Collaborative Research Center TRR 257
P3H
Particle Physics Phenomenology after the Higgs Discovery

Research Training Group
Physics of the Heaviest
Particles at the LHC

TTK
Institut für
Theoretische
Teilchenphysik und
Kosmologie

RWTH AACHEN
UNIVERSITY

Top-Bottom Interference Contribution to Fully-Inclusive Higgs Production

Felix Eschment

with M. Czakon, M. Niggetiedt, R. Poncelet, T. Schellenberger

based on *Phys.Rev.Lett*. 132 (2024) 21, 211902

19 July 2024, ICHEP 2024

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and 2407.12413

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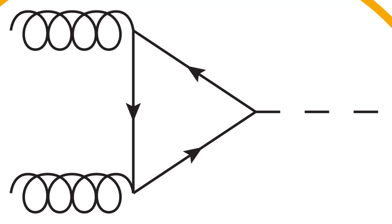
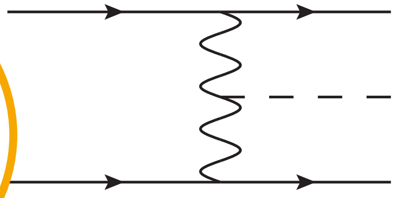
Motivation

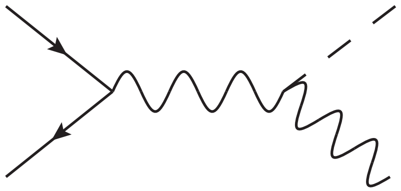
- Higgs production cross section central observable in Higgs physics
- HL-LHC anticipates $\mathcal{O}(2\%)$ uncertainty



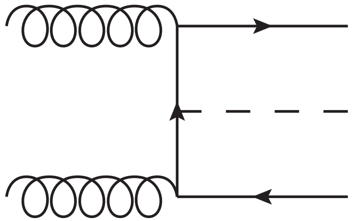
pp → H (gluon fusion)



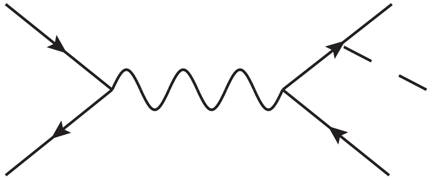
pp → qqH (VBF)



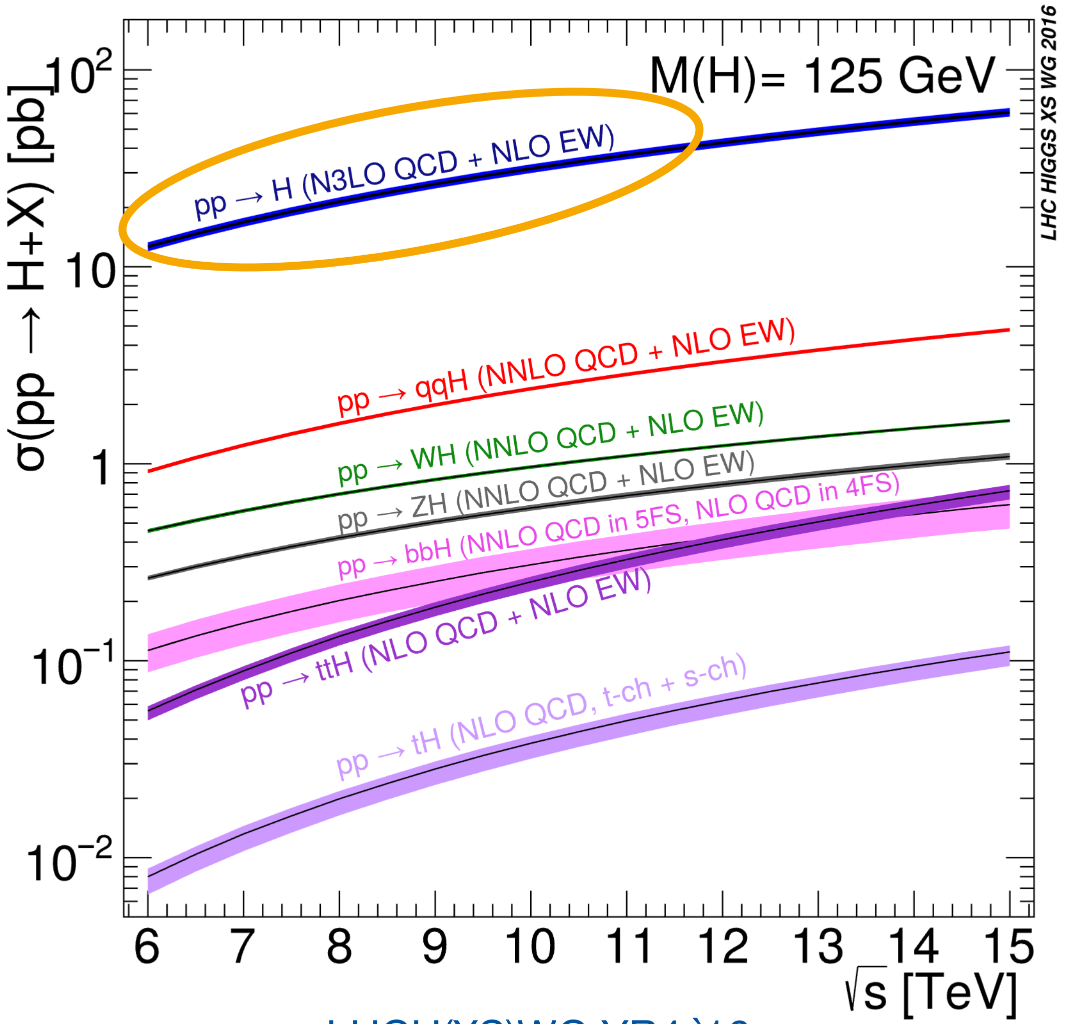
pp → WH / ZH (Higgs Strahlung)



pp → ttH/bbH



pp → tH

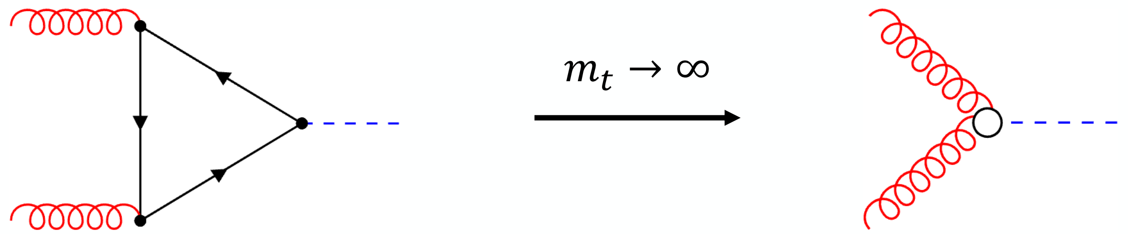


LHCH(XS)WG YR4 `16

→ Crucial to reduce theory uncertainty on gluon-fusion cross section as much as possible

Gluon fusion

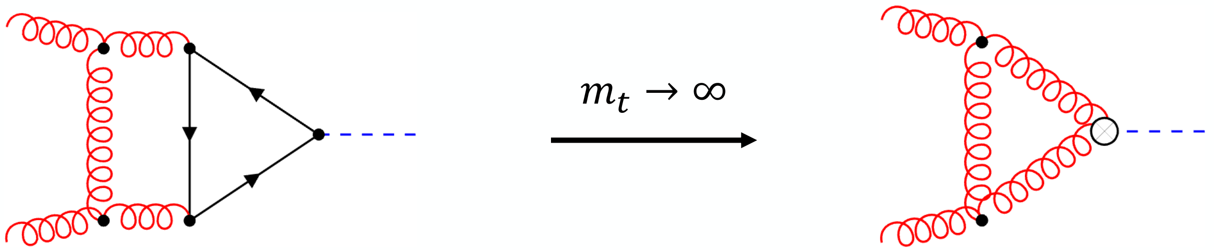
- Loop induced process, LO known for almost 50 years



Heavy-top limit (HTL): one fewer loop, one fewer scale
→ Effective field theory (EFT)

Georgi, Glashow, Machacek, Nanopoulos `78

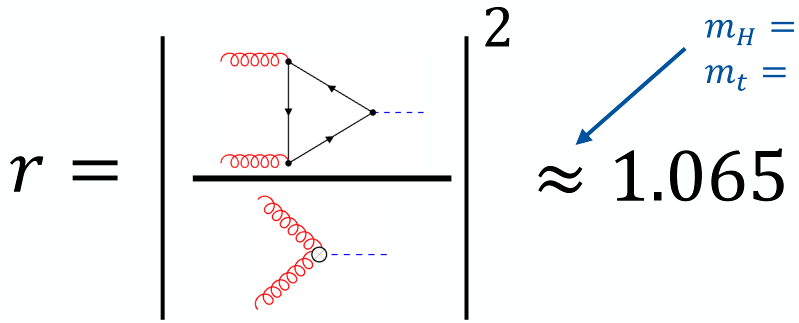
- NLO:



Graudenz, Spira, Zerwas `93

Dawson `91
Djouadi, Spira, Zerwas `91

Better agreement with **rescaling**: $\sigma_{\mathrm{HEFT}} = r\, \sigma_{\mathrm{HTL}}$



$m_H = 125$ GeV
$m_t = 173.055$ GeV

→ rEFT, HEFT (Higgs effective field theory)

Gluon fusion (YR4 `16)

$\sqrt{S}$	13 TeV
m_h	125 GeV
PDF	`PDF4LHC15_nnlo_100`
$\alpha_s(m_Z)$	0.118
$m_t(m_t)$	162.7 GeV ($\overline{\text{MS}}$)
$m_b(m_b)$	4.18 GeV ($\overline{\text{MS}}$)
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$\mu = \mu_R = \mu_F$	62.5 GeV ($= m_H/2$)

- Yellow report from 2016 (LHCH(XS)WG YR4 `16), following Anastasiou, Duhr, Dulat, et al. `16

$$\boxed{\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ \text{(theory)} \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF+}\alpha_s)\,.}$$

$48.58\,\text{pb} =$	$16.00\,\text{pb}$	$(+32.9\%)$	(LO, rEFT)	Georgi, Glashow, Machacek, Nanopoulos `78
	$+\,20.84\,\text{pb}$	$(+42.9\%)$	(NLO, rEFT)	Dawson `91; Djouadi, Spira, Zerwas `91
	$-\;\;2.05\,\text{pb}$	(-4.2%)	$((t,b,c)$, exact NLO)	Graudenz, Spira, Zerwas `93
	$+\;\;9.56\,\text{pb}$	$(+19.7\%)$	(NNLO, rEFT)	Ravindran, Smith, Van Neerven, `02; Harlander, Kilgore `02; Anastasiou, Melnikov `02
	$+\;\;0.34\,\text{pb}$	$(+0.7\%)$	(NNLO, $1/m_t$)	Harlander, Ozeren, `09; Pak, Rogal, Steinhauser `10; Harlander, Mantler, Marzani, Ozeren, `10
	$+\;\;2.40\,\text{pb}$	$(+4.9\%)$	(EW, QCD-EW)	Aglietti, Bonciani, Degrassi, Vicini, `04; Actis, Passarino, Sturm, Uccirati, `08; Anastasiou, Boughezal, Petriello, `09
	$+\;\;1.49\,\text{pb}$	$(+3.1\%)$	(N^{3}LO, rEFT)	Anastasiou, Duhr, Dulat, Herzog, Mistlberger `15

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Our Goal

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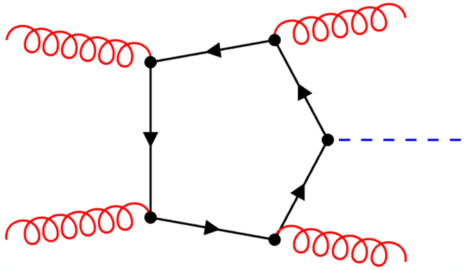
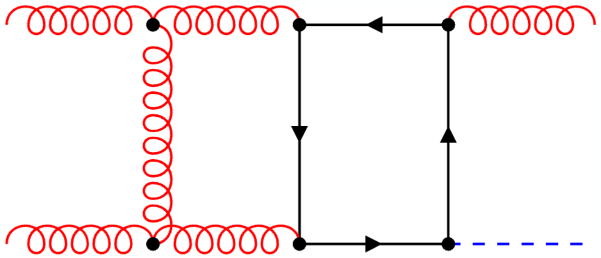
Computation

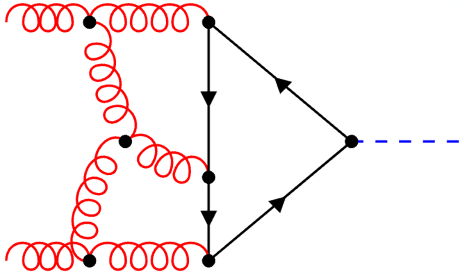
Ingredients for NNLO



double-real



real-virtual



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We use calculation from Budge, Campbell, De Laurentis, et al. `20
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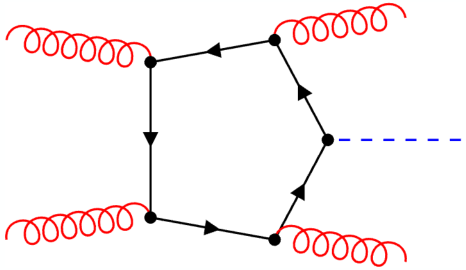

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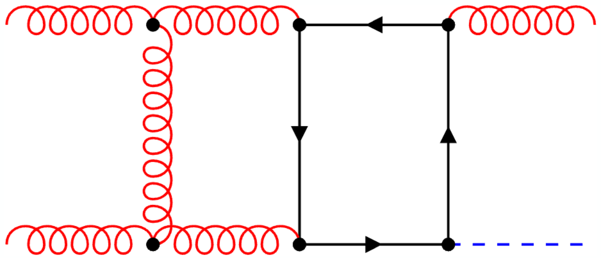

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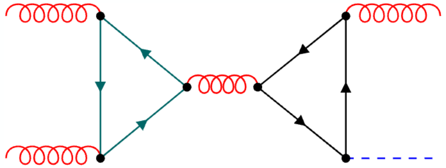
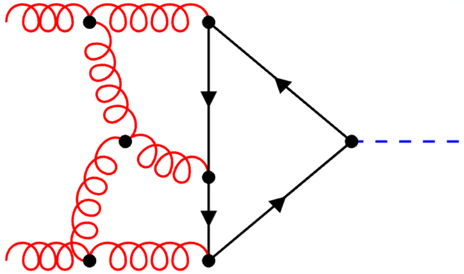
Single massive flavor: Interpolation of numerical grid of regulated amplitude; Analytical expression for IR counterterm

Two massive loops with different masses:
→ Always factorized into one-loop contributions





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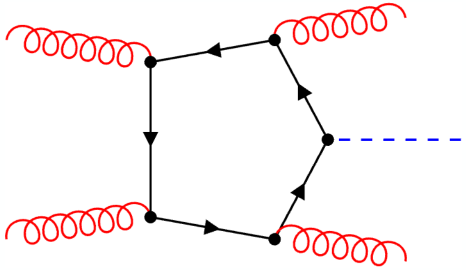

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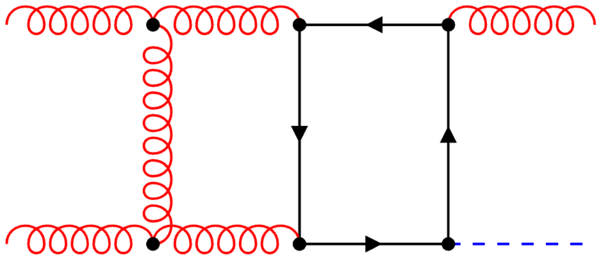

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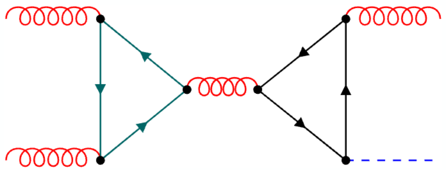
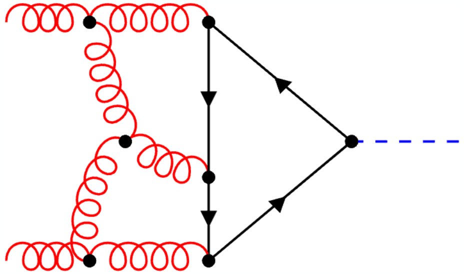
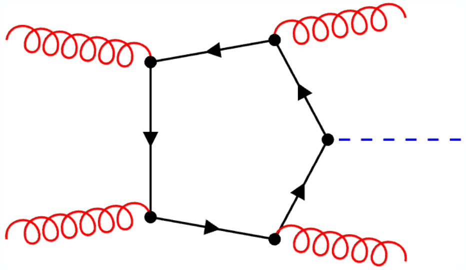




double-virtual

Deep asymptotic expansion in m_H^2/m_t^2, m_b^2/m_H^2:
- Single massive quark flavor: Czakon, Niggetiedt `20
- Two massive quark flavors: Niggetiedt, Usovitsch `23

Ingredients for NNLO



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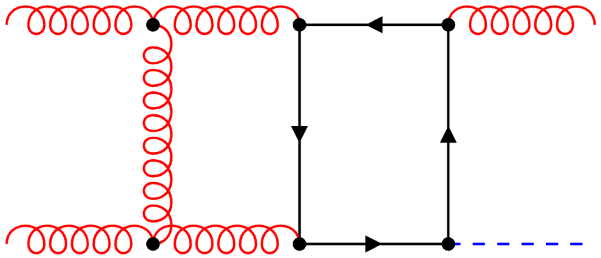

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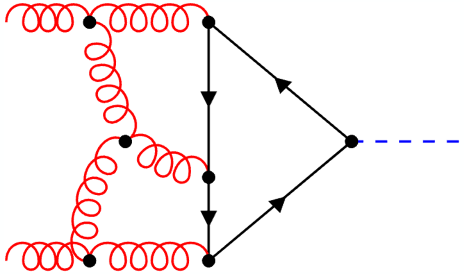

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Phase-space integration with sector-improved residue subtraction (Czakon `10) as implemented in C++ code Stripper

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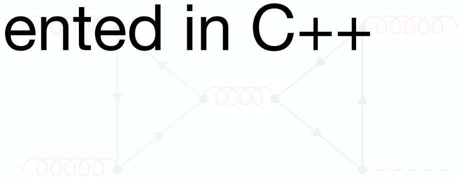
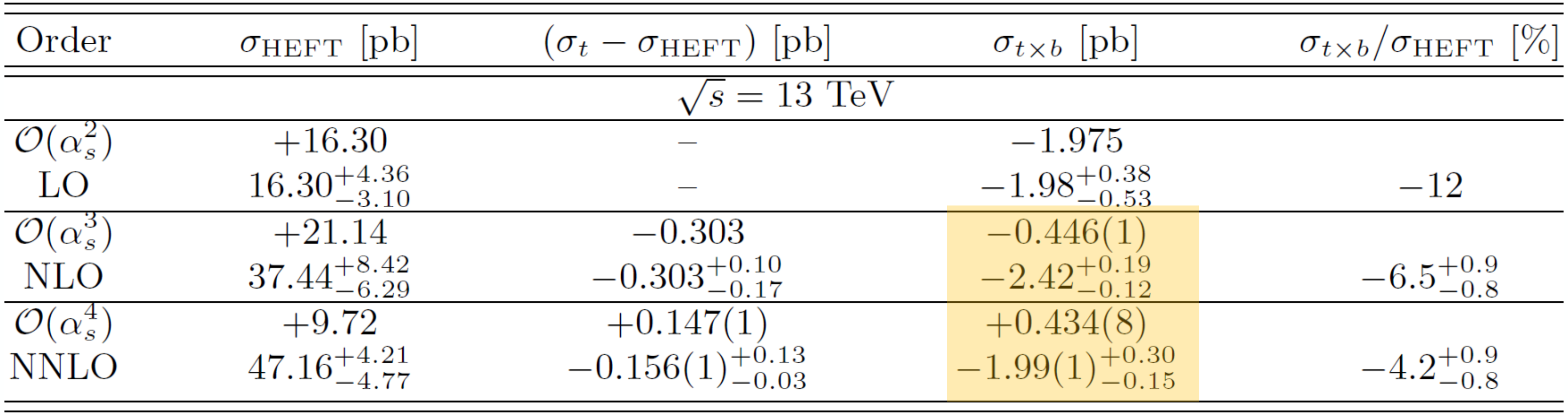
→ Alwa[illegible] e-loop contributions



Deep asymptotic expansion in m_H^2/m_t^2, m_b^2/m_H^2
- Single massive quark flavor: Czakon, Niggetiedt `20
- Two massive quark flavors: Niggetiedt, Usovitsch `23

Results: Part I

- Effects of interference of top- and bottom-quark amplitudes on Higgs production in gluon fusion at the LHC
  - PDF set: **NNPDF31_nnlo_as_0118** NNPDF Collaboration `17
  - $\mu_R = \mu_F = m_H/2$ (central scale), uncertainties with seven-point variation
  - m_H = 125 GeV ⇒ $m_t \approx$ 173.055 GeV and $m_b \approx$ 4.779 GeV (both in OS-scheme)
  - HEFT values obtained with SusHi Harlander, Liebler, Mantler `16



Order	σ_{HEFT} [pb]	$(\sigma_t - \sigma_{\mathrm{HEFT}})$ [pb]	$\sigma_{t\times b}$ [pb]	$\sigma_{t\times b}/\sigma_{\mathrm{HEFT}}$ [%]
		$\sqrt{s} = 13$ TeV		
$\mathcal{O}(\alpha_s^2)$	$+16.30$	–	-1.975	
LO	$16.30^{+4.36}_{-3.10}$	–	$-1.98^{+0.38}_{-0.53}$	-12
$\mathcal{O}(\alpha_s^3)$	$+21.14$	-0.303	$-0.446(1)$	
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$	$-2.42^{+0.19}_{-0.12}$	$-6.5^{+0.9}_{-0.8}$
$\mathcal{O}(\alpha_s^4)$	$+9.72$	$+0.147(1)$	$+0.434(8)$	
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$	$-1.99(1)^{+0.30}_{-0.15}$	$-4.2^{+0.9}_{-0.8}$

Czakon, FE, Niggetiedt, Poncelet, Schellenberger `23

- Interference effects much larger than pure top mass effect
- Interference effect at NNLO cancels against NLO
- Interference effect at NNLO larger than NLO scale variation (similar in HEFT but less severe)
- Interference NNLO scale variation increases compared to NLO
- Similar effects for different top quark mass ($m_t \approx$ 170.979 GeV)

Mass renormalization

On-shell to $\overline{\text{MS}}$ mass renormalization

$$Z_m^{\overline{\text{MS}}}\overline{m} = Z_m^{\text{OS}} m^{\text{OS}} \quad \Longrightarrow \quad m^{\text{OS}} = \overline{m}\left(1 + c_1\frac{\alpha_s}{\pi} + c_2\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha^3)\right)$$

Gray, Broadhurst, Grafe, Schilcher \`90

$$\mathcal{M}^{\overline{\text{MS}}} = \mathcal{M}^{\text{OS}} + \delta\mathcal{M}$$

$$\delta\mathcal{M}^{(1)} = \overline{m}c_1\frac{\alpha_s}{\pi}\frac{\mathrm{d}\mathcal{M}^{\text{OS},(0)}}{\mathrm{d}m}\bigg|_{m=\overline{m}}$$

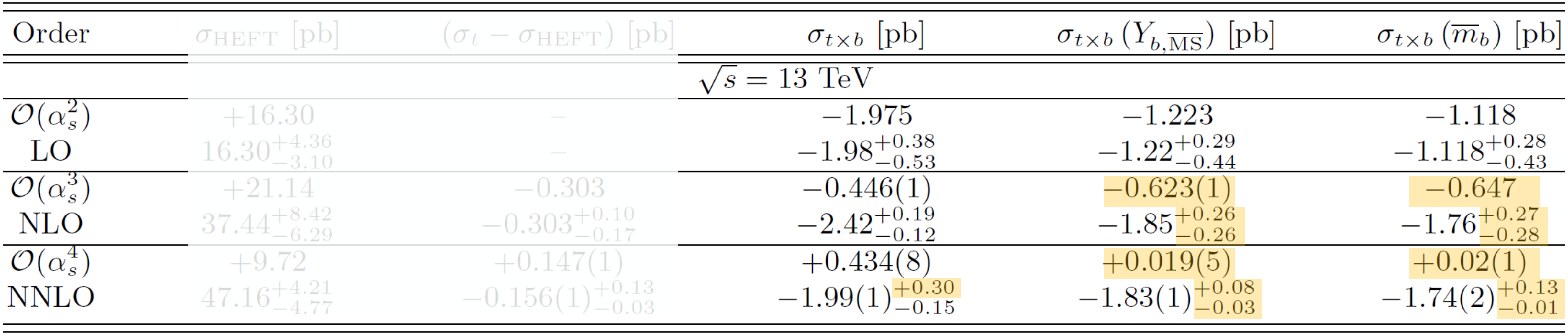
$$\delta\mathcal{M}^{(2)} = \overline{m}\left[c_1\frac{\alpha_s}{\pi}\frac{\mathrm{d}\mathcal{M}^{\text{OS},(1)}}{\mathrm{d}m}\bigg|_{m=\overline{m}} + c_2\left(\frac{\alpha_s}{\pi}\right)^2\frac{\mathrm{d}\mathcal{M}^{\text{OS},(0)}}{\mathrm{d}m}\bigg|_{m=\overline{m}}\right] + \frac{1}{2}\left(\overline{m}c_1\frac{\alpha_s}{\pi}\right)^2\frac{\mathrm{d}^2\mathcal{M}^{\text{OS},(0)}}{\mathrm{d}m^2}\bigg|_{m=\overline{m}}$$

Two possibilities:

1. Only Yukawa coupling in $\overline{\text{MS}}$ → Derivatives trivial since amplitude linear
2. Bottom mass always in $\overline{\text{MS}}$ → Requires some work (derivatives, different mass value in integrals)

Results: Part II

$\overline{m}_b(\overline{m}_b) = 4.18$ GeV



Order	σ_{HEFT} [pb]	$(\sigma_t - \sigma_{\mathrm{HEFT}})$ [pb]	$\sigma_{t\times b}$ [pb]	$\sigma_{t\times b}\,(Y_{b,\overline{\mathrm{MS}}})$ [pb]	$\sigma_{t\times b}\,(\overline{m}_b)$ [pb]
			$\sqrt{s} = 13$ TeV		
$\mathcal{O}(\alpha_s^2)$	$+16.30$	–	-1.975	-1.223	-1.118
LO	$16.30^{+4.36}_{-3.10}$	–	$-1.98^{+0.38}_{-0.53}$	$-1.22^{+0.29}_{-0.44}$	$-1.118^{+0.28}_{-0.43}$
$\mathcal{O}(\alpha_s^3)$	$+21.14$	-0.303	$-0.446(1)$	$-0.623(1)$	-0.647
NLO	$37.44^{+8.42}_{-6.29}$	$-0.303^{+0.10}_{-0.17}$	$-2.42^{+0.19}_{-0.12}$	$-1.85^{+0.26}_{-0.26}$	$-1.76^{+0.27}_{-0.28}$
$\mathcal{O}(\alpha_s^4)$	$+9.72$	$+0.147(1)$	$+0.434(8)$	$+0.019(5)$	$+0.02(1)$
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.156(1)^{+0.13}_{-0.03}$	$-1.99(1)^{+0.30}_{-0.15}$	$-1.83(1)^{+0.08}_{-0.03}$	$-1.74(2)^{+0.13}_{-0.01}$

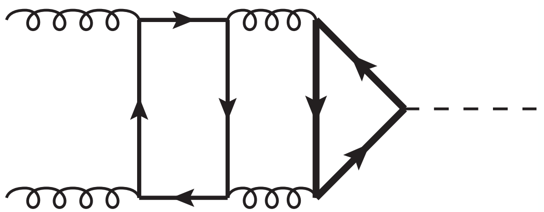
Czakon, FE, Niggetiedt, Poncelet, Schellenberger `23, `24

- Much better convergence behavior in $\overline{\mathrm{MS}}$
- Using $\overline{\mathrm{MS}}$ only for coupling vs. everywhere leads to similar results
- At NNLO, values for the first time compatible between $\overline{\mathrm{MS}}$ and OS

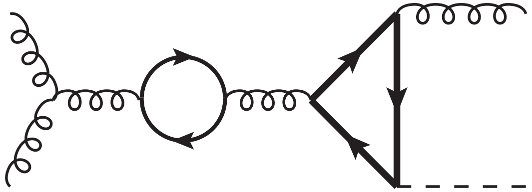
Flavor scheme

5-flavor scheme vs. 4-flavor scheme

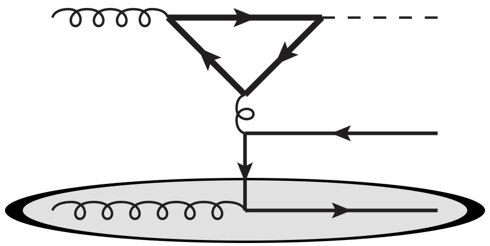
- So far: b-quarks not coupled to the Higgs assumed as massless (5FS)
- While gauge-invariant, testing the effect of the neglected mass desirable → 4FS



double-virtual



real-virtual



double-real

Calculation of amplitudes equivalent to 5FS case

Amplitudes with Recola (Actis, Denner, Hofer, et al. `16)
Phase-space integration as separate calculation of LO Hbb production

- Exclude b-quark from initial state
- Infrared singularities regulated by finite m_b, only sum of contributions free of $\log(m_b)$ divergences

HEFT in the 4-flavor scheme

- Effects of 4FS concern b-quark *not* coupled to Higgs → also studiable within HEFT
- More stable calculation → can numerically test pole cancellation for small m_b

Order	σ_{HEFT} [pb]			
	$\sqrt{s} = 13$ TeV			
	5FS	4FS $(m_b = 0.01$ GeV$)$	4FS $(m_b = 0.1$ GeV$)$	4FS $(m_b = 4.78$ GeV$)$
$\mathcal{O}(\alpha_s^2)$	$+16.30$	$+16.27$	$+16.27$	$+16.27$
LO	$16.30^{+4.36}_{-3.10}$	$+16.27^{+4.63}_{-3.22}$	$+16.27^{+4.63}_{-3.22}$	$+16.27^{+4.63}_{-3.22}$
$\mathcal{O}(\alpha_s^3)$	$+21.14$	$+20.08(3)$	$+20.08(3)$	$+20.08(3)$
NLO	$37.44^{+8.42}_{-6.29}$	$+36.35(3)^{+8.57}_{-6.32}$	$+36.35(3)^{+8.57}_{-6.32}$	$+36.35(3)^{+8.57}_{-6.32}$
$\mathcal{O}(\alpha_s^4)$	$+9.72$	$+10.8(4)$	$+11.2(4)$	$+9.5(2)$
NNLO	$47.16^{+4.21}_{-4.77}$	$47.2(4)^{+5.4}_{-5.4}$	$+47.5(4)^{+5.4}_{-5.5}$	$+45.9(2)^{+4.3}_{-4.9}$

Czakon, FE, Niggetiedt, Poncelet, Schellenberger `24

4FS PDF set: NNPDF31_nnlo_as_0118_nf_4

- Convergence for small m_b, seems like numbers approach 5FS value
- Effect of finite m_b is ~3%, order of magnitude as estimated in Pietrulewicz, Stahlhofen `23

Results: Part III

- Top-bottom interference in the 5FS vs. 4FS

Order	$\sigma_{t\times b}$ [pb]			
	$\sqrt{s} = 13$ TeV			
	5FS $m_t = 173.06$ GeV $\overline{m}_b(\overline{m}_b) = 4.18$ GeV	5FS $m_t = 173.06$ GeV $m_b = 4.78$ GeV	5FS $m_t(m_t) = 162.7$ GeV $\overline{m}_b(\overline{m}_b) = 4.18$ GeV	4FS $m_t = 173.06$ GeV $\overline{m}_b(\overline{m}_b) = 4.18$ GeV
$\mathcal{O}(\alpha_s^2)$	-1.11	-1.98	-1.12	-1.15
LO	$-1.11^{+0.28}_{-0.43}$	$-1.98^{+0.38}_{-0.53}$	$-1.12^{+0.28}_{-0.42}$	$-1.15^{+0.29}_{-0.45}$
$\mathcal{O}(\alpha_s^3)$	-0.65	-0.44	-0.64	-0.66
NLO	$-1.76^{+0.27}_{-0.28}$	$-2.42^{+0.19}_{-0.12}$	$-1.76^{+0.27}_{-0.28}$	$-1.81^{+0.28}_{-0.30}$
$\mathcal{O}(\alpha_s^4)$	$+0.02$	$+0.43$	-0.02	-0.02
NNLO	$-1.74(2)^{+0.13}_{-0.03}$	$-1.99(2)^{+0.29}_{-0.15}$	$-1.78(1)^{+0.15}_{-0.03}$	$-1.83(2)^{+0.14}_{-0.03}$

Czakon, FE, Niggetiedt, Poncelet, Schellenberger `24

4FS PDF set: NNPDF31_nnlo_as_0118_nf_4

- Difference between schemes below scale uncertainties
- Small effect on already small contribution → Use of (simpler) 5FS justified

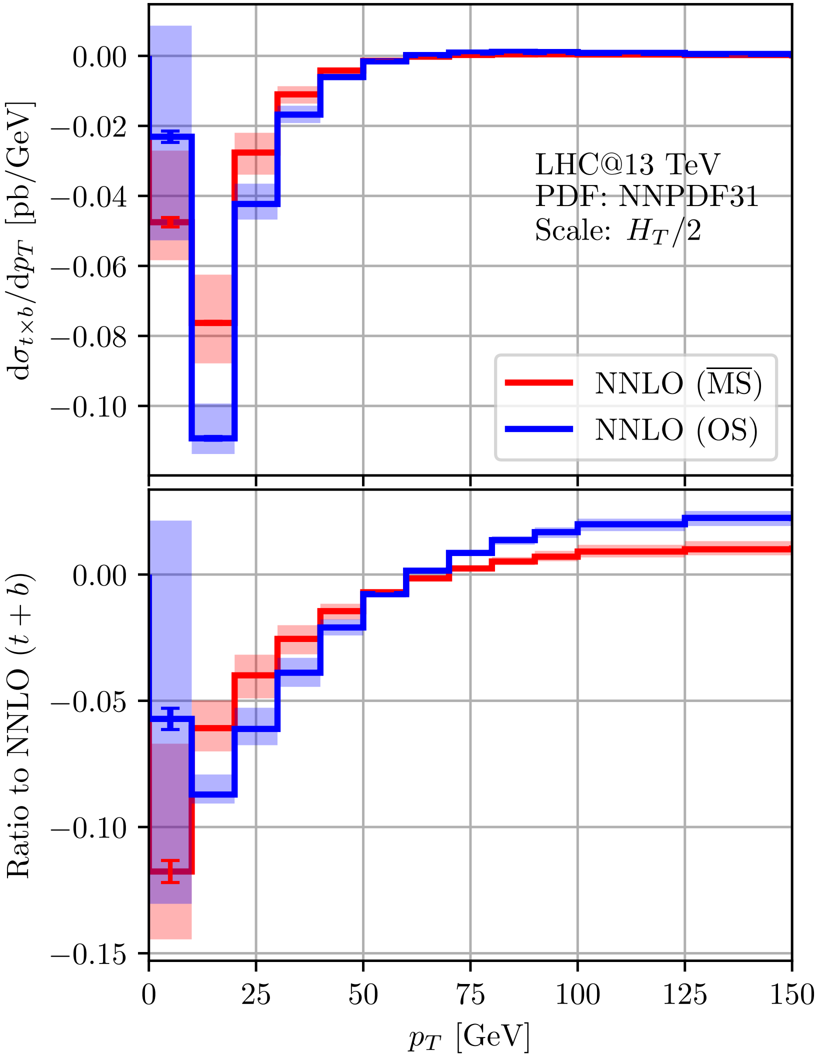
Results Part IV: Distributions

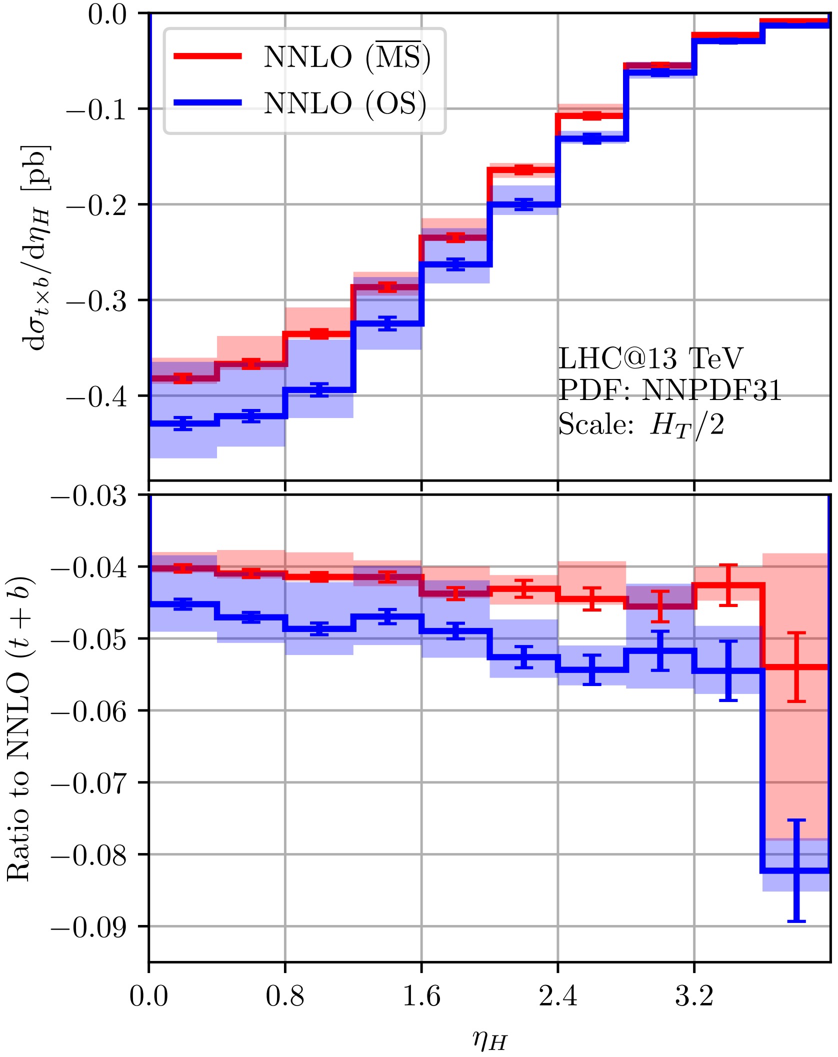
Top-Bottom Interference Effects

Czakon, FE, Niggetiedt, Poncelet, Schellenberger `24

$\mu_R = \mu_F = H_T/2 = (\sqrt{m_H^2 + p_T^2} + \sum_i |p_{i,T}|)/2$ (central scale), seven-point variation

m_H = 125 GeV, $m_t^{OS} \approx$ 173.055 GeV, and $m_b(m_b)$ = 4.18 GeV





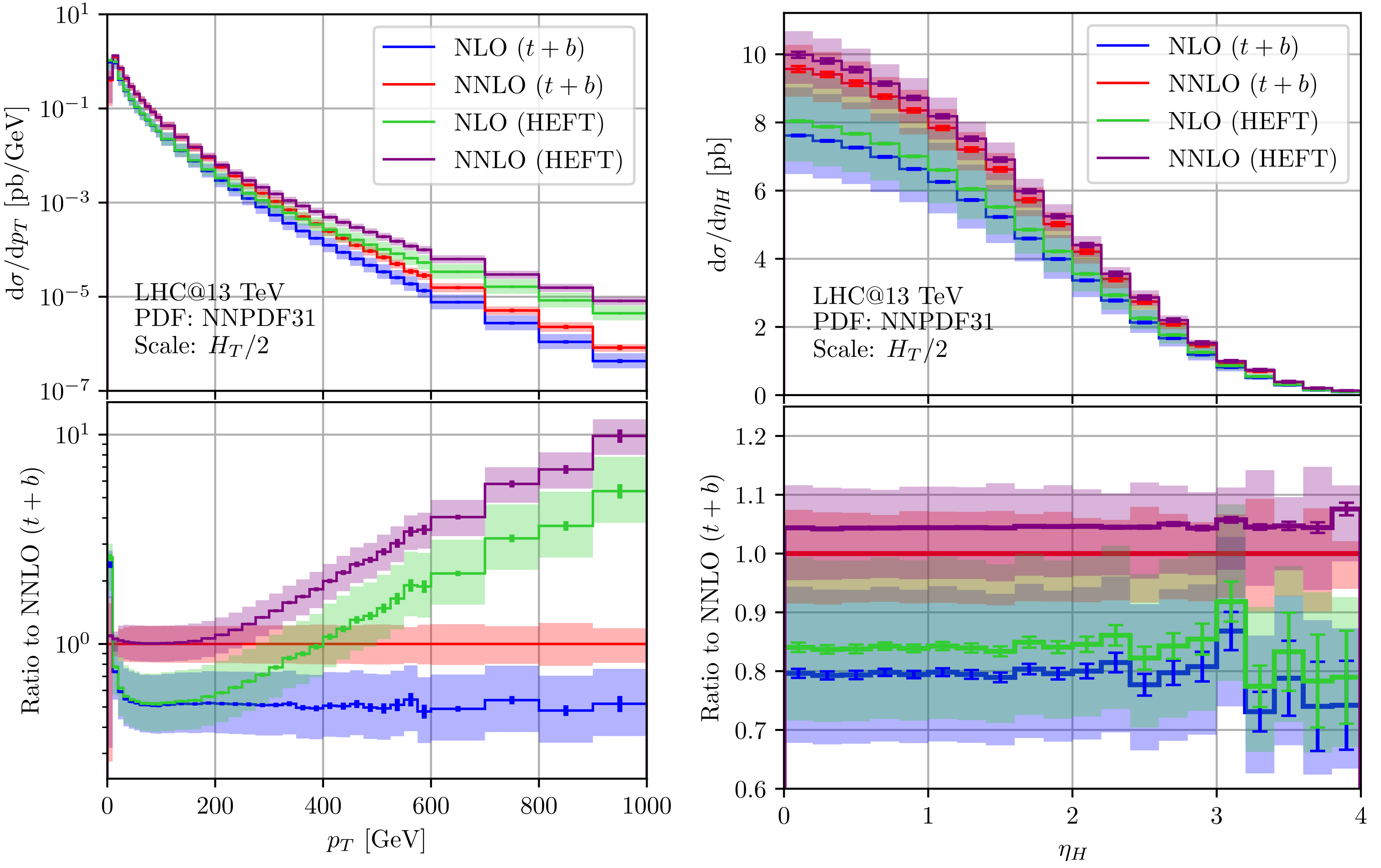
- **p_T distribution**: Except for first bin (only there three-loop is needed), known from previous calculations of quark-mass effects on Higgs+jet at NLO
  Lindert, Melnikov, Tancredi, Wever `17
  Caola, Lindert, Melnikov, et al. `18
  Bonciani, Del Duca, Frellesvig, et al. `22
- Find agreement with known results
- **Rapidity**: New result, all bins contain genuine three-loop corrections
- Mass-effects barely affect shape of distribution, but shift it downwards

Distributions: HEFT vs. Full Theory

Czakon, FE, Niggetiedt, Poncelet, Schellenberger `24

$\mu_R = \mu_F = H_T/2 = (\sqrt{m_H^2 + p_T^2} + \sum_i |p_{i,T}|)/2$ (central scale), seven-point variation

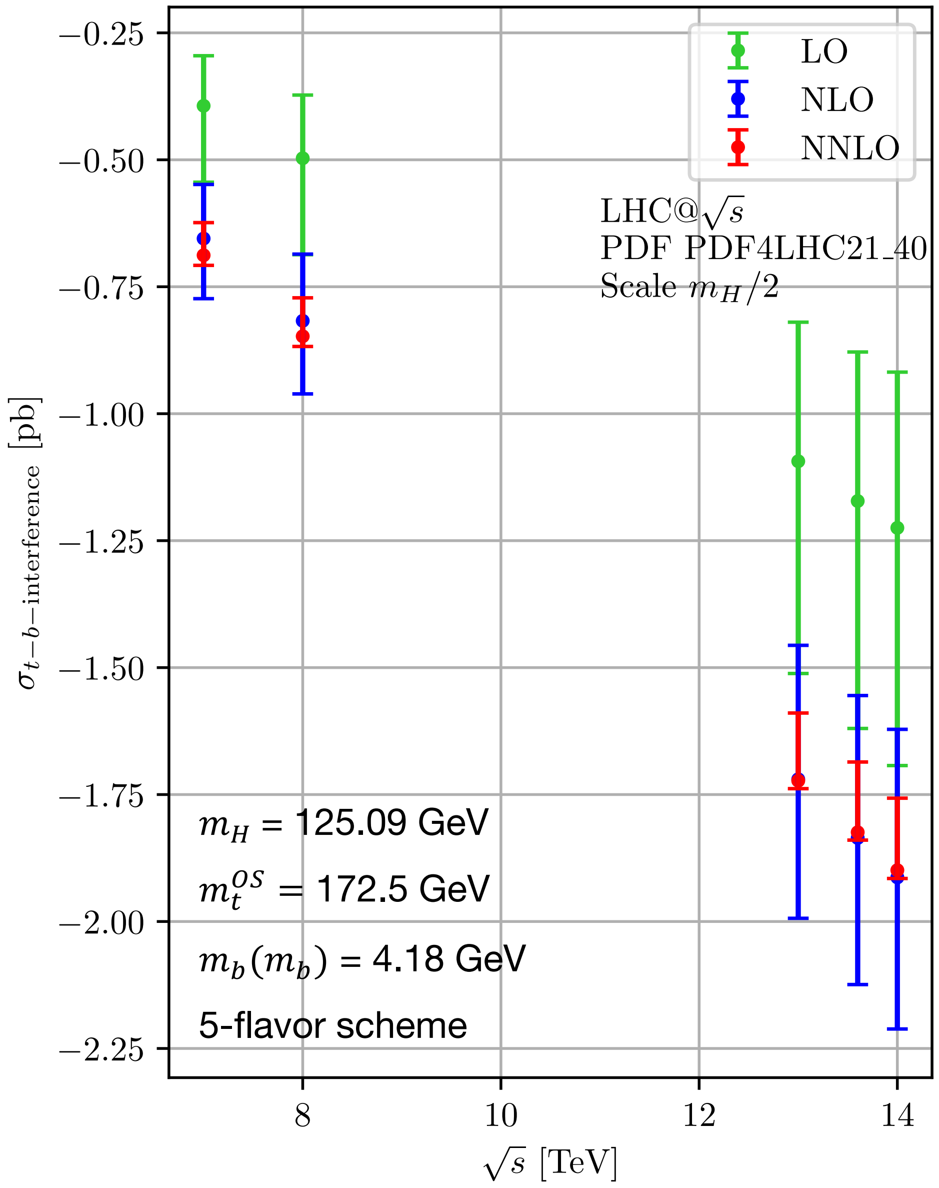
m_H = 125 GeV, $m_t^{OS} \approx$ 173.055 GeV, and $m_b(m_b)$ = 4.18 GeV



- Full theory: At very high p_T, only scale is p_T → $d\sigma/dp_T^2 \sim 1/p_T^4$
- Effective theory: dimensionful coupling → $d\sigma/dp_T^2 \sim 1/(v^2 p_T^2)$

Conclusions

- Complete analysis of top-bottom-interference effects on the Higgs production cross section at NNLO
- Addresses one of the leading theory uncertainties
- $\overline{MS}$ scheme shows better perturbative convergence than OS scheme
- Good agreement between 4- and 5-flavor scheme
- Differential distributions, including novel rapidity spectra



Conclusions

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- Addresses one of the leading theory uncertainties
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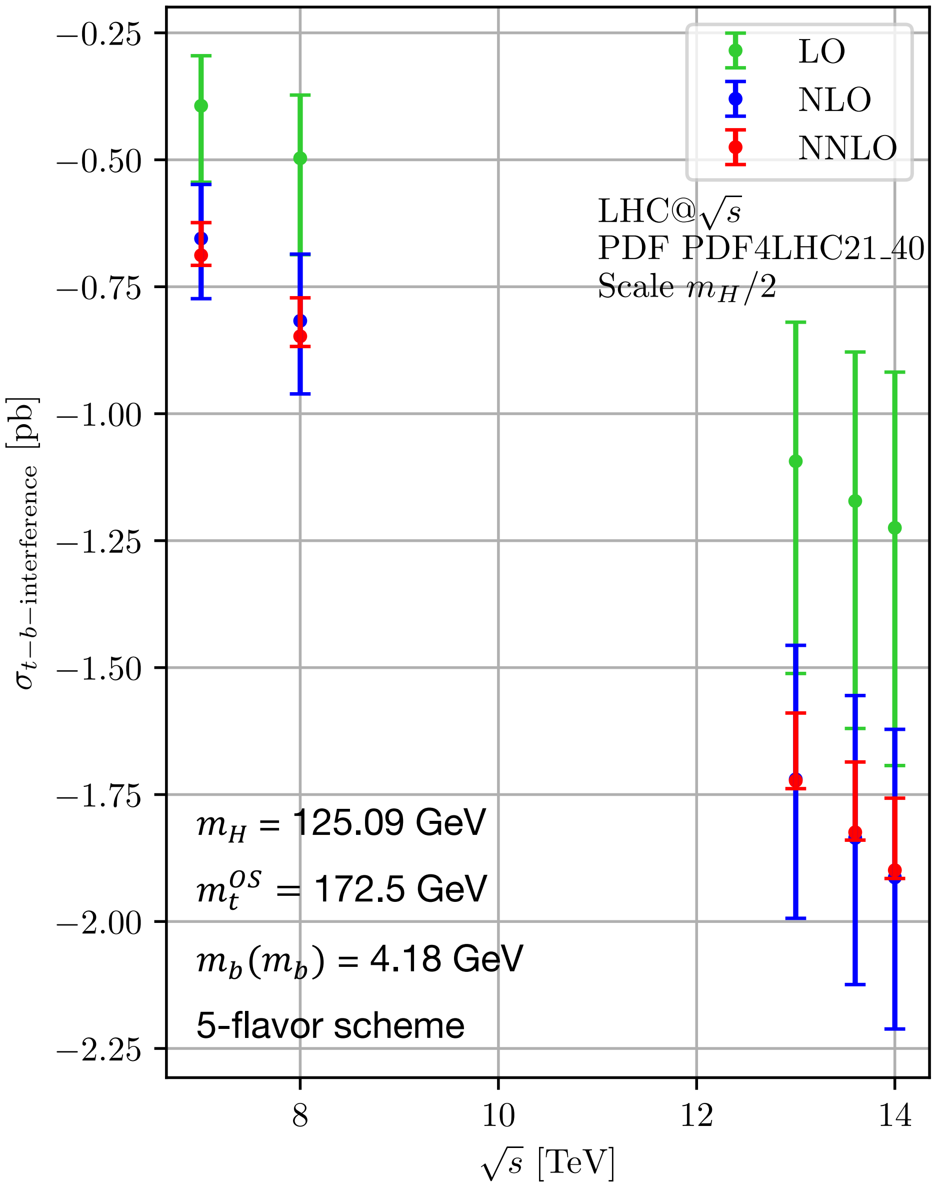
- Good agreement between 4- and 5-flavor scheme
- Differential distributions, including novel rapidity spectra

Thank you for your attention!

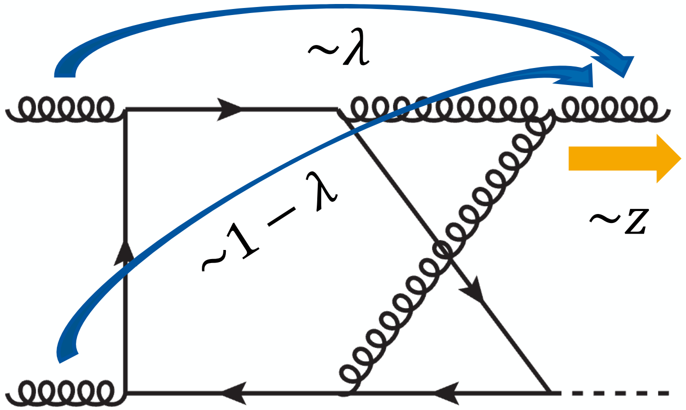


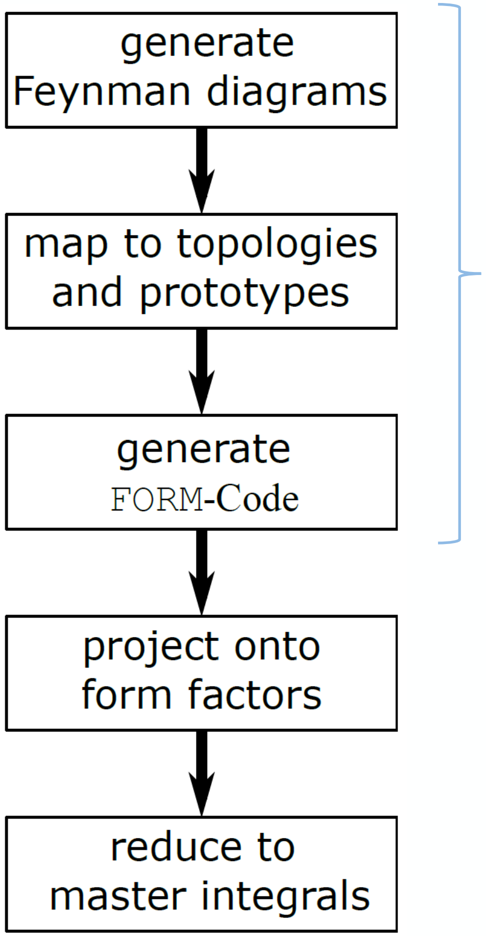
Backup

Real-virtual corrections

- Variables: $\hat{s}, \hat{t}, \hat{u}, m_H^2, m_q^2$
- Introduce dimensionless variables and <u>fix ratio</u> $m_q^2 \,/\, m_H^2$
  - z parametrizes soft limit
  - λ parametrizes collinear limit

$$\boxed{\begin{aligned}\hat{t}/\hat{s} &= z\,\lambda \\ \hat{u}/\hat{s} &= z\,(1\text{-}\lambda)\end{aligned}} \rightleftarrows \boxed{\begin{aligned}z &= 1\text{-}m_H^2/\hat{s} \\ \lambda &= \hat{t}/(\hat{t}+\hat{u})\end{aligned}}$$





DiaGen
Czakon (private)

FORM
Vermaseren `00

Kira⊕FireFly
Klappert, Lange, et al. `20

→ Solve master integrals with differential equations in $m_q^2/m_H^2, z$, and λ

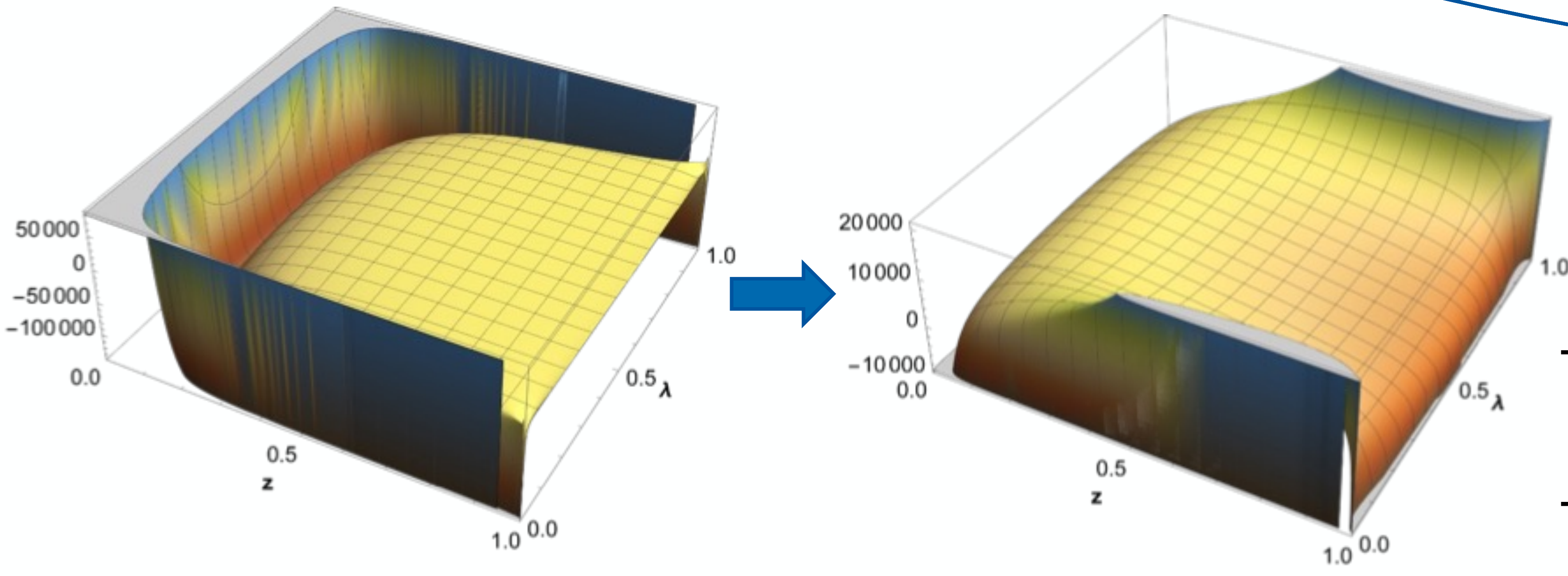
→ Boundary conditions: $\frac{m_q^2}{m_H^2} \to \infty$ with large-mass expansion

Real-virtual corrections

- Create grid with numerical values of squared amplitude
- Subtract IR singularities:

$$\langle M^{(1)}_{\text{exact}}|M^{(2)}_{\text{exact}}\rangle\big|_{\text{regulated}} \equiv \langle M^{(1)}_{\text{exact}}|M^{(2)}_{\text{exact}}\rangle - \left[\langle M^{(1)}_{\text{HEFT}}|M^{(2)}_{\text{HEFT}}\rangle + \frac{8\pi\alpha_s}{\hat{t}}\left\langle P^{(0)}_{gg}\left(\frac{\hat{s}}{\hat{s}+\hat{u}}\right)\right\rangle\langle F^{(1)}|\left(F^{(2)}_{\text{exact}} - F^{(2)}_{\text{HEFT}}\right)\rangle\right]$$

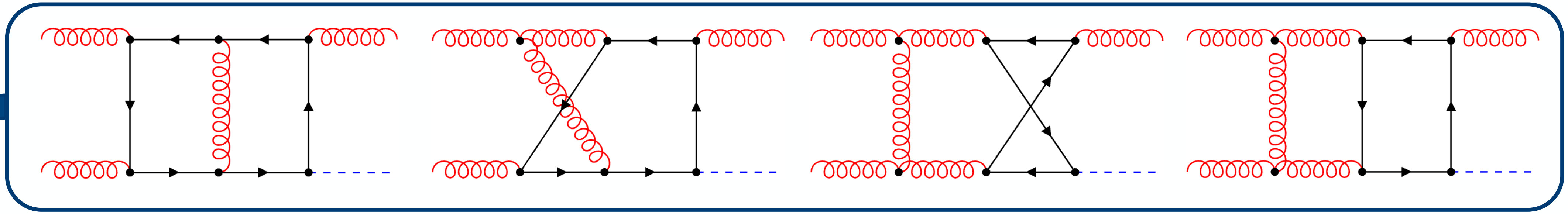
$$\boxed{r = \frac{\sigma^{\text{LO}}_{t\times b}}{\sigma^{\text{LO}}_{\text{HTL}}} \approx -0.129}$$

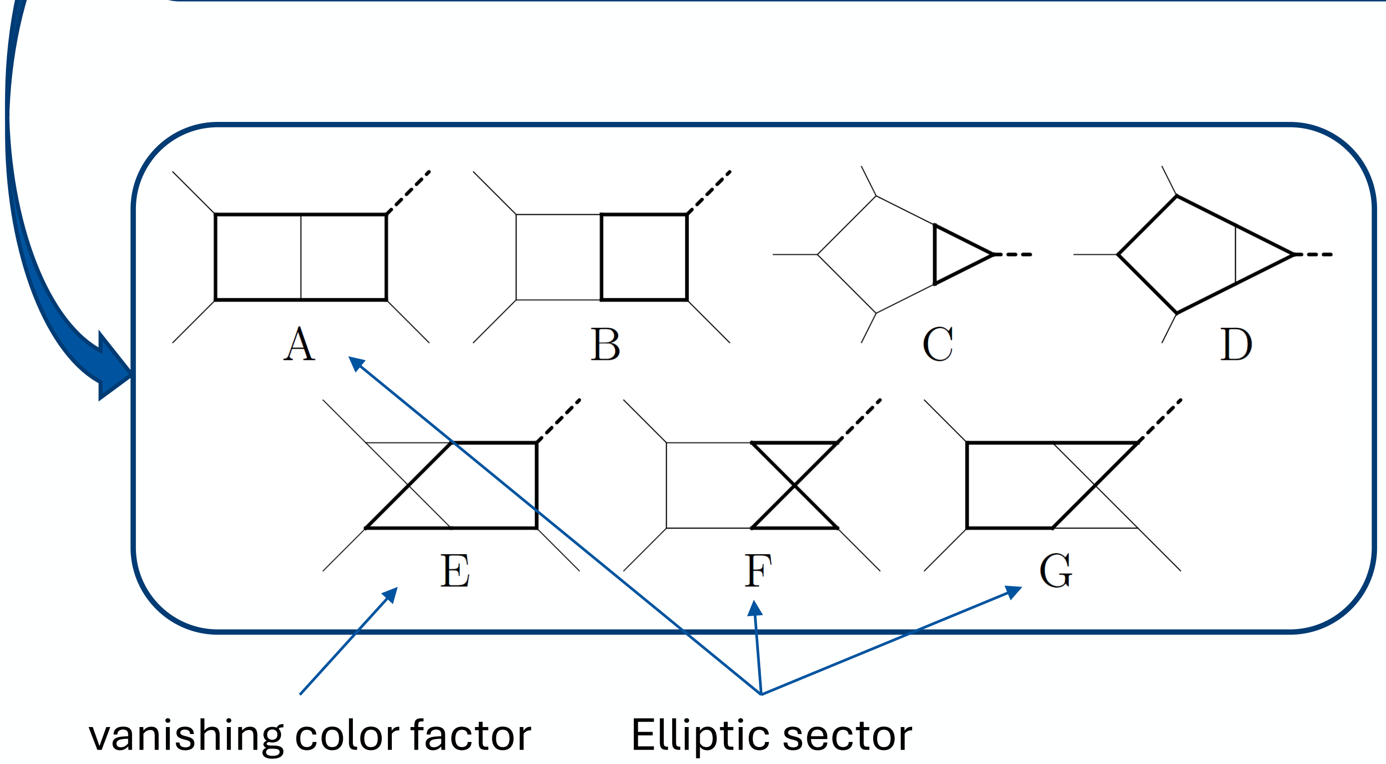


→ Interpolate to any phase-space point with cubic splines

→ Add back subtracted terms using analytical expression

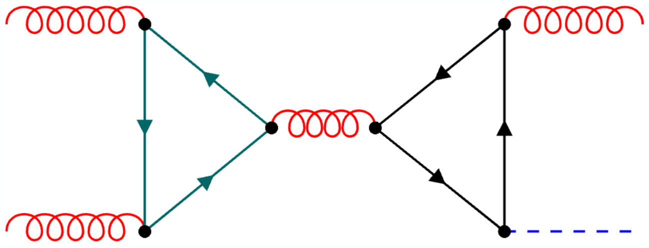
Real-virtual corrections





A,B,C,D: Bonciani, Del Duca, Frellesvig, et al. `16
F: Bonciani, Del Duca, Frellesvig, et al. `19
G: Frellesvig, Hidding, Maestri, et al. `19

Contributions with two closed fermion chains are always factorizable:



Parametrization

- Variables: $\hat{s}, \hat{t}, \hat{u}, m_H^2, m_q^2$
- Introduce dimensionless variables and fix ratio m_q^2/m_H^2
  - ➢ z parametrizes soft limit
  - ➢ λ parametrizes collinear limit

$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_t^2/m_H^2 = 23/12$

$m_b^2/m_H^2 = 1/684$

Range of parameters:

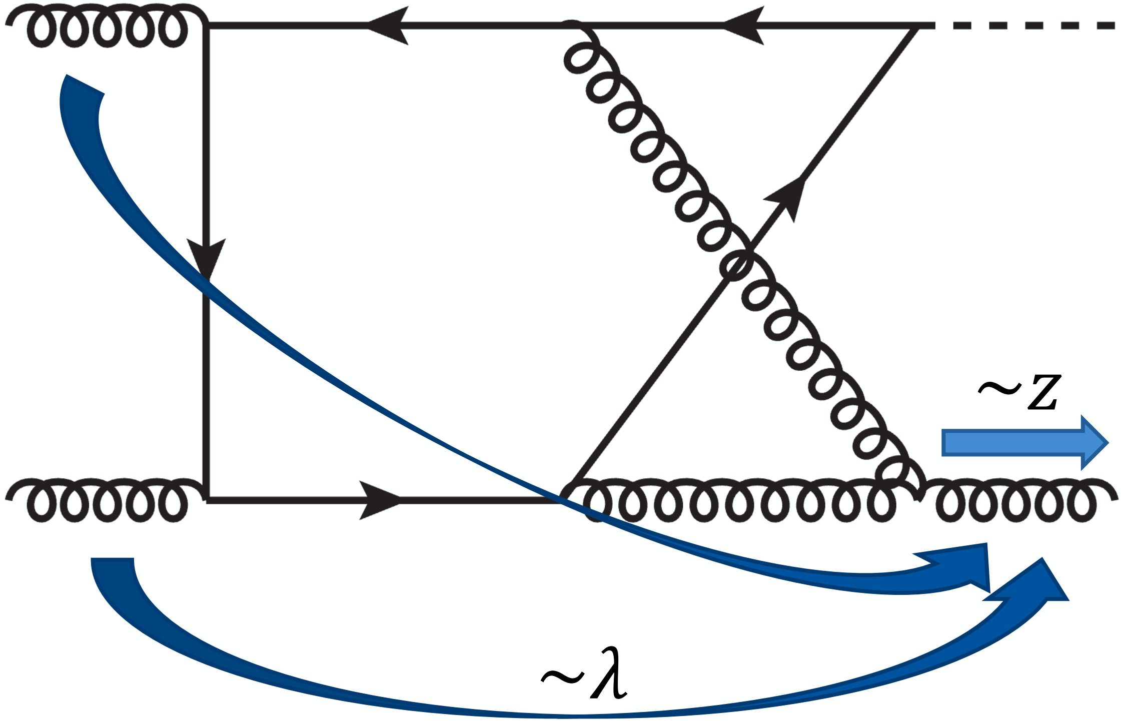
- $\lambda \in (0,1)$
- $z \in (0,1)$

$\hat{t}/\hat{s} = z\,\lambda$

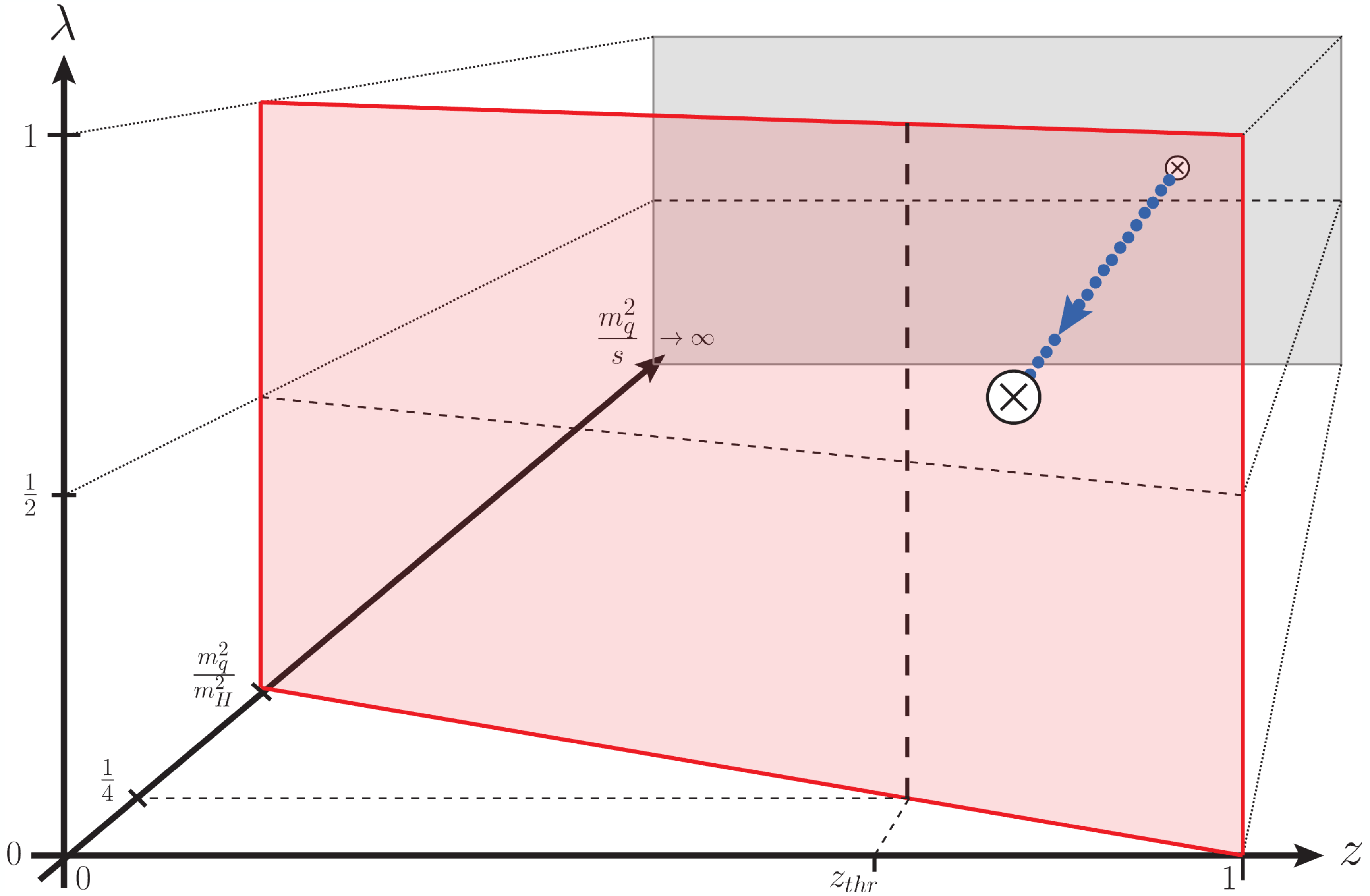
$\hat{u}/\hat{s} = z\,(1-\lambda)$

$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$



Evolution of differential equations



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_t^2/m_H^2 = 23/12$

Range of parameters:
- $\lambda \in (0,1)$
- $z \in (0,1)$

$z_{thr} = 1 - \frac{m_H^2}{4m_q^2}$

For $m_t^2/m_H^2 = 23/12$: $z_{thr} = 20/23 \approx 0.87$

Evolution in the (z,λ)-plane

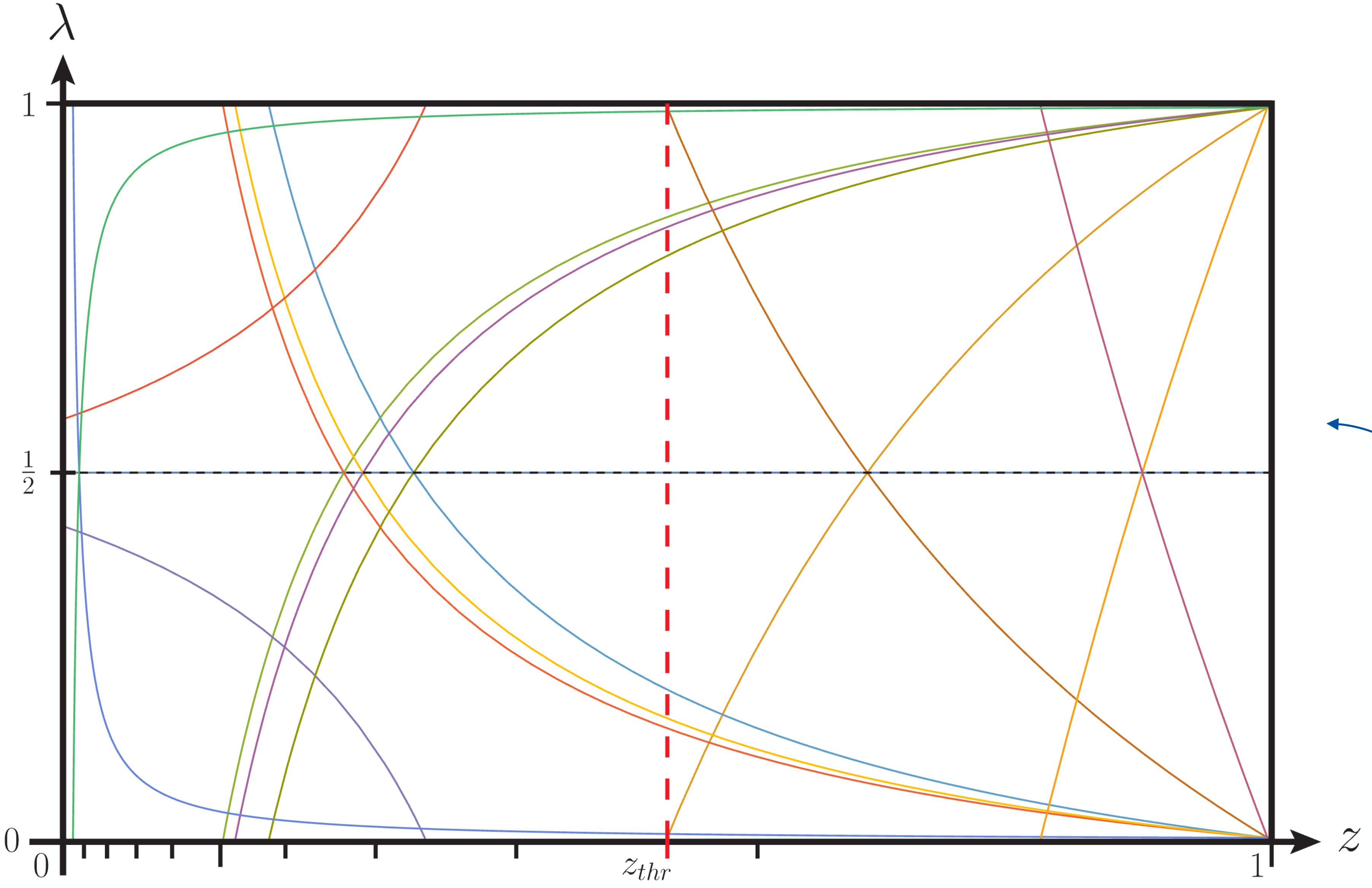
z = 1-$m_H^2/\hat{s}$

λ = $\hat{t}/(\hat{t}+\hat{u})$

m_t^2/m_H^2 = 23/12

Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$



Poles of differential equations in λ

Evolution in the (z,λ)-plane

z = 1-$m_H^2/\hat{s}$

λ = $\hat{t}/(\hat{t}+\hat{u})$

m_t^2/m_H^2 = 23/12

Range of parameters:
- $\lambda \in (0,1)$
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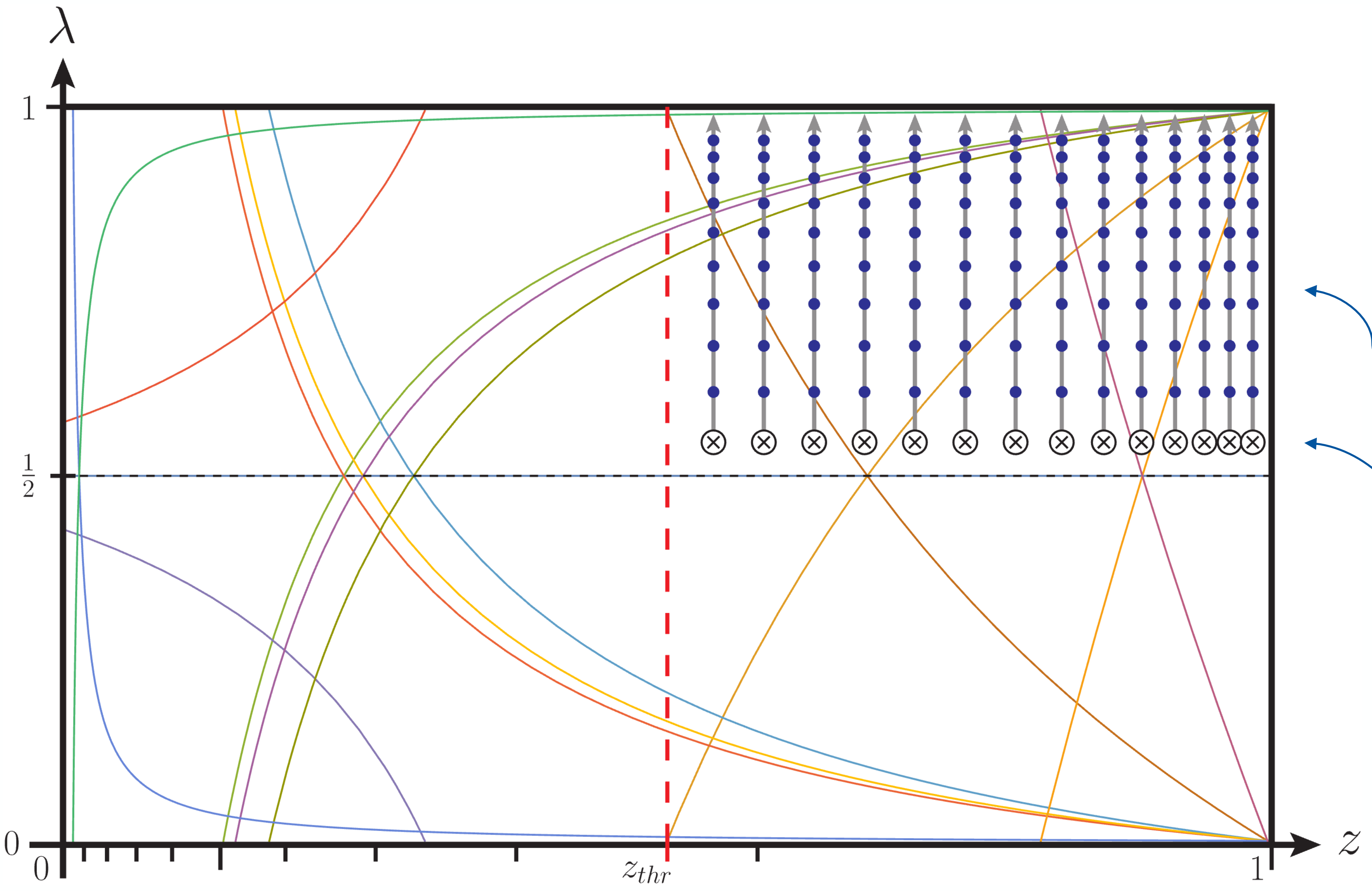
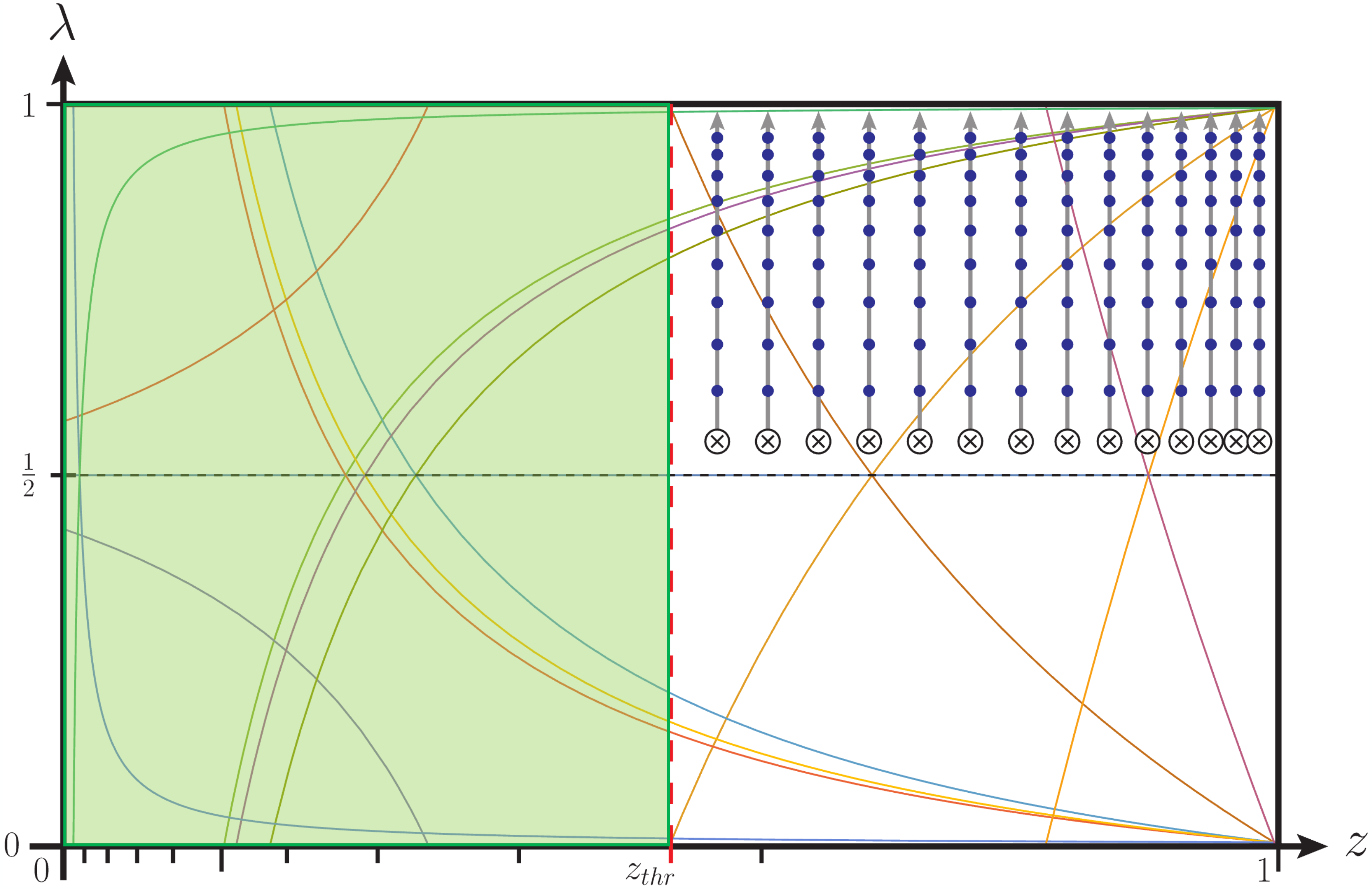


Collect numerical samples for MI along straight integration contours

Boundaries from numerical integration in the mass

Evolution in the (z,λ)-plane



z = 1-$m_H^2/\hat{s}$

λ = $\hat{t}/(\hat{t}+\hat{u})$

m_t^2/m_H^2 = 23/12

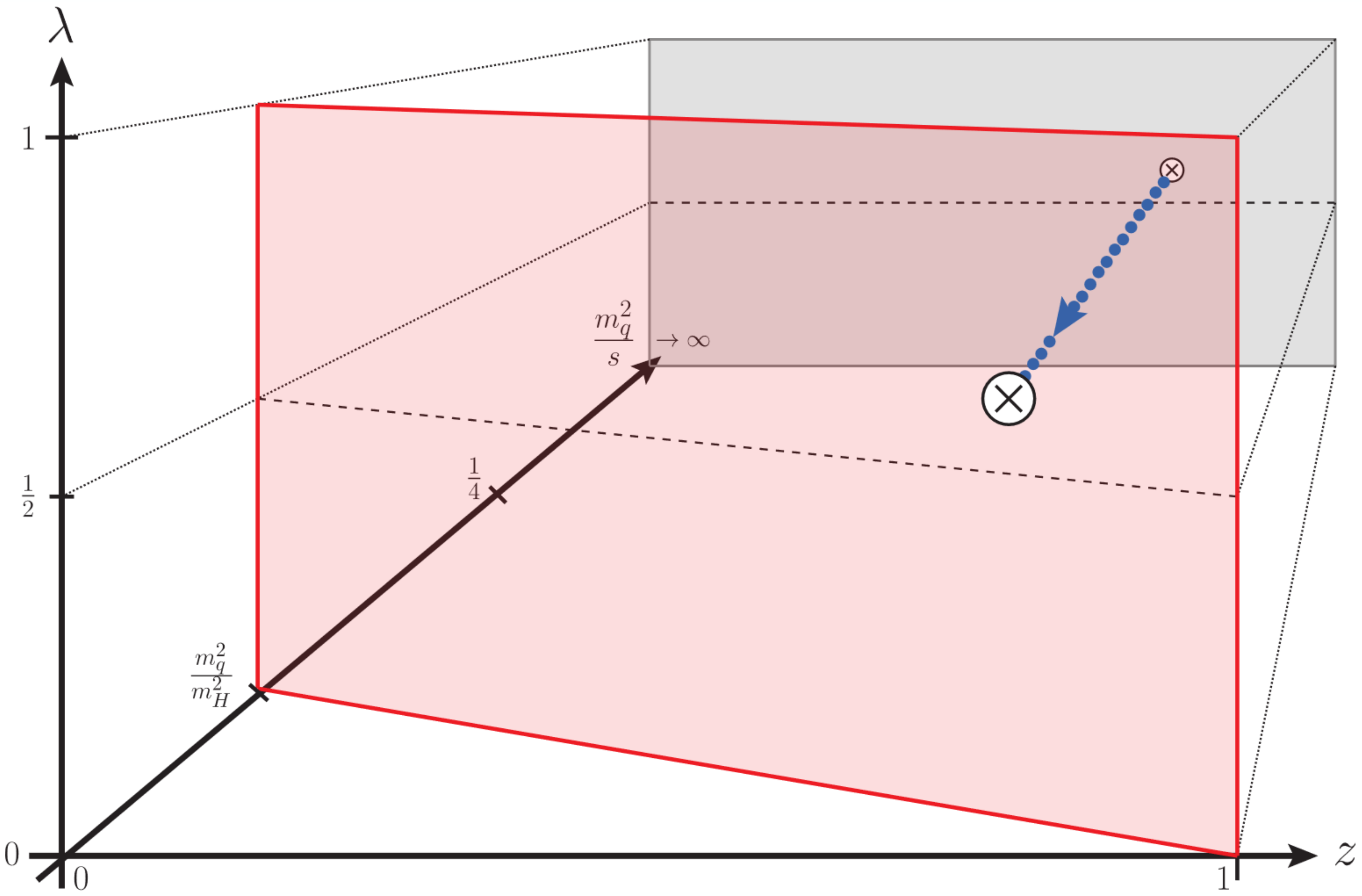
Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Region below threshold covered by LME

LME $\mathcal{O}((1/m_q^2)^{40})$

Evolution of differential equations



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

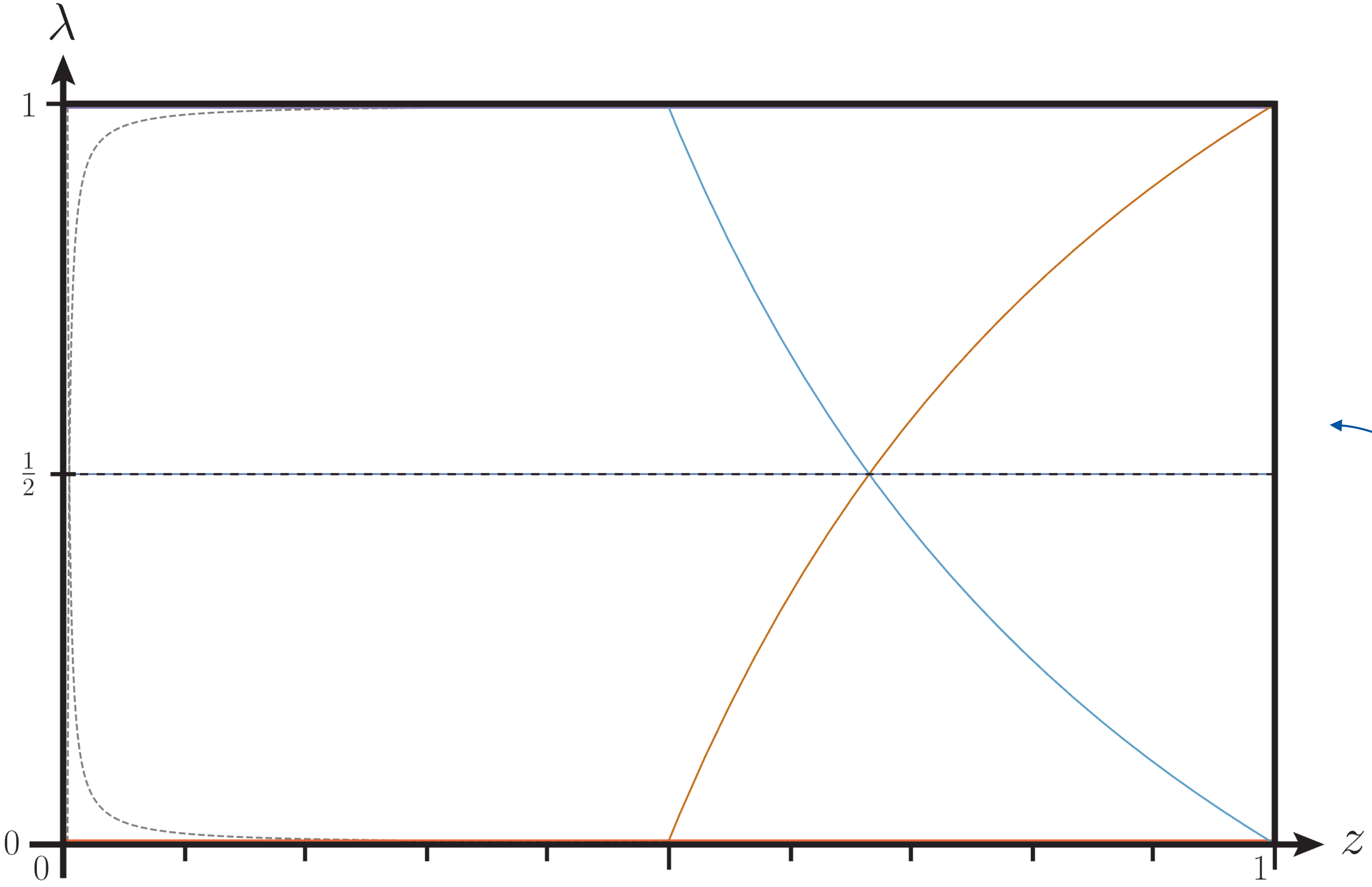
$m_b^2/m_H^2 = 1/684$

Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

$z_{thr} = 1 - \frac{m_H^2}{4m_q^2} < 0$

Evolution in the (z,λ)-plane



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

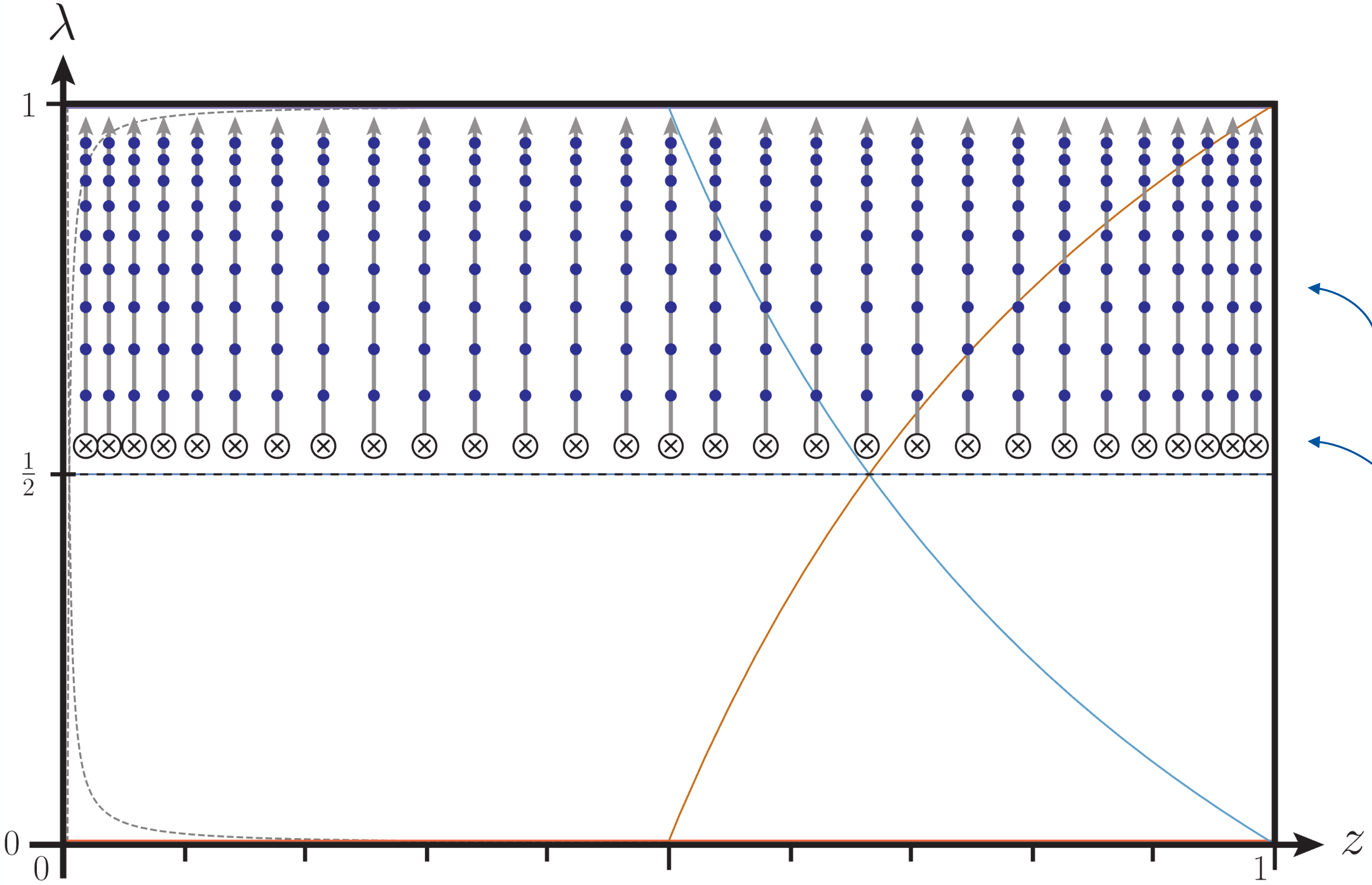
$m_b^2/m_H^2 = 1/684$

Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Poles of differential equations in λ

Evolution in the (z,λ)-plane



$z = 1 - m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_b^2/m_H^2 = 1/684$

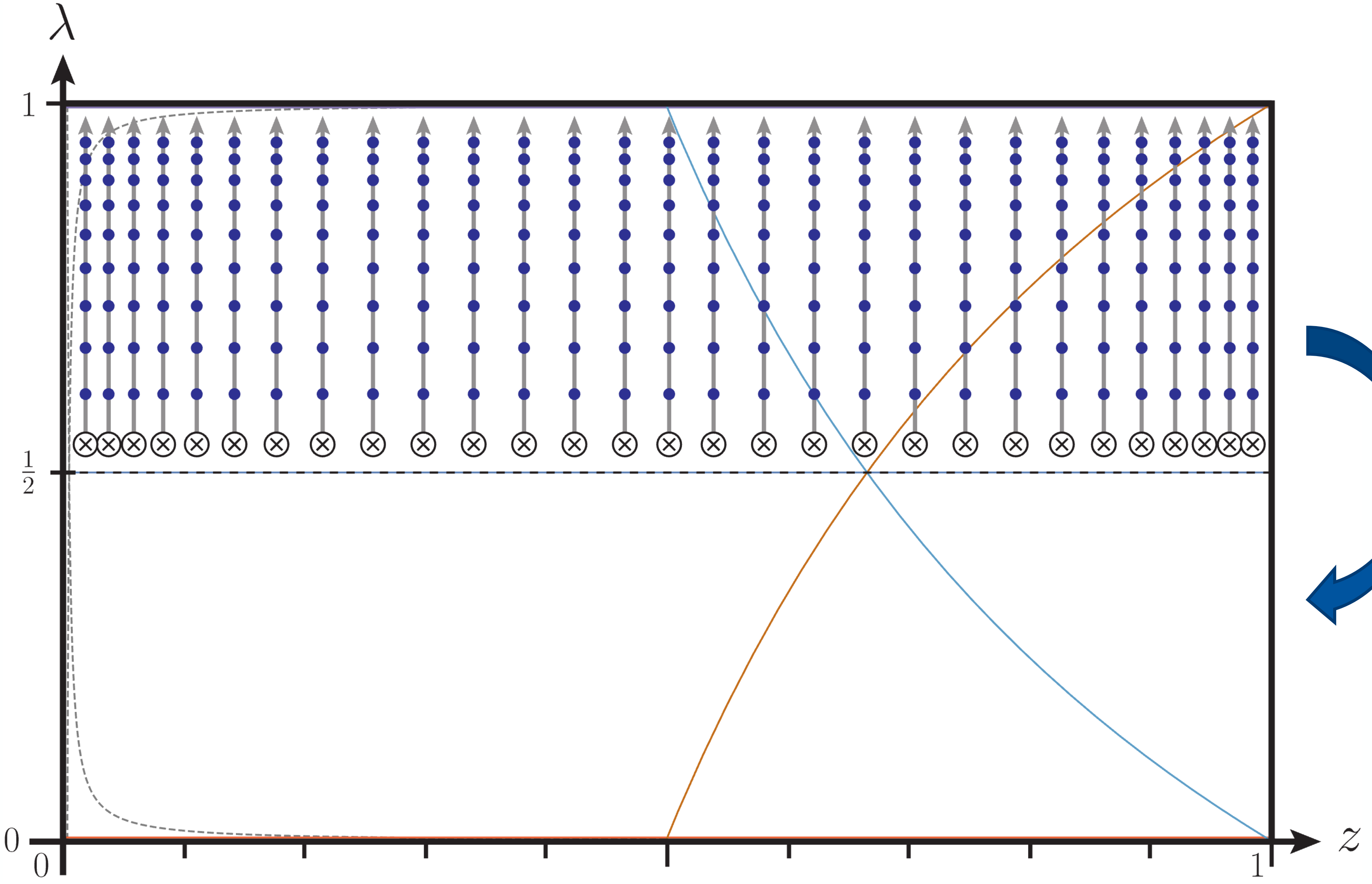
Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Collect numerical samples for MI along straight integration contours

Boundaries from numerical integration in the mass

Evolution in the (z,λ)-plane



$z = 1\text{-}m_H^2/\hat{s}$

$\lambda = \hat{t}/(\hat{t}+\hat{u})$

$m_b^2/m_H^2 = 1/684$

Range of parameters:

- $\lambda \in (0,1)$
- $z \in (0,1)$

Exploit symmetry of the problem at the amplitude level!

Construction of amplitudes

- Collected 2×10^6 numerical samples for MIs at m_t^2/m_H^2 by evaluation of the LME and numerical evolution above threshold
- Collected 1×10^6 numerical samples for MIs at m_b^2/m_H^2 via numerical evolution in the entire phase space

Insert into form factors and construct helicity amplitudes

$$\mathcal{M}_{+++} = \frac{1}{\sqrt{2}}\frac{s_{12}s_{23}}{\langle12\rangle\langle23\rangle\langle31\rangle}\left(F_1+\frac{s_{13}}{s_{23}}F_2+\frac{s_{13}}{s_{12}}F_3+\frac{s_{13}}{2}F_4\right)$$

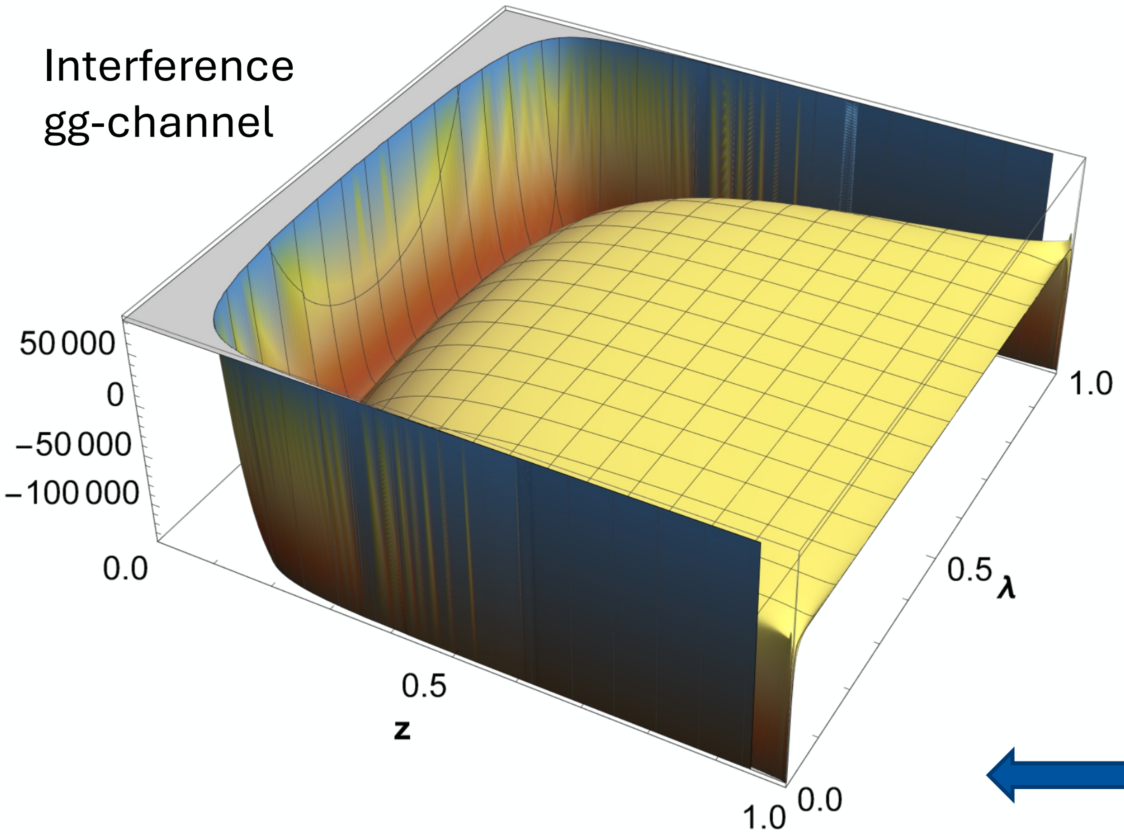
$$\mathcal{M}_{---} = \mathcal{M}_{+++}\Big|_{\langle ij\rangle\leftrightarrow[ji]} = -\frac{1}{\sqrt{2}}\frac{s_{12}s_{23}}{[12][23][31]}\left(F_1+\frac{s_{13}}{s_{23}}F_2+\frac{s_{13}}{s_{12}}F_3+\frac{s_{13}}{2}F_4\right)$$

$$\mathcal{M}_{++-} = \frac{1}{\sqrt{2}}\frac{[12]^3}{[13][23]}\frac{s_{23}}{s_{12}}\left(F_1+\frac{s_{13}}{2}F_4\right) \qquad \mathcal{M}_{--+} = \mathcal{M}_{++-}\Big|_{\langle ij\rangle\leftrightarrow[ji]}$$

$$\mathcal{M}_{+-+} = \frac{1}{\sqrt{2}}\frac{[13]^3}{[12][23]}\frac{s_{12}}{s_{13}}\left(F_2+\frac{s_{23}}{2}F_4\right) \qquad \mathcal{M}_{-+-} = \mathcal{M}_{+-+}\Big|_{\langle ij\rangle\leftrightarrow[ji]}$$

$$\mathcal{M}_{-++} = \frac{1}{\sqrt{2}}\frac{[23]^3}{[12][13]}\frac{s_{13}}{s_{23}}\left(F_3+\frac{s_{12}}{2}F_4\right) \qquad \mathcal{M}_{+--} = \mathcal{M}_{-++}\Big|_{\langle ij\rangle\leftrightarrow[ji]}$$

$$|\mathcal{M}|^2 = \sum_{h\in Hel.}|\mathcal{M}_h|^2$$



Interference gg-channel