Dispersive determination of neutrino mass ordering

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Presented at FPCP2024, May 28, 2024

2306.03463

Unsolved issues in neutrino physics

- Today's talk will try to answer:
- Neutrino mass ordering

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (7.55^{+0.20}_{-0.16}) \times 10^{-5}\ \text{eV}^2 \qquad \Delta m_{32}^2 \equiv m_3^2 - m_2^2 = (2.424 \pm 0.03) \times 10^{-3}\ \text{eV}^2$$

but normal ordering or inverted ordering?

- Why small mixing in quark sector, but large mixing in lepton sector?

CKM: θ_{12} = 13.04° ±0.05°, θ_{13} = 0.201° ±0.011°, θ_{23} = 2.38° ±0.06°

Pontecorvo–Maki–Nakagawa–Sakata: $\boldsymbol{\theta_{12} = 33.41^{\circ}{}^{+0.75^\circ}_{-0.72^\circ}}$ $\theta_{13} = 8.54^{\circ}{}^{+0.11^\circ}_{-0.12^\circ}$

- Why lepton mixing has maximal angle $\theta_{23} \approx 45^\circ$?

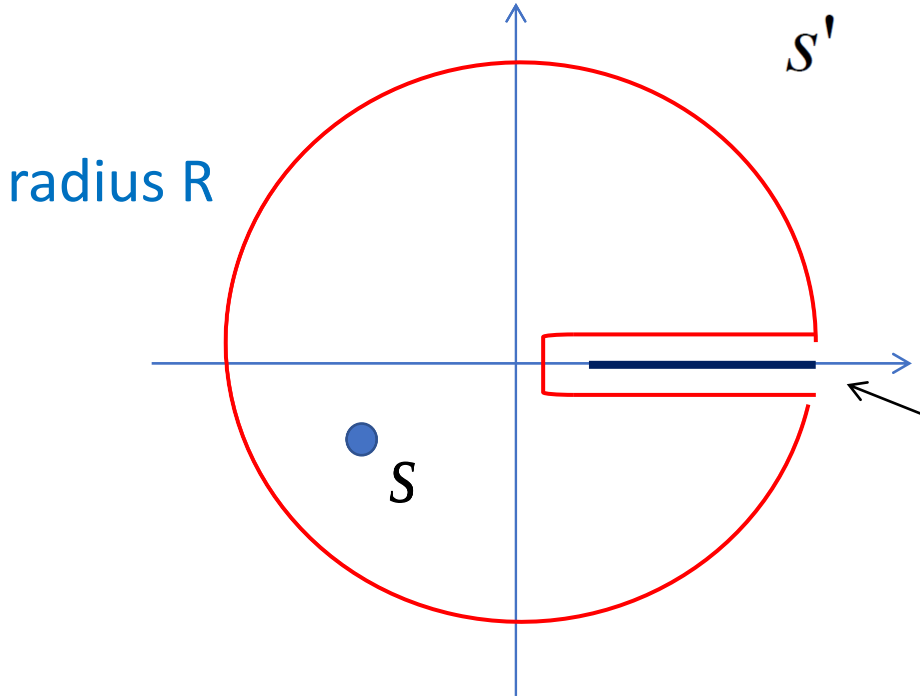
Dispersion relation

- $\mu^- e^+$-$\mu^+ e^-$ mixing amplitude $\Pi(s) \equiv M(s) - i\Gamma(s)/2$

$$M(s) = \frac{1}{2\pi}\int^{R} ds' \frac{\Gamma(s')}{s-s'} + \frac{1}{2\pi i}\int_{C_R} ds' \frac{\Pi(s')}{s'-s}$$

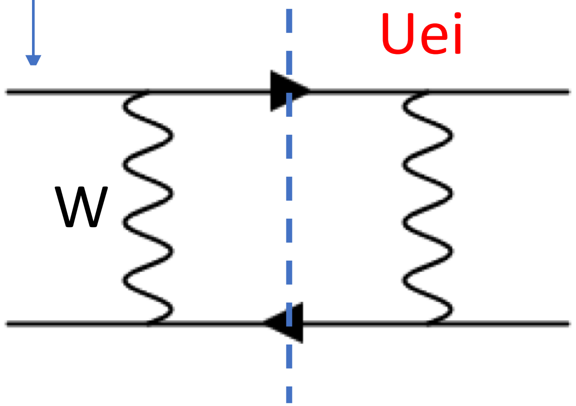
so far, it's identity for physical observable based on analyticity

real part imaginary part



invariant mass squared s

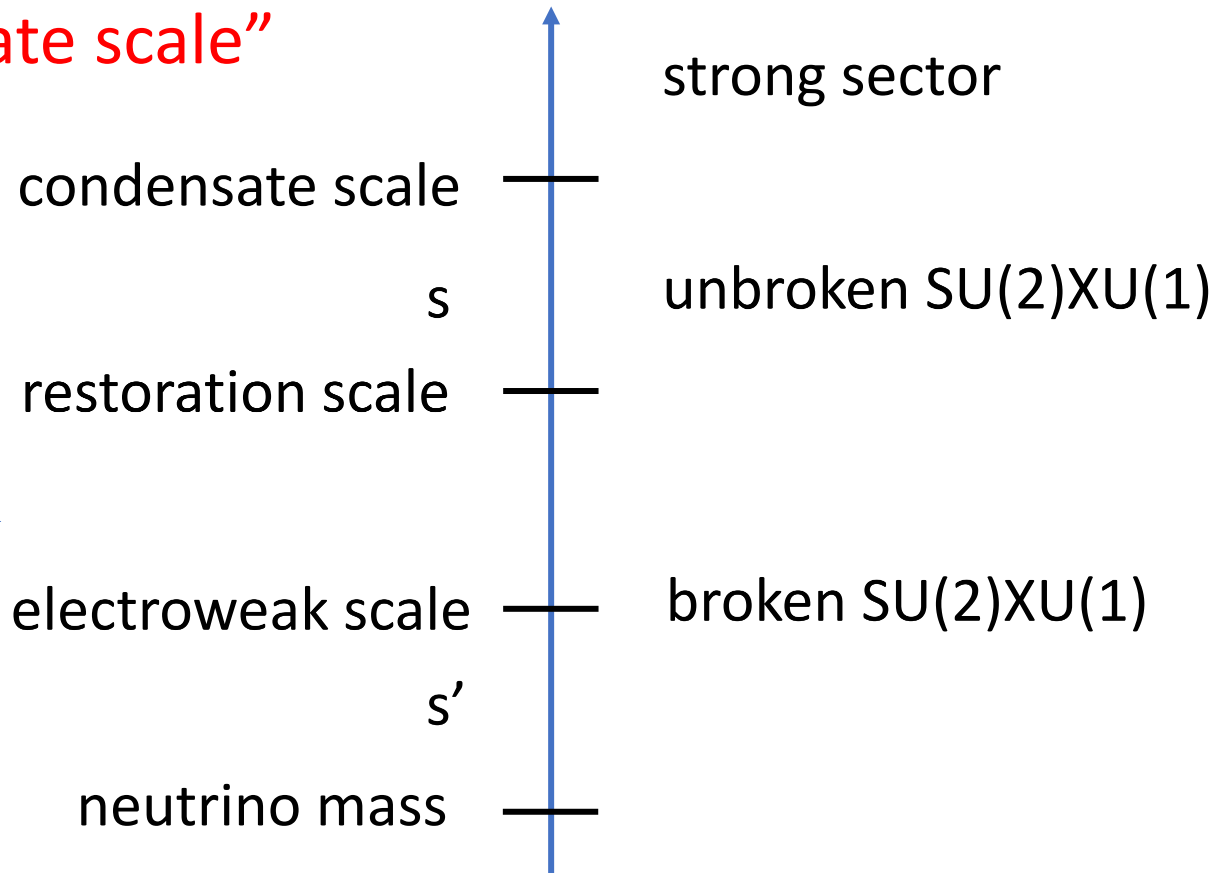
PMNS matrix element



branch cut caused by Intermediate real states

What if EW symmetry restored at high energy?

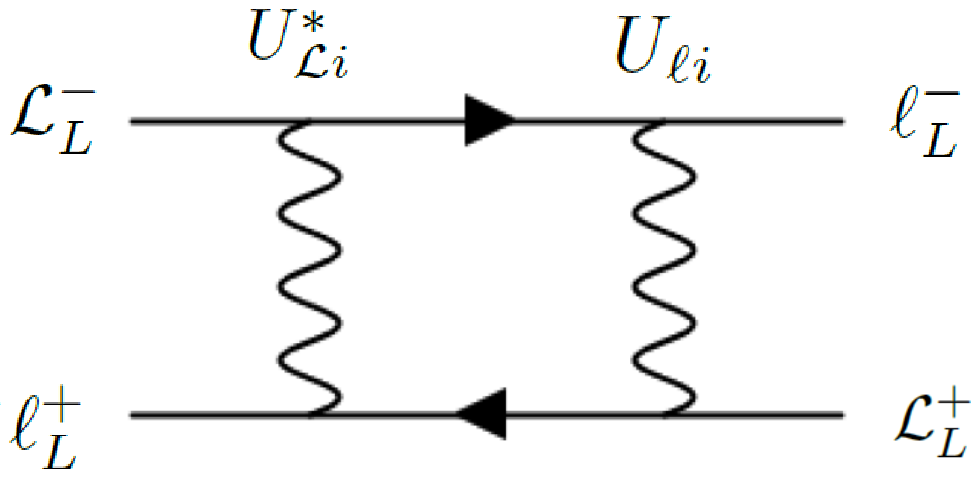
- Composite Higgs model, Kaplan and Georgi, Phys. Lett. B136, 183 (1984):
- “The electroweak group is broken at a scale much smaller than the condensate scale”
- Hyperquark condensates misalign with SU(2)XU(1) vacuum owing to Yukawa couplings



LO mixing in symmetric phase

Li, 2306.03463

- Internal particles massless
- All intermediate channels give same contribution
- Sum over all channels vanishes due to unitarity $\sum_i U^*_{\mathcal{L}i} U_{\ell i} = 0$
- Mixing phenomenon disappears!



restoration scale

$$M(s) = \frac{1}{2\pi}\int^{\Lambda} ds' \frac{\Gamma(s')}{s-s'} \approx 0$$

$s > \Lambda$

EW symmetry broken at low energy;
constrains fermion masses and mixing angles

Box diagram in broken phase

- s' can be low, so $\Gamma(s')$ depends on PMNS matrix elements and intermediate neutrino masses in broken phase.
- Box-diagram contribution

Cheng 1982
Buras et al 1984

$$\Gamma(s) \propto \sum_{i,j=1}^{3} \lambda_i \lambda_j \Gamma_{ij}(s), \qquad \lambda_i = U_{\mathcal{L}i} U_{\ell i}$$

$$\Gamma_{ij}(s) = \frac{1}{s^2} \frac{\sqrt{s^2 - 2s(m_i^2 + m_j^2) + (m_i^2 - m_j^2)^2}}{(m_W^2 - m_i^2)(m_W^2 - m_j^2)} \times \left\{ \left(m_W^4 + \frac{m_i^2 m_j^2}{4} \right) [2s^2 - 4s(m_i^2 + m_j^2) + 2(m_i^2 - m_j^2)^2] + 3m_W^2 s(m_i^2 + m_j^2)(m_i^2 + m_j^2 - s) \right\}$$

Constraints

- How to diminish dispersive integral $\int^{\Lambda} ds' \frac{\Gamma(s')}{s-s'}$?
- Asymptotic expansion

$$\Gamma_{ij}(s') \approx \Gamma_{ij}^{(1)} s' + \Gamma_{ij}^{(0)} + \frac{\Gamma_{ij}^{(-1)}}{s'} + \cdots$$

$$\Gamma_{ij}^{(1)} = \frac{4m_W^4 - 6m_W^2(m_i^2+m_j^2) + 4m_i^2 m_j^2}{2(m_W^2 - m_i^2)(m_W^2 - m_j^2)}, \Rightarrow \Lambda^2/s$$

$$\Gamma_{ij}^{(0)} = -\frac{3(m_i^2+m_j^2)\left[4m_W^4 - 4m_W^2(m_i^2+m_j^2) + m_i^2 m_j^2\right]}{2(m_W^2 - m_i^2)(m_W^2 - m_j^2)} \Rightarrow (m_i^2+m_j^2)\Lambda/s$$

$$\Gamma_{ij}^{(-1)} = \frac{3(m_i^4+m_j^4)\left[4m_W^4 - 2m_W^2(m_i^2+m_j^2) + m_i^2 m_j^2\right]}{2(m_W^2 - m_i^2)(m_W^2 - m_j^2)}. \Rightarrow (m_i^4+m_j^4)\ln\Lambda/s$$

to have finite integral

$$\sum_{i,j} \lambda_i \lambda_j \Gamma_{ij}^{(m)} \approx 0, \quad m = 1, 0, -1$$

$$\int ds' \frac{\Gamma_{12}(s')}{s-s'} \approx \frac{1}{s}\sum_{i,j}\lambda_i\lambda_j g_{ij} \qquad g_{ij} \equiv \int_{t_{ij}}^{\infty} ds' \left[\Gamma_{ij}(s') - \Gamma_{ij}^{(1)} s' - \Gamma_{ij}^{(0)} - \frac{\Gamma_{ij}^{(-1)}}{s'}\right]$$

to diminish integral

$$\sum_{i,j} \lambda_i \lambda_j g_{ij} \approx 0$$

These four conditions constrain neutrino masses and mixing angles!

Test quark mixing first---constrain quark masses and CKM matrix elements

for D mixing $\lambda_i \equiv V_{ci}^* V_{ui} \qquad i, j = d, s, b$

Minimization

- Use unitarity to eliminate λ_b and to rewrite constraints

$$r^2 R_{dd}^{(m)} + 2r R_{ds}^{(m)} + 1 \approx 0, , \quad m = 1, 0, -1, i$$

← refer to finite integral g_{ij}

$$R_{dd}^{(m)} = \frac{\Gamma_{dd}^{(m)} - 2\Gamma_{db}^{(m)} + \Gamma_{bb}^{(m)}}{\Gamma_{ss}^{(m)} - 2\Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}}, \quad R_{ds}^{(m)} = \frac{\Gamma_{ds}^{(m)} - \Gamma_{db}^{(m)} - \Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}}{\Gamma_{ss}^{(m)} - 2\Gamma_{sb}^{(m)} + \Gamma_{bb}^{(m)}} \quad m = 1, 0, -1$$

- Expression for $m = i$ similar, but with g_{ij}
- Ratio of CKM elements

$$r = \frac{\lambda_d}{\lambda_s} = \frac{V_{cd}^* V_{ud}}{V_{cs}^* V_{us}} \equiv u + iv,$$

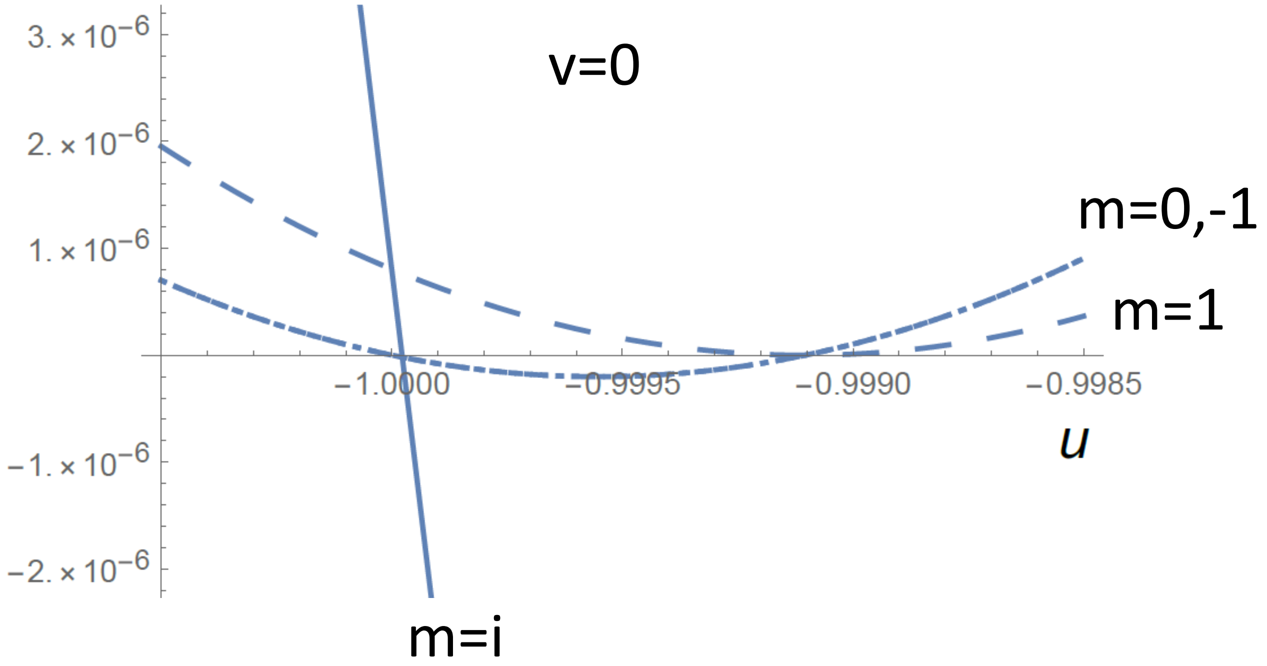
- Tune u and v to minimize the sum (real parts of constraints)

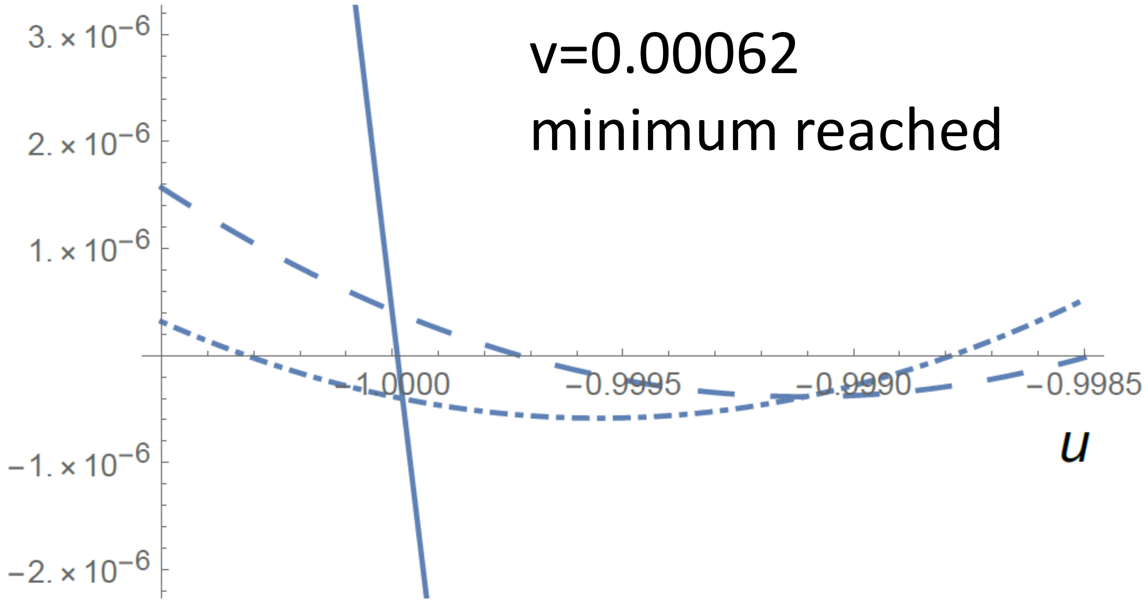
$$\sum_{m=1,-1,i} \left[(u^2 - v^2) R_{dd}^{(m)} + 2u R_{ds}^{(m)} + 1 \right]^2$$

then imaginary parts also small

Results

$m_d = 0.005\ \mathrm{GeV} \quad m_s = 0.12\ \mathrm{GeV} \quad m_b = 4.0\ \mathrm{GeV} \quad m_W = 80.377\ \mathrm{GeV}$





$$r = \frac{V_{cd}^* V_{ud}}{V_{cs}^* V_{us}} = -1.0 + (6.2^{+1.2}_{-1.0}) \times 10^{-4} i$$

variation of ms by 0.01 GeV

PDG

$$u = -1.00029 \pm 0.00002, \quad v = 0.00064 \pm 0.00002$$

they agree well; CP phase must exist

Global fits

experimental discrimination of NO, IO difficult

	Ref. [188] w/o SK-ATM		Ref. [188] w SK-ATM		Ref. [189] w SK-ATM		Ref. [190] w SK-ATM	
NO	Best Fit Ordering		Best Fit Ordering		Best Fit Ordering		Best Fit Ordering	
Param	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.04^{+0.14}_{-0.13}$	$2.65 \to 3.46$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.46^{+0.87}_{-0.88}$	$30.98 \to 36.03$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.58^{+0.20}_{-0.33}$	$4.27 \to 6.09$	$5.63^{+0.18}_{-0.24}$	$4.33 \to 6.09$	$5.51^{+0.19}_{-0.80}$	$4.30 \to 6.02$	$5.47^{+0.20}_{-0.30}$	$4.45 \to 5.99$
$\theta_{23}/^\circ$	$48.3^{+1.2}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$47.9^{+1.1}_{-4.0}$	$41.0 \to 50.9$	$47.7^{+1.2}_{-1.7}$	$41.8 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.241^{+0.066}_{-0.065}$	$2.046 \to 2.440$	$2.237^{+0.066}_{-0.065}$	$2.044 \to 2.435$	$2.14^{+0.09}_{-0.07}$	$1.90 \to 2.39$	$2.160^{+0.083}_{-0.069}$	$1.96 \to 2.41$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.41^{+0.18}_{-0.14}$	$7.9 \to 8.9$	$8.45^{+0.16}_{-0.14}$	$8.0 \to 8.9$
$\delta_{CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	221^{+39}_{-28}	$144 \to 357$	238^{+41}_{-33}	$149 \to 358$	218^{+38}_{-27}	$157 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$	$2.449^{+0.032}_{-0.030}$	$2.358 \to 2.544$	$2.454^{+0.029}_{-0.031}$	$2.362 \to 2.544$	$2.419^{+0.035}_{-0.032}$	$2.319 \to 2.521$	2.424 ± 0.03	$2.334 \to 2.524$
IO	$\Delta\chi^2 = 6.2$		$\Delta\chi^2 = 10.4$		$\Delta\chi^2 = 9.5$		$\Delta\chi^2 = 11.7$	
$\frac{\sin^2\theta_{12}}{10^{-1}}$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.10^{+0.13}_{-0.12}$	$2.75 \to 3.50$	$3.03^{+0.14}_{-0.13}$	$2.64 \to 3.45$	$3.20^{+0.20}_{-0.16}$	$2.73 \to 3.79$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$	$33.40^{+0.87}_{-0.81}$	$30.92 \to 35.97$	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
$\frac{\sin^2\theta_{23}}{10^{-1}}$	$5.63^{+0.19}_{-0.26}$	$4.30 \to 6.12$	$5.65^{+0.17}_{-0.22}$	$4.36 \to 6.10$	$5.57^{+0.17}_{-0.24}$	$4.44 \to 6.03$	$5.51^{+0.18}_{-0.30}$	$4.53 \to 5.98$
$\theta_{23}/^\circ$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$	$48.2^{+1.0}_{-1.4}$	$41.8 \to 50.9$	$47.9^{+1.0}_{-1.7}$	$42.3 \to 50.7$
$\frac{\sin^2\theta_{13}}{10^{-2}}$	$2.261^{+0.067}_{-0.064}$	$2.066 \to 2.461$	$2.259^{+0.065}_{-0.065}$	$2.064 \to 2.457$	$2.18^{+0.08}_{-0.07}$	$1.95 \to 2.43$	$2.220^{+0.074}_{-0.076}$	$1.99 \to 2.44$
$\theta_{13}/^\circ$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$	$8.49^{+0.15}_{-0.14}$	$8.0 \to 9.0$	$8.53^{+0.14}_{-0.15}$	$8.1 \to 9.0$
$\delta_{CP}/^\circ$	285^{+24}_{-26}	$205 \to 354$	282^{+23}_{-25}	$205 \to 348$	247^{+26}_{-27}	$193 \to 346$	281^{+23}_{-27}	$202 \to 349$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.34^{+0.17}_{-0.14}$	$6.92 \to 7.91$	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.24$
$\frac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$	$-2.509^{+0.032}_{-0.032}$	$-2.603 \to -2.416$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$	$-2.478^{+0.035}_{-0.033}$	$-2.577 \to -2.375$	$-2.50\pm^{+0.04}_{-0.03}$	$-2.59 \to -2.39$

Neutrino mass orderings

- Apply to lepton $\mu^- e^+$-$\mu^+ e^-$ mixing with intermediate neutrino channels
- Normal ordering (NO) $m_1^2 = 10^{-6}\ \mathrm{eV}^2$ (as long as it is small enough)

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (7.55^{+0.20}_{-0.16}) \times 10^{-5}\ \mathrm{eV}^2 \qquad \Delta m_{32}^2 \equiv m_3^2 - m_2^2 = (2.424 \pm 0.03) \times 10^{-3}\ \mathrm{eV}^2$$

de Salas et al, 2018

- Predict

$$r = \frac{U_{\mu 1}^* U_{e1}}{U_{\mu 2}^* U_{e2}} \approx \underline{-1.0} - 0.02i$$

global fit

$$r = -\underline{(0.738^{+0.050}_{-0.048})} - (0.179^{+0.136}_{-0.125})i$$

- Be reminded that it is LO analysis
- Inverted ordering (IO) $r \approx -1.0 - O(10^{-5})i$ $\qquad r = -(1.03^{+0.05}_{-0.16}) - (0.356^{+0.015}_{-0.048})i$

dramatically different

- NO and observed PMNS matrix satisfy constraint at order of magnitude

Mixing patterns

- Insert u=-1 into m=1 constraint to get analytical expression of v

$$v \approx \frac{(m_W^2 - m_b^2)(m_s^2 - m_d^2)}{(m_W^2 - m_s^2)(m_b^2 - m_d^2)} \approx \frac{m_s^2}{m_b^2}$$

- In terms of Wolfenstein parameters $v = A^2\lambda^4\eta$ Ahn et al, 2011
- Produce well-known empirical relation (Cheng, Sher 1987)

$$\lambda = V_{us} \approx (A^2\eta)^{-1/4}\sqrt{\frac{m_s}{m_b}} \approx \sqrt{\frac{m_s}{m_b}}$$

$$A \approx 0.826 \qquad \eta \approx 0.348$$

$$(A^2\eta)^{-1/4} \approx 1.43 \sim O(1)$$

Belfatto et al, 2023

- Chau-Keung parametrization $V_{us} \approx s_{12}$
- Larger mixing angles in lepton sector due to $\frac{m_2^2}{m_3^2} \approx 3.1 \times 10^{-2} \gg \frac{m_s^2}{m_b^2} \approx 9.0 \times 10^{-4}$
- Indeed, $\sqrt{m_s/m_b}/\sqrt{m_2/m_3} \approx s_{12}^{CKM}/s_{12}^{PNMS} \approx 0.42$

Mixing of generations 1-3

- Heavy lepton could be μ or τ , same intermediate neutrinos
- $\tau^- e^+$-$\tau^+ e^-$ and $\mu^- e^+$-$\mu^+ e^-$ satisfy same constraints?
- Magnitude of PMNS matrix elements

$$|U| = \begin{bmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{bmatrix} = \begin{bmatrix} 0.803 \sim 0.845 & 0.514 \sim 0.578 & 0.142 \sim 0.155 \\ 0.233 \sim 0.505 & 0.460 \sim 0.693 & 0.630 \sim 0.779 \\ 0.262 \sim 0.525 & 0.473 \sim 0.702 & 0.610 \sim 0.762 \end{bmatrix}$$

NuFIT, 2023

These two rows are indeed similar

Maximal mixing angle θ_{23}

- Recall v has two solutions with opposite signs, so one for $\mu^- e^+$-$\mu^+ e^-$ another for $\tau^- e^+$-$\tau^+ e^-$?
- Check data

	$U^*_{\tau 1}U_{e1}/(U^*_{\tau 2}U_{e2})$	$r = U^*_{\mu 1}U_{e1}/(U^*_{\mu 2}U_{e2})$	
	$-(1.231^{+0.078}_{-0.186}) + (0.204^{+0.085}_{-0.138})i$	$r = -(0.738^{+0.050}_{-0.048}) - (0.179^{+0.136}_{-0.125})i$	de Salas et al, 2018
	$-(1.139^{+0.139}_{-0.207}) + (0.266^{+0.050}_{-0.124})i$	$r = -(0.801^{+0.219}_{-0.097}) - (0.265^{+0.090}_{-0.145})i$	Capozzi et al, 2018

- Implication: $\theta_{23} \approx 45^\circ$

roughly equal

$$\frac{U^*_{\mu 1}U_{e1}}{U^*_{\mu 2}U_{e2}} = -\frac{c_{12}}{s_{12}}\frac{c_{12}s_{12}(c^2_{23} - s^2_{13}s^2_{23}) + c_{23}s_{13}s_{23}c_\delta(c^2_{12} - s^2_{12}) - c_{23}s_{13}s_{23}s_\delta i}{(c_{12}c_{23} - s_{12}s_{13}s_{23})^2 + 2c_{12}c_{23}s_{12}s_{13}s_{23}(1 - c_\delta)}$$

roughly equal

$$\frac{U^*_{\tau 1}U_{e1}}{U^*_{\tau 2}U_{e2}} = -\frac{c_{12}}{s_{12}}\frac{c_{12}s_{12}(s^2_{23} - c^2_{23}s^2_{13}) - c_{23}s_{13}s_{23}c_\delta(c^2_{12} - s^2_{12}) + c_{23}s_{13}s_{23}s_\delta i}{(c_{12}s_{23} + c_{23}s_{12}s_{13})^2 - 2c_{12}c_{23}s_{12}s_{13}s_{23}(1 - c_\delta)}$$

$$(c^2_{12} + s^2_{12}s^2_{13})(c^2_{23} - s^2_{23}) \approx 0$$

Summary

- Neutrino mass ordering? --- normal ordering
- NO and IO discriminated by CP phases (~180 vs 270 degrees; the former favored by dispersive constraint)
- Small (large) mixing in quark (lepton) sector? --- different mass ratios ~ s_{12}

$$m_2^2/m_3^2 \gg m_s^2/m_b^2$$

- Lepton has maximal angle θ_{23}? --- mixings of generations 1-2 and 1-3 obey same dispersive constraints
- Only assumption: EW symmetry restored at high energy --- direction to model building of new physics?