Neutrino masses and mixing in an inverse seesaw (2,3) model augmented with S_4 modular flavor symmetry

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Plan of the talk

- Introduction
- Motivation
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- Summary

Introduction

Brief ideas about neutrinos

Some Experiments on Neutrino Physics

Some Open Questions on Neutrino Physics

Neutrino oscillations

Brief overview of Standard Model

Brief ideas about neutrinos

- The history of neutrino began with the investigation of β-decay.
- To remove the discrepancies in the conservation of energy, momentum etc. **Wolfgang Pauli** proposed an uncharged and **spin** $\frac{1}{2}$ particle known as neutrino.
- There are three flavours of neutrinos: electron neutrino, muon neutrino and tau neutrino.
- The sources of neutrinos are of two types. One is naturally occurring: Sun, Supernova, Big Bang. The other is man made or artificial sources: particle accelerators or nuclear reactors.
- According to the Standard Model, neutrino has no mass but from various experiments it was proved that neutrino has extremely small mass.

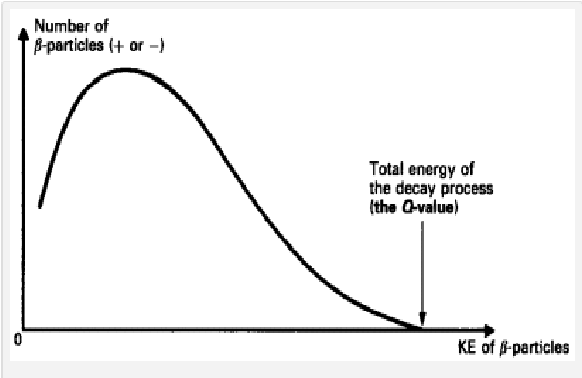


Fig-1: Energy Spectrum of Beta Decay

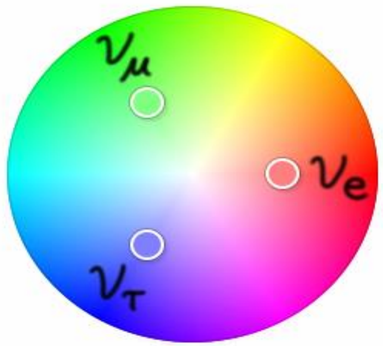


Fig-2: Three Flavours of Neutrino

Some Experiments on Neutrino Physics

- Various experiments were performed from time to time to detect neutrinos like Homestake experiment, Kamiokande and Superkamiokande experiment etc.
- From Homestake experiment the observed rate of solar neutrino was found to be smaller than the rate predicated by SSM, which is called **"Solar Neutrino Problem"**.
- The detectors like Kamiokande, IMB etc. were used to detect atmospheric neutrinos and **atmospheric neutrino anomaly** was found.

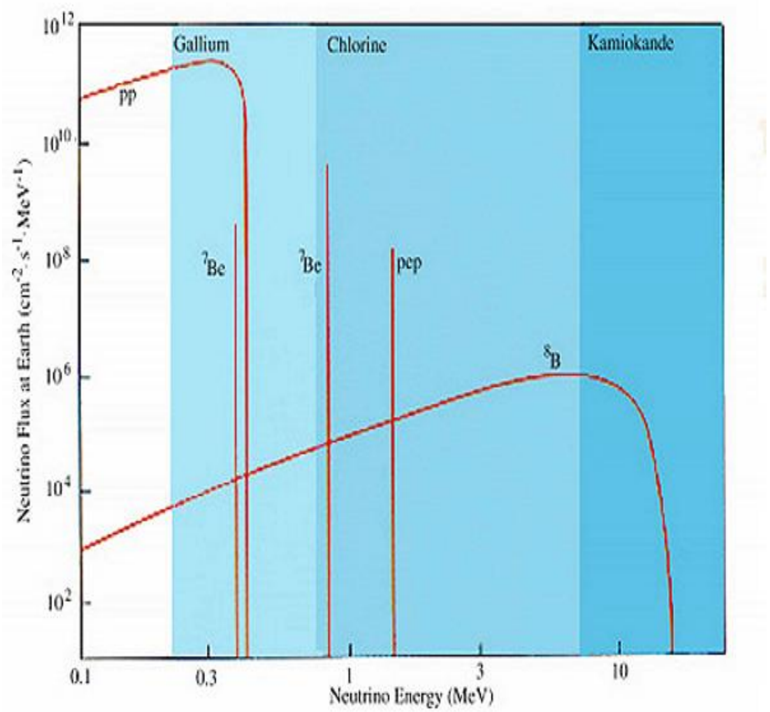


Fig-3: Neutrino flux production of SSM[1]

[1] J.N. Bahcall and R.K. Ulrich, Rev. Mod. Phys. 60, 297(1988)

Some Open Questions on Neutrino Physics

- Why are the masses of neutrinos considerably smaller than the masses of corresponding charged leptons?
- Are massive neutrinos Dirac or Majorana particles?
- What is the absolute scale of neutrino masses?
- Is there any CP violation in leptonic sector?
- Does sterile neutrino exist?
- Is the mass hierarchy of neutrino normal or inverted?
- Why are the pattern of lepton mixing so different from that of the quarks?

Neutrino oscillations

- The idea of neutrino oscillations was first postulated by **Bruno Pontecorvo**.
- Neutrino Oscillations can occur in both vacuum and matter during transition.
- The idea of neutrino oscillations came from the study of solar neutrino problem and atmospheric neutrino anomaly.
- The experimental discovery of neutrino oscillations was done by **SuperKamiokande Observatory**.

In case of neutrino oscillations, we can write flavor eigenstate as a linear combination of mass eigenstates as

$$|\nu_\alpha(0)\rangle = \sum_{a=1}^{3} U_{a\alpha}^{*} \, |\nu_a(0)\rangle$$

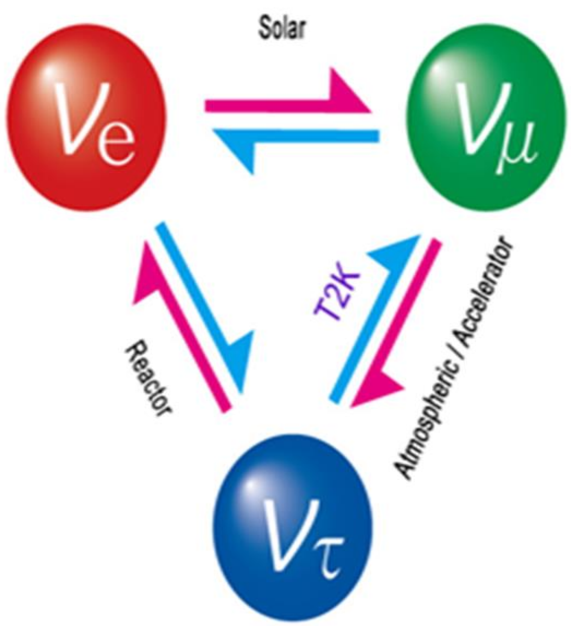


Fig-4: Neutrino Oscillations between three generations

With further calculations, the probability of conversion is found to be

$$P_{(\nu_e\to\nu_\mu;L)} = \sin^2 2\theta \sin^2 \frac{\Delta m_{21}^2 t}{4E} = \sin^2 2\theta \sin^2 \frac{\pi L}{L_0}$$

There are six neutrino oscillation parameters viz. $\Delta m_{21}^2, \Delta m_{31}^2, \theta_{12}, \theta_{13}, \theta_{23}, \delta$. The experimental ranges of these parameters are given in the following table:

Oscillation Parameters	Normal Ordering		Inverted Ordering	
	best fit $\pm$ 1σ	3σ range	best fit $\pm$ 1σ	3σ range
$\Delta m_{21}^2/10^{-5}eV^2$	$7.42^{+0.21}_{-0.20}$	$6.82-8.04$	$7.42^{+0.21}_{-0.20}$	$6.82-8.04$
$\lvert\Delta m_{3l}^2\rvert/10^{-3}eV^2$	$2.510^{+0.027}_{-0.027}$	$2.430-2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574--2.410$
θ_{12}°	$33.45^{+0.77}_{-0.75}$	$31.27-35.87$	$33.45^{+0.78}_{-0.75}$	$31.27-35.87$
θ_{23}°	$42.1^{+1.1}_{-0.9}$	$39.7-50.9$	$49.0^{+0.9}_{-1.9}$	$39.8-51.6$
θ_{13}°	$8.62^{+0.12}_{-0.12}$	$8.25-8.98$	$8.61^{+0.14}_{-0.12}$	$8.24-9.02$
δ_{CP}°	230^{+36}_{-25}	$144-350$	278^{+22}_{-30}	$194-345$

Table-1: Current neutrino oscillation parameters from global fits [2]

[2] Gonzalez-Garcia, Maria Concepcion, Michele Maltoni, and Thomas Schwetz. "NuFIT: three-flavour global analyses of neutrino oscillation experiments." Universe 7, no. 12 (2021): 459

A brief overview of Standard Model

- The Standard Model explains three of the four fundamental forces that govern the Universe: electromagnetic force, strong force, and weak force.
- The fourth fundamental force is the gravitational force, which is not adequately explained by the Standard Model.
- The Standard Model of Particle Physics describes the most basic building blocks of the Universe.
- Also SM could not explain the masses of neutrinos.

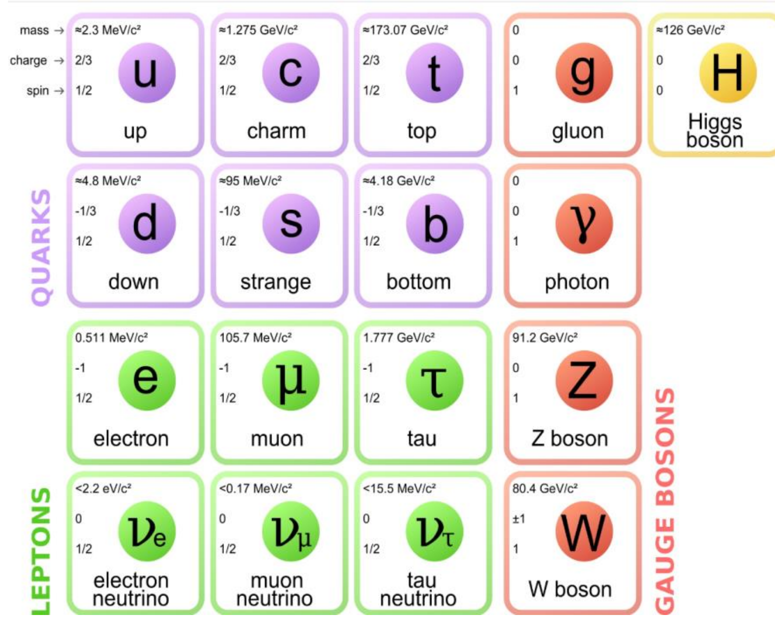


Fig-5: Standard Model of elementary particles

In Standard Model, each interaction is described as an exchange of particles known as **force carriers**.

Motivation

In particle physics, symmetries have always played an important role.

In recent days, the modular symmetry has gained a lot of importance because of its property to minimize the extra particle called 'flavon' while analyzing a model with respect to a particular symmetry group.

By imposing this type of symmetry, we can reduce the number of flavon fields in the model which can enhance the predictability of the model.

Neutrino masses and mixing angles are simultaneously constrained by this modular symmetry.

Model Framework

- In the present work, we will be using $\Gamma(4)$ which is isomorphic to the discrete symmetry group S_4. Here, we focus on modular forms[3] of level 4.
- The five linearly independent **weight 3** and **level-4** modular forms are given by

$$Y_1(\tau) = \frac{i}{8}\left(8\frac{\eta'\left(\tau+\frac{1}{2}\right)}{\eta\left(\tau+\frac{1}{2}\right)} + 32\frac{\eta'(4\tau)}{\eta(4\tau)} - \frac{\eta'\left(\frac{\tau}{4}\right)}{\eta\left(\frac{\tau}{4}\right)} - \frac{\eta'\left(\frac{\tau+1}{4}\right)}{\eta\left(\frac{\tau+1}{4}\right)} - \frac{\eta'\left(\frac{\tau+2}{4}\right)}{\eta\left(\frac{\tau+2}{4}\right)} - \frac{\eta'\left(\frac{\tau+3}{4}\right)}{\eta\left(\frac{\tau+3}{4}\right)}\right)$$

$$Y_2(\tau) = \frac{i\sqrt{3}}{8}\left(\frac{\eta'\left(\frac{\tau}{4}\right)}{\eta\left(\frac{\tau}{4}\right)} - \frac{\eta'\left(\frac{\tau+1}{4}\right)}{\eta\left(\frac{\tau+1}{4}\right)} + \frac{\eta'\left(\frac{\tau+2}{4}\right)}{\eta\left(\frac{\tau+2}{4}\right)} - \frac{\eta'\left(\frac{\tau+3}{4}\right)}{\eta\left(\frac{\tau+3}{4}\right)}\right)$$

$$Y_3(\tau) = i\left(\frac{\eta'\left(\tau+\frac{1}{2}\right)}{\eta\left(\tau+\frac{1}{2}\right)} - 4\frac{\eta'(4\tau)}{\eta(4\tau)}\right)$$

[3] Feruglio, Ferruccio. ”Are neutrino masses modular forms?.” In From My Vast Repertoire. . . Guido Altarelli’s Legacy, pp. 227-266. 2019

$$Y_4(\tau) = \frac{i}{4\sqrt{2}}\left(-\frac{\eta'\left(\frac{\tau}{4}\right)}{\eta\left(\frac{\tau}{4}\right)} + i\frac{\eta'\left(\frac{\tau+1}{4}\right)}{\eta\left(\frac{\tau+1}{4}\right)} + \frac{\eta'\left(\frac{\tau+2}{4}\right)}{\eta\left(\frac{\tau+2}{4}\right)} - i\frac{\eta'\left(\frac{\tau+3}{4}\right)}{\eta\left(\frac{\tau+3}{4}\right)}\right)$$

$$Y_5(\tau) = \frac{i}{4\sqrt{2}}\left(-\frac{\eta'\left(\frac{\tau}{4}\right)}{\eta\left(\frac{\tau}{4}\right)} - i\frac{\eta'\left(\frac{\tau+1}{4}\right)}{\eta\left(\frac{\tau+1}{4}\right)} + \frac{\eta'\left(\frac{\tau+2}{4}\right)}{\eta\left(\frac{\tau+2}{4}\right)} + i\frac{\eta'\left(\frac{\tau+3}{4}\right)}{\eta\left(\frac{\tau+3}{4}\right)}\right)$$

Here, $\eta(\tau)$ is the Dedekind eta-function and it is defined as

$$\eta(\tau) = q^{\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^n) \quad \text{and } q \equiv e^{2\pi i\tau}$$

Within the present model, the complete neutrino mass matrix is given by

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_{NS} \\ 0 & M_{NS}^T & \mu \end{pmatrix}$$

Here, M_D is at **electroweak scale**, M_{NS} at **TeV scale**, and μ at **keV scale** to get SM neutrinos at **sub-eV scale**[4].

Now, the inverse seesaw formula for light neutrinos is given by

$$M_\nu = M_D M_{NS}^{-1}\mu(M_{NS}^{-1})^T M_D^T$$

[4] Deppisch, F., and J. W. F. Valle. "Enhanced lepton flavor violation in the supersymmetric inverse seesaw model." Physical Review D 72, no. 3 (2005): 036001

The particle content and the group charges for the model is given in Table-2. Here, the Yukawa couplings have non-trivial group transformation.

Fields	L	l_R	N_R	S_i	H_u	H_d	Φ
$SU(2)_L$	3	1	1	1	2	2	1
S_4	3	1,	2	3	1	1	1
k_I	-2	-2	-2	-2	0	0	0

Table-2: Particle content of the model and their charges

Now, the modular weight of the Yukawa coupling and its transformation under S_4 symmetry is given in Table-3.

Yukawa coupling	S_4	k_I
Y	$3,3'$	2,4

Table-3: The modular weight of the Yukawa coupling and its transformation under S_4 symmetry

Based on the above charge assignments and symmetries, the Lagrangian of the model for the leptonic sector is given by

$$\mathcal{L}_{lepton} = \mathcal{L}_L + \mathcal{L}_D + \mathcal{L}_{NS} + \mathcal{L}_S$$

The Lagrangian for the charged leptons is given by

$$\mathcal{L}_L = \alpha\left(Ll_{R_1}^c\right)_{3'}(H_d)_1 Y_{3'} + \beta\left(Ll_{R_2}^c\right)_3 (H_d)_1 Y_3^{(4)} + \gamma\left(Ll_{R_3}^c\right)_{3'}(H_d)_1 Y_{3'}^{(4)}$$

Hence, the charged lepton mass matrix is found to be

$$M_L = v_d \begin{pmatrix} \alpha Y_3 & -2\beta Y_2 Y_3 & 2\gamma Y_1 Y_3 \\ \alpha Y_5 & \beta\left(\sqrt{3} Y_1 Y_4 + Y_2 Y_5\right) & \gamma\left(\sqrt{3} Y_2 Y_4 - Y_1 Y_5\right) \\ \alpha Y_2 & \beta\left(\sqrt{3} Y_1 Y_5 + Y_2 Y_4\right) & \gamma\left(\sqrt{3} Y_2 Y_5 - Y_1 Y_4\right) \end{pmatrix}$$

The Lagrangian for the Dirac mass term is given by

$$\mathcal{L}_D = (LN_R)_3 (H_u)_1 Y_3^{(4)} + (LN_R)_{3'} (H_u)_1 Y_{3'}^{(4)}$$

Now, the Dirac mass matrix for the neutrinos can be written as

$$M_D = v_u \begin{pmatrix} -2\alpha_D Y_2 Y_3 & -2\beta_D Y_1 Y_3 \\ -\alpha_D\left(\frac{\sqrt{3}}{2} Y_2 Y_4 + \frac{1}{2} Y_2 Y_5\right) + \beta_D\left(\frac{3}{2} Y_2 Y_5 - \frac{\sqrt{3}}{2} Y_1 Y_4\right) & \alpha_D\left(\frac{3}{2} Y_1 Y_5 + \frac{\sqrt{3}}{2} Y_2 Y_4\right) + \beta_D\left(\frac{3}{2} Y_1 Y_5 + \frac{\sqrt{3}}{2} Y_2 Y_4\right) \\ -\alpha_D\left(\frac{\sqrt{3}}{2} Y_1 Y_5 + \frac{1}{2} Y_2 Y_4\right) + \beta_D\left(\frac{3}{2} Y_2 Y_4 - \frac{\sqrt{3}}{2} Y_1 Y_5\right) & \alpha_D\left(\frac{3}{2} Y_1 Y_4 + \frac{\sqrt{3}}{2} Y_2 Y_5\right) + \beta_D\left(\frac{3}{2} Y_1 Y_4 + \frac{\sqrt{3}}{2} Y_2 Y_5\right) \end{pmatrix}$$

The Lagrangian for the mixing between right-handed neutrinos and gauge singlet neutral fermions is given by

$$\mathcal{L}_{NS} = \alpha_{NS}\left(Y_3^{(4)} N_R\right)_3 (S_1)_3 \Phi + \beta_{NS}\left(Y_{3'}^{(4)} N_R\right)_3 (S_2)_{3'} \Phi$$

The resultant mass matrix is found to be

$$M_{NS} = v_{NS}\begin{pmatrix} -2\alpha_{NS} Y_2 Y_3 & -\frac{\alpha_{NS}}{2}(\sqrt{3}Y_1Y_4 + Y_2Y_5) + \frac{\sqrt{3}}{2}\beta_{NS}(\sqrt{3}Y_2Y_5 - Y_1Y_4) & -\frac{\alpha_{NS}}{2}(\sqrt{3}Y_1Y_5 + Y_2Y_5) + \frac{\sqrt{3}}{2}\beta_{NS}(\sqrt{3}Y_2Y_4 - Y_1Y_5) \\ -2\beta_{NS} Y_1 Y_3 & \frac{\sqrt{3}}{2}\alpha_{NS}(\sqrt{3}Y_1Y_5 + Y_2Y_4) + \frac{\beta_{NS}}{2}(\sqrt{3}Y_2Y_4 - Y_1Y_5) & \frac{\sqrt{3}}{2}\alpha_{NS}(\sqrt{3}Y_1Y_4 + Y_2Y_5) + \frac{\beta_{NS}}{2}(\sqrt{3}Y_2Y_5 - Y_1Y_4) \end{pmatrix}$$

The Lagrangian for the Majorana mass term can be written as

$$\mathcal{L}_S = \mu_0 SSY_3^{(4)}$$

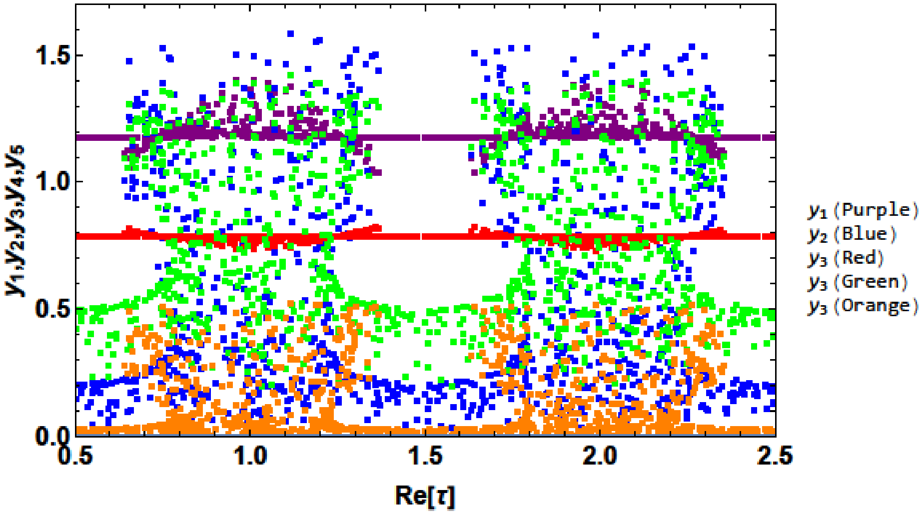
The Majorana mass matrix is given by

$$M_S = \mu_0\begin{pmatrix} 0 & -(\sqrt{3}Y_1Y_5 + Y_2Y_4) & \sqrt{3}Y_1Y_4 + Y_2Y_5 \\ -(\sqrt{3}Y_1Y_5 + Y_2Y_4) & 2Y_2Y_3 & 0 \\ \sqrt{3}Y_1Y_4 + Y_2Y_5 & 0 & -2Y_2Y_3 \end{pmatrix}$$

Results

To satisfy the current neutrino oscillation data, we can choose the following ranges of the free parameters for the model:

$$\mathrm{Re}\,[\tau] \in [0.5,2.5]\,,\ \mathrm{Im}\,[\tau] \in [0.75,1.5]\,,\{\alpha_D, \beta_D\} \in [0.1,10]\,,\{\alpha_{NS}, \beta_{NS}\} \in [0.1,1]\,,$$
$$v_{NS} = [1,10]\,\mathrm{TeV},\ \mu_0 = [10^2, 10^3]\,\mathrm{eV}$$



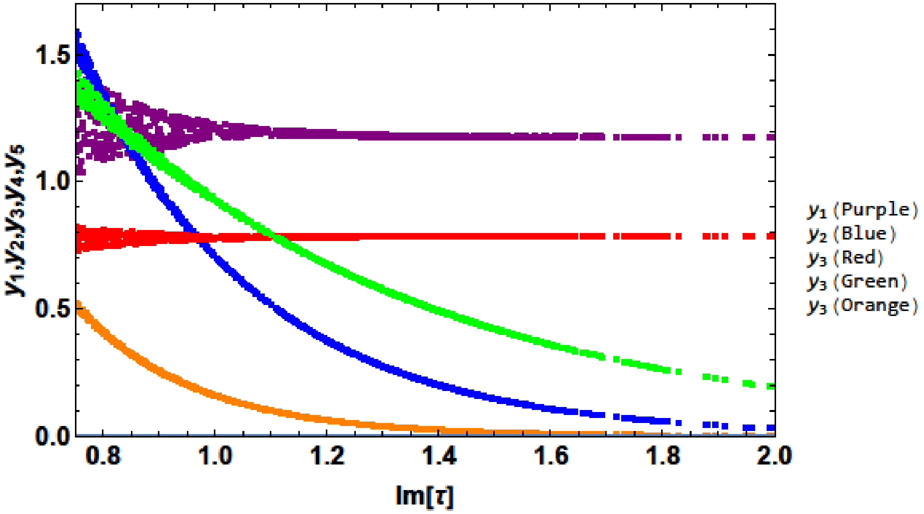


Fig-6: Variation of Yukawa couplings as a function of real and imaginary part of τ for NO.

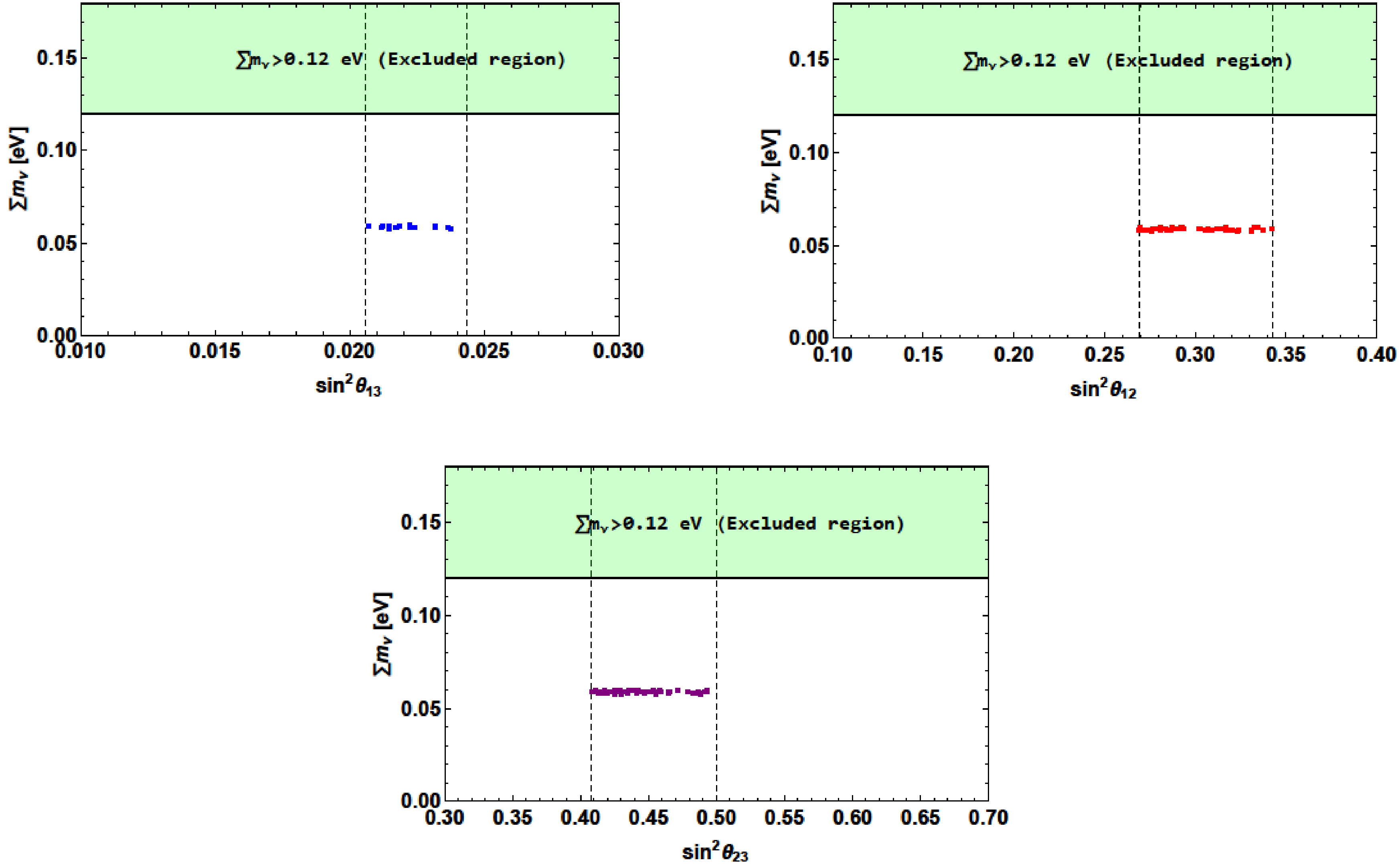


Fig-7: The correlation between sum of neutrino masses (m_ν) and $\sin^2\theta_{13}$ (left panel), $\sin^2\theta_{12}$ (right panel), $\sin^2\theta_{23}$ (bottom plot) for NO

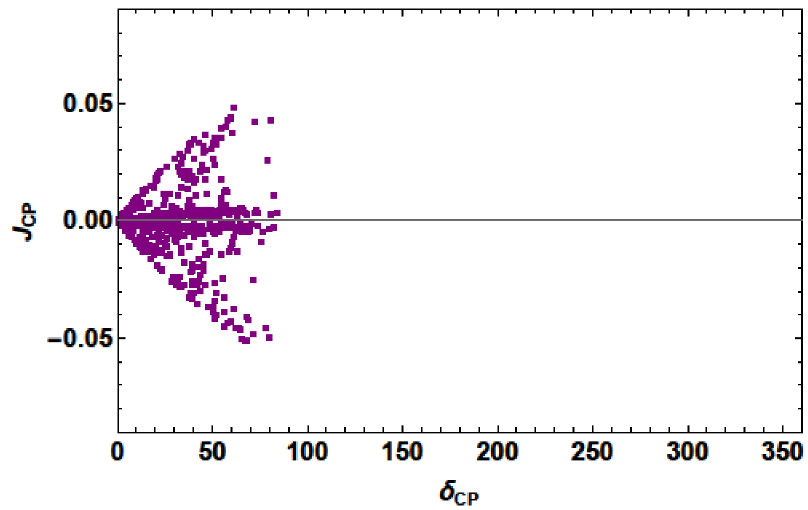


Fig-8: Correlation between Jarsklog invariant (J_{CP}) with Dirac CP phase (δ_{CP}) for NO

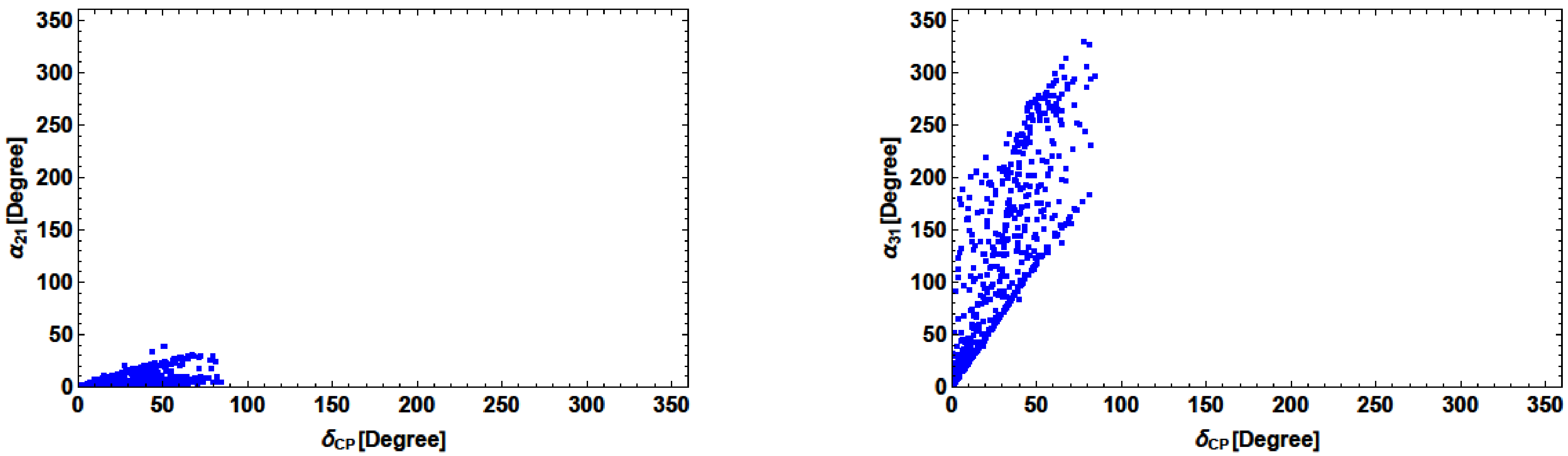


Fig-9: Correlation between Majorana phases with Dirac CP phase (δ_{CP}) for NO

Summary

- This symmetry restricts the use of multiple flavon fields which enhances the predictability of the model at current and future collider experiments.
- The neutrino mass matrix for the model has been formulated which is characterized by the modulus τ and free parameters for the model.
- The lower bound for sum of the neutrino masses is around 0.06 eV.
- The Dirac CP phase is restricted in the region $0°$ to $90°$.
- The J_{CP} is found to be of the order of 10^{-2}.
- The Majorana phases are found to be in the ranges $0° \leq \alpha_{21} \leq 50°$ and $0° \leq \alpha_{31} \leq 340°$.

Thank You!