Theory on CKM and heavy quark decay

Oliver Witzel

Universität Siegen / TP1 Theoretical Particle Physics / CPPS Center for Particle Physics Siegen

22nd Conference on Flavor Physics and CP Violation (FPCP 2024)

Bangkok, Thailand · May 28, 2024

Cabibbo-Kobayashi-Maskawa (CKM) matrix

- Masses and quark mixing in the SM arise from Yukawa interactions with the Higgs condensate
- CKM matrix describes probability for transition of flavor j to flavor i
- Fundamental parameters of the SM
- Extracted combining experiment and theory
- PDG [Workman et al. PTEP (2022) 083C01]

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 0.97370(14) & 0.2245(8) & 0.00382(24) \\ 0.221(4) & 0.987(11) & 0.0408(14) \\ 0.0080(3) & 0.0388(11) & 1.013(30) \end{bmatrix}$$

$$\frac{|\delta V_{CKM}|}{|V_{CKM}|} = \begin{bmatrix} 0.014 & 0.35 & 6.3 \\ 1.8 & 1.1 & 3.4 \\ 3.8 & 2.8 & 3.0 \end{bmatrix} \%$$

Cabibbo-Kobayashi-Maskawa (CKM) matrix

- CKM matrix in the SM is a 3×3 unitary matrix
  - → Different parametrizations: e.g. 3 mixing angles and a CP-violating phase

$$V_{\text{CKM}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

- Exploit experimental observation: $s_{13} \ll s_{23} \ll s_{12} \ll 1$
- Wolfenstein parametrization

$$V_{\text{CKM}} = \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

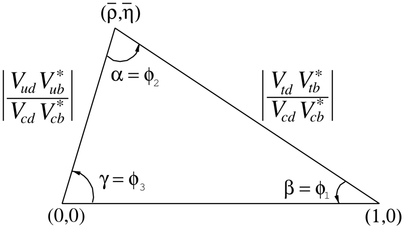
with $s_{12} = \lambda = \frac{|V_{us}|}{|V_{ud}|^2+|V_{us}|^2}$ $\qquad s_{23} = A\lambda^2 = \lambda\left|\frac{V_{cb}}{V_{us}}\right|$

$$s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar\rho+i\bar\eta)\sqrt{1-A^2\lambda^4}}{\sqrt{1-\lambda^2}(1-A^2\lambda^4(\bar\rho+i\bar\eta))} \qquad (\bar\rho + i\bar\eta) = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

→ Unitary in all order of λ

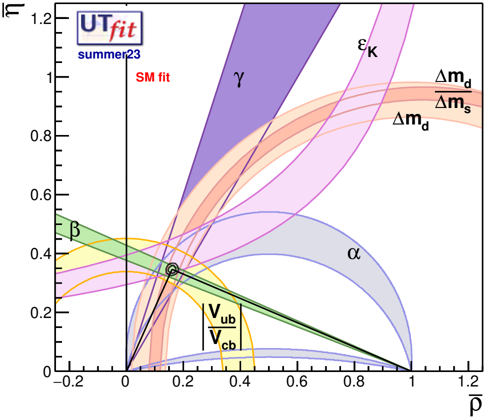
Unitarity triangle

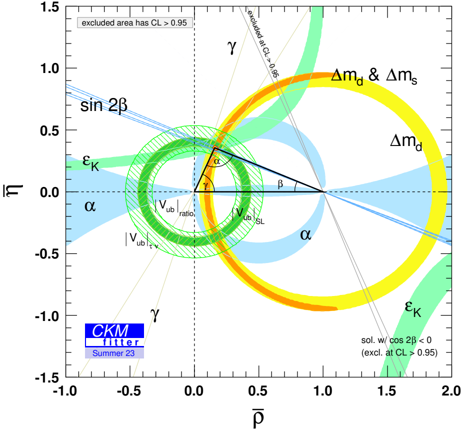
- Can define six different unitarity triangles
  - → Most commonly used: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}v_{tb}^* = 0$
  - → Divide all sides by $V_{cd}V_{cb}^*$
- Vertices are exactly $(0,0)$, $(1,0)$, $(\bar\rho, \bar\eta)$
- Overconstrain CKM elements to test and constrain the SM



[PDG, Workman et al. PTEP (2022) 083C01]

Unitarity triangle





[http://www.utfit.org]

[http://ckmfitter.in2p3.fr]

→ Dividing all sides by $V_{cd} V_{cb}^*$

V_{cd}

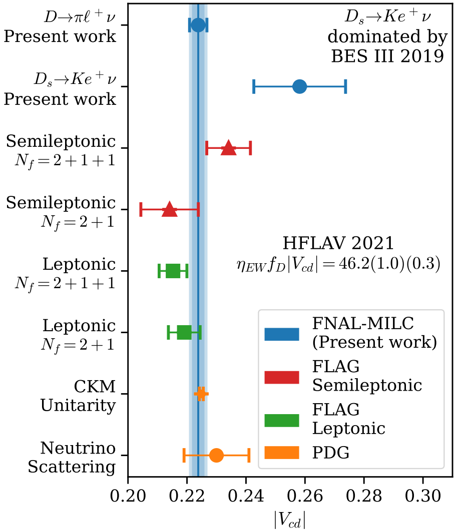
Determination of V_{cd}

- PDG averages three different determinations [PDG, Workman et al. PTEP (2022) 083C01]
  - → Earlier determination based on neutrino scattering data
    $|V_{cd}|^{\nu}_{PDG} = 0.230 \pm 0.011$
  - → Leptonic $D^+ \to \{\mu^+\nu_\mu, \tau^+\nu_\tau\}$ decays: LQCD (FNAL/MILC, ETMC) + experiment (BESIII, CLEO)
    $|V_{cd}|^{f_D}_{PDG} = 0.2181 \pm 0.0050$
  - → Semileptonic $D \to \pi\ell\nu$: LQCD (ETMC) + experiment (BaBar, BESIII, CLEO-c, Belle)
    $|V_{cd}|^{D\pi(0)}_{PDG} = 0.2330 \pm 0.014$

- $|V_{cd}|_{PDG} = 0.221 \pm 0.004$

- New $D \to \pi\ell\nu$ and $D_s \to K\ell\nu$ determinations by FNAL/MILC exploiting full q^2 dependence
  $|V_{cd}|^{D\pi}_{FNAL/MILC} = 0.2238 \pm 0.0029$ (with BaBar, BESIII, CLEO-c, Belle data)
  $|V_{cd}|^{D_sK}_{FNAL/MILC} = 0.258 \pm 0.015$ (with BESIII data)
  [FNAL/MILC PRD107(2023)094516]

Determination of V_{cd}

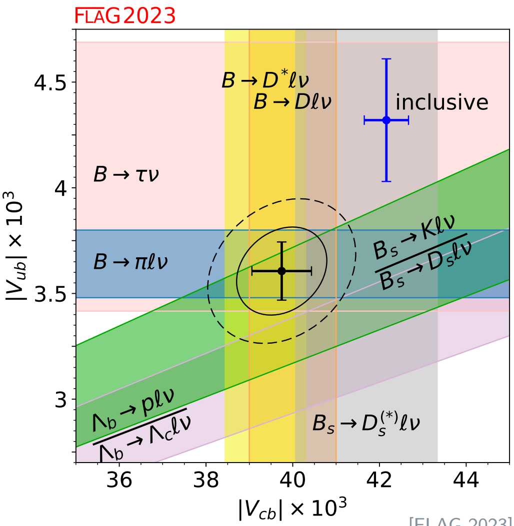


[FNAL/MILC PRD107(2023)094516]

V_{cb}

How to determine V_{cb}?

- Leptonic $B_c \to \tau\nu_\tau$ decays
  - → Experimentally very challenging
- Semileptonic decays
  - → B or B_s initial state
  - → Inclusive decays
  - → Exclusive decays
    - hadronic pseudoscalar final state
    - hadronic vector final state
- Long standing $2-3\sigma$ discrepancy between inclusive and exclusive



[FLAG 2023]

Inclusive and exclusive decays

- Study $b \to c\ell\nu_\ell$ transition via
- Charged SM decay with W exchange
- $\ell = \mu, e$ because τ more sensitive to new physics effects

- Inclusive decays

$$B \to X_c \ell\nu_\ell$$

→ e^+e^- B factories operating at $\Upsilon(4s)$
→ HQE and fit to lepton moments
→ New LQCD developments
[Hashimoto PTEP(2017)053B03]
[Hansen, Meyer, Robaina PRD96(2017)094513]
[Bailas et al. PTEP(2020)043B07]
[Gambino, Hashimoto PRL 125(2020)032001]
[Barone et al. arXiv:2305.14092]...

- Exclusive decays

$$B \to D^{(*)}\ell\nu_\ell \text{ or } B_s \to D_s^{(*)}\ell\nu_\ell$$

→ e^+e^- B factories and LHCb
→ Form factors (LQCD, LCSR)

Inclusive decays

- Systematic expansion (OPE) of the total semileptonic decay width in powers of Λ/m_b

  → $m_b \gg \Lambda_{\text{QCD}}$

$$\mathcal{B} = |V_{cb}|^2 \left[\Gamma(b \to c\ell\nu_\ell) + \frac{1}{m_{c,b}} + \alpha_s + \ldots \right]$$

- OPE does not allow point-by-point prediction

  → Converges however if integrated over large phase space

$$\int d\Phi\; w^n(\nu, p_\ell, p_\nu) \frac{d\Gamma}{d\Phi} \quad \text{with} \quad \nu = p_B/m_B$$

- Weight functions

$w = (p_\ell + p_\nu)^2 = q^2$	4-momentum transfer squared
$w = (m_B\nu - q)^2 = M_X^2$	invariant mass squared
$w = (\nu \cdot p_\ell) = E_\ell^B$	lepton energy

Determining V_{cb}^{incl}

- Established inclusive determination is based on spectral moments (hadronic mass moments, lepton energy moments, ...)

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}\left(\frac{1}{m_b^4}\right)$$

→ $d\Gamma$ calculated perturbatively up to $\mathcal{O}(\alpha_s^3)$ [Fael, Schönwald, Steinhauser PRD 104 (2021) 016003]

→ μ_π^2, μ_G^2, ρ_D^3, ρ_{LS}^3 nonperturbative dynamics fitted from data

→ Large number of HQE parameters

- Semileptonic fit to experimentally measured spectral moments yields

→ Including 3-loop α_s corrections [Bordone, Capdevila, Gambino PLB 822 (2021) 136679]

$$|V_{cb}| = (42.16 \pm 0.51) \cdot 10^{-3} \qquad (1.2\% \text{ uncertainty})$$

- Challenging to improve due to large number of higher order terms

Determining V_{cb}^{incl} using q^2 moments

- Reduce number of terms exploiting reparametrization invariance (RPI) [Fael, Mannel, Vos JHEP 02 (2019) 177]
  → Not all observables are RPI, $\langle (q^2)^n \rangle$ moments are RPI (no dependence on ν)

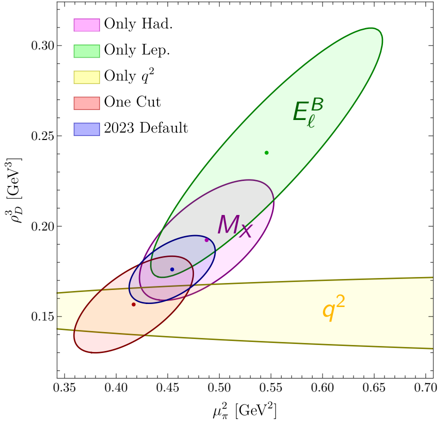
- Measurements of q^2 moments by [Belle PRD 104 (2021) 112011] and [Belle II PRD 107 (2023) 072002]

- First determination of V_{cb} using q^2 moments
  [Bernlochner et al. JHEP 10 (2022) 068]
  → Including contributions up to $1/m_b^4$, correction up to α_s

$$|V_{cb}| = (41.69 \pm 0.63) \cdot 10^{-3} \qquad \text{(1.5\% uncertainty)}$$

- Simultaneous extraction using all moments
  [Finauri, Gambino JHEP 02 (2024) 206]

$$|V_{cb}| = (41.97 \pm 0.48) \cdot 10^{-3} \qquad \text{(1.1\% uncertainty)}$$



Determining $|V_{cb}|^{\text{excl}}$

- Heavy-to-heavy transition ⇝ HQET relations

- Available channels
  → $B \to D\ell\nu$
  → $B_s \to D_s\ell\nu$
  → $B \to D^*\ell\nu$
  → $B_s \to D_s^*\ell\nu$

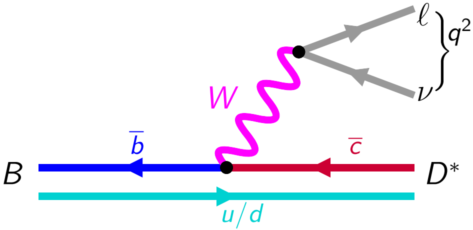
pseudoscalar final states

vector final states

- D^* and D_s^* suitable for using the narrow width approximation
  → Treat as QCD-stable particle

- $B \to D^*\ell\nu$ experimentally preferred (BaBar, Belle, Belle II, LHCb)
  → B_s decays only measured at LHCb

Exclusive semi-leptonic decays on the lattice



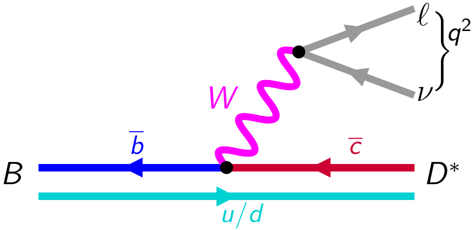
$$q^2 = M_B^2 + M_{D^*}^2 - 2M_B E_{D^*}$$

- Treat D^* as QCD-stable particle (narrow-width approximation)
- Conventionally parametrized placing the B meson at rest in terms of

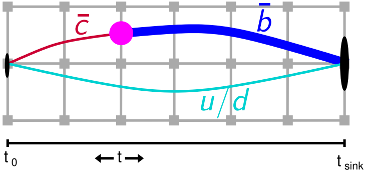
$$\frac{d\Gamma(B \to D^*\ell\nu)}{dq^2} = \mathcal{K}_{D^*}(q^2, m_\ell) \cdot |\mathcal{F}(q^2)|^2 \cdot |V_{cb}|^2$$

experiment known theory input (nonperturbative) CKM

Exclusive semi-leptonic decays on the lattice



$$q^2 = M_B^2 + M_{D^*}^2 - 2 M_B E_{D^*}$$



- Nonperturbative input
  - → Parametrizes interactions due to the (nonperturbative) strong force
  - → Use operator product expansion (OPE) to identify short distance contributions
  - → Calculate the flavor changing currents as point-like operators using lattice QCD
- Calculate hadronic matrix elements for the flavor changing currents V^μ and A^μ in terms of the form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, and $A_2(q^2)$

Exclusive semi-leptonic decays: $B \to D^*\ell\nu$

$$\langle D^*(k,\varepsilon_\nu)|\mathcal{V}^\mu|B(p)\rangle = V(q^2)\frac{2i\varepsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma}{M_B + M_D^*}$$

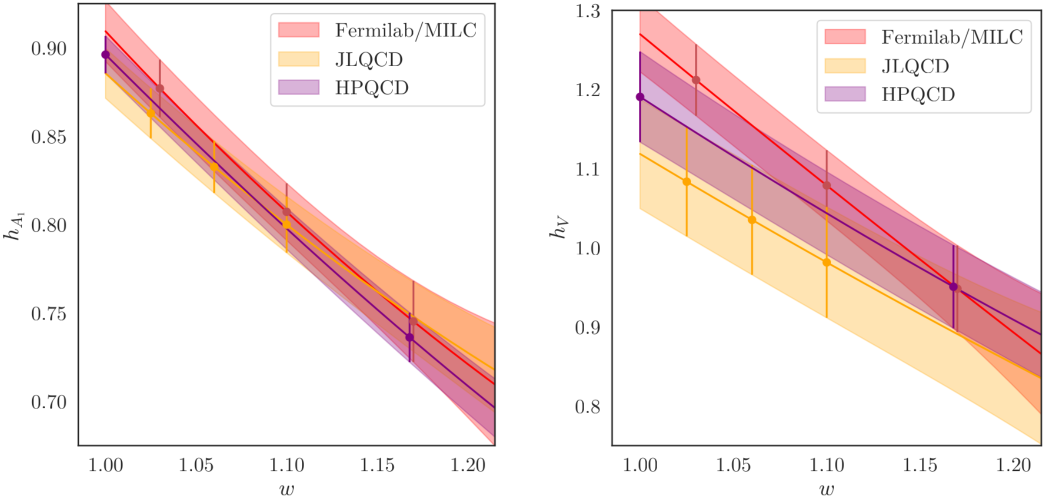
$$\langle D^*(k,\varepsilon_\nu)|\mathcal{A}^\mu|B(p)\rangle = A_0(q^2)\frac{2M_D^*\varepsilon^*\cdot q}{q^2}q^\mu + A_1(q^2)(M_B + M_{D^*})\left[\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] - A_2(q^2)\frac{\varepsilon^*\cdot q}{M_B + M_D^*}\left[k^\mu + p^\mu - \frac{M_B^2 - M_{D^*}^2}{q^2}q^\mu\right]$$

- Determine the four form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$ or in HQE convention $h_V(w)$, $h_{A_0}(w)$, $h_{A_1}(w)$, $h_{A_2}(w)$ with $w = v_{D^*}\cdot v_B$

First lattice calculations over the full q^2 range

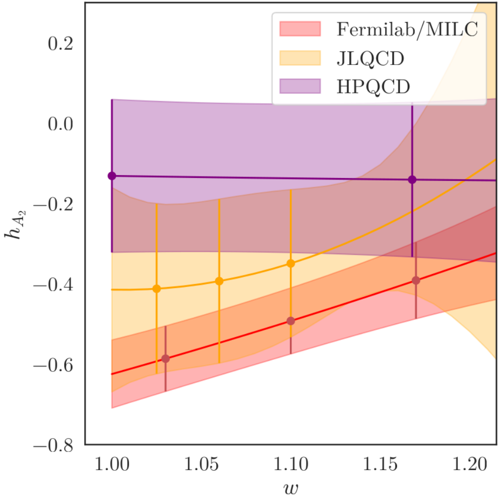
- $B \to D^*\ell\nu$
  - → 2021 Fermilab/MILC [Bazavov et al. EPJC 82 (2022) 1141]
  - → 2023 HPQCD [Harrison, Davies, arXiv:2304.03137]
  - → 2023 JLQCD [Y. Aoki et al. PRD 109 (2024) 074503]
  - → Preliminary LANL/SWME [Jang et al. PoS Lattice2019 (2020) 056]
- $B_s \to D_s^*\ell\nu$
  - → 2022 HPQCD [Harrison, Davies PRD105(2022) 094506][arXiv:2304.03137]
- Some tension in the shape of the form factors
  - → Further scrutiny required

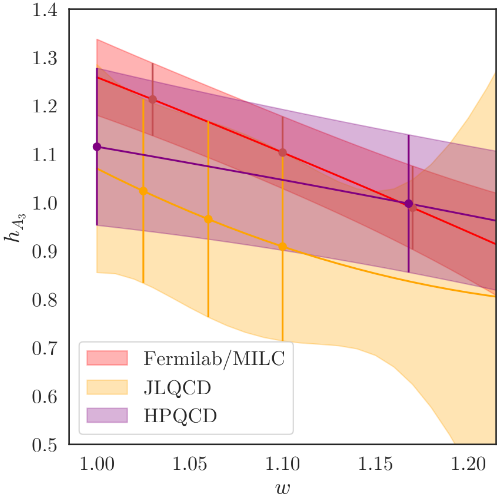
Comparison LQCD $B \to D^*\ell\nu_\ell$ form factors



[Plots/fits: A. Vaquero CKM 2023]

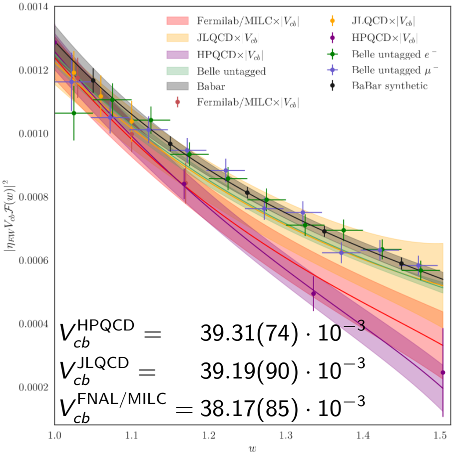
Comparison LQCD $B \to D^* \ell \nu_\ell$ form factors



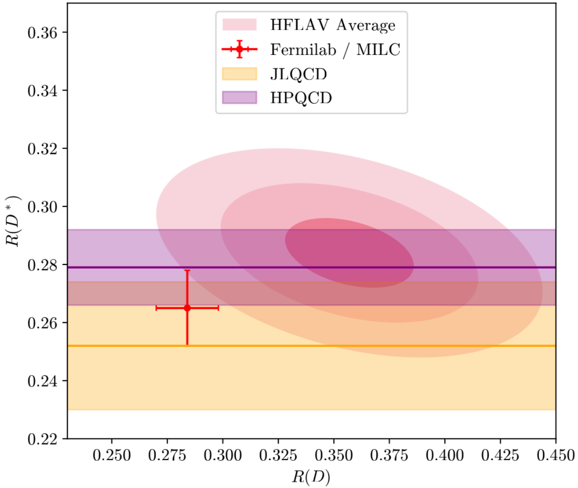


[Plots/fits: A. Vaquero CKM 2023]

Comparison LQCD $B \to D^*\ell\nu_\ell$ form factors



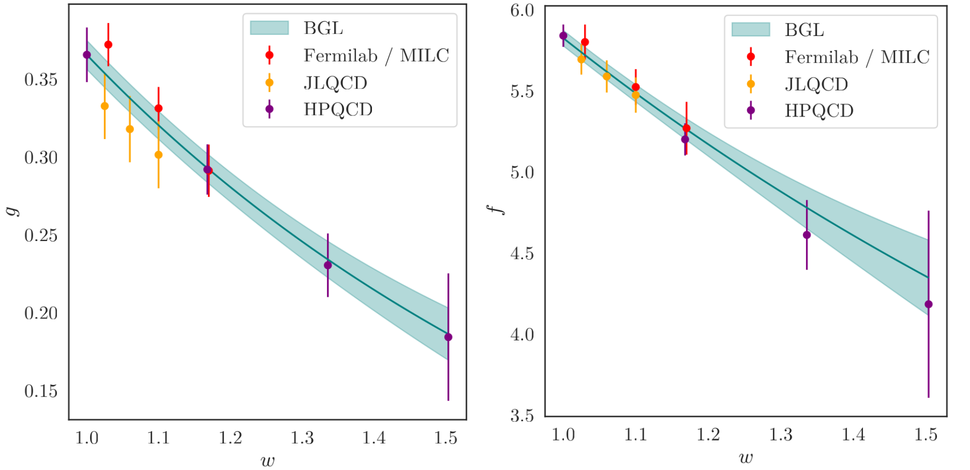
$$V_{cb}^{\mathrm{HPQCD}} = 39.31(74) \cdot 10^{-3}$$
$$V_{cb}^{\mathrm{JLQCD}} = 39.19(90) \cdot 10^{-3}$$
$$V_{cb}^{\mathrm{FNAL/MILC}} = 38.17(85) \cdot 10^{-3}$$



- Fitting Belle untagged data (p-value $\gtrsim 0.5$)
- Including BaBar (p-value ≈ 0.04)

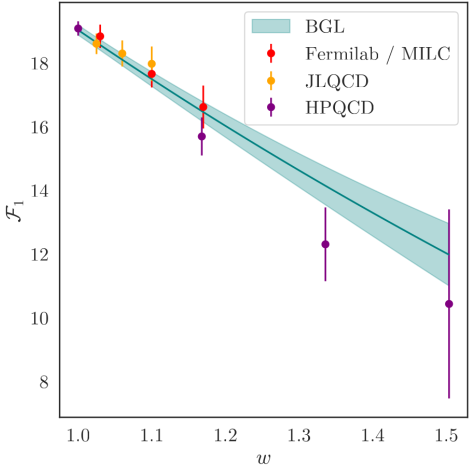
[Plots/fits: A. Vaquero CKM 2023]

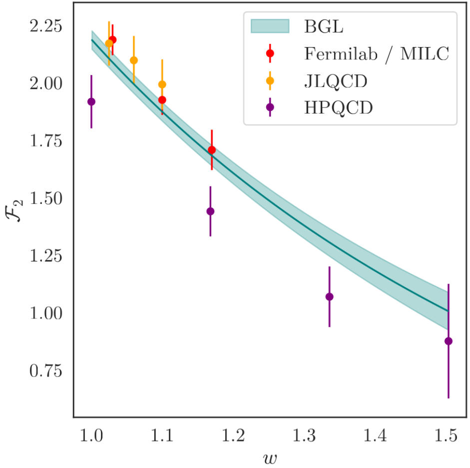
Joint analysis LQCD $B \to D^* \ell \nu_\ell$ form factors [Plots/fits: A. Vaquero CKM 2023]



- Slight tension between Fermilab/MILC and JLQCD

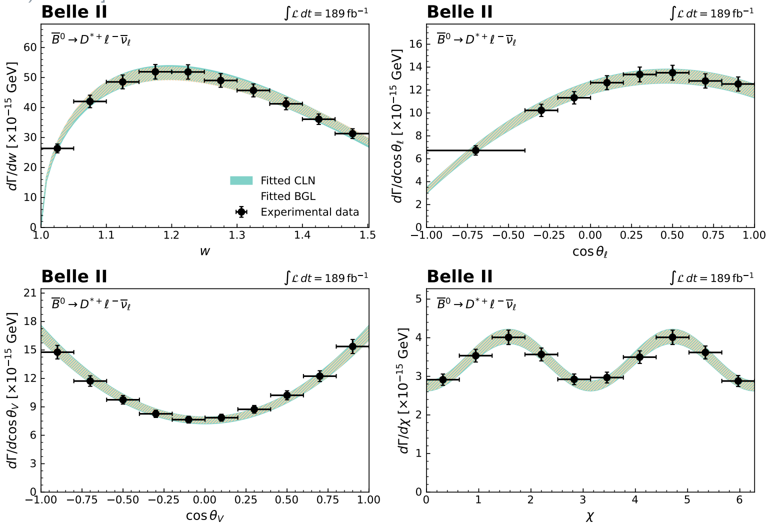
Joint analysis LQCD $B \to D^* \ell \nu_\ell$ form factors [Plots/fits: A. Vaquero CKM 2023]



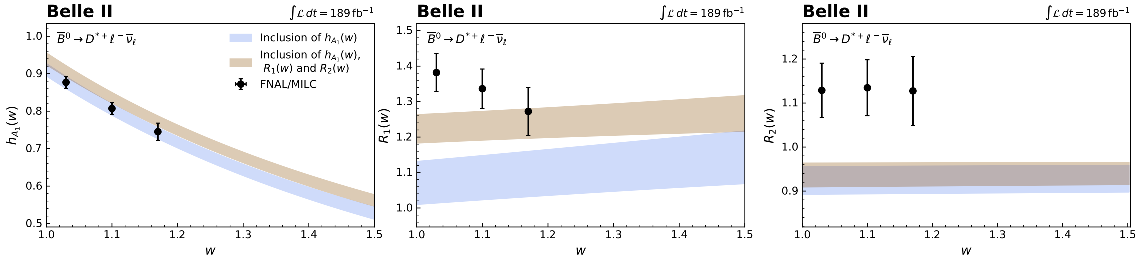


- Different slopes in lattice data
- Tension between {Fermilab/MILC, JLQCD} and HPQCD

Belle II 2023 [Belle II PRD 108 (2023) 092013]



Belle II 2023: comparison to Fermilab/MILC [Belle II PRD 108 (2023) 092013]



$$|V_{cb}|^{\mathrm{BGL}} = 40.57(1.16)$$
$$|V_{cb}|^{\mathrm{CLN}} = 40.13(1.13)$$

$$R_1(w) = \frac{h_v(w)}{h_{A1}(w)}$$

$$R_2(w) = \frac{\frac{m_{D*}}{m_B} h_{A_2(w)} + h_{A3}(w)}{h_{A1}(w)}$$

Status V_{cb}

- PDG: $40.8(1.4) \cdot 10^{-3}$ [PDG, Workman et al. PTEP (2022) 083C01]

- Inclusive decays

$$|V_{cb}|^{E_\ell^B, M_X} = 42.16(51) \cdot 10^{-3}$$
$$|V_{cb}|^{q^2} = 41.69(63) \cdot 10^{-3}$$
$$|V_{cb}|^{E_\ell^B, M_X, q^2} = 41.97(48) \cdot 10^{-3}$$

- Exclusive decays (LQCD)

$$V_{cb}^{\text{HPQCD}} = 39.31(74) \cdot 10^{-3}$$
$$V_{cb}^{\text{JLQCD}} = 39.19(90) \cdot 10^{-3}$$
$$V_{cb}^{\text{FNAL/MILC}} = 38.17(85) \cdot 10^{-3}$$

D^* (Belle)

$$V_{cb}^{\text{FNAL/MILC}} = 40.6(1.2) \cdot 10^{-3}$$

D^* (Belle II, BGL)

$$V_{cb}^{\text{HPQCD}} = 42.2(2.3) \cdot 10^{-3}$$

D_s^* [PRD 105 (2022) 094506]

$$V_{cb}^{\text{HPQCD}} = 42.3(1.7) \cdot 10^{-3}$$

D_s [PRD 101 (2020) 074513]

$$V_{cb}^{\text{FNAL/MILC}} = 39.7(1.7) \cdot 10^{-3}$$
$$V_{cb}^{\text{HPQCD}} = 40.2(2.1) \cdot 10^{-3}$$

D [PRD 92 (2015) 034506] [PRD 92 (2015) 054510]

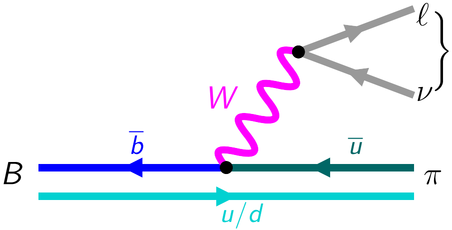
- BGL z-parametrization preferred over CLN

  [Boyd, Grinstein, Lebed PRL 74 (1995) 4603] [Caprini, Lellouch, Neubert NPB 530 (1995) 153]

V_{ub}

(least precisely known CKM matrix elements)

Exclusive semi-leptonic decays on the lattice



$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

- Conventionally parametrized placing the B meson at rest

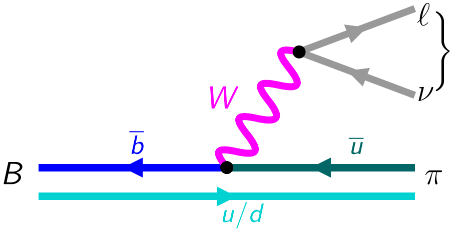
$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}\,\frac{(q^2-m_\ell^2)^2\sqrt{E_\pi^2-M_\pi^2}}{q^4 M_B^2}$$

experiment — CKM — known

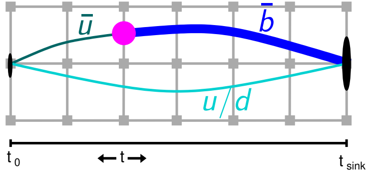
$$\times\left[\left(1+\frac{m_\ell^2}{2q^2}\right)M_B^2(E_\pi^2-M_\pi^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(M_B^2-M_\pi^2)^2|f_0(q^2)|^2\right]$$

nonperturbative input

Exclusive semi-leptonic decays on the lattice



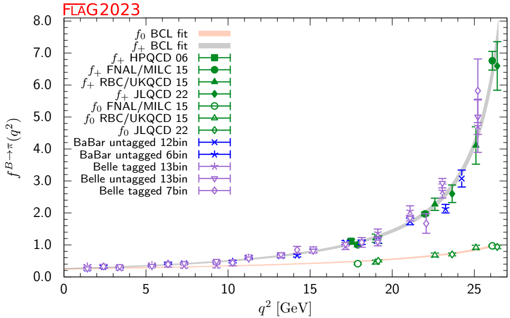
$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

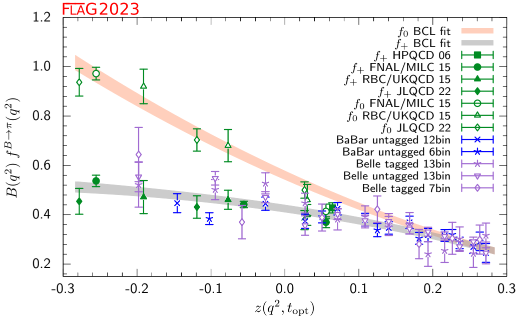


- To determine nonperturbative input, perform OPE, and calculate hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle\pi|V^\mu|B\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{M_B^2-M_\pi^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2-M_\pi^2}{q^2}q^\mu$$

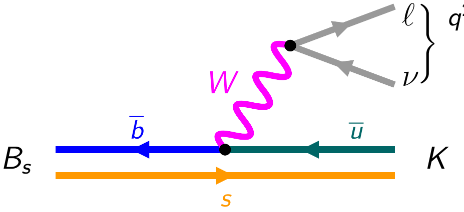
Exclusive semi-leptonic decays on the lattice



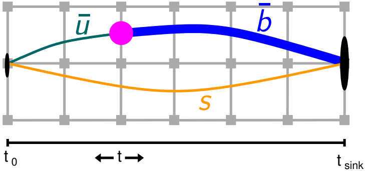


- FLAG average: [FLAG 21, web update 2024]
  [Fermilab/MILC PRD92(2015)014024] [RBC/UKQCD PRD 91 (2015) 074510] [JLQCD PRD 106 (2022) 054502]
  → Shown in addition [HPQCD PRD73(2006)074502][PRD75(2007)119906]
- Experimental data:
  [BaBar PRD 83 (2011) 032007][PRD 86 (2012) 092004] [Belle PRD 83 (2011) 071101][PRD 88 (2013) 032005]
- $V_{ub}^{\rm excl} = 3.64(16)\cdot 10^{-3}$

Exclusive semi-leptonic decays on the lattice



$q^2 = M_{B_s}^2 + M_K^2 - 2M_{B_s}E_K$



- To determine nonperturbative input, perform OPE, and calculate hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K|V^\mu|B_s\rangle = f_+(q^2)\left(p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_{B_s}^2 - M_K^2}{q^2}q^\mu$$

- Alternatively consider a strange quark as spectator and study $B_s \to K\ell\nu$

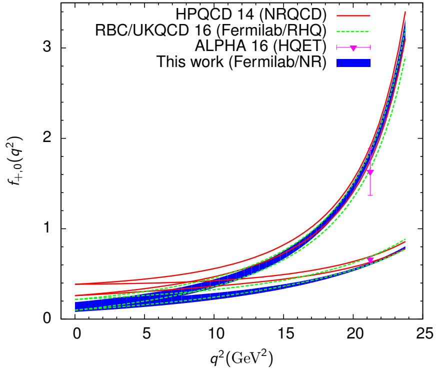
$B_s \to K\ell\nu$ vs. $B \to \pi\ell\nu$

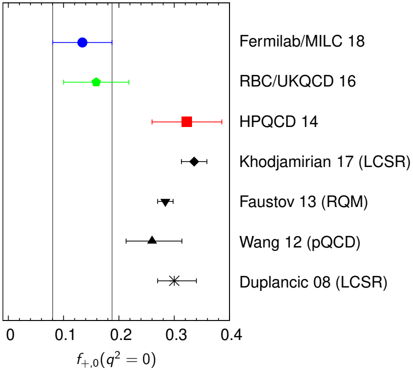
- Experimentally not ideal for B factories which prefer $B \to \pi\ell\nu$
  - → Running at $\Upsilon(5s)$ is less efficient in creating $B_s\bar{B}_s$ pairs
- Abundantly created in pp collisions at the LHC ⇝ LHCb
  - → Normalization not straight forward at LHCb, better to consider (double-)ratios
  - → Determine $|V_{cb}|/|V_{ub}|$ from $B_s \to D_s\ell\nu/B_s \to K\ell\nu$
  - → Alternative: $\Lambda_b \to \Lambda_c\ell\nu/\Lambda \to p\ell\nu$ [Detmold, Lehner, Meinel, PRD92 (2015) 034503]
- Compare:

$$M_B = 5279 \text{ MeV} : M_\pi = 138 \text{ MeV} \sim 38,\ q^2 \text{ range} \sim [m_\ell^2, 27] \text{ GeV}^2$$
$$M_{B_s} = 5367 \text{ MeV} : M_K = 494 \text{ MeV} \sim 11,\ q^2 \text{ range} \sim [m_\ell^2, 24] \text{ GeV}^2$$

⇝ cheaper and more precise to compute with LQCD

$B_s \to K\ell\nu$ tensions (< 2023) [Bazavov et al. PRD100(2019)034501]

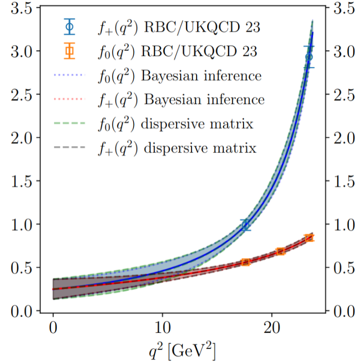


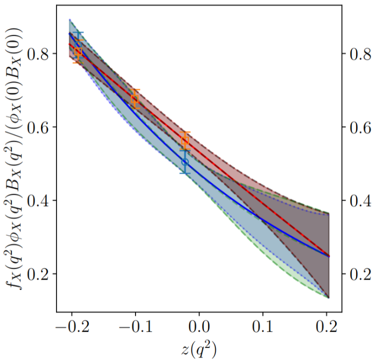


- HPCQD, RBC-UKQCD, ALPHA, Fermilab/MILC
  [Bouchard et al. PRD90(2014)054506] [Flynn et al. PRD91(2015)074510] [Bahr et al. PLB757(2016)473] [Bazavov et al. PRD100(2019)034501]
- Lattice form factors differ at $q^2 = 0$

RBC/UKQCD 2023: z-expansion

[Flynn et at. PRD 107 (2023) 114512] [Flynn, Jüttner, Tsang JHEP 12 (2023) 175]





$q^2 = 0$	
Duplancic, Melic	$0.30(^{+4}_{-3})$
Khodjamirian, Rusov	0.336(23)
Faustov, Galkin	0.284(14)
Wang, Xiao	$0.26(^{+4}_{-3})(2)$
RBC/UKQCD 23	0.25(11)
Fermilab/MILC 18	0.13(5)
HPQCD 14	0.32(6)

- Consistent with result of dispersive matrix method by Martinelli, Simula, Vitorio et al.
- $|V_{ub}|$ from combination with LHCb [PRL 126 (2021) 081804] and $\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)$ [PRD 101 (2020) 072004]
  low bin: 0.0041(24) high bin: 0.00376(55) full: 0.00378(61)

Status V_{ub}

- PDG: $3.82(20) \cdot 10^{-3}$ [PDG, Workman et al. PTEP (2022) 083C01]

- Inclusive decays

$$|V_{ub}|^{\text{avg}} = 4.13(26) \cdot 10^{-3}$$

- Exclusive decays

$$V_{ub}^{\text{FLAG 24}} = 3.64(16) \cdot 10^{-3} \qquad B \to \pi\ell\nu$$

$$V_{ub}^{\text{RBC/UKQCD}} = 3.78(61) \cdot 10^{-3} \qquad B_s \to K\ell\nu$$

Summary

- V_{cd}
  - → Consistent determinations using neutrino scattering, leptonic or semileptonic decays
  - → New FNAL/MILC results for semileptonic decays will reduce error of the average
- V_{cb}
  - → Inclusive determinations self-consistent, improved due to new method using q^2 moments
  - → Three lattice collaborations have determined $B \to D^* \ell\nu$ form factors over the full q^2 range
  - → Tensions among lattice data and lattice data to experimental data need better understanding
  - → Tension between inclusive and exclusive determination persists
- V_{ub}
  - → CKM matrix element with the largest uncertainty
  - → Only incremental updates for exclusive determinations recently

Heavy flavors on the lattice

- Quark masses

up ~ 0.002 GeV

charm ~ 1.25 GeV

top ~ 175 GeV

down ~ 0.005 GeV

strange ~ 0.095 GeV

bottom ~ 4.2 GeV

- Lattice simulations have a cutoff a^{-1}
  - → Fully relativistic quarks require $am \ll 1$ i.e. $m \ll a^{-1}$
  - → Typically $a^{-1} \gtrsim 2$ GeV $\Rightarrow m_{\text{charm}} \lesssim a^{-1} \lesssim m_{\text{bottom}}$
  - → Charm but in particular bottom quarks require special considerations

×10

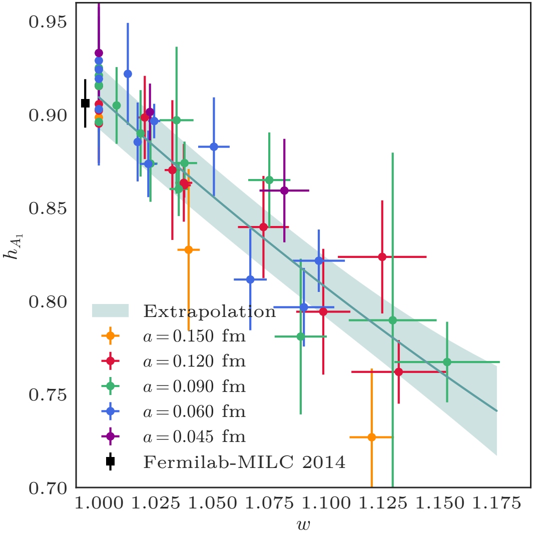
Simulating heavy flavors

- Traditionally: simulate charm and bottom using effective actions
  → Heavy quark effective Theory (HQET), Non-Relativistic QCD, Relativistic Heavy Quark (RHQ, Fermilab, Tsukuba)
  → Allows to simulate charm and bottom quarks on coarser lattices
  → Additional systematic uncertainties, partly perturbative renormalization, ...
  → Few percent total errors

- State-of-the-art: fully relativistic simulations at $a^{-1} > 2$ GeV
  → Heavy Highly Improved Staggered Quarks (HISQ), Heavy Domain-Wall Fermions (DWF), ...
  → Same action for light (up/down/strange) as for heavy (charm/bottom) quarks
    ⇝ Simulate heavier than charm and extrapolate
  → Fully nonperturbative renormalization straight-forward, reduced systematic uncertainties
  → Sub-percent precision feasible ⇝ QED effects become relevant

Fermilab/MILC 2021: $B \to D^* \ell \nu$ [Bazavov et al. EPJC 82 (2022) 1141]

- Effective Fermilab action for c and b quarks
  - → ASQTAD light/strange
  - → Perturbatively tuned parameters
  - → Directly simulating physical c and b quarks
  - → Mostly nonperturbative renormalization
- Perform calculation at high q^2 and use z expansion to extrapolate to low q^2
- $a \approx 0.15 - 0.045$ fm
- $M_\pi \gtrsim 180$ MeV

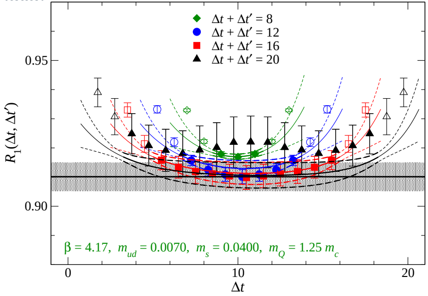


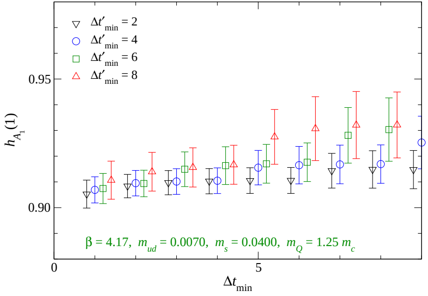
JLQCD 2023: $B \to D^* \ell \nu$ [Y. Aoki et al. PRD 109 (2024) 074503]

- Unitary setup
  - → MDWF light/strange and heavy quarks with $am_c \leq am_Q \leq 2.44 \cdot am_c$
  - → Additional extrapolation in the heavy quark mass to reach m_b
  - → Fully nonperturbative renormalization

- $a \approx 0.044$ fm, 0.055 fm, 0.080 fm
- $M_\pi \gtrsim 230$ MeV

- Carefully checking for excited state contamination using multiple source sink separations (e.g. for h_{A_1})

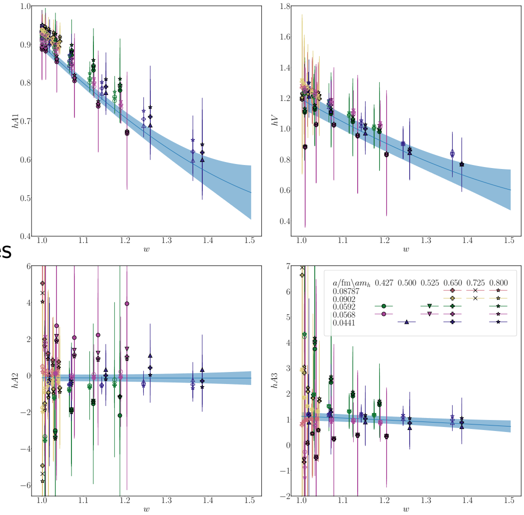




HPQCD 2023: $B_{(s)} \to D^*_{(s)} \ell\nu$ [Harrison, Davies, arXiv:2304.03137]

- All-HISQ setup
  - → Updating [Harrison, Davies PRD105(2022).094506]
  - → Fully non-perturbative renormalization
  - → Simulate heavier-than-charm → close-to-bottom
  - → Directly cover most of the allowed q^2 range at the finest lattice spacing
  - ⇝ Parametrize pole mass for different charm masses in a combined chiral, heavy quark, continuum, kinematical extra-/interpolation
  - → Also analyzing tensor BSM operators

- h_V, h_{A1}, h_{A2}, h_{A3} for $B \to D^* \ell\nu$

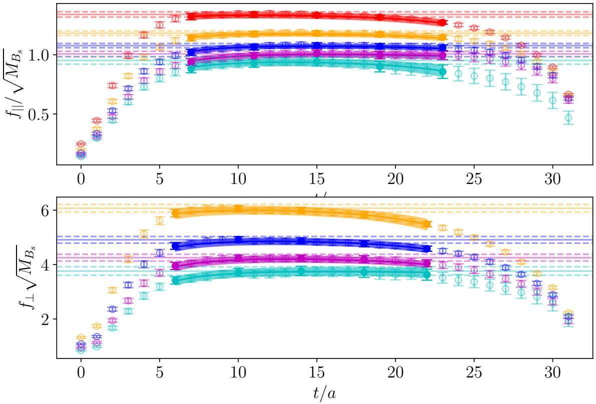


RBC/UKQCD 2023: Update $B_s \to K\ell\nu$ [Flynn et at. PRD 107 (2023) 114512]

- Effective RHQ action for b quarks
  - → SDWF light/strange
  - → Nonperturbatively tuned RHQ parameters
  - → Directly simulating physical b quarks
  - → Mostly nonperturbative renormalization

- Perform calculation at high q^2 and use z expansion to extrapolate to low q^2

- $a \approx 0.11, 0.08, 0.07$ fm
- $M_\pi \gtrsim 250$ MeV



- $a \approx 0.07$ fm, $M_\pi = 250$ GeV

Kinematical z-expansion (BGL) [Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]

- Map complex q^2 plane with cut $q^2 > t_*$ onto the unit disk in z

$$z(q^2, t_*, t_0) = \frac{\sqrt{t_* - q^2} - \sqrt{t_* - t_0}}{\sqrt{t_* - q^2} + \sqrt{t_* - t_0}}$$

with

$$t_* = (M_B + M_\pi)^2 \qquad \text{(two-particle production threshold)}$$
$$t_\pm = (M_{B_s} \pm M_K)^2 \qquad \text{(with } t_- = q^2_{max}\text{)}$$
$$t_0 \equiv t_{\rm opt} = t_* - \sqrt{t_*(t_* - t_-)} \qquad \text{(symmetrize range of } z\text{)}$$

- BGL express form factors $f_X = f_+,\ f_0$ as

$$f_X(q^2) = \frac{1}{B_X(q^2)\phi_X(q^2, t_0)} \sum_{n\geq 0} a_{X,n}(t_0) z^n$$

- With outer function $\phi_X(q^2, t_0)$ and Blaschke factors $B_X(q^2)$
- Account for cut differing from pair-production threshold [Gubernari, van Dyk, Virto JHEP02 (2021) 088] [Gubernari, van Dyk, Reboud, Virto JHEP09 (2022) 133]

Bayesian inference for form factors [Flynn, Jüttner, Tsang JHEP 12 (2023) 175]

- Compute z expansion coefficients as expectation values: $\langle g(a)\rangle = N\int da\, g(a)\pi(a|f,C_f)\pi_a$
- Probability for parameters given model and data

$$\pi(a|f,C_f) \propto \exp\left\{-\frac{1}{2}\chi^2(a,f)\right\} \qquad \chi^2(a,f) = (f-Za)^T C_f^{-1}(f-Za)$$

and prior knowledge from unitarity constraint $\pi_a \propto \theta(1-|a_+|_\alpha^2)\theta(1-|a_0|_\alpha^2)$

- Perform Monte Carlo integration using multivariate distribution a but drop samples incompatible with unitarity

- To increase probability modify expression and correct with an accept-reject step

$$\pi(a|f_p,C_{f_p})\pi_a(a_p|M) \propto \theta(a)\exp\left\{-\frac{1}{2}(f_p-Za)^T C_{f_p}^{-1}(f_p-Za) - \frac{1}{2}a^T\frac{M}{\sigma^2}a\right\}$$

with $\quad p \leq \dfrac{\exp\left\{-1/\sigma^2\right\}}{\exp\left\{-a^T\frac{M}{2\sigma^2}a\right\}}$