_2(\bar{K}\pi) \approx 0.82 e^{-i(151)^\circ}$$

naive factorization —
$$a_1 \simeq 1.09 \quad \text{and} \quad a_2 \simeq -0.11$$

- Both C and E can be enhanced by rescattering from T.



Cheng, CWC 2010

The $D \to \pi^+\pi^-$ and K^+K^- Decays

- These two SCS decay modes are closely related:

opposite in sign

➠ opposite in CPA

$$
\begin{aligned}
A_{\pi^+\pi^-} &= \frac{1}{2}(\lambda_d - \lambda_s)(T + E + \Delta P)_{\pi\pi} - \frac{1}{2}\lambda_b(T + E + \Sigma P)_{\pi\pi} \\
&\to \lambda_d(T + E) - \lambda_b \Sigma P \qquad [\mathrm{SU(3)\ limit}] \\
A_{K^+K^-} &= \frac{1}{2}(\lambda_s - \lambda_d)(T + E - \Delta P)_{KK} - \frac{1}{2}\lambda_b(T + E + \Sigma P)_{KK} \\
&\to \lambda_s(T + E) - \lambda_b \Sigma P \qquad [\mathrm{SU(3)\ limit}]
\end{aligned}
$$

where the penguin amplitudes can be safely ignored in BR calculations, and

$$
\begin{aligned}
\Sigma P &= (P + PE + PA)_d + (P + PE + PA)_s \\
\Delta P &= (P + PE + PA)_d - (P + PE + PA)_s \\
\lambda_q &= V_{cq}^* V_{uq}
\end{aligned}
$$

— sum of d- and s- penguins

— difference between d- and s- penguins

the quark involved in the penguin loop

- In the SU(3) limit, their amplitudes are the same in magnitude and their CPA's are opposite. ➠ sum rule: the sum of individual CPA's vanishes

Improved Sum Rule and Data

- In the U-spin limit, there is an improved sum rule:

$$\frac{a_{\rm CP}^{\rm dir}(D^0\to\pi^+\pi^-)}{a_{\rm CP}^{\rm dir}(D^0\to K^+K^-)} = -\frac{\Gamma(D^0\to K^+K^-)}{\Gamma(D^0\to\pi^+\pi^-)} < 0$$

Grossman, Kagan, Nir 2007
Pirtskhalava, Uttayarat 2012
Grossman, Robinson 2013

while data show that it is broken:

$$3.01^{+0.95}_{-5.95} \neq -2.81 \pm 0.06\ !$$

LHCb 2023
Schacht 2023

- The latest HFLAV data,

$$a_{\rm CP}^{\rm dir}(D^0\to\pi^+\pi^-) = (2.30\pm0.59)\times10^{-3}$$
$$a_{\rm CP}^{\rm dir}(D^0\to K^+K^-) = (0.44\pm0.54)\times10^{-3}$$

HFLAV 2023

also supports the same-sign CPA’s.

- Such a disagreement is beyond the naive SU(3) symmetry breaking of ~30%.

Large SU(3) Breaking

- For a long time, the BR's of $D \to \pi^+\pi^-, K^+K^-$ are known to deviate significantly from naive expectations: with negligible penguin amplitudes, the two modes have identical decay strength, but with different phase spaces.
  ➠ expect $\mathscr{B}(\pi^+\pi^-) > \mathscr{B}(K^+K^-)$
- Empirically, however, the ratio of their decay rates

$$\frac{\Gamma(K^+K^-)}{\Gamma(\pi^+\pi^-)} \simeq 2.8$$

  is noticeably larger than 1 in the SU(3) limit (neglecting phase space effects).
- It has been argued that such a ratio can be explained by SU(3) breaking on the amplitude of the order of

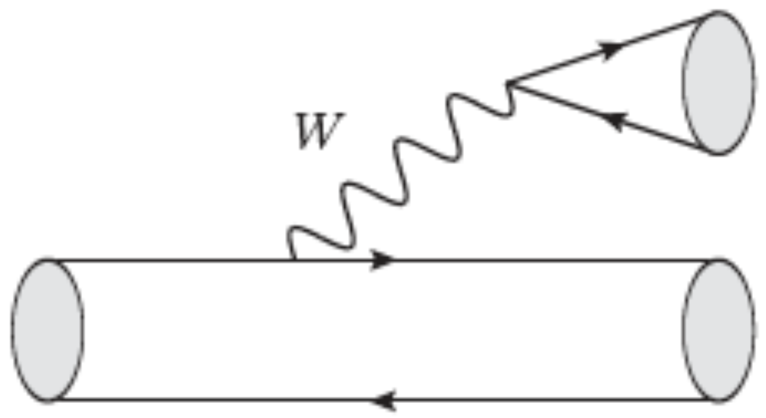
$$\varepsilon = \frac{m_s - m_d}{\Lambda_{\mathrm{QCD}}} \sim 0.3$$

Schacht 2023

Large SU(3) Breaking

- SU(3) breaking in T:

$$\frac{T(K^+K^-)}{T(\pi^+\pi^-)} \simeq \frac{f_K}{f_\pi}\frac{F_+^{DK}(m_K^2)}{F_+^{D\pi}(m_\pi^2)} \simeq 1.32$$



➠ however, not the complete story

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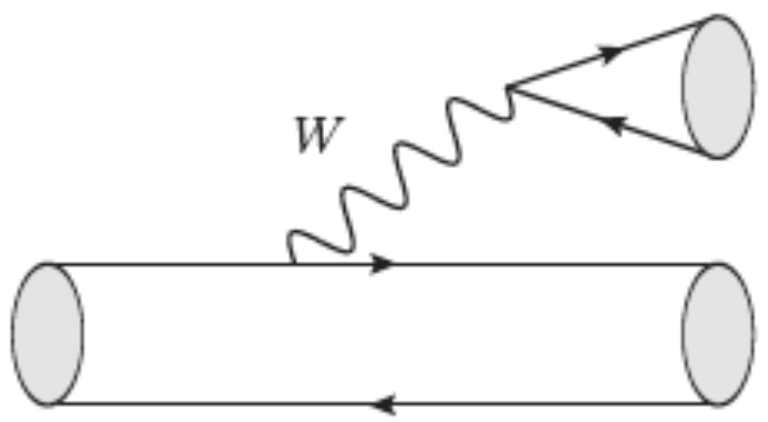


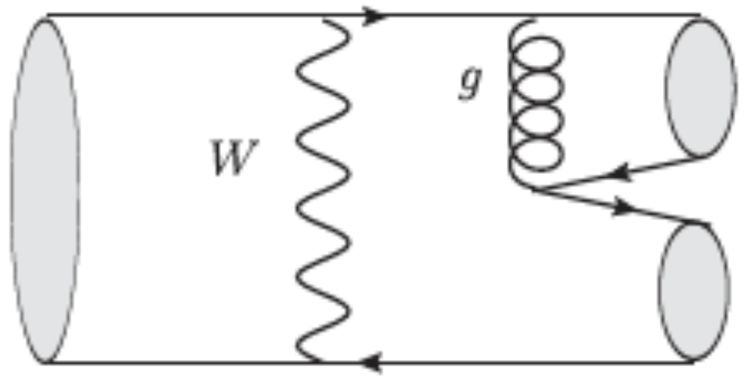
➠ however, not the complete story

- SU(3) breaking in E:

opposite sign between them

vanishing in SU(3) limit — $A(D \to K^0\overline{K^0}) = \lambda_d\,(E_d + 2PA_d) + \lambda_s\,(E_s + 2PA_s)$

diagrams of $c\bar{u} \to q\bar{q}$ $(q = d, s)$



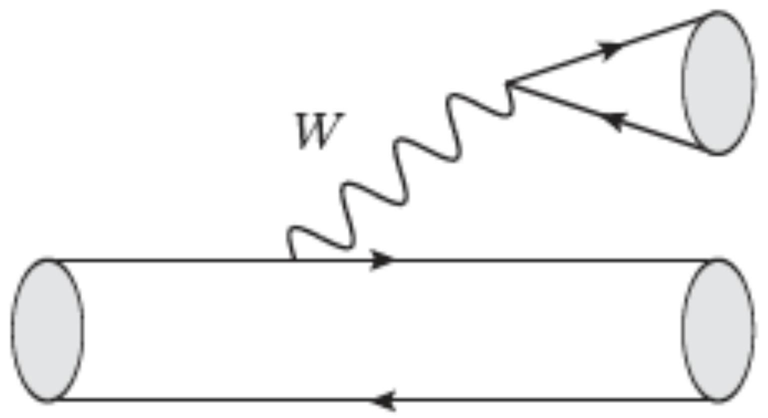
➠ needs different E_d and E_s to explain the nonzero rate:

$$\begin{cases} \mathrm{I}: & E_d = 1.10e^{i15.1^\circ}E, \quad E_s = 0.62e^{-i19.7^\circ}E \\ \mathrm{II}: & E_d = 1.10e^{i15.1^\circ}E, \quad E_s = 1.42e^{-i13.5^\circ}E \end{cases}$$

two possible solutions for symmetry breaking

Large SU(3) Breaking

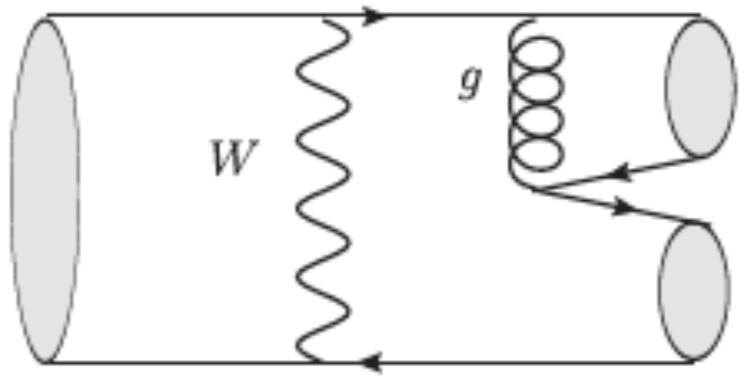
- SU(3) breaking in T:



$$\frac{T(K^+K^-)}{T(\pi^+\pi^-)} \simeq \frac{f_K}{f_\pi}\frac{F_+^{DK}(m_K^2)}{F_+^{D\pi}(m_\pi^2)} \simeq 1.32$$

➠ however, not the complete story

- SU(3) breaking in E:



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two possible solutions for symmetry breaking

- Agglutination of these effects leads to apparently large SU(3) breaking in the observed rates of K^+K^-, $\pi^+\pi^-$, $\pi^0\pi^0$ and $K^0\overline{K^0}$.

Results

- Our fit results are (in units of 10^{-3})

Cheng, CWC 2019

Decay mode	$\pi^+\pi^-$	$\pi^0\pi^0$	K^+K^-	K_SK_S
$\mathcal{B}_{\mathrm{SU(3)}}$	2.28 ± 0.02	1.50 ± 0.03	1.91 ± 0.02	0
$\mathcal{B}_{\cancel{\mathrm{SU(3)}}}$	1.47 ± 0.02	0.82 ± 0.02	4.03 ± 0.03	0.141 ± 0.007
$\mathcal{B}_{\mathrm{exp}}$	1.455 ± 0.024	0.826 ± 0.025	4.08 ± 0.06	0.141 ± 0.005

- With naive QCDF and tree-level matrix elements using LCSR, the perturbation approach gives

$$\mathcal{B}(\pi^+\pi^-)\big|_{\mathrm{LCSR}} = \left(1.40^{+1.53}_{-1.06}\right) \times 10^{-3}$$
$$\mathcal{B}(K^+K^-)\big|_{\mathrm{LCSR}} = \left(3.67^{+3.90}_{-2.69}\right) \times 10^{-3}$$

Lenz, Piscopo, Rusov 2024

also in agreement with data, though with large theory uncertainties coming from conservative estimates of missing contributions.

- The branching ratio puzzle can be resolved in either approach.

Tree-level CPV

- Direct CPA's in hadronic charm decays can occur even at the tree level, through the interference between T and C, C and E, or even between E's.

Chau, Cheng 1984
Cheng, CWC 2012

- As an example, $A(D \to K^0\overline{K^0}) = \lambda_d E_d + \lambda_s E_s$ (vanishing in the SU(3) limit) when neglecting the PA amplitudes, leading to the prediction

$$a_{\rm CP}^{\rm dir}(K_S K_S) = \frac{2\,{\rm Im}\,(\lambda_d \lambda_s^*)}{|\lambda_d|^2}\frac{{\rm Im}\,(E_d^* E_s)}{|E_d - E_s|^2} = 1.3\times 10^{-3}\frac{|E_d E_s|}{|E_d - E_s|^2}\sin\delta_{ds}$$

$$= \begin{cases} -1.05\times 10^{-3} \\ -1.99\times 10^{-3} \end{cases}$$

from the two possible solutions of $E_{d,s}$; a precise measurement of it will resolve the ambiguity

which are virtually unchanged when PA contributions are also included.

- In contrast, the factorization-assisted topological-amplitude (FAT) approach predicts 1.11×10^{-3}, opposite in sign.

Li, Lü, Yu 2012

- Latest HFLAV gives $a_{\rm CP}^{\rm dir}(K_S K_S) = -0.019 \pm 0.010$.

HFLAV 2023

Penguin-induced CPV

- Direct CPV does not occur at the tree level in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$.
  ➠ CPA arises from the interference between tree and penguin amplitudes

$$\Delta a_{\rm CP}^{\rm dir} = -1.30 \times 10^{-3} \left(\left| \frac{P_d + PE_d + PA_d}{T + E - \Delta P} \right|_{KK} \sin\delta_{KK} + \left| \frac{P_s + PE_s + PA_s}{T + E + \Delta P} \right|_{\pi\pi} \sin\delta_{\pi\pi} \right)$$

negligible

strong phases of the numerator relative to the denominator

- For the QCD penguin amplitudes, people employ QCDF, pQCD and LCSR:

$$\left(\frac{P}{T}\right)_{\pi\pi}^{\rm QCDF} \approx 0.23 e^{-i150^\circ} \qquad \left(\frac{P}{T}\right)_{KK}^{\rm QCDF} \approx 0.22 e^{-i150^\circ}$$

Cheng, CWC 2019

$$\left(\frac{P}{T}\right)_{\pi\pi}^{\rm pQCD} \approx 0.30 e^{i110^\circ} \qquad \left(\frac{P}{T}\right)_{KK}^{\rm pQCD} \approx 0.24 e^{i110^\circ}$$

Li, Lü, Yu 2012

a factor of ~3 smaller than above

$$\left|\frac{P}{T}\right|_{\pi\pi}^{\rm LCSR} = 0.089^{+0.042}_{-0.037} \qquad \left|\frac{P}{T}\right|_{KK}^{\rm LCSR} = 0.066^{+0.031}_{-0.029}$$

Khodjamirian, Petrov 2017
Lenz, Piscopo, Rusov 2024

showing some difference between LCSR and QCD-inspired approaches.

Estimates of $\Delta a_{\rm CP}^{\rm dir}$ Without FSI's

- The CPA difference is measured to be

$$\Delta a_{\rm CP}^{\rm dir}\big|_{\rm exp} = (-15.7 \pm 2.9) \times 10^{-4}$$

LHCb 2019

- Taking the above-mentioned $|P/T|$ ratios from LCSR and varying strong phases, one obtains an upper bound of

$$|\Delta a_{\rm CP}^{\rm dir}| \leq 2.4 \times 10^{-4}$$

Lenz, Piscopo, Rusov 2024

6 times smaller than the above measurement.
➠ violation of quark-hadron duality used in the calculation is hard to estimate

- Using QCDF for penguin amplitudes, we also have

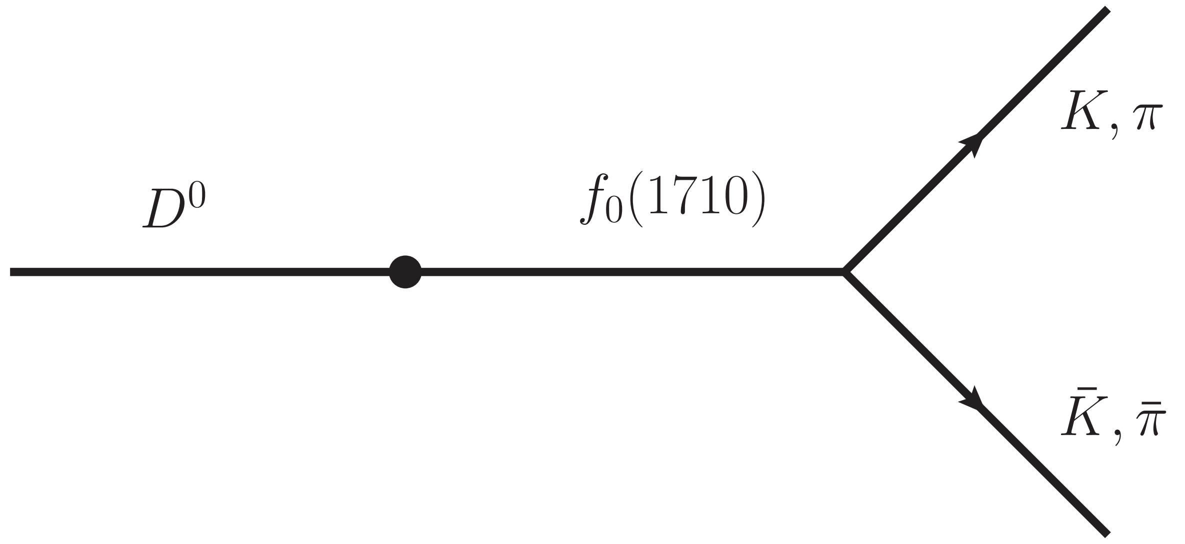
$$|\Delta a_{\rm CP}^{\rm dir}| \sim \mathcal{O}(10^{-4})$$

Cheng, CWC 2019

after further including PE and PA amplitudes, making the penguin strong phase even closer to $180°$.

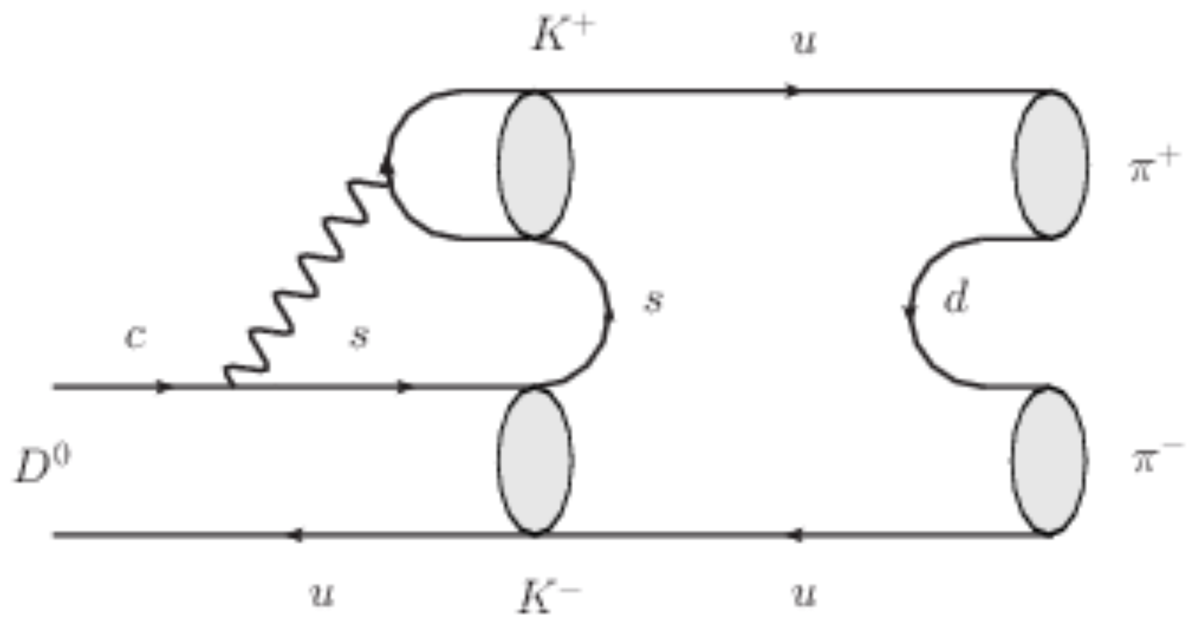
Final-State Interactions

- An isospin amplitude analysis incorporates nearby scalar resonances of $f_0(1710)$ and/or $f_0(1790)$ claims to be able to explain the $\Delta a_{\rm CP}^{\rm dir}$ data.
  ➠ need more precise experimental information on the BR's of such scalars to KK and $\pi\pi$ to verify the dynamical mechanism



Cheng, CWC 2010
Schacht, Soni 2022

- Although short-distance PE and PA amplitudes are negligibly small ($|PE/T| \sim 0.04$ and $|PA/T| \sim 0.02$), large long-distance contributions to $P + PE$ can possibly arise from $D^0 \to K^+K^-$ followed by a resonance-like final-state interaction (FSI) rescattering.



topologically $P + PE$-like

Cheng, CWC 2013

- It is thus conceivable to have $(P + PE)^{\rm LD} \sim E$, in both strength and strong phase. ➠ take $(P + PE)^{\rm LD}_{d,s} \approx (1.48 \pm 0.30)e^{i(120.9 \pm 30.0)^\circ}$

$\Delta a_{\rm CP}^{\rm dir}$ in Symmetry-based Approach

Cheng, CWC 2019

- Due to mainly the enhancement of $(PE)^{\rm LD}_{d,s}$, we have

$$\left(\frac{P_s + PE_s + PA_s + PE_s^{\rm LD}}{T + E^d + \Delta P}\right)_{\pi\pi} = 0.77 e^{i114^\circ}$$

— closer to 90°

$$\left(\frac{P_d + PE_d + PA_d + PE_d^{\rm LD}}{T + E^s - \Delta P}\right)_{KK} = \begin{cases} 0.45 e^{i117^\circ} & \text{solution I} \\ 0.45 e^{i110^\circ} & \text{solution II} \end{cases}$$

not so huge enhancement in magnitudes in comparison with QCDF calculation

Note that no attempt has been made to fit CPA's!

leading to the prediction

$$a_{\rm CP}^{\rm dir}(\pi^+\pi^-) = (0.80 \pm 0.22) \times 10^{-3}$$

$$a_{\rm CP}^{\rm dir}(K^+K^-) = \begin{cases} (-0.33 \pm 0.14) \times 10^{-3} & \text{Solution I} \\ (-0.44 \pm 0.12) \times 10^{-3} & \text{Solution II} \end{cases}$$

— still predict opposite-sign CPA's

$$\Rightarrow \quad \Delta a_{\rm CP}^{\rm dir} = \begin{cases} (-1.14 \pm 0.26) \times 10^{-3} & \text{Solution I} \\ (-1.25 \pm 0.25) \times 10^{-3} & \text{Solution II} \end{cases}$$

More About Final-State Interactions

- Based upon general arguments of CPT invariance and considering the S-wave S-matrix for the rescatttering of the two coupled channels of $\pi^+\pi^-$ and K^+K^- purely within SM, it is derived that

Bediaga, Frederico, Magalhães 2023

$$\sum_{f=\pi\pi,KK}\left(\left|\mathcal{A}_{D^0\to f}\right|^2-\left|\mathcal{A}_{\bar{D}^0\to f}\right|^2\right)=0$$

— due to CPT invariance for the two-channel scattering

also predicting opposite CPA's

$$a_{\rm CP}^{\rm dir}(\pi\pi)=-\frac{\Delta a_{\rm CP}^{\rm dir}\mathcal{B}(D^0\to K^+K^-)}{\sum\mathcal{B}}=(1.135\pm0.021)\times10^{-3}$$

$$a_{\rm CP}^{\rm dir}(KK)=\frac{\Delta a_{\rm CP}^{\rm dir}\mathcal{B}(D^0\to \pi^+\pi^-)}{\sum\mathcal{B}}=-(0.405\pm0.077)\times10^{-3}$$

$$\text{where } \sum\mathcal{B}\equiv\mathcal{B}(D^0\to K^+K^-)+\mathcal{B}(D^0\to\pi^+\pi^-)$$

$$\Delta a_{\rm CP}^{\rm dir}=(-1.31\pm0.20)\times10^{-3}$$

— using a particular set of parameter and strong phases

More About Final-State Interactions

- Although the importance of rescattering effects has been recognized, it is argued that a full picture should be obtained by employing dispersive relations.

Pich, Solomonidi, Silva 2023

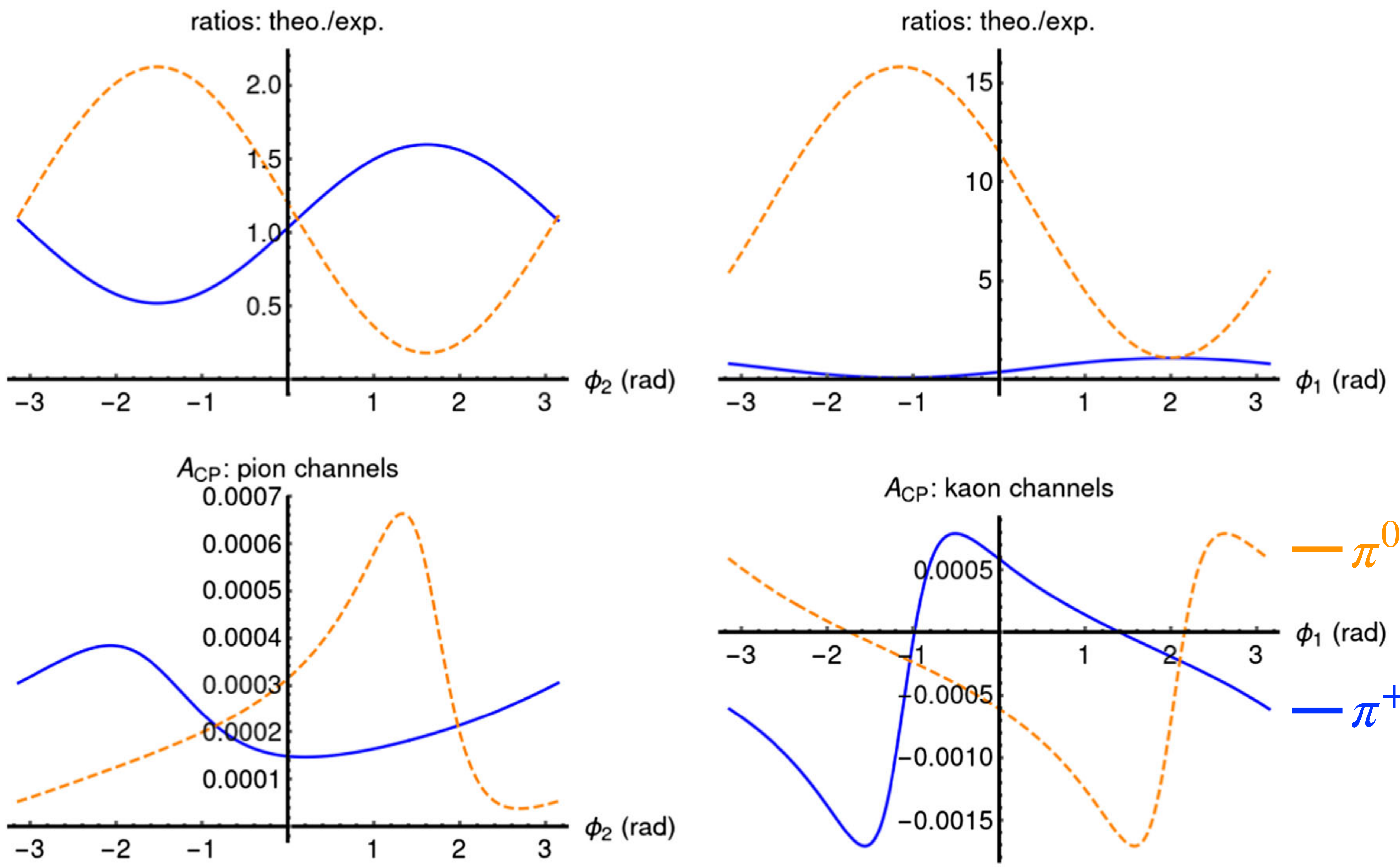
Only pion and kaon pairs are included in the analysis. There is a plan to include further inelasticities.

pion channels

kaon channels

ratios of theory and experimental BR's —

predicted CPA's are much below the experimental values —



— $\pi^0\pi^0$ or $K^0\overline{K^0}$ mode

— $\pi^+\pi^-$ or K^+K^- mode

- Rescattering effects turn out producing insufficient enhancement for CPA's.

Predictions / Data on BR's / CPA's of SCS PP Decays

Cheng, CWC 2019

	Mode	$\mathcal{B}$ ours	$\mathcal{B}$ exp	$a_{\rm CP}^{\rm dir}$ ours	$a_{\rm CP}^{\rm dir}$ Buccella+ 2019	$a_{\rm CP}^{\rm dir}$ HFLAV
$D^0 \to$	$\pi^+\pi^-$	1.47 ± 0.02	1.455 ± 0.024	0.80 ± 0.22	$1.17 \pm 0.20/1.18 \pm 0.20$	2.30 ± 0.59
	$\pi^0\pi^0$	0.82 ± 0.02	0.826 ± 0.025	0.82 ± 0.30	$0.04 \pm 0.09/0.79 \pm 0.10$	-0.3 ± 6.4
	$\pi^0\eta$	0.92 ± 0.02	0.63 ± 0.06	-0.05 ± 0.28		
	$\pi^0\eta'$	1.36 ± 0.03	0.92 ± 0.10	-0.15 ± 0.17		
	$\eta\eta$	1.82 ± 0.04	2.11 ± 0.19	-0.52 ± 0.07		
		2.11 ± 0.04		-0.65 ± 0.07		
	$\eta\eta'$	0.69 ± 0.03	1.01 ± 0.19	0.29 ± 0.21		
		1.63 ± 0.08		0.22 ± 0.15		
	K^+K^-	4.03 ± 0.03	4.08 ± 0.06	-0.33 ± 0.14	$-0.47 \pm 0.08/-0.46 \pm 0.08$	0.44 ± 0.54
		4.05 ± 0.05		-0.44 ± 0.12		
	K_SK_S	0.141 ± 0.007	0.141 ± 0.005	-1.05	$0.43 \pm 0.07/0.38 \pm 0.07$	-19 ± 10
		0.141 ± 0.007		-1.99		
$D^+ \to$	$\pi^+\pi^0$	0.93 ± 0.02	1.247 ± 0.033	0		
	$\pi^+\eta$	4.08 ± 0.16	3.77 ± 0.09	-0.63 ± 0.23		
	$\pi^+\eta'$	4.69 ± 0.08	4.97 ± 0.19	0.11 ± 0.18		
	K^+K_S	4.25 ± 0.10	3.04 ± 0.09	-0.30 ± 0.18	$-0.40 \pm 0.07/-0.26 \pm 0.05$	
$D_s^+ \to$	π^+K_S	1.27 ± 0.04	1.22 ± 0.06	0.42 ± 0.24	$-0.40 \pm 0.07/-0.36 \pm 0.07$	
	π^0K^+	0.56 ± 0.02	0.63 ± 0.21	0.91 ± 0.27	$0.48 \pm 0.06/-0.03 \pm 0.04$	
	$K^+\eta$	0.86 ± 0.03	1.77 ± 0.35	-0.81 ± 0.08		
	$K^+\eta'$	1.49 ± 0.08	1.8 ± 0.6	0.07 ± 0.25		

all in units of 10^{-3}

Predictions / Data on BR's / CPA's of SCS PP Decays

Cheng, CWC 2019

	Mode	$\mathcal{B}$ ours	$\mathcal{B}$ exp	a_{CP}^{dir} ours	a_{CP}^{dir} Buccella+ 2019	a_{CP}^{dir} HFLAV
$D^0 \to$	$\pi^+\pi^-$	1.47 ± 0.02	1.455 ± 0.024	0.80 ± 0.22	$1.17 \pm 0.20/1.18 \pm 0.20$	2.30 ± 0.59
	$\pi^0\pi^0$	0.82 ± 0.02	0.826 ± 0.025	0.82 ± 0.30	$0.04 \pm 0.09/0.79 \pm 0.10$	-0.3 ± 6.4
	$\pi^0\eta$	0.92 ± 0.02	0.63 ± 0.06	-0.05 ± 0.28		
	$\pi^0\eta'$	1.36 ± 0.03	0.92 ± 0.10	-0.15 ± 0.17		
	$\eta\eta$	1.82 ± 0.04	2.11 ± 0.19	-0.52 ± 0.07		
		2.11 ± 0.04		-0.65 ± 0.07		
	$\eta\eta'$	0.69 ± 0.03	1.01 ± 0.19	0.29 ± 0.21		
		1.63 ± 0.08		0.22 ± 0.15		
	K^+K^-	4.03 ± 0.03	4.08 ± 0.06	-0.33 ± 0.14	$-0.47 \pm 0.08/-0.46 \pm 0.08$	0.44 ± 0.54
		4.05 ± 0.05		-0.44 ± 0.12		
	K_SK_S	0.141 ± 0.007	0.141 ± 0.005	-1.05	$0.43 \pm 0.07/0.38 \pm 0.07$	-19 ± 10
		0.141 ± 0.007		-1.99		
$D^+ \to$	$\pi^+\pi^0$	0.93 ± 0.02	1.247 ± 0.033	0		
	$\pi^+\eta$	4.08 ± 0.16	3.77 ± 0.09	-0.63 ± 0.23		
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	π^0K^+	0.56 ± 0.02	0.63 ± 0.21	0.91 ± 0.27	$0.48 \pm 0.06/-0.03 \pm 0.04$	
	$K^+\eta$	0.86 ± 0.03	1.77 ± 0.35	-0.81 ± 0.08		
	$K^+\eta'$	1.49 ± 0.08	1.8 ± 0.6	0.07 ± 0.25		

modes of interest, with sufficiently large BR's and CPA's.

all in units of 10^{-3}

Predictions / Data on BR's / CPA's of SCS PP Decays

Cheng, CWC 2019

	Mode	$\mathcal{B}$ ours	$\mathcal{B}$ exp	$a_{\rm CP}^{\rm dir}$ ours	$a_{\rm CP}^{\rm dir}$ Buccella+ 2019	$a_{\rm CP}^{\rm dir}$ HFLAV
$D^0 \to$	$\pi^+\pi^-$	1.47 ± 0.02	1.455 ± 0.024	0.80 ± 0.22	$1.17 \pm 0.20/1.18 \pm 0.20$	2.30 ± 0.59
	$\pi^0\pi^0$	0.82 ± 0.02	0.826 ± 0.025	0.82 ± 0.30	$0.04 \pm 0.09/0.79 \pm 0.10$	-0.3 ± 6.4
	$\pi^0\eta$	0.92 ± 0.02	0.63 ± 0.06	-0.05 ± 0.28		
	$\pi^0\eta'$	1.36 ± 0.03	0.92 ± 0.10	-0.15 ± 0.17		
	$\eta\eta$	1.82 ± 0.04	2.11 ± 0.19	-0.52 ± 0.07		
		2.11 ± 0.04		-0.65 ± 0.07		
	$\eta\eta'$	0.69 ± 0.03	1.01 ± 0.19	0.29 ± 0.21		
		1.63 ± 0.08		0.22 ± 0.15		
	K^+K^-	4.03 ± 0.03	4.08 ± 0.06	-0.33 ± 0.14	$-0.47 \pm 0.08/-0.46 \pm 0.08$	0.44 ± 0.54
		4.05 ± 0.05		-0.44 ± 0.12		
	K_SK_S	0.141 ± 0.007	0.141 ± 0.005	-1.05	$0.43 \pm 0.07/0.38 \pm 0.07$	-19 ± 10
		0.141 ± 0.007		-1.99		
$D^+ \to$	$\pi^+\pi^0$	0.93 ± 0.02	1.247 ± 0.033	0		
	$\pi^+\eta$	4.08 ± 0.16	3.77 ± 0.09	-0.63 ± 0.23		
	$\pi^+\eta'$	4.69 ± 0.08	4.97 ± 0.19	0.11 ± 0.18		
	K^+K_S	4.25 ± 0.10	3.04 ± 0.09	-0.30 ± 0.18	$-0.40 \pm 0.07/-0.26 \pm 0.05$	
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	π^0K^+	0.56 ± 0.02	0.63 ± 0.21	0.91 ± 0.27	$0.48 \pm 0.06/-0.03 \pm 0.04$	
	$K^+\eta$	0.86 ± 0.03	1.77 ± 0.35	-0.81 ± 0.08		
	$K^+\eta'$	1.49 ± 0.08	1.8 ± 0.6	0.07 ± 0.25		

modes of interest, with sufficiently large BR's and CPA's.

predict $\Delta a_{\rm CP}^{\rm dir}(K^+K^- - \pi^+\pi^-)$ to be $(-1.14 \pm 0.26) \times 10^{-3}$ or $(-1.25 \pm 0.25) \times 10^{-3}$

all in units of 10^{-3}

Topological Diagrams for $D \to VP$ Decays

- In the case of $D \to VP$ decays, the spectator quark in the D meson may end up in P or V meson in the final state. Though of the same flavor topology, these two types of diagrams have no relation *a priori* and should be distinguished.
  - Less significant SU(3) breaking for $T_{V,P}$ and $C_{P,V}$. For example,

$$\frac{|T_V + E_P|_{\pi^+\rho^-}}{|T_V + E_P|_{K^+K^{*-}}} \simeq 1.08 \qquad \frac{|T_P + E_V|_{\pi^-\rho^+}}{|T_P + E_V|_{K^-K^{*+}}} \simeq 0.91$$

  - $A_{V,P}$ roughly in phase for small $\mathscr{B}(\pi^+\rho)$ and large $\mathscr{B}(\pi^+\omega)$ of D_s^+.
  - $C_{V,P}$ roughly in phase for small $\mathscr{B}(\pi^0\omega)$, sizable $\mathscr{B}(\eta\omega)$, and large $\mathscr{B}(\pi^0\rho^0)$ of D^0.
  - Analogous to the PP sector, SU(3) breaking is required in the d, s-type $E_{V,P}$ in order to explain nonzero rates of $D^0 \to K^0\overline{K^{*0}}$ and $\overline{K^0}K^{*0}$.

Predictions / Data on BR's / CPA's of SCS VP Decays

		$\mathcal{B}$			$a_{\rm CP}^{\rm dir}$		
	Mode	ours	Qin+ 2014	exp	ours	Qin+ 2014	HFLAV
$D^0 \to$	$\pi^+\rho^-$	5.12 ± 0.29	$4.74/4.66$	5.15 ± 0.25	0.77 ± 0.22	-0.03	
	$\pi^-\rho^+$	10.21 ± 0.91	$10.2/10.0$	10.1 ± 0.4	-0.14 ± 0.04	-0.01	
	$\pi^0\rho^0$	3.90 ± 0.26	$3.55/3.83$	3.86 ± 0.23	0.37 ± 0.15	-0.03	
	K^+K^{*-}	1.68 ± 0.11	$1.72/1.73$	1.65 ± 0.11	-0.75 ± 0.37	-0.01	
	K^-K^{*+}	4.43 ± 0.31	$4.37/4.37$	4.56 ± 0.21	0.15 ± 0.04	0	
	$K^0\bar{K}^{*0}$	0.27 ± 0.06	$1.1/1.1$	0.246 ± 0.048	-0.15 ± 0.21	-0.7	
	$\bar{K}^0K^{*0}$	0.32 ± 0.09	$1.1/1.1$	0.336 ± 0.063	-0.34 ± 0.16	-0.7	
	$\pi^0\omega$	0.12 ± 0.05	$0.85/0.18$	0.117 ± 0.035	-2.14 ± 0.95	0.02	
	$\pi^0\phi$	1.22 ± 0.04	$1.11/1.11$	1.20 ± 0.04	0	-0.0002	
	$\eta\omega$	2.25 ± 0.14	$2.4/2.0$	1.98 ± 0.18	-0.38 ± 0.10	-0.1	
	$\eta'\omega$	0.01 ± 0.00	$0.04/0.02$	$\ldots$	0.96 ± 0.66	2.2	
	$\eta\phi$	0.16 ± 0.02	$0.19/0.18$	0.167 ± 0.034	0	0.003	$-19 \pm 44 \pm 6$
	$\eta\rho^0$	0.59 ± 0.07	$0.54/0.45$	$\ldots$	0.10 ± 0.30	1.0	
	$\eta'\rho^0$	0.06 ± 0.01	$0.21/0.27$	$\ldots$	0.16 ± 0.22	-0.1	
$D^+ \to$	$\pi^+\rho^0$	0.61 ± 0.10	$0.42/0.58$	0.83 ± 0.15	2.20 ± 1.38	0.5	
	$\pi^0\rho^+$	4.53 ± 0.64	$2.7/2.5$	$\ldots$	0.49 ± 0.37	0.2	
	$\pi^+\omega$	0.26 ± 0.07	$0.95/0.80$	0.28 ± 0.06	0.74 ± 2.03	-0.05	
	$\pi^+\phi$	6.29 ± 0.20	$5.65/5.65$	5.68 ± 0.11	0	-0.0001	
	$\eta\rho^+$	1.02 ± 0.34	$0.7/2.2$	$\ldots$	1.78 ± 0.69	-0.6	
	$\eta'\rho^+$	1.03 ± 0.11	$0.7/0.8$	$\ldots$	0.08 ± 0.11	0.5	
	$K^+\bar{K}^{*0}$	3.82 ± 0.25	$3.61/3.60$	$3.83^{+0.14}_{-0.21}$	-1.06 ± 0.30	0.2	
	$\bar{K}K^{*+}$	9.80 ± 0.41	$11/11$	34 ± 16	0.10 ± 0.04	0.04	
D_s^+	π^+K^{*0}	3.65 ± 0.24	$2.52/2.35$	2.13 ± 0.36	1.05 ± 0.30	-0.1	
	π^0K^{*+}	1.02 ± 0.07	$0.8/1.0$	$\ldots$	1.15 ± 0.40	-0.2	
	$K^+\rho^0$	2.10 ± 0.10	$1.9/2.5$	2.5 ± 0.4	-0.08 ± 0.07	0.3	
	$K^0\rho^+$	11.47 ± 0.48	$9.1/9.6$	$\ldots$	-0.08 ± 0.04	0.3	
	ηK^{*+}	0.64 ± 0.20	$0.2/0.2$	$\ldots$	0.10 ± 0.48	1.1	
	$\eta' K^{*+}$	0.33 ± 0.02	$0.2/0.2$	$\ldots$	-0.12 ± 0.13	-0.5	
	$K^+\omega$	2.12 ± 0.10	$0.6/0.07$	0.87 ± 0.25	0.01 ± 0.08	-2.3	
	$K^+\phi$	0.12 ± 0.02	$0.166/0.166$	0.182 ± 0.041	0	-0.8	

Cheng, CWC 2019

all in units of 10^{-3}

Cheng, CWC 2019

modes of interest, with sufficiently large BR's and CPA's.

	Mode	$\mathcal{B}$ ours	$\mathcal{B}$ Qin+ 2014	$\mathcal{B}$ exp	a_{CP}^{dir} ours	a_{CP}^{dir} Qin+ 2014	a_{CP}^{dir} HFLAV
$D^0 \to$	$\pi^+\rho^-$	5.12 ± 0.29	$4.74/4.66$	5.15 ± 0.25	0.77 ± 0.22	-0.03	
	$\pi^-\rho^+$	10.21 ± 0.91	$10.2/10.0$	10.1 ± 0.4	-0.14 ± 0.04	-0.01	
	$\pi^0\rho^0$	3.90 ± 0.26	$3.55/3.83$	3.86 ± 0.23	0.37 ± 0.15	-0.03	
	K^+K^{*-}	1.68 ± 0.11	$1.72/1.73$	1.65 ± 0.11	-0.75 ± 0.37	-0.01	
	K^-K^{*+}	4.43 ± 0.31	$4.37/4.37$	4.56 ± 0.21	0.15 ± 0.04	0	
	$K^0\bar{K}^{*0}$	0.27 ± 0.06	$1.1/1.1$	0.246 ± 0.048	-0.15 ± 0.21	-0.7	
	$\bar{K}^0K^{*0}$	0.32 ± 0.09	$1.1/1.1$	0.336 ± 0.063	-0.34 ± 0.16	-0.7	
	$\pi^0\omega$	0.12 ± 0.05	$0.85/0.18$	0.117 ± 0.035	-2.14 ± 0.95	0.02	
	$\pi^0\phi$	1.22 ± 0.04	$1.11/1.11$	1.20 ± 0.04	0	-0.0002	
	$\eta\omega$	2.25 ± 0.14	$2.4/2.0$	1.98 ± 0.18	-0.38 ± 0.10	-0.1	
	$\eta'\omega$	0.01 ± 0.00	$0.04/0.02$	$\dots$	0.96 ± 0.66	2.2	
	$\eta\phi$	0.16 ± 0.02	$0.19/0.18$	0.167 ± 0.034	0	0.003	$-19 \pm 44 \pm 6$
	$\eta\rho^0$	0.59 ± 0.07	$0.54/0.45$	$\dots$	0.10 ± 0.30	1.0	
	$\eta'\rho^0$	0.06 ± 0.01	$0.21/0.27$	$\dots$	0.16 ± 0.22	-0.1	
$D^+ \to$	$\pi^+\rho^0$	0.61 ± 0.10	$0.42/0.58$	0.83 ± 0.15	2.20 ± 1.38	0.5	
	$\pi^0\rho^+$	4.53 ± 0.64	$2.7/2.5$	$\dots$	0.49 ± 0.37	0.2	
	$\pi^+\omega$	0.26 ± 0.07	$0.95/0.80$	0.28 ± 0.06	0.74 ± 2.03	-0.05	
	$\pi^+\phi$	6.29 ± 0.20	$5.65/5.65$	5.68 ± 0.11	0	-0.0001	
	$\eta\rho^+$	1.02 ± 0.34	$0.7/2.2$	$\dots$	1.78 ± 0.69	-0.6	
	$\eta'\rho^+$	1.03 ± 0.11	$0.7/0.8$	$\dots$	0.08 ± 0.11	0.5	
	$K^+\bar{K}^{*0}$	3.82 ± 0.25	$3.61/3.60$	$3.83^{+0.14}_{-0.21}$	-1.06 ± 0.30	0.2	
	$\bar{K}K^{*+}$	9.80 ± 0.41	$11/11$	34 ± 16	0.10 ± 0.04	0.04	
D_s^+	π^+K^{*0}	3.65 ± 0.24	$2.52/2.35$	2.13 ± 0.36	1.05 ± 0.30	-0.1	
	π^0K^{*+}	1.02 ± 0.07	$0.8/1.0$	$\dots$	1.15 ± 0.40	-0.2	
	$K^+\rho^0$	2.10 ± 0.10	$1.9/2.5$	2.5 ± 0.4	-0.08 ± 0.07	0.3	
	$K^0\rho^+$	11.47 ± 0.48	$9.1/9.6$	$\dots$	-0.08 ± 0.04	0.3	
	ηK^{*+}	0.64 ± 0.20	$0.2/0.2$	$\dots$	0.10 ± 0.48	1.1	
	$\eta'K^{*+}$	0.33 ± 0.02	$0.2/0.2$	$\dots$	-0.12 ± 0.13	-0.5	
	$K^+\omega$	2.12 ± 0.10	$0.6/0.07$	0.87 ± 0.25	0.01 ± 0.08	-2.3	
	$K^+\phi$	0.12 ± 0.02	$0.166/0.166$	0.182 ± 0.041	0	-0.8	

all in units of 10^{-3}

Cheng, CWC 2019

	Mode	$\mathcal{B}$ ours	$\mathcal{B}$ Qin+ 2014	$\mathcal{B}$ exp	a_{CP}^{dir} ours	a_{CP}^{dir} Qin+ 2014	a_{CP}^{dir} HFLAV
$D^0 \to$	$\pi^+\rho^-$	5.12 ± 0.29	$4.74/4.66$	5.15 ± 0.25	0.77 ± 0.22	-0.03	
	$\pi^-\rho^+$	10.21 ± 0.91	$10.2/10.0$	10.1 ± 0.4	-0.14 ± 0.04	-0.01	
	$\pi^0\rho^0$	3.90 ± 0.26	$3.55/3.83$	3.86 ± 0.23	0.37 ± 0.15	-0.03	
	K^+K^{*-}	1.68 ± 0.11	$1.72/1.73$	1.65 ± 0.11	-0.75 ± 0.37	-0.01	
	K^-K^{*+}	4.43 ± 0.31	$4.37/4.37$	4.56 ± 0.21	0.15 ± 0.04	0	
	$K^0\bar{K}^{*0}$	0.27 ± 0.06	$1.1/1.1$	0.246 ± 0.048	-0.15 ± 0.21	-0.7	
	$\bar{K}^0K^{*0}$	0.32 ± 0.09	$1.1/1.1$	0.336 ± 0.063	-0.34 ± 0.16	-0.7	
	$\pi^0\omega$	0.12 ± 0.05	$0.85/0.18$	0.117 ± 0.035	-2.14 ± 0.95	0.02	
	$\pi^0\phi$	1.22 ± 0.04	$1.11/1.11$	1.20 ± 0.04	0	-0.0002	
	$\eta\omega$	2.25 ± 0.14	$2.4/2.0$	1.98 ± 0.18	-0.38 ± 0.10	-0.1	
	$\eta'\omega$	0.01 ± 0.00	$0.04/0.02$	$\ldots$	0.96 ± 0.66	2.2	
	$\eta\phi$	0.16 ± 0.02	$0.19/0.18$	0.167 ± 0.034	0	0.003	$-19 \pm 44 \pm 6$
	$\eta\rho^0$	0.59 ± 0.07	$0.54/0.45$	$\ldots$	0.10 ± 0.30	1.0	
	$\eta'\rho^0$	0.06 ± 0.01	$0.21/0.27$	$\ldots$	0.16 ± 0.22	-0.1	
$D^+ \to$	$\pi^+\rho^0$	0.61 ± 0.10	$0.42/0.58$	0.83 ± 0.15	2.20 ± 1.38	0.5	
	$\pi^0\rho^+$	4.53 ± 0.64	$2.7/2.5$	$\ldots$	0.49 ± 0.37	0.2	
	$\pi^+\omega$	0.26 ± 0.07	$0.95/0.80$	0.28 ± 0.06	0.74 ± 2.03	-0.05	
	$\pi^+\phi$	6.29 ± 0.20	$5.65/5.65$	5.68 ± 0.11	0	-0.0001	
	$\eta\rho^+$	1.02 ± 0.34	$0.7/2.2$	$\ldots$	1.78 ± 0.69	-0.6	
	$\eta'\rho^+$	1.03 ± 0.11	$0.7/0.8$	$\ldots$	0.08 ± 0.11	0.5	
	$K^+\bar{K}^{*0}$	3.82 ± 0.25	$3.61/3.60$	$3.83^{+0.14}_{-0.21}$	-1.06 ± 0.30	0.2	
	$\bar{K}K^{*+}$	9.80 ± 0.41	$11/11$	34 ± 16	0.10 ± 0.04	0.04	
D_s^+	π^+K^{*0}	3.65 ± 0.24	$2.52/2.35$	2.13 ± 0.36	1.05 ± 0.30	-0.1	
	π^0K^{*+}	1.02 ± 0.07	$0.8/1.0$	$\ldots$	1.15 ± 0.40	-0.2	
	$K^+\rho^0$	2.10 ± 0.10	$1.9/2.5$	2.5 ± 0.4	-0.08 ± 0.07	0.3	
	$K^0\rho^+$	11.47 ± 0.48	$9.1/9.6$	$\ldots$	-0.08 ± 0.04	0.3	
	ηK^{*+}	0.64 ± 0.20	$0.2/0.2$	$\ldots$	0.10 ± 0.48	1.1	
	$\eta'K^{*+}$	0.33 ± 0.02	$0.2/0.2$	$\ldots$	-0.12 ± 0.13	-0.5	
	$K^+\omega$	2.12 ± 0.10	$0.6/0.07$	0.87 ± 0.25	0.01 ± 0.08	-2.3	
	$K^+\phi$	0.12 ± 0.02	$0.166/0.166$	0.182 ± 0.041	0	-0.8	

modes of interest, with sufficiently large BR's and CPA's.

predict $\Delta a_{CP}^{dir}(K^+K^{*-} - \pi^+\rho^-)$ to be $(-1.52 \pm 0.43) \times 10^{-3}$

all in units of 10^{-3}

Summary

- The theory community has been facing serious challenges in the charm sector, mainly due to the lack of a good effective theory.
- A reliable, first-principle calculation of long-distance, nonperturbative dynamics is missing, same for both perturbation- and symmetry-based approaches.
- Flavor SU(3) symmetry-breaking effects are introduced in a data-driven way.
- It is still an open question whether the observed CPA's can be accommodated within the SM or not.
- "New physics" in the CP violation of the D mesons here could mean either new mechanisms within the SM or physics beyond the SM.
- At the moment, precision measurements for the CPA's of more decay modes (also the BR's of some less well-determined modes) will check theory predictions and tell us which theoretical approach is closer to the true story.

Enjoy FPCP
(Fun Puzzles in Charm Physics)!