Exploring Constraints on Simplified Dark Matter Model Through Flavor and Electroweak Observables

Lipika Kolay

IIT Guwahati

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In Collaboration with: Dr. Soumitra Nandi

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Why BSM?

Standard Model prediction is validated to high precision, however

1. Matter-Anti matter asymmetry
   - $\eta = \frac{n_B - \bar{n}_B}{\gamma} \sim 10^{-9}$
2. Neutrino oscillation
   - Non-zero mass of neutrino
3. Dark Matter
   - $\sim$ 25% of total energy budget is dark matter
   - Observational Evidances: Galaxy Rotation Curve, Gravitational lensing, CMBR
   - WIMP, SIMP, FIMP ....
4. and many more.....

The Simplified Model

Model Lagrangian:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi + \frac{1}{2}\partial_\mu S \partial^\mu S - \bar{\chi}(c_{s\chi} + i c_{p\chi}\gamma_5)\chi S - \bar{\psi}(c_{s\psi} + i c_{p\psi}\gamma_5)\psi S - V(S,H) + \mathcal{L}_{VVS}\,.$$

Where,

$$\mathcal{L}_{VVS} = -c'_w\, W^+_\mu W^{\mu -} S - \frac{c'_z}{2}\, Z_\mu Z^\mu S$$

$$V(S,H) = \mu_S^2 S^2 + \frac{\lambda_4}{4!} S^4 + \frac{\lambda_3}{3!} S^3 + \lambda_1 S H^\dagger H + \lambda_2 S^2 H^\dagger H\,,$$

Under $\mathcal{Z}_2 :: \chi \to -\chi\,.$

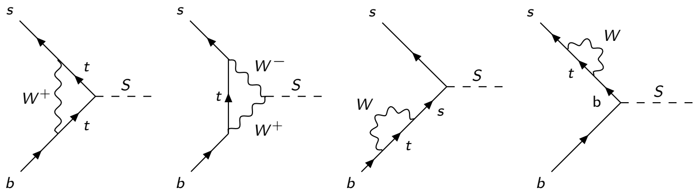
Yukawa coupling of **S**-ψ-ψ re-scaled as (MFV):

$$\bar{\psi}(c_{s\psi} + i c_{p\psi}\gamma_5)\psi S = m_\psi \bar{\psi}(c_s + i c_p \gamma_5)\psi S\,.$$

where, $c_s = \frac{\sqrt{2} g_s}{v}, c_p = \frac{\sqrt{2} g_p}{v}$.
Gauge couplings are re-scaled as: $c'_V = 2 m_V^2 c_G$.

Contribution to FCNC Vertex

- $b - s - S$ will be modified as:
  - vertex correction
  - external leg correction



- Charm and up-quark mediated diagrams are also possible.
- $b \to d\ S$ and $s \to d\ S$ vertices can be drawn in similar way.

b-s-S vertex correction

Total diagram contribution:

$$\mathcal{L}_{eff}^{bsS} = \frac{2\sqrt{2}G_F M_W^2}{16\pi^2}\left(C_1\left[\bar{b}(m_b P_L + m_s P_R)s\right] + C_2\left[\bar{b}(m_b P_L - m_s P_R)s\right]\right)$$

- Scalar and pseudoscalar operators
- $C_1, C_2 \Rightarrow$ loop function$\Rightarrow$ have divergences.
- Divergences can be absorbed by RGE of coupling (in LLA):

$$\begin{aligned} C_1(\Lambda) &= I_1 + \frac{3m_t^2}{2m_w^2}\left(c_s + c_G \log\frac{m_t^2}{m_W^2}\right) log\frac{\Lambda^2}{m_t^2}, \\ C_2(\Lambda) &= I_2 + \frac{m_t^2}{2m_w^2}(-ic_p) log\frac{\Lambda^2}{m_t^2}. \end{aligned}$$

$I_1, I_2 \rightarrow$ Finite part of loop integration, $\Lambda \rightarrow$ cutoff scale.

- We also have $b-d-S, s-d-S$ FCNC vertices in this model.

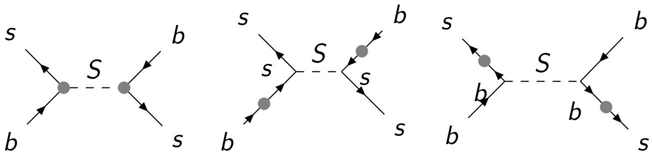


FIGURE: Feynman diagrams contributing to meson mixing

Generated in **1-loop box diagram in SM**: [arXiv:hep-ex/0103016v1]

Mixing observable : mass difference

$$\Delta M^q_{SM} = \frac{|\mathcal{M}|}{m_{B_q}} = \frac{G_F^2}{6\pi^2} m_{B_q} f^2_{B_q} B_{B_q} \eta_B m_W^2 |V^*_{td} V_{tb}|^2 f\left(\frac{m_t^2}{m_W^2}\right)$$

$$\Delta M_{tot} = \Delta M_{SM} + \Delta M_{NP},$$

$$\Delta = \frac{\Delta M_{NP}}{\Delta M_{SM}}, \text{ with } \Delta M_{NP} = \frac{|\mathcal{M}_{NP}|}{m_B}$$

$$\Delta M_{tot} = \Delta M_{SM}(1 + \Delta)$$

LHCb'22

$B^0_s - \bar{B}^0_s \approx 9\%$ NP contribution.

$B_0 - \bar{B}_0 \approx 7.5\%$ NP contribution.

10% NP is considered.

Wilson Coeffs, Rare and Semileptonic Decays

The low energy effective Hamiltonian for $b \to s\ell\ell$: [arXiv:1205.5811]

$$\mathcal{H}_{eff} = -\frac{4\,G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1}^{6} C_i(\mu)\mathcal{O}_i(\mu) + \sum_{i=7,8,9,10,P,S,T,T5} \left(C_i(\mu)\mathcal{O}_i + C_i'(\mu)\mathcal{O}_i' \right) \right]$$

$$\mathcal{O}_S = \frac{e^2}{16\pi^2}(\bar{s}P_R b)(\bar{\ell}\ell), \qquad \mathcal{O}_S' = \frac{e^2}{16\pi^2}(\bar{s}P_L b)(\bar{\ell}\ell),$$

$$\mathcal{O}_P = \frac{e^2}{16\pi^2}(\bar{s}P_R b)(\bar{\ell}\gamma_5\ell), \qquad \mathcal{O}_P' = \frac{e^2}{16\pi^2}(\bar{s}P_L b)(\bar{\ell}\gamma_5\ell),$$

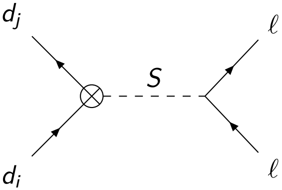
Wilson Coefficients:

$$C_S' = \frac{1}{e^2}\frac{m_W^2 m_\ell}{M_S^2}\frac{m_s c_s}{m_b}\left(C_1(\Lambda) + C_2(\Lambda)\right), \quad C_P' = \frac{1}{e^2}\frac{m_W^2 m_\ell}{M_S^2}\frac{m_s(ic_p)}{m_b}\left(C_1(\Lambda) + C_2(\Lambda)\right),$$

$$C_S = \frac{1}{e^2}\frac{m_W^2}{M_S^2} m_\ell c_s \left(C_1(\Lambda) - C_2(\Lambda)\right), \quad C_P = \frac{1}{e^2}\frac{m_W^2}{M_S^2} m_\ell (ic_p)\left(C_1(\Lambda) - C_2(\Lambda)\right).$$

- From $b \to s\ell\ell$ process: branching ratios, Isospin asymmetries, LFUV observables, angular observables.
- Performed a global fit, taking updated values of R_K, R_{K^*} by LHCb. [arXiv:2212.09152]

Rare Decays



$$\mathcal{B}r(B_0^s \to \mu^+\mu^-) = (3.09^{+0.46\ +0.15}_{-0.43\ -0.11}) \times 10^{-9}$$
$$\mathcal{B}r(B_0 \to \mu^+\mu^-) = (0.12^{+0.08}_{-0.07} \pm 0.01) \times 10^{-9}$$
$$\mathcal{B}r(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$$
$$\mathcal{B}r(K_S \to \mu^+\mu^-) < 2.1 \times 10^{-10}$$

[PTEP 2022 (2022) 083C01, PhysRevLett.128.041801, PhysRevLett.125.231801]

- Sensitive to scalar and pseudoscalar operators.

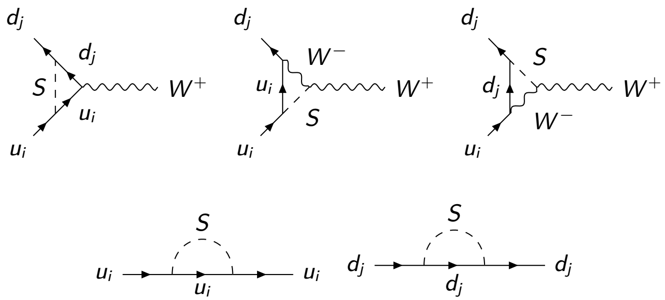
Branching Ratio in BSM scenario [1205.5811]

$$Br(B_0 \to \mu^+\mu^-)^{tot} = \tau_{B_0} f_B^2 m_B^3 \frac{G_F^2 \alpha^2}{64\pi^3} |V_{ts}^* V_{tb}|^2 \beta_\mu(m_B^2) [\frac{m_B^2}{m_b^2} |\mathbf{C_s} - \mathbf{C_s'}|^2 (1 - \frac{4m_\mu^2}{m_B^2}) + |\frac{m_B}{m_b}(\mathbf{C_p} - \mathbf{C_p'}) + 2\frac{m_\mu}{m_B}(C_{10} - C_{10}')|^2]$$

$\beta_\ell(q^2) = \sqrt{1 - 4m_\ell^2/q^2}$, $f_B \to$ decay constant of B_0 meson.

Contribution to FCCC Vertex

- Vertex correction
- Counter term



Loop Contribution:

$$\mathcal{L}^{eff}_{u_i \to d_j W} = \frac{-gV^*_{ij}}{\sqrt{2}} \left[C_{VL} \bar{d}_j \gamma_\mu (1-\gamma_5) u_i + C_{VR} \bar{d}_j \gamma_\mu (1+\gamma_5) u_i \right] W^\mu .$$

Anomalous couplings : $t \to bW_\mu$ decay

The general Lagrangian for this decay: [arXiv:1707.05393]

$$\mathcal{L}_{tbW} = -\frac{g}{\sqrt{2}}\bar{b}\gamma_\mu(V_L P_L + V_R P_R)tW_\mu - \frac{g}{\sqrt{2}}\bar{b}\frac{i\sigma_{\mu\nu}q_\nu}{m_W}(g_L P_L + g_R P_R)tW_\mu + h.c$$

- $V_L = V_{tb}$ in SM.
- V_R, g_L and g_R : purely NP.

In this model:

$$V_L = V_{tb}^* \, (1 + C_{VL}) \, ,$$
$$V_R = V_{tb}^* \; C_{VR} \, .$$

95% CL interval			
Coupling	ATLAS	CMS	ATLAS+CMS combination
$\text{Re}(V_\text{R})$	$[-0.17, 0.25]$	$[-0.12, 0.16]$	$[-0.11, 0.16]$
$\text{Re}(g_\text{L})$	$[-0.11, 0.08]$	$[-0.09, 0.06]$	$[-0.08, 0.05]$
$\text{Re}(g_\text{R})$	$[-0.03, 0.06]$	$[-0.06, 0.01]$	$[-0.04, 0.02]$
V_L	0.995 ± 0.021		

TABLE: Experimental limits on the coefficients of left and right handed vector, tensor current in $t \to bW_\mu^-$ decay.

[PDG:2022,2005.03799,2004.12181,1612.02577]

FCCC Process: Semileptonic decays

Differential decay rate

$$\frac{d\Gamma(P \to M\ell\nu_\ell)}{dq^2} = \frac{G_F^2 |V_{u_i d_j}|^2}{\pi^3 m_P^3} q^2 \sqrt{\lambda_M(q^2)} \left(1 - \frac{m_\ell^2}{q^2}\right) |1 + C_{VL} + C_{VR}|^2 \left\{ \left(1 + \frac{m_\ell^2}{2q^2}\right) {H_{V,0}^s}^2 + \frac{3}{2} \frac{m_\ell^2}{q^2} {H_{V,t}^s}^2 \right\}.$$

Branching fraction

$$\mathcal{B}(P \to \ell\nu_\ell) = \frac{\tau_P}{8\pi} m_P m_\ell^2 f_P^2 G_F^2 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 \left| V_{u_i d_j}(1 + C_{VL} - C_{VR})\right|^2.$$

- Will impact the overall normalisation, not the q^2 shapes.

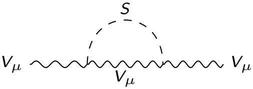
CKM element modifies as:

$$V'_{u_i d_j} = V_{u_i d_j}(1 + C_{VL} \pm C_{VR})$$

- Performed CKM fit using inputs from those semileptonic and leptonic decays.

Electroweak observables: W mass anomaly

$m_W^{SM} = 80357 \pm 6$ MeV



$$m_W^{CDF} = 80.4335 \pm 0.0094 \text{GeV}$$
$$m_W^{LHCb} = 80.354 \pm 0.030 \text{GeV}$$
$$m_W^{ATLAS} = 80.360 \pm 0.016 \text{GeV}$$
$$m_W^{D0} = 80.367 \pm 0.023 \text{GeV}$$

[Science 376(2022)170,2109.01113,ATLAS'23,1203.0293]

Loop contribution

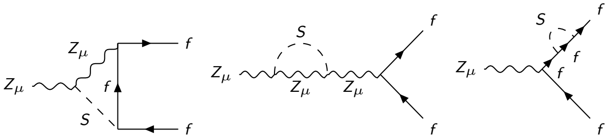
$$\Sigma_V(q^2) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Sigma_{V,T}(q^2) + \frac{q^\mu q^\nu}{q^2}\Sigma_{V,L}(q^2)\,.$$

$$\Delta r^{\delta\rho} = -\frac{c_w^2}{s_w^2}\Delta\rho = -\frac{c_w^2}{s_w^2}\left(\frac{\Sigma_{Z,T}(0)}{M_Z^2} - \frac{\Sigma_{W,T}(0)}{M_W^2}\right) = 1 - \frac{\pi\alpha_{em}}{\sqrt{2}G_F}\frac{1}{M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right)}.$$

Observable considered

$$\delta(\Delta r) = (\Delta r)_{exp} - (\Delta r)_{SM}$$

Electroweak observables: Z-pole Observables



Decay width

$$\Gamma_{tot}(Z \to f\bar{f}) = \frac{N_c^b}{48}\frac{\alpha}{s_W^2 c_W^2} m_Z \sqrt{1-\mu_f^2}\left(|g_{af}|^2(1-\mu_f^2) + |g_{vf}|^2(1+\frac{\mu_f^2}{2})\right)(1+\delta_f^0)(1+\delta_b)$$
$$(1+\delta_{QCD})(1+\delta_{QED})(1+\delta_\mu^f)\,,$$

Loop corrected couplings:

$$g_{af} \to a_f + \Delta a_f^{NP}\,,$$
$$g_{vf} \to v_f + \Delta v_f^{NP}$$

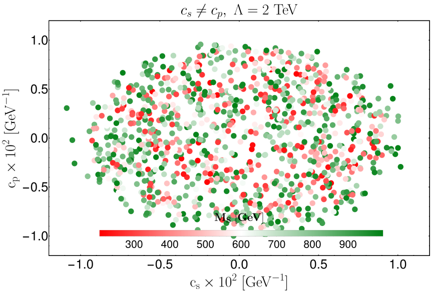
Observables:

$$R_\ell = \frac{\Gamma_{had}}{\Gamma_\ell}; \quad R_c = \frac{\Gamma_c}{\Gamma_{had}}; \quad R_b = \frac{\Gamma_b}{\Gamma_{had}}; \quad A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

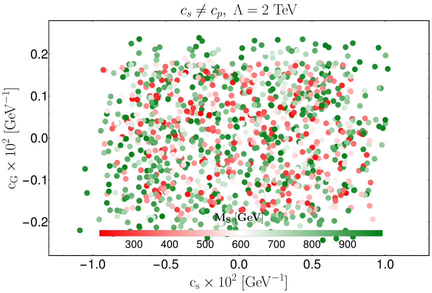
$$\Gamma_\ell = \frac{1}{3}(\Gamma_e + \Gamma_\mu + \Gamma_\tau)$$

$$\Gamma_{had} = \Gamma_u + \Gamma_d + \Gamma_s + \Gamma_c + \Gamma_b.$$

Combined Constraint on c_s, c_p and M_S



(A)



(B)

FIGURE: The allowed parameter space in the c_s-c_p and c_p-c_G planes for $\Lambda = 2$ TeV is determined by considering all flavour and electroweak observables.

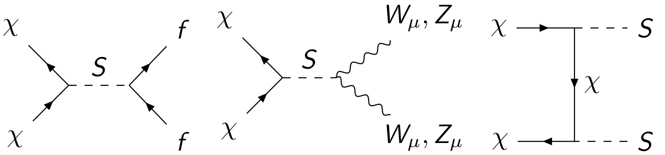
Combined Constraint on c_s, c_p, c_G and c_G

Λ [TeV]	M_S [GeV]	$c_s \times 10^2$ [GeV^{-1}]	c_p [GeV^{-1}]	$c_G \times 10^3$ [GeV^{-1}]	$\Delta M_W \times 10^2$ [GeV]
1	250	0.027 ± 1.734	0.0 ± 0.018	-0.820 ± 1.413	0.409 ± 1.408
	500	-0.048 ± 1.953	0.0 ± 0.020	-1.013 ± 1.755	0.406 ± 1.405
	800	-0.172 ± 2.239	0.0 ± 0.022	-1.333 ± 2.275	0.410 ± 1.399
2	250	0.038 ± 1.457	0.0 ± 0.015	-0.684 ± 1.174	0.410 ± 1.409
	500	0.005 ± 1.558	0.0 ± 0.016	-0.782 ± 1.353	0.407 ± 1.408
	800	0.032 ± 1.662	0.0 ± 0.017	0.899 ± 1.546	0.405 ± 1.392
	1000	0.058 ± 1.726	0.0 ± 0.017	0.982 ± 1.690	0.404 ± 1.391

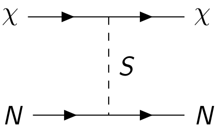
TABLE: Fit results of the parameters c_s, c_p and c_G for different combinations of Λ and M_S, when all the observables of table are considered along with weighted mean data of $\delta(\Delta r)$ observable from LHCb, ATLAS and D0 experiment. The p-value for the fit is $\sim 30.49\%$ with 42 d.o.f. The last column shows the prediction of the ΔM_W value from this fit result for each case.

DM Phenomenology

- $\chi \Rightarrow$ Fermionic DM
- **DM + DM** $\rightarrow$ Annihilation channels:

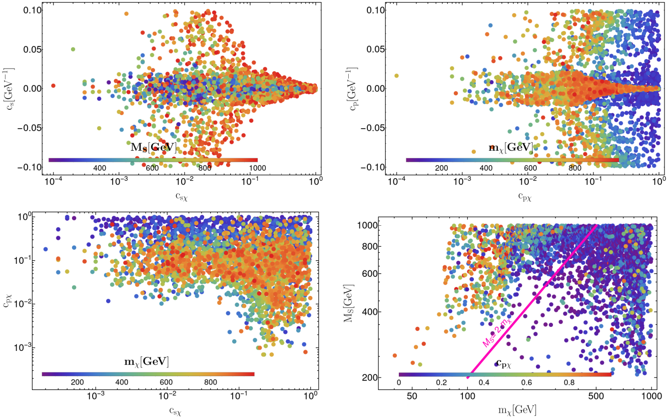


- Direct Detection:



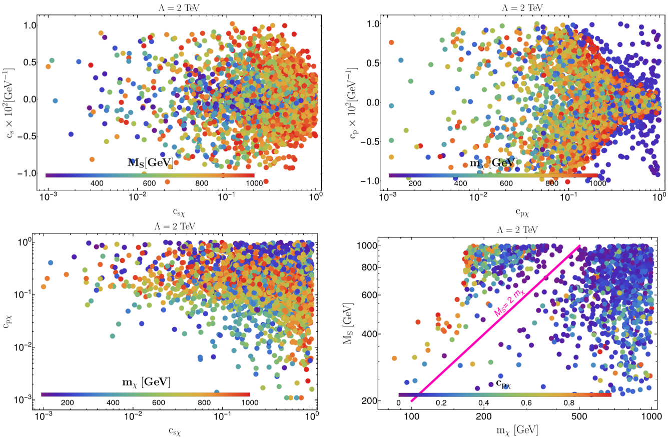
- Observed relic density : $\Omega h^2 = 0.012 \pm 0.001$ [Planck 2018]
- Spin Independent Direct Detection crossection (LZ-2022, PandaX-4T, XENON1T)

DM Phenomenology



- For $c_{s\chi} > 0.1$, c_s very small and $M_S \geq 600\mathrm{GeV}$. For $c_{s\chi} < 0.1$ and $M_S < 500\mathrm{GeV}$, $c_s \sim 10^{-3}$, from DD.
- For $m_\chi \geq 500\mathrm{GeV}$ and $c_{p\chi} > 0.3$, $c_p \sim 10^{-3}$, from observed relic density.
- For $c_{s\chi} < 0.1$, $c_{p\chi} \sim 0.1$, for $m_\chi > 400\mathrm{GeV}$. and $c_{p\chi} \sim 1$ for $m_\chi \leq 350\mathrm{GeV}$.
- For $c_{p\chi} \leq 0.4$, $M_S > 400, m_\chi > 300\mathrm{GeV}$ and for $c_{p\chi} \geq 0.4$, $M_S \geq 500, m_\chi < 300\mathrm{GeV}$.

Correlation: DM + Flavor Parameter Space



- For $c_{s\chi} \gtrsim 0.3$, $|c_s| \leq 0.005\mathrm{GeV}^{-1}$. $M_S < 400\mathrm{GeV}$, $c_s < 0.005\mathrm{GeV}^{-1}$ for even larger $c_{s\chi}$.
- $M_S > 500\mathrm{GeV}$, c_s approaches to $0.001 GeV^{-1}$ for $c_{p\chi}$ approaches to 1.
- $c_{s\chi}$, $c_{p\chi}$, range, correlation remains same, except lesser number of allowed points.
- Depletion in the $M_S = 2m_\chi$ region.

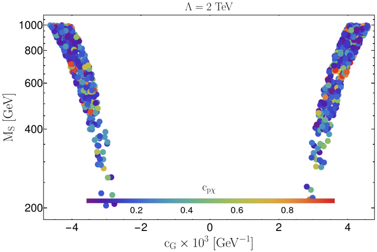
Summary

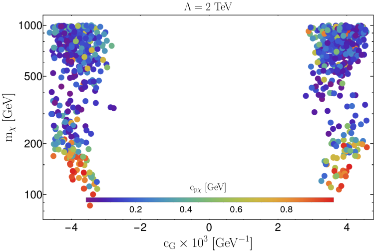
1. Fermionic DM (χ) model with spin-0 mediator (S) is considered.
2. Effective FCNC and FCCC vertices are obtained from loop calculations.
3. Constraint on mediator-fermion couplings c_s, c_p and c_G obtained from FCNC, FCCC and EW processes : $c_s \leq 0.01\mathrm{GeV}^{-1}, \;\; \mathrm{c_p} \leq 0.01\mathrm{GeV}^{-1}$ and $c_G \leq 0.002\mathrm{GeV}^{-1}$.
4. Correlation among free parameters studied from the DM-phenomenology.
5. Obtained **final parameter space** allowed by all the low energy and other processes together with DM constraints.

Thank You!

DM+Flav PArameter Space with CDF data included

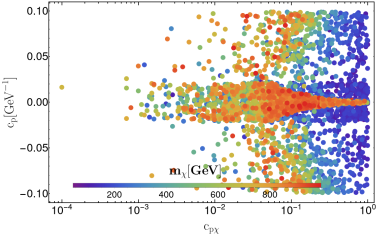
- Allowed parameter space if we add CDF data too:

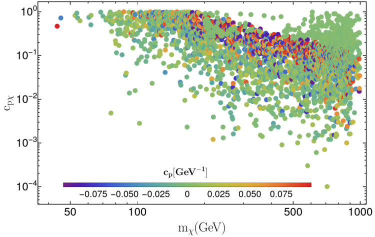




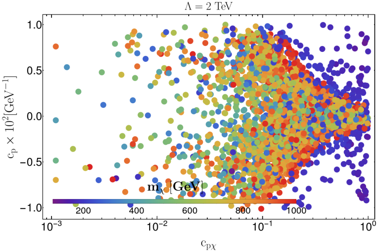
Other Plot of DM and DM+Flav

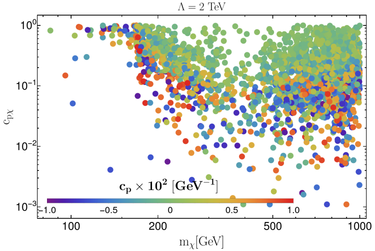
- Other DM plots:





- Other DM+Flavor plots:





Combined Fit with CDF data included

Λ [TeV]	M_S [GeV]	$c_s \times 10^2$ [GeV^{-1}]	c_p [GeV^{-1}]	$c_G \times 10^3$ [GeV^{-1}]	$\Delta M_W \times 10^2$ [GeV]
1	250	-0.090 ± 1.747	0.0 ± 0.018	2.770 ± 0.259	4.657 ± 0.869
	500	0.162 ± 2.014	0.0 ± 0.020	3.432 ± 0.321	4.654 ± 0.869
	800	0.566 ± 3.616	0.0 ± 0.033	4.487 ± 0.419	4.645 ± 0.869
2	250	-0.130 ± 1.493	0.0 ± 0.015	2.305 ± 0.215	4.658 ± 0.869
	500	-0.018 ± 1.560	0.0 ± 0.015	2.646 ± 0.247	4.656 ± 0.869
	800	0.107 ± 1.693	0.0 ± 0.016	3.048 ± 0.285	4.653 ± 0.869
	1000	0.197 ± 1.829	0.0 ± 0.017	3.330 ± 0.311	4.651 ± 0.869

TABLE: Fit results of the parameters c_s, c_p and c_G for different combinations of Λ and M_S, all the observables of FCCC, FCNC, EW along with $\delta(\Delta r)$ (CDF, LHCb, ATLAS, D0) are taken into account, with a p-value of $\sim 3\%$ for 45 d.o.f. The last column shows the prediction of the value of ΔM_W from this fit result for each case.

$b \to s\ell^+\ell^-$ Fit

Λ[TeV]	M_S[GeV]	c_s[GeV^{-1}]	c_p[GeV^{-1}]	c_G[GeV^{-1}]
1	250	0.43(181)	-0.51(44)	-0.26(122)
	500	0.87(363)	-1.02(886)	-0.52(245)
	800	-1.39(581)	1.63(142)	0.84(392)
2	250	0.37(152)	-0.43(38)	-0.24(113)
	500	0.73(307)	-0.86(75)	-0.49(227)
	800	1.17(491)	-1.38(120)	-0.78(363)
	1000	-1.47(615)	1.72(150)	0.97(454)

TABLE: Fit results of the couplings c_s, c_p and c_G from a fit to the available data on $B \to K^{(*)}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ decays and on $R(K^{(*)})$.

Backup

$$\Delta m_w = -\frac{1}{2} m_w \frac{s_w^2}{c_w^2 - s_w^2} \delta(\Delta r) \tag{1}$$

$$\Delta r^{\delta\rho} = -\frac{c_w^2}{s_w^2} \Delta\rho \tag{2}$$

$$\rho = \frac{G_{NC}}{G_{CC}} \tag{3}$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{m_z^2} - \frac{\Sigma_W(0)}{m_w^2} \tag{4}$$

Measured Value M_W	Reference	Δr	$\delta(\Delta r)$
80.357 ± 0.006 GeV	SM [?]	-0.03068 ± 0.00040	–
80.4335 ± 0.0094 GeV	CDF [?]	-0.03526 ± 0.00059	$(-4.58316 \pm 0.66899) \times 10^{-3}$
80.354 ± 0.030 GeV	LHCb[?]	-0.03050 ± 0.00190	$(0.17915 \pm 1.92090) \times 10^{-3}$
80.360 ± 0.016 GeV	ATLAS[?]	-0.03086 ± 0.00097	$(-0.17919 \pm 1.02076) \times 10^{-3}$
80.367 ± 0.023 GeV	D0 [?]	-0.03128 ± 0.00156	$(-0.59747 \pm 1.42068) \times 10^{-3}$

Possible Higher Dimensional Models

The fermion-mediator coupling can be derived from

$$\mathcal{L}_{dim5} = -\frac{C}{\Lambda}[\bar{\psi}_L i\gamma_5 H\psi_R S] - y_f[\bar{\psi}_L H\psi_R] + h.c \quad (5)$$

The couplings will then be:

$$\left(-\frac{Cv}{\Lambda\sqrt{2}}\cos\theta + (\frac{Cu}{\Lambda\sqrt{2}} + \frac{y_\psi\alpha}{\sqrt{2}})\sin\theta\right)[\bar{\psi}i\gamma_5\psi s] + \left(\frac{Cv\alpha}{\Lambda\sqrt{2}}\cos\theta + (\frac{Cu\alpha}{\Lambda\sqrt{2}} - \frac{y_\psi}{\sqrt{2}})\sin\theta\right)[\bar{\psi}\psi s] \quad (6)$$

The gauge-couplings can be derived from:

$$\mathcal{L}_{gauge} = \frac{C_V}{\Lambda}S|D_\mu H|^2 \quad (7)$$

with

$$c'_w = 2m_W^2\left(\frac{1}{v}\sin\theta + \frac{C_V}{2\Lambda}\cos\theta\right)$$
$$c'_z = 2m_Z^2\left(\frac{1}{v}\sin\theta + \frac{C_V}{2\Lambda}\cos\theta\right) \quad (8)$$

Backup

$$\Gamma_{tot}(Z \to f\bar{f}) = \frac{N_c^b}{48} \frac{\alpha}{s_w^2 c_w^2} m_Z \sqrt{1-\mu_b^2} \left(|g_{af}|^2 (1-\mu_b^2) + |g_{vf}|^2 (1+\frac{\mu_b^2}{2}) \right) (1+\delta_f^0)(1+\delta_b)(1+\delta_{QCD})(1+\delta_{QED})(1+\delta_\mu^f) \quad (9)$$

$$R_\ell^{exp} = 20.767 \pm 0.025, \quad R_c^{exp} = 0.1721 \pm 0.0030, \quad R_b^{exp} = 0.21629 \pm 0.00066$$

$$R_\ell^{SM} = 20.751 \pm 0.005, \quad R_c^{SM} = 0.17223 \pm 0.00005, \quad R_b^{SM} = 0.21580 \pm 0.00015$$

Backup

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$A_{FB}^{0,f} = \frac{3}{4} A_e A_f, \quad A_e = 2\frac{g_{v\ell} g_{a\ell}}{(g_{v\ell})^2 + (g_{a\ell})^2}, \quad A_f = 2\frac{g_{vf} g_{af}}{(g_{vf})^2 + (g_{af})^2} \tag{10}$$

If we write this as: $A_{FB}^{0,f} = A_{FB}^{0,f(SM)} + \delta A_{FB}^{0,f}$, then

$$\delta A_{FB}^{0,f} = \frac{3}{4}[\delta A_\ell A_f^{SM} + A_\ell^{SM} \delta A_f] \tag{11}$$

$A_{FB} = \frac{N_f - N_B}{N_f + N_B}$
$N_f \rightarrow$ Number of events with f going to forward hemisphere.
$N_B \rightarrow$ backwords. $A_e \rightarrow$ creation of Z boson in e^+e^- annihilation. A_f decay of Z to $f\bar{f}$.

CKM-fit Observables

Observable	Value	Reference
$\|V_{ud}\|$ (nucl)	$0.97373 \pm 0.00009 \pm 0.00053$	[?]
$\|V_{us}\|f_+^{K\to\pi}(0)$	0.2165 ± 0.0004	[?]
$\|V_{cd}\|_{\nu N}$	0.230 ± 0.011	[?]
$\|V_{cs}\|_{W\to c\bar{s}}$	$0.94^{+0.32}_{-0.26} \pm 0.13$	[?]
$\|V_{ub}\|_{excl}$	$(3.91 \pm 0.13) \times 10^{-3}$	[?]
$\|V_{cb}\|_{B\to D}$	$(40.84 \pm 1.15) \times 10^{-3}$	[?]
$\mathcal{B}(\Lambda_p \to p\mu^-\bar{\nu}_\mu)_{q^2>15}/\mathcal{B}(\Lambda_p \to \Lambda_c\mu^-\bar{\nu}_\mu)_{q^2>7}$	$(0.947 \pm 0.081) \times 10^{-2}$	[?]
$\mathcal{B}(B^- \to \tau^-\bar{\nu}_\tau)$	$(1.09 \pm 0.24) \times 10^{-4}$	[?]
$\mathcal{B}(D_s^- \to \mu^-\bar{\nu}_\mu)$	$(5.51 \pm 0.16) \times 10^{-3}$	[?]
$\mathcal{B}(D_s^- \to \tau^-\bar{\nu}_\tau)$	$(5.52 \pm 0.24) \times 10^{-2}$	[?]
$\mathcal{B}(D^- \to \mu^-\bar{\nu}_\mu)$	$(3.77 \pm 0.18) \times 10^{-4}$	[?]
$\mathcal{B}(D^- \to \tau^-\bar{\nu}_\tau)$	$(1.20 \pm 0.27) \times 10^{-3}$	[?]
$\mathcal{B}(K^- \to e^-\bar{\nu}_e)$	$(1.582 \pm 0.007) \times 10^{-5}$	[?]
$\mathcal{B}(K^- \to \mu^-\bar{\nu}_\mu)$	0.6356 ± 0.0011	[?]
$\mathcal{B}(\tau^- \to K^-\bar{\nu}_\tau)$	$(0.6986 \pm 0.0085) \times 10^{-2}$	[?]
$\mathcal{B}(K^- \to \mu^-\bar{\nu}_\mu)/\mathcal{B}(\pi^- \to \mu^-\bar{\nu}_\mu)$	1.3367 ± 0.0029	[?]
$\mathcal{B}(\tau^- \to K^-\bar{\nu}_\tau)/\mathcal{B}(\tau^- \to \pi^-\bar{\nu}_\tau)$	$(6.467 \pm 0.84) \times 10^{-2}$	[?]
$\mathcal{B}(B_s \to \mu^+\mu^-)$	$(3.09^{+0.46}_{-0.43}\,{}^{+0.15}_{-0.11}) \times 10^{-9}$	[?]

Observable	Value	Reference
$\lvert V_{cd}\rvert f_+^{D\to\pi}(0)$	0.1426 ± 0.0018	[?]
$\lvert V_{cs}\rvert f_+^{D\to K}(0)$	0.7180 ± 0.0033	[?]
$\lvert\varepsilon_K\rvert$	$(2.228 \pm 0.011) \times 10^{-3}$	[?]
Δm_d	(0.5065 ± 0.0019) ps^{-1}	[?]
Δm_s	(17.7656 ± 0.021) ps^{-1}	[?]
sin 2β	0.699 ± 0.017	[?]
ϕ_s	-0.057 ± 0.021	[?]
α	$(85.2^{+4.8}_{-4.3})^\circ$	[?]
γ	$(66.2^{+3.4}_{3.6})^\circ$	[?]
V_L	0.995 ± 0.021	[?]
V_R	$[-0.11, 0.16]$	[?, ?, ?, ?]
M_W	$80.4335 \pm 0.0094 GeV$	[?]
R_b^0	0.21629 ± 0.00066	[?]
R_ℓ	20.767 ± 0.025	[?]
$A_{FB}^{0,b}$	0.0992 ± 0.0016	[?]
A_b	0.923 ± 0.020	[?]
A_c	0.670 ± 0.027	[?]

$b \to s\ell^+\ell^-$ Processes

The effective Hamiltonian describing the $b \to s\ell^+\ell^-$ transitions at low energy is [arXiv:1205.5811]

$$\mathcal{H}_{eff} = -\frac{4\,G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1}^{6} C_i(\mu)\mathcal{O}_i(\mu) + \sum_{i=7,8,9,10,P,S,T,T5} \left(C_i(\mu)\mathcal{O}_i + C_i'(\mu)\mathcal{O}_i' \right) \right]$$

The operator basis :

$$\mathcal{O}_7 = \frac{e}{g^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}, \qquad \mathcal{O}_7' = \frac{e}{g^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu},$$

$$\mathcal{O}_8 = \frac{1}{g} m_b (\bar{s}\sigma_{\mu\nu} T^a P_R b) G^{\mu\nu\, a}, \qquad \mathcal{O}_8' = \frac{1}{g} m_b (\bar{s}\sigma_{\mu\nu} T^a P_L b) G^{\mu\nu\, a},$$

$$\mathcal{O}_9 = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell), \qquad \mathcal{O}_9' = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell),$$

$$\mathcal{O}_{10} = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell), \qquad \mathcal{O}_{10}' = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),$$

$$\mathcal{O}_S = \frac{e^2}{16\pi^2} (\bar{s} P_R b)(\bar{\ell}\ell), \qquad \mathcal{O}_S' = \frac{e^2}{16\pi^2} (\bar{s} P_L b)(\bar{\ell}\ell),$$

$$\mathcal{O}_P = \frac{e^2}{16\pi^2} (\bar{s} P_R b)(\bar{\ell}\gamma_5 \ell), \qquad \mathcal{O}_P' = \frac{e^2}{16\pi^2} (\bar{s} P_L b)(\bar{\ell}\gamma_5 \ell),$$

$$\mathcal{O}_T = \frac{e^2}{16\pi^2} (\bar{s}\sigma_{\mu\nu} b)(\bar{\ell}\sigma^{\mu\nu} \ell), \qquad \mathcal{O}_{T5} = \frac{e^2}{16\pi^2} (\bar{s}\sigma_{\mu\nu} b)(\bar{\ell}\sigma^{\mu\nu} \gamma_5 \ell),$$