Chulalongkorn University

Study of neutrino mass matrices with vanishing trace and one vanishing minor

Sangeeta Dey

Department of Physics, Cotton University, Guwahati, India

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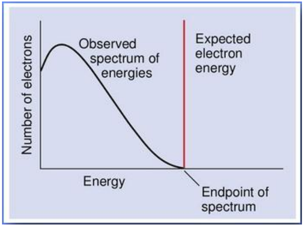
phy1891006_sangeeta@cottonuniversity.ac.in

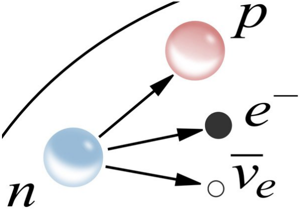
Plan of Talk

- Introduction
- Motivation
- Framework of the Neutrino Mass Matrix
- Texture Analysis
- Symmetry realization
- Results and Discussion
- References

Introduction

- Neutrino physics originated from β-decay when it was established that the average energy of the electrons produced in the β-decay is significantly smaller than the total energy released.





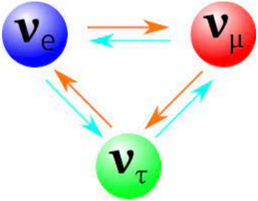
Introduction

- **Pauli** suggested that some of the energy has been taken away by a new particle neutrino which is emitted in the decay process, which carries energy and have **spin** $\frac{1}{2}$, but which is **massless, neutral and weak interacting.**
- From the standard model of particle physics, we find that the neutrinos ν_e, ν_μ and ν_τ are **massless.**
- In 1956, **Frederick Reines and Clyde L. Cowan, Jr.** first discovered electron neutrino in a nuclear fission reactor.

Introduction

Neutrino Oscillation

- However **Super-Kamiokande experiment** carried out in 1996 confirmed that neutrinos undergo oscillation giving the concept of neutrino mass.



Introduction

- Masses and mixing of three flavors of neutrino (ν_e , ν_μ, ν_τ) can be described by a (3×3) mass matrix M_ν.

$$M_\nu = \begin{pmatrix} \mathrm{M}_{ee} & \mathrm{M}_{e\mu} & \mathrm{M}_{e\tau} \\ \mathrm{M}_{e\mu} & \mathrm{M}_{\mu\mu} & \mathrm{M}_{\mu\tau} \\ \mathrm{M}_{\tau e} & \mathrm{M}_{\tau\mu} & \mathrm{M}_{\tau\tau} \end{pmatrix} \tag{1}$$

- We can rewrite the neutrino mass matrix as

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T \tag{2}$$

- V is the diagonalizing PMNS matrix, parametrized as $V = UP_\nu$ and (m_1, m_2, m_3) are the neutrino mass eigenvalues.

Introduction

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \quad (3)$$

$$P_\nu = diag(1, e^{i\alpha}, e^{i(\beta+\delta)})$$

- M_ν is parametrized by a total of nine parameters $(m_1, m_2, m_3, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha, \beta)$.
- Out of these nine parameters only five of them are measured by neutrino oscillation experiments. They are the three mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and two mass squared differences $(\Delta m^2_{21}, \Delta m^2_{32})$.

Motivation

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- Inspite of the tremendous effort and progress, the neutrino mass matrix has not been fully determined, by any of the conceivable set of feasible experiments.
- Although there is increasing information on the numerical values of these parameters, the origin of leptonic flavor structure is still a mystery.
- The smallness of the neutrino mass, mass hierarchy, origin of CP violation are some of the striking questions whose answers are still lacking in the standard model of particle physics.

Framework of neutrino mass matrix

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- At first we reconstructed the neutrino mass matrix M_ν as

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T \tag{4}$$

Here (m_1, m_2, m_3) are the neutrino mass eigenvalues and V is the diagonalising PMNS matrix.

$$M_\nu = U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T, \tag{5}$$

- where $\lambda_1 = m_1$, $\lambda_2 = m_2 e^{2i\alpha}$, $\lambda_3 = m_3 e^{2i(\beta+\delta)}$.

Framework of neutrino mass matrix

- We have six cases of one vanishing minor.
- The minors of the off-diagonal elements can be given by:

$$\begin{aligned} C_{mn} = (-1)^{m+n}((M_\nu)_{(m+1,n-1)}(M_\nu)_{(m+2,n+1)} \\ -(M_\nu)_{(m+1,n+1)}(M_\nu)_{(m+2,n+2)}) \end{aligned} \tag{6}$$

and for diagonal elements:

$$\begin{aligned} C_{mm} = (-1)^{2m}((M_\nu)_{(m+1,m+1)}(M_\nu)_{(m+2,m+2)} \\ -(M_\nu)_{(m+1,m+2)}(M_\nu)_{(m+2,m+1)}) \end{aligned} \tag{7}$$

- For $m+l$, $n+l > 3$, we have to take the values $(m+l)-3$, $(n+l)-3$. Here m, n can take values (1, 2, 3) and $l = 1, 2$.

Framework of neutrino mass matrix

- The condition for vanishing minor

$$C_{mn} = 0, \quad C_{mm} = 0 \tag{8}$$

- Solving Eq(8) we get

$$m_1 m_2 e^{2i\alpha} A_3 + m_2 m_3 e^{2i(\alpha+\beta+\delta)} A_1 + m_3 m_1 e^{2i(\beta+\delta)} A_2 = 0 \tag{9}$$

$$A_i = (U_{pj}U_{qj}U_{rk}U_{sk} - U_{tj}U_{uj}U_{vk}U_{wk}) + (j \longleftrightarrow k) \tag{10}$$

here (i,j,k) is a cyclic permutation of (1,2,3). Therefore the two constraint equations becomes

Framework of neutrino mass matrix

$$\lambda_1\lambda_2 A_3 + \lambda_2\lambda_3 A_1 + \lambda_3\lambda_1 A_2 = 0 \tag{11}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 0 \tag{12}$$

- Considering $\lambda_1 > 0$ and defining $X = \frac{\lambda_2}{\lambda_1}$ and $Y = \frac{\lambda_3}{\lambda_1}$

$$XA_3 + XYA_1 + YA_2 = 0 \tag{13}$$

$$1 + X + Y = 0 \tag{14}$$

Framework of neutrino mass matrix

On solving Eqs.13 and 14 we have

$$X_{\pm} = \frac{(A_3 - A_1 - A_2) \pm \sqrt{(A_3 - A_1 - A_2)^2 - 4A_1A_2}}{2A_1} \tag{15}$$

$$Y_{\pm} = \frac{(A_2 - A_1 - A_3) \pm \sqrt{(A_3 - A_1 - A_2)^2 - 4A_1A_2}}{2A_1} e^{-2i\delta} \tag{16}$$

Framework of neutrino mass matrix

- For the solution pairs (X_+, Y_-) and (X_-, Y_+) we get the Majorana phases as

$$\alpha = \frac{1}{2}Arg[\frac{(A_3 - A_1 - A_2) \pm \sqrt{(A_3 - A_1 - A_2)^2 - 4A_1A_2}}{2A_1}] \qquad (17)$$

$$\beta = \frac{1}{2}Arg[\frac{(A_2 - A_1 - A_3) \pm \sqrt{(A_3 - A_2 - A_1)^2 - 4A_1A_2}}{2A_1}e^{-2i\delta}] \qquad (18)$$

- The solution pairs (X_+, Y_+) and (X_-, Y_-) satisfy the constraint equations under the condition $(A_3 - A_1 - A_2)^2 - 4A_1A_2 = 0$.

$$\alpha = \frac{1}{2} Arg[\frac{(A_3 - A_1 - A_2)}{2A_1}], \quad (19)$$

$$\beta = \frac{1}{2} Arg[\frac{(A_2 - A_1 - A_3)}{2A_1} e^{-2i\delta}]. \quad (20)$$

Framework of neutrino mass matrix

- The ratios of the neutrino masses are

$$\rho = |\frac{m_2}{m_1} e^{2i\alpha}| \quad and \quad \sigma = |\frac{m_3}{m_1} e^{2i\beta}| \tag{21}$$

- We have checked the viability of the model by calculating the ratio of the solar and atmospheric mass squared difference (R_ν).

$$R_\nu = \frac{\delta m^2}{\Delta m^2} = \frac{2(\rho^2 - 1)}{|2\sigma^2 - \rho^2 - 1|} \tag{22}$$

where $\delta m^2 = m_2^2 - m_1^2$ is the solar mass splitting and $\Delta m^2 = |m_3^2 - \frac{1}{2}(m_2^2 + m_1^2)|$ the atmospheric mass splitting. For NH, $R_\nu = \frac{2\epsilon}{\sigma^2}$, if we consider $\rho = 1 + \epsilon$ for m_1 and m_2 being very close to each other with the values $0.013 < \epsilon < 0.017$ on 3σ [nufit2021]. For IH, $R_\nu = \frac{2(\rho^2-1)}{\rho^2+1}$.

Framework of neutrino mass matrix

- The range of δ is further checked by plotting the atmospheric mixing angle θ_{23} against δ.
- Finally we have calculated the allowed range of the Majorana Phases for the allowed cases for the viable range of δ .
- We have studied the Neutrinoless double beta decay and Jarlskog Invariant for all the viable cases.

$$|m_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}| \tag{23}$$

$$J_{cp} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \tag{24}$$

Framework of neutrino mass matrix

TABLE I. Neutrino oscillation parameters from global fits [nufit2021]. $\Delta m^2_{3l} = \Delta m^2_{31} > 0$ for normal hierarchy and $\Delta m^2_{3l} = \Delta m^2_{32} < 0$ for inverted hierarchy.

Parameter	Normal Ordering		Inverted Ordering	
	best fit $\pm 1\sigma$	3σ range	best fit $\pm 1\sigma$	3σ range
θ°_{12}	$33.45^{+0.77}_{-0.75}$	$31.27 - 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 - 35.87$
θ°_{23}	$42.1^{+1.1}_{-0.9}$	$39.7 - 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 - 51.6$
θ°_{13}	$8.62^{+0.12}_{-0.12}$	$8.25 - 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 - 9.02$
δ°_{cp}	230^{+36}_{-25}	$144 - 350$	278^{+22}_{-30}	$194 - 345$
$\Delta m^2_{21}/10^{-5}eV^2$	$7.42^{+0.21}_{-0.20}$	$6.82 - 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 - 8.04$
$\lvert\Delta m^2_{3l}\rvert/10^{-3}eV^2$	$2.510^{+0.027}_{-0.027}$	$2.430 - 2.593$	$2.490^{+0.026}_{-0.028}$	$-2.574 - -2.410$

Texture Analysis

Case C_{11}

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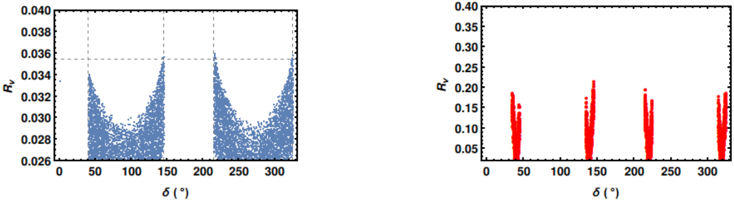
$$A_1 = c_{12}^2 c_{13}^2, \quad A_2 = s_{12}^2 c_{13}^2, \quad A_3 = s_{13}^2 e^{2i\delta} \tag{25}$$

- Now we consider the pair (X_+, Y_-) and plot R_ν for both NH and IH.

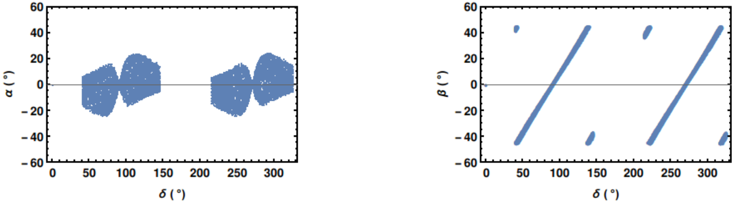


(a) (b)

FIG. 1. R_ν plots (a) for NH and (b) for IH.

Case C_{11}

- Plot for α and β for this solution pair for the allowed ranges of δ.



(a)

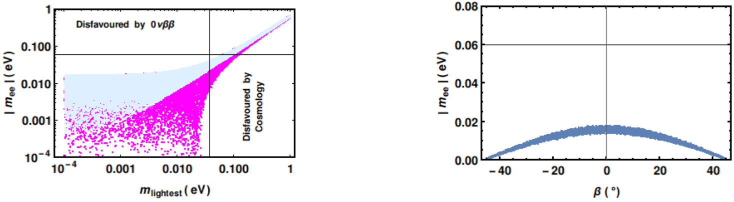
(b)

FIG. 2. α and β plots for NH for the pair (X_+, Y_-) with $\delta = (50^\circ, 150^\circ) \oplus (220^\circ, 320^\circ)$.

Case C_{11}

- Similar procedure is followed for the solution pairs (X_-, Y_+),(X_+, Y_+) and (X_-, Y_-) of the texture but plots show that R_ν acquires values far beyond the experimental range. Hence all these solutions of the texture are ruled out.
- Now to explore further phenomenology of the texture, $|m_{ee}|$ - $m_{lightest}$ and $|m_{ee}|$-β are plotted for neutrinoless double beta decay where the mass of the lightest neutrino is bound within 0.037 eV and 0.042 eV for NH and IH respectively at 95$\%$ confidence.

Case C_{11}



(a)

(b)

FIG. 3. $|m_{ee}|$ plots for (X_+, Y_-) for NH versus $m_{lightest}$ and β.

- From Fig.3 we observe that $|m_{ee}|$ lies within the experimental bounds.

Case C_{11}

- Now we plot J_{cp}-δ for CP violation.

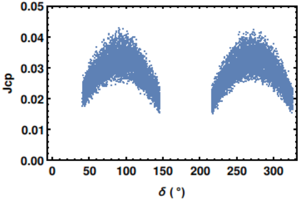


FIG. 4. J_{cp} plot for NH for the solution pair (X_+, Y_-).

- In Fig.4, we find J_{cp} within the range $(0.018 - 0.04)$. Thus case $C_{11} = 0$ is viable under the phenomenological study for normal mass ordering in case of the solution pair (X_+, Y_-) of the texture.

- All the remaining textures $C_{12} = 0$, $C_{13} = 0$, $C_{22} = 0$, $C_{23} = 0$ and $C_{33} = 0$ have been examined following our procedure of analysis.

TABLE II. Viable cases under normal hierarchy, inverted hierarchy and neutrinoless double beta decay.

Case	(X_+, Y_-)		(X_-, Y_+)		$(X_+, Y_+)/(X_-, Y_-)$		Neutrinoless Double Beta Decay
	NH	IH	NH	IH	NH	IH	
C_{11}	✓	x	x	x	x	x	All the viable cases are allowed
C_{12}	✓	x	x	✓	✓	x	
C_{13}	x	x	x	x	✓	x	
C_{22}	✓	x	✓	x	x	x	
C_{23}	x	x	x	x	x	x	
C_{33}	✓	x	x	x	x	x	

TABLE III. Allowed ranges of CP phases δ, α, β, $\lvert m_{ee}\rvert$ and J_{cp} for the viable cases.

Case	(X_+, Y_-)		(X_-, Y_+)		$(X_+, Y_+)/(X_-, Y_-)$	
	NH	IH	NH	IH	NH	IH
C_{11}	$\delta = (50^{\circ}, 150^{\circ}) \oplus (220^{\circ}, 320^{\circ})$	-	-	-	-	-
	$\alpha = (-25^{\circ}, 25^{\circ})$	-	-	-	-	-
	$\beta = (-45^{\circ}, 45^{\circ})$	-	-	-	-	-
	$\lvert m_{ee}\rvert = (0, 0.02)$ eV	-	-	-	-	-
	$J_{cp} = (0.018, 0.04)$	-	-	-	-	-
C_{12}	$\delta = (20^{\circ}, 31^{\circ}) \oplus (45^{\circ}, 55^{\circ})$	-	-	$\delta = (82^{\circ}, 92^{\circ})$	$\delta = (0, 360^{\circ})$	-
	$\oplus(128^{\circ}, 135^{\circ}) \oplus (148^{\circ}, 152^{\circ})$			$\oplus(270^{\circ}, 276^{\circ})$		
	$\oplus(225^{\circ}, 231^{\circ}) \oplus (300^{\circ}, 315^{\circ})$					
	$\oplus(331^{\circ}, 342^{\circ})$					
	$\alpha = (-6^{\circ}, 6^{\circ})$	-	-	$\alpha = (3^{\circ}, 6^{\circ})$	$\alpha = (-10^{\circ}, 10^{\circ})$	-
		-	-	$\oplus(-6^{\circ}, -3^{\circ})$		-
	$\beta = (-45^{\circ}, -20^{\circ}) \oplus (0, 45^{\circ})$	-	-	$\beta = (-45^{\circ}, -35^{\circ})$	$\beta = (-50^{\circ}, 50^{\circ})$	-
		-	-	$\oplus(35^{\circ}, 45^{\circ})$		-
	$\lvert m_{ee}\rvert = (0, 0.02)$ eV	-	-	$\lvert m_{ee}\rvert = (0.03, 0.035)$ eV	$\lvert m_{ee}\rvert = (0, 0.02)$ eV	-
	$J_{cp} = (0.01, 0.04)$	-	-	$J_{cp} = (0.023, 0.04)$	$J_{cp} = (0, 0.04)$	-

Case	(X_+, Y_-)		(X_-, Y_+)		$(X_+, Y_+)/(X_-, Y_-)$	
	NH	IH	NH	IH	NH	IH
C_{13}		-	-	-	$\delta = (55^{\circ}, 130^{\circ})$	-
		-	-	-	$\oplus(230^{\circ}, 310^{\circ})$	-
	-	-	-	-	$\alpha = (-9^{\circ}, -3^{\circ})$	-
	-	-	-	-	$\oplus(3^{\circ}, 9^{\circ})$	-
	-	-	-	-	$\beta = (-50^{\circ}, 50^{\circ})$	-
	-	-	-	-	$\lvert m_{ee}\rvert = (0, 0.02)$ eV	-
	-	-	-	-	$J_{cp} = (0, 0.04)$	-
C_{22}	$\delta = (40^{\circ}, 90^{\circ}) \oplus (230^{\circ}, 279^{\circ})$	-	$\delta = (0, 30^{\circ}) \oplus (196^{\circ}, 210^{\circ})$	-	-	-
	$\oplus(310^{\circ}, 350^{\circ})$		$\oplus(290^{\circ}, 335^{\circ})$	-	-	-
	$\alpha = (-45^{\circ}, -35^{\circ}) \oplus (0, 45^{\circ})$	-	$\alpha = (-45^{\circ}, 45^{\circ})$	-	-	-
	$\beta = (-45^{\circ}, 45^{\circ})$	-	$\beta = (-50^{\circ}, 50^{\circ})$	-	-	-
	$\lvert m_{ee}\rvert = (0, 0.02)$ eV	-	$\lvert m_{ee}\rvert = (0, 0.02)$ eV	-	-	-
	$J_{cp} = (0, 0.04)$	-	$J_{cp} = (0, 0.04)$	-	-	-
C_{33}	$\delta = (90^{\circ}, 265^{\circ})$	-	-	-	-	-
	$\alpha = \beta = (-45^{\circ}, 45^{\circ})$	-	-	-	-	-
	$\lvert m_{ee}\rvert = (0, 0.02)$ eV	-	-	-	-	-
	$J_{cp} = (0, 0.04)$	-	-	-	-	-

Symmetry Realization

- We implement Z_5 Abelian flavor symmetry group to realize all the viable textures. A Z_5 consists of the group elements

$$(1, \omega, \omega^2, \omega^3, \omega^4)$$

where $\omega = e^{\frac{i2\pi}{5}}$ is the generator of the group.

- The Lagrangian can be written as

$$\mathcal{L} = \left(\frac{<\Phi>}{\Lambda}\right)^{Q_{D_{Li}}+Q_{l_{Rj}}} Y_{ij}^{(k)} \overline{D}_{Li}\phi_k l_{Rj} + \left(\frac{<\Phi>}{\Lambda}\right)^{Q_{D_{Li}}+Q_{\nu_{Rj}}} Y_{ij}^{(k)} \overline{D}_{Li}\tilde{\phi}_k \nu_{Rj} + \left(\frac{<\Phi>}{\Lambda}\right)^{Q_{\nu_{Ri}}+Q_{\nu_{Rj}}} Y_{ij}^{(k)} \chi_k \overline{\nu}_{Ri}\nu_{Rj} + h.c. \tag{26}$$

The $Q_\alpha(\alpha = D_{Li}, l_{Rj}, \nu_{Rj})$ are the FN charges for the SM fermion ingredients.

Symmetry Realization

- For all the cases, we assign the FN charges for the Lepton sector as

$$\overline{D}_{1,2,3} : (a+1, a, a), l_{R1,2,3} : (0,1,2), \nu_{R1,2,3} : (d, c, b). \quad (27)$$

Here $D_{Li}, l_{Rj}, \nu_{Ri}, (i, j = 1, 2, 3)$ represents the $SU(2)_L$ doublets, the RH $SU(2)_L$ singlets and the RH neutrino singlets respectively.

Symmetry Realization

Case C_{11}

- M_R and M_D for vanishing minor of the element M_{11}.

$$M_R = \begin{pmatrix} 0 & \xi & \zeta \\ \xi & \eta & \upsilon \\ \zeta & \upsilon & \kappa \end{pmatrix}, \; M_D = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix},$$

$$M_\nu = -M_D M_R^{-1} M_D^T = \frac{1}{\Gamma} \begin{pmatrix} (-\upsilon^2 + \eta\kappa)x^2 & (\zeta - \xi y)xy & (-\zeta\eta + \xi\upsilon)xz \\ (\zeta - \xi y)xy & -\zeta^2 y^2 & \xi\zeta yz \\ (-\zeta\eta + \xi\upsilon)xz & \xi\zeta yz & -\xi^2 z^2 \end{pmatrix}, \tag{28}$$

where $\Gamma = -\zeta\eta^2 + 2\xi\zeta\upsilon - \xi^2\kappa$.

- On implementing Z_5 symmetry, the fields of the relevant particles transform as:

$$\nu_{R1} \to \omega^3 \nu_{R1}, \qquad \overline{D}_{L1} \to \omega^2 \overline{D}_{L1}, \qquad l_{R1} \to \omega^3 l_{R1} \tag{29}$$

$$\nu_{R2} \to \omega^2 \nu_{R2}, \qquad \overline{D}_{L2} \to \omega^3 \overline{D}_{L2}, \qquad l_{R2} \to \omega^2 l_{R2} \tag{30}$$

$$\nu_{R3} \to \nu_{R3}, \qquad \overline{D}_{L3} \to \overline{D}_{L3}, \qquad l_{R3} \to l_{R3} \tag{31}$$

Symmetry Realization

Case C_{11}

- Forming the required bilinears dictated by Z_5 symmetry we obtain

$$\nu_{Ri}^T\nu_{Rj} = \begin{pmatrix} \omega & 1 & \omega^3 \\ 1 & \omega^4 & \omega^2 \\ \omega^3 & \omega^2 & 1 \end{pmatrix}, \quad \overline{D}_{Li}\nu_{Rj} = \begin{pmatrix} 1 & \omega^4 & \omega^2 \\ \omega & 1 & \omega^3 \\ \omega^3 & \omega^2 & 1 \end{pmatrix}, \tag{32}$$

$$\overline{D}_{Li}l_{Rj} = \begin{pmatrix} 1 & \omega^4 & \omega^2 \\ \omega & 1 & \omega^3 \\ \omega^3 & \omega^2 & 1 \end{pmatrix}. \tag{33}$$

- We consider the transformation of the singlet scalars $\chi_k (k = 1, 2, 3)$ which is responsible for the Majorana neutrino mass matrix M_R and SM-like doublet scalar ϕ which is responsible for the Dirac neutrino mass matrix M_D and the lepton mass matrix M_l under Z_5 transformation as

$$\chi_1 \to \omega^2\chi_1, \ \chi_2 \to \omega^3\chi_2, \chi_3 \to \omega\chi_3 \tag{34}$$

$$\phi \to \phi \tag{35}$$

Symmetry Realization

Case C_{11}

- Now the lagrangian dictated by Z_5 is

$$\begin{aligned}\mathcal{L}_M^{Z_5} = {} & \epsilon^{d+c} m_{12}\nu_{R1}^T c^{-1}\nu_{R2} + \epsilon^{d+b} Y^{(1)}_{\chi 13}\chi_1 \nu_{R1}^T c^{-1}\nu_{R3} + \epsilon^{2c} Y^{(3)}_{\chi 22}\chi_3\nu^T_{R2}c^{-1}\nu_{R2} \\ & + \epsilon^{c+b}Y^{(2)}_{\chi 23}\chi_2\nu^T_{R2}c^{-1}\nu_{R3} + \epsilon^{2b} m_{33}\nu^T_{R3}c^{-1}\nu_{R3} + \epsilon^{a+d+1}Y_{D_{11}}\overline{D}_{L1}\tilde{\phi}\nu_{R1} \\ & + \epsilon^{a+c}Y_{D_{22}}\overline{D}_{L2}\tilde{\phi}\nu_{R2} + \epsilon^{a+b}Y_{D_{33}}\overline{D}_{L3}\tilde{\phi}\nu_{R3} + \epsilon^{a+1}Y_{l_{11}}\overline{D}_{L1}\phi l_{R1} \\ & + \epsilon^{a+1}Y_{l_{22}}\overline{D}_{L2}\phi l_{R2} + \epsilon^{a+2}Y_{l_{33}}\overline{D}_{L3}\phi l_{R3}. \end{aligned} \tag{36}$$

- Now we construct the mass matrix M_R, M_D and M_l as

$$\mathrm{M}_R = \begin{pmatrix} 0 & \epsilon^{d+c}m_{12} & y^{(1)}_{\chi 13}\chi_1\epsilon^{d+b} \\ \epsilon^{d+c}m_{12} & y^{(3)}_{\chi 22}\chi_3\epsilon^{2c} & y^{(2)}_{\chi 23}\chi_2\epsilon^{c+b} \\ y^{(1)}_{\chi 13}\chi_1\epsilon^{d+b} & y^{(2)}_{\chi 23}\chi_2\epsilon^{c+b} & \epsilon^{2b}m_{33} \end{pmatrix}, \tag{37}$$

$$\mathrm{M}_D = \begin{pmatrix} y_{D_{11}}\tilde{\phi}\epsilon^{a+d+1} & 0 & 0 \\ 0 & y_{D_{22}}\tilde{\phi}\epsilon^{a+c} & 0 \\ 0 & 0 & y_{D_{33}}\tilde{\phi}\epsilon^{a+b} \end{pmatrix}, M_l = \begin{pmatrix} y_{l_{11}}\phi\epsilon^{a+1} & 0 & 0 \\ 0 & y_{l_{22}}\phi\epsilon^{a+1} & 0 \\ 0 & 0 & y_{l_{33}}\phi\epsilon^{a+2} \end{pmatrix}. \tag{38}$$

Symmetry Realization

Case C_{11}

- We get the effective neutrino mass matrix M_ν using seesaw mechanism $M_\nu = -M_D M_R^{-1} M_D^T$ as

$$\mathsf{M}_\nu = \Omega \begin{pmatrix} \epsilon^2\tilde{\phi}^2 y_{D_{11}}^2 ({y_{\chi_{23}}^{(2)}}^2 \chi_2^2 - m_{33} y_{\chi_{22}}^{(3)} \chi_3) & \epsilon\tilde{\phi}^2 y_{D_{11}} y_{D_{22}} (m_{11} m_{33} - y_{\chi_{13}}^{(1)} y_{\chi_{23}}^{(2)} \chi_1 \chi_2) & -\epsilon\tilde{\phi}^2 y_{D_{11}} y_{D_{33}} (m_{11} y_{\chi_{23}}^{(2)} \chi_2 - y_{\chi_{13}}^{(1)} y_{\chi_{22}}^{(3)} \chi_1 \chi_3) \\ \epsilon\tilde{\phi}^2 y_{D_{11}} y_{D_{22}} (m_{11} m_{33} - y_{\chi_{13}}^{(1)} y_{\chi_{23}}^{(2)} \chi_1 \chi_2) & \tilde{\phi}^2 y_{D_{22}}^2 {y_{\chi_{13}}^{(1)}}^2 \chi_1^2 & -m_{11}\tilde{\phi}^2 y_{D_{22}} y_{D_{33}} Y_{\chi_{13}}^{(1)} \chi_1 \\ -\epsilon\tilde{\phi}^2 y_{D_{11}} y_{D_{33}} (m_{11} y_{\chi_{23}}^{(2)} \chi_2 - y_{\chi_{13}}^{(1)} y_{\chi_{22}}^{(3)} \chi_1 \chi_3) & -\tilde{\phi}^2 y_{D_{22}} y_{D_{33}} Y_{\chi_{13}}^{(1)} \chi_1 m_{11} & \tilde{\phi}^2 y_{D_{33}}^2 m_{11}^2 \end{pmatrix}, \tag{39}$$

where $\Omega = \dfrac{\epsilon^{2a}}{m_{11}^2 m_{33} - y_{\chi_{13}}^{(1)} \chi_1 (2 m_{11} y_{\chi_{23}}^{(2)} \chi_2 - y_{\chi_{13}}^{(1)} y_{\chi_{22}}^{(3)} \chi_1 \chi_3)}$.

Results and Discussion

- The textures $C_{11} = 0$, $C_{13} = 0$, $C_{22} = 0$ and $C_{33} = 0$ have been found viable for normal hierarchy only and the case $C_{12} = 0$ has been found viable for both normal and inverted hierarchies.
- Again the case $C_{23} = 0$ is completely ruled out.
- Interestingly the solution pair (X_+, Y_-) supports all the cases except the case $C_{13} = 0$.
- The solution pairs (X_+, Y_+) and (X_-, Y_-) support the cases $C_{12} = 0$ and $C_{13} = 0$ for normal hierarchy.
- Further the solution pair (X_-, Y_+) supports $C_{12} = 0$ for inverted hierarchy and $C_{22} = 0$ for normal hierarchy.

Results and Discussion

- The Majorana phase α for the textures $C_{12} = 0$ and $C_{13} = 0$ is vanishingly small and the range is highly constrained.
- For all the viable textures, the atmospheric mixing angle θ_{23} lies in the range $(40^{\circ}, 45^{\circ})$. Thus the phenomenology of these textures favors the first quadrant for atmospheric mixing.
- For all the cases both the neutrinoless double beta decay rate, $|m_{ee}|$ and the strength of the Dirac CP violation, J_{CP} remain within the experimental bounds.
- We have found that all the cases favour normal hierarchy of neutrino mass pattern using FN mechanism.

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Thank You