NLP Correction to H+jet Production

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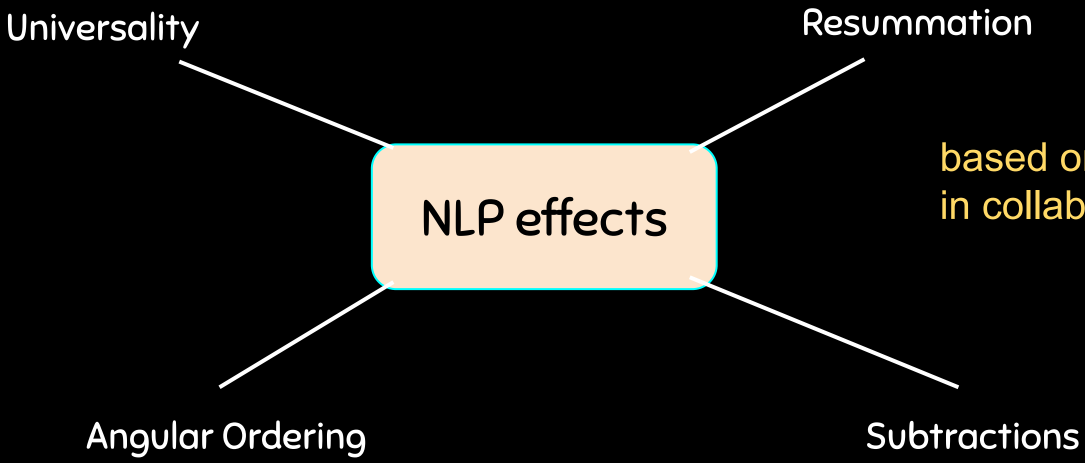

based on 2405.06444, 2309.08343,
in collaboration with Satyajit Seth

What is NLP?

Scattering cross-section in threshold expansion

$$\frac{d\sigma}{d\xi} \approx \sum_{n=0}^{\infty} \alpha_s^n \left\{ \sum_{m=0}^{2n-1} C_{nm} \left(\frac{\log^m \xi}{\xi} \right)_+ + d_n \delta(\xi) + \sum_{m=0}^{2n-1} D_{nm} \log^m \xi \right\} .$$

Next-to-Leading Power (NLP) terms,

Obtained using next-to-soft approximation.

Not very well Known in literature

No general method of resummation is known

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Del Duca, Laenen et. al.

Coloured processes: photon +jet 1905.08741

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Higgs+ jet production: SP, Seth

PRD 109, 114018 (2024), 2309.08343

arXiv: 2405.06444

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Next-to-soft gluon radiation: Amplitude Square

$$\mathcal{A}^2_{\mathrm{LP+NLP}} = \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \mathcal{A}^2_{\mathrm{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)$$

$$\delta p_1 = -\frac{1}{2}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} p_1 - \frac{p_1 \cdot k}{p_1 \cdot p_2} p_2 + k\right), \quad \delta p_2 = -\frac{1}{2}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} p_2 - \frac{p_2 \cdot k}{p_1 \cdot p_2} p_1 + k\right)$$

Shifts in momentum produce NLP terms

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Can we do this at the colour order helicity amplitudes?

Outline

Spinor Shifts to calculate NLP Amplitude

Soft quark formalism

Gluon fusion to H+1 jet production

Soft quark contributions : How different are they from gluon corrections

Universality

Soft and next-to-soft gluon radiation: shifts

General n-particle amplitude

1411.1669, 1404.5551
Strominger et. al.

$$\mathcal{A} = \mathcal{A}_n\left(\{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\} \right)$$

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Emission of a soft gluon $\quad p_k, |k\rangle \to \lambda|k\rangle, |k] \to |k]$

$$\mathcal{A}_{n+1}\Big(\{\lambda|k\rangle, |k]\}, \{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\} \Big) = \Big(S^{(0)} + S^{(1)} \Big) \mathcal{A}_n\Big(\{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\} \Big).$$

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LP and NLP contributions

$$S^{(0)} = \frac{\langle n1\rangle}{\langle k1\rangle \langle nk\rangle} \equiv \frac{1}{\lambda^2} \qquad S^{(1)} = \frac{1}{\langle k1\rangle} |k] \frac{\partial}{\partial |1]} - \frac{1}{\langle kn\rangle} |k] \frac{\partial}{\partial |n]} \equiv \frac{1}{\lambda}$$

H +1 jet production via gluon fusion

Simplest case in SM to test a method with a massive colourless particle.

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G\, H\, \text{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

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$$\mathcal{A}_n(p_i, h_i, c_i) = i \left(\frac{\alpha_s}{6\pi v}\right) g_s^{n-2} \sum_{\sigma \in \mathcal{S}_{n'}} \mathrm{Tr}\left(\mathbf{T}^{c_1} \mathbf{T}^{c_2} \ldots \mathbf{T}^{c_n}\right) A_n^{\{c_i\}}\left(h_1\, h_2\, h_3 \ldots h_n; H\right) .$$

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Gluon fusion: $g(p_1) + g(p_2) \to H(-p_3) + g(-p_4)$

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$$\mathcal{A}^{124}_{+++} = \frac{m_H^4}{\langle 12\rangle \langle 24\rangle \langle 41\rangle}, \qquad \mathcal{A}^{124}_{-++} = \frac{[24]^3}{[12][14]}$$

Next-to-soft contributions

$$\mathcal{A}^{1245}_{h_1 h_2 h_4 +} = \frac{\langle 14\rangle}{\langle 15\rangle\langle 45\rangle}\mathcal{A}^{1'2\,4'}_{h_1 h_2 h_4} \qquad |1'] = |1] + \frac{\langle 45\rangle}{\langle 41\rangle}|5],\ |4'] = |4] + \frac{\langle 15\rangle}{\langle 14\rangle}|5]$$

SP, Seth, 2309.08343

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$$\mathcal{A}^{1254}_{h_1 h_2 h_4 +} = \frac{\langle 24\rangle}{\langle 25\rangle\,\langle 45\rangle}\mathcal{A}^{1\,2'\,4'}_{h_1 h_2 h_4} \qquad |2'] = |2] + \frac{\langle 45\rangle}{\langle 42\rangle}|5],\ |4'] = |4] + \frac{\langle 25\rangle}{\langle 24\rangle}|5]$$

$$\mathcal{A}^{1524}_{h_1 h_2 h_4 +} = \frac{\langle 12\rangle}{\langle 15\rangle\,\langle 52\rangle}\mathcal{A}^{1'\,2'\,4}_{h_1 h_2 h_4} \qquad |1'] = |1] + \frac{\langle 25\rangle}{\langle 21\rangle}|5],\ |2'] = |2] + \frac{\langle 15\rangle}{\langle 12\rangle}|5]$$

SP, Seth, 2309.08343

No NMHV contribution@NLP

$$\mathcal{A}^{124}_{-++} = \frac{[24]^3}{[12][14]}$$

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Matches with full calculation@NLP

Born	Helicity of extra emission	NLP
$\mathcal{A}_{+++}$	$+$	$\mathcal{A}_{++++}\vert_{\mathrm{NLP}}$
	-	$\mathcal{A}_{+++-}\vert_{\mathrm{NLP}}$
$\mathcal{A}_{-++}$	$+$	$\mathcal{A}_{-+++}\vert_{\mathrm{NLP}}$
	-	0
$\mathcal{A}_{+-+}$	$+$	$\mathcal{A}_{+-++}\vert_{\mathrm{NLP}}$
	-	0
$\mathcal{A}_{++-}$	$+$	$\mathcal{A}_{++-+}\vert_{\mathrm{NLP}}$
	-	0

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NLP logarithms

$$s_{12}^2 \frac{d^2\sigma_{++++}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{16\pi\left(s_{12}\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)+2\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)+16\pi\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\times\frac{1}{m_H^2}\mathcal{A}^2_{+++}\,.$$

$$s_{12}^2 \frac{d^2\sigma_{-+++}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{13}}-\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)+4\pi\left(\frac{3}{s_{13}}-\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{-++}\,.$$

$$s_{12}^2 \frac{d^2\sigma_{++-+}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)-4\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{++-}\,.$$

$$s_{12}^2 \frac{d^2\sigma_{+-++}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{23}}-\frac{1}{s_{13}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)+4\pi\left(\frac{3}{s_{23}}-\frac{1}{s_{13}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{+-+}\,.$$

$$s_{12}^2 \frac{d^2\sigma_{+++-}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}} = \mathcal{F}\left\{-16\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right)-4\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{+++}+s_{12}^2\frac{d^2\sigma_{++++}}{ds_{13}ds_{23}}\bigg|_{\text{NLP-LL}}\,.$$

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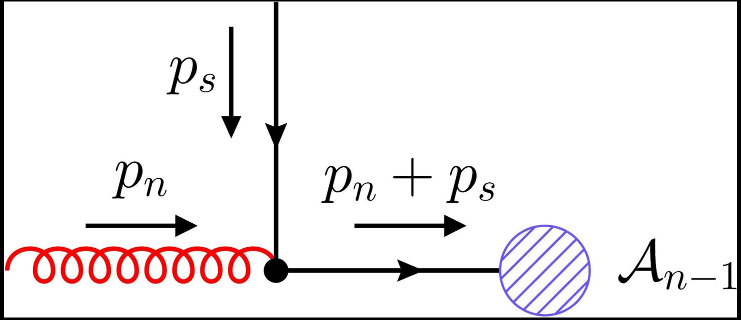
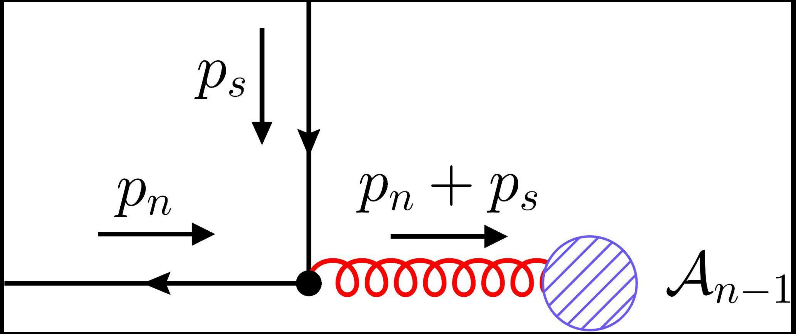
Soft quark contributions

Soft fermions do not contribute at Leading Power (LP): Power Suppressed





$$\bar{u}_{h_s}(p_s)\not{\epsilon}\,\frac{\not{p}_n}{2p_n\cdot p_s}\mathcal{A}_n \approx \mathcal{O}\left(\frac{1}{\sqrt{p_s}}\right)$$

Contributes at Next-to-leading power.

Soft Quark operators in helicity basis

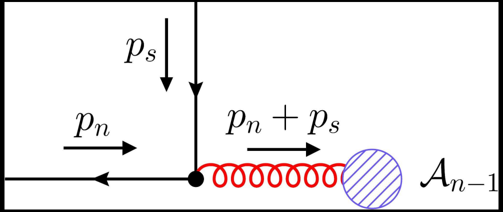
Only two operators are sufficient

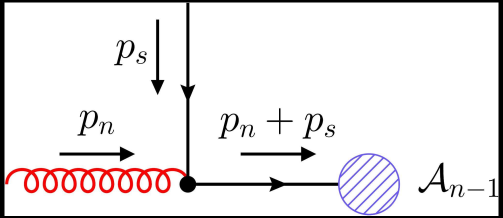
SP, Seth

Act on Colour ordered amplitudes.

2405.06444

Soft Quark Operator	Explicit Form
$\mathcal{Q}^{q_s^+ \bar{q}_h^- \to g_c^-}$	$-\frac{1}{\langle sh \rangle}$
$\mathcal{Q}^{g_h^+ q_s^+ \to q_c^+}$	$-\frac{1}{\langle sh \rangle}$





Simple and general!

Sample result: $g(p_1) + q(p_2) + H(-p_3) + \bar{q}(-p_4) + g(-p_5) \to 0$

Soft anti-quark:

$$s_{12}^2 \frac{d^2\sigma^{++-+}_{1_g\,2_q\,4_{\bar{q}(s)}\,5_g}}{ds_{13}\,ds_{23}}\Bigg|_{\text{NLP-LL}} = \mathcal{F}_{qg}\left\{4\pi N\left(\frac{1}{s_{13}}\right)\log\left(\frac{s_{45}}{\mu^2}\right)\right\}\left|\mathcal{A}^{+++}_{1_g\,2_g\,5_g}\right|^2,$$

$$s_{12}^2 \frac{d^2\sigma^{-+-+}_{1_g\,2_q\,4_{\bar{q}(s)}\,5_g}}{ds_{13}\,ds_{23}}\Bigg|_{\text{NLP-LL}} = \mathcal{F}_{qg}\left\{2\pi N\left[\left(\frac{3}{s_{13}}\right)\log\left(\frac{s_{45}}{\mu^2}\right) + \frac{2}{s_{13}}\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right] - \frac{2\pi}{N}\left(\frac{1}{s_{13}}\right)\log\left(\frac{s_{45}}{\mu^2}\right)\right\}\left|\mathcal{A}^{-++}_{1_g\,2_g\,5_g}\right|^2,$$

Soft gluon

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SP, Seth, 2405.06444

Universality?

NLP amplitude analysis:

Pseudo-scalar Higgs and Higgs NLP corrections differ only in prefactors.

Logarithms are same

Does it indicate universal nature? Need to be explored for many more processes.

SP, Seth, 2405.06444

Conclusion

Developed a method to calculate NLP corrections using modern amplitude methods.

Works at the level of colour order amplitudes.

Phase-space parametrisation and integrations are simple after squaring the amplitude.

Soft quarks are simple to handle in colour ordered helicity amplitudes.

Universal structure of logarithms for scalar and pseudo-scalar Higgs production.

Thank You

Backup slides

Next-to-soft gluon radiation: Amplitude

$$\mathcal{A}^{\sigma}_{\rm NLP} = \sum_{i=1}^{2} \mathbf{T}_i \left(\frac{2\,p_i^\sigma - k^\sigma}{2\,p_i\cdot k} - \frac{i k_\alpha \Sigma_i^{\sigma\alpha}}{p_i\cdot k} - \frac{i k_\alpha L_i^{\sigma\alpha}}{p_i\cdot k} \right) \otimes \mathcal{A}_{\rm LO}$$

$$\mathcal{O}\left(\frac{1}{k}\right) + \mathcal{O}(k^0) \qquad \mathcal{O}(k^0) \qquad \mathcal{O}(k^0)$$

Low 1958,
Knoll and Burnell 1968,
Del-Duca 1990