New Physics Prospects in Semileptonic $\Lambda_b \to \Lambda_c^*$ Decays

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Outline

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- Theoretical Framework for the $\Lambda_b \to \Lambda_c^* \tau \bar{\nu}_\tau$ decays
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Motivation

Flavor Anomalies

- Flavor anomalies in b-hadron decays - BSM physics.
- Discrepancies seen in decays: $b \to s\ell^+\ell^-$
  $b \to c\tau^-\overline{\nu}_\tau$
- Lepton flavor universality (LFU)

$$R(H_s) = \frac{\mathcal{B}(H_b \to H_s\mu^+\mu^-)}{\mathcal{B}(H_b \to H_s e^+e^-)}$$

$$R(H_c) = \frac{\mathcal{B}(H_b \to H_c\tau\bar{\nu}_\tau)}{\mathcal{B}(H_b \to H_c\ell\bar{\nu}_\ell)}$$

Flavor-changing neutral currents ($b \to s\ell^+\ell^-$)

- The ratio,

$$R_{K^{(*)}} = \frac{\mathcal{B}(\bar{B} \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(\bar{B} \to K^{(*)}e^+e^-)}$$

- Earlier , $R^{Expt}_{K^{(*)}} < R^{\rm SM}_{K^{(*)}}$. The latest LHCb results predict $R_{K^{(*)}} \simeq 1$, in agreement with the SM.[1]

  Tensions with the SM in :
- Angular $B \to K^*\mu^+\mu^-$ observable P_5'.[2]
- Total branching ratio and angular observables in $B_s \to \phi\mu^+\mu^-$ and $\mathcal{B}(B \to K\mu^+\mu^-)$.[3,4]

- New Physics (NP) models: Z', Leptoquarks.

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[1] R. Aaij et al. [LHCb]; Phys. Rev. Lett. 131, no.5, 051803 (2023)
[2] Sebastien Descotes-Genon et al.; JHEP 01, 048 (2013)
[3] R. Aaij et al. (LHCb); JHEP 09, 179 (2015)
[4] R. Aaij et al. (LHCb); JHEP 06, 133 (2014)

Flavor-changing charged currents ($b \to c\tau^- \overline{\nu}_\tau$)

- The world average value of R_D and R_{D^*} reported by HFLAV[5],

$$R_D^{Expt} = \frac{\mathcal{B}(\bar{B} \to D\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D\ell^-\bar{\nu}_\ell)} = 0.357 \pm 0.029$$

$$R_{D^*}^{Expt} = \frac{\mathcal{B}(\bar{B} \to D^*\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^*\ell^-\bar{\nu}_\ell)} = 0.284 \pm 0.012$$

  deviate from their SM predictions, $R_D^{\rm SM} = 0.298 \pm 0.004$, $R_{D^*}^{\rm SM} = 0.254 \pm 0.005$ by 2σ and 2.2σ, respectively.

- LHCb[6] measured the value of $R_{J/\psi}$

$$R_{J/\psi}^{Expt} = \frac{\mathcal{B}(B_c \to J/\psi\tau\bar{\nu}_\tau)}{\mathcal{B}(B_c \to J/\psi\mu\bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18$$

  and $R_{J/\psi}^{\rm SM} = 0.289 \pm 0.01$.

[5] https://hflav-eos.web.cern.ch/hflav-eos/semi/summer23/html/RDsDsstar/RDRDs.html.

[6] R. Aaij et al. (LHCb Collaboration); Phys. Rev. Lett. 120, 121801 (2018).

- Belle measured the longitudinal polarization of $\tau(P_\tau^{D^*})$[7] and of $D^*(F_L^{D^*})$[8] in $B \to D^* \tau \overline{\nu}_\tau$ decay,

$$P_\tau^{D^*} = \frac{\Gamma(\lambda_\tau = 1/2) - \Gamma(\lambda_\tau = -1/2)}{\Gamma(\lambda_\tau = 1/2) + \Gamma(\lambda_\tau = -1/2)} = -0.38 \pm 0.51^{+0.21}_{-0.16}$$

$$F_L^{D^*} = \frac{\Gamma_{\lambda_{D^*}=0}(B \to D^* \tau \overline{\nu}_\tau)}{\Gamma(B \to D^* \tau \overline{\nu}_\tau)} = 0.60 \pm 0.08 \pm 0.04$$

and their SM predictions are

$$P_\tau^{D^*} = -0.497 \pm 0.013 \quad , \quad F_L^{D^*} = 0.457 \pm 0.010.$$

- NP models: W', Leptoquarks.

- Leptoquarks as possible NP candidates to explain flavor anomalies.

[7] S. Hirose et al. (Belle); Phys. Rev. D 97, 012004 (2018).

[8] A.Abdesselam et al.(Belle),in 10th International Workshop on the CKM Unitarity Triangle(2019).

A. Angelescu et al., "Closing the window on single leptoquark solutions to the B-physics anomalies", JHEP 10 (2018) 183

Model	$R_{K^{(*)}}$	$R_{D^{(*)}}$	$R_{K^{(*)}}$ & $R_{D^{(*)}}$
S_1	✗*	✓	✗*
R_2	✗*	✓	✗
$\widetilde{R}_2$	✗	✗	✗
S_3	✓	✗	✗
U_1	✓	✓	✓
U_3	✓	✗	✗

- Only U_1 can simultaneously accomodate $R_{K^{(*)}}$ and $R_{D^{(*)}}$.
- We examine the impact of the presence of U_1 leptoquark in the $\Lambda_b \to \Lambda_c^*(2595, 2625)\tau^-\bar{\nu}_\tau$ decays.

Theoretical Framework for the $\Lambda_b \to \Lambda_c^* \tau \bar{\nu}_\tau$ decays

Effective Hamiltonian

The most general effective Hamiltonian for $b \to c \ell \nu_\ell$ transition including NP contributions[9]

$$\mathcal{H}_{eff}^{b\to c\ell\nu} = \frac{4G_F V_{cb}}{\sqrt{2}} \Big[O_{V_L} + \sum_i C_i O_i \Big],$$

$C_i \longrightarrow C_{V_{L,R}}, C_{S_{L,R}}, C_T$ Wilson coefficients.

The fermionic operators are:

$$\begin{aligned}
\mathcal{O}_{V_L} &= (\bar{c}_L \gamma^\mu b_L) \left(\bar{\ell}_L \gamma_\mu \nu_{\ell_L}\right), \quad \mathcal{O}_{V_R} = (\bar{c}_R \gamma^\mu b_R) \left(\bar{\ell}_L \gamma_\mu \nu_{\ell_L}\right) \\
\mathcal{O}_{S_L} &= (\bar{c}_R b_L) \left(\bar{\ell}_R \nu_{\ell_L}\right), \quad \mathcal{O}_{S_R} = (\bar{c}_L b_R) \left(\bar{\ell}_R \nu_{\ell_L}\right) \\
\mathcal{O}_T &= (\bar{c}_R \sigma^{\mu\nu} b_L) \left(\bar{\ell}_R \sigma_{\mu\nu} \nu_{\ell_L}\right)
\end{aligned}$$

9 C. Murgui, A. Peñuelas, M. Jung and A. Pich ; JHEP 09, 103 (2019).

U_1 leptoquark model

- Interaction Lagrangian describing the coupling between U_1 and SM fermions can be described by,

$$\mathcal{L}_{U_1} = h_L^{ij} \bar{Q}_i \gamma_\mu U_1^\mu L_j + h_R^{ij} \bar{d}_{Ri} \gamma_\mu U_1^\mu \ell_{Rj} + h.c.$$

  where $h_{L,R}^{ij}$ are 3×3 complex matrices.

- Integrating out the heavy U_1, the couplings contributing to $b \to c\tau\bar{\nu}_\tau$ are,

$$\begin{aligned} C_{V_L}(\mu_{\rm LQ}) &= \frac{1}{2\sqrt{2} G_F V_{cb}} \frac{(V_{\rm CKM} h_L)^{23} (h_L^{33})^*}{M_{U_1}^2} \\ C_{S_R}(\mu_{\rm LQ}) &= \frac{-1}{\sqrt{2} G_F V_{cb}} \frac{(V_{\rm CKM} h_L)^{23} (h_R^{33})^*}{M_{U_1}^2} \end{aligned}$$

Hadronic matrix elements :

$$B_1(p_1, M_1) \longrightarrow B_2^{(*)}(p_2, M_2) + \ell(p_\ell, m_\ell) + \nu_\ell(p_\nu, 0)$$

The hadronic matrix elements can be parametrized in terms of various form factors.

For $1/2^+ \to 1/2^-$ transition $(\Lambda_b \to \Lambda_c^*(2595)\tau\bar{\nu}_\tau)$,

$$\begin{aligned} M_\mu^V &= \langle B_2, \lambda_2|\bar{c}\gamma_\mu b|B_1, \lambda_1\rangle \\ &= \overline{u}_{B_2}(p_2, \lambda_2)\Big[\gamma_\mu F_1^V(q^2) - i\sigma_{\mu\nu}\frac{q_\nu}{M_1}F_2^V(q^2) + \frac{q_\mu}{M_1}F_3^V(q^2)\Big]\gamma_5 u_{B_1}(p_1, \lambda_1) \end{aligned}$$

$$\begin{aligned} M_\mu^A &= \langle B_2, \lambda_2|\bar{c}\gamma_\mu\gamma_5 b|B_1, \lambda_1\rangle \\ &= \overline{u}_{B_2}(p_2, \lambda_2)\Big[\gamma_\mu F_1^A(q^2) - i\sigma_{\mu\nu}\frac{q_\nu}{M_1}F_2^A(q^2) + \frac{q_\mu}{M_1}F_3^A(q^2)\Big]u_{B_1}(p_1, \lambda_1) \end{aligned}$$

$q^\mu \longrightarrow$ four-momentum transfer,
$\lambda_1, \lambda_2 \longrightarrow$ helicities of the parent and daughter baryon,
$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$.

For $1/2^+ \to 3/2^-$ transition $(\Lambda_b \to \Lambda_c^*(2625)\tau\bar{\nu}_\tau)$,

$$\begin{aligned} M_\mu^V &= \langle B_2^*, \lambda_2 | \bar{c}\gamma_\mu b | B_1, \lambda_1 \rangle \\ &= \overline{u}_{B_2^*}(p_2, \lambda_2) \Big[g_{\alpha\mu} F_1^V(q^2) + \gamma_\mu \frac{p_{1\alpha}}{M_1} F_2^V(q^2) + \frac{p_{1\alpha} p_{2\mu}}{M_1^2} F_3^V(q^2) \\ &\quad + \frac{p_{1\alpha} q_\mu}{M_1^2} F_4^V(q^2) \Big] u_{B_1}(p_1, \lambda_1) \end{aligned}$$

$$\begin{aligned} M_\mu^A &= \langle B_2^*, \lambda_2 | \bar{c}\gamma_\mu \gamma_5 b | B_1, \lambda_1 \rangle \\ &= \overline{u}_{B_2^*}(p_2, \lambda_2) \Big[g_{\alpha\mu} F_1^A(q^2) + \gamma_\mu \frac{p_{1\alpha}}{M_1} F_2^A(q^2) + \frac{p_{1\alpha} p_{2\mu}}{M_1^2} F_3^A(q^2) \\ &\quad + \frac{p_{1\alpha} q_\mu}{M_1^2} F_4^A(q^2) \Big] \gamma_5 u_{B_1}(p_1, \lambda_1) \end{aligned}$$

- We use the form factors obtained in the framework of Lattice QCD.[10,11]

[10] S. Meinel and G. Rendon; Phys. Rev. D 103, 094516 (2021)

[11] S. Meinel and G. Rendon; Phys. Rev. D 105, 054511 (2022)

Helicity Amplitudes

- The helicity amplitudes are defined as :

$$H^{V/A}_{\lambda_2,\lambda_W} = M^{V/A}_{\mu}(\lambda_2)\overrightarrow{\epsilon}^{*\mu}(\lambda_W).$$

$\lambda_2 \longrightarrow$ helicity of the daughter baryon,
$\lambda_W \longrightarrow$ helicity of the $W^-_{off-shell}$,
$\epsilon^{\mu} \longrightarrow$ polarization of the $W^-_{off-shell}$.

For $1/2^+ \to 1/2^-$ transition,

$$\begin{aligned} H^{V,A}_{\frac{1}{2}t} &= (1 + C_{V_L} \pm C_{V_R})\sqrt{\frac{Q_\mp}{q^2}}\left(F^{V,A}_1(M_1 \pm M_2) \mp F^{V,A}_3\frac{q^2}{M_1}\right) \\ H^{V,A}_{\frac{1}{2}0} &= (1 + C_{V_L} \pm C_{V_R})\sqrt{\frac{Q_\pm}{q^2}}\left(F^{V,A}_1(M_1 \mp M_2) \mp F^{V,A}_2\frac{q^2}{M_1}\right) \\ H^{V,A}_{\frac{1}{2}1} &= (1 + C_{V_L} \pm C_{V_R})\sqrt{2Q_\pm}\left(-F^{V,A}_1 \pm F^{V,A}_2\frac{M_1 \mp M_2}{M_1}\right) \end{aligned}$$

$$H^{S,P}_{\frac{1}{2}0} = (C_{S_L} \pm C_{S_R}) \frac{\sqrt{Q_\mp}}{m_b \mp m_c} \left[-F_1^{V,A}(M_1 \pm M_2) \pm F_3^{V,A} \frac{q^2}{M_1} \right]$$

where $Q_\pm = (M_1 \pm M_2)^2 - q^2$.

- The scalar and pseudoscalar Helicity amplitudes $H^{S,P}_{\frac{1}{2}0}$ are obtained using equations of motion.

For $1/2^+ \to 3/2^-$ transition,

$$\begin{aligned} H^{V,A}_{\frac{1}{2}t} &= (1 + C_{V_L} \pm C_{V_R}) \Big\{ \pm \sqrt{\frac{2}{3} \frac{Q_\mp}{q^2}} \frac{Q_\pm}{2M_1M_2} \Big(F_1^{V,A} M_1 \pm F_2^{V,A}(M_1 \mp M_2) \\ &+ F_3^{V,A} \frac{(M_1+M_2)(M_1-M_2) - q^2}{2M_1} + F_4^{V,A} \frac{q^2}{M_1} \Big) \Big\} \\ H^{V,A}_{\frac{1}{2}0} &= (1 + C_{V_L} \pm C_{V_R}) \Big\{ \pm \sqrt{\frac{2}{3} \frac{Q_\pm}{q^2}} \Big(F_1^{V,A} \frac{(M_1+M_2)(M_1-M_2) - q^2}{2M_2} \\ &\pm F_2^{V,A} \frac{Q_\mp (M_1 \pm M_2)}{2M_1M_2} + F_3^{V,A} \frac{|\mathbf{p_2}|^2}{M_2} \Big) \Big\} \end{aligned}$$

$$
\begin{aligned}
H^{V,A}_{\frac{1}{2}1} &= (1 + C_{V_L} \pm C_{V_R})\sqrt{\frac{Q_\pm}{3}}\left(F_1^{V,A} - F_2^{V,A}\frac{Q_\mp}{M_1 M_2}\right) \\
H^{V,A}_{\frac{3}{2}1} &= (1 + C_{V_L} \pm C_{V_R})\Big\{\pm\sqrt{Q_\pm}F_1^{V,A}\Big\} \\
H^{S,P}_{\frac{1}{2}0} &= (C_{S_L} \pm C_{S_R})\sqrt{\frac{2}{3}}\frac{\sqrt{Q_\mp}}{(m_b \mp m_c)}\frac{Q_\pm}{2M_2}\Big\{F_1^{V,A} \pm F_2^{V,A}\frac{(M_1 \mp M_2)}{M_1} \\
&+ F_3^{V,A}\frac{(M_1+M_2)(M_1-M_2)-q^2}{2M_1^2} + F_4^{V,A}\frac{q^2}{M_1^2}\Big\}
\end{aligned}
$$

- The total left-handed helicity amplitude is

$$
\begin{aligned}
H_{\lambda_2,\lambda_W} &= H^V_{\lambda_2,\lambda_W} - H^A_{\lambda_2,\lambda_W}, \\
H^{SP}_{\lambda_2,0} &= H^S_{\lambda_2,0} - H^P_{\lambda_2,0}.
\end{aligned}
$$

Angular decay distribution

The twofold angular distribution for the $B_1 \to B_2^{(*)} \ell^- \bar{\nu}_\ell$ decay [12],

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = \frac{G_F^2 |V_{cb}|^2 q^2 |\mathbf{p}_2|}{512\pi^3 m_{B_1}^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[A_1 + \frac{m_\ell^2}{q^2} A_2 + 2A_3 + \frac{4m_\ell}{\sqrt{q^2}} A_4\right],$$

$$\begin{aligned}
A_1 &= 2\sin^2\theta_\ell \left(H^2_{\frac{1}{2},0} + H^2_{-\frac{1}{2},0}\right) + (1-\cos\theta_\ell)^2 \left(H^2_{\frac{1}{2},1} + H^2_{\frac{3}{2},1}\right) \\
&\quad + (1+\cos\theta_\ell)^2 \left(H^2_{-\frac{1}{2},-1} + H^2_{-\frac{3}{2},-1}\right), \\
A_2 &= 2\cos^2\theta_\ell \left(H^2_{\frac{1}{2},0} + H^2_{-\frac{1}{2},0}\right) + \sin^2\theta_\ell \left(H^2_{\frac{1}{2},1} + H^2_{-\frac{1}{2},-1} + H^2_{\frac{3}{2},1} + H^2_{-\frac{3}{2},-1}\right) \\
&\quad + 2\left(H^2_{\frac{1}{2},t} + H^2_{-\frac{1}{2},t}\right) - 4\cos\theta_\ell \left(H_{\frac{1}{2},t} H_{\frac{1}{2},0} + H_{-\frac{1}{2},t} H_{-\frac{1}{2},0}\right), \\
A_3 &= \left(H^{SP}_{\frac{1}{2},0}\right)^2 + \left(H^{SP}_{-\frac{1}{2},0}\right)^2, \\
A_4 &= -\cos\theta_\ell \left(H_{\frac{1}{2},0} H^{SP}_{\frac{1}{2},0} + H_{-\frac{1}{2},0} H^{SP}_{-\frac{1}{2},0}\right) + \left(H_{\frac{1}{2},t} H^{SP}_{\frac{1}{2},0} + H_{-\frac{1}{2},t} H^{SP}_{-\frac{1}{2},0}\right).
\end{aligned}$$

[12] S. Shivashankara, W. Wu, A. Datta; Phys.Rev.D91, 115003 (2015).

Differential decay rate

On integrating out $\cos\theta_\ell$,

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{cb}|^2 q^2 |\mathbf{p}_2|}{192\pi^3 m_{B_1}^2}\left(1-\frac{m_l^2}{q^2}\right)^2 H_{\frac{1}{2}\to\frac{1}{2}(\frac{3}{2})},$$

where

$$\begin{aligned}
H_{\frac{1}{2}\to\frac{1}{2}} = {} & \left(H^2_{\frac{1}{2}0}\right) + \left(H^2_{-\frac{1}{2}0}\right) + \left(H^2_{\frac{1}{2}1}\right) + \left(H^2_{-\frac{1}{2}-1}\right) + \frac{m_l^2}{2q^2}\Big[\left(H^2_{\frac{1}{2}0}\right) \\
& + \left(H^2_{-\frac{1}{2}0}\right) + \left(H^2_{\frac{1}{2}1}\right) + \left(H^2_{-\frac{1}{2}-1}\right) + 3\left(H^2_{\frac{1}{2}t} + H^2_{-\frac{1}{2}t}\right)\Big] \\
& + \frac{3}{2}\Big[\left(H^{SP}_{\frac{1}{2}0}\right)^2 + \left(H^{SP}_{-\frac{1}{2}0}\right)^2\Big] + \frac{3m_l}{\sqrt{q^2}}\Big[H_{\frac{1}{2}t}H^{SP}_{\frac{1}{2}0} + H_{-\frac{1}{2}t}H^{SP}_{-\frac{1}{2}0}\Big] \\
H_{\frac{1}{2}\to\frac{3}{2}} = {} & \left(H^2_{\frac{1}{2}0}\right) + \left(H^2_{-\frac{1}{2}0}\right) + \left(H^2_{\frac{1}{2}1}\right) + \left(H^2_{-\frac{1}{2}-1}\right) + \left(H^2_{\frac{3}{2}1}\right) + \left(H^2_{-\frac{3}{2}-1}\right) \\
& + \frac{m_l^2}{2q^2}\Big[\left(H^2_{\frac{1}{2}0}\right) + \left(H^2_{-\frac{1}{2}0}\right) + \left(H^2_{\frac{1}{2}1}\right) + \left(H^2_{-\frac{1}{2}-1}\right) + \left(H^2_{\frac{3}{2}1}\right) \\
& + \left(H^2_{-\frac{3}{2}-1}\right) + 3\left(H^2_{\frac{1}{2}t} + H^2_{-\frac{1}{2}t}\right)\Big] + \frac{3}{2}\Big[\left(H^{SP}_{\frac{1}{2}0}\right)^2 + \left(H^{SP}_{-\frac{1}{2}0}\right)^2\Big] \\
& + \frac{3m_l}{\sqrt{q^2}}\Big[H_{\frac{1}{2}t}H^{SP}_{\frac{1}{2}0} + H_{-\frac{1}{2}t}H^{SP}_{-\frac{1}{2}0}\Big]
\end{aligned}$$

q^2- dependent observables

- Differential branching fraction

$$DBR(q^2) = \tau_{B_1}\left(\frac{d\Gamma}{dq^2}\right).$$

- Ratio of branching fractions

$$R(q^2) = \frac{\frac{d\Gamma}{dq^2}(B_1 \to B_2^{(*)}\tau\overline{\nu}_\tau)}{\frac{d\Gamma}{dq^2}(B_1 \to B_2^{(*)}\ell\overline{\nu}_\ell)}$$

- Forward-backward asymmetry of the charged lepton

$$A^\ell_{FB}(q^2) = \frac{\left(\int_0^1 - \int_{-1}^0\right)\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell}d\cos\theta_\ell}{\left(\int_0^1 + \int_{-1}^0\right)\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell}d\cos\theta_\ell},$$

- Convexity parameter

$$C_F^\tau(q^2) = \frac{1}{d\Gamma/dq^2}\frac{d^2}{d(\cos\theta_\ell)^2}\left(\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell}\right),$$

- Longitudinal polarization of the charged lepton

$$P_L^\tau(q^2) = \frac{d\Gamma^{\lambda_\tau=1/2}/dq^2 - d\Gamma^{\lambda_\tau=-1/2}/dq^2}{d\Gamma^{\lambda_\tau=1/2}/dq^2 + d\Gamma^{\lambda_\tau=-1/2}/dq^2}$$

Constraints on New Couplings $(h_L^{23}, h_L^{33}, h_R^{33})$

- U_1 LQ couplings,

$$\begin{aligned} C_{V_L} &\propto h_L^{23}(h_L^{33})^* \\ C_{S_R} &\propto h_L^{23}(h_R^{33})^* \end{aligned}$$

- NP couplings are assumed to be real.
- The new couplings are constrained using experimental measurements of $R_{D^{(*)}}$, $R_{J/\psi}$, $F_L^{D^*}$, $P_\tau^{D^*}$ and $\mathcal{B}(B_c^+ \to \tau^+ \nu_\tau)$.
- χ^2 fitting is performed to obtain the best-fit values.

$$\chi^2(C_k) = \sum_{ij}^{N_{obs}} [O_i^{exp} - O_i^{th}(C_k)] V_{ij}^{-1} [O_j^{exp} - O_j^{th}(C_k)]$$

$O_i^{exp} \longrightarrow$ experimental values of the observables
$O_i^{th} \longrightarrow$ theoretical predictions
$V \longrightarrow$ covariance matrix considering the correlation of R_D and R_{D^*}

Experimental and SM theoretical values of the observables

Observable	Experimental value	SM value
R_D	0.357 ± 0.029	0.298 ± 0.004
R_{D^*}	0.284 ± 0.012	0.254 ± 0.005
$R_{J/\psi}$	$0.71 \pm 0.17 \pm 0.18$	0.289 ± 0.01
$F_L^{D^*}$	$0.60 \pm 0.08 \pm 0.04$	0.457 ± 0.010
$P_\tau^{D^*}$	$-0.38 \pm 0.51^{+0.21}_{-0.16}$	-0.497 ± 0.013

- The experimental upper limit on $\mathcal{B}(B_c^+ \to \tau^+\nu_\tau)$ is[13]

$$\mathcal{B}(B_c^+ \to \tau^+\nu_\tau) \lesssim 30\%$$

[13] R. Alonso, B. Grinstein, and J. Martin Camalich; Phys. Rev. Lett. 118, 081802 (2017).

- The LQ mass has been directly constrained as $M_{LQ} \gtrsim 1.5$ TeV from the LQ pair production searches [14,15].
- We take $M_{LQ} = 2$ TeV.

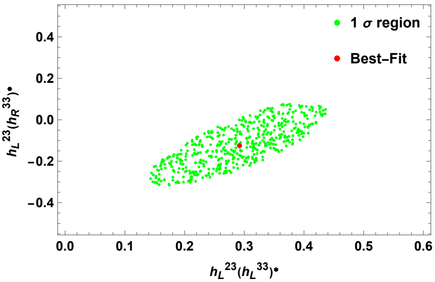

Figure 1: The allowed parameter space for the Leptoquark couplings.

	Best-fit value	$\chi^2_{\rm min}$
SM	$C_i = 0$	22.593
U_1	$(h_L^{23}(h_L^{33})^*, h_L^{23}(h_R^{33})^*) = (0.292, -0.125)$	4.822

[14] ATLAS, G. Aad et al.; J. High Energy Phys. 06, 188 (2023).
[15] CMS, A. Hayrapetyan et al., (2023), arXiv:2308.07826 [hep-ex]

Predictions for various q^2- dependent observables in

$\Lambda_b \to \Lambda_c^*(2595)\tau^-\bar{\nu}_\tau$ Decay

$(\ 1/2^+ \to 1/2^-\ )$

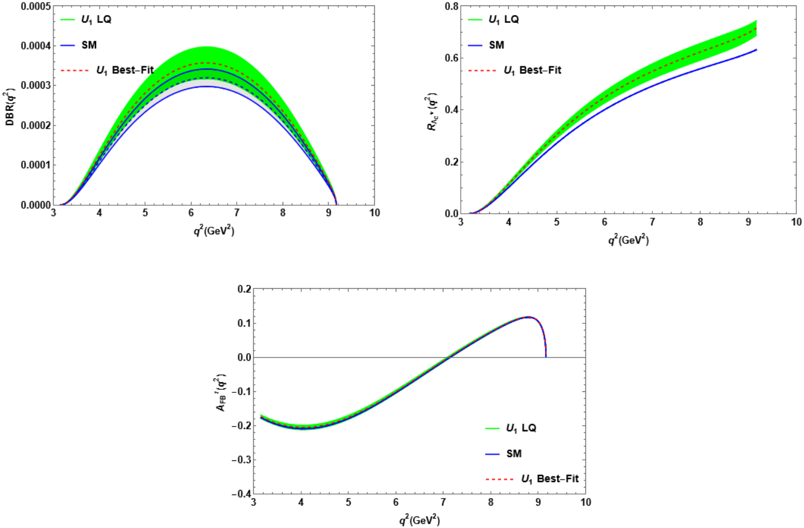

Figure 2: q^2- dependence of $DBR(q^2)$, $R_{\Lambda_c^*}(q^2)$ and $A_{FB}^{\tau}(q^2)$ in SM and in U_1 LQ scenario for $\Lambda_b \to \Lambda_c^*(2595)\tau^-\bar{\nu}_\tau$ decay.

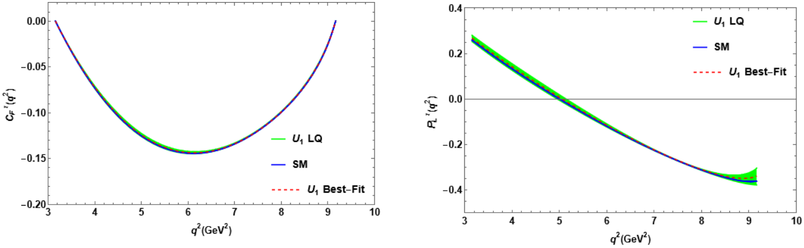

Figure 3: q^2- dependence of $C_F^\tau(q^2)$ and $P_L^\tau(q^2)$ in SM and in U_1 LQ scenario for $\Lambda_b \to \Lambda_c^*(2595)\tau^-\bar{\nu}_\tau$ decay.

Predictions for various q^2- dependent observables in

$\Lambda_b \to \Lambda_c^*(2625)\tau^-\bar{\nu}_\tau$ Decay

$(\ 1/2^+ \to 3/2^- \ )$

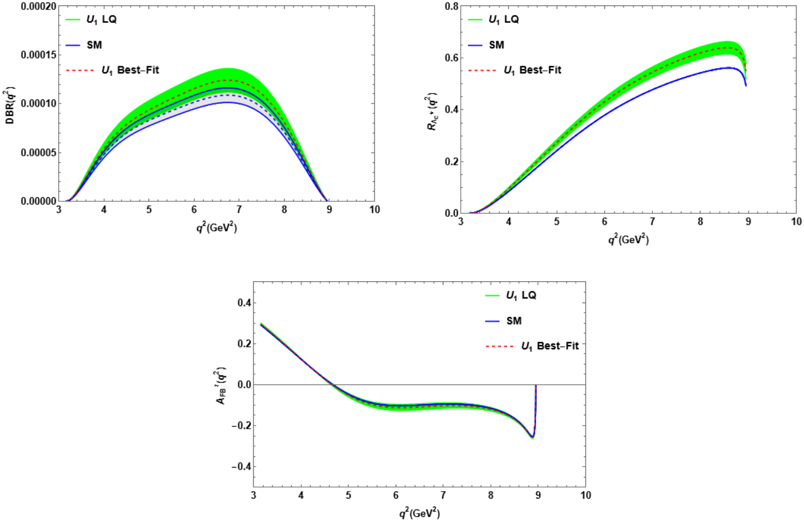

Figure 4: q^2- dependence of $DBR(q^2)$, $R_{\Lambda_c^*}(q^2)$ and $A_{FB}^{\tau}(q^2)$ in SM and in U_1 LQ scenario for $\Lambda_b \to \Lambda_c^*(2625)\tau^-\bar{\nu}_\tau$ decay.

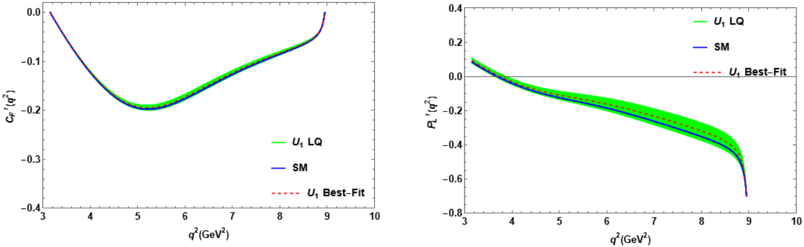

Figure 5: q^2- dependence of $C_F^{\tau}(q^2)$ and $P_L^{\tau}(q^2)$ in SM and in U_1 LQ scenario for $\Lambda_b \to \Lambda_c^*(2625)\tau^- \bar{\nu}_\tau$ decay.

Summary

- $\Lambda_b \to \Lambda_c^* \tau \bar{\nu}_\tau$ decay modes are analyzed within the SM and in U_1 leptoquark scenario.
- The best-fit values of the NP couplings are obtained using a χ^2 analysis.
- Predictions for various q^2 dependent observables such as $DBR(q^2)$, $R_{\Lambda_c^*}(q^2)$, $A_{FB}^\tau(q^2)$, $C_F^\tau(q^2)$, and $P_L^\tau(q^2)$ are presented in the SM and in U_1 LQ model.
- The observables are sensitive to U_1 LQ effects. Deviations from the SM prediction is more pronounced in $R_{\Lambda_c^*}(q^2)$ and $P_L^\tau(q^2)$ in both the decay modes.
- The LFU ratio $R_{\Lambda_c^*}(q^2)$ displays a prominent deviation from the SM in the higher q^2 region. Measurement of this observable can further substantiate the observed anomalies in $b-$ decays.
- The $b-$baryon decay modes mediated by $b \to c \ell \nu_\ell$ can act as complementary decay channels to $b-$meson decays with regards to NP.
- Search for physics beyond SM will be enriched by studies of such modes also.

Thank You

Back up

Form Factors

The matrix elements when parametrized in terms of form factors from lattice QCD [16]:

For $1/2^+ \to 1/2^-$ transition,

$$\begin{aligned}
\langle B_2|V^\mu|B_1\rangle &= \overline{u}_{B_2}(p_2,s_2)\Big[f_0^{(\frac{1}{2}^-)}(M_1+M_2)\frac{q^\mu}{q^2} \\
&+ f_+^{(\frac{1}{2}^-)}\frac{M_1-M_2}{s_-}\left(p_1^\mu+p_2^\mu-(M_1^2-M_2^2)\frac{q^\mu}{q^2}\right) \\
&+ f_\perp^{(\frac{1}{2}^-)}\left(\gamma^\mu+\frac{2M_2}{s_-}p_1^\mu-\frac{2M_1}{s_-}p_2^\mu\right)\Big]\gamma_5 u_{B_1}(p_1,s_1) \\
\langle B_2|A^\mu|B_1\rangle &= \overline{u}_{B_2}(p_2,s_2)\Big[-g_0^{(\frac{1}{2}^-)}(M_1-M_2)\frac{q^\mu}{q^2} \\
&- g_+^{(\frac{1}{2}^-)}\frac{M_1+M_2}{s_+}\left(p_1^\mu+p_2^\mu-(M_1^2-M_2^2)\frac{q^\mu}{q^2}\right) \\
&- f_\perp^{(\frac{1}{2}^-)}\left(\gamma^\mu-\frac{2M_2}{s_+}p_1^\mu-\frac{2M_1}{s_+}p_2^\mu\right)\Big]u_{B_1}(p_1,s_1)
\end{aligned}$$

where $s_\pm = (M_1 \pm M_2)^2 - q^2$.

[16] S. Meinel and G. Rendon; Phys. Rev. D 103, 094516 (2021)

For $1/2^+ \to 3/2^-$ transition,

$$\begin{aligned}
\langle B_2^*|V^\mu|B_1\rangle &= \overline{u}_{B_2^*}(p_2,s_2)\Big[f_0^{(\frac{3}{2}^-)}\frac{M_2}{s_+}\frac{(M_1-M_2)p^\lambda q^\mu}{q^2} \\
&+ f_+^{(\frac{3}{2}^-)}\frac{M_2}{s_-}\frac{(M_1+M_2)p^\lambda(q^2(p_1^\mu+p_2^\mu)-(M_1^2-M_2^2)q^\mu)}{q^2 s_+} \\
&+ f_\perp^{(\frac{3}{2}^-)}\frac{M_2}{s_-}\left(p^\lambda\gamma^\mu-\frac{2p^\lambda(M_1p_2^\mu+M_2p_1^\mu)}{s_+}\right) \\
&+ f_{\perp'}^{(\frac{3}{2}^-)}\frac{M_2}{s_-}\left(p^\lambda\gamma^\mu-\frac{2p^\lambda p_2^\mu}{M_2}+\frac{2p^\lambda(M_1p_2^\mu+M_2p_1^\mu)}{s_+}+\frac{s_-g^{\lambda\mu}}{M_2}\right)\Big] \\
&\times u_{B_1}(p_1,s_1)
\end{aligned}$$

$$
\begin{aligned}
\langle B_2^*|A^\mu|B_1\rangle &= \overline{u}_{B_2^*}(p_2,s_2)\Big[-g_0^{(\frac{3}{2}^-)}\frac{M_2}{s_-}\frac{(M_1+M_2)p^\lambda q^\mu}{q^2} \\
&- g_+^{(\frac{3}{2}^-)}\frac{M_2}{s_+}\frac{(M_1-M_2)p^\lambda(q^2(p_1^\mu+p_2^\mu)-(M_1^2-M_2^2)q^\mu)}{q^2 s_-} \\
&- g_\perp^{(\frac{3}{2}^-)}\frac{M_2}{s_+}\left(p^\lambda\gamma^\mu-\frac{2p^\lambda(M_1p_2^\mu-M_2p_1^\mu)}{s_-}\right) \\
&- g_{\perp'}^{(\frac{3}{2}^-)}\frac{M_2}{s_+}\left(p^\lambda\gamma^\mu+\frac{2p^\lambda p_2^\mu}{M_2}+\frac{2p^\lambda(M_1p_2^\mu-M_2p_1^\mu)}{s_-}-\frac{s_+g^{\lambda\mu}}{M_2}\right)\Big] \\
&\times \gamma_5 u_{B_1}(p_1,s_1)
\end{aligned}
$$

The two sets of form factors are related as follows.

$$
\begin{aligned}
F_1^{V(\frac{1}{2}^-)} &= \sigma_G^{(\frac{1}{2}^-)} \frac{(M_1 - M_2)^2 (f_\perp^{(\frac{1}{2}^-)} - f_+^{(\frac{1}{2}^-)})}{s_-} - f_\perp^{(\frac{1}{2}^-)}, \\
F_2^{V(\frac{1}{2}^-)} &= \sigma_G^{(\frac{1}{2}^-)} \frac{M_1 (M_1 - M_2)(f_\perp^{(\frac{1}{2}^-)} - f_+^{(\frac{1}{2}^-)})}{s_-}, \\
F_3^{V(\frac{1}{2}^-)} &= \sigma_G^{(\frac{1}{2}^-)} \frac{M_1 (M_1 + M_2)(s_- f_0^{(\frac{1}{2}^-)} + q^2 f_\perp^{(\frac{1}{2}^-)} - (M_1 - M_2)^2 f_+^{(\frac{1}{2}^-)})}{q^2 s_-}, \\
F_1^{A(\frac{1}{2}^-)} &= \sigma_G^{(\frac{1}{2}^-)} \frac{(M_1 + M_2)^2 (g_\perp^{(\frac{1}{2}^-)} - g_+^{(\frac{1}{2}^-)})}{s_+} - g_\perp^{(\frac{1}{2}^-)}, \\
F_2^{A(\frac{1}{2}^-)} &= -\sigma_G^{(\frac{1}{2}^-)} \frac{M_1 (M_1 + M_2)(g_\perp^{(\frac{1}{2}^-)} - g_+^{(\frac{1}{2}^-)})}{s_+}, \\
F_3^{A(\frac{1}{2}^-)} &= \sigma_G^{(\frac{1}{2}^-)} \frac{M_1 (M_1 - M_2)(-s_+ g_0^{(\frac{1}{2}^-)} - q^2 g_\perp^{(\frac{1}{2}^-)} + (M_1 + M_2)^2 g_+^{(\frac{1}{2}^-)})}{q^2 s_+},
\end{aligned}
$$

$$
\begin{aligned}
F_1^{V(\frac{3}{2}^-)} &= \sigma_G^{(\frac{3}{2}^-)} f_{\perp'}^{(\frac{3}{2}^-)}, \\
F_2^{V(\frac{3}{2}^-)} &= \sigma_G^{(\frac{3}{2}^-)} \frac{M_1 M_2}{s_-}[f_{\perp}^{(\frac{3}{2}^-)} + f_{\perp'}^{(\frac{3}{2}^-)}], \\
F_3^{V(\frac{3}{2}^-)} &= \sigma_G^{(\frac{3}{2}^-)} \frac{2M_1^2}{s_- s_+}[-M_2(M_1+M_2) f_{\perp}^{(\frac{3}{2}^-)} + M_2(M_1+M_2)(f_{\perp'}^{(\frac{3}{2}^-)} + f_{+}^{(\frac{3}{2}^-)}) \\
&- s_+ f_{\perp'}^{(\frac{3}{2}^-)}], \\
F_4^{V(\frac{3}{2}^-)} &= \sigma_G^{(\frac{3}{2}^-)} \frac{M_1^2 M_2}{q^2 s_+ s_-}[(M_1 - M_2) s_- f_0^{(\frac{3}{2}^-)} - 2M_2 q^2 (f_{\perp}^{(\frac{3}{2}^-)} - f_{\perp'}^{(\frac{3}{2}^-)}) \\
&- (M_1+M_2)(M_1^2 - M_2^2 - q^2) f_{+}^{(\frac{3}{2}^-)}]
\end{aligned}
$$

$$
\begin{aligned}
F_1^{A(\frac{3}{2}^-)} &= \sigma_G^{(\frac{3}{2}^-)} g_{\perp'}^{(\frac{3}{2}^-)}, \\
F_2^{A(\frac{3}{2}^-)} &= \sigma_G^{(\frac{3}{2}^-)} \frac{M_1 M_2}{s_+}[g_{\perp}^{(\frac{3}{2}^-)} + g_{\perp'}^{(\frac{3}{2}^-)}], \\
F_3^{A(\frac{3}{2}^-)} &= \sigma_G^{(\frac{3}{2}^-)} \frac{2M_1^2}{s_- s_+}[M_2(M_1 - M_2)(g_{\perp}^{(\frac{3}{2}^-)} - g_{\perp'}^{(\frac{3}{2}^-)} - g_{+}^{(\frac{3}{2}^-)}) - s_- g_{\perp'}^{(\frac{3}{2}^-)}], \\
F_4^{A(\frac{3}{2}^-)} &= \sigma_G^{(\frac{3}{2}^-)} \frac{M_1^2 M_2}{q^2 s_+ s_-}[-(M_1+M_2) s_+ g_0^{(\frac{3}{2}^-)} - 2M_2 q^2 (g_{\perp}^{(\frac{3}{2}^-)} - g_{\perp'}^{(\frac{3}{2}^-)}) \\
&+ (M_1 - M_2)(M_1^2 - M_2^2 - q^2) g_{+}^{(\frac{3}{2}^-)}]
\end{aligned}
$$

Form factor parametrization

The lattice results for the form factors are extrapolated to the continuum limit and the physical pion mass using the model

$$\begin{aligned} f(q^2) &= F^f \left[1 + C^f \frac{m_\pi^2 - m_{\pi,phys}^2}{(4\pi f_\pi)^2} + D^f a^2 \Lambda^2\right] \\ &+ A^f \left[1 + \tilde{C}^f \frac{m_\pi^2 - m_{\pi,phys}^2}{(4\pi f_\pi)^2} + \tilde{D}^f a^2 \Lambda^2\right] (\omega - 1) \end{aligned}$$

with fit parameters $F^f, A^f, C^f, D^f, \tilde{C}^f, \tilde{D}^f$ for each form factor f, and using the kinematic variable ,

$$w(q^2) = v.v^{'} = \frac{M_1^2 + M_2^2 - q^2}{2M_1 M_2}$$

In the physical limit $m_\pi = m_{\pi,phys}, a = 0$, the functions reduce to

$$f(q^2) = F^f + A^f(\omega - 1).$$

The parameters describing the $\Lambda_b \to \Lambda_c^*(2595)$ and $\Lambda_b \to \Lambda_c^*(2625)$ form factors at the physical pion mass and in the continuum limit are given in Table 1.

Table 1: Form factor Parameters[17]

f	F^f	A^f
$f_0^{(\frac{1}{2}^-)}$	0.545(64)	-2.21(66)
$f_+^{(\frac{1}{2}^-)}$	0.1628(90)	1.16(31)
$f_\perp^{(\frac{1}{2}^-)}$	0.1690(79)	0.57(25)
$g_0^{(\frac{1}{2}^-)}$	0.221(11)	0.94(33)
$g_+^{(\frac{1}{2}^-)}$	0.582(64)	-2.24(65)
$g_\perp^{(\frac{1}{2}^-)}$	1.22(16)	-6.1(1.9)
$f_0^{(\frac{3}{2}^-)}$	4.29(67)	-27.3(8.7)
$f_+^{(\frac{3}{2}^-)}$	0.0498(70)	1.28(27)
$f_\perp^{(\frac{3}{2}^-)}$	-0.073(14)	2.52(35)
$f_{\perp'}^{(\frac{3}{2}^-)}$	0.0687(40)	-0.280(89)
$g_0^{(\frac{3}{2}^-)}$	0.0027(35)	1.23(21)
$g_+^{(\frac{3}{2}^-)}$	3.46(58)	-24.7(8.1)
$g_\perp^{(\frac{3}{2}^-)}$	3.47(57)	-22.6(7.8)
$g_{\perp'}^{(\frac{3}{2}^-)}$	-0.062(38)	0.62(57)

[17] S. Meinel and G. Rendon; Phys. Rev. D 105, 054511 (2022)

Leptoquarks

- LQs are coloured bosons which couple to both quarks and leptons.
- The LQ interactions are invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$.
- They have well-defined fermion number $F = 3B + L$ where B and L are baryon and lepton numbers, respectively.

All LQs are classified by:

- Fermion number $|F| = 0, 2$
- Spin 0 (scalar) or Spin 1 (vector)
- Electric charge, $Q = I_3 + Y$, $|Q| = 1/3, 2/3, 4/3, 5/3$

LQ states naturally emerge in various extensions of the SM, such as,

- $SU(5)$ GUT model
- Pati-Salam color $SU(4)$
- Composite models
- Technicolor models

Experimental evidence searched:

- Direct : Production cross sections at colliders
- Indirect : LQ-induced four-fermion interactions

List of scalar and vector leptoquarks

LQ Symbol	$(SU(3), SU(2), U(1))$	Spin	$F = 3B + L$
S_1	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	0	-2
$\tilde{S}_1$	$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	0	-2
S_3	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	0	-2
V_2	$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$	1	-2
$\tilde{V}_2$	$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	1	-2
R_2	$(\bar{\mathbf{3}}, \mathbf{2}, 7/6)$	0	0
$\tilde{R}_2$	$(\bar{\mathbf{3}}, \mathbf{2}, 1/6)$	0	0
U_1	$(\bar{\mathbf{3}}, \mathbf{1}, 2/3)$	1	0
$\tilde{U}_1$	$(\bar{\mathbf{3}}, \mathbf{1}, 5/3)$	1	0
U_3	$(\bar{\mathbf{3}}, \mathbf{3}, 2/3)$	1	0

Helicity Amplitude

- For $\frac{1}{2}^+ \to \frac{1}{2}^-$,

  $H^{V,A}_{-\lambda_2,-\lambda_W} = \mp H^{V,A}_{\lambda_2,\lambda_W}$ and $H^{S,P}_{-\lambda_2,-\lambda_W} = \mp H^{S,P}_{\lambda_2,\lambda_W}$.

- For $\frac{1}{2}^+ \to \frac{3}{2}^-$,

  $H^{V,A}_{-\lambda_2,-\lambda_W} = \pm H^{V,A}_{\lambda_2,\lambda_W}$ and $H^{S,P}_{-\lambda_2,-\lambda_W} = \pm H^{S,P}_{\lambda_2,\lambda_W}$.

Theoretical predictions for observables

The differential decay rate of $\bar{B} \to D\tau^- \bar{\nu}_\tau$ is given by [18],

$$\begin{aligned}\frac{d\Gamma(\bar{B} \to D\tau^- \bar{\nu}_\tau)}{dq^2} &= \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} q^2 \sqrt{\lambda_D(q^2)} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \\ &\times \Bigg\{ |1 + C_{V_L} + C_{V_R}|^2 \Bigg[\left(1 + \frac{m_\tau^2}{2q^2}\right) \\ &\times H_{V,0}^2 + \frac{3}{2}\frac{m_\tau^2}{q^2} H_{V,t}^2 \Bigg] + \frac{3}{2}|C_{S_R} + C_{S_L}|^2 H_S^2 \\ &+ 3Re\Big[(1 + C_{V_L} + C_{V_R})(C_{S_R}^* + C_{S_L}^*) \Big] \frac{m_\tau}{\sqrt{q^2}} H_S H_{V,t} \Bigg\},\end{aligned}$$

where $\lambda_D(q^2) = [(m_B - m_D)^2 - q^2][(m_B + m_D)^2 - q^2]$.

The amplitudes depend upon HQET form factors, and in this work, we adopt the Caprini *et al.* parametrization for these form factors [19]. The parameters of the $\bar{B} \to D$ HQET form factors are extracted from lattice QCD calculations [20].

[18] Y. Sakaki, M. Tanaka, A. Tayduganov, R. Watanabe; Phys. Rev. D 88, 094012 (2013).
[19] I. Caprini, L. Lellouch, M. Neubert; Nucl. Phys. B 530 (1998).
[20] M. Okamoto, et al.; Nucl. Phys. B, Proc. Suppl. 140 (2005).

The differential decay rate of $\bar{B} \to D^* \tau^- \bar{\nu}_\tau$ $(B_c \to J/\psi \tau^- \bar{\nu}_\tau)$ is given by [21,22],

$$\begin{aligned}
\frac{d\Gamma(\bar{B}(B_c) \to D^*(J/\psi)\tau^-\bar{\nu}_\tau)}{dq^2} &= \frac{G_F^2|V_{cb}|^2}{192\pi^3 m_{B(B_c)}^3} q^2 \sqrt{\lambda_{D^*(J/\psi)}(q^2)} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \\
&\times \Bigg\{ (|1 + C_{V_L}|^2 + |C_{V_R}|^2) \Bigg[\left(1 + \frac{m_\tau^2}{2q^2}\right) \\
&\times \left(H_{V,0}^2 + H_{V,+}^2 + H_{V,-}^2\right) + \frac{3}{2}\frac{m_\tau^2}{q^2} H_{V,t}^2 \Bigg] \\
&-2Re\left[\left(1 + C_{V_L}\right) C_{V_R}^*\right] \\
&\times \left[\left(1 + \frac{m_\tau^2}{2q^2}\right) \left(H_{V,0}^2 + 2H_{V,+}H_{V,-}\right) \right. \\
&\left. + \frac{3}{2}\frac{m_\tau^2}{q^2} H_{V,t}^2 \right] + \frac{3}{2}|C_{S_R} - C_{S_L}|^2 H_S^2 \\
&+3Re\Big[\left(1 + C_{V_L} - C_{V_R}\right)\big(C_{S_R}^* \\
&-C_{S_L}^*\big)\Big] \frac{m_\tau}{\sqrt{q^2}} H_S H_{V,t} \Bigg\},
\end{aligned}$$

[21] Y. Sakaki, M. Tanaka, A. Tayduganov, R. Watanabe; Phys. Rev. D 88, 094012 (2013).
[22] R. Watanabe; Phys. Lett. B 776, 5 (2018).

The fitted parameters for $\bar{B} \to D^*$ HQET form factors are obtained from HFLAV [23]. The form factors for $B_c \to J/\psi \tau^- \bar{\nu}_\tau$ are obtained using perturbative QCD[24].

The longitudinal polarization of τ ($P_\tau^{D^*}$) and of $D^*(F_L^{D^*})$ in $B \to D^* \tau^- \bar{\nu}_\tau$ are respectively given by,

$$P_\tau^{D^*} = \frac{\Gamma(\lambda_\tau = 1/2) - \Gamma(\lambda_\tau = -1/2)}{\Gamma(\lambda_\tau = 1/2) + \Gamma(\lambda_\tau = -1/2)},$$

$$F_L^{D^*} = \frac{\Gamma_{\lambda_{D^*}=0}(B \to D^* \tau^- \bar{\nu}_\tau)}{\Gamma(B \to D^* \tau^- \bar{\nu}_\tau)},$$

where λ_τ and λ_{D^*} denote the helicity of τ and D^*, respectively.

[23] Y.S. Amhis, et al.; Phys. Rev. D 107(5) (2023) 052008.

[24] W.-F. Wang, Y.-Y. Fan, Z.-J. Xiao; Chin. Phys. C 37 (2013) 093102

The branching fraction of $B_c^+ \to \tau^+\nu_\tau$ is given by,

$$\mathcal{B}(B_c^+ \to \tau^+\nu_\tau) = \frac{G_F^2|V_{cb}|^2 m_\tau^2}{8\pi}\tau_{B_c} m_{B_c} f_{B_c}^2 \left(1 - \frac{m_\tau^2}{m_{B_c}^2}\right)^2 \times \left|(1 + C_{V_L} - C_{V_R}) + \frac{m_{B_c}^2}{m_\tau(m_b + m_c)}(C_{S_R} - C_{S_L})\right|^2.$$

where $f_{B_c} = 489 \pm 4 \pm 3$ MeV. [25]

[25] T.-W. Chiu, T.-H. Hsieh, C.-H. Huang, K. Ogawa; Phys. Lett. B 651 (2007).