



Neutrino oscillation with global data analysis

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Ref: JHEP01(2021)124; JHEP03(2021)019;

Phys.Rev.D 105 (2022) 7, 075022; Phys.Rev.D 105 (2022) 9, 096029;

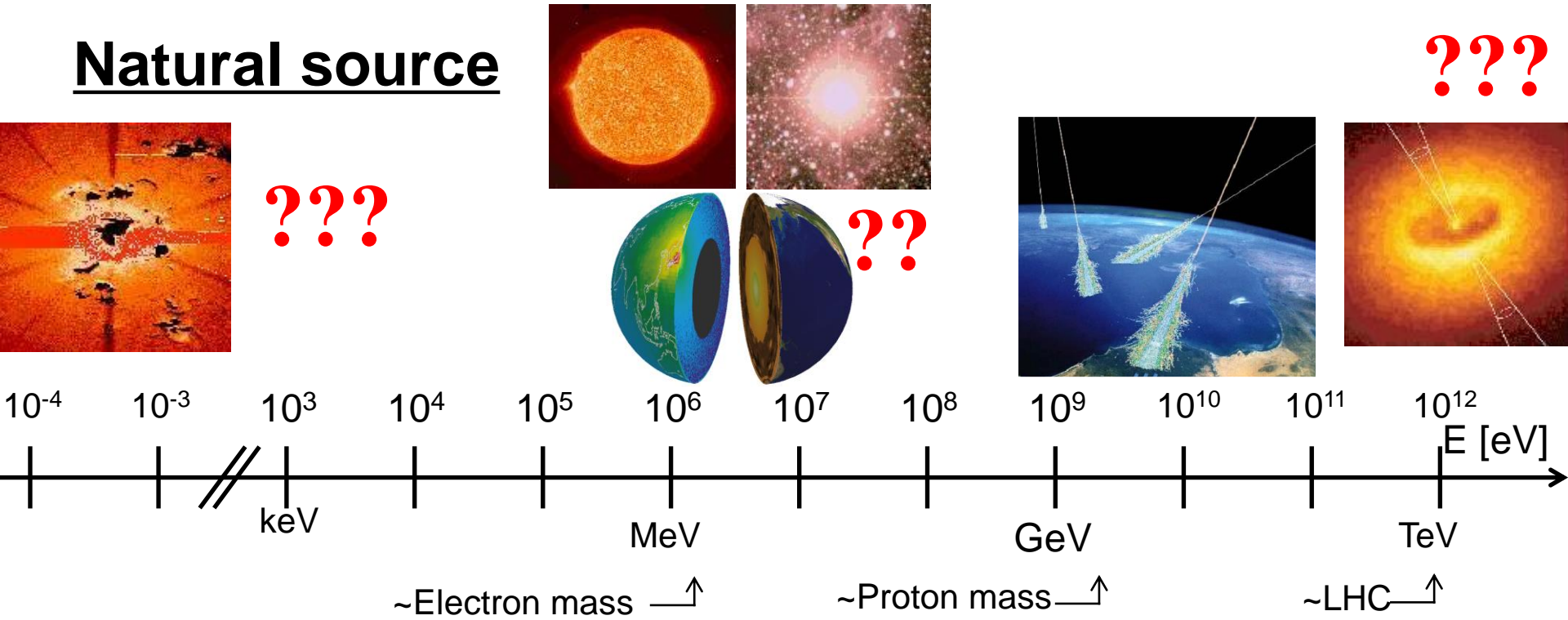
Phys.Rev.D 108 (2023) 6, 062004; arXiv:2312.11704; arXiv: 2403.05819



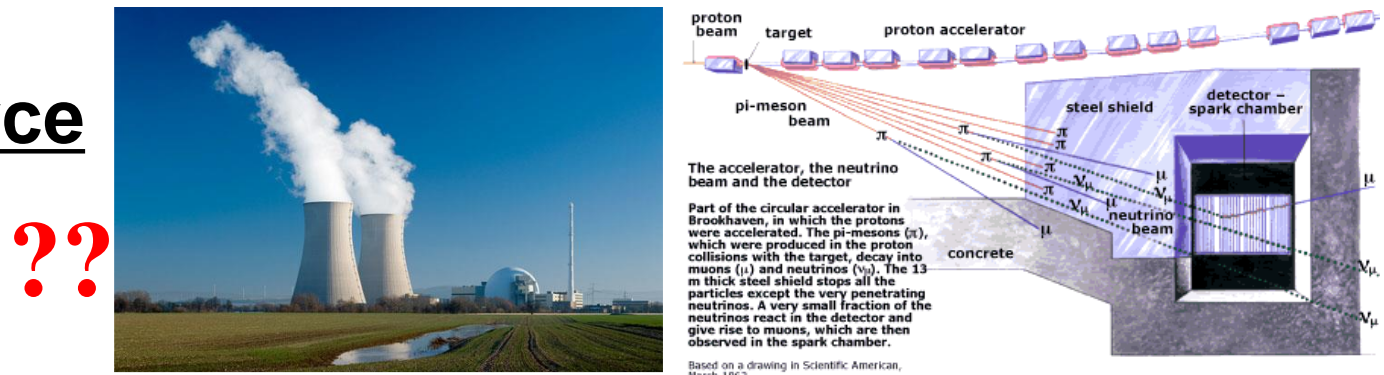
- Motivations
- Non-unitarity(NU) v.s non-standard interactions(NSIs)
- Global fits of mixing parameters without unitarity
- Constraints of NSIs based on SMEFT
- Summary

Where are neutrinos from?

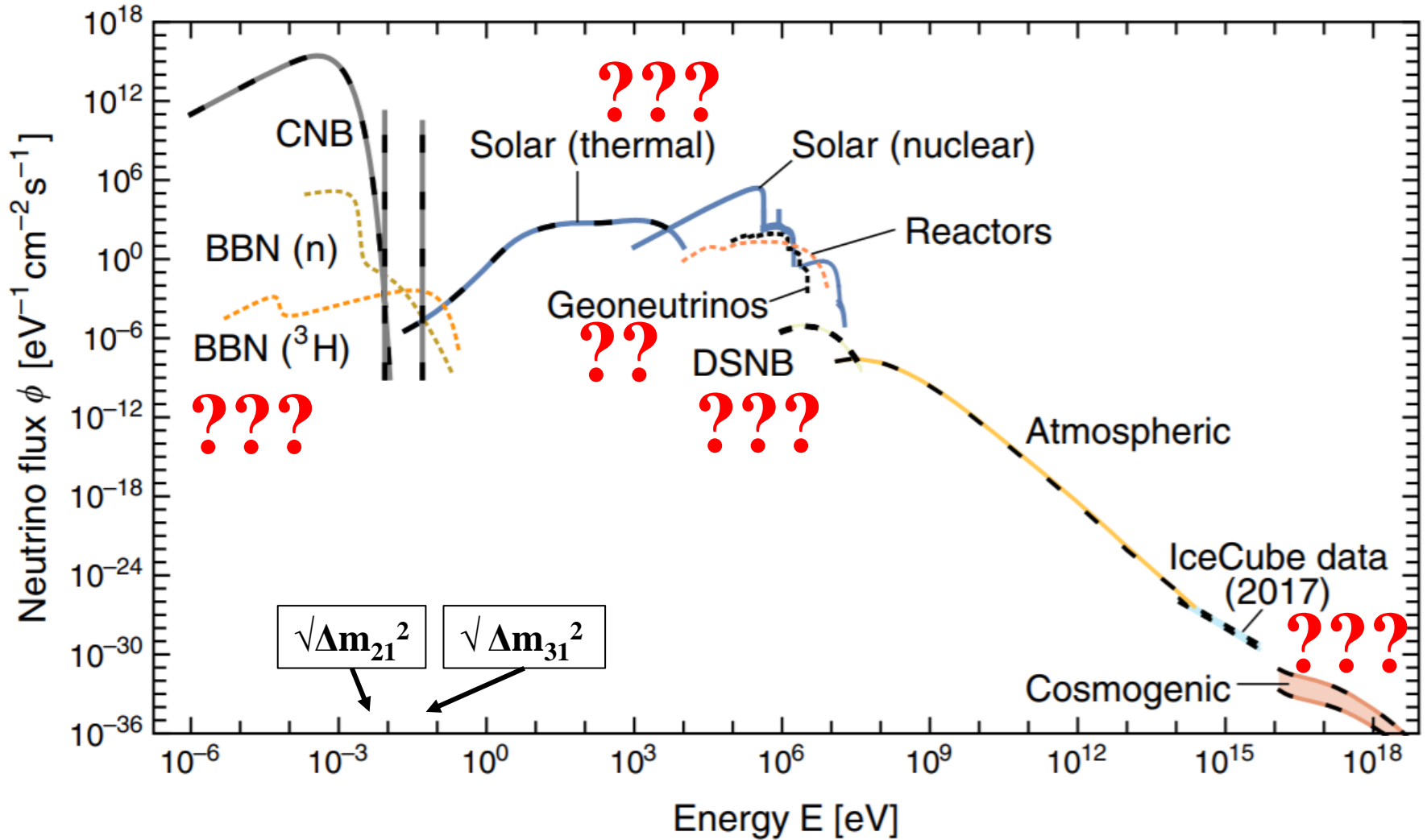
Natural source



Artificial source

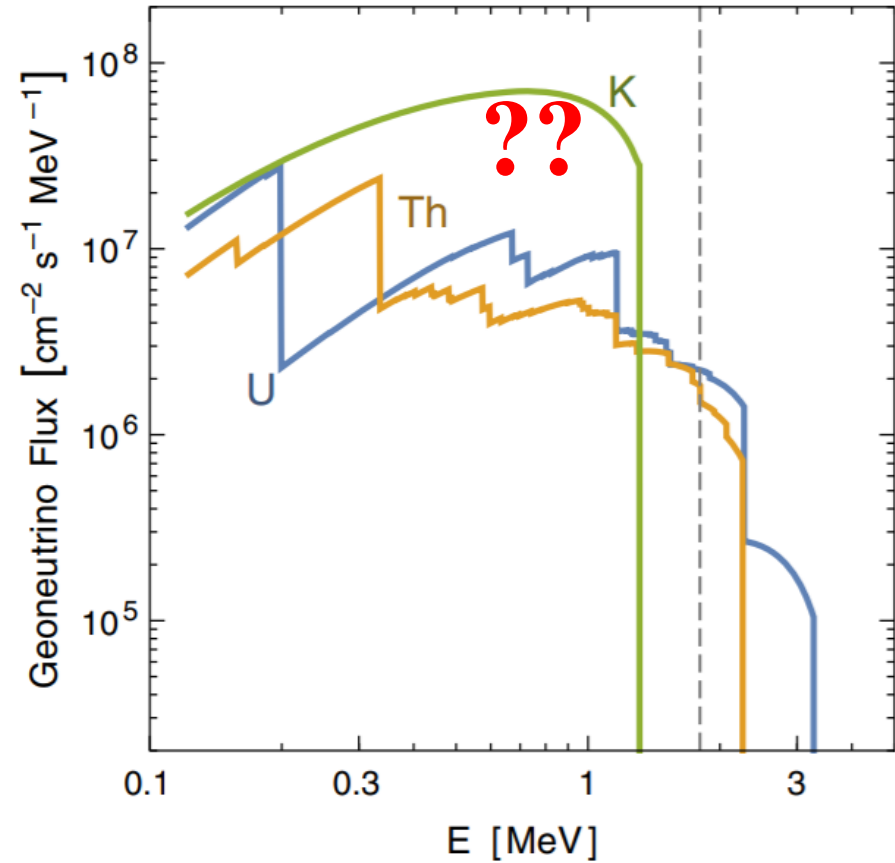
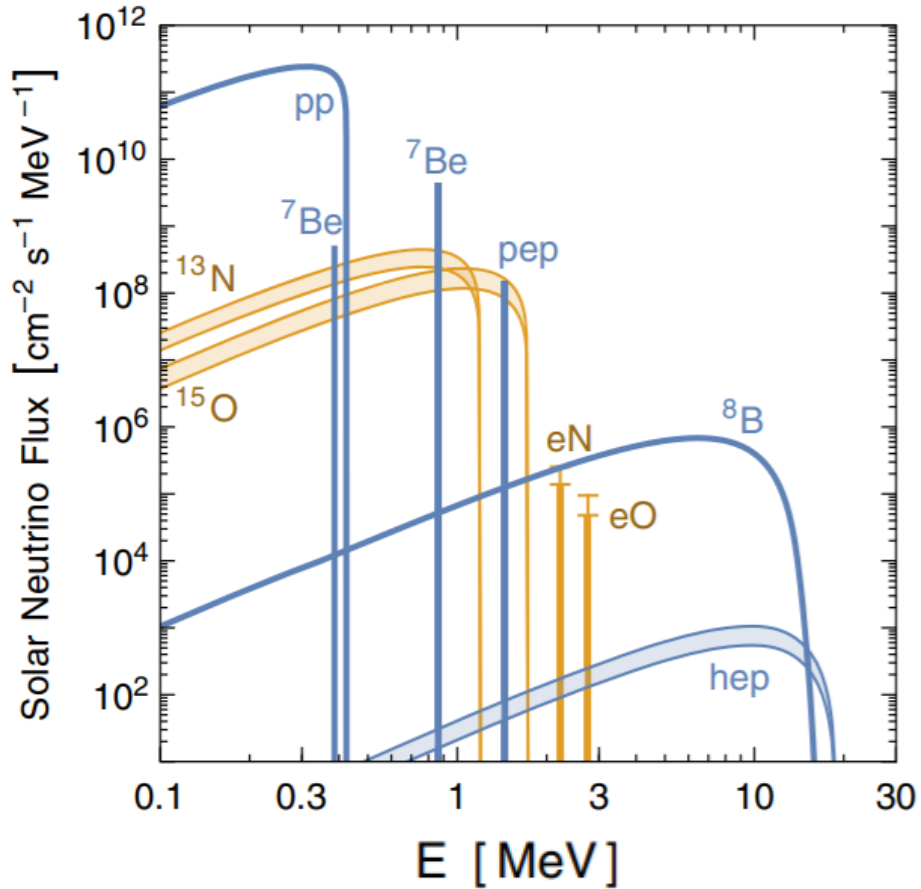


Energy spectra of various neutrino sources



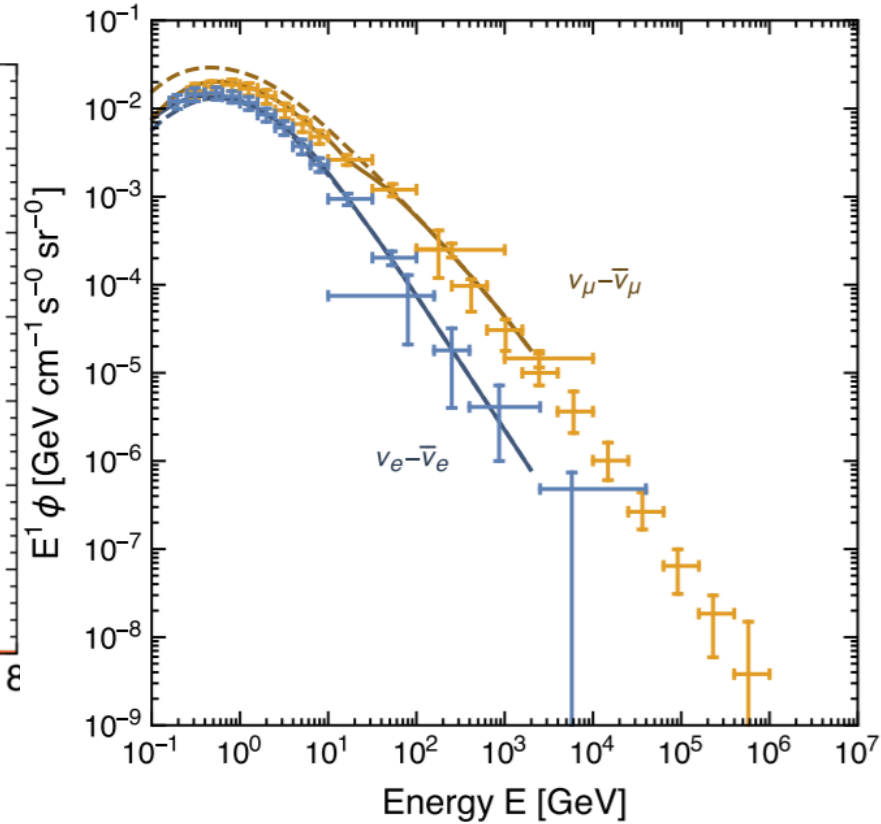
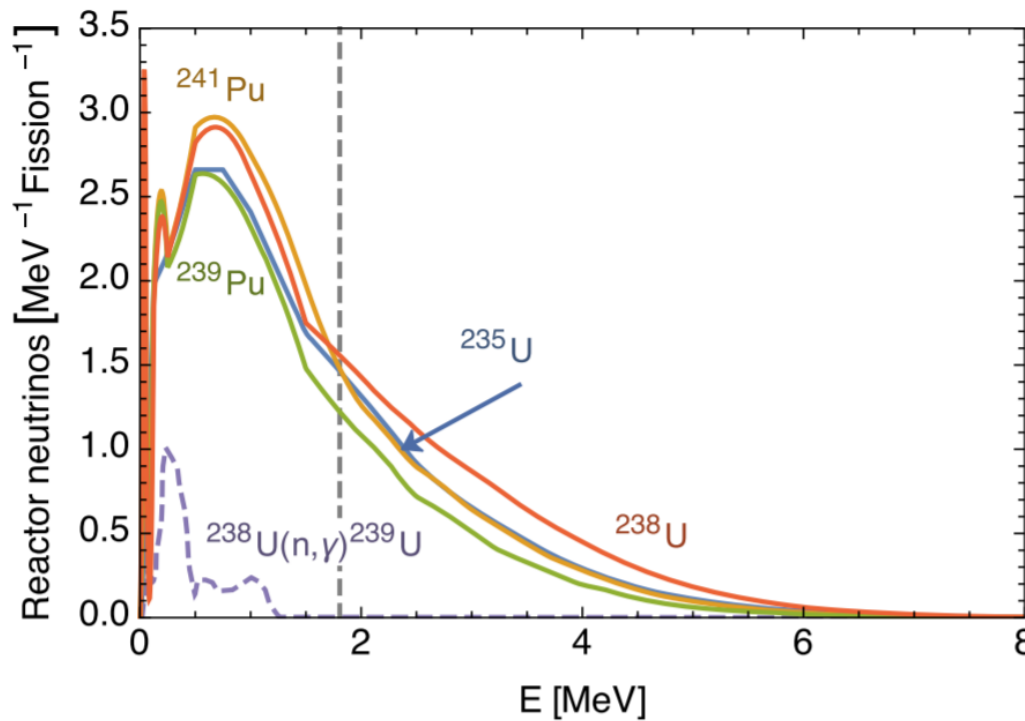
Ref: Edoardo Vitagliano, Irene Tamborra, and Georg Raffelt. *Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components*. *Rev. Mod. Phys.*, 92:45006, 2020.

Energy spectra of solar&geo-neutrino sources



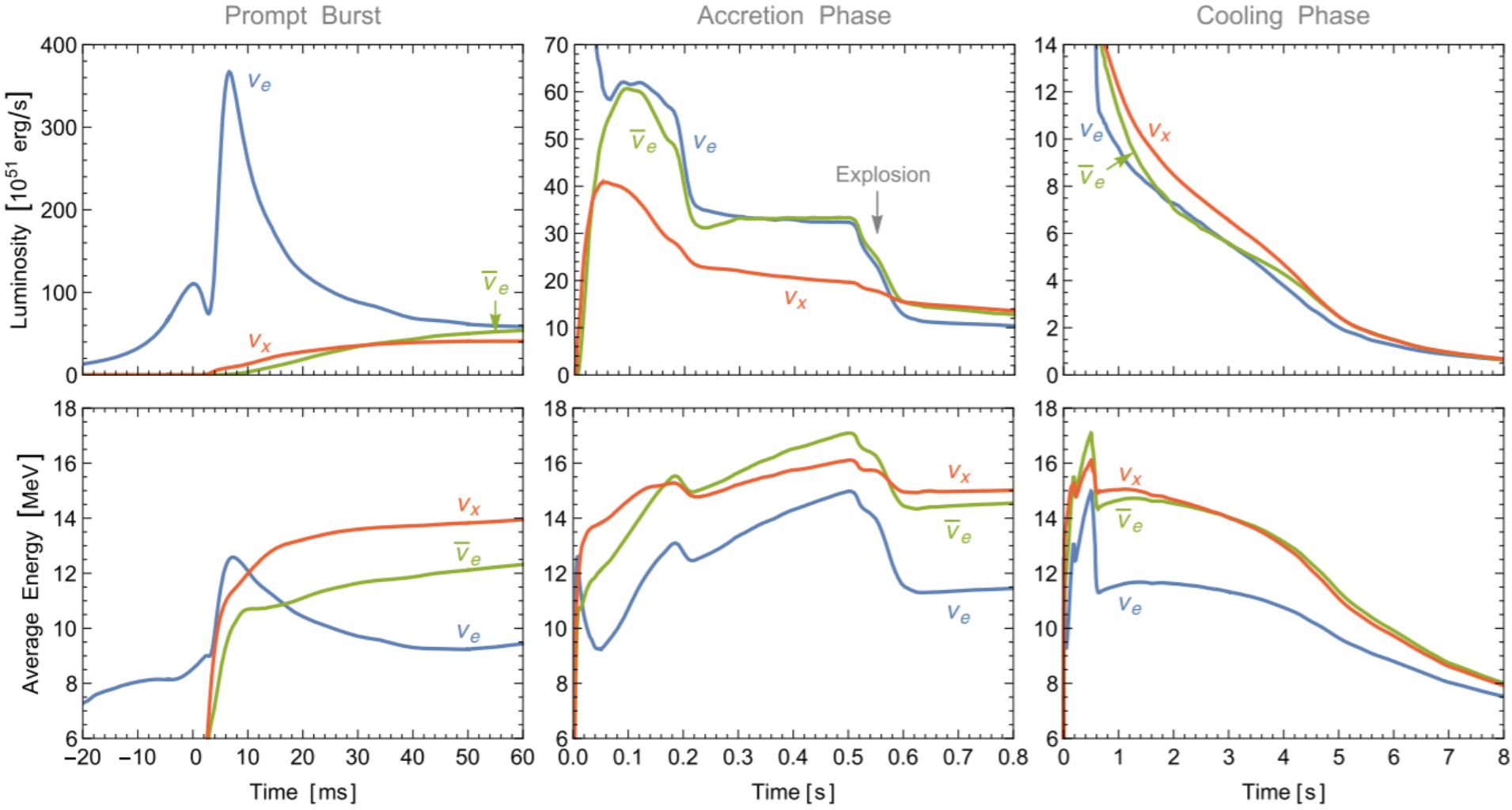
Ref: Edoardo Vitagliano, Irene Tamborra, and Georg Raffelt. Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components. *Rev. Mod. Phys.*, 92:45006, 2020.

Energy spectra of reactor & atm. neutrinos



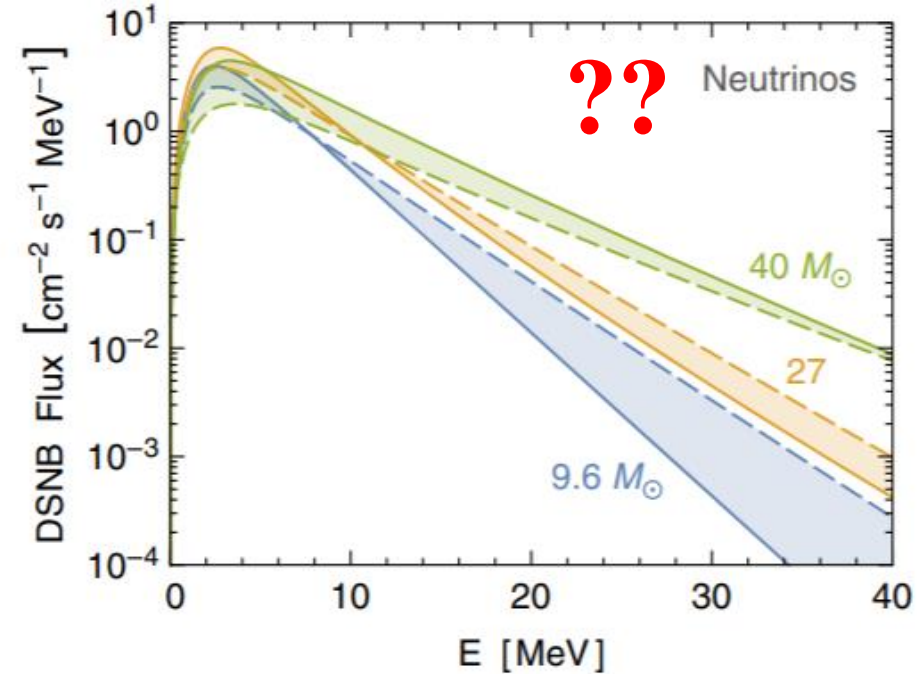
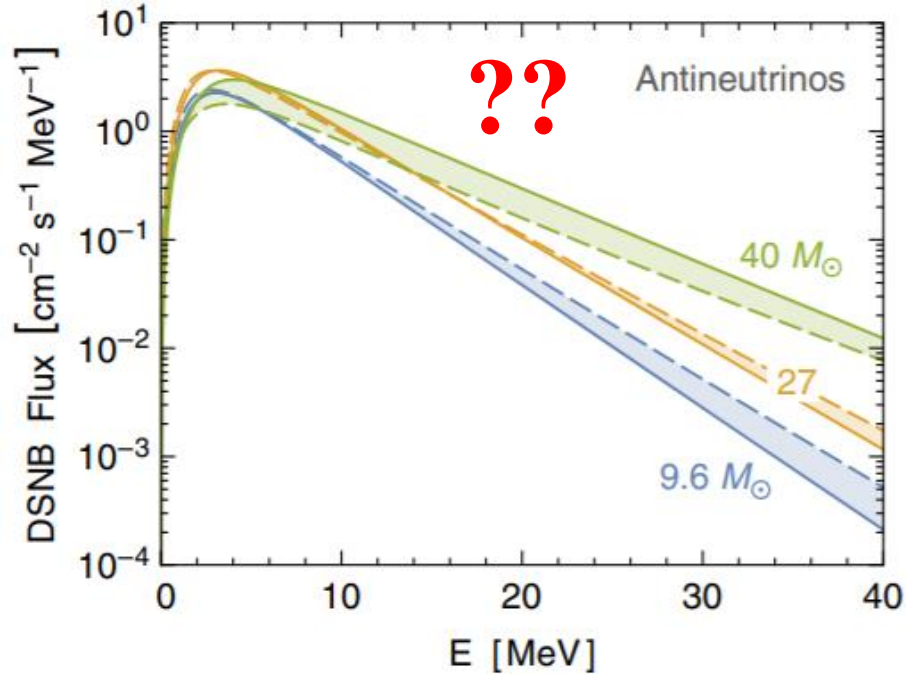
Ref: Edoardo Vitagliano, Irene Tamborra, and Georg Raffelt. Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components. *Rev. Mod. Phys.*, 92:45006, 2020.

Energy spectra of SN neutrino sources



Ref: Edoardo Vitagliano, Irene Tamborra, and Georg Raffelt. *Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components*. *Rev. Mod. Phys.*, 92:45006, 2020.

Energy spectra of DSNB neutrino sources



Ref: Edoardo Vitagliano, Irene Tamborra, and Georg Raffelt. Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components. *Rev. Mod. Phys.*, 92:45006, 2020.

Three-generation neutrino oscillations

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$= \left(\text{Image of neutrino beams} \right) \times \left(\text{Image of Daya Bay reactor with } \theta_{13} \text{ label} \right) \times \left(\text{Image of the Sun} \right)$$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i < j}^n \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin^2 \left(\frac{\Delta_{ij}}{2} \right) + 2 \sum_{i < j} \text{Im}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin(\Delta_{ij})$$

$$U^\dagger U = 1$$

$$\frac{\Delta_{ij}}{2} = 1.27 \frac{m_i^2 - m_j^2}{\text{eV}^2} \frac{L/E}{\text{Km/GeV}}$$

- $\Delta m_{ij}^2 = m_i^2 - m_j^2$ The mass differences
- $U_{\alpha j}$ The mixing angles (and Dirac phases)

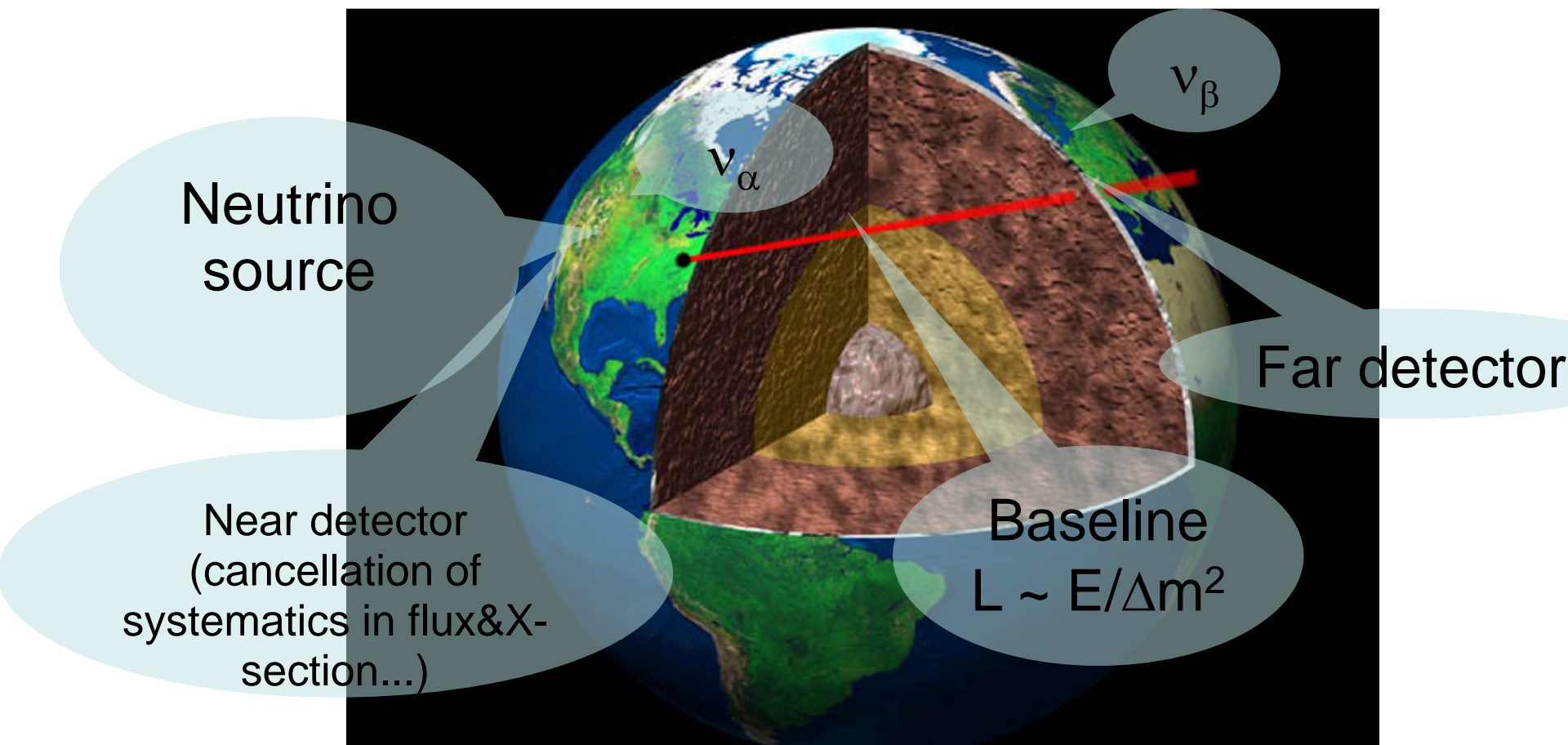
- E The neutrino energy
- L Distance ν source to detector

• For 3 ν framework, we have **6+1** free parameters in neutrino oscillations:

$$P(\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{21}^2, \Delta m_{31}^2) + \rho \text{ (in matter)}$$

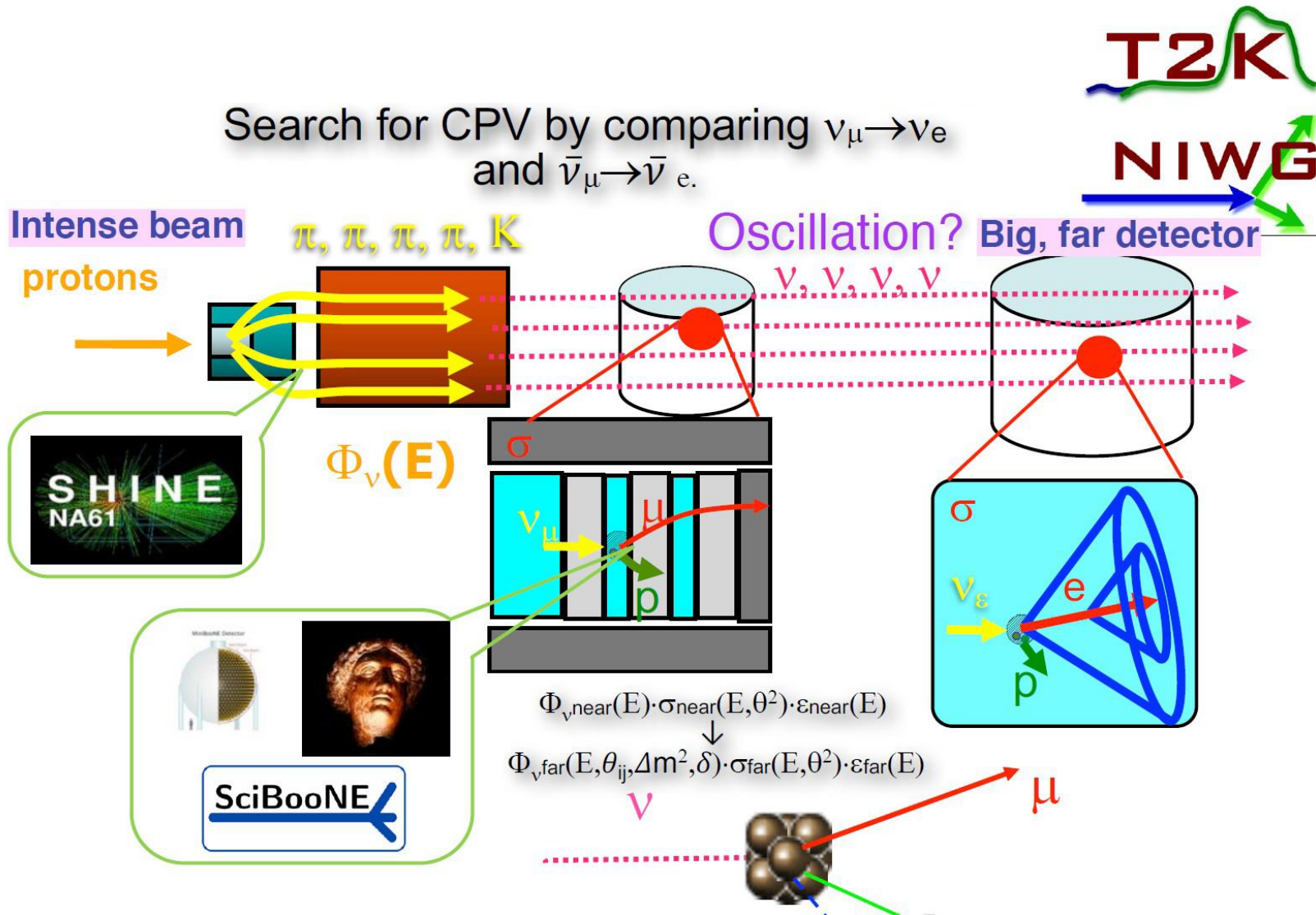
$\sqrt{\quad} \quad \sqrt{\quad} \quad \sqrt{\quad} \quad ? \quad \sqrt{\quad} \quad \sqrt{?}$

Working principles of neutrino oscillation experiments



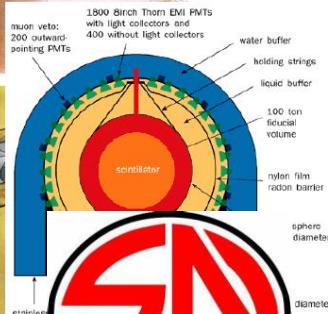
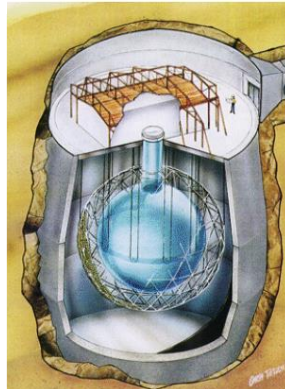
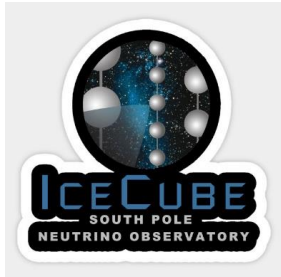
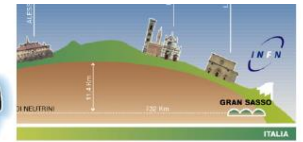
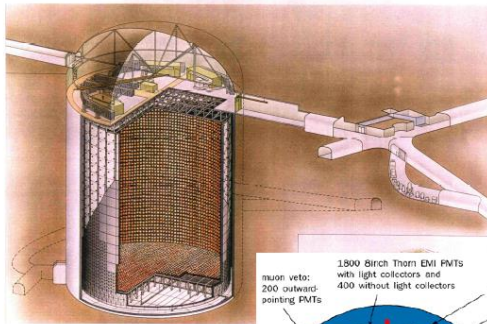
- Get the neutrino source as clean as possible. Muon decay v.s pion decay beams.
- Deploy the best detector to reconstruct the oscillated neutrino spectra: Gd-WC, LAr TPC, scintillator detector with flavour&charge identifications...
- Data mining: precision measurement & discovery of new physics...

An example: T2K



K. McFarland, Neutrino Interaction Uncertainties @ NNN2018

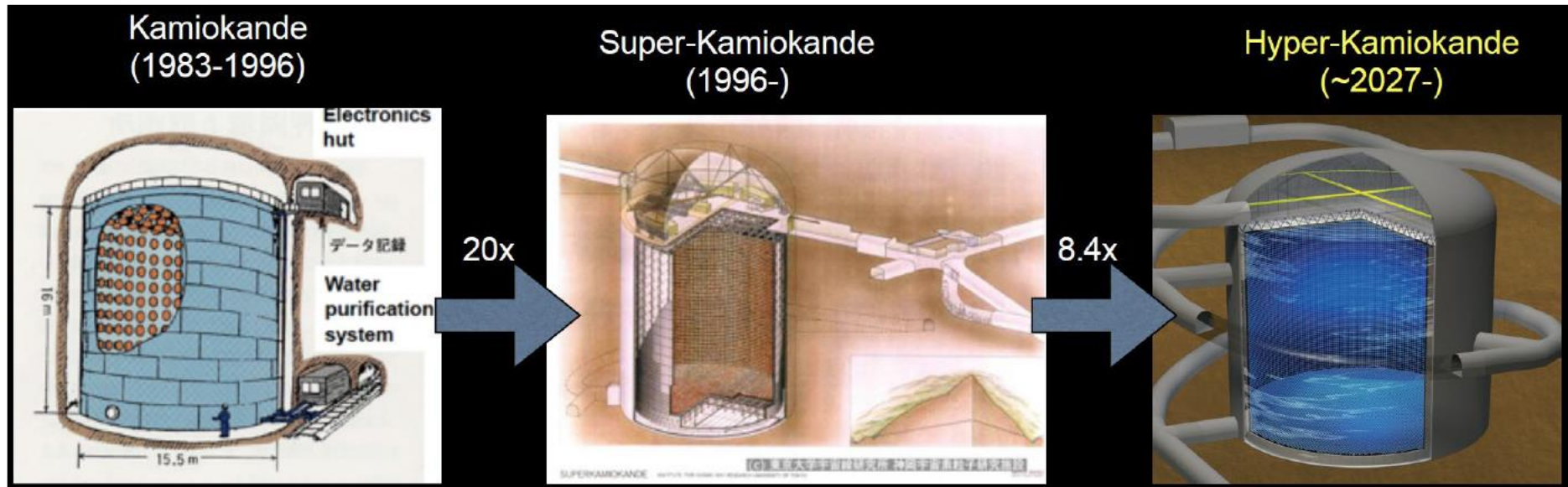
Lots of neutrino experiments to get a new horizon



More are coming...

WC detectors: Kamiokande → SK → HyperK

- Bigger than bigger! PMT technology revolutionized!



3 kton WC 20% coverage
with 20'' PMT

50 kton WC 40% coverage
with 20'' PMT

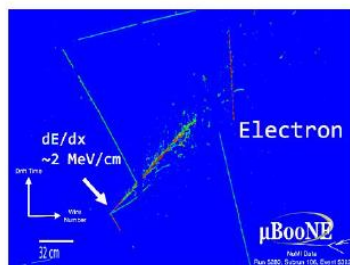
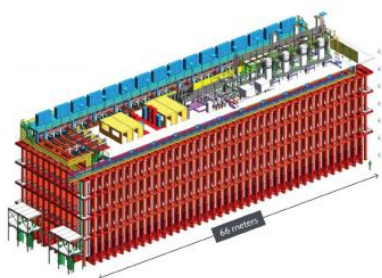
260 kton WC 40% coverage
with 20''+HQE PMT

- Construction started in **2020**.
- Data taking from **2027**.
- J-PARC neutrino beam will be upgraded from 0.7 to **1.3 MW**

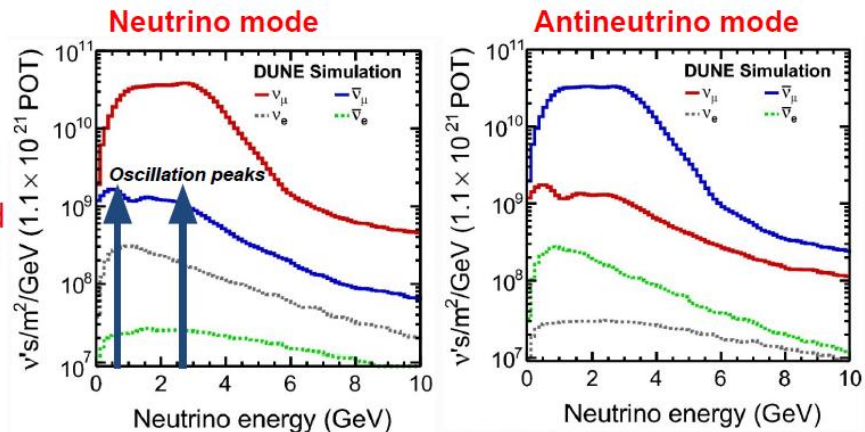
LAr TPC: DUNE



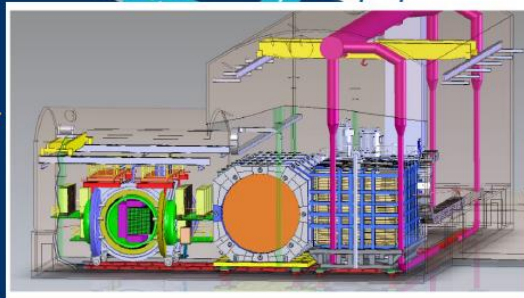
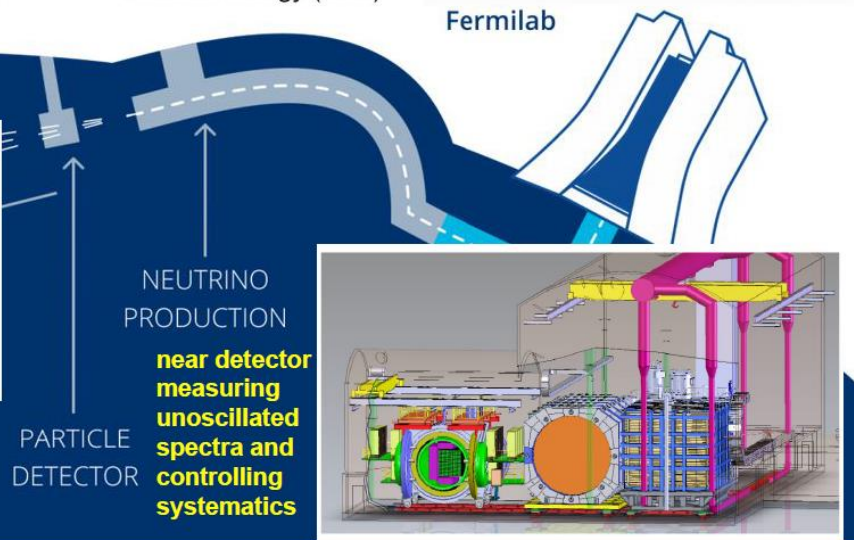
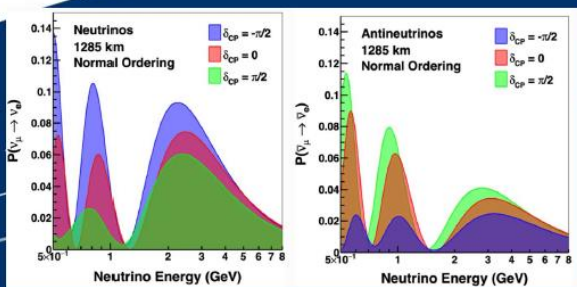
Courtesy: Guang Yang@NuFact2021



Broad band
neutrino and
antineutrino
beam

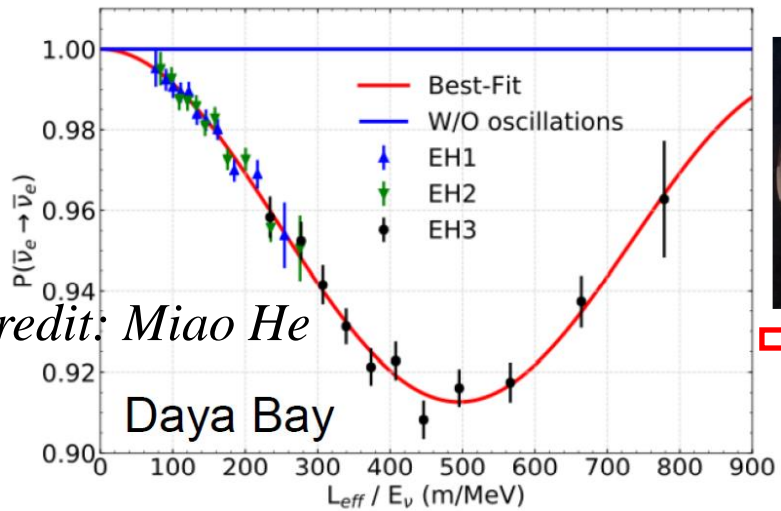


70 kt liquid
argon far
detector
with four
modules

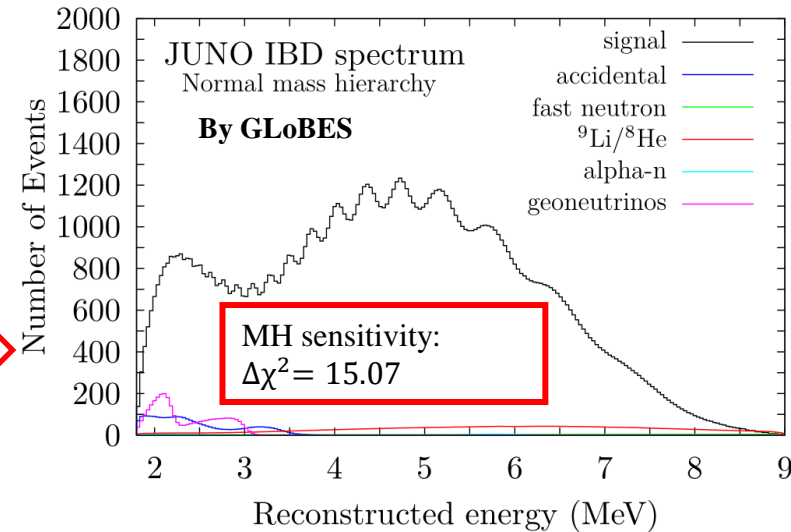


LSc: DYB (20 t*8) → JUNO (20 kt)

- Daya Bay neutrino experiment: mission completed!



MCP-PMT
20''+HQE





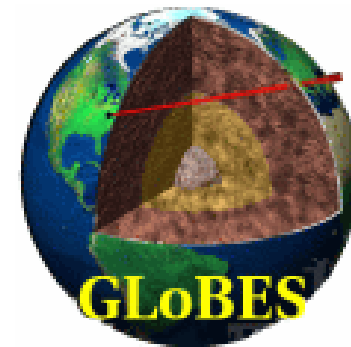
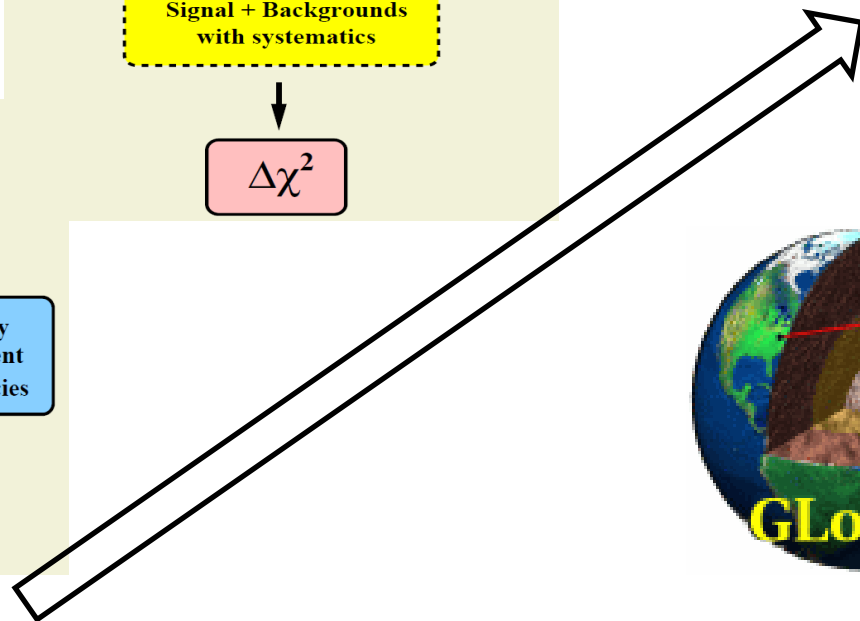
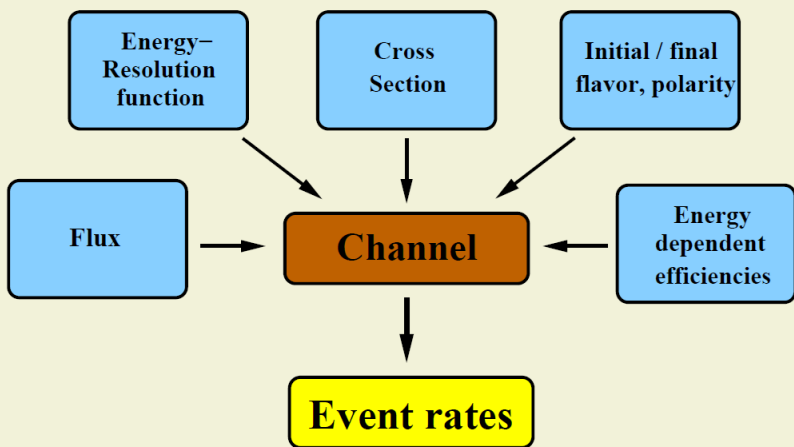
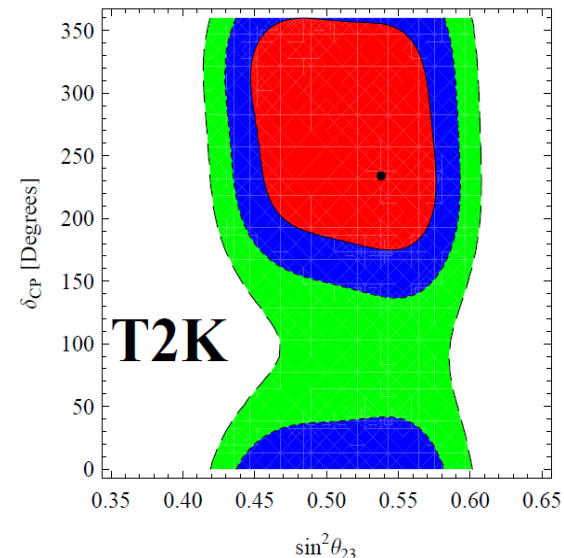
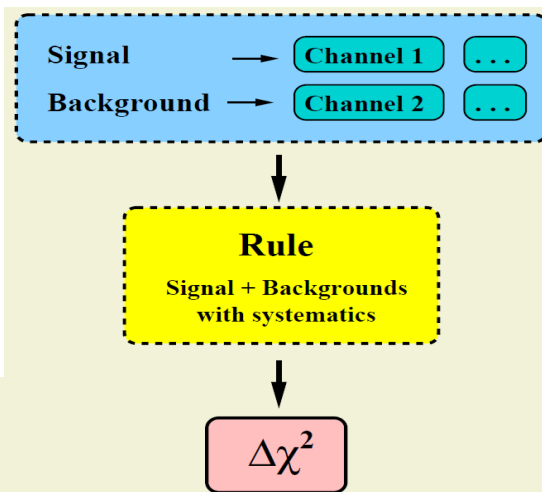
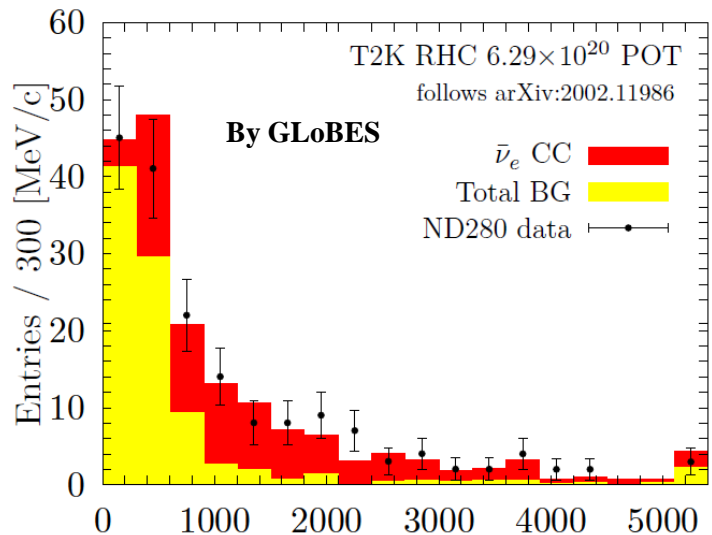
Hot topics in neutrino oscillation physics

- What is the neutrino mass ordering?
- Are there CP violations in the lepton sector?
- How much precision shall we reach to tell new physics?
- What are the current and future systematic limitations on precision measurement and how to address them?
- Is the neutrino mixing matrix unitary?
- Are there non-standard neutrino interactions?
- Are there more than three-flavor neutrinos?
- Reactor antineutrino flux deficit seems resolved. How about Gallium anomaly?
- How consistent are results from NOvA and T2K?
- How to examine neutrino mass models based on flavor symmetries (A4, S4, Modular...)?
- What's next even after neutrino mass ordering and Dirac CP phase?
- ...

Simulations of neutrino oscillations w/o new physics



$$N = \text{time} \times \# \text{ of targets} \times \text{efficiency} \times \int_{E_i}^{E_f} dE_\nu \frac{d\phi(E_\nu)}{dE_\nu} \sigma(E_\nu)$$

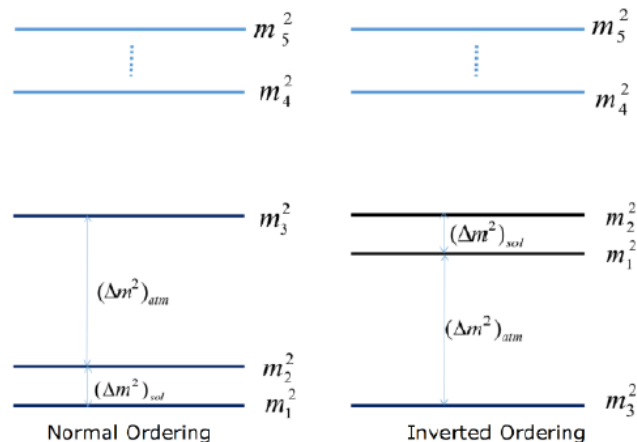




- Motivations
- **Non-unitarity(NU) v.s non-standard interactions(NSIs)**
- Global fits of mixing parameters without unitarity
- Constraints of NSIs based on SMEFT
- Future prospects

Non-unitary neutrino mixings (NU)

- Light sterile neutrino anomaly (eV scale)
- Heavy sterile neutrinos from see-saw model (GeV scale)
- Dark matter candidate (keV scale)
- IUUV (indirect unitary violation) by heavy sterile neutrinos
- DUV (direct unitary violation) by light sterile neutrinos: oscillation with active ones



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

- Simplifying the mixing matrix to deal with DUV and IUUV, Phys. Lett., B718:1447-1453, 2013
- Perturbation study of oscillation probabilities for DUV and IUUV, Phys. Rev., D93(3):033008

Non-Standard Interactions (NSIs)

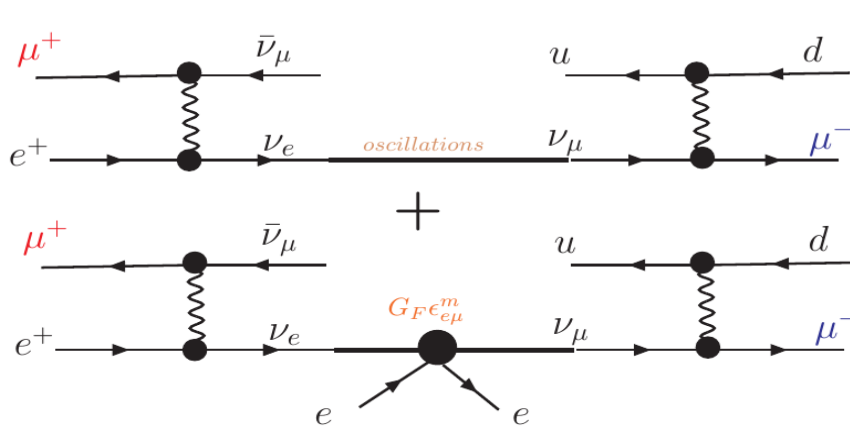
- **New physics beyond SM: new particles, new couplings, new phenomenon...**

- Flavor violating interactions with neutrinos:

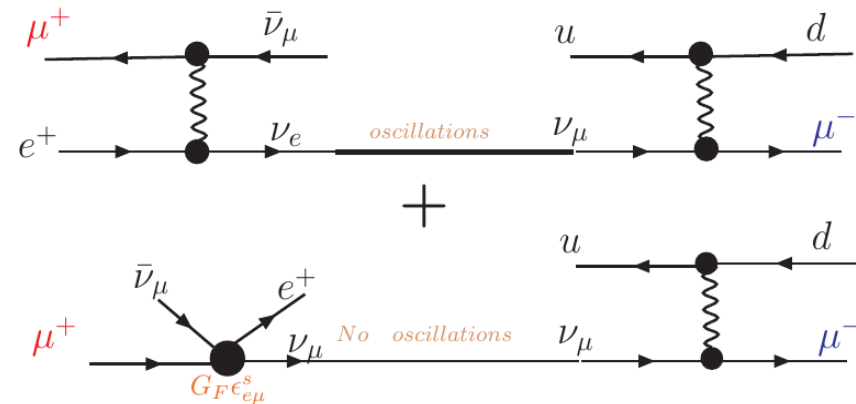
$$\nu_\alpha f \rightarrow \nu_\beta f, l_\alpha^- \rightarrow \nu_\beta e^- \bar{\nu}_e \dots$$

- 4-fermion vertices:

$$L_{\text{eff}} = 2\sqrt{2}G_F(\epsilon^{L/R})_{\beta\delta}^{\alpha\gamma}(\bar{\nu}^\beta\gamma^\rho P_L\nu_\alpha)(\bar{\ell}^\delta\gamma^\rho P_{L/R}\ell_\gamma)$$



NSI happens to neutrino propagation in matter

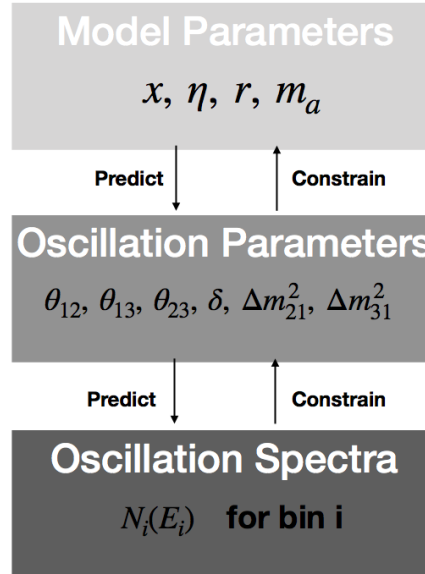


NSI at neutrino productions

Constraints on flavor-symmetry neutrino models



Flavour Symmetry Embedded -- GLOBES
(FaSE-GLOBES) arXiv: [2006.14886](https://arxiv.org/abs/2006.14886)

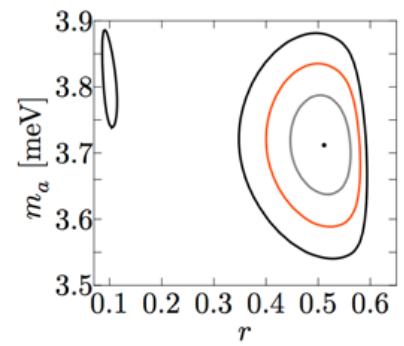
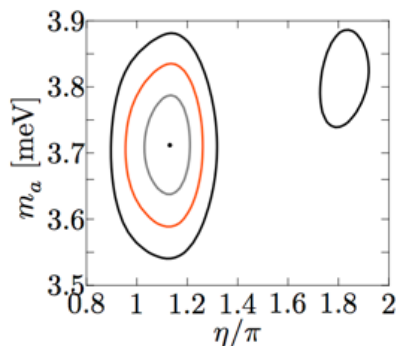
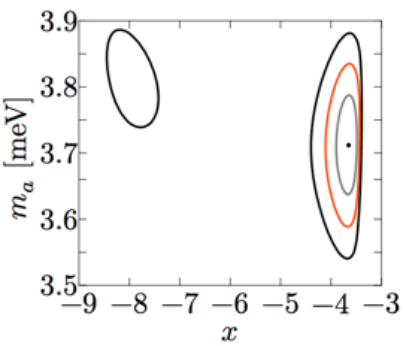
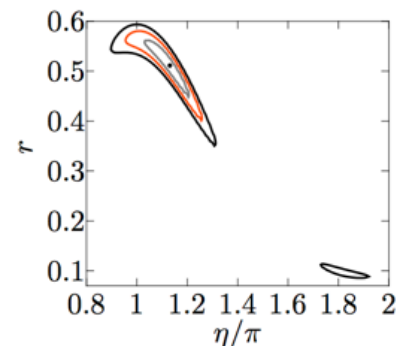
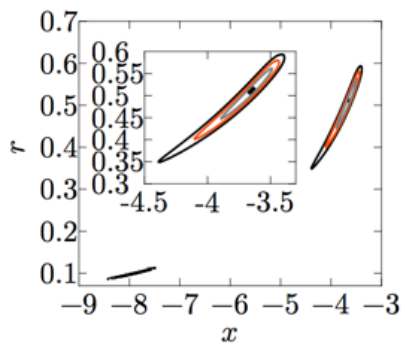
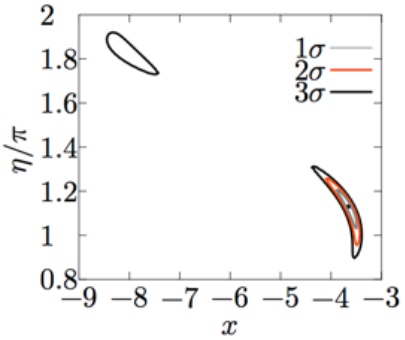


$$\vec{\mathcal{M}} = \{x, \eta, m_a, r\}$$

$$\chi^2(\vec{\mathcal{M}}) = \sum_{i=1}^N \frac{[\mu_i(\vec{\mathcal{O}}(\vec{\mathcal{M}})) - n_i]^2}{\sigma_i^2}$$

$$\vec{\mathcal{O}} = \{\theta_{12}, \theta_{13}, \theta_{23}, \delta_{\text{CP}}, \Delta m_{21}^2, \Delta m_{31}^2\}$$

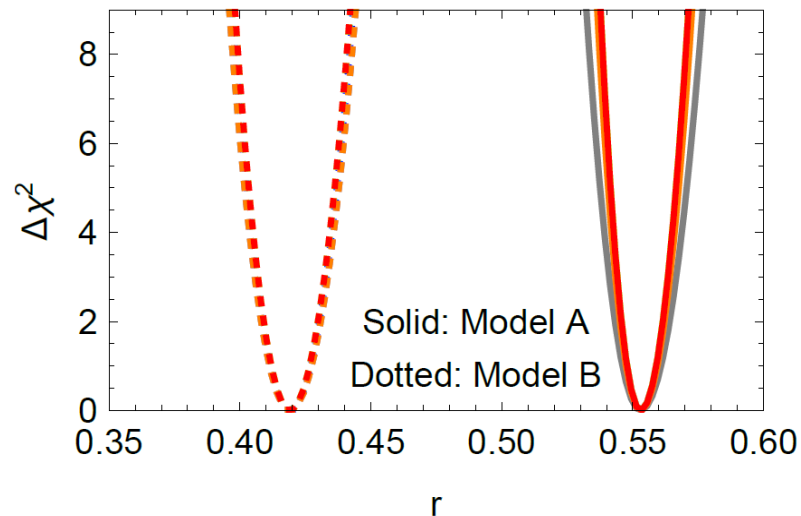
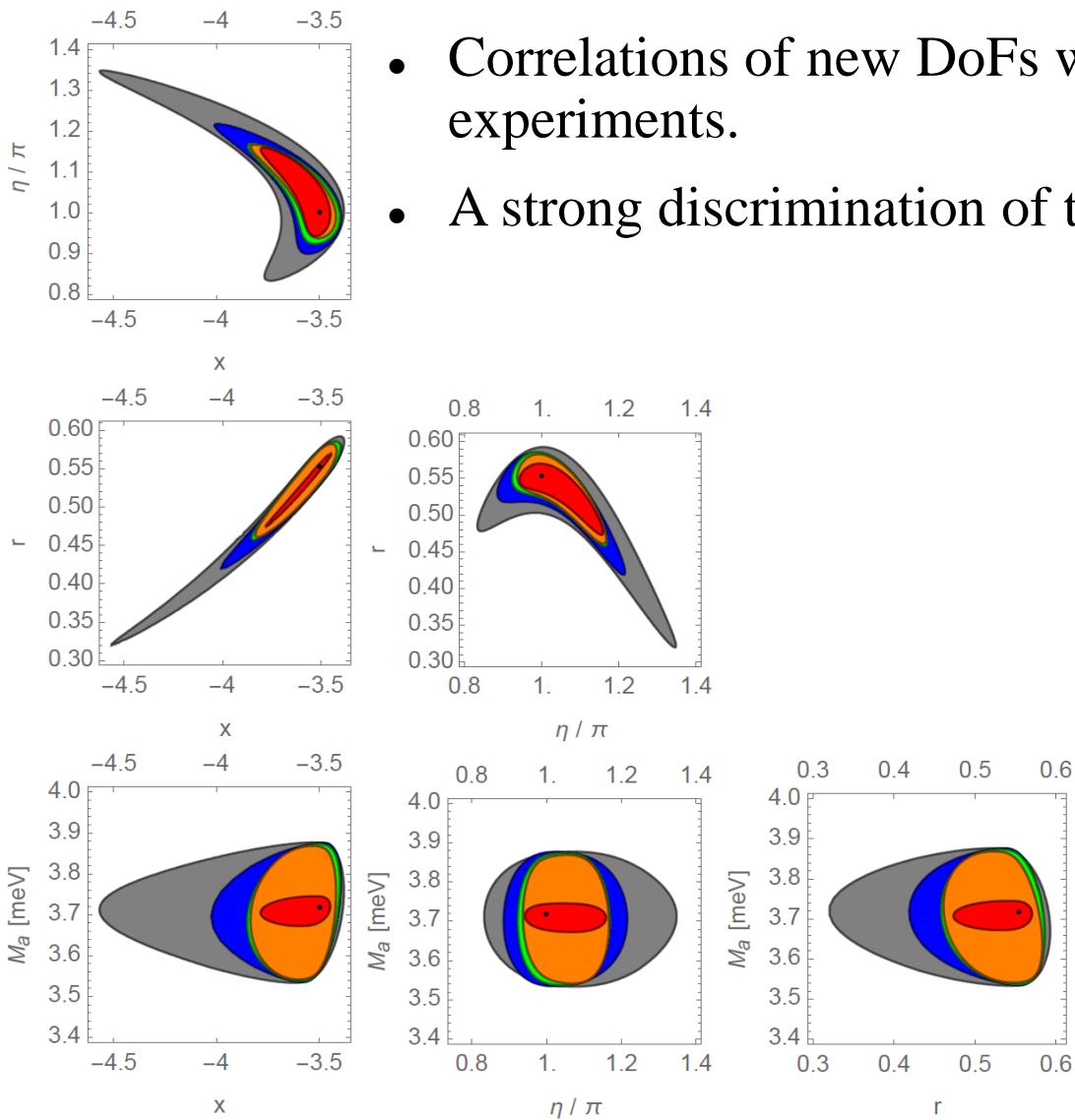
$$\chi^2(\vec{\mathcal{O}}) = \sum_{i=1}^N \frac{[\mu_i(\vec{\mathcal{O}}) - n_i]^2}{\sigma_i^2}$$



[Ref:1907.01371](https://arxiv.org/abs/1907.01371)

Test of neutrino models based on flavor symmetries

- Correlations of new DoFs well constrained by future neutrino experiments.
- A strong discrimination of tri-direct CP symmetry models.



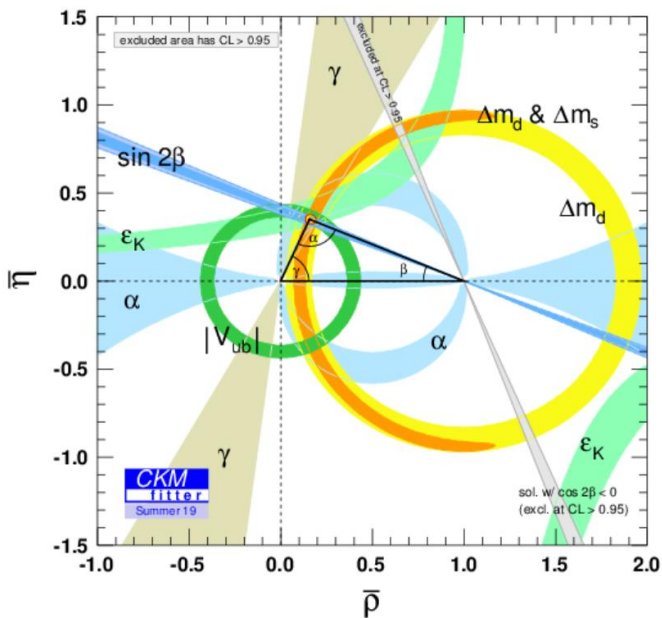
[Ref: 1905.12939](#)



- Motivations
- Non-unitarity(NU) v.s non-standard interactions(NSIs)
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- Constraints of NSIs based on SMEFT
- Future prospects

What if there is non-unitary mixing?

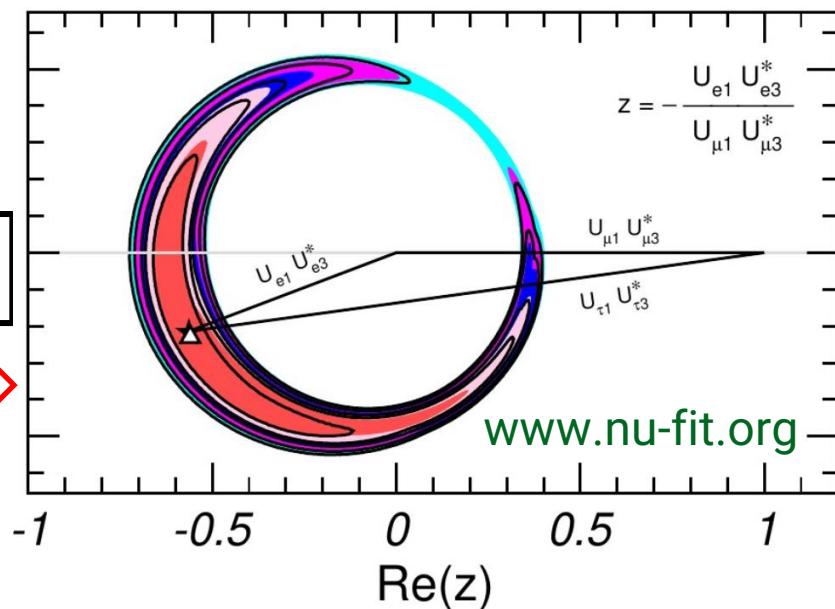
Quark mixing



$$U^\dagger U \stackrel{?}{=} 1$$



Lepton mixing



No unitarity assumption here!

$$U_{e1}U_{\mu2}^* + U_{e2}U_{\mu2}^* + U_{e3}U_{\mu3}^* = 0$$

- Reduced constraints without the unitary assumption in the quark mixing.
- Let's keep the democratic way in quark and lepton mixing?

Simple mathematics w/o unitarity assumption



Unitarity

ROWS: $\alpha = (e, \mu, \tau)$ $j = (1, 2, 3)$

COLUMNS:

$$UU^\dagger = 1$$

$$U^\dagger U = 1$$

$$\sum_j |U_{\alpha j}|^2 = 1 \quad \text{row normalizations}$$

$$\sum_\alpha |U_{\alpha j}|^2 = 1 \quad \text{column normalizations}$$

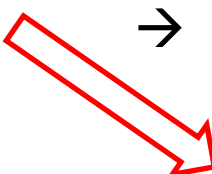
$$\sum_j U_{\alpha j} U_{\beta j}^* = 0 \quad \alpha \neq \beta \quad \text{row orthogonality}$$

$$\sum_\alpha U_{\alpha j} U_{\alpha k}^* = 0 \quad j \neq k \quad \text{column orthogonality}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Non-unitarity (NU)

18 real parameters in linear algebra
→ **-5** by rephrasing fields in physics


$$U^{\text{NU}} = \begin{pmatrix} |U_{e1}|e^{i\phi_{e1}} & |U_{e2}|e^{i\phi_{e2}} & |U_{e3}| \\ |U_{\mu1}|e^{i\phi_{\mu1}} & |U_{\mu2}|e^{i\phi_{\mu2}} & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}| & |U_{\tau3}| \end{pmatrix}$$

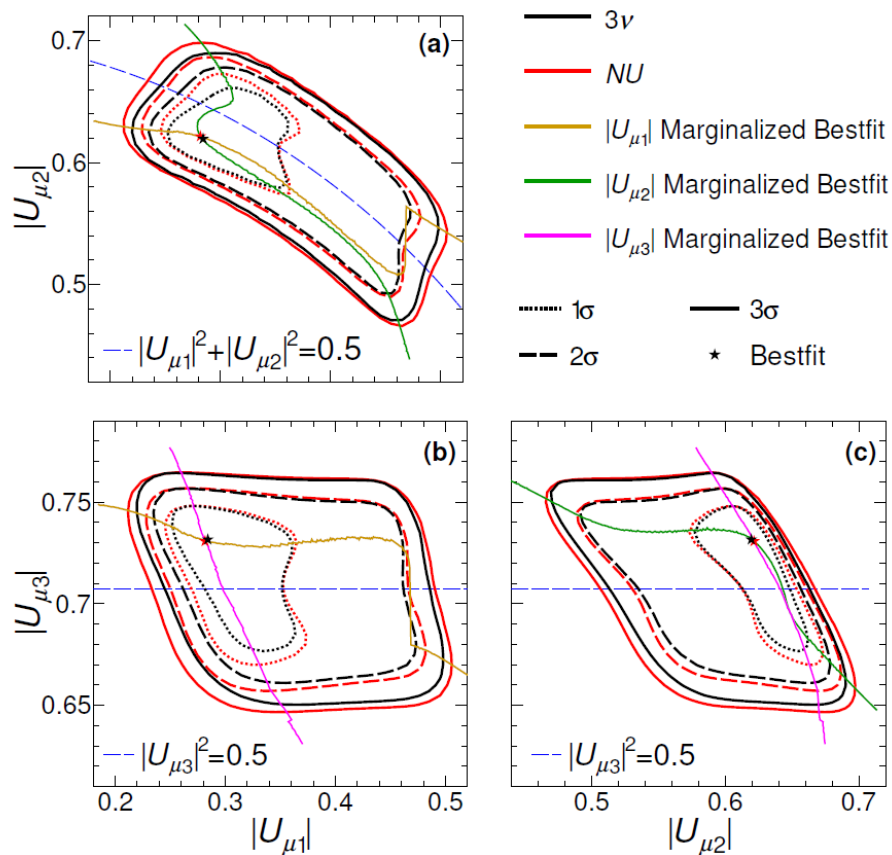
- Only 4 real parameters with unitarity.
- Now we have 13 real parameters after rephrasing fields for NU!

What if there is non-unitary mixing?

$$U^{\text{NU}} = \begin{pmatrix} |U_{e1}|e^{i\phi_{e1}} & |U_{e2}|e^{i\phi_{e2}} & |U_{e3}| \\ |U_{\mu1}|e^{i\phi_{\mu1}} & |U_{\mu2}|e^{i\phi_{\mu2}} & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}| & |U_{\tau3}| \end{pmatrix} \quad P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{NU}} = \left| \sum_{i=1}^3 U_{\beta i}^* U_{\alpha i} \right|^2 - 4 \sum_{i < j} \Re(U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*) \sin^2 \left(\frac{\Delta m_{ji}^2 L}{4E_\nu} \right) \pm 2 \sum_{i < j} \Im(U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*) \sin \left(\frac{\Delta m_{ji}^2 L}{2E_\nu} \right),$$

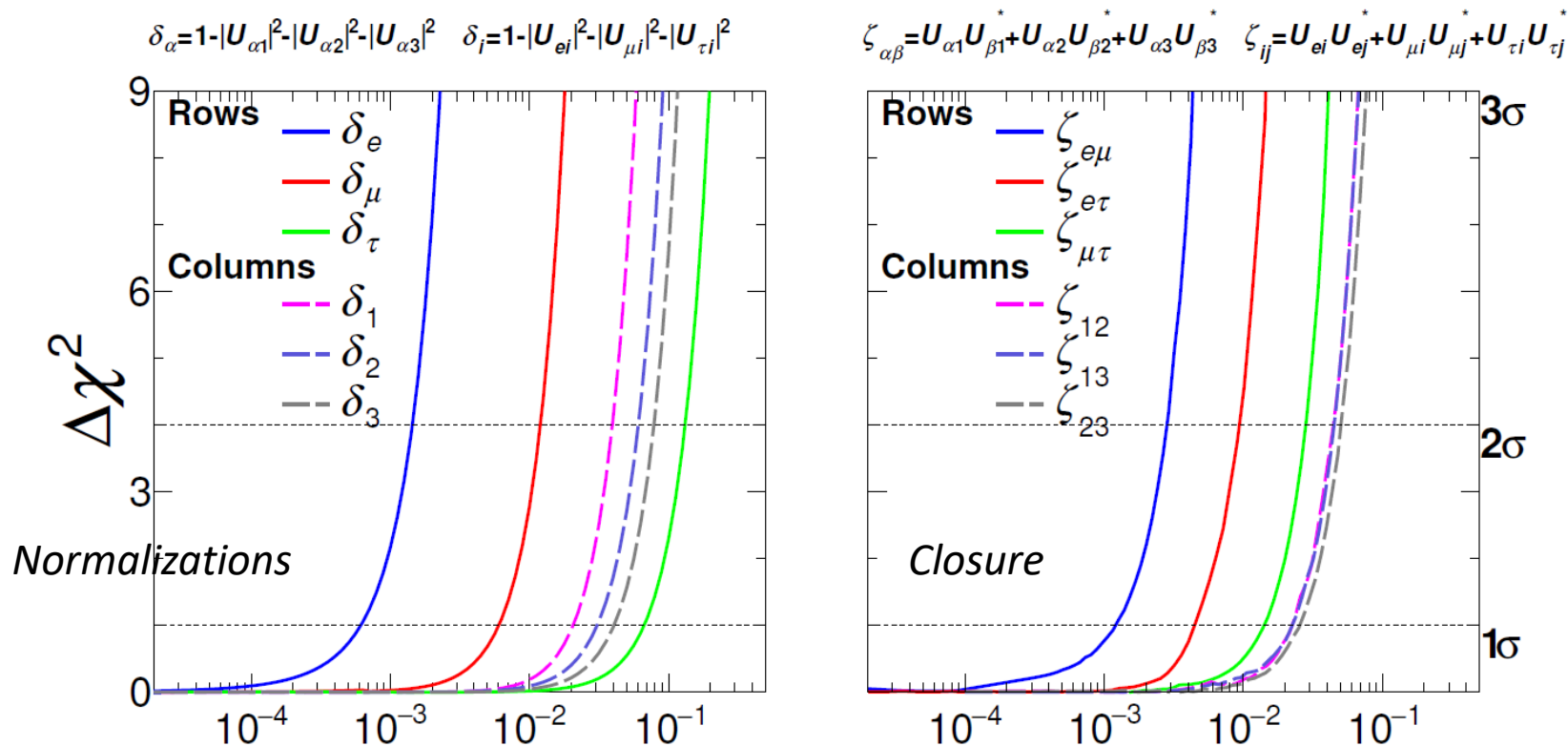
Type	Exps	Measurement
MBL Reactor	RENO, Daya Bay Double Chooz	$4 U_{e3} ^2(U_{e1} ^2 + U_{e2} ^2)$
LBL Reactor	KamLAND	$4 U_{e1} ^2 U_{e2} ^2$
Solar	SNO	$ U_{e2} ^2$
LBL Accelerator ($\nu_\mu \rightarrow \nu_\mu$)	NOvA, T2K	$4 U_{\mu3} ^2(U_{\mu1} ^2 + U_{\mu2} ^2)$
LBL Accelerator ($\nu_\mu \rightarrow \nu_e$)	NOvA, T2K	$4\Re[U_{e3}U_{\mu3}^*(U_{e1}U_{\mu1}^* + U_{e2}U_{\mu2}^*)]$
LBL Accelerator ($\nu_\mu \rightarrow \nu_\tau$)	OPERA	$4\Re[U_{\tau3}U_{\mu3}^*(U_{\tau1}U_{\mu1}^* + U_{\tau2}U_{\mu2}^*)]$

- Correlations between 3nu mixing matrix elements without unitarity assumption.
- Octant degeneracies get worse for NU.



What if there is non-unitary mixing?

$$U^{\text{NU}} = \begin{pmatrix} |U_{e1}|e^{i\phi_{e1}} & |U_{e2}|e^{i\phi_{e2}} & |U_{e3}| \\ |U_{\mu1}|e^{i\phi_{\mu1}} & |U_{\mu2}|e^{i\phi_{\mu2}} & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}| & |U_{\tau3}| \end{pmatrix} \quad P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{NU}} = \left| \sum_{i=1}^3 U_{\beta i}^* U_{\alpha i} \right|^2 - 4 \sum_{i < j} \Re(U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*) \sin^2 \left(\frac{\Delta m_{ji}^2 L}{4E_\nu} \right) \pm 2 \sum_{i < j} \Im(U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta i}^*) \sin \left(\frac{\Delta m_{ji}^2 L}{2E_\nu} \right),$$

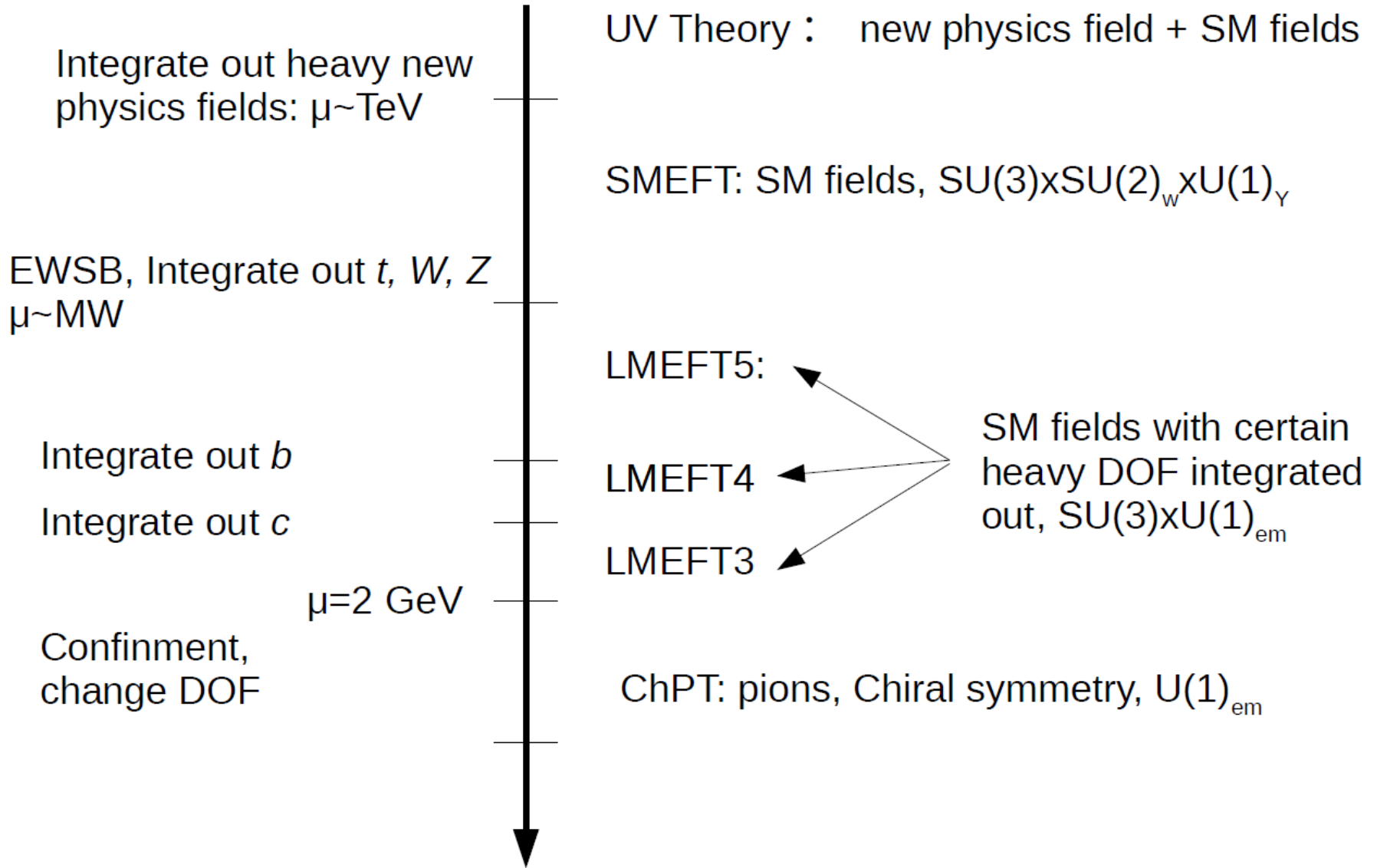


- Nice precision in the first and second row/column.
- Tau neutrino physics are to be improved for better constraints on 3rd row/column!

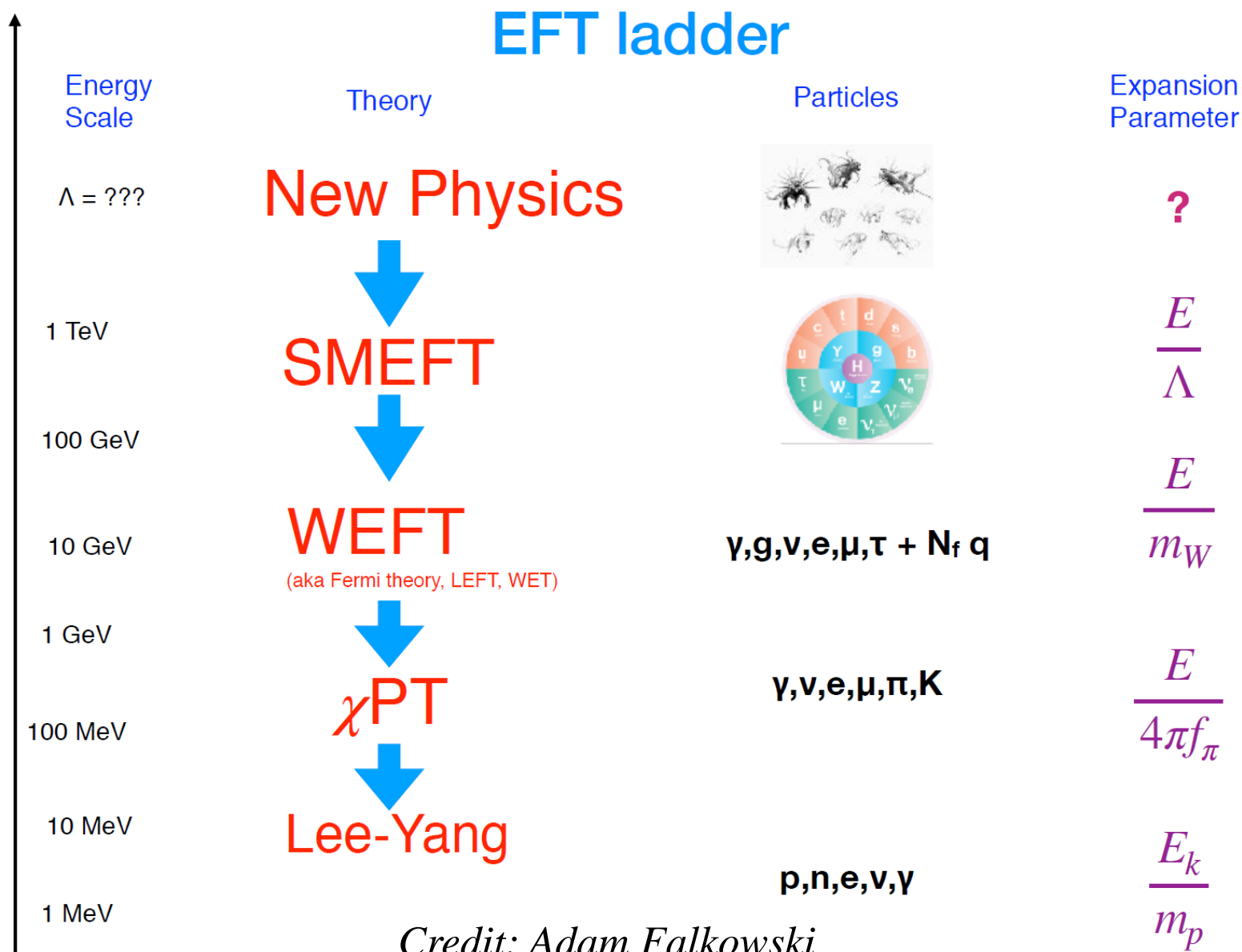


- Motivations
- Non-unitarity(NU) v.s non-standard interactions(NSIs)
- Global fits of mixing parameters without unitarity
- **Constraints of NSIs based on SMEFT**
- Future prospects

Working principle of effective field theory (EFT)



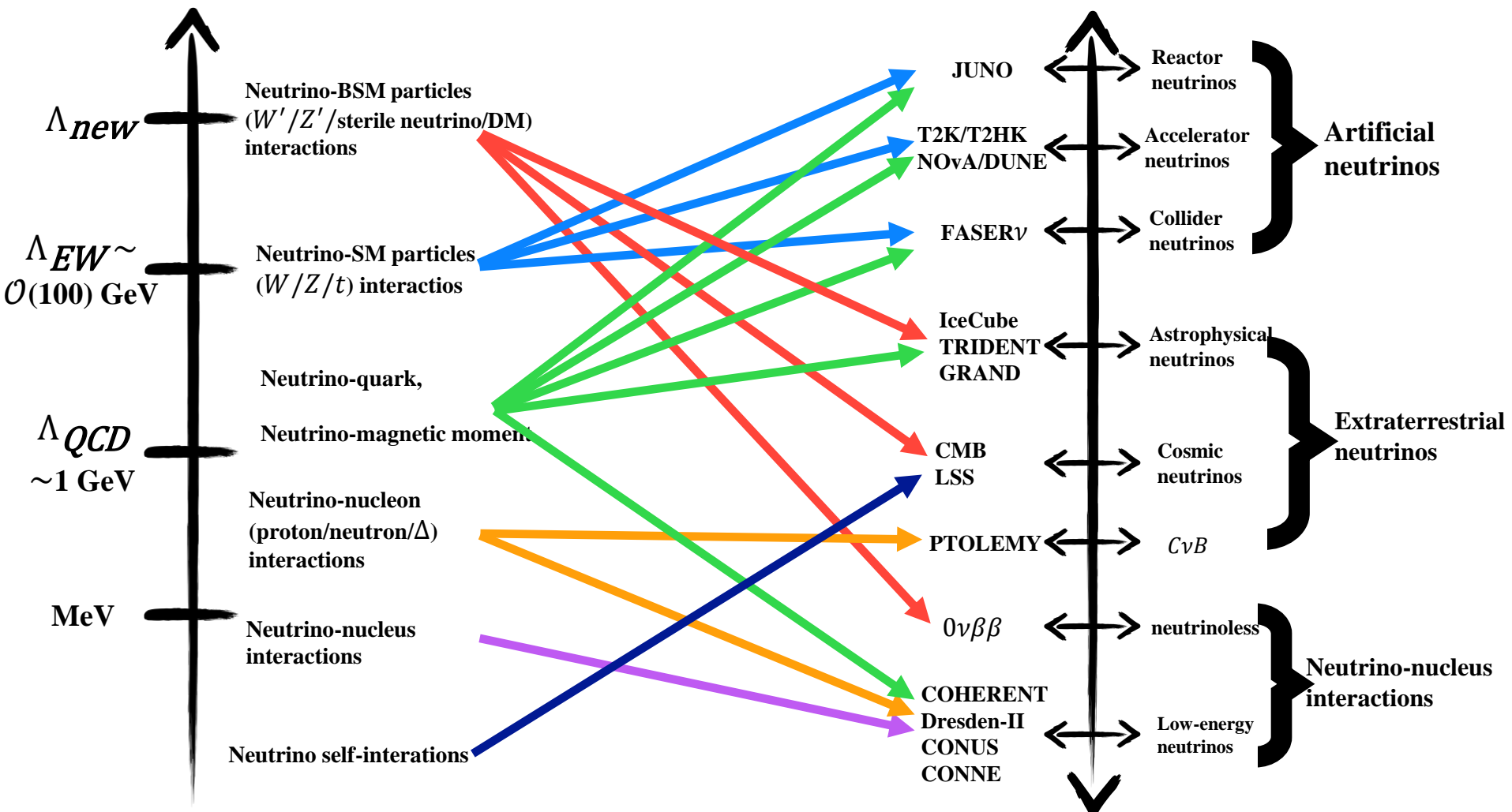
EFT: connecting low-energy phenomenon to high-energy scale



Credit: Adam Falkowski

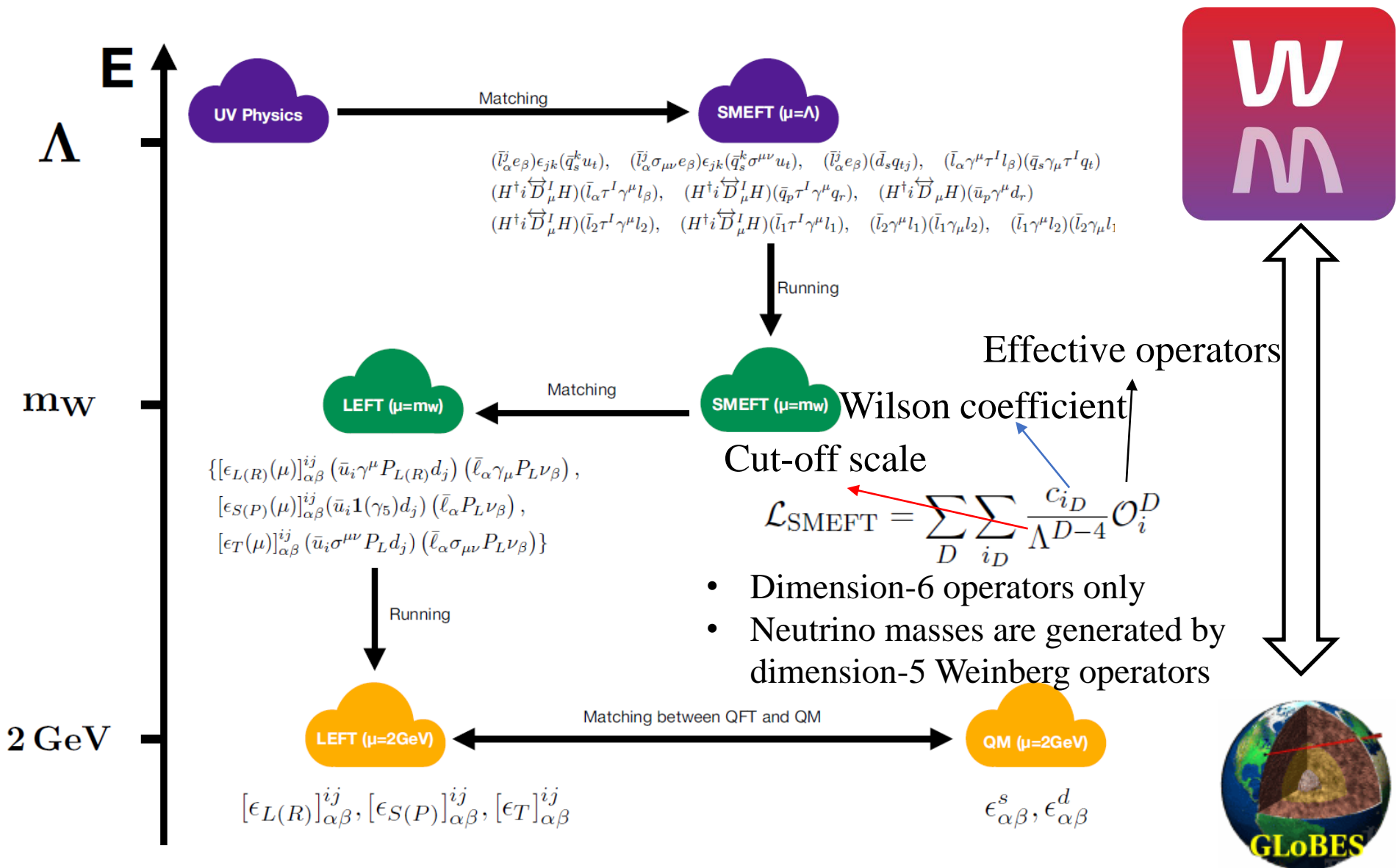


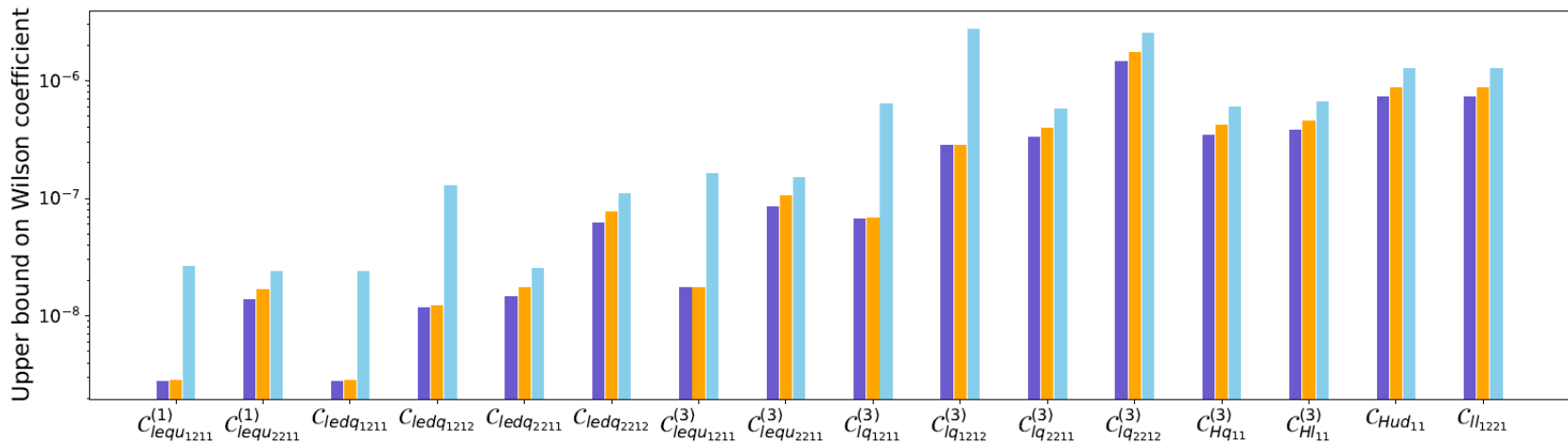
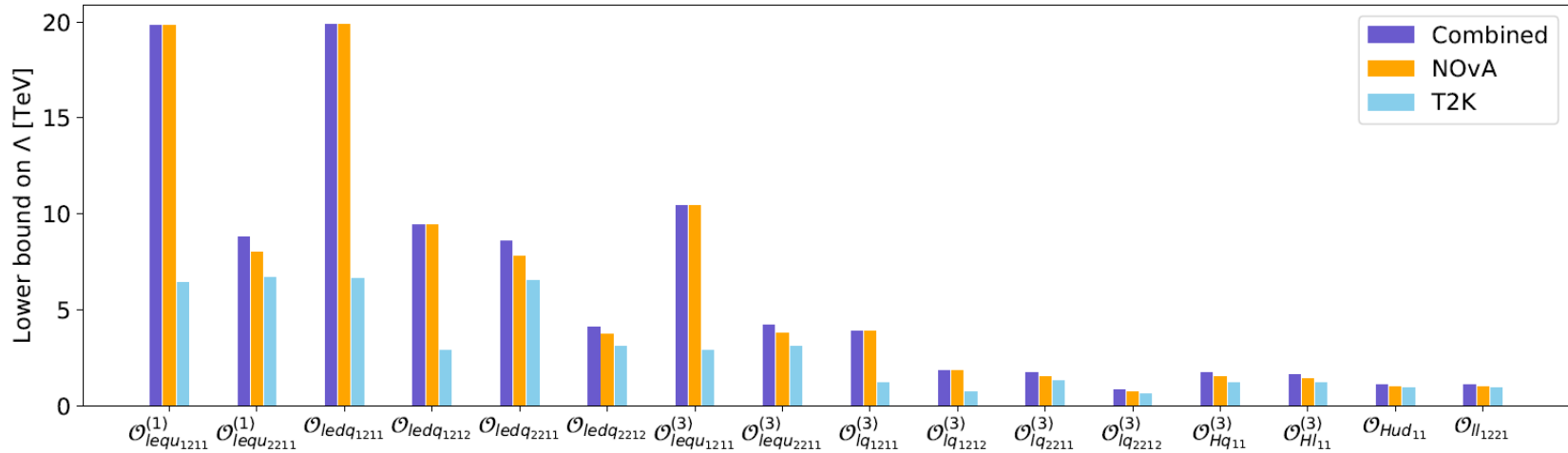
EFT: connecting low-energy phenomenon to high-energy scale



Funded project in collaboration with **Jiang-Hao Yu**, **Ning-Qiang Song** and **Guang Li**!

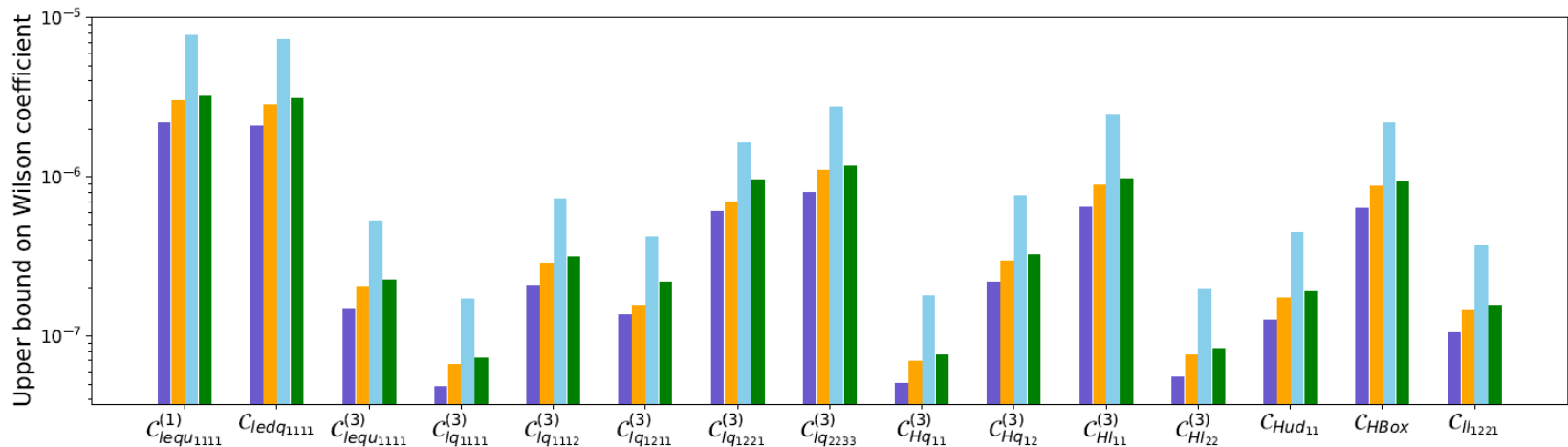
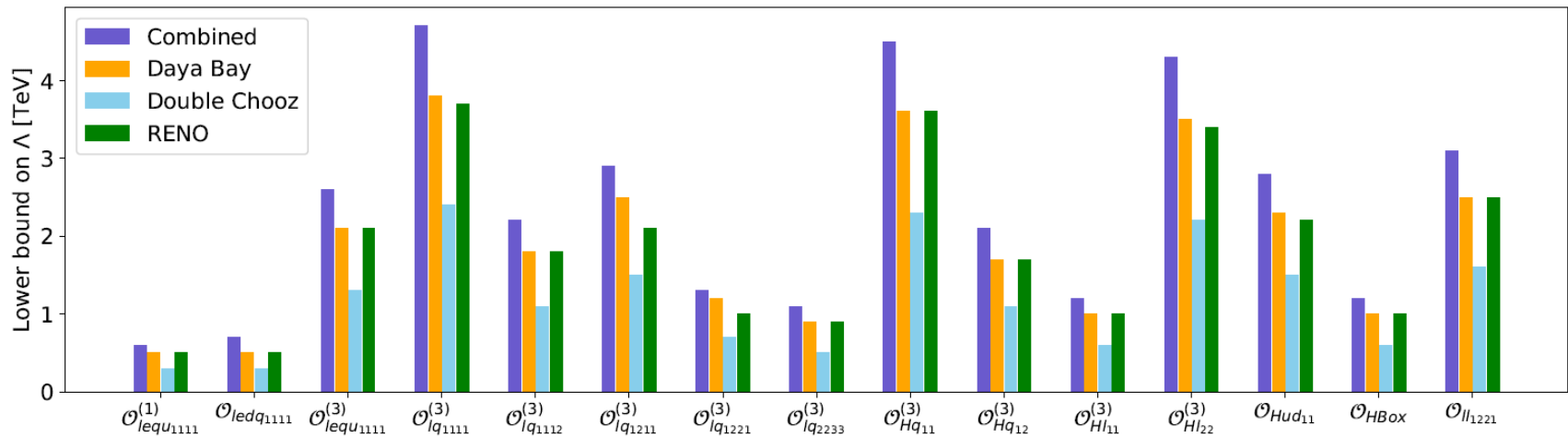
EFT: connecting low-energy phenomenon to high-energy scale





- T2K and NOvA are already sensitive to new physics around 20 TeV.
- Correlations among different dimension-6 operators play important roles.

SMEFT-NSIs by reactor neutrino experiments



- Reactor neutrino experiments are sensitive to new physics around 5 TeV.
- Complementarity between LBL and reactor expts due to different sets of operators



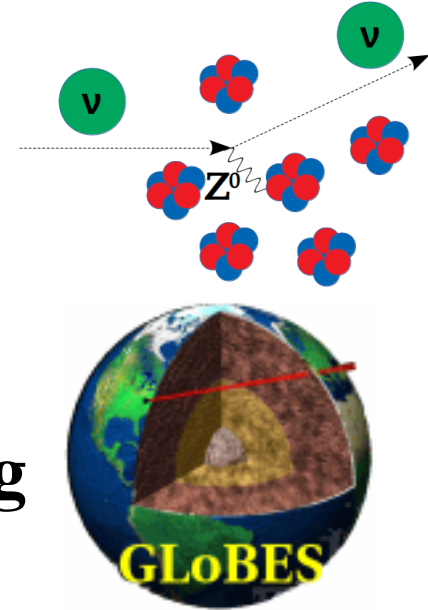
- Motivations
- Non-unitarity(NU) v.s non-standard interactions(NSIs)
- Global fits of mixing parameters without unitarity
- Constraints of NSIs based on SMEFT
- **Future prospects on SMEFT-NSIs**

Democratic treatment of CKM and PMNS mixing



flavio

Smelli



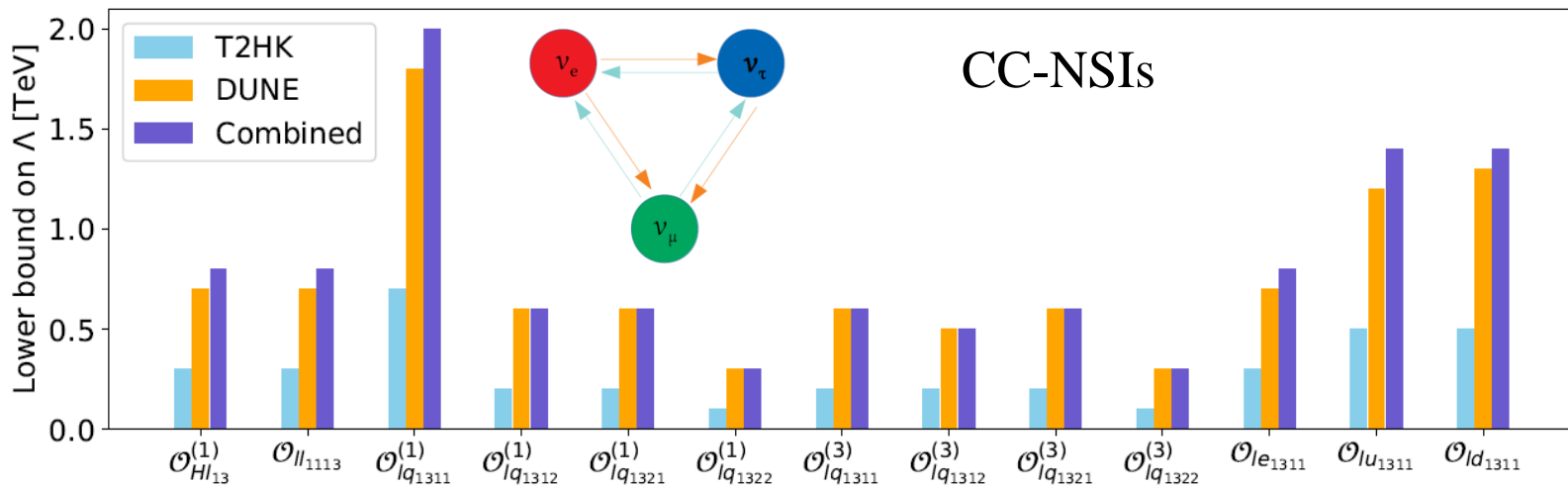
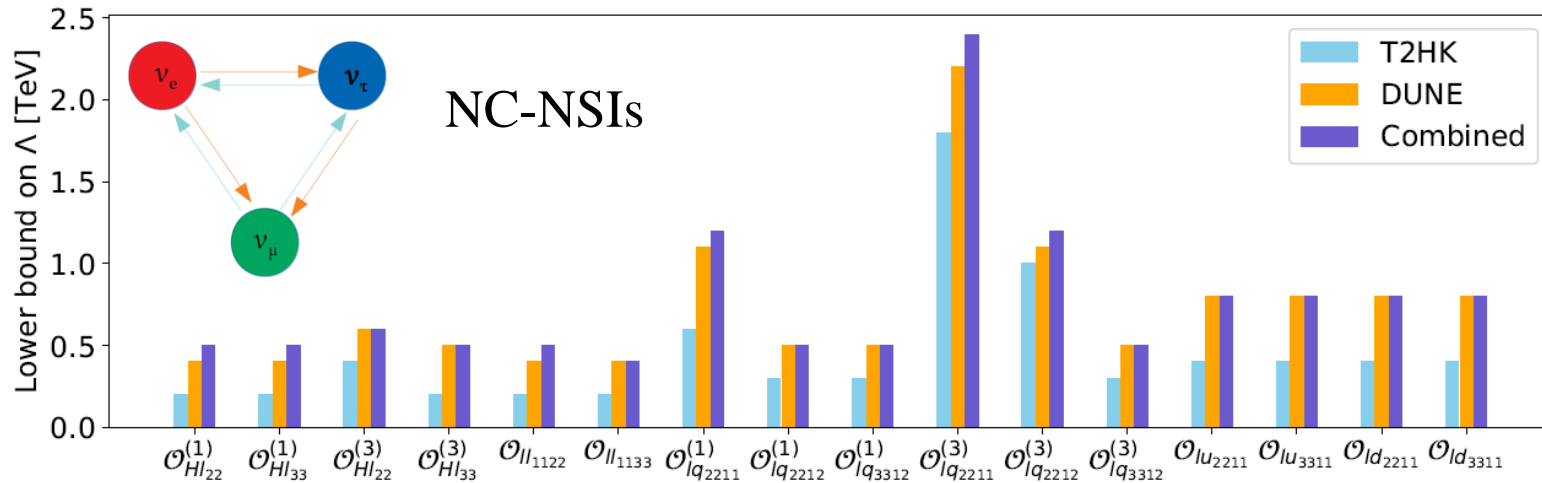
Quark mixing **v.s** neutrino mixing/scattering



$$\mathcal{O}_{1,2,3,4}^{\text{CKM}} = \Gamma(K \rightarrow \mu\nu_\mu) / \Gamma(\pi \rightarrow \mu\nu_\mu), \text{Br}(B \rightarrow X_c e \nu), \text{Br}(B^+ \rightarrow \tau \nu), \Delta M_d / \Delta M_s.$$

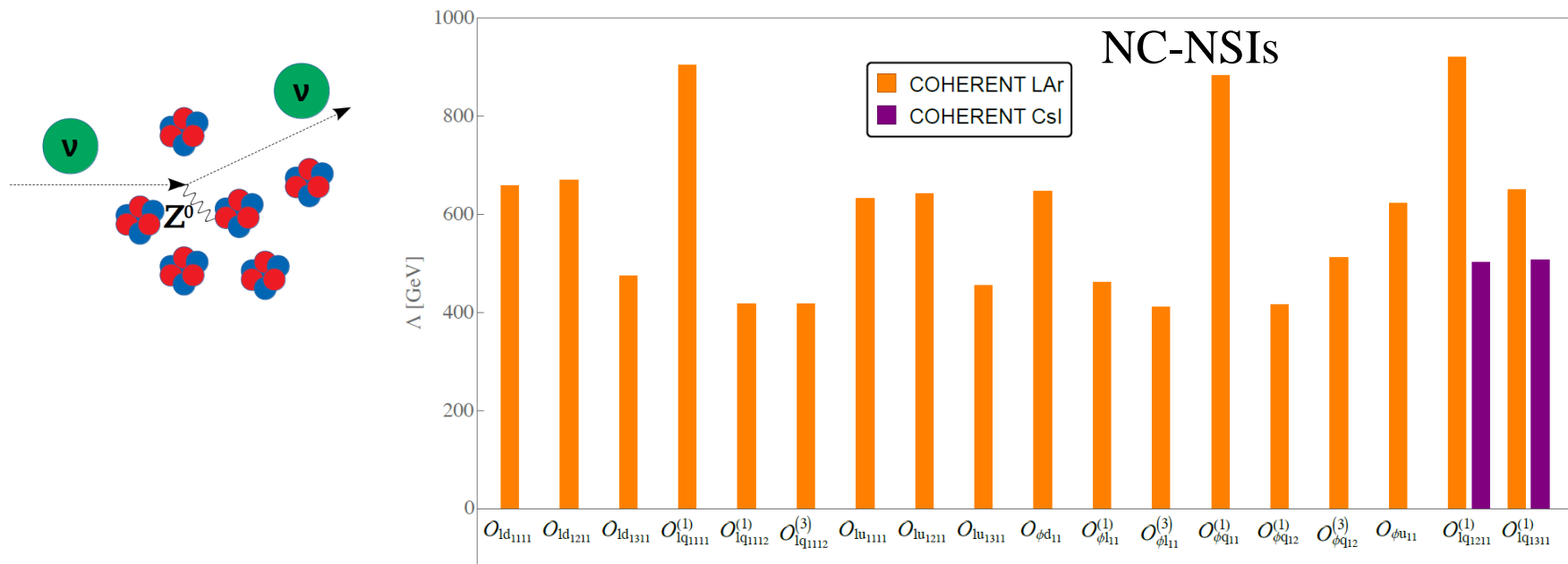
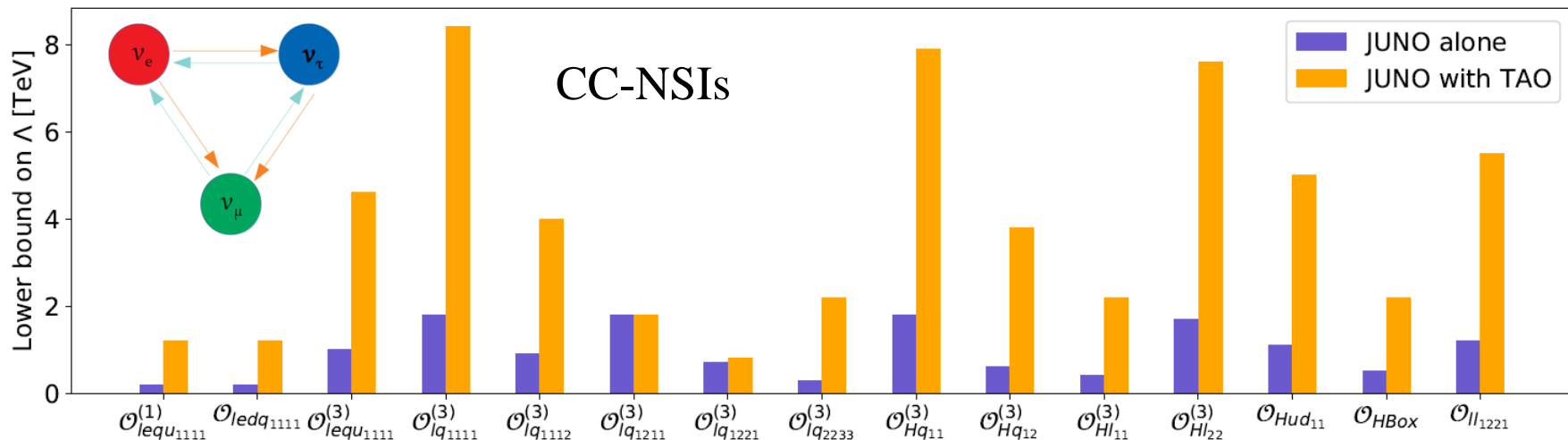
Operator	$\mathcal{O}_{qq1313}^{(1)}$	$\mathcal{O}_{qq2323}^{(1)}$	$\mathcal{O}_{qq1313}^{(3)}$	$\mathcal{O}_{qq2323}^{(3)}$	\mathcal{O}_{dd1313}	\mathcal{O}_{dd2323}
Λ valid (TeV)	> 365	> 51	> 365	> 51	> 383	> 53
Operator	$\mathcal{O}_{qd1213}^{(1)}$	$\mathcal{O}_{qd1313}^{(1)}$	$\mathcal{O}_{qd2323}^{(1)}$	$\mathcal{O}_{qd1213}^{(8)}$	$\mathcal{O}_{qd1313}^{(8)}$	$\mathcal{O}_{dd2323}^{(8)}$
Λ valid (TeV)	> 23	> 1383	> 178	> 25	> 1466	> 188

SMEFT-NSIs by T2HK and DUNE

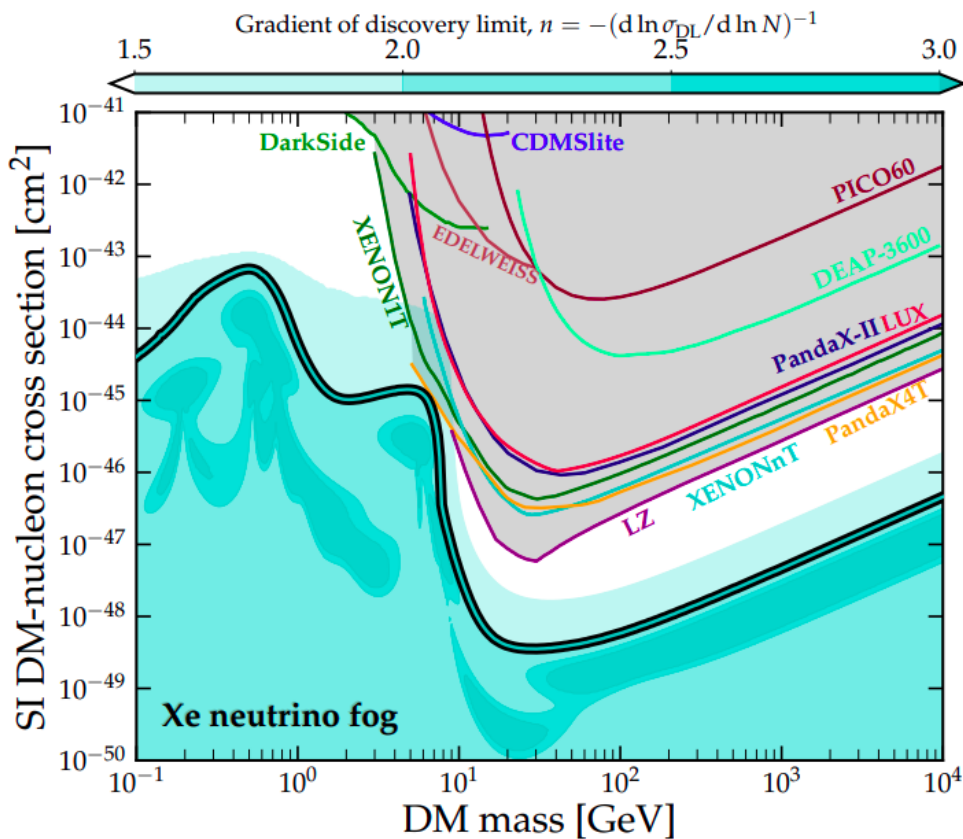


- Cover 1635 SMEFT operators in dimension-6. ND is important to constraint SMEFT-NSIs.
- DUNE has better sensitivity than T2HK due to the longer baseline.

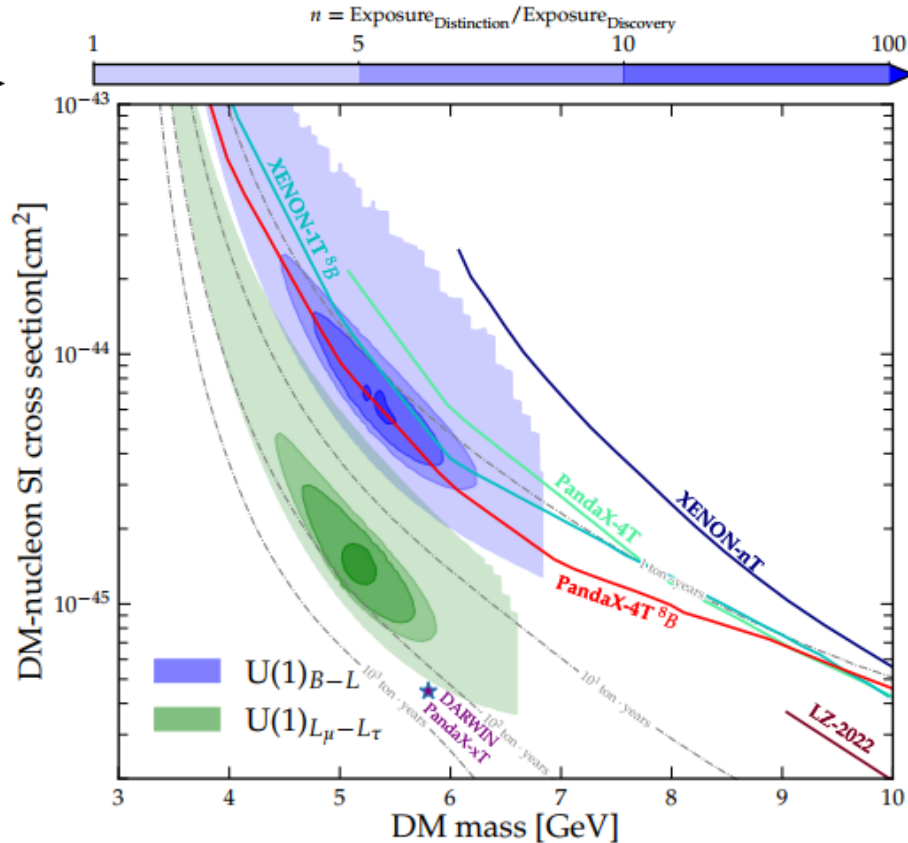
SMEFT-NSIs by JUNO w/o TAO and COHERENT



One stone, two birds: DM & neutrino NSIs



J. Tang, B. L. Zhang, Phys.Rev.D 108 (2023) 6, 062004



J. Tang, B. L. Zhang, arXiv: [2403.05819](https://arxiv.org/abs/2403.05819),

<https://github.com/zhangblong/DistinctionLimit>

- Neutrino oscillation is the first direct evidence BSM.
- Discovery of CPV & determination of MH is around the corner. Neutrino will be used for new physics searches.
- New physics might be hidden in the uncertainties.
- Unitary mixing should not be taken by default, as tau neutrino-related part is yet to be improved.
- We can have better knowledge of underlying theory by RGE running and matching in different scales to connect the low-energy neutrino oscillation experiments and the UV completion model in SMEFT.
- Let's work together to discover new physics with neutrinos...

News—Two Postdoc positions opening in SYSU



1. One to work on neutrino physics, such as neutrino scattering or data analysis for JUNO OSIRIS with German colleagues.
 2. The other to work on muon physics and its applications
- ◆ Salary: ~50 k Euros/year + bonus + on-campus apartment.
 - ◆ Application packages: CV, publication list, research statement, two reference letters to email: tangjian5@mail.sysu.edu.cn
 - ◆ Remote interview might happen soon after a complete application package is received.
 - ◆ Deadline: June 30, 2024.

The background features a large, light green watermark of the Tsinghua University logo. The logo is circular and contains the university's name in Chinese characters '清華大學' at the top and 'TSINGHUA UNIVERSITY' at the bottom. In the center of the logo is a detailed illustration of a building with a central tower and the year '1924' at the base. Two dark green rectangular shapes are positioned on the left and right sides of the slide, partially overlapping the watermark.

THANK YOU

An example for demonstration

- Start from a UV theory

$$\Delta\mathcal{L} = (D_\mu H_2^\dagger)(D^\mu H_2) - M^2|H_2|^2 - Y\bar{L}_i H_{2i} e_R - Y^* \bar{e}_R H_{2i}^\dagger L_i$$

- Matching by covariant derivative expansion, EoM

$$(D^2 H_2^\dagger)_i + M^2 H_{2i}^\dagger = -Y\bar{L}_i e_R$$

$$(D^2 H_2)_i + M^2 H_{2i} = -Y^* \bar{e}_R L_i$$

$$(D_{ij}^2 + M^2 \delta_{ij}) H_{2j}^\dagger = -Y\bar{L}_i e_R$$

$$(D_{ij}^2 + M^2 \delta_{ij}) H_{2j} = -Y^* \bar{e}_R L_i$$

- Solve for classical solution

$$\begin{aligned} H_{c,2i} &= -(D_{ij}^2 + M^2 \delta_{ij})^{-1} Y^* \bar{e}_R L_j \\ &= -\frac{1}{M^2} \left(1 + \frac{D^2}{M^2}\right)_{ij}^{-1} Y^* \bar{e}_R L_j \\ &= -\frac{1}{M^2} Y^* \bar{e}_R L_i + \mathcal{O}\left(\frac{1}{M^4}\right) \end{aligned}$$

$$H_{c,2i}^\dagger = -\frac{1}{M^2} Y \bar{L}_i e_R + \mathcal{O}\left(\frac{1}{M^4}\right)$$

An example for demonstration

- Put the classical solution back to Lagrangian density

$$(D_\mu H_{2,c}^\dagger)(D^\mu H_{2,c}) = -H_{2,c}^\dagger D^2 H_{2,c} \sim \mathcal{O}\left(\frac{1}{M^4}\right)$$

$$-M^2 |H_{2,c}|^2 = -\frac{|Y|^2}{M^2} \bar{L}_i e_R \bar{e}_R L_i$$

$$-Y \bar{L}_i H_{2i,c} e_R - Y^* \bar{e}_R H_{2i,c}^\dagger L_i = \frac{2|Y|^2}{M^2} \bar{L}_i e_R \bar{e}_R L_i$$

$$\mathcal{L}_{eff}^{dim-6} = \frac{|Y|^2}{M^2} \bar{L}_i e_R \bar{e}_R L_i$$

Not in
Warsaw Basis

- Fierz transformation to Warsaw basis

$$\frac{|Y|^2}{M^2} \bar{L}_i e_R \bar{e}_R L_i = \frac{|Y|^2}{2M^2} (\bar{L}_i \gamma^\mu L_i) (\bar{e}_R \gamma^\mu e_R)$$

$$C_{le} = \frac{|Y|^2}{2M^2}$$

$$Q_{le}$$

SMEFT at scale M

An example for demonstration

- Run down from scale M to electroweak scale v

$$\mu \frac{d}{d\mu} C_i = \sum_j \gamma_{ij} C_j$$

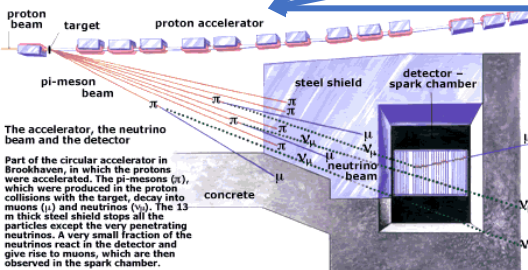
- Probably generate non-zero Wilson coefficients other than C_{le}
- Matching at electroweak scale to LEFT

$$L_p = \begin{bmatrix} \nu_p \\ e_p \end{bmatrix}$$

$$\begin{aligned} \frac{Y_{ps} Y_{tr}^\dagger}{2M^2} (\bar{L}_i^p \gamma^\mu L_i^r) (\bar{e}_R^s \gamma^\mu e_R^t) &= \frac{Y_{ps} Y_{tr}^\dagger}{2M^2} (\bar{\nu}_L^p \gamma^\mu \nu_L^r + \bar{e}_L^p \gamma^\mu e_L^r) (\bar{e}_R^s \gamma^\mu e_R^t) \\ &= \frac{Y_{ps} Y_{tr}^\dagger}{2M^2} \left[(\bar{\nu}_L^p \gamma^\mu \nu_L^r) (\bar{e}_R^s \gamma^\mu e_R^t) + (\bar{e}_L^p \gamma^\mu e_L^r) (\bar{e}_R^s \gamma^\mu e_R^t) \right] \\ &\quad \downarrow \qquad \qquad \qquad \downarrow \\ &\quad \mathcal{O}_{\nu e}^{V,LR} \qquad \qquad \qquad \mathcal{O}_{ee}^{V,LR} \end{aligned}$$

Matching between QM and QFT NSIs

$$H = \frac{1}{2E_\nu} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + A \begin{pmatrix} 1 + \epsilon_{ee}^m & \epsilon_{e\mu}^m & \epsilon_{e\tau}^m \\ \epsilon_{e\mu}^{m*} & \epsilon_{\mu\mu}^m & \epsilon_{\mu\tau}^m \\ \epsilon_{e\tau}^{m*} & \epsilon_{\mu\tau}^{m*} & \epsilon_{\tau\tau}^m \end{pmatrix} \right]$$



QM NSIs	Relations to QFT NSIs
$\epsilon_{e\beta}^s$ (β decay)	$\left[\epsilon_L - \epsilon_R - \frac{g_T}{g_A} \frac{m_e}{f_T(E_\nu)} \epsilon_T \right]_{e\beta}^*$
$\epsilon_{\beta e}^d$ (inverse β decay)	$\left[\epsilon_L + \frac{1-3g_A^2}{1+3g_A^2} \epsilon_R - \frac{m_e}{E_\nu - \Delta} \left(\frac{g_S}{1+3g_A^2} \epsilon_S - \frac{3g_A g_T}{1+3g_A^2} \epsilon_T \right) \right]_{e\beta}$
$\epsilon_{\mu\beta}^s$ (pion decay)	$\left[\epsilon_L - \epsilon_R - \frac{m_\pi^2}{m_\mu(m_u+m_d)} \epsilon_P \right]_{\mu\beta}^*$
$\epsilon_{\mu\beta}^s$ (muon decay)	$\left[g_{22} + \frac{3m_e m_\mu (m_\mu - 2E_\nu)}{16m_\mu E_\nu^2 + 6m_\mu (m_\mu^2 + m_e^2) - 4E_\nu (5m_\mu^2 + m_e^2)} h_{21} \right]_{\mu\beta}^*$
$\epsilon_{e\beta}^s$ (muon decay)	$\left[g_{22} + \frac{m_e}{4(m_\mu - 2E_\nu)} h_{21} \right]_{e\beta}^*$

- QM NSIs are fully independent without much info about the underlying theory.
- QFT NSIs allow their correlations.