



Theoretical interpretations of exotic hadrons

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OUTLINE

- Introduction
- Theoretical pictures and models
- A group theory approach and compact multiquark picture in constituent quark models
- Summary and outlook

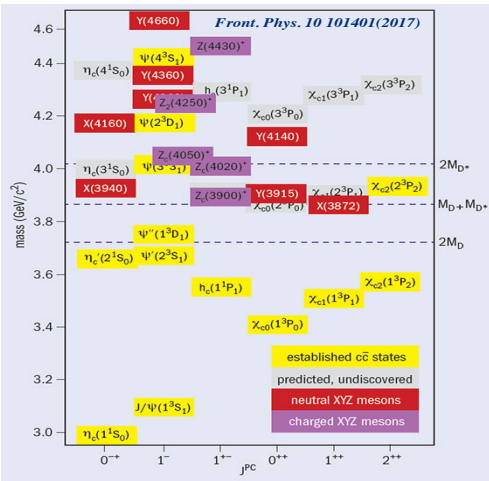
Quantum numbers for $q\bar{q}$ and exotic mesons

- Allowed quantum numbers for $q\bar{q}$ meson.

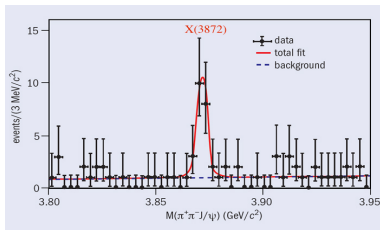
L	S	J^{PC}	L	S	J^{PC}	L	S	J^{PC}
0	0	0^{-+}	1	0	1^{+-}	2	0	2^{-+}
0	1	1^{--}	1	1	0^{++}	2	1	1^{--}
			1	1	1^{++}	2	1	2^{--}
			1	1	2^{++}	2	1	3^{--}

- Mesons with quantum numbers $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}$ are not possible for $q\bar{q}$, like $\pi_1(1400)$, $\pi_1(1600)$, and $\eta_1(1855)$ with $J^{PC} = 1^{-+}$
- Mesons with ordinary quantum numbers but unlikely to be $q\bar{q}$ states, like scalar mesons with $J^{PC} = 0^{++}$: $f_0(500)$ ($\pi\pi$), $K_0^*(800)$, $a_0(980)$, $f_0(980)$ ($K\bar{K}$), and $a_1(1420)$ with $J^{PC} = 1^{++}$.
- $d^*(2380)(6q)$ with $I(J^P) = 0(3^+)$.

Charmonium-like exotic hadrons



- Charged charmonium-like Z_c and Z_{cS} states are observed
- Neutral charmonium-like X and Y states are very unlikely to be pure $c\bar{c}$ states.
 $X(3872) \rightarrow J/\psi\pi^+\pi^-$ first seen by BELLE(2003), also seen by CDF, D0(2004) and BaBar(2005).

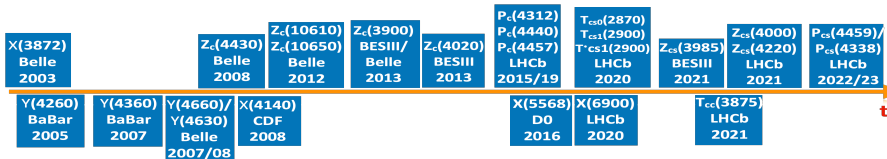


From PRL 91 (2003) 262001

Timeline of the discovery of exotic hadrons

- History of exotic states from experiments:

- Heavy-quark exotics:



- Light-quark exotics:



Possible configurations of exotic hadrons

- QCD does not rule out the existence of more complex hadrons



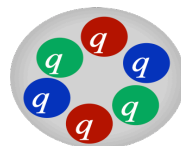
baryon



tetraquark



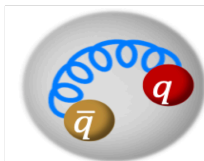
pentaquark



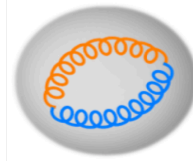
dibaryon



meson



hybrid



glueball

Theoretical pictures of exotic hadron states

- Compact multiquark picture:
 Pentaquark: $q^4\bar{q}$, diquark-diquark-antiquark $q^2q^2\bar{q}$ configurations
 Tetraquark: diquark-antidiquark $q^2\bar{q}^2$, meson-meson $q\bar{q}q\bar{q}$ configurations
 6q: dibaryon q^3q^3 , Hexaquark $q^3\bar{q}^3$
- Hadronic molecular picture:
 Pentaquark: $(q^3)(q\bar{q})$ configurations
 Tetraquark: $(q\bar{q})(q\bar{q})$ configurations
 6q: dibaryon $(q^3)(q^3)$, Hexaquark $(q\bar{q})(q\bar{q})(q\bar{q})$, $(q^3)(\bar{q}^3)$ configurations
- Hybrid picture:
 hybrid baryon q^3G , hybrid meson $q\bar{q}G$

Theoretical models in some reviews

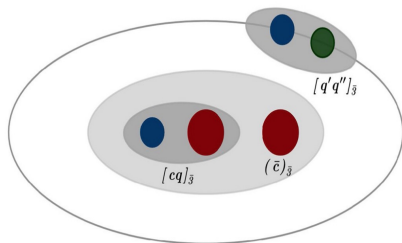
- The hidden-charm pentaquark and tetraquark states, Phys. Rept. **639** (2016) 1
- Exotic hadrons with heavy flavors: X, Y, Z, and related states, Prog. Theor. Exp. Phys. **2016** (2016) 062C01
- Heavy-quark QCD exotica, Progress in Particle and Nuclear Physics **93** (2017)143
- Multiquark resonances, Phys. Rept. **668** (2017) 1
- Hadronic molecules, Rev. Mod. Phys. **90** (2018) 015004
- Exotics: Heavy pentaquarks and tetraquarks, Progress in Particle and Nuclear Physics **97** (2017) 123
- Nonstandard heavy mesons and baryons: Experimental evidence, Rev. Mod. Phys. **90** (2018) 015003
- Pentaquark and tetraquark states, Progress in Particle and Nuclear Physics**107** (2019) 237
- The XYZ states: experimental and theoretical status and perspectives, Phys. Rept. **873** (2020) 154
- Threshold cusps and triangle singularities in hadronic reactions, Progress in Particle and Nuclear Physics **112** (2020) 103757
- Tetra- and pentaquark structures in the constituent quark model, Symmetry **12** (2020) 1869
- Chiral perturbation theory for heavy hadrons and chiral effective field theory for heavy hadronic molecules, Phys. Rept. **1019** (2023) 2266
- An updated review of the new hadron states, Rep. Prog. Phys. **86** (2023) 026201
- Tetraquarks and Pentaquarks from Quark Model Perspective. Symmetry **2023**, 15, 1298.

Theoretical models

- Quark models:
 - Constituent quark models
 - Chiral quark model
 - Color flux-tube model
- Effective field theories
- Dynamical models:
 - coupled channel effect
 - triangle singularity
- QCD sum rules
- Lattice QCD simulations
- Heavy-ion collisions simulations

Constituent quark models

- Phenomenological models to explain hadron mass spectra, hadron structure, resonance excitations and decays, interactions between hadron-hadron
 - Highly model-dependent, depends on assumptions and model parameters
- Hyperfine forces (confinement potential): One-gluon exchange (OGE)(De Rújula et al., 1975), Goldstone-boson exchange (GBE) (Glozman and Riska, 1996), instanton-induced interaction (INS)(t' Hooft, 1976).
- Applied in all compact multiquark, molecule, hybrid and hadrocharmonium pictures.
- Example: In a diquark-diquark-antiquark picture, [PLB **793**, 365 (2019).]
 - all spin-spin interactions between the constituents of the light diquark and heavy triquark are considered to predict s-wave and p-wave hidden-charm pentaquark mass spectra



Other quark models

- Chiral Quark Model: Chiral symmetry is applied+ GBE
 - Mostly applied to explain the hadron-hadron interaction and hadron spectrum.
 - $qqq\bar{q}Q$ states in compact pentaquark and molecular pictures [EPJC **81** (2021) 224].
 - d^* (2380) mass, wave functions, strong decay and form factors, [PRC **91** (2015) 064002; PRC **94** (2016) 014003].
- Color flux-tube model: massless quarks lie at the end of the string and the string tension leads to the mass of the hadrons as its potential energy, applied to compact multiquark and hybrid spectrum.
 - $Y(4626)$ as a P-wave compact tetraquark state $cs\bar{c}\bar{s}$, [PRD **101** (2020) 054039]
 - Applied to $q\bar{q}G$ [PRD **31** (1985) 2910] and q^3G [PRD **85** (2012) 054016]
 - Do not respect local color gauge invariance.

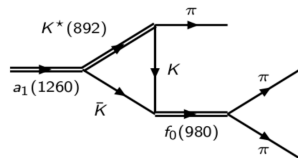


Effective field theories

- General effective models to explain hadron-hadron interactions, dynamical properties - quarks as point-like particles, only focus on interactions, ignore spectrum
- Chiral perturbation theory: mainly for low-energy interactions of hadrons, the two-pole structure of $\Lambda(1405)$ in the hadronic molecular picture [PLB **846**, 138264 (2023).]
- Effective Hamiltonian in a hadrocharmonium picture to describe P_c states and decay widths [MPLA **35**, 2050151 (2020).]
- In a contact-range effective field theory incorporating heavy-quark spin symmetry, P_c states are interpreted as $\Sigma_c \bar{D}^{(*)}$ molecules. [PRL **122**, 242001 (2019).]
- Chiral effective interaction Lagrangians + Covariant confined quark model (CCQM), $a_1(1420)$ interpreted as a $K^* \bar{K}$ molecule. [PRD **96**, 114004 (2017).]
- Applied to $X(3872)$, $Z_c(3900)$, $Z(4430)$, $X(5568)$, $Z_b(10610)$ and $Z_b(10650)$ states.

Dynamical models

- Coupled channel effects: formation and decay of hadronic resonances, hadron structure.
 - highly depends on interacting potentials and coupling constants, hard to disentangle the contribution of individual channel
 - two pole structure of $\Lambda(1405)$, derive the width and shape [EPJC**81** (2021) 582].
 - 9 scalar mesons f_0 except $f_0(500)$ as scalar glueball [PLB**816** (2021) 136227].
- Poles could be created when one or more particles in the triangle loop on shell.
 - Interpretation of $a_1(1420)$ as a triangle singularity in the decay of $a_1(1260)$ to $K^* \bar{K}$ [PRD **91** (2015) 094015; PRD **94**, 096015 (2016)].

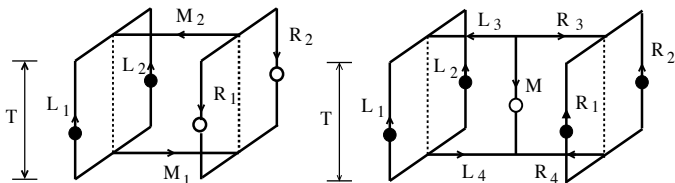


QCD sum rules

- A non-perturbative tool to extract hadron properties, hadrons are represented by their interpolating quark currents taken at large virtualities.
- Obtain mass spectrum, dynamical properties, and hadron structures for certain quantum numbers, determine strong coupling constant
- $a_1(1420)$ with $J^{PC} = 1^{++}$ as a tetraquark state.[PRD 91(2015) 094022]
- Mass and strong decay properties of P_c states in the hadronic molecular picture with QCD sum rules, [PRD **95** (2017) 094016; PLB **782**(2018) 694.]

Lattice QCD

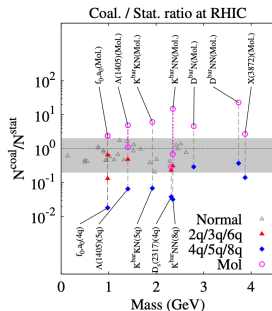
- Lattice QCD methods (LQCD): A non-perturbative tool to provide numerical results describing color confinement between two static quarks. (Not a model)
 - Applied to compact multiquark and hybrid pictures, obtain mass spectrum, hadron structures, and dynamical properties
 - faces computational limitations (multi-channel, 3 or more body decay)
- Tetraquark and Pentaquark with OGE + color flux tube confining potential [PRD **72** (2005) 014505; PRL **94**, 192001],



- Predictions of exotic hybrid mesons: [PRD **88** (2013) 094505] light exotic hybrid around $1.8 \rightarrow 2.1$ GeV and hybrid baryons q^3G : [PRD **85** (2012) 054016]

Heavy-ion collision simulations

- Not a model, but a promising approach to identify the hadronic structures
- Production rates (Yield) of exotic hadrons in different structures are expected to be very different.
 - Ratios of the yields at RHIC in the coalescence model to those in the statistical model [PRL 106(2011) 212001]
- Yield of P_c states in the pentaquark, nucleuslike (hadrocharmonium) and molecular pictures [PRD 91(2022) 094022].



Construction of the wave-function of multi-quark system

The construction of q^3 and $q^4\bar{q}$ state is guided by two main rules.

- The q^3 , $q^4\bar{q}$ and $q^2\bar{q}^2$ wave function should be a color singlet;
- The q^3 , $q^4\bar{q}$ and $q^2\bar{q}^2$ wave function should be antisymmetric under any permutation of any identical quark configuration.

Group theory techniques:

- Yamanouchi basis approach is a powerful method for constructing multi-quark states.
- Permutation symmetry are applied to derive the spatial color spin and flavor wave function for all multi-quark system $q\bar{q}$, q^3 , $q^2\bar{q}^2$, $q^4\bar{q}$, $q^3Q\bar{Q}$ ($q = u, d, s$ quark in SU(3) flavor symmetry; $Q = c, b$ quark), $q^3\bar{q}^3$ (q^6) and so on.

Simple explanation: Pentaquark young tabloid representations in SU(3) flavor symmetry

- $q^4 \bar{q}$ configuration young tabloid construction

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (q^4 \bar{q}) = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (q^4) \otimes \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array} (\bar{q})$$

$$\psi_A = \frac{1}{\sqrt{3}} \left(\psi_{[211]_\lambda}^c \psi_{[31]_\rho}^{osf} - \psi_{[211]_\rho}^c \psi_{[31]_\lambda}^{osf} + \psi_{[211]_\eta}^c \psi_{[31]_\eta}^{osf} \right) \quad (1)$$

- $q^3 Q \bar{Q}$ ($q^3 c \bar{c}, q^3 b \bar{b}$) color singlet and octet young tabloid construction

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (q^3 Q \bar{Q}) = \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} (q^3) \otimes \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} (Q \bar{Q})$$

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (q^3 Q \bar{Q}) = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (q^3) \otimes \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (Q \bar{Q})$$

$$\psi_A = \frac{1}{\sqrt{2}} \left(\psi_{[21]_\lambda}^c \psi_{[21]_\rho}^{osf} - \psi_{[21]_\rho}^c \psi_{[21]_\lambda}^{osf} \right) \quad (2)$$

Tetraquark young tabloid representations in SU(3) flavor symmetry

- $qc\bar{q}\bar{c}$ configuration young tabloid construction

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (qc\bar{q}\bar{c}) = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (\bar{c}\bar{q}) \otimes \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array} (qc)$$

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (qc\bar{q}\bar{c}) = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (\bar{c}\bar{q}) \otimes \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (qc)$$

$$\psi_{[2]_6^6 [22]_6^6}^{qc\bar{q}\bar{c}} = \frac{1}{\sqrt{6}} \sum_{i=1}^6 \psi_{[2]_6^6}^{qc} \psi_{[22]_6^6}^{\bar{q}\bar{c}}, \quad \psi_{[11]_3^3 [211]_3^3}^{qc\bar{q}\bar{c}} = \frac{1}{\sqrt{3}} \sum_{i=1}^3 \psi_{[11]_3^3}^{qc} \psi_{[211]_3^3}^{\bar{q}\bar{c}} \quad (3)$$

- Color parts for two quarks and two antiquarks young tabloid

$$\begin{array}{|c|} \hline \\ \hline \end{array} (q) \otimes \begin{array}{|c|} \hline \\ \hline \end{array} (c) = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} \oplus \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array} (qc)$$

$$\begin{array}{|c|} \hline \\ \hline \\ \hline \end{array} (\bar{q}) \otimes \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array} (\bar{c}) = \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} (\bar{q}\bar{c})$$

Constituent quark model with Cornell-like potential

- The realistic Hamiltonian:

$$\begin{aligned}
 H &= H_0 + H_{hyp}^{OGE} + H_{q^3}^{SO}, \\
 H_0 &= \sum_{k=1}^N \left(m_k + \frac{p_k^2}{2m_k} \right) + \sum_{i<j}^N \left(-\frac{3}{8} \lambda_i^C \cdot \lambda_j^C \right) \left(A_{ij} r_{ij} - \frac{B_{ij}}{r_{ij}} \right), \\
 H_{hyp}^{OGE} &= -C_{OGE} \sum_{i<j} \frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j,
 \end{aligned} \tag{4}$$

$$A_{ij} = a \sqrt{\frac{m_{ij}}{m_u}}, \quad B_{ij} = b \sqrt{\frac{m_u}{m_{ij}}} \tag{5}$$

- The 3 model coupling constants and 4 constituent quark masses are fitted,

$$\begin{aligned}
 m_u = m_d &= 327 \text{ MeV}, & m_s &= 498 \text{ MeV}, \\
 m_c &= 1642 \text{ MeV}, & m_b &= 4960 \text{ MeV}, \\
 C_m &= 18.3 \text{ MeV}, & a &= 49500 \text{ MeV}^2, & b &= 0.75
 \end{aligned} \tag{6}$$

Mass of ground state $q^4\bar{q}$ pentaquarks.

J^P	$q^3 s\bar{q}$ configurations	$(S^{q^3s}, S^{\bar{q}}, S)$	$M^{EV} (q^3 s\bar{q})$
$\frac{5}{2}^-$	$\Psi_{[31]_F[4]_S}^{sf} (q^4\bar{q})$	(2,1/2,5/2)	2269
$\frac{3}{2}^-$	$\Psi_{[4]_F[31]_S}^{sf} (q^4\bar{q})$	(1,1/2,3/2)	2269
	$\left(\begin{array}{l} \Psi_{[31]_F[4]_S}^{sf} (q^4\bar{q}) \\ \Psi_{[31]_F[31]_S}^{sf} (q^4\bar{q}) \end{array} \right)^{mix1}$	(2,1/2,3/2) (1,1/2,3/2)	(1805) (2269)
	$\Psi_{[22]_F[31]_S}^{sf} (q^4\bar{q})$	(1,1/2,3/2)	2049
$\frac{1}{2}^-$	$\Psi_{[4]_F[31]_S}^{sf} (q^4\bar{q})$	(1,1/2,1/2)	2562
	$\left(\begin{array}{l} \Psi_{[31]_F[31]_S}^{sf} (q^3 s\bar{q}) \\ \Psi_{[31]_F[22]_S}^{sf} (q^4\bar{q}) \end{array} \right)^{mix2}$	(1,1/2,1/2) (0,1/2,1/2)	(1986) (2162)
	$\Psi_{[22]_F[31]_S}^{sf} (q^4\bar{q})$	(1,1/2,1/2)	1683

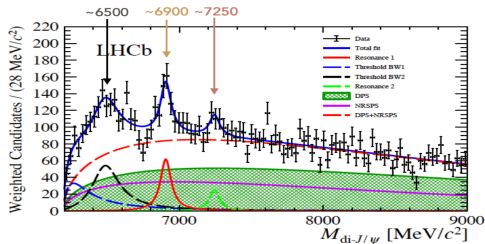
- An isospin 1/2 narrow resonance $N^+(1685)$ ($\Gamma \leq 30$ MeV) firstly reported in GRAAL [PLB **647**: 23-29 (2007)]. Confirmed in A2@Mainz, CBELSA/TAPS, and LNS-Sendai.
- $N^+(1685)$ cannot be accommodated in the q^3 picture.
- The lowest compact pentaquark state?? [PRD. **101**, 076025 (2020)]

Mass of fully-charm tetraquark states

- Compact $cc\bar{c}\bar{c}$ tetraquark states of various configurations.

$cc\bar{c}\bar{c}$ configurations	$I^G J^{PC}$	M(1S)	M(2S)	M(3S)
$\Psi^{cs}_{(6\otimes\bar{6})}[(cc)s=0\otimes(\bar{c}\bar{c})s=0]S=0$	0^+0^{++}	6514	6840	7098
$\Psi^{cs}_{(\bar{3}\otimes 3)}[(cc)s=0\otimes(\bar{c}\bar{c})s=0]S=0$	0^+0^{++}	6466	6883	7225
$\Psi^{cs}_{(\bar{3}\otimes 3)}[(cc)s=1\otimes(\bar{c}\bar{c})s=1]S=1$	0^-1^{+-}	6494	6911	7253
$\Psi^{cs}_{(\bar{3}\otimes 3)}[(cc)s=1\otimes(\bar{c}\bar{c})s=1]S=2$	0^+2^{++}	6551	6968	7310

- $X(6900)$ might be the first radial excited state with $J^{PC} = 1^{+-}$ in the $\bar{3}_c \otimes 3_c$ configuration [PRD. **103**, 116027 (2021)]



Summary and outlook

- The nature of any exotic hadron still remains an open question with diverged interpretations. Our understanding is still very limited.
- Look forward more experimental data from LHCb, BESIII, Belle II, JLab, EIC, PANDA, ... to reject theoretical models.

Thank You Very Much For Your Attentions!

Mass of charmoniumlike tetraquark states

- The compact $qc\bar{q}\bar{c}$ and $cc\bar{c}\bar{c}$ tetraquark states of different configurations are also estimated in a constituent quark model[Phys. Rev. D. **103**, 116027 (2021)]

$qc\bar{q}\bar{c}$ configurations	$\psi_{0,\otimes_3}^{S=0}$		$\psi_{1,\otimes_1}^{S=0}$		$\psi_{1,\otimes_3}^{S=1}$		$\psi_{1,\otimes_1}^{S=1}$		$\psi_{1,\otimes_1}^{S=2}$	
	0 ⁺ 0 ⁺⁺ /1 ⁻ 0 ⁺⁺		0 ⁺ 0 ⁺⁺ /1 ⁻ 0 ⁺⁺		0 ⁻ 1 ⁺⁻ /1 ⁺ 1 ⁺⁻		0 ⁻ 1 ⁺⁻ /1 ⁺ 1 ⁺⁻		0 ⁺ 2 ⁺⁺ /1 ⁻ 2 ⁺⁺	
	Ours	Data	Ours	Data	Ours	Data	Ours	Data	Ours	Data
$\Psi_{6_c\otimes\bar{6}_c}^c(1S)$	4202	$Z_c(4250)$	3925	$X(3915)$	4162	$Z_c(4200)$	4024	$Z_c(4020)/Z_c(4055)$	4221	...
$\Psi_{6_c\otimes\bar{6}_c}^c(2S)$	4566	...	4289	$X(4350)$	4526	$Z_c(4430)$	4388	...	4584	...
$\Psi_{\bar{3}_c\otimes\bar{3}_c}^c(1S)$	4033	$Z_c(4050)$	4114	$Z_c(4100)$	4113	$X(4160)$	4154	$X(4160)$	4233	...
$\Psi_{\bar{3}_c\otimes\bar{3}_c}^c(2S)$	4434	...	4516	...	4514	...	4555	...	4634	...

Spatial and spin-flavor wave functions

- The spatial wave functions are in symmetric type for ground state pentaquarks. And we constructed the spatial wave functions to high orders in the harmonic oscillator interaction as complete bases.
- All possible spin-flavor $[31]$ configurations of q^4 cluster

$$[31]_{FS}$$

$$[31]_{FS}[31]_F[22]_S \quad [31]_{FS}[31]_F[31]_S \quad [31]_{FS}[31]_F[4]_S \quad [31]_{FS}[211]_F[22]_S$$

$$[31]_{FS}[211]_F[31]_S \quad [31]_{FS}[22]_F[31]_S \quad [31]_{FS}[4]_F[31]_S$$

- All possible spin-flavor $[21]$ configurations of q^3 cluster

$$[21]_{FS}$$

$$[21]_{FS}[21]_F[3]_S \quad [21]_{FS}[3]_F[21]_S \quad [21]_{FS}[21]_F[21]_S$$

Constituent quark model

- The constituent quark model describes the observed meson spectrum as $q\bar{q}$ states. [Review "Quark Model", RPP(2023)]

n	$2s+1\ell_J$	J^{PC}	$l = 1$ $u\bar{d}, \bar{u}d, \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$l = \frac{1}{2}$ $u\bar{s}, \bar{d}s; \bar{d}s, -\bar{u}s$	$l = 0$ f'	$l = 0$ f	θ_{quad} [°]	θ_{lin} [°]
1	1S_0	0^{-+}	π	K	η	$\eta'(958)$	-11.4	-24.5
1	3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.1	36.4
1	1P_1	1^{+-}	$b_1(1235)$	K_{1B}^\dagger	$h_1(1380)$	$h_1(1170)$		
1	3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
1	3P_1	1^{++}	$a_1(1260)$	K_{1A}^\dagger	$f_1(1420)$	$f_1(1285)$		
1	3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	32.1	30.5
1	1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
1	3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
1	3D_2	2^{--}		$K_2(1820)$				
1	3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
1	3F_4	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
1	3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
1	3H_6	6^{++}	$a_6(2450)$			$f_6(2510)$		
2	1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
2	3S_1	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		

Partial decay widths of $I = \frac{1}{2}$ mixing pentaquark states

- There are six mass eigenstates of isospin 1/2 below the mass threshold. One may name them $X(4298)$, $X(4426)$, $X(4444)$, $X(4457)$, $X(4378)$ and $X(4509)$.
- We varied the mass of both the compact pentaquark states and the hadronic molecules to check the stability of X states.

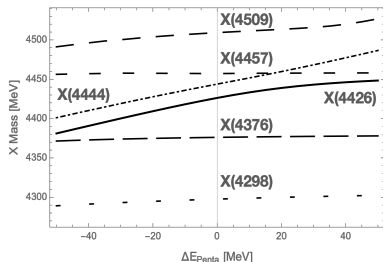


Figure: X mass dependence on the mass of pentaquark components

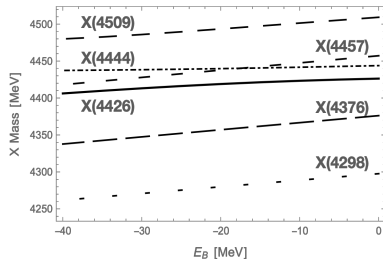


Figure: X mass dependence on the binding energy of molecular states.

Partial decay widths of $I = \frac{1}{2}$ mixing pentaquark states

- Partial decay width ratios of $I = 1/2$ mixing pentaquark states, normalized to 4457 MeV state.

J	Threshold	Mass	Eigenvector ²	Total	$p\eta_c$	pJ/ψ	$\Sigma_c^* \bar{D}$	$\Sigma_c \bar{D}$	$\Lambda_c^+ \bar{D}$	$\Sigma_c^* \bar{D}^*$	$\Sigma_c \bar{D}^*$	$\Lambda_c^+ \bar{D}^*$
	$\Sigma_c \bar{D}(4322)$	4298	(0.88,0.05,0.02,0.06)	0.57	0.21	0.11						0.25
$\frac{1}{2}$	$\Sigma_c \bar{D}^*(4462)$	4444	(0.22,0.38,0.10,0.30)	14.71	0.01	0.13		9.99	2.74			1.85
	$\Sigma_c \bar{D}^*(4462)$	4426	(0.24,0.10,0.47,0.20)	17.53	0.01	0.15		10.61	1.63			5.13
	$\Sigma_c^* \bar{D}^*(4526)$	4509	(0.77,0.10,0.13,0)	1.87		0.28	0.08				0.43	1.08
$\frac{3}{2}$	$\Sigma_c^* \bar{D}(4386)$	4376	(0.95,0.05,0,0)	1.06		0.35						0.71
	$\Sigma_c \bar{D}^*(4462)$	4457	(0.95,0.02,0.01,0.02)	1.00		0.09	0.61					0.31

- $X(4298)$, $X(4457)$, $X(4378)$ and $X(4509)$ are dominantly hadronic molecules while $X(4426)$ has considerable both the molecular and compact pentaquark components.
- $P_c(4312)$, $P_c(4457)$ and $P_c(4380)$ resonances might be mainly $\Sigma_c \bar{D}$, $\Sigma_c \bar{D}^*$ and $\Sigma_c^* \bar{D}$ hadronic molecules respectively, and $P_c(4440)$ might include sizable pentaquark components.