

Role of the Right-Handed Neutrino in $B_c^+ \rightarrow B_s \mu^+ \nu_\mu$ Decay

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Introduction

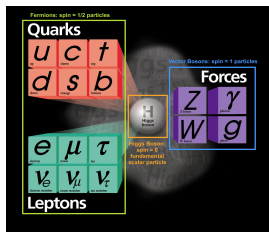
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- The SM is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.



Hints for New Physics

- Neutrino masses
- Matter Anti-matter Asymmetry
- Lepton Flavor Universality Violation
- Dark Matter

- Right-Handed Neutrinos

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- Right-Handed Neutrinos
- The inclusion of Right-Handed Neutrinos can play an important role in the search of new physics [[JHEP 08 \(2020\) 022](#); [JHEP10\(2021\)122](#)].
- Our goal is to find the impact of RHN on exclusive semileptonic $B_c^+ \rightarrow B_s \mu^+ \nu$ decay induced at quark level by $c \rightarrow s \mu^+ \nu$.
- The extension of the low-energy effective Hamiltonian with the inclusion of the dimension-six operator gives a platform to analyze the effects of New Physics.

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Effective Hamiltonian

The low energy dimension-six effective Hamiltonian is

$$H_{\text{eff}} = \frac{4G_F V_{cs}}{\sqrt{2}} \left[O_{LL}^V + \sum_{\substack{i=S,V,T \\ A,B=L,R}} C_{AB}^i O_{AB}^i \right] \quad (1)$$

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with ten fermion operators

- $O_{AB}^V = (\bar{s}\gamma_\mu P_{AC})(\bar{\nu}_\mu\gamma_\mu P_{B\mu})$
- $O_{AB}^S = (\bar{s}P_{AC})(\bar{\nu}_\mu P_{B\mu})$
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The Wilson Coefficients of New Physics are constrained by using the available experimental measurements of $D_{(s)}^{0(+)}$ mesons [[Physical Review D, 103\(7\):075019, 2021](#)].

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Experimental Constraint

- To constrain the NP effect, we considered the six leptonic and semi-leptonic charm transitions.

Mode	SM	Experiment
$D_S^+ \rightarrow \mu^+ \nu_\mu$	$(5.28 \pm 0.08) \times 10^{-3}$	$(5.50 \pm 0.23) \times 10^{-3}$
$D^0 \rightarrow K^- \mu^+ \nu_\mu$	$(3.40 \pm 0.22) \times 10^{-2}$	$(3.41 \pm 0.04) \times 10^{-2}$
$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	$(8.70 \pm 0.73) \times 10^{-2}$	$(8.76 \pm 0.19) \times 10^{-2}$
$D^0 \rightarrow K^{*-} \mu^+ \nu_\mu$	$(1.81 \pm 0.16) \times 10^{-2}$	$(1.89 \pm 0.24) \times 10^{-2}$
$D^+ \rightarrow \bar{K}^{*0} \mu^+ \nu_\mu$	$(4.75 \pm 0.42) \times 10^{-2}$	$(5.27 \pm 0.15) \times 10^{-2}$
$D_S^+ \rightarrow \phi \mu^+ \nu_\mu$	$(2.33 \pm 0.40) \times 10^{-2}$	$(1.90 \pm 0.50) \times 10^{-2}$

Table: Branching ratios of leptonic and semi-leptonic decays calculated in the SM [PRD, 96(5):054514, 2017; JMPA, 21(30):6125–6172,2006] and comparison with the currently available experimental values [PDG, 2020(8):083C01, 2020]

χ^2 Minimization

- For best fit values of Wilson coefficients the χ^2 minimization [JHEP 09 (2018) 152] methodology is used:

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$$\chi^2(C_i^{eff}) = \sum_n \frac{[\mathcal{O}_m^{th}(C_i^{eff}) - \mathcal{O}_m^{exp}]^2}{\sigma_{\mathcal{O}_m^{exp}}^2} \quad (2)$$

Here $\mathcal{O}_m^{th}(C_i^{eff})$ are the theoretical prediction value which depend upon the NP WCs C_i^{eff} . \mathcal{O}_m^{exp} are the experimental measurement and $\sigma_{\mathcal{O}_m^{exp}}$ are the corresponding uncertainty.

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- To obtain the most likely value of WCs the minimization is done by MINUIT library [Comput. Phys. Commun. 10, 343 (1975)].

Wilson Coefficient Values

- The best fit values are:

Coefficient (s)	Value (s)	χ^2
C_{LL}^V	$(-3.95 \pm 4.69) \times 10^{-3}$	10.81
C_{RL}^V	$(-1.26 \pm 0.48) \times 10^{-2}$	04.44
C_{LR}^V	0.0 ± 0.07	11.53
C_{RR}^V	0.0 ± 0.07	11.53
C_{LL}^S	$(-0.53 \pm 0.77) \times 10^{-3}$	11.07
C_{RL}^S	$(-7.42 \pm 0.07) \times 10^{-2}$	11.53
C_{LR}^S	$(5.90 \pm 4.93) \times 10^{-3}$	11.18
C_{RR}^S	$(5.90 \pm 4.93) \times 10^{-3}$	11.18
C_{LL}^T	$(-3.66 \pm 0.97) \times 10^{-2}$	01.76
C_{RR}^T	$(6.35 \pm 1.41) \times 10^{-2}$	06.53

Table: Best fit values of Wilson coefficients. For the SM $\chi_{SM}^2 = 11.534$.

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Differential Decay Width

The decay width in terms of angular dependence is given by

$$\frac{d\Gamma(B_c^+ \rightarrow B_s \mu^+ \nu_\mu)}{dq^2 d\cos\theta_\mu} = \frac{G_F^2 V_{cs}^2}{256 m_{B_c}^3 \pi^3} q^2 \lambda_{B_s}^{1/2}(q^2) \left(1 - \frac{m_\mu^2}{q^2}\right)^2 \times \{J_0(q^2) + J_1(q^2) \cos\theta_\mu + J_2(q^2) \cos^2\theta_\mu\} \quad (3)$$

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where, $q^2 = (p_{\mu^+} + p_{\nu})$ the square momentum transferred to the muon pair, θ_μ is the polar angle of the muon momentum in the rest frame of the $\mu^+ \nu$ pair, w.r.to the z-axis defined by the B_c meson momentum. The differential angular coefficients are given as

$$J_0(q^2) = \left| \mathcal{W}_0^L - \frac{2m_\mu}{\sqrt{q^2}} \mathcal{W}_T^L \right|^2 + \frac{m_\mu^2}{q^2} \left| \mathcal{W}_t^L + \frac{\sqrt{q^2}}{m_\mu} \mathcal{W}_S^L \right|^2 + (L \leftrightarrow R),$$

$$J_1(q^2) = \frac{2m_\mu^2}{q^2} \text{Re} \left[\left(\mathcal{W}_0^L - \frac{2\sqrt{q^2}}{m_\mu} \mathcal{W}_T^L \right) \left(\mathcal{W}_t^{L*} + \frac{\sqrt{q^2}}{m_\mu} \mathcal{W}_S^{L*} \right) \right] + (L \leftrightarrow R), \quad (4)$$

$$J_2(q^2) = - \left(1 - \frac{m_\mu^2}{q^2} \right) \left(|\mathcal{W}_0^L|^2 - 4 |\mathcal{W}_T^L|^2 \right) + (L \leftrightarrow R),$$

Hadronic Amplitudes

$$\begin{aligned}
 \mathcal{W}_0^L &= (1 + C_{LL}^V + C_{RL}^V) H_{V,0}^S, & \mathcal{W}_0^R &= (C_{LR}^V + C_{RR}^V) H_{V,0}^S, \\
 \mathcal{W}_t^L &= (1 + C_{LL}^V + C_{RL}^V) H_{V,t}^S, & \mathcal{W}_t^R &= (C_{LR}^V + C_{RR}^V) H_{V,t}^S, \\
 \mathcal{W}_S^L &= (C_{RL}^S + C_{LL}^S) H_S^S, & \mathcal{W}_S^R &= (C_{RR}^S + C_{LR}^S) H_S^S, \\
 \mathcal{W}_T^L &= 2C_{LL}^T H_T^S, & \mathcal{W}_T^R &= 2C_{RR}^T H_T^S.
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 \mathcal{W}_T^L &= 2C_{LL}^T H_T^S, & \mathcal{W}_T^R &= 2C_{RR}^T H_T^S.
 \end{aligned} \tag{5}$$

The hadronic amplitudes in terms of F_0 , F_+ and F_T form factors are:

$$\begin{aligned}
 H_{V,0}^S(q^2) &\equiv H_{V_L,0}^S(q^2) = H_{V_R,0}^S(q^2) = \sqrt{\frac{\lambda_{B_s}(q^2)}{q^2}} F_+(q^2), \\
 H_{V,t}^S(q^2) &\equiv H_{V_L,t}^S(q^2) = H_{V_R,t}^S(q^2) = \frac{m_{B_c}^2 - m_{B_s}^2}{\sqrt{q^2}} F_0(q^2), \\
 H_S^S(q^2) &\equiv H_{S_L}^S(q^2) = H_{S_R}^S(q^2) \simeq \frac{m_{B_c}^2 - m_{B_s}^2}{m_c - m_s} F_0(q^2), \\
 H_T^S(q^2) &= H_{T_{L+}}^S = H_{T_{L0t}}^S = -H_{T_{R+}}^S = H_{T_{R0t}}^S = -\frac{\sqrt{\lambda_{B_s}(q^2)}}{m_{B_c} + m_{B_s}} F_T(q^2),
 \end{aligned} \tag{6}$$

Differential Decay Width

$$\begin{aligned}
 \frac{d\Gamma}{dq^2}(B_c^+ \rightarrow B_s \mu^+ \nu) = & \frac{G_F^2 V_{cs}^2}{192 m_{B_c}^3 \pi^3} q^2 \lambda_{B_s}^{1/2}(q^2) \left(1 - \frac{m_\mu^2}{q^2}\right)^2 \\
 & \times \left\{ \left(\left|1 + C_{LL}^V + C_{RL}^V\right|^2 + \left|C_{LR}^V + C_{RR}^V\right|^2 \right) \right. \\
 & \left[\left(H_{V,0}^S\right)^2 \left(\frac{m_\mu^2}{2q^2} + 1\right) + \frac{3m_\mu^2}{2q^2} \left(H_{V,t}^S\right)^2 \right] \\
 & + \frac{3}{2} \left(H_S^S\right)^2 \left(\left|C_{RL}^S + C_{LL}^S\right|^2 + \left|C_{RR}^S + C_{LR}^S\right|^2 \right) \\
 & + 8 \left(\left|C_{LL}^T\right|^2 + \left|C_{RR}^T\right|^2 \right) \left(H_T^S\right)^2 \left(1 + \frac{2m_\mu^2}{q^2}\right) \\
 & + 3\text{Re} \left[\left(1 + C_{LL}^V + C_{RL}^V\right) \left(C_{RL}^S + C_{LL}^S\right)^* \right. \\
 & \left. + \left(C_{LR}^V + C_{RR}^V\right) \left(C_{RR}^S + C_{LR}^S\right)^* \right] \frac{m_\mu}{\sqrt{q^2}} H_S^S H_{V,t}^S \\
 & \left. - 12\text{Re} \left[\left(1 + C_{LLL}^V + C_{RL}^V\right) C_{LLL}^{T*} + \left(C_{RR}^V + C_{LR}^V\right) C_{RR}^{T*} \right] \frac{m_\mu}{\sqrt{q^2}} H_T^S H_{V,0}^S \right\}.
 \end{aligned} \tag{7}$$

Form Factor

- The computation of the form factors is evaluated over the full range of q^2 ; $0 < q^2 < (M_{B_c^+} - M_{B_s})^2$ by chain fit of the results from HISQ method and NQCD [[Physical Review D, 102\(1\):014513, 2020](#)]:

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$$f(q^2) = P(q^2) \sum_n^N A_n Z_\rho(q^2)^n \quad (8)$$

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$$f(q^2) = P(q^2) \sum_n^N A_n z_p(q^2)^n \quad (8)$$

where, $z_p(q^2) = \frac{z(q^2)}{|z(M_p^2)|}$,

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_-}}{\sqrt{t_+ - q^2} + \sqrt{t_-}} \text{ with } t_+ = (m_{B_c} + m_{B_s})^2$$

$$P(q^2) = (1 - q^2/M_{\text{res}}^2)^{-1} \text{ and } A_n \text{ is the covariance matrix.}$$

Scenarios

Spin	Q.N.	Nature	ν_L	ν_R
0	$S_1 \sim (\bar{3}, 1, 1/3)$	LQ	$C_{LL}^V, C_{LL}^S, C_{LL}^T$	$C_{RR}^V, C_{RR}^S, C_{RR}^T$
0	$\Phi \sim (1, 2, 1/2)$	SB	C_{LL}^S, C_{RL}^S	C_{LR}^S, C_{RR}^S
0	$R_2 \sim (3, 2, 1/6)$	LQ	-	C_{RR}^S, C_{RR}^T
1	$U_1^\mu \sim (3, 1, 2/3)$	LQ	C_{LL}^V, C_{RL}^S	C_{RR}^V, C_{LR}^S
1	$V_2^\mu \sim (\bar{3}, 2, -1/6)$	LQ	-	C_{LR}^S
1	$V^\mu \sim (1, 1, -1)$	VB	-	C_{RR}^V

Table: Spin, $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers and nature (LQ = leptoquark, SB = scalar boson, VB = vector boson) of the possible candidates to mediate $c \rightarrow s$ transitions [[JHEP 08 \(2020\) 022](#)].

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New Physics

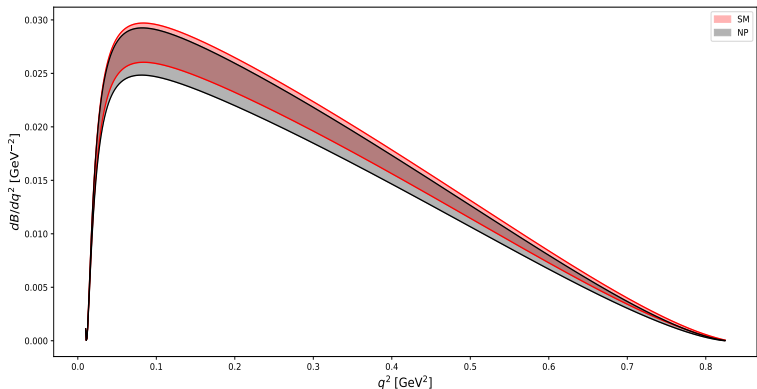


Figure: q^2 spectrum for all new physics contribution both the SM and NP includes the uncertainty of the form factors.

RHN scenario

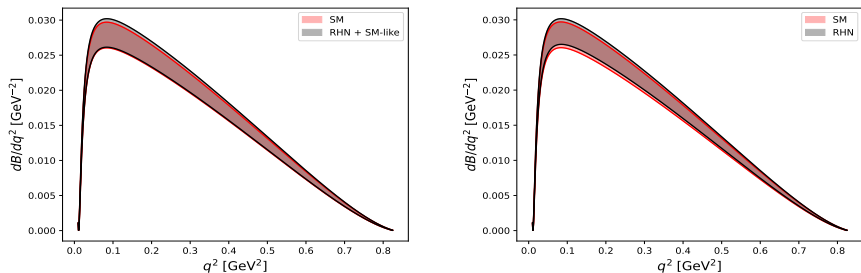


Figure: q^2 spectrum for the RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$ (left side), and RHN: $O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$ (right side).

Vector Boson (1,1,-1) & Leptoquark ($\bar{3},2,-1/6$) scenario

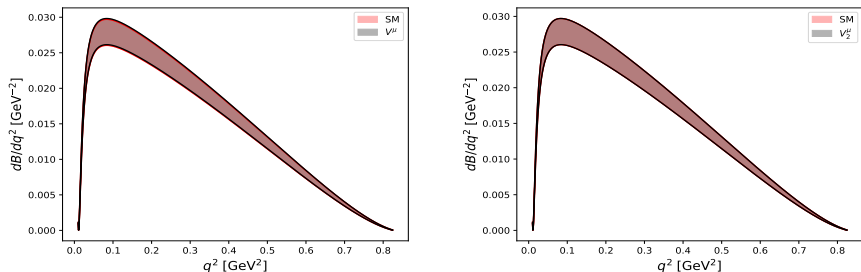


Figure: q^2 spectrum for the Vector Boson (V^μ): O_{RR}^V (left), and Leptoquark (V_2^μ): O_{LR}^S (right).

Scalar Boson (1,2,1/2) scenario

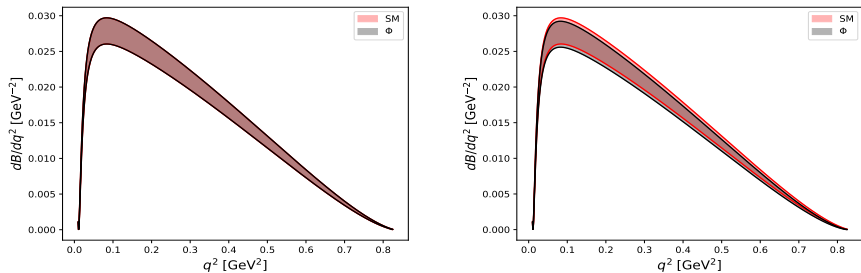


Figure: q^2 spectrum for Φ (1,2,1/2): O_{LR}^S, O_{RR}^S (left) included only right-handed operators; O_{LL}^S, O_{RL}^S and O_{RR}^S, O_{LR}^S (right) included both left and right-handed operators.

Leptoquark (3,1,2/3) scenario

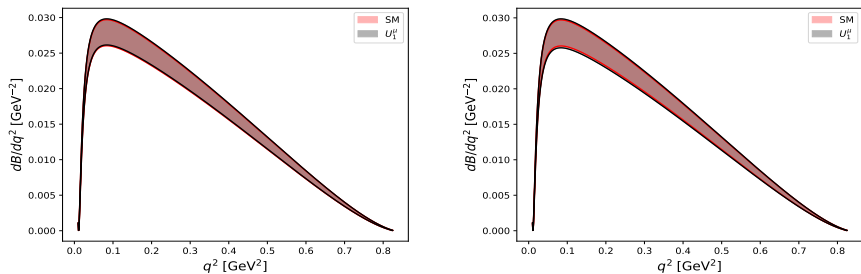


Figure: q^2 spectrum for U_1^μ (3,1,2/3): O_{RR}^V, O_{LR}^S (left) with right-handed operators only; O_{LL}^V, O_{RL}^S and O_{RR}^V, O_{LR}^S (right) with both left and right-handed operators.

Leptoquark (3,2,1/6) scenario

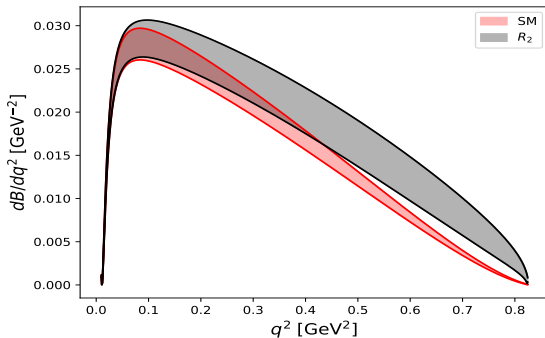


Figure: q^2 spectrum for R_2 (3,2,1/6): O_{RR}^S, O_{RR}^T with $C_{RR}^S = 4rC_{RR}^T$ where $r = 2$ at b-quark scale.

Leptoquark ($\bar{3}, 1, 1/3$) scenario

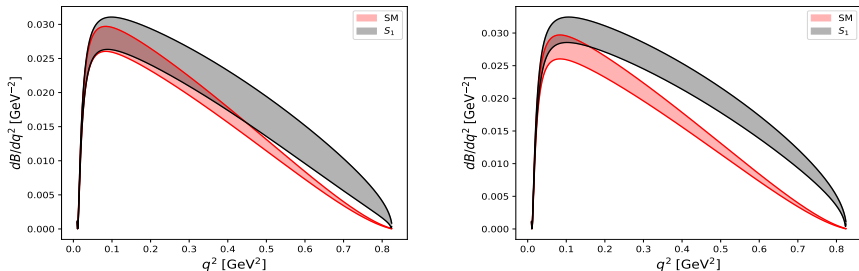


Figure: q^2 spectrum for S_1 ($\bar{3}, 1, 1/3$): $O_{RR}^V, O_{RR}^S, O_{RR}^T$ with $C_{RR}^S = -4rC_{RR}^T$ (left) for right-handed operators only; $O_{LL}^V, O_{LL}^S, O_{LL}^T$ and $O_{RR}^V, O_{RR}^S, O_{RR}^T$ with $C_{LL}^S = -4rC_{LL}^T$ and $C_{RR}^S = -4rC_{RR}^T$ (right) for both left and right-handed operators.

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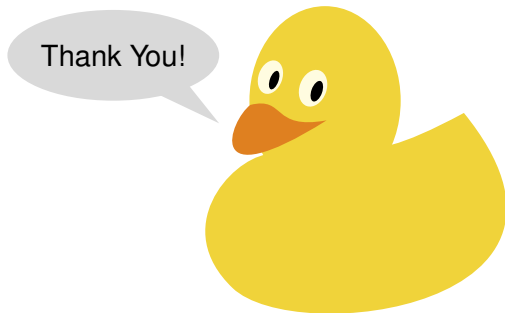
- Using the EFT approach, the impact of NP operator's on the $B_C^+ \rightarrow B_S \mu^+ \nu_\mu$ decay is analyzed.
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- The NP Wilson Coefficients are constrained by using the leptonic and semileptonic charm decays.
- We have focused on the NP operators which can arise due to the presence of RHN.
- Among all the scenarios, the scenario 6 and 7 gives a small deviation in Branching fraction from the SM.
- Other observable like A_{FB} should be investigated in order to see any effects of NP.



Backup Slides

Differential Decay Width

- Differential decay distribution can be written as

$$\frac{d\Gamma(B_c^+ \rightarrow B_s \mu^+ \nu_\mu)}{dq^2 d \cos \theta_\mu} = \frac{G_F^2 V_{cs}^2}{256 m_{B_c}^3 4\pi^3} q^2 \lambda_{B_s}^{1/2}(q^2) \left(1 - \frac{m_\mu^2}{q^2}\right)^2 L_{\mu\nu} H^{\mu\nu} \quad (9)$$

where the short-hand notation for the Kallen triangle function is

$$\lambda_{B_s}(q^2) \equiv \lambda(m_{B_c}^2, m_{B_s}^2, q^2) = m_{B_c}^4 + m_{B_s}^4 + q^4 - 2m_{B_c}^2 m_{B_s}^2 - 2m_{B_s}^2 q^2 - 2m_{B_c}^2 q^2$$

Leptonic Amplitude

The lepton helicity amplitudes are defined as

$$L_{V\mp A,\lambda}^{\lambda_\mu, \overset{L}{R}}(q^2, \theta_\mu, \phi) = \epsilon_\alpha(\lambda) \langle \mu^+ (\lambda_\mu) \nu (\lambda_\nu) | \bar{\nu} \gamma^\mu (1 \mp \gamma_5) \mu | 0 \rangle$$

$$L_{S\mp P,\lambda}^{\lambda_\mu, \overset{L}{R}}(q^2, \theta_\mu, \phi) = \langle \mu^+ (\lambda_\mu) \nu (\lambda_\nu) | \bar{\nu} \gamma^\mu (1 \mp \gamma_5) \mu | 0 \rangle$$

$$L_{T\mp T5,\lambda}^{\lambda_\mu, \overset{L}{R}}(q^2, \theta_\mu, \phi) = -L_{T\mp T5,\lambda}^{\lambda_\mu, \overset{L}{R}}(q^2, \theta_\mu, \phi) = -i\epsilon_\alpha(\lambda)\epsilon_\beta(\lambda') \langle \mu^+ (\lambda_\mu) \nu (\lambda_\nu) |$$

where λ_μ is the muon helicity and $\epsilon(\lambda)$ are the polarization vector of the intermediate vector boson.

Leptonic: $D_s^+ \rightarrow \mu^+ \nu_\mu$

The branching fraction for leptonic is

$$B = \frac{G_F^2}{8\pi} |V_{cs}|^2 f_{D_s^+}^2 M_{D_s^+} m_\mu^2 \left[\left| 1 + C_{LL}^V - C_{RL}^V + \frac{m_{D_s^+}^2}{m_\mu(m_c + m_s)} (C_{RL}^S - C_{LL}^S) \right|^2 + \left| C_{RR}^V - C_{LR}^V + \frac{m_{D_s^+}^2}{m_\mu(m_c + m_s)} \right|^2 \right] \quad (10)$$

where $\tau_{D_s^+}$ is the lifetime of the D_s^+ meson, $M_{D_s^+}$ and m_μ are the masses of the D_s^+ meson and muon lepton respectively, and the decay constant is

$$f_{D_s^+} = (248.0 \pm 1.6) \text{ MeV}$$

Semi-leptonic: $D \rightarrow P \mu^+ \nu_\mu$

$$\begin{aligned}
\frac{d\mathcal{B}}{dq^2}(D \rightarrow P \mu^+ \nu) &= \frac{G_F^2 V_{CS}^2}{192 m_D^3 \pi^3} q^2 \lambda_P^{1/2}(q^2) \left(q^2 \left(1 - \frac{m_\mu^2}{q^2} \right) \right)^2 \tau_D \\
&\times \left\{ \left(\left| 1 + C_{LL}^V + C_{RL}^V \right|^2 + \left| C_{LR}^V + C_{RR}^V \right|^2 \right) \right. \\
&\left[\left(H_{V,0}^P \right)^2 \left(\frac{m_\mu^2}{2q^2} + 1 \right) + \frac{3m_\mu^2}{2q^2} \left(H_{V,s}^P \right)^2 \right] \\
&+ \frac{3}{2} \left(H_{S,s}^P \right)^2 \left(\left| C_{RL}^S + C_{LL}^S \right|^2 + \left| C_{RR}^S + C_{LR}^S \right|^2 \right) \\
&+ 8 \left(\left| C_{LL}^T \right|^2 + \left| C_{RR}^T \right|^2 \right) \left(H_T^P \right)^2 \left(1 + \frac{2m_\mu^2}{q^2} \right) \\
&+ 3\text{Re} \left[\left(1 + C_{LL}^V + C_{RL}^V \right) \left(C_{RL}^S + C_{LL}^S \right)^* \right. \\
&\left. + \left(C_{LR}^V + C_{RR}^V \right) \left(C_{RR}^S + C_{LR}^S \right)^* \right] \frac{m_\mu}{\sqrt{q^2}} H_{S,s}^P H_{V,s}^P \\
&\left. - 12\text{Re} \left[\left(1 + C_{LL}^V + C_{RL}^V \right) C_{LL}^{T*} + \left(C_{RR}^V + C_{LR}^V \right) C_{RR}^{T*} \right] \frac{m_\mu}{\sqrt{q^2}} H_T^P H_{V,0}^P \right\}.
\end{aligned} \tag{11}$$

Semi-leptonic: $D \rightarrow V \mu^+ \nu_\mu$

$$\begin{aligned}
\frac{dB}{dq^2}(D \rightarrow V \mu^+ \nu) &= \frac{G_F^2 V_{CS}^2}{192 m_D^3 \pi^3} q^2 \lambda_V^{1/2}(q^2) \left(1 - \frac{m_\mu^2}{q^2}\right)^2 \tau_D \\
&\times \left\{ \left(|1 + C_{LL}^V|^2 + |C_{RL}^V|^2 + |C_{LR}^V|^2 + |C_{RR}^V|^2 \right) \right. \\
&\left[(H_{V,+}^2 + H_{V,-}^2 + H_{V,0}^2) \left(\frac{m_\mu^2}{2q^2} + 1 \right) + \frac{3m_\mu^2}{2q^2} (H_{V,t}^2) \right] \\
&+ \left((1 + C_{LL}^V) C_{RL}^{V*} + C_{LR}^V C_{RR}^{V*} \right) \left[(H_{V,0}^2 + 2H_{V,+}H_{V,-}) \left(\frac{m_\mu^2}{2q^2} + 1 \right) + \frac{3m_\mu^2}{2q^2} (H_{V,t}^2) \right] \\
&+ \frac{3}{2} H_S^2 \left(|C_{RL}^S - C_{LL}^S|^2 + |C_{RR}^S - C_{LR}^S|^2 \right) \\
&+ 8 \left(|C_{LL}^T|^2 + |C_{RR}^T|^2 \right) (H_{T,+}^2 + H_{T,-}^2 + H_{T,0}^2) \left(1 + \frac{2m_\mu^2}{q^2} \right) \\
&+ 3 \text{Re} \left[(1 + C_{LL}^V - C_{RL}^V) (C_{RL}^S - C_{LL}^S)^* + (C_{LR}^V - C_{RR}^V) (C_{RR}^S - C_{LR}^S)^* \right] \frac{m_\mu}{\sqrt{q^2}} H_S H_{V,t} \\
&- 12 \text{Re} \left[(1 + C_{LL}^V) C_{LL}^{T*} + C_{RR}^V C_{RR}^{T*} \right] \frac{m_\mu}{\sqrt{q^2}} (H_{T,0} H_{V,0} + H_{T,+} H_{V,+} + H_{T,-} H_{V,-}) \\
&+ 12 \text{Re} \left[C_{RL}^V C_{LL}^{T*} + C_{LR}^V C_{RR}^{T*} \right] \frac{m_\mu}{\sqrt{q^2}} (H_{T,0} H_{V,0} + H_{T,+} H_{V,-} - H_{T,-} H_{V,+}) \left. \right\} \quad (12)
\end{aligned}$$

Scenarios

- The following scenarios [JHEP 08 (2020) 022] are analyzed:
 - ① RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$

Scenarios

- The following scenarios [JHEP 08 (2020) 022] are analyzed:
 - 1 RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - 2 RHN: $O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$

Scenarios

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 - 1 RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - 2 RHN: $O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - 3 Vector Boson nature: $V^\mu (1,1,-1)$: O_{RR}^V

Scenarios

- The following scenarios [JHEP 08 (2020) 022] are analyzed:
 - ① RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ② RHN: $O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ③ Vector Boson nature: $V^\mu (1,1,-1)$: O_{RR}^V
 - ④ Scalar Boson nature
 - ① $\varphi (1,2,1/2)$: O_{LR}^S, O_{RR}^S
 - ② $\varphi (1,2,1/2)$: O_{LL}^S, O_{RL}^S and O_{LR}^S, O_{RR}^S

Scenarios

- The following scenarios [JHEP 08 (2020) 022] are analyzed:
 - ① RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ② RHN: $O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ③ Vector Boson nature: $V^\mu (1,1,-1)$: O_{RR}^V
 - ④ Scalar Boson nature
 - ① $\varphi (1,2,1/2)$: O_{LR}^S, O_{RR}^S
 - ② $\varphi (1,2,1/2)$: O_{LL}^S, O_{RL}^S and O_{LR}^S, O_{RR}^S
 - ⑤ Leptoquark nature
 - ① $U_1^\mu (3,1,2/3)$: O_{RR}^V, O_{LR}^S
 - ② $U_1^\mu (3,1,2/3)$: O_{LL}^V, O_{RL}^S and O_{RR}^V, O_{LR}^S

Scenarios

- The following scenarios [JHEP 08 (2020) 022] are analyzed:
 - ① RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ② RHN: $O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ③ Vector Boson nature: $V^\mu (1,1,-1)$: O_{RR}^V
 - ④ Scalar Boson nature
 - ① $\varphi (1,2,1/2)$: O_{LR}^S, O_{RR}^S
 - ② $\varphi (1,2,1/2)$: O_{LL}^S, O_{RL}^S and O_{LR}^S, O_{RR}^S
 - ⑤ Leptoquark nature
 - ① $U_1^\mu (3,1,2/3)$: O_{RR}^V, O_{LR}^S
 - ② $U_1^\mu (3,1,2/3)$: O_{LL}^V, O_{RL}^S and O_{RR}^V, O_{LR}^S
 - ⑥ Leptoquark nature: $R_2 (3,2,1/6)$: O_{RR}^S, O_{RR}^T with $C_{RR}^S = 4rC_{RR}^T$

Scenarios

- The following scenarios [JHEP 08 (2020) 022] are analyzed:
 - ① RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ② RHN: $O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ③ Vector Boson nature: $V^\mu (1,1,-1)$: O_{RR}^V
 - ④ Scalar Boson nature
 - ① $\varphi (1,2,1/2)$: O_{LR}^S, O_{RR}^S
 - ② $\varphi (1,2,1/2)$: O_{LL}^S, O_{RL}^S and O_{LR}^S, O_{RR}^S
 - ⑤ Leptoquark nature
 - ① $U_1^\mu (3,1,2/3)$: O_{RR}^V, O_{LR}^S
 - ② $U_1^\mu (3,1,2/3)$: O_{LL}^V, O_{RL}^S and O_{RR}^V, O_{LR}^S
 - ⑥ Leptoquark nature: $R_2 (3,2,1/6)$: O_{RR}^S, O_{RR}^T with $C_{RR}^S = 4rC_{RR}^T$
 - ⑦ Leptoquark nature
 - ① $S_1 (\bar{3},1,1/3)$: $O_{RR}^V, O_{RR}^S, O_{RR}^T$ with $C_{RR}^S = -4rC_{RR}^T$
 - ② $S_1 (\bar{3},1,1/3)$: $O_{LL}^V, O_{LL}^S, O_{LL}^T$ and $O_{RR}^V, O_{RR}^S, O_{RR}^T$ with $C_{LL}^S = -4rC_{LL}^T$ and $C_{RR}^S = -4rC_{RR}^T$

Scenarios

- The following scenarios [JHEP 08 (2020) 022] are analyzed:
 - ① RHN + SM-like contribution: $O_{LL}^V, O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ② RHN: $O_{LR}^V, O_{RR}^V, O_{LR}^S, O_{RR}^S, O_{RR}^T$
 - ③ Vector Boson nature: $V^\mu (1,1,-1)$: O_{RR}^V
 - ④ Scalar Boson nature
 - ① $\varphi (1,2,1/2)$: O_{LR}^S, O_{RR}^S
 - ② $\varphi (1,2,1/2)$: O_{LL}^S, O_{RL}^S and O_{LR}^S, O_{RR}^S
 - ⑤ Leptoquark nature
 - ① $U_1^\mu (3,1,2/3)$: O_{RR}^V, O_{LR}^S
 - ② $U_1^\mu (3,1,2/3)$: O_{LL}^V, O_{RL}^S and O_{RR}^V, O_{LR}^S
 - ⑥ Leptoquark nature: $R_2 (3,2,1/6)$: O_{RR}^S, O_{RR}^T with $C_{RR}^S = 4rC_{RR}^T$
 - ⑦ Leptoquark nature
 - ① $S_1 (\bar{3},1,1/3)$: $O_{RR}^V, O_{RR}^S, O_{RR}^T$ with $C_{RR}^S = -4rC_{RR}^T$
 - ② $S_1 (\bar{3},1,1/3)$: $O_{LL}^V, O_{LL}^S, O_{LL}^T$ and $O_{RR}^V, O_{RR}^S, O_{RR}^T$ with $C_{LL}^S = -4rC_{LL}^T$ and $C_{RR}^S = -4rC_{RR}^T$
 - ⑧ Leptoquark nature: $V_2^\mu (\bar{3},2,-1/6)$: O_{LR}^S