Unintegrated gluon distributions in the BGK saturation model

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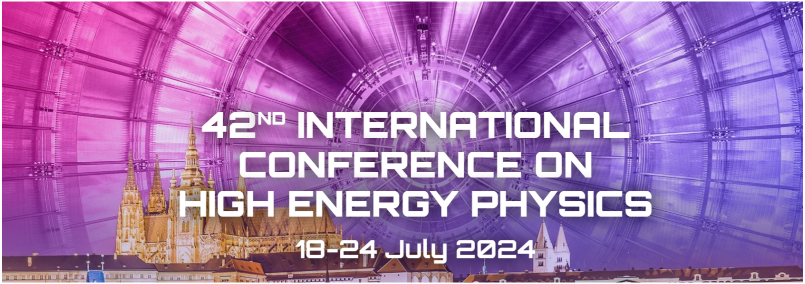

Introduction

1. We evaluate the unintegrated gluon distribution of the proton starting from a parametrization of the color dipole cross section including Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution and saturation effects.
2. We perform the Fourier-Bessel transform of $\sigma(x,r)/\alpha(r)$. At large transverse momentum of gluons we match the so-obtained distribution to the logarithmic derivative of the collinear gluon distribution.
3. We check our approach by calculating the proton structure function $F_L(x,Q^2)$ finding good agreement with HERA data.

A. Łuszczak, M. Łuszczak, W. Schafer,
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Unintegrated gluon distributions from the dipole cross section

Bartels–Golec-Biernat–Kowalski (BGK) approach

$$\sigma(x, r) = \sigma_0 \left(1 - \exp\left[- \frac{\sigma_{\rm DGLAP}(x,r)}{\sigma_0} \right]\right) ,$$

for small dipole sizes, $\sigma(x, r)$ will be proportional to the DGLAP -evolving collinear gluon distribution of the proton, $\sigma(x, r) \to \sigma_{\rm DGLAP}(x, r)$:

$$\sigma_{\rm DGLAP}(x, r) = \frac{\pi^2}{N_c} \alpha(r) r^2 x g_{\rm DGLAP}\left(x, \frac{C}{r^2} + \mu_0^2\right) ,$$

with $N_c = 3$, and

$$\alpha(r) \equiv \alpha_s \left(\frac{C}{r^2} + \mu_0^2 \right) .$$

for large dipole sizes, the dipole cross section approaches a constant

$$\sigma(x, r) \to \sigma_0 ,$$

while the strong coupling freezes at large r:

$$\alpha(r) \to \alpha_{\rm fr} = \alpha_s(\mu_0^2) .$$

The behaviour in the infrared is thus governed by σ_0 and μ_0 which are parameters to be fitted to an appropriate experimental observable. The dimensionless number C is fixed as $C = 4$.

Unintegrated gluon distributions from the dipole cross section

The gluon density, which is parametrized at the starting scale μ_0^2, is evolved to larger scales, μ^2 using NLO DGLAP evolution.

We consider the following form of the gluon density:

$$xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1 - x)^{C_g},$$

the *soft* ansatz, as used in the original BGK model:

- The free parameters for this model are σ_0 and the parameters for gluon A_g, λ_g, C_g. Their values are obtained by a fit to the data. It is also possible to vary the parameter μ_0^2. However, to assure that the evolution is performed in the perturbative region and to be compatible with the standard pdf fits, we took as a starting scale $\mu_0^2 = 1.9\,\mathrm{GeV}^2$. In the BGK model, the μ_0^2 scale is the same as the Q_0^2 scale of the standard QCD pdf fits.
- These values were chosen in **A. Luszczak and H. Kowalski, Phys. Rev. D 95, 014030 (2017), 1611.10100** as giving the best fit of HERA data. Lower values $\mu_0^2 = 1.1\,\mathrm{GeV}^2$ give similar results but a somewhat worse fit.

Unintegrated gluon distributions from the dipole cross section

The color dipole cross section is related to the UGD of the target as (Nikolaev,1994):

$$\sigma(x,r) = \frac{4\pi}{N_c}\,\alpha(r)\int \frac{d^2}{^4}\mathcal{F}(x,)\Big[1-\exp(i\boldsymbol{r})\Big]$$
$$= \frac{\pi^2}{N_c}\alpha(r)r^2\int_0^\infty \frac{d^2}{^2}\,\frac{4[1-J_0(||r)]}{^2 r^2}\mathcal{F}(x,)\,.$$

Before we proceed to inverting the Fourier transform (FT) of below eq. we introduce:

$$f(x,\kappa) \equiv \frac{4\pi}{N_c}\,\frac{\mathcal{F}(x,\kappa)}{\kappa^4}\,.$$

Then, by performing the FT we obtain:

$$\int d^2\boldsymbol{r}\exp[-i\boldsymbol{kr}]\,\frac{\sigma(x,r)}{\alpha(r)} = (2\pi)^2\delta^{(2)}(\boldsymbol{k})\int d^2 f(x,) - (2\pi)^2 f(x,\boldsymbol{k}),$$

or, alternatively

$$f(x,\boldsymbol{k}) = \frac{\sigma_0}{\alpha_{\rm fr}}\int\frac{d^2\boldsymbol{r}}{(2\pi)^2}e^{-i\boldsymbol{kr}}\left(1-\frac{\sigma(x,r)}{\sigma_0}\,\frac{\alpha_{\rm fr}}{\alpha(r)}\right).$$

Unintegrated gluon distributions from the dipole cross section

We can finally write for the unintegrated glue $\mathcal{F}(x, \boldsymbol{k})$ as:

$$\mathcal{F}(x, \boldsymbol{k}) \quad = \quad \boldsymbol{k}^4 \frac{\sigma_0}{\alpha_{\rm fr}} \frac{N_c}{8\pi^2} \int_0^\infty r dr \, J_0(|\boldsymbol{k}| r) \Big(1 - \frac{\sigma(x, r)}{\sigma_0} \frac{\alpha_{\rm fr}}{\alpha(r)} \Big)$$

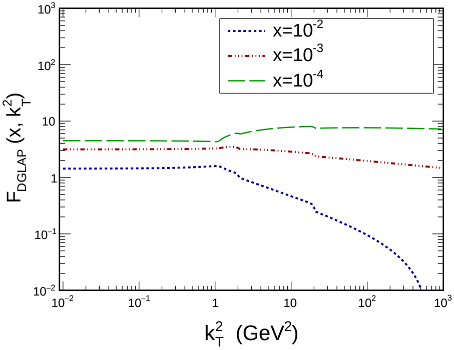
- The so defined unintegrated glue depends only on the gluon longitudinal momentum fraction and transverse momentum. This is a general feature of k_T-factorization, especially of approaches typically used at small-x.
- In order to be able to extend the UGD to large $\boldsymbol{k}^2$, we match the FT with the logarithmic derivative of the collinear glue of the BGK fit:

$$\mathcal{F}_{\rm DGLAP}(x, \boldsymbol{k}) = \frac{\partial x g_{\rm DGLAP}(x, \boldsymbol{k}^2)}{\partial \log \boldsymbol{k}^2}$$

We find, that by choosing $\boldsymbol{k}^2_{\rm match} = 20 \, {\rm GeV}^2$, we can find an excellent reproduction of the original $\sigma(x, r)$ when inserting the so-matched UGD back into dipole eq.

Numerical results

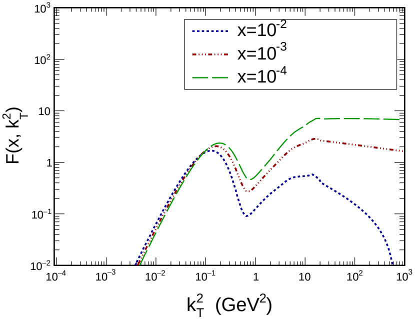
Unintegrated glue in the DGLAP approximation



- we show the logarithmic $\boldsymbol{k^2}$-derivative of the input collinear glue as a function of $\boldsymbol{k^2}$ for three values of x
- we observe a hard tail at $\boldsymbol{k^2} > 1\,\mathrm{GeV}^2$, which is rapidly increasing at small x
- there is also a substantial x-dependence of the soft plateau at $\boldsymbol{k^2} < 1\,\mathrm{GeV}^2$

Unintegrated glue from the Fourier transform

merged with the DGLAP UGD at $\boldsymbol{k}^2_{\rm match} = 20\,\mathrm{GeV}^2$.



- Our main result, the UGD related to the BGK dipole cross section.
- A quite peculiar shape emerges, with a pronounced bump in the soft region and a local mimimum in the $\boldsymbol{k^2} \sim 1\,\mathrm{GeV}^2$ region. While the fall-off for $\boldsymbol{k^2} \to 0$ has only a very weak x-dependence, the latter is stronger in the region between the local maximum and minimum in the soft region. At perturbatively large $\boldsymbol{k^2}$, there is a substantial low-x growth of the UGD. The tail at $\boldsymbol{k^2} > \boldsymbol{k^2_{\rm match}} = 20\,\mathrm{GeV}^2$ finally is inherented from the log-derivative of the collinear glue.

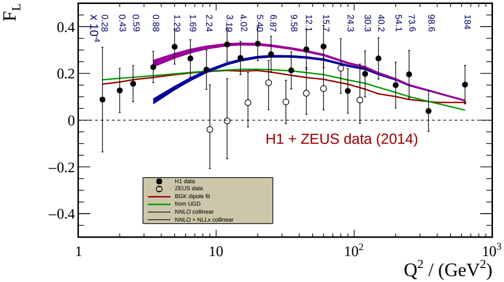


Figure: Our results for the structure function $F_L(x, Q^2)$ using the dipole approach (red curve) and the k_T-factorization (green curve). The blue and magenta curves show the collinear factorization results without and with small-x resummation respectively

- our results for $F_L(x, Q^2)$ compared to the HERA data of the H1 and ZEUS collaborations

Unintegrated gluon distributions from the dipole cross section

We show numerical results following two methodologies. The longitudinal structure function F_L is related to the virtual photoabsorption cross section for longitudinally polarized photons as

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\rm em}} \sigma_L(x, Q^2)$$

We evaluate $\sigma_L(x, Q^2)$ in two different methodologies.

Firstly, from the dipole approach:

$$\sigma_L(x, Q^2) = \sum_{f\bar{f}} \int_0^1 dz \int d^2\boldsymbol{r} \left| \Psi_{\gamma^*}^{(f\bar{f})}(z, \boldsymbol{r}, Q^2) \right|^2 \sigma(x, r)$$

Secondly, from the k_T-factorization approach :

$$\sigma_L(x, Q^2) = \frac{\alpha_{\rm em}}{\pi} \int_0^1 dz \int d^2\boldsymbol{k} \int \frac{d^2}{4} \alpha_s(q^2) \mathcal{F}(x_g, \not{x}) \; 4Q^2 z^2 (1-z)^2 \left(\frac{1}{\boldsymbol{k}^2 + \varepsilon^2} - \frac{1}{(\boldsymbol{k}-)^2 + \varepsilon^2} \right)^2 .$$

Conclusions

- We have presented an unintegrated gluon distribution obtained from a BGK-type fit of the color dipole cross section to HERA structure function data. We argued that the Fourier transform to momentum space is best performed on $\sigma(x, r)/\alpha(r)$ instead of the dipole cross section itself.
- The so-obtained UGD was then matched to a perturbative tail calculated from the DGLAP-evolved collinear glue that enters the BGK construction. We have obtained a good agreement of the proton structure function F_L, using both the color dipole factorization and k_T-factorization.
- An interesting future extension would be to study the impact parameter dependent gluon distribution.