Hadronic Decays of Charmed Baryons in the Topological Diagrammatic Approach

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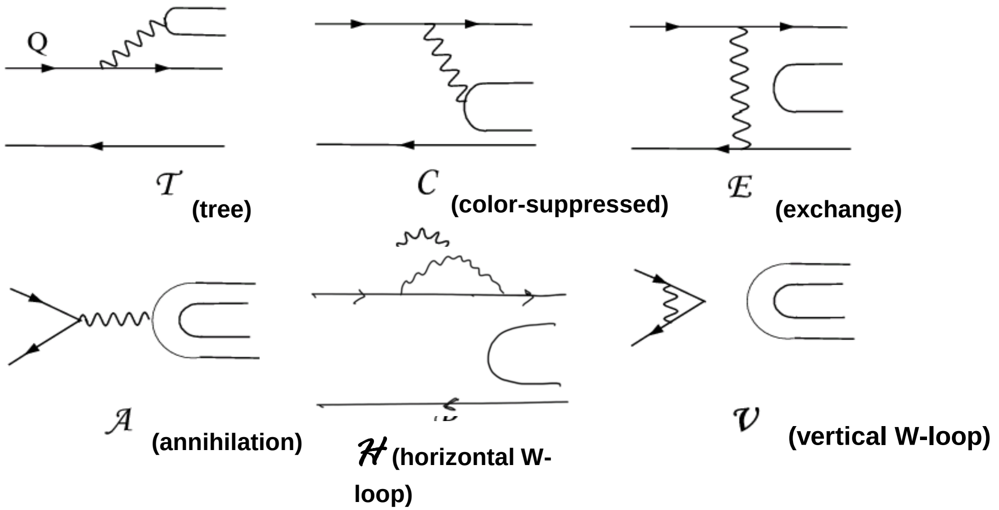
Topological Diagrammatic Approach (TDA)

All two-body hadronic decays of charmed mesons can be expressed in terms of six distinct topological diagrams [Chau ('80,'83); Chau, HYC ('86)]



The great merit & strong point of this approach ⇒ magnitude and strong phase of each topological tree amplitude are determined

⇒ **Direct CPV at tree level can be reliably estimated, e.g. $D_s^+ \to K^+\eta$**

DCPV induced by penguin e.g. $D^0 \to \pi^+\pi^-, K^+K^-$ can be estimated via $P^{LD} = E$

D. Wang ('22)

Is the TDA applicable to charmed baryon sector?

- Kohara ('91): use wave functions $\psi^k(8)_{A_{12}}$ & $\psi^k(8)_{S_{12}}$ for octet baryons
- Chau, Cheng, Tseng ('96): use wave functions $\psi^k(8)_{A_{12}}$ & $\psi^k(8)_{A_{23}}$ for octet baryons
- He, Shi, Wang ('19): general TDA amplitudes
- Zhao, Wang, Hsiao, Yu ('20); Hsiao, Yi, Cai, Zhao ('20)
- Hsiao, Wang, Zhao ('22): followed Kohara closely, but didn't treat S- & P-waves separately

Unlike IRA (irreducible SU(3) approach), global fits to the measured rates and decay asymmetries in TDA were still absent

Although IRA is very popular in describing charmed baryon decays, TDA is more intuitive, graphic and easier to implement model calculations.

Wave function of octet baryon

1. $|\mathcal{B}^{m,k}(8)\rangle = a|\chi^m(1/2)_{A_{12}}\rangle\underline{|\psi^k(8)_{A_{12}}\rangle} + b|\chi^m(1/2)_{S_{12}}\rangle\underline{|\psi^k(8)_{S_{12}}\rangle}$

$$|\psi^k(8)_{A_{12}}\rangle = \sum_{q_a,q_b,q_c} |[q_a q_b]q_c\rangle\langle[q_a q_b]q_c|\psi^k(8)_{A_{12}}\rangle,$$ **antisymmetric at first two quarks**

$$|\psi^k(8)_{S_{12}}\rangle = \sum_{q_a,q_b,q_c} |\{q_a q_b\}q_c\rangle\langle\{q_a q_b\}q_c|\psi^k(8)_{S_{12}}\rangle$$ **symmetric at first two quarks**

$\psi^k(8)_{A_{12}}$ & $\psi^k(8)_{S_{12}}$ **are orthogonal**

2. $|\tilde{\mathcal{B}}^{m,k}(8)\rangle = \alpha|\chi^m(1/2)_{A_{12}}\rangle\underline{|\psi^k(8)_{A_{12}}\rangle} + \beta|\chi^m(1/2)_{A_{23}}\rangle\underline{|\psi^k(8)_{A_{23}}\rangle}$

$\psi^k(8)_{A_{12}}$ & $\psi^k(8)_{A_{23}}$ **are not orthogonal**

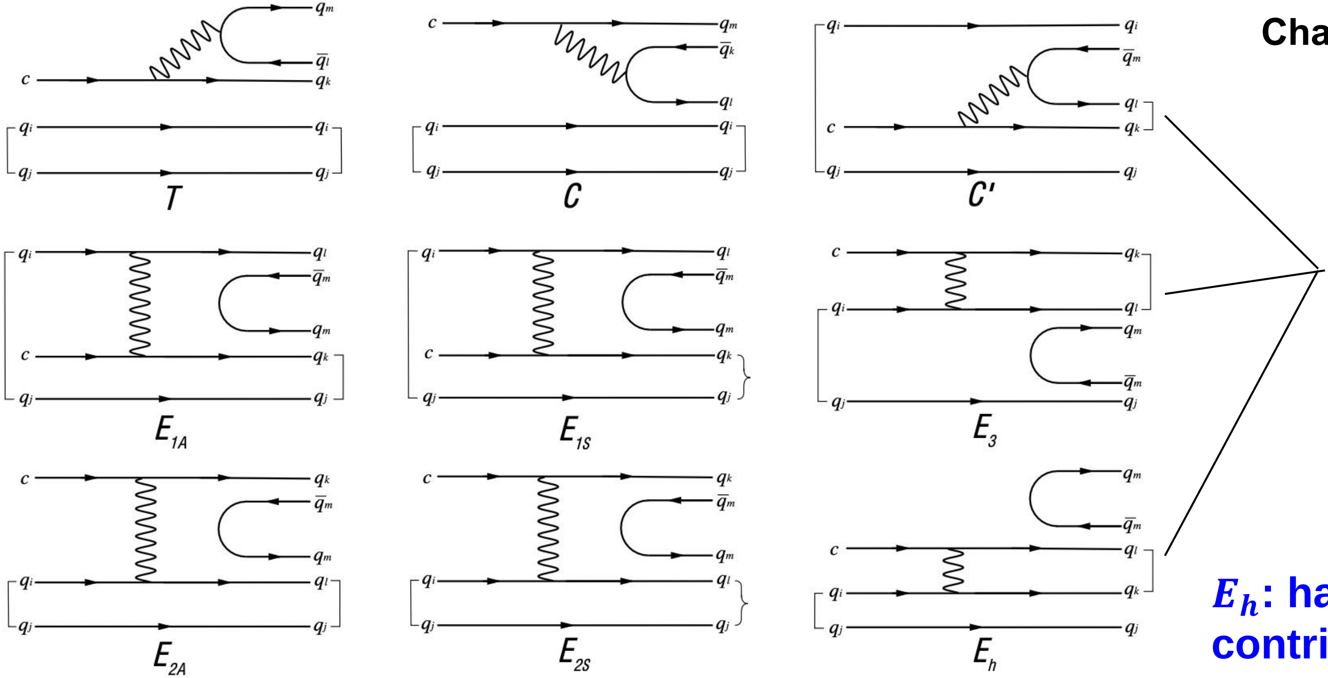
Kohara ('97)

Nevertheless, physics is independent of the convention one chooses

In this work we follow CCT ('96) to use the $\psi^k(8)_{A_{12}}$ & $\psi^k(8)_{S_{12}}$ basis

Weak decays of antitriplet charmed baryons: $\mathcal{B}_c(\overline{3}) \to \mathcal{B}(8)M(8+1)$

Chau, Cheng, Tseng ('96)



KPW theorem

$E_{2A} = -E_{1A}$, $E_{2S} = -E_{1S}$

E_h: hairpin diagram contributing to SU(3)-singlet η_1

Korner-Pati-Woo (KPW) theorem: quark pair in a baryon produced by weak interactions is required to be antisymmetric in flavor

$$\begin{aligned}\mathcal{A}_{\rm TDA} = \;& T(\mathcal{B}_c)^{ij} H^{km}_l (\mathcal{B}_8)_{ijk} M^l_m + C(\mathcal{B}_c)^{ij} H^{ml}_k (\mathcal{B}_8)_{ijl} M^k_m + C'(\mathcal{B}_c)^{ij} H^{kl}_m (\mathcal{B}_8)_{klj} M^m_i \\ & + E_{1A}(\mathcal{B}_c)^{ij} H^{kl}_i (\mathcal{B}_8)_{jkm} M^m_l + E_{1S}(\mathcal{B}_c)^{ij} H^{kl}_i M^m_l \left[(\mathcal{B}_8)_{jmk} + (\mathcal{B}_8)_{kmj}\right] \\ & + E_{2A}(\mathcal{B}_c)^{ij} H^{kl}_i (\mathcal{B}_8)_{jlm} M^m_k + E_{2S}(\mathcal{B}_c)^{ij} H^{kl}_i M^m_k \left[(\mathcal{B}_8)_{jml} + (\mathcal{B}_8)_{lmj}\right] \\ & + E_3(\mathcal{B}_c)^{ij} H^{kl}_i (\mathcal{B}_8)_{klm} M^m_j + E_h(\mathcal{B}_c)^{ij} H^{kl}_i (\mathcal{B}_8)_{klj} M^m_m ,\end{aligned}$$

$$(\mathcal{B}_c)^{ij} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \quad (\mathcal{B}_8)^i_j = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix} \quad (\mathcal{B}_8)_{ijk} = \epsilon_{ijl}(\mathcal{B}_8^T)^l_k$$

KPW theorem $\Longrightarrow$ $E_{2A} = -E_{1A}$, $E_{2S} = -E_{1S}$

There still exist 2 redundant degrees of freedom through redefinitions

$$\begin{aligned}&\tilde{T} = T - E_{1S}, \quad \tilde{C} = C + E_{1S}, \quad \tilde{C}' = C' - 2E_{1S} \\ &\tilde{E}_1 = E_{1A} + E_{1S} - E_3, \quad \tilde{E}_h = E_h + 2E_{1S}.\end{aligned}$$

CCT ('96)

$\Longrightarrow$ 5 independent topological diagrams and TDA amplitudes

Recall that there are also 5 independent tensor invariants in IRA

$$A(\Lambda_c^+ \to \Lambda\pi^+) = \frac{1}{3}(-a - c - d_3 + d_4 + e) \qquad \text{Kohara ('91)}$$

$$= \frac{1}{2\sqrt{6}}(2A_A + B_A - C_{1A} + C_{2A}) - \frac{1}{2\sqrt{2}}C_{2S}. \qquad \text{CCT ('96)}$$

$$= \frac{1}{\sqrt{6}}(-T + C' + E_{1A} + 3E_{1S} - E_3) \qquad \text{This work ('24)}$$

$$= \frac{1}{\sqrt{6}}(-4\widetilde{T} + \widetilde{C}' + \widetilde{E}_1) \qquad \text{This work ('24)}$$

Although these TDA amplitudes are equivalent, it is important to use the minimum set of TDA to fit to the data

Global Fit

Five TDA parameters: $\tilde{T}, \tilde{C}, \tilde{C}', \tilde{E}_1, \tilde{E}_h$ ⇒ 19 unknown parameters

$$|\tilde{T}|_S e^{i\delta_S^{\tilde{T}}}, \quad |\tilde{C}|_S e^{i\delta_S^{\tilde{C}}}, \quad |\tilde{C}'|_S e^{i\delta_S^{\tilde{C}'}}, \quad |\tilde{E}_1|_S e^{i\delta_S^{\tilde{E}_1}}, \quad |\tilde{E}_h|_S e^{i\delta_S^{\tilde{E}_h}},$$

$$|\tilde{T}|_P e^{i\delta_P^{\tilde{T}}}, \quad |\tilde{C}|_P e^{i\delta_P^{\tilde{C}}}, \quad |\tilde{C}'|_P e^{i\delta_P^{\tilde{C}'}}, \quad |\tilde{E}_1|_P e^{i\delta_P^{\tilde{E}_1}}, \quad |\tilde{E}_h|_P e^{i\delta_P^{\tilde{E}_h}}$$

Observable	PDG [42]	BESIII	Belle	Average
$10^2\mathcal{B}(\Lambda_c^+ \to \Lambda^0\pi^+)$	1.29 ± 0.05			1.29 ± 0.05
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+)$	1.27 ± 0.06			1.27 ± 0.06
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^0)$	1.25 ± 0.09			1.25 ± 0.09
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\eta)$	0.44 ± 0.20		0.314 ± 0.044 [49]	0.32 ± 0.04 [42, 49]
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\eta')$	1.5 ± 0.6		0.416 ± 0.086 [49]	0.44 ± 0.15 [42, 49]
$10^2\mathcal{B}(\Lambda_c^+ \to \Xi^0K^+)$	0.55 ± 0.07			0.55 ± 0.07
$10^4\mathcal{B}(\Lambda_c^+ \to \Lambda^0K^+)$	6.0 ± 0.5	$6.21 \pm 0.61^*$ [50]	6.57 ± 0.40 [51]	6.35 ± 0.31 [42, 51]
$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^0K^+)$	4.9 ± 0.6	$4.7 \pm 0.95^*$ [52]	3.58 ± 0.28 [51]	3.82 ± 0.51 [42, 51]
$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^+K_S)$	4.7 ± 1.4	$4.8 \pm 1.5^*$ [52]		4.7 ± 1.4
$10^4\mathcal{B}(\Lambda_c^+ \to n\pi^+)$	6.6 ± 1.3			6.6 ± 1.3
$10^4\mathcal{B}(\Lambda_c^+ \to p\pi^0)$	< 0.8	$1.56^{+0.72}_{-0.58} \pm 0.20$ [47]		$1.56^{+0.75}_{-0.61}$ [47]
$10^2\mathcal{B}(\Lambda_c^+ \to pK_S)$	1.59 ± 0.07			1.59 ± 0.07
$10^3\mathcal{B}(\Lambda_c^+ \to p\eta)$	1.41 ± 0.11	1.63 ± 0.33 [47], 1.57 ± 0.12 [53]		1.49 ± 0.08 [42, 47, 53]
$10^4\mathcal{B}(\Lambda_c^+ \to p\eta')$	4.9 ± 0.9			4.9 ± 0.9
$10^2\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)$	1.43 ± 0.32		$1.80 \pm 0.52^*$ [46]	1.80 ± 0.52 [46]
$10^2\frac{\mathcal{B}(\Xi_c^0\to\Xi^-K^+)}{\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)}$	2.75 ± 0.57			2.75 ± 0.57
$10^2\frac{\mathcal{B}(\Xi_c^0\to\Lambda K_S^0)}{\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)}$	22.5 ± 1.3		$22.9 \pm 1.4^*$ [48]	22.9 ± 1.4 [48]
$10^2\frac{\mathcal{B}(\Xi_c^0\to\Sigma^0 K_S^0)}{\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)}$	3.8 ± 0.7			3.8 ± 0.7
$10^2\frac{\mathcal{B}(\Xi_c^0\to\Sigma^+K^-)}{\mathcal{B}(\Xi_c^0\to\Xi^-\pi^+)}$	12.3 ± 1.2			12.3 ± 1.2
$10^2\mathcal{B}(\Xi_c^+ \to \Xi^0\pi^+)$	1.6 ± 0.8			1.6 ± 0.8
$\alpha(\Lambda_c^+ \to \Lambda^0\pi^+)$	-0.84 ± 0.09		-0.755 ± 0.006 [51]	-0.76 ± 0.01 [42, 51]
$\alpha(\Lambda_c^+ \to \Sigma^0\pi^+)$	-0.73 ± 0.18		-0.463 ± 0.018 [51]	-0.47 ± 0.03 [42, 51]
$\alpha(\Lambda_c^+ \to pK_S)$	0.18 ± 0.45			0.18 ± 0.45
$\alpha(\Lambda_c^+ \to \Sigma^+\pi^0)$	-0.55 ± 0.11		-0.48 ± 0.03 [49]	-0.49 ± 0.03 [42, 49]
$\alpha(\Xi_c^0 \to \Xi^-\pi^+)$	-0.64 ± 0.05			-0.64 ± 0.05
$\alpha(\Lambda_c^+ \to \Sigma^+\eta)$			-0.99 ± 0.06 [49]	-0.99 ± 0.06 [49]
$\alpha(\Lambda_c^+ \to \Sigma^+\eta')$			-0.46 ± 0.07 [49]	-0.46 ± 0.07 [49]
$\alpha(\Lambda_c^+ \to \Lambda^0K^+)$			-0.585 ± 0.052 [51]	-0.585 ± 0.052 [51]
$\alpha(\Lambda_c^+ \to \Sigma^0K^+)$			-0.55 ± 0.20 [51]	-0.55 ± 0.20 [51]
$\alpha(\Lambda_c^+ \to \Xi^0K^+)$		0.01 ± 0.16 [33]		0.01 ± 0.16 [33]

⇐ **fit to 30 data inputs of BFs and $\alpha's$**

$$\Gamma \propto \left(|A|^2 + \kappa^2|B|^2\right)$$

$$\alpha = \frac{2\kappa|A^*B|\cos(\delta_P - \delta_S)}{|A|^2 + \kappa^2|B|^2}$$

	$\lvert X_i\rvert_S$ $(10^{-2}G_F\ \mathrm{GeV}^2)$	$\lvert X_i\rvert_P$	$\delta_S^{X_i}$ (in radian)	$\delta_P^{X_i}$
$\tilde{T}$	2.37 ± 0.41	16.56 ± 0.69	–	2.76 ± 0.32
$\tilde{C}$	1.04 ± 1.08	13.82 ± 0.58	-1.97 ± 0.79	-0.37 ± 0.44
$\tilde{C}'$	2.59 ± 0.95	24.97 ± 1.67	0.29 ± 0.19	2.86 ± 0.36
$\tilde{E}_1$	4.10 ± 0.20	2.56 ± 2.21	1.18 ± 0.38	-0.96 ± 0.43
$\tilde{E}_h$	1.54 ± 1.22	19.16 ± 3.00	-1.35 ± 0.60	0.37 ± 0.41

Channel	$10^2\mathcal{B}$	α	$\|A\|$	$\|B\|$	$\delta_P-\delta_S$	$10^2\mathcal{B}_{\rm exp}$	$\alpha_{\rm exp}$
$\Lambda_c^+\to\Lambda^0\pi^+$	1.31 ± 0.05	-0.76 ± 0.01	2.76 ± 0.25	16.96 ± 0.39	-2.92 ± 0.29	1.29 ± 0.05	-0.76 ± 0.01
$\Lambda_c^+\to\Sigma^0\pi^+$	1.26 ± 0.05	-0.48 ± 0.02	4.07 ± 0.86	15.48 ± 2.29	2.08 ± 0.04	1.27 ± 0.06	-0.47 ± 0.03
$\Lambda_c^+\to\Sigma^+\pi^0$	1.27 ± 0.05	-0.48 ± 0.02	4.07 ± 0.86	15.48 ± 2.29	2.08 ± 1.15	1.25 ± 0.09	-0.49 ± 0.03
$\Lambda_c^+\to\Sigma^+\eta$	0.33 ± 0.04	-0.93 ± 0.05	2.30 ± 0.35	9.48 ± 1.16	-2.80 ± 0.16	0.32 ± 0.04	-0.99 ± 0.06
$\Lambda_c^+\to\Sigma^+\eta'$	0.39 ± 0.11	-0.45 ± 0.07	3.81 ± 1.44	23.04 ± 3.84	-4.25 ± 0.08	0.44 ± 0.15	-0.46 ± 0.07
$\Lambda_c^+\to\Xi^0K^+$	0.41 ± 0.03	-0.16 ± 0.13	3.89 ± 0.19	2.43 ± 2.10	-2.15 ± 0.65	0.55 ± 0.07	0.01 ± 0.16
$\Lambda_c^+\to\Lambda^0K^+$	0.0639 ± 0.0030	-0.56 ± 0.05	1.09 ± 0.18	3.32 ± 0.59	2.17 ± 0.06	0.0635 ± 0.0031	-0.585 ± 0.052
$\Lambda_c^+\to\Sigma^0K^+$	0.0376 ± 0.0032	-0.54 ± 0.08	0.40 ± 0.15	3.86 ± 0.26	2.56 ± 0.44	0.0382 ± 0.0051	-0.55 ± 0.20
$\Lambda_c^+\to\Sigma^+K_S$	0.0377 ± 0.0032	-0.54 ± 0.08	0.40 ± 0.15	3.86 ± 0.26	2.56 ± 0.44	0.047 ± 0.014	
$\Lambda_c^+\to n\pi^+$	0.063 ± 0.009	-0.78 ± 0.13	1.01 ± 0.14	2.43 ± 0.38	3.81 ± 0.30	0.066 ± 0.013	
$\Lambda_c^+\to p\pi^0$	0.0176 ± 0.0034	-0.11 ± 0.75	0.64 ± 0.13	0.94 ± 0.66	4.59 ± 1.70	$0.0156^{+0.0075}_{-0.0061}$	
$\Lambda_c^+\to pK_S$	1.57 ± 0.07	0.01 ± 0.31	1.41 ± 1.51	18.68 ± 0.79	1.54 ± 0.82	1.59 ± 0.07	0.18 ± 0.45
$\Lambda_c^+\to p\eta$	0.151 ± 0.008	0.07 ± 0.37	1.01 ± 0.53	5.46 ± 0.67	1.48 ± 0.45	0.149 ± 0.008	
$\Lambda_c^+\to p\eta'$	0.052 ± 0.009	-0.54 ± 0.19	0.77 ± 0.30	4.72 ± 0.73	2.29 ± 0.13	0.049 ± 0.009	
$\Xi_c^0\to\Xi^-\pi^+$	2.83 ± 0.10	-0.72 ± 0.03	4.51 ± 0.79	31.47 ± 1.31	2.76 ± 0.32	1.80 ± 0.52	-0.64 ± 0.05
$\Xi_c^+\to\Xi^0\pi^+$	0.9 ± 0.2	-0.93 ± 0.07	2.27 ± 0.30	8.21 ± 1.16	-3.5 ± 0.23	1.6 ± 0.8	
Channel	$10^2\mathcal{R}_X$	α	$\|A\|$	$\|B\|$	$\delta_P-\delta_S$	$10^2(\mathcal{R}_X)_{\rm exp}$	$\alpha_{\rm exp}$
$\Xi_c^0\to\Xi^-K^+$	4.10 ± 0.05	-0.76 ± 0.03	1.04 ± 0.18	7.25 ± 0.30	2.76 ± 0.32	2.75 ± 0.57	
$\Xi_c^0\to\Lambda K_S^0$	24.0 ± 1.0	-0.23 ± 0.19	2.06 ± 0.87	13.59 ± 1.13	1.89 ± 0.32	22.9 ± 1.4	
$\Xi_c^0\to\Sigma^0K_S^0$	3.9 ± 0.7	0.01 ± 0.65	1.92 ± 0.43	3.47 ± 2.00	4.72 ± 1.67	3.8 ± 0.7	
$\Xi_c^0\to\Sigma^+K^-$	13.0 ± 1.1	-0.21 ± 0.17	3.89 ± 0.19	2.43 ± 2.10	4.13 ± 0.65	12.3 ± 1.2	

$\Lambda_c^+ \to \Xi^0 K^+$

- Theory in 1990s ⇒ small BF & zero α due to smallness of S wave
- BF was measured, not that small, $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+) = (0.55 \pm 0.07)\%$
- In recent studies based on SU(3) ⇒ α is predicted to be of order unity
- The puzzle with α was resolved by BESIII ⇒ $\alpha_{\Xi^0 K^+} = 0.01 \pm 0.16$.
  Phase shifts must be taken into account

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$$\delta_P - \delta_S = -1.55 \pm 0.25 \pm 0.05 \quad \text{or} \quad 1.59 \pm 0.25 \pm 0.05 \text{ rad.}$$

Hence, $\alpha_{\Xi^0 K^+} \propto \cos(\delta_P - \delta_S) \sim 0.02$

- Smallness of $\alpha_{\Xi^0 K^+}$ can be accommodated recently in both IRA and TDA

IRA: Geng et al. 2310.05491

TDA: Zhong et al. 2401.15926, 2404.01359

1. $\Lambda_c^+ \to \Xi^0 K^+$

$$\Gamma \propto \left(|A|^2 + \kappa^2|B|^2\right)$$

$$\alpha = \frac{2\kappa|A^*B|\cos(\delta_P - \delta_S)}{|A|^2 + \kappa^2|B|^2}$$

BF & longitudinal decay asymmetry

- **BESIII**

  I. $\begin{cases} |A| = 1.6^{+1.9}_{-1.6} \pm 0.4\,, \\ |B| = 18.3 \pm 2.8 \pm 0.7\,, \end{cases}$ II. $\begin{cases} |A| = 4.3^{+0.7}_{-0.2} \pm 0.4 \\ |B| = 6.7^{+8.3}_{-6.7} \pm 1.6 \end{cases}$

$$\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+) = (0.55 \pm 0.07)\%$$

$$\alpha_{\Xi^0 K^+} = 0.01 \pm 0.16$$

$$\delta_P - \delta_S = -1.55 \pm 0.25 \pm 0.05 \quad \text{or} \quad 1.59 \pm 0.25 \pm 0.05 \text{ rad.}$$

- **Our fit**

$$|A| = 3.89 \pm 0.19,\ |B| = 2.43 \pm 2.12$$

Solution II preferred

$$\alpha_{\Xi^0 K^+} = -0.16 \pm 0.13$$

$$\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+) = (0.41 \pm 0.03)\%$$

$$\delta_P - \delta_S = -2.15 \pm 0.65 \text{ rad}$$

2. $\Xi_c^0 \to \Xi^- \pi^+$

BF = $(1.80 \pm 0.52)\%$ by Belle, $(1.43 \pm 0.32)\%$ by PDG

Our fit

Channel	$10^2\mathcal{B}$	α	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P - \delta_S$
$\Xi_c^0 \to \Xi^- \pi^+$	2.83 ± 0.10	-0.72 ± 0.03	4.51 ± 0.79	31.47 ± 1.31	2.76 ± 0.32

$(2.72 \pm 0.09)\%$ by Geng et al. (2310.05491)

support from the sum rule

$$\frac{\tau_{\Lambda_c^+}}{\tau_{\Xi_c^0}}\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = 3\mathcal{B}(\Lambda_c^+ \to \Lambda \pi^+) + \mathcal{B}(\Lambda_c^+ \to \Sigma^0 \pi^+) - \frac{1}{\sin^2\theta_C}\mathcal{B}(\Lambda_c^+ \to n\pi^+)$$

$$\Rightarrow \quad \mathcal{B}\left(\Xi_c^0 \to \Xi^- \pi^+\right) = (2.85 \pm 0.30)\%$$

Improved measurement of $\Xi_c^0 \to \Xi^- \pi^+$ is urgently needed

Conclusion

- TDA approach for charmed baryon decays is established
- Magnitudes of S- and P-wave amplitudes and their phase shifts are presented, very crucial for the future study of CP violation
- The smallness of α in $\Lambda_c^+ \to \Xi^0 K^+$ is accommodated
- Equivalence between TDA and IRA is established
- Improved measurement of $\Xi_c^0 \to \Xi^- \pi^+$ is urgently needed

Back-up slides

Irreducible SU(3) Approach (IRA)

$$\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}\sum_{q_1,q_2}^{d,s}V_{cq_1}V_{uq_2}(c_+O_+^{q_1q_2}+c_-O_-^{q_1q_2})+h.c.=\mathcal{H}_{\rm eff}^{\mathbf{6}}+\mathcal{H}_{\rm eff}^{\overline{\mathbf{15}}}$$

$$\begin{aligned}\mathcal{A}_{\rm IRA}= \;& a_1\,(\mathcal{B}_c)_i\,(H_6)^{ik}_j\,(\mathcal{B}_8)^j_k\,M^l_l+a_2\,(\mathcal{B}_c)_i\,(H_6)^{ik}_j\,(\mathcal{B}_8)^l_k\,M^j_l+a_3\,(\mathcal{B}_c)_i\,(H_6)^{ik}_j\,(\mathcal{B}_8)^j_l\,M^l_k\\&+a_4\,(\mathcal{B}_c)_i\,(H_6)^{jk}_l\,(\mathcal{B}_8)^i_j\,M^l_k+a_5\,(\mathcal{B}_c)_i\,(H_6)^{jk}_l\,(\mathcal{B}_8)^l_j\,M^i_k\\&+a_6\,(\mathcal{B}_c)_i\,(H_{\overline{15}})^{ik}_j\,(\mathcal{B}_8)^j_k\,M^l_l+a_7\,(\mathcal{B}_c)_i\,(H_{\overline{15}})^{ik}_j\,(\mathcal{B}_8)^l_k\,M^j_l+a_8\,(\mathcal{B}_c)_i\,(H_{\overline{15}})^{ik}_j\,(\mathcal{B}_8)^j_l\,M^l_k\\&+a_9\,(\mathcal{B}_c)_i\,(H_{\overline{15}})^{jk}_l\,(\mathcal{B}_8)^i_j\,M^l_k+a_{10}\,(\mathcal{B}_c)_i\,(H_{\overline{15}})^{jk}_l\,(\mathcal{B}_8)^l_j\,M^i_k.\end{aligned}$$

He, Shi, Wang ('20)

One of the five terms with H_6 is redundant

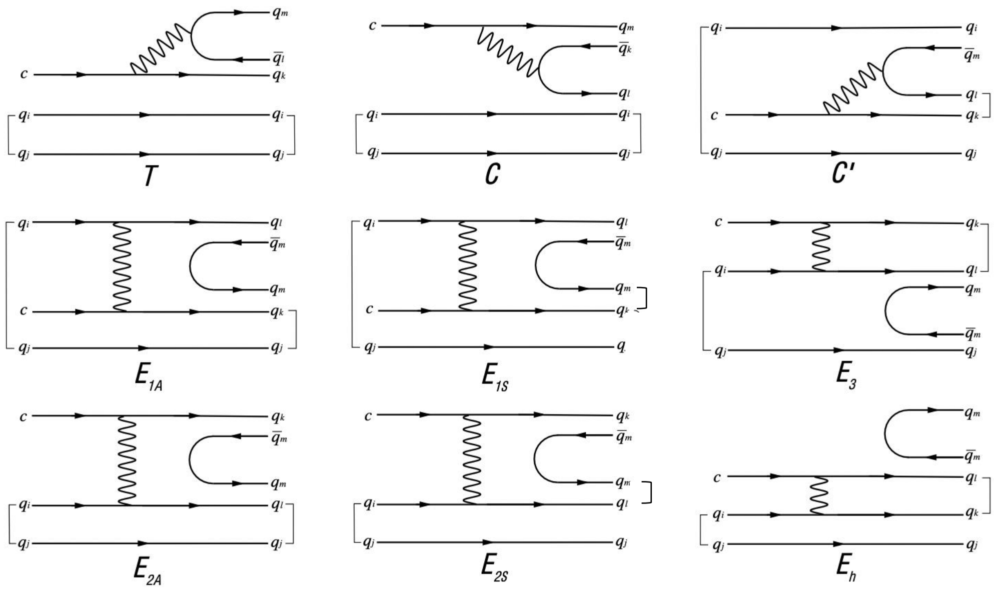
$$a_1'=a_1-a_5,\quad a_2'=a_2+a_5,\quad a_3'=a_3+a_5,\quad a_4'=a_4+a_5$$

Four of the five terms with $H_{\overline{15}}$ are prohibited by the KPW theorem

$$a_6=a_7=a_8=a_{10}=0$$

⇒ **five independent SU(3) tensor invariants $a_1', a_2', a_3', a_4', a_9',$**

⇒ **equivalence of TDA and IRA is established**



$$
\begin{aligned}
\tilde{\mathcal{A}}_{\rm TDA} = \quad & T(\mathcal{B}_c)^{ij} H^{km}_l \, (\mathcal{B}_8)_{ijk} \, M^l_m + C(\mathcal{B}_c)^{ij} H^{ml}_k \, (\mathcal{B}_8)_{ijl} \, M^k_m \\
& + C'(\mathcal{B}_c)^{ij} H^{kl}_m \, (\mathcal{B}_8)_{klj} \, M^m_i + E_{1A}(\mathcal{B}_c)^{ij} H^{kl}_i \, (\mathcal{B}_8)_{jkm} \, M^m_l \\
& + E'_{1A}(\mathcal{B}_c)^{ij} H^{kl}_i \, (\mathcal{B}_8)_{kmj} \, M^m_l + E_{2A}(\mathcal{B}_c)^{ij} H^{kl}_i \, (\mathcal{B}_8)_{jlm} \, M^m_k \\
& + E'_{2A}(\mathcal{B}_c)^{ij} H^{kl}_i \, (\mathcal{B}_8)_{lmj} \, M^m_k + E_3(\mathcal{B}_c)^{ij} H^{kl}_i \, (\mathcal{B}_8)_{klm} \, M^m_j \\
& + E_h(\mathcal{B}_c)^{ij} H^{kl}_i \, (\mathcal{B}_8)_{klj} \, M^m_m .
\end{aligned}
$$

Hsiao, Wang, Zhao ('22)