42nd International Conference on High Energy Physics

Measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section in the centre-of-mass range 0.62 to 3.5 GeV at Belle II

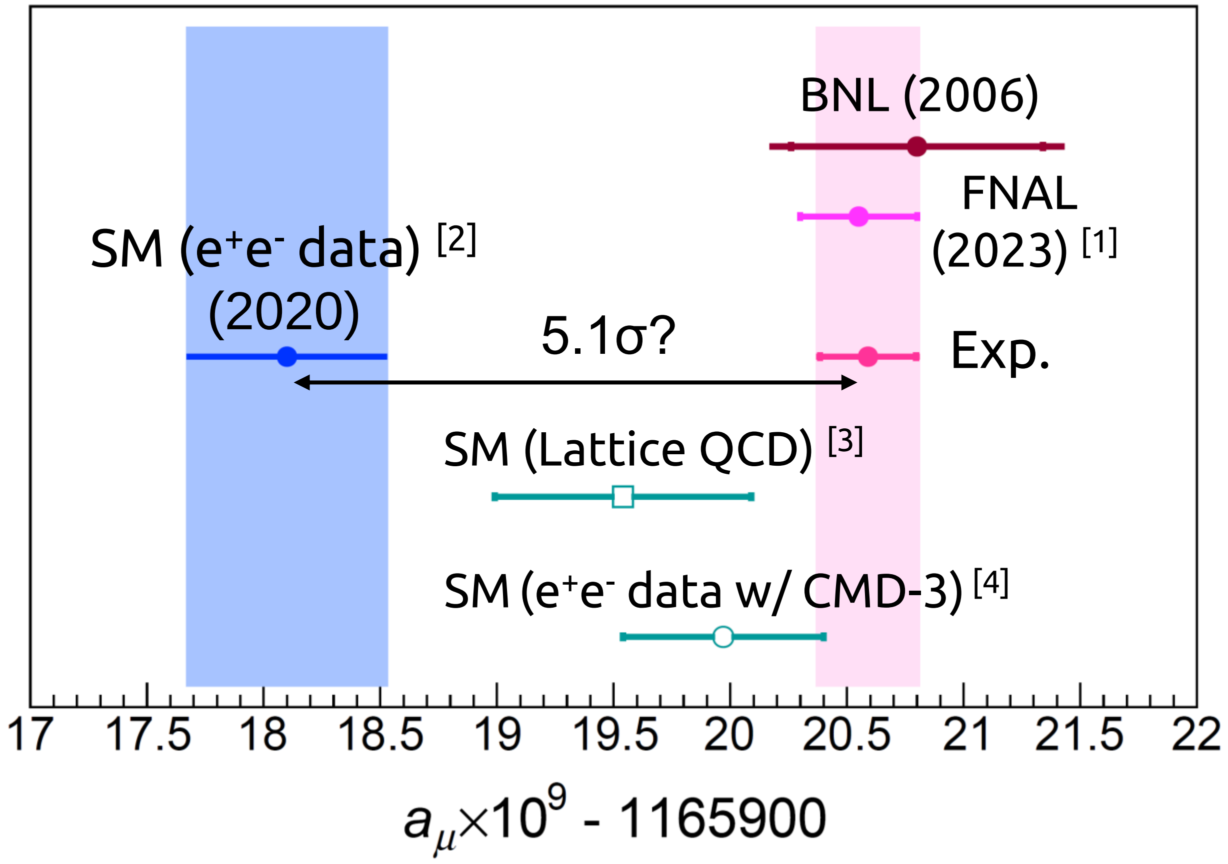
Yuki Sue, Nagoya University
on behalf of the Belle II collaboration

18 July 2024

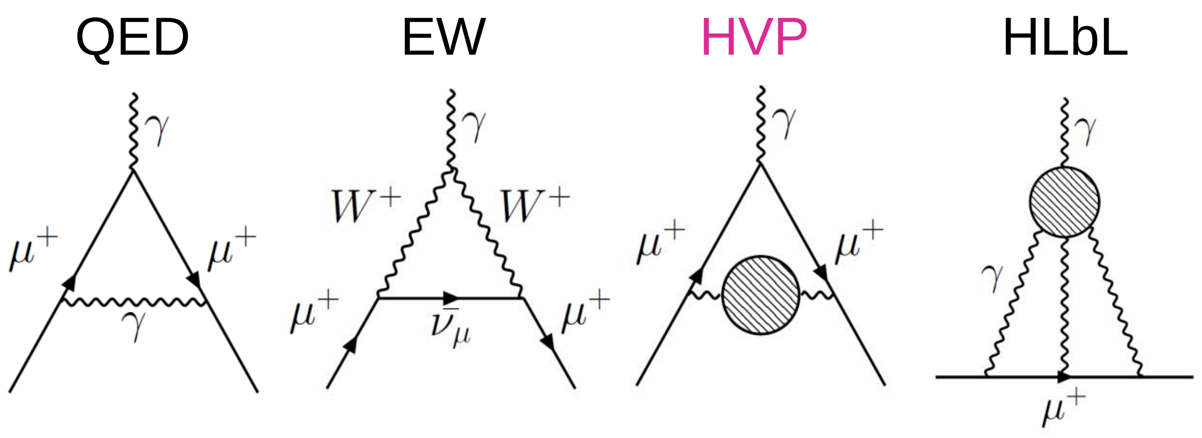
arXiv:2404.04915

Present status of the muon (g-2) SM calculation

- 5σ (or 1-2σ) difference with new direct measurements by Fermilab experiment
- Non-negligible uncertainty in theoretical predictions
- Major uncertainty (~80%) is derived from Hadronic Vacuum Polarization (HVP) term
- Validation by independent experiments is important to understand HVP situation



$$a_\mu^{\mathrm{SM}} = \frac{g_\mu - 2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$



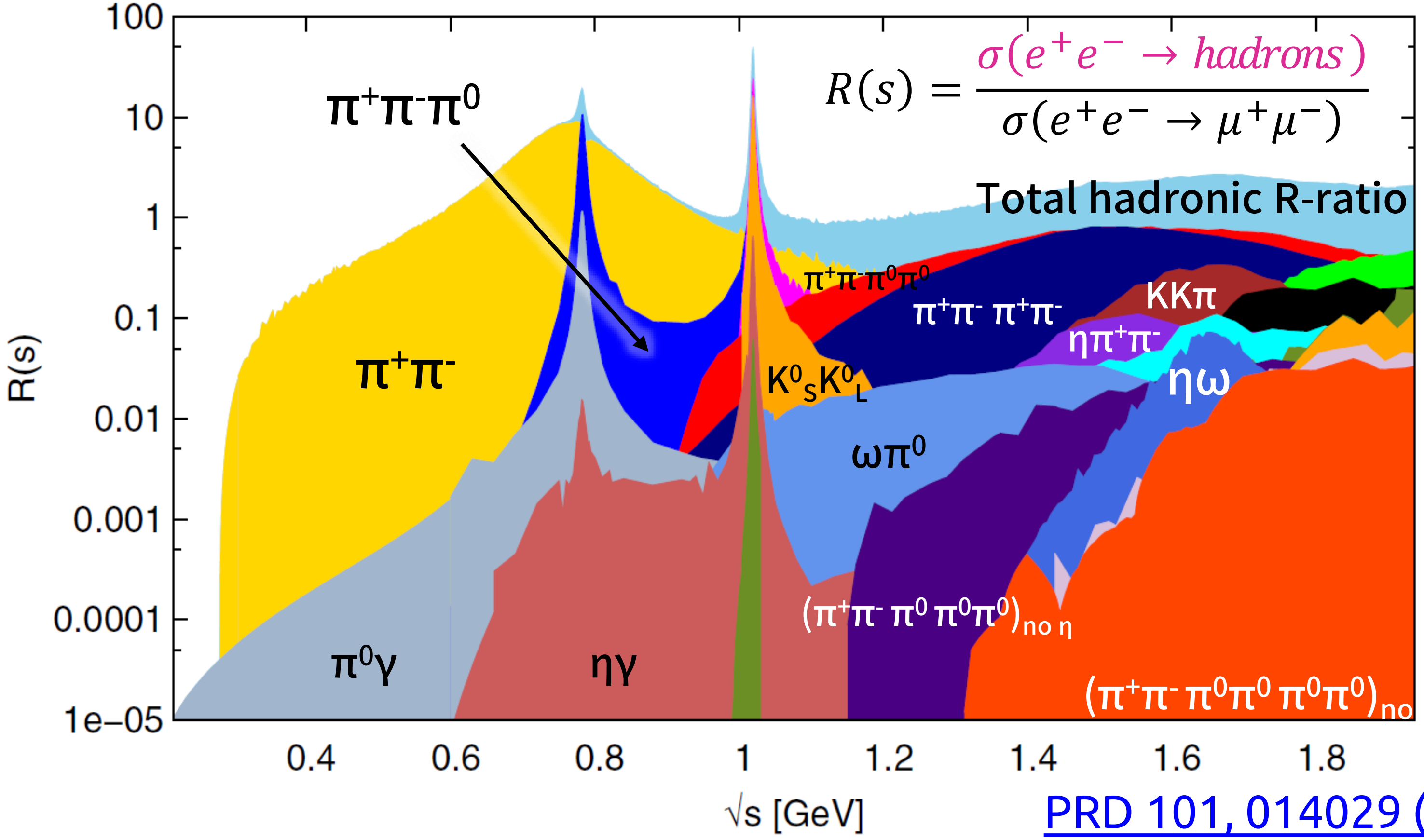
[1] PRL 131, 161802 (2023)
[2] Phys. Rept. 887, 1 (2020)
[3] Nature 593, 7857 (2021)
[4] PRL 132, 231903 (2024)

Cross section measurements of exclusive channels

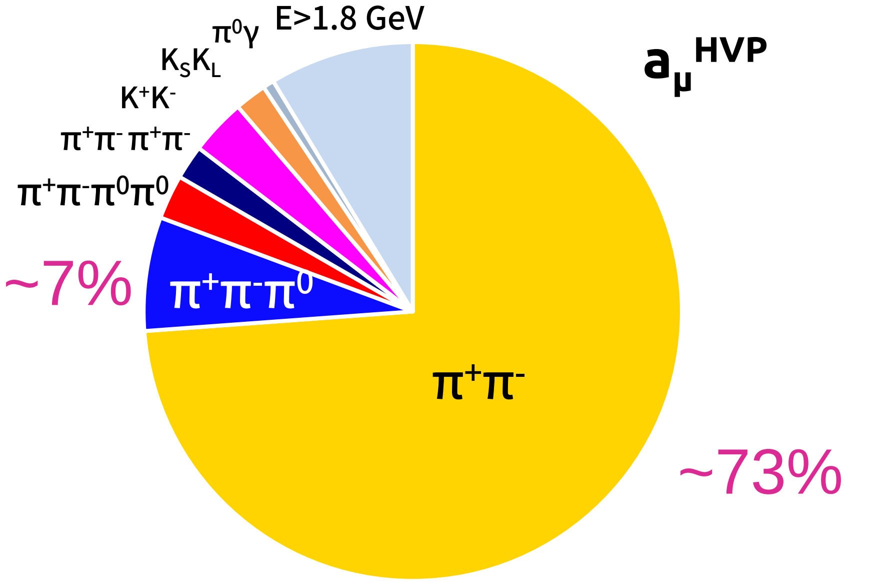
Leading order HVP contribution

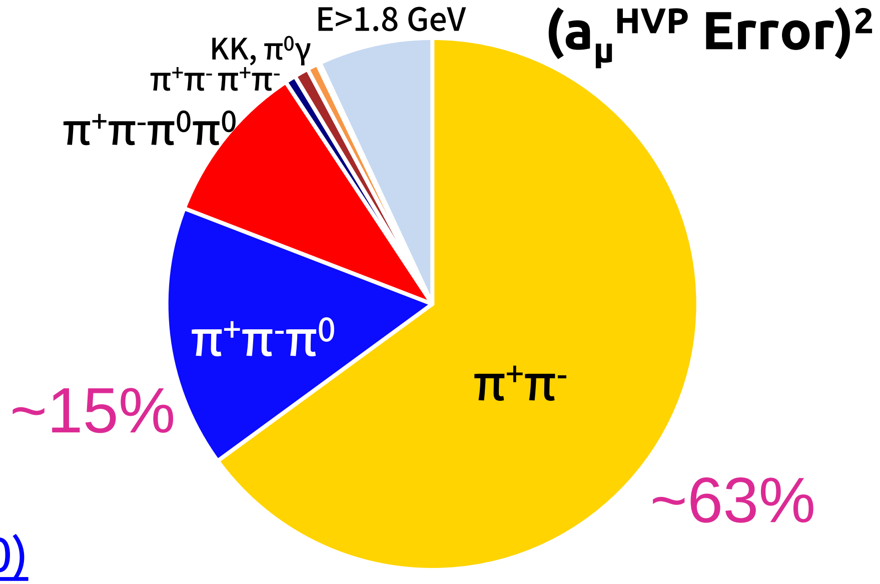
$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{\widehat{K}(s)}{s^2} \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} ds$$

- Verify cross sections at Belle II
- As a first step, we begin with $e^+e^- \to \pi^+\pi^-\pi^0$ channel

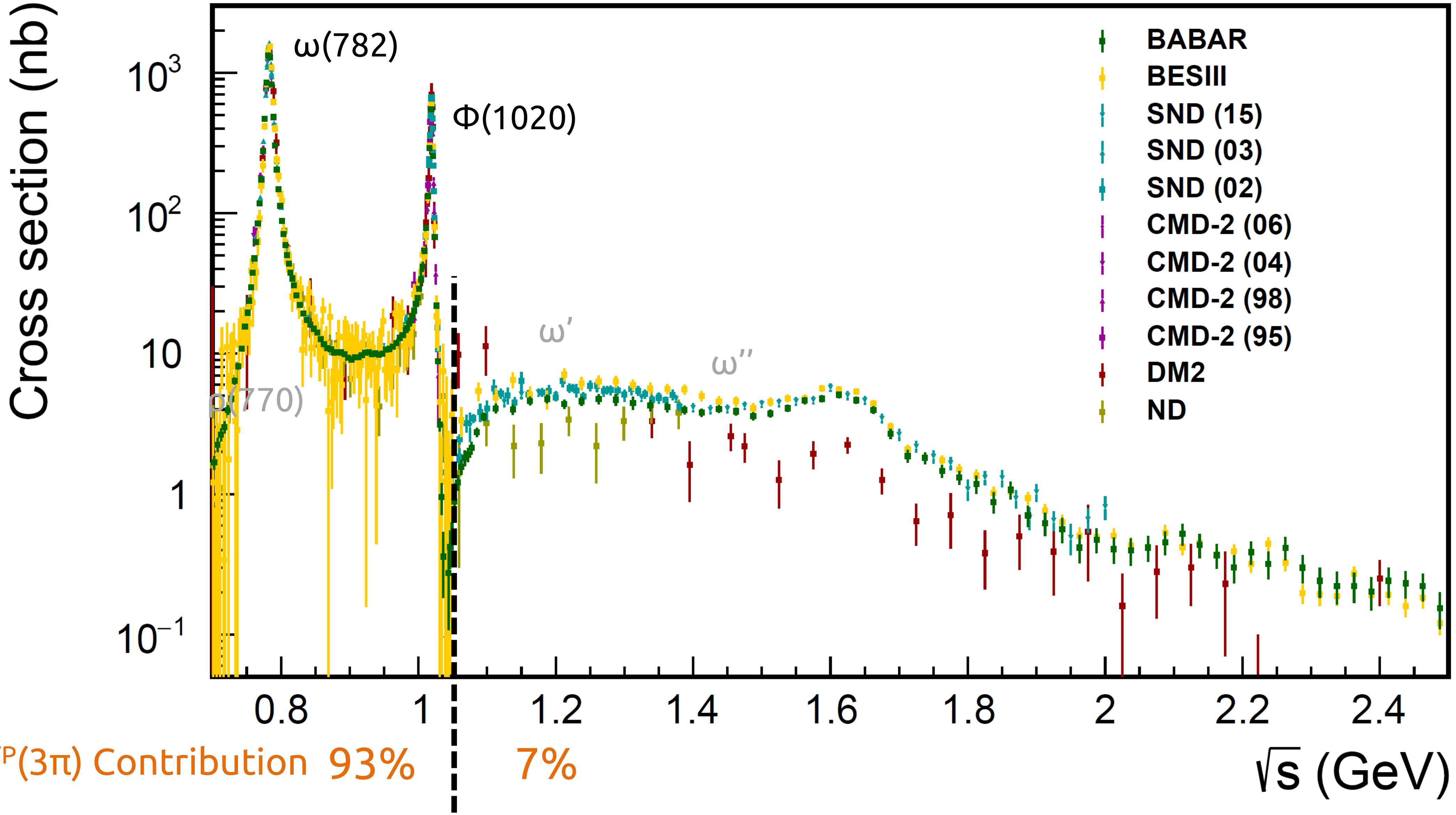


PRD 101, 014029 (2020)





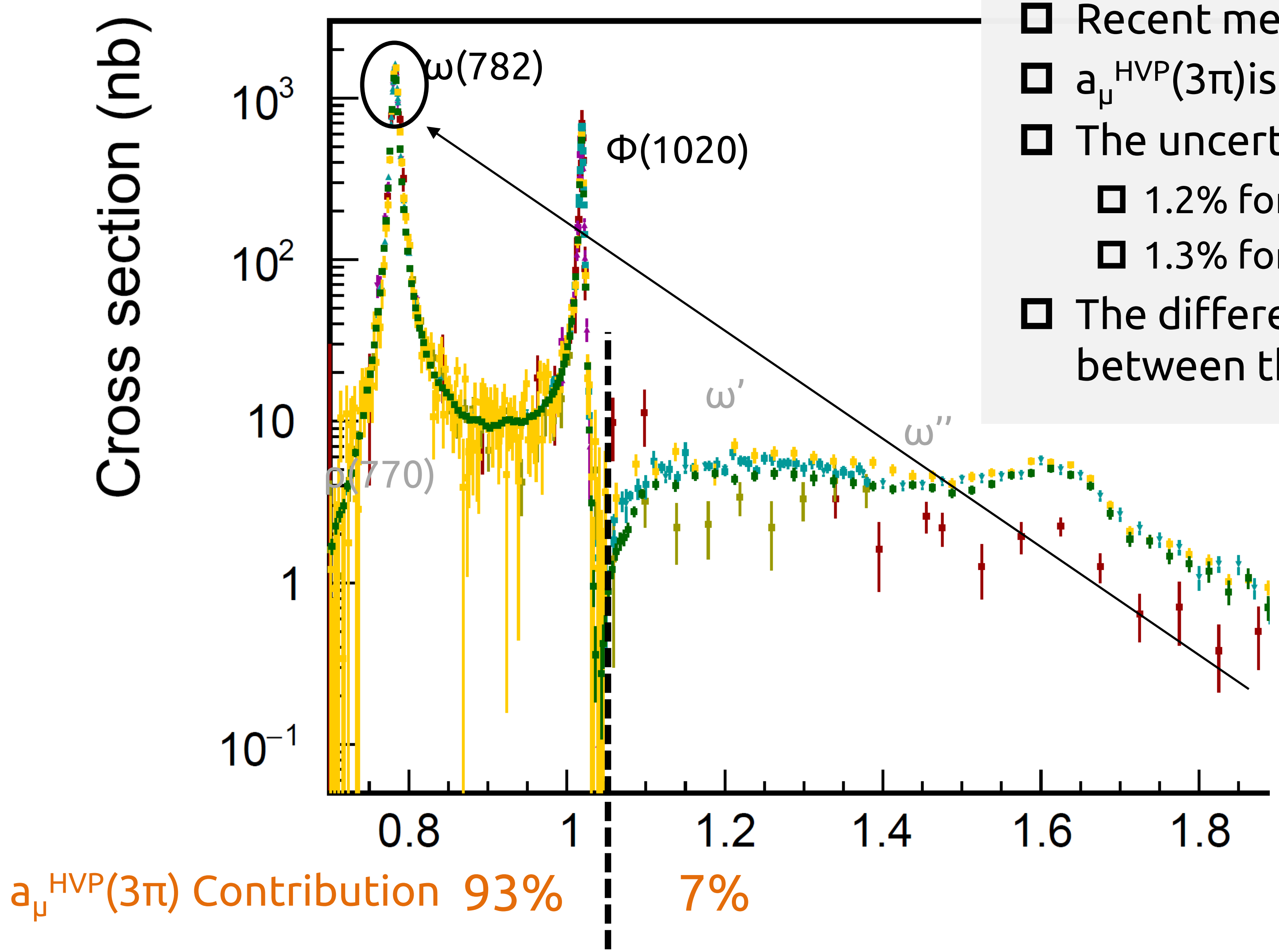
Previous measurements for $e^+e^- \to \pi^+\pi^-\pi^0$

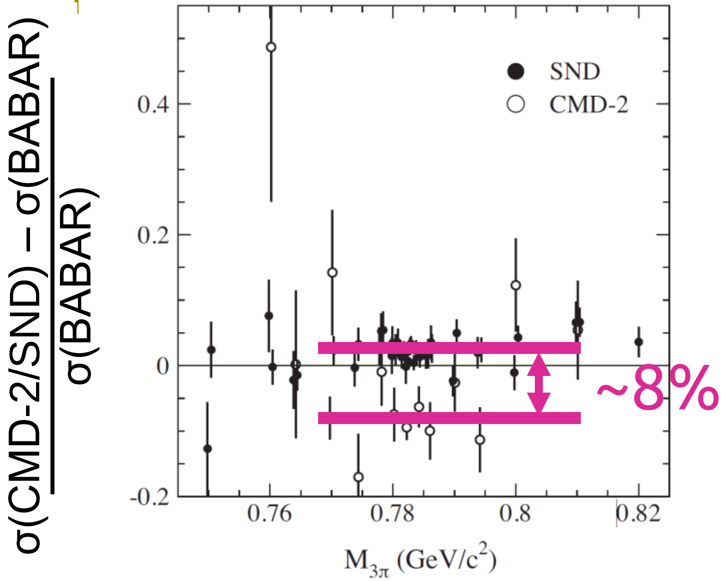


$a_\mu^{HVP}(3\pi)$ Contribution 93% 7%

Previous measurements for $e^+e^- \to \pi^+\pi^-\pi^0$

- Recent measurements: BaBar, SND, CMD-2…
- $a_\mu^{HVP}(3\pi)$ is dominated by ω and Φ resonances
- The uncertainty of $a_\mu(3\pi)$:
  - 1.2% for the global fit
  - 1.3% for BABAR alone
- The difference in the cross section between the experiments below 1.1 GeV

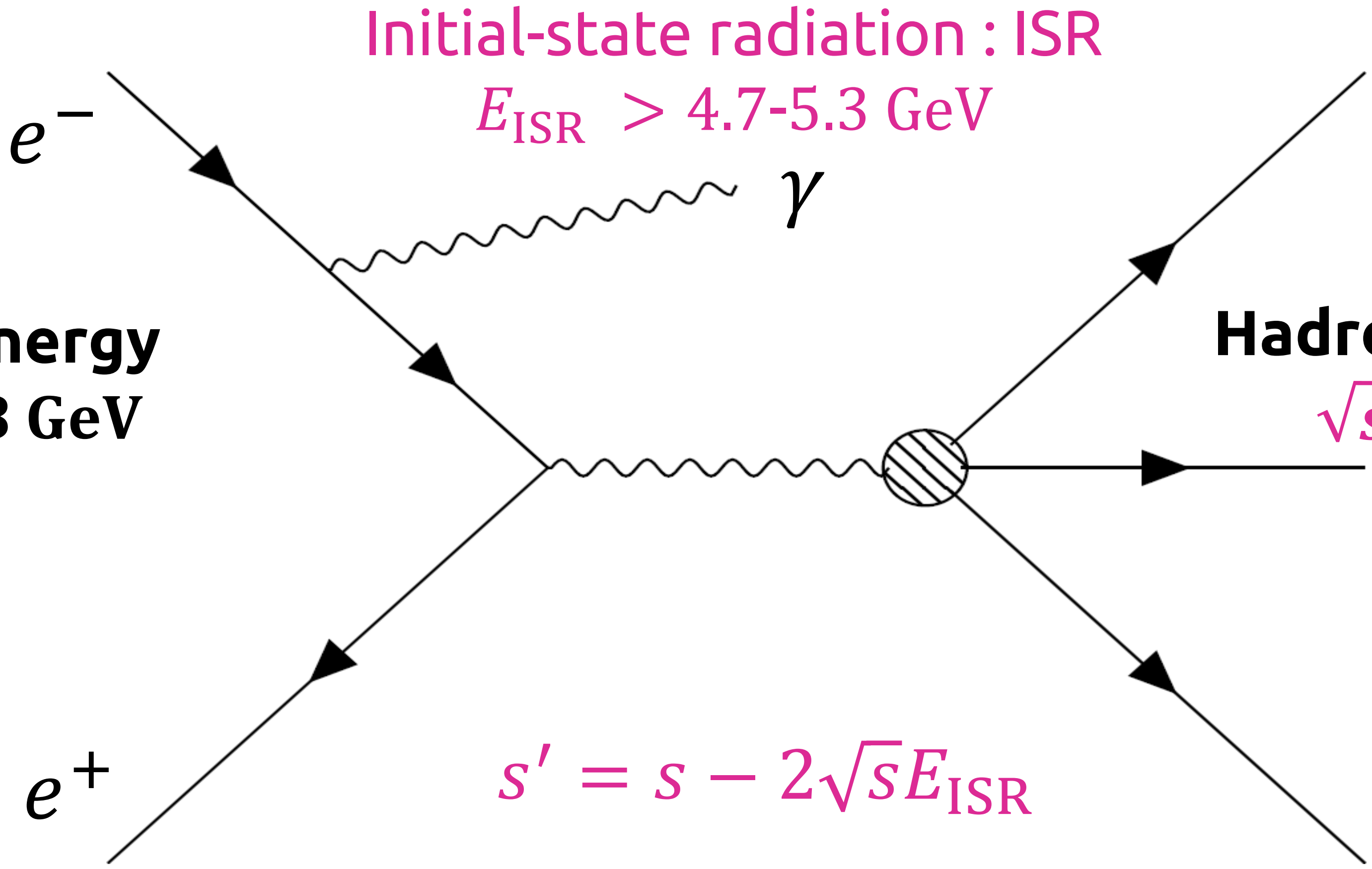




Radiative return method

- Use a process associated with energetic ISR emission
- Measure the cross section $e^+e^-\to$hadrons in the energy range 0.4-3.5 GeV in e^+e^- collision at 10.58 GeV

Initial e^+e^- energy
$\sqrt{s} = 10.58$ GeV

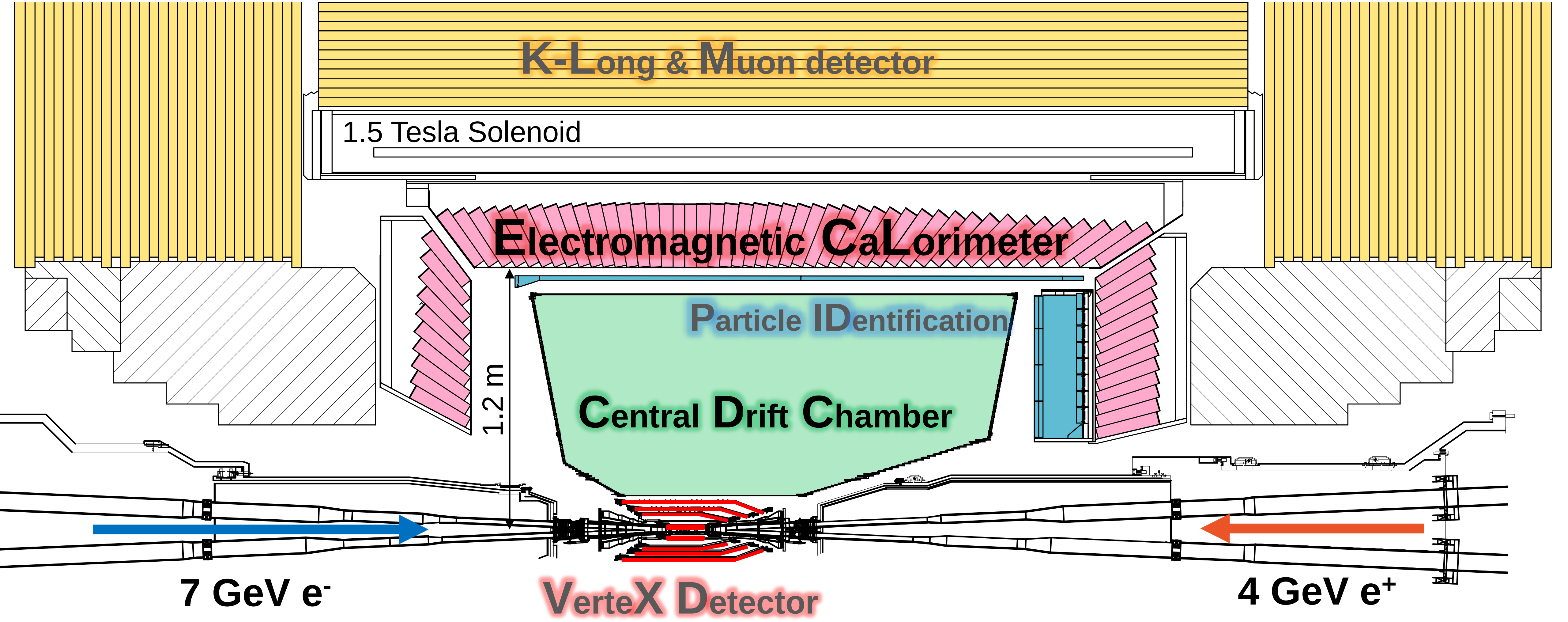


Hadronic system energy
$\sqrt{s'} = 0.4\text{-}3.5$ GeV

Belle II detector

Trigger & DAQ

New calorimeter-based trigger enables light-hadron cross section measurements



Analysis overview

Signal spectrum

Cross section

$$\sigma_{3\pi}(M(3\pi)) = \frac{N_{\text{signal}}}{\varepsilon(M(3\pi)) \cdot L_{\text{eff}}(M(3\pi))}$$

3π mass **Efficiency** **Integrated luminosity**

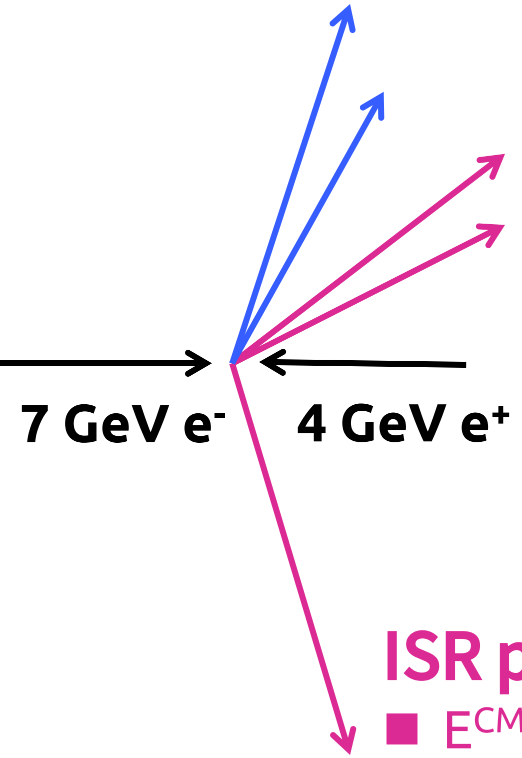
- Target : $\delta a_\mu^{3\pi}/a_\mu^{3\pi}$ ~2% with 191 fb^{-1} data
- Key items
  - Robust event selection to extract $e^+e^-\to\pi^+\pi^-\pi^0\gamma_{ISR}$
  - Background suppression and background determination (≤1% at ω)
  - Precise determination of the efficiency in ≤1%
  - Unfolding the spectrum to remove detector resolution effects
- Blind analysis
  - The data are examined after all selections and corrections are determined.

$e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR}$ selection

Reconstruct two tracks + three photons : $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR} \to \pi^+\pi^-\gamma\gamma\gamma_{ISR}$

$\pi^\pm$

- From the interaction point
- Exact two tracks in an event



π^0-decay photons

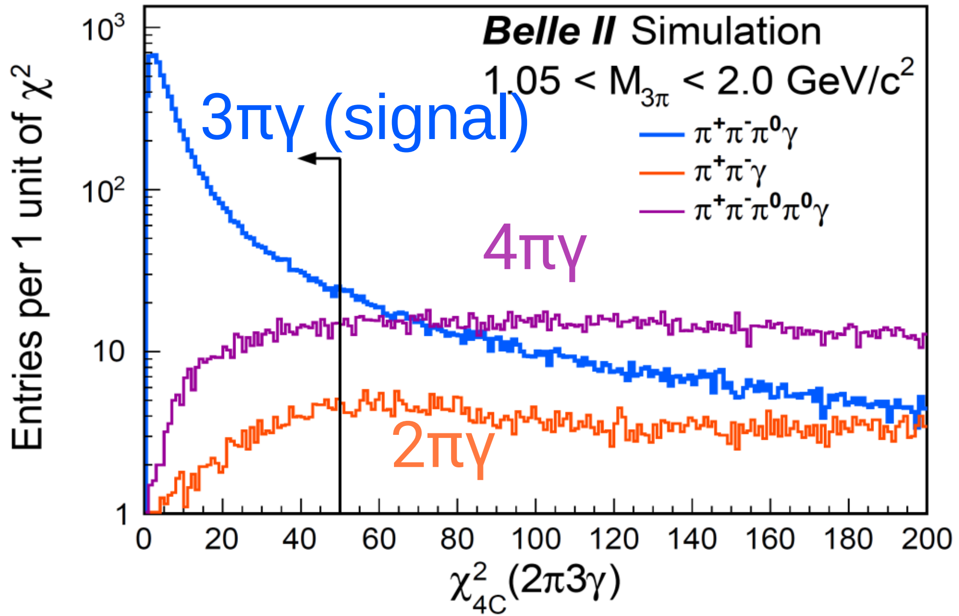
- E > 100 MeV
- $M(\gamma\gamma) < 1\ \mathrm{GeV}/c^2$
  Wide range for π^0 mass fit

ISR photon

- $E^{CMS} > 2$ GeV
- In barrel ECL for trigger

- New calorimeter trigger using ISR photon
  - > 99% efficiency
- Four-momentum conservation kinematic fit
- Residual background suppression
  - Reduce ~90% background losing 10% signal

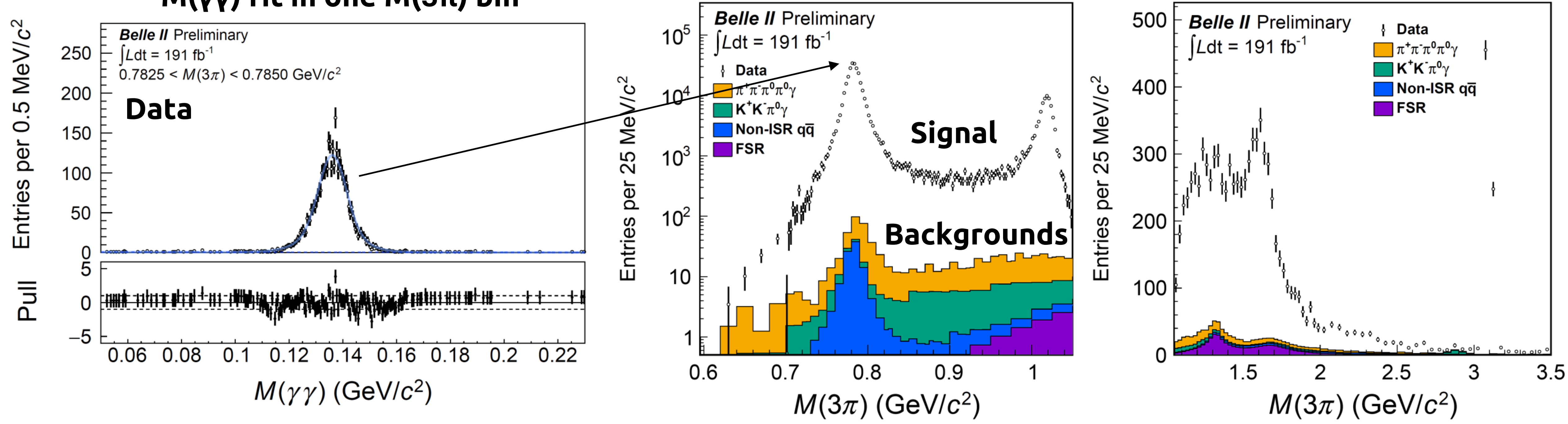
4C-Kfit χ^2 distribution (MC)



Signal extraction

- Fit M(γγ) in each M(3π) bin to extract π^0 signal
- Test residual backgrounds using data control samples

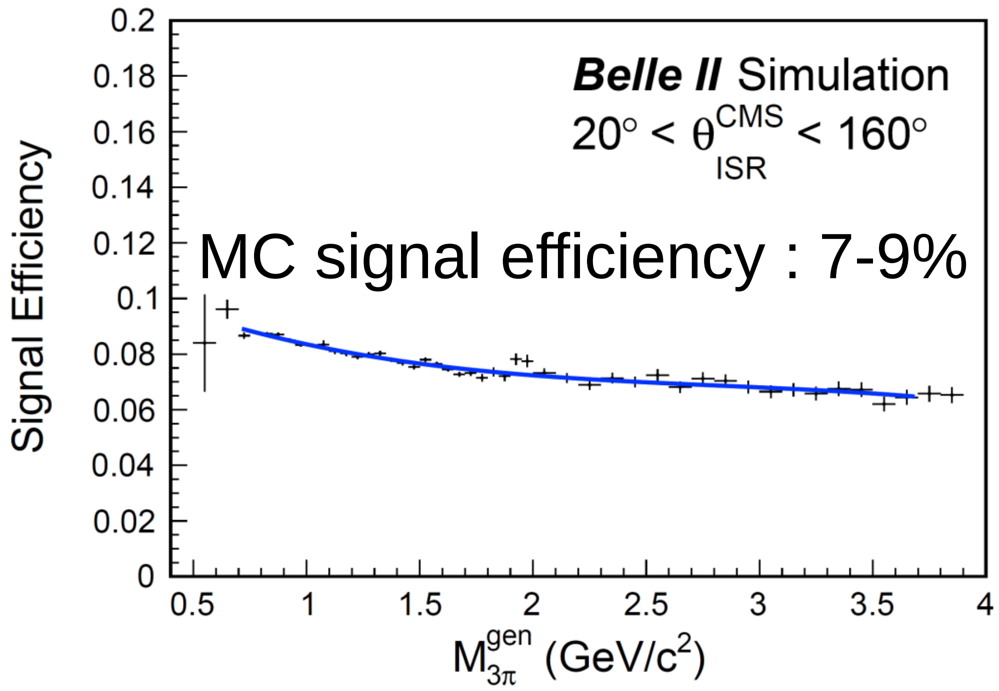
M(γγ) fit in one M(3π) bin



Signal efficiency and data-MC corrections

Efficiency $\varepsilon = \varepsilon_{\mathrm{MC}} \prod_i (1 + \eta_i)$ Data-MC correction η_i ~ O(1)%

- 1st order signal efficiency is estimated using MC of the x10 larger statistics
- Possible differences between data and MC are studied in data-driven way using several control samples



Sources	Efficiency correction η_i (%)
Trigger	-0.1±0.1
ISR photon detection	0.2±0.7
Tracking	-1.4±0.8
π^0 detection	-1.4±1.0
Background suppression	-1.9±0.2
χ^2 distribution	0.0±0.6
Total correction	-4.6±2.0

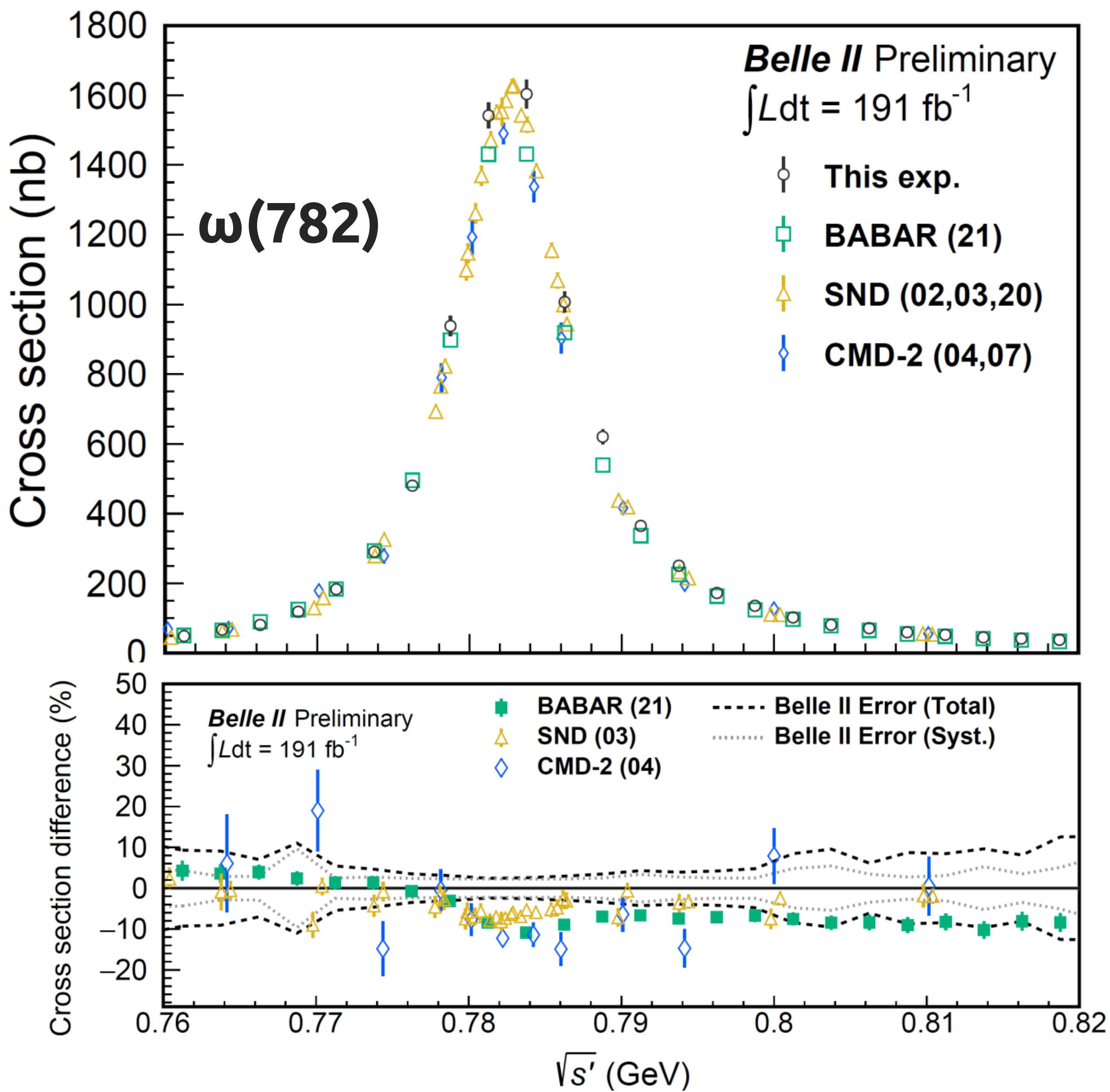
Systematic uncertainty for $e^+e^-\to\pi^+\pi^-\pi^0$ cross section

- Luminosity is measured with Bhabha events and confirmed with $e^+e^-\to\gamma\gamma$ and $\mu^+\mu^-$ processes
- Major systematic uncertainty comes from MC generator, and π^0 efficiency

Source	Systematic uncertainty (%)	
	$\sqrt{s}$ < 1.05 GeV[2]	$\sqrt{s}$ > 1.05 GeV
Trigger efficiency	0.1	0.2
ISR photon efficiency	0.7	0.7
Tracking efficiency	0.8	0.8
π^0 efficiency	1.0	1.0
χ^2 criteria efficiency	0.6	0.3
Background suppression efficiency	0.2	1.9
MC generator (due to missing NNLO MC)	1.2	1.2
Radiative correction	0.5	0.5
Integrated luminosity	0.6	0.6
Total systematics	2.2	2.8

Result: cross section at the ω resonance

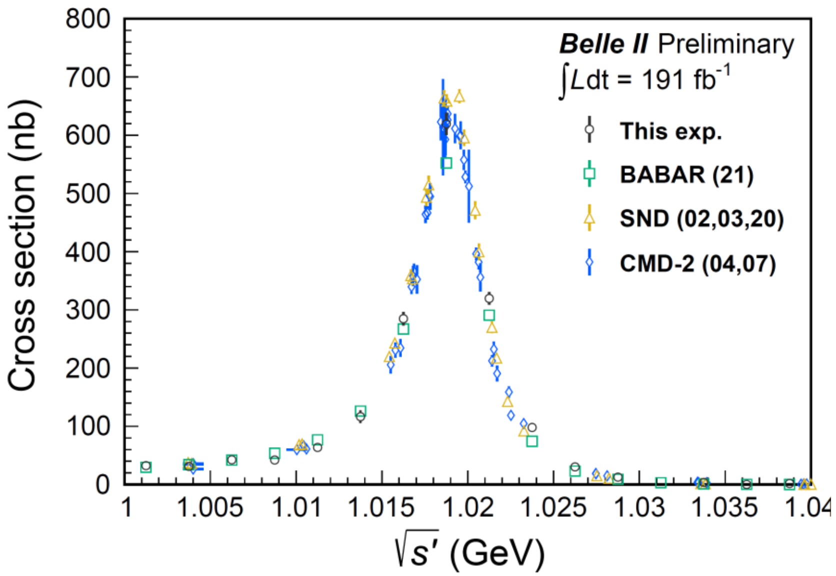
- ω resonance has a large cross section and a large contribution to $a_\mu(3\pi)$
- Measured cross section at ω is 5-10% higher than BABAR, SND, and CMD-2



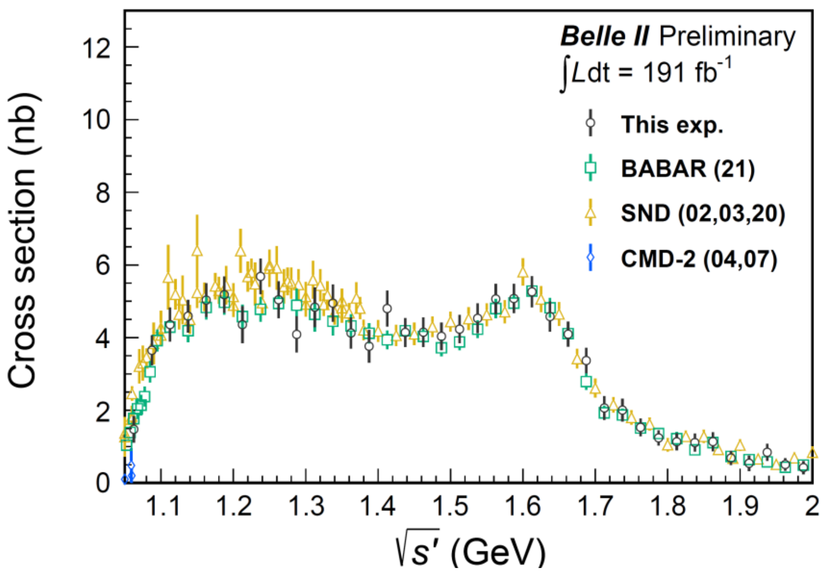
Result: cross section in higher energy

- Cross section in √s' > 1.05 GeV is in good agreement with BABAR result

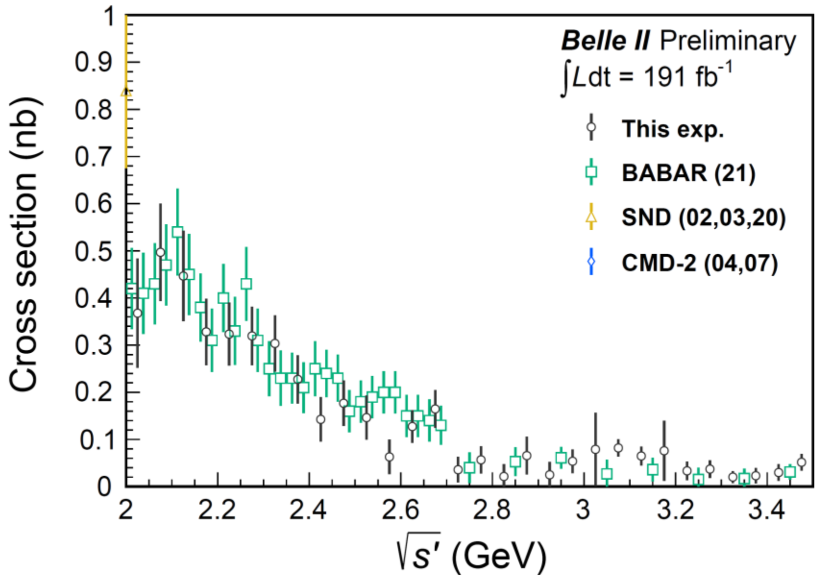
Φ(1020)



1.05-2.00 GeV



2.00-3.50 GeV



Results: 3π contribution to a_μ HVP

Contribution to 3π LO HVP using solely our result

$$a_\mu^{\mathrm{LO,HVP},3\pi}(0.62\text{-}\ 1.8\ \mathrm{GeV}) = \left(48.91 \pm 0.25_{\mathrm{stat}} \pm 1.07_{\mathrm{syst}}\right) \times 10^{-10}$$

	$a_\mu(3\pi)\times10^{10}$	Difference$\times10^{10}$
BABAR alone [PRD 104, 11 (2021)]	45.86 ± 0.14 ± 0.58	3.2±1.3 (6.9%)
Global fit* [JHEP 08, 208 (2023)]	45.91 ± 0.37 ± 0.38	3.0±1.2 (6.5%)

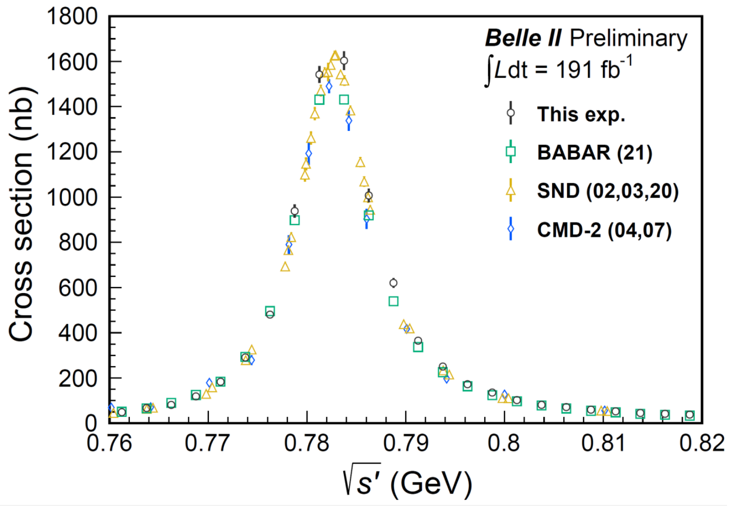
* Not includes BESIII preliminary result [arXiv:1912:11208]

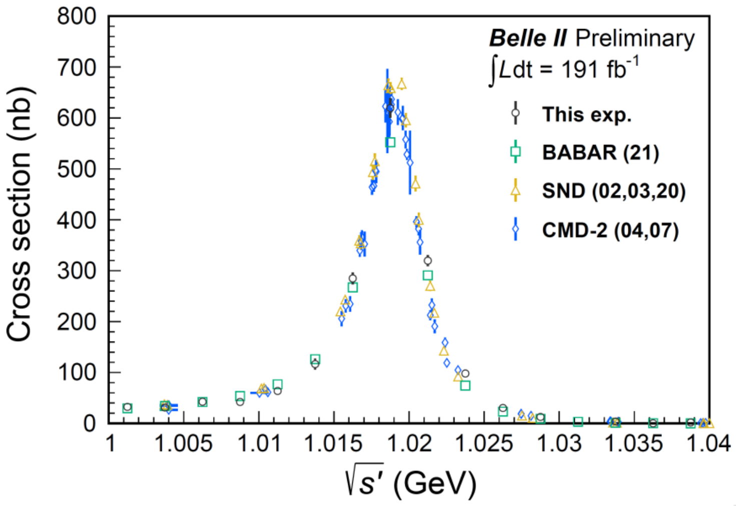
- 6.5% higher than the global fit result with 2.5σ significance
- This difference $3\text{x}10^{-10}$ corresponds 10% of $\Delta a_\mu = a_\mu(\mathrm{Exp}) - a_\mu(\mathrm{SM}) = 25\text{x}10^{-10}$

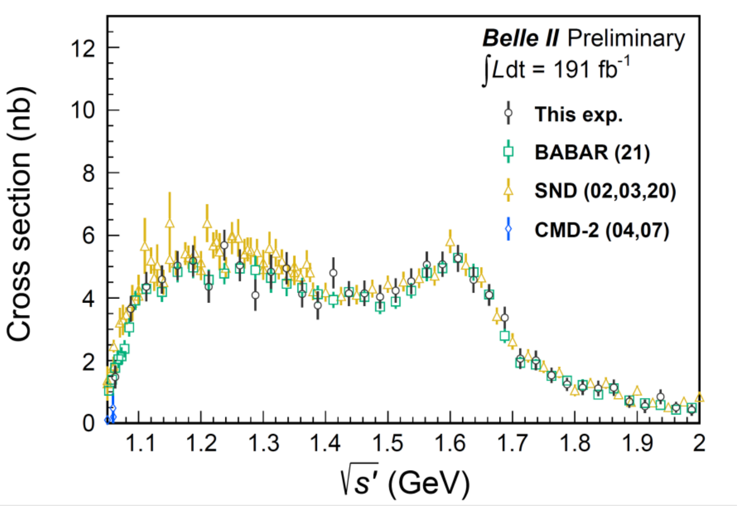
Summary

- Cross-section measurements are ongoing at the SuperKEKB/Belle II experiment
  - High trigger efficiency well determined by the comparison of independent trigger mode rates
- We measured the $e^+e^-\to\pi^+\pi^-\pi^0$ cross section with systematic uncertainty of 2.2%
  - This is the first $e^+e^-\to$hadrons cross section measurement at Belle II
  - Experimental systematic uncertainty is well-understood
  - The remaining largest uncertainty is from the MC generator due to missing NNLO QED generator
- Our results are about 2.5σ greater than BABAR and global fit
  - $a_\mu^{\mathrm{LO,HVP,}}(3\pi) = \left(48.91 \pm 0.25_{\mathrm{stat}} \pm 1.07_{\mathrm{syst}}\right) \times 10^{-10}$

Submitted to PRD [arXiv:2404.04915]







Next: $e^+e^- \to \pi^+\pi^-$ at Belle II

- Further analyses of the hadronic channels via ISR are on going
- Target precision for $e^+e^- \to \pi^+\pi^-$: 0.5% of $a_\mu(2\pi)$
- Trying to follow BABAR methods as a baseline
- Systematics uncertainty dominant analysis
  - BABAR : 232 fb^{-1} [PRD 86 032013 (2012)]
  - We can use a larger dataset to control systematic uncertainties
- Design of data-driven efficiency corrections for tracking, trigger and π/μ/K ID is ongoing

Backup

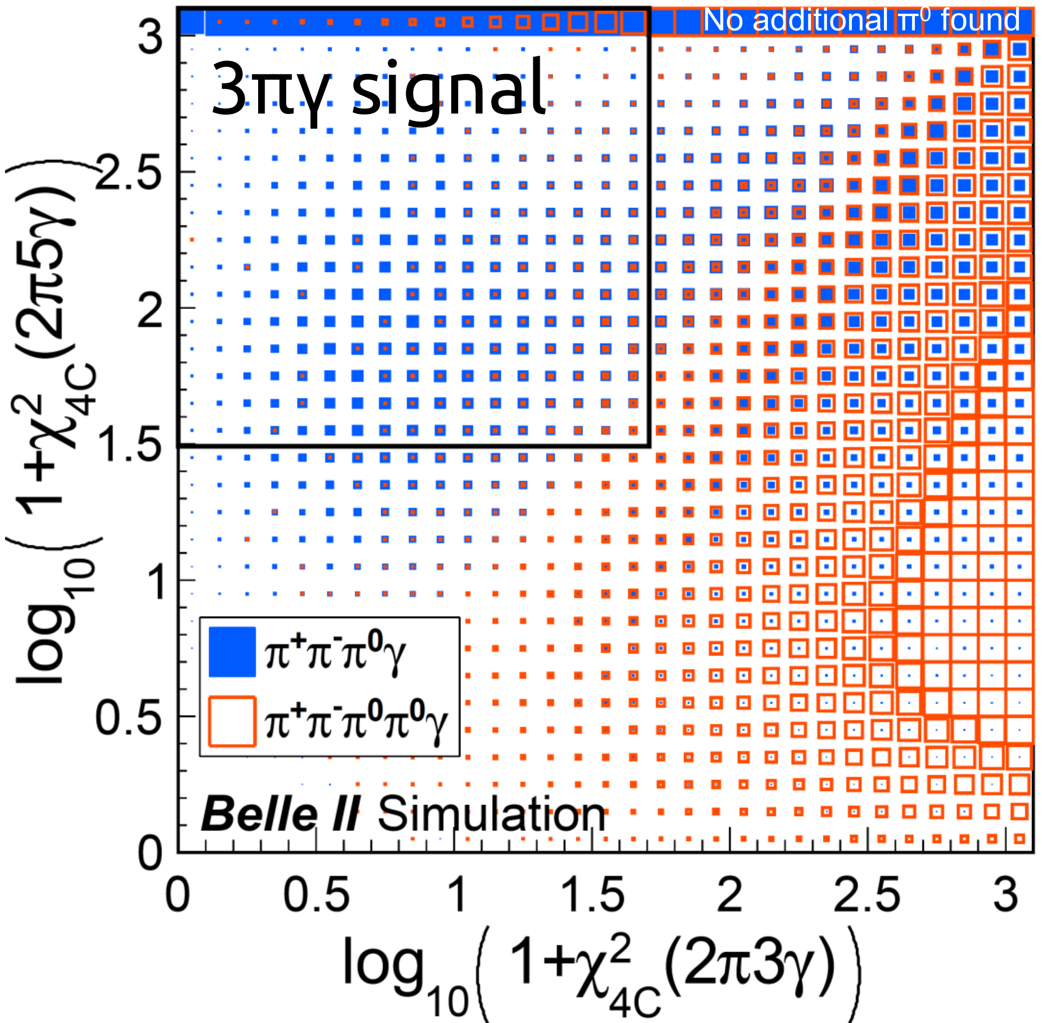
Background suppression (1)

A) Background not containing real π^0 : $e^+e^- \to e^+e^-\gamma$, $\pi^+\pi^-\gamma$, $\mu^+\mu^-\gamma$
   - Pion/Electron ID > 0.1
   - $M^2_{recoil}(\pi^+\pi^-) > 4\ GeV^2/c^4$

B) Charged kaon : $e^+e^- \to K^+K^-\pi^0\gamma$
   - Pion/Kaon ID $L(\pi/K) > 0.1$

C) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$
   - Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ (with additional π^0)
   - 4C kinematic fit under $\pi^+\pi^-\pi^0\pi^0\gamma$ ($2\pi5\gamma$) hypothesis, and $\chi^2_{4C}(2\pi5\gamma) > 30$

$\chi^2_{4C}(2\pi3\gamma)$ versus $\chi^2_{4C}(2\pi5\gamma)$



Background suppression (2)

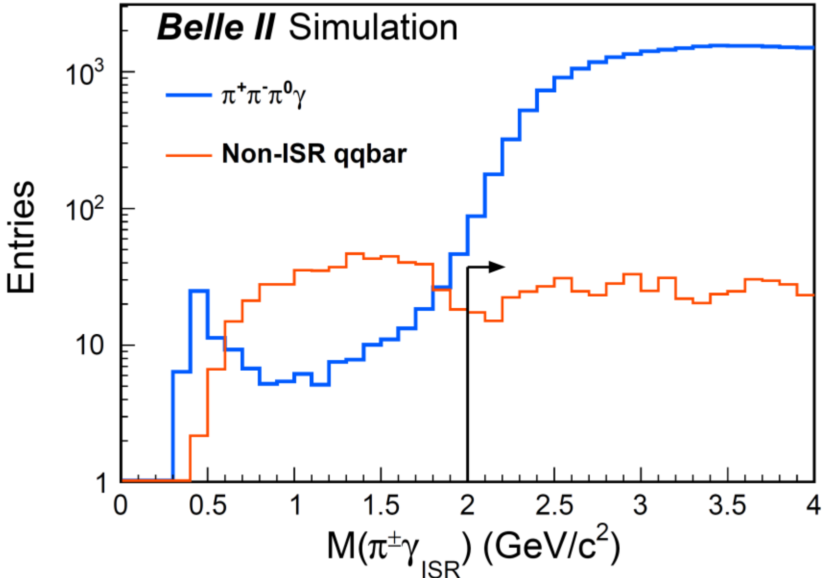
D) Background not containing real ISR : Non-ISR qqbar (dominated by $\pi^+\pi^-\pi^0\pi^0$) and $\tau^+\tau^-$

i. $M(\pi^\pm\gamma_{ISR})$ > 2 GeV/c^2 to reduce high momentum $\rho^\pm \to \pi^+\pi^0$

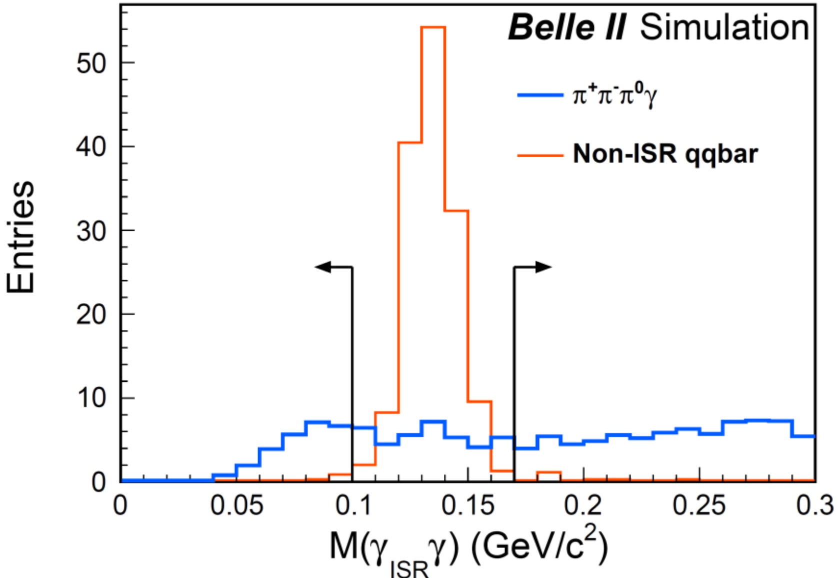
ii. $M(\gamma_{ISR}\gamma)$ cut to reduce ISR candidate from π^0-decay photon

iii. Cluster shape cut to reduce ISR-like photon in which two photons from of π^0 are merged

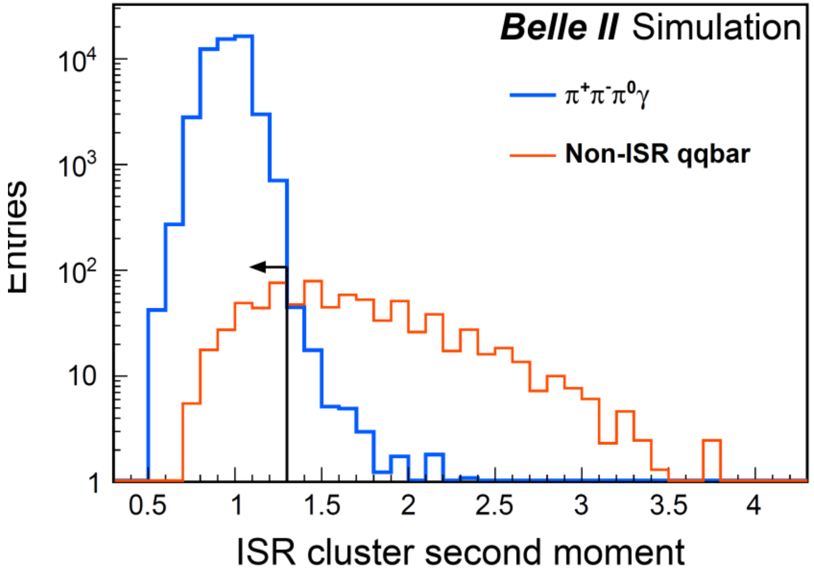
i) $M(\pi^\pm\gamma_{ISR})$ cut



ii) $M(\gamma_{ISR}\gamma)$ cut



iii) ISR photon cluster shape cut

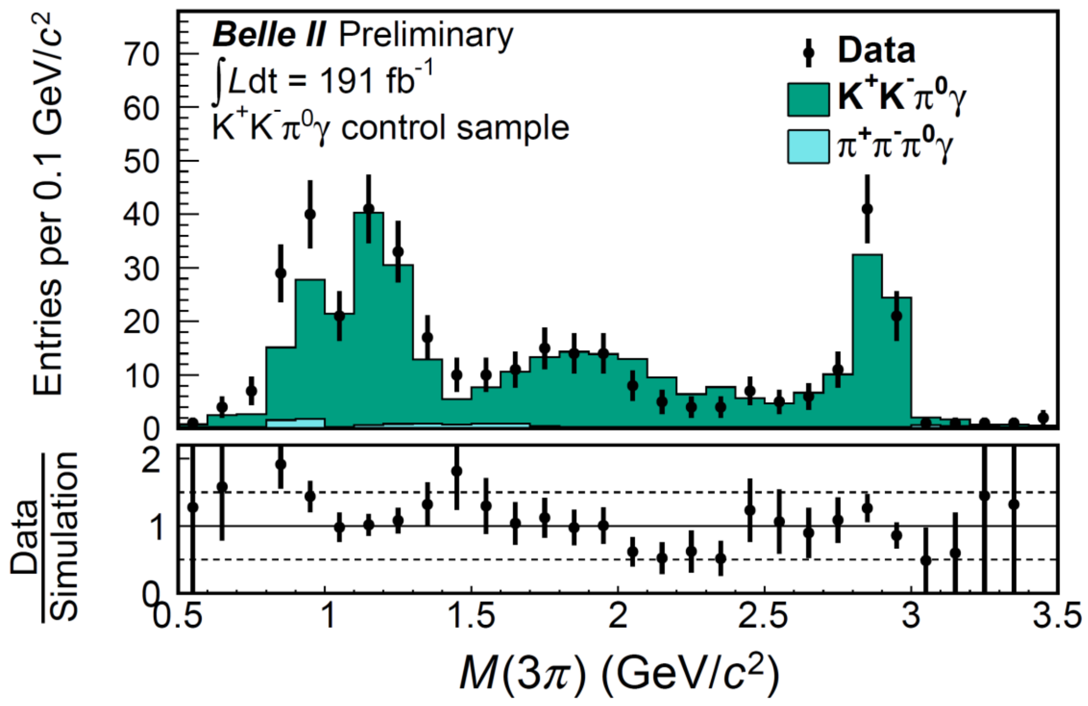


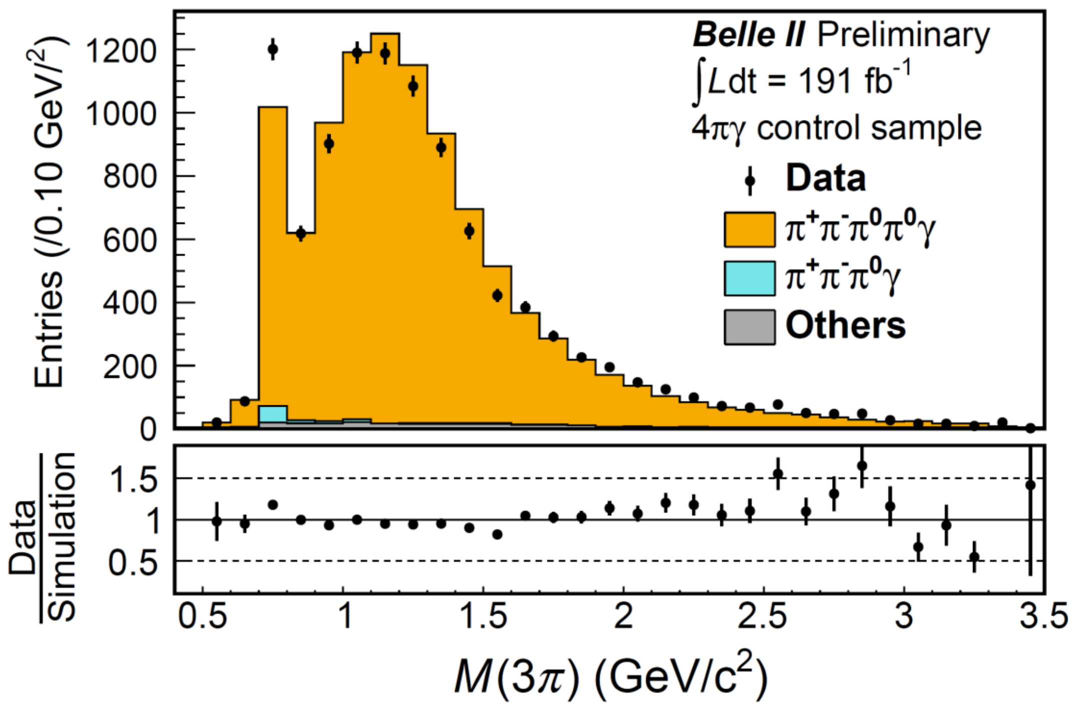
Background estimation

Estimate by determining a mass-dependent data-MC scale factor using a control sample.

$$N^{\mathrm{data}}_{\mathrm{Signal}} = N^{\mathrm{MC}}_{\mathrm{Signal}} \cdot \frac{N^{\mathrm{data}}_{\mathrm{Control}}}{N^{\mathrm{MC}}_{\mathrm{Control}}}$$

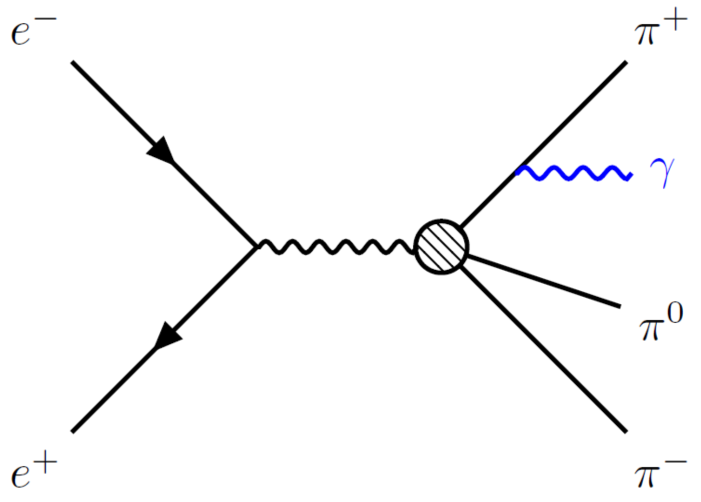
- $e^+e^- \to K^+K^-\pi^0\gamma$: Invert π/K-ID $L(\pi/K) > 0.1 \Rightarrow L(\pi/K) < 0.1$
- $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$: Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2(4\pi\gamma) < 30$
- Non-ISR qqbar : $0.10 < M(\gamma_{ISR}\gamma) < 0.17$ GeV / large cluster second moment





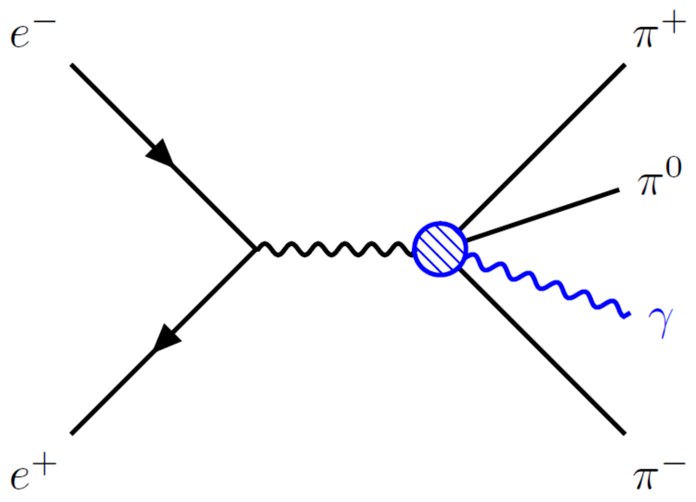
Final-state radiation background

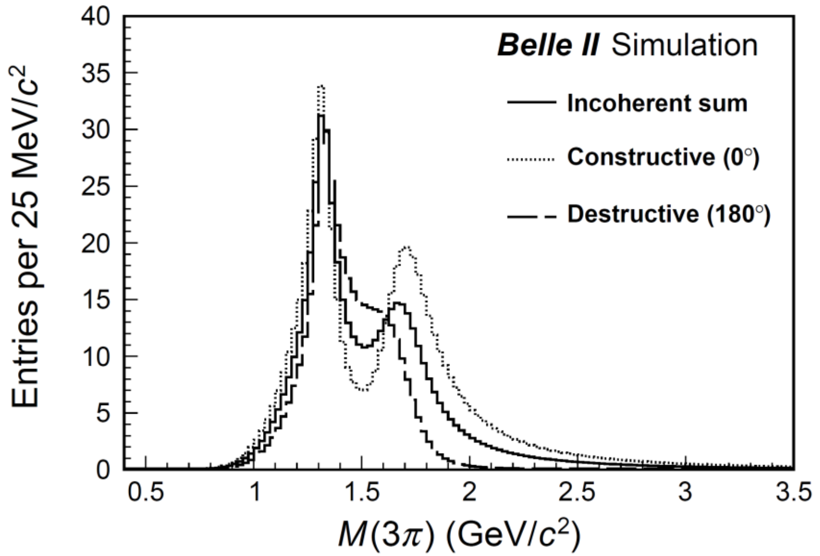
- Difficult to reject FSR background or extract control sample
- Estimate FSR background using pQCD prediction based on the BABAR previous analysis [PRD112003]



FSR emission from final-state pions

~ 0.001fb → < 1 event occur



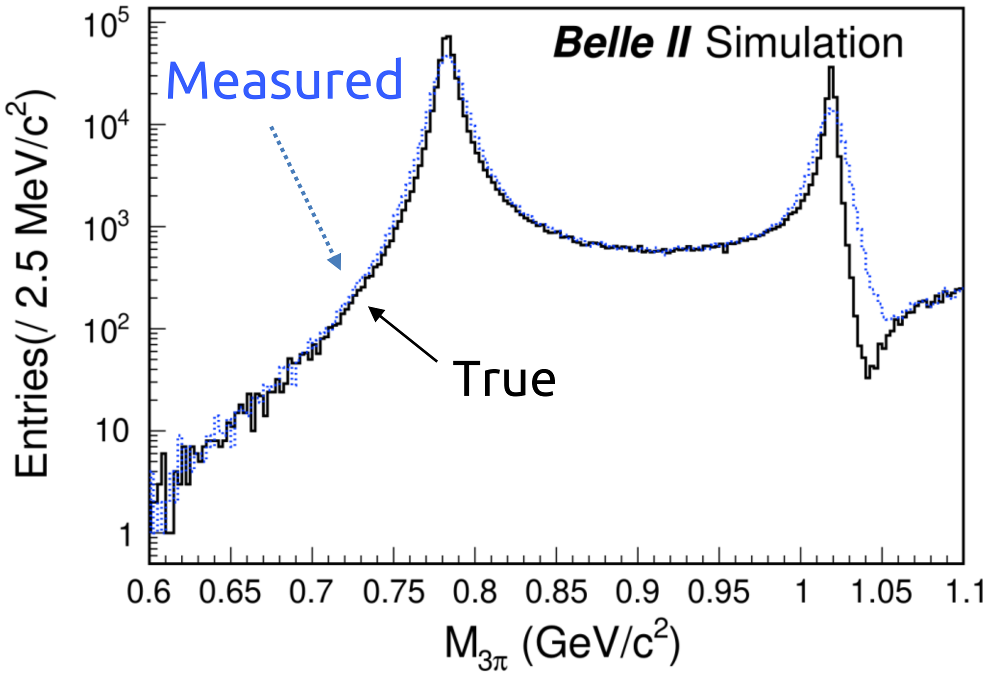


FSR emission from the quark legs

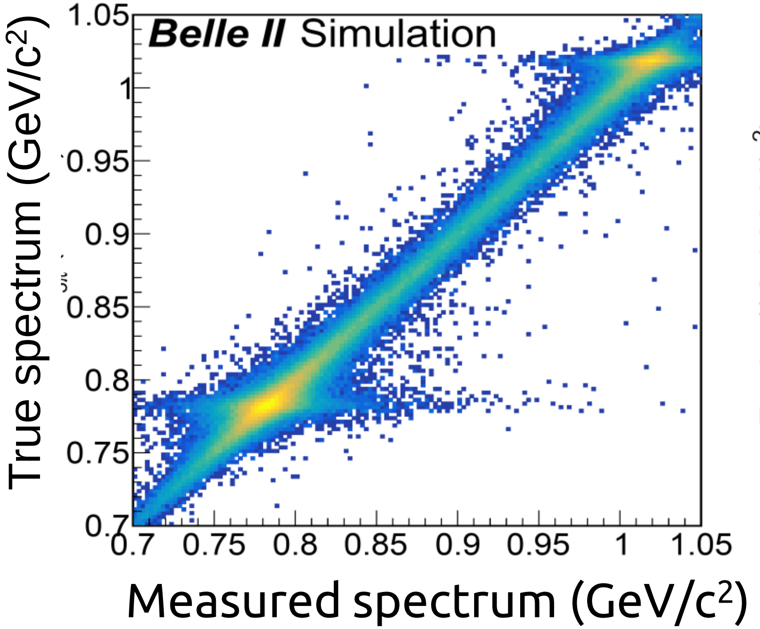
- $e^+e^- \to M\gamma_{FSR} \to \pi^+\pi^-\pi^0\gamma_{FSR}$;
  M= η, $a_1(1260)$, $a_2(1320)$, $a_1(1640)$, $a_2(1700)$, $a_1(1930)$, $a_2(2030)$

Unfolding

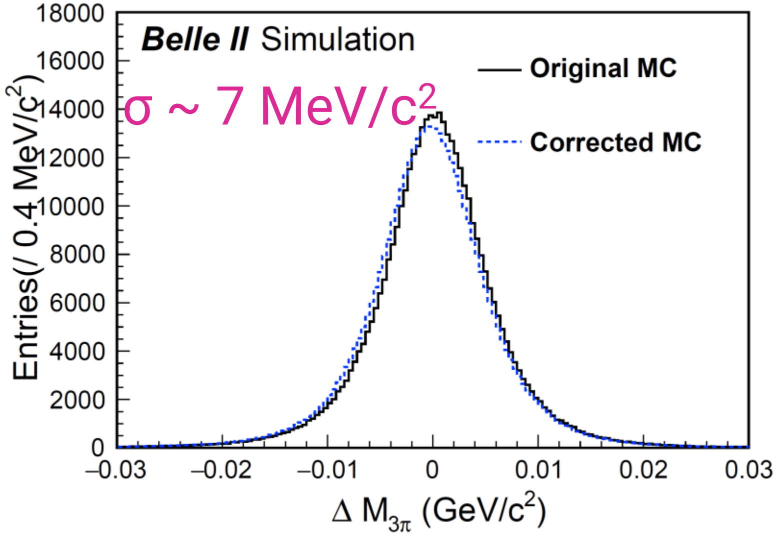
- The signal spectrum is unfolded to mitigate the effect of detector resolution
  - Typically with a mass resolution around 7-10 MeV/c^2
- The data-MC difference of mass bias and resolution is determined by a Gaussian convolution fit to the ω, Φ, and J/ψ resonances
  - Mass bias of 0.5-1.5 MeV/c^2, and resolution of about 1 MeV/c^2 is corrected



Transfer matrix



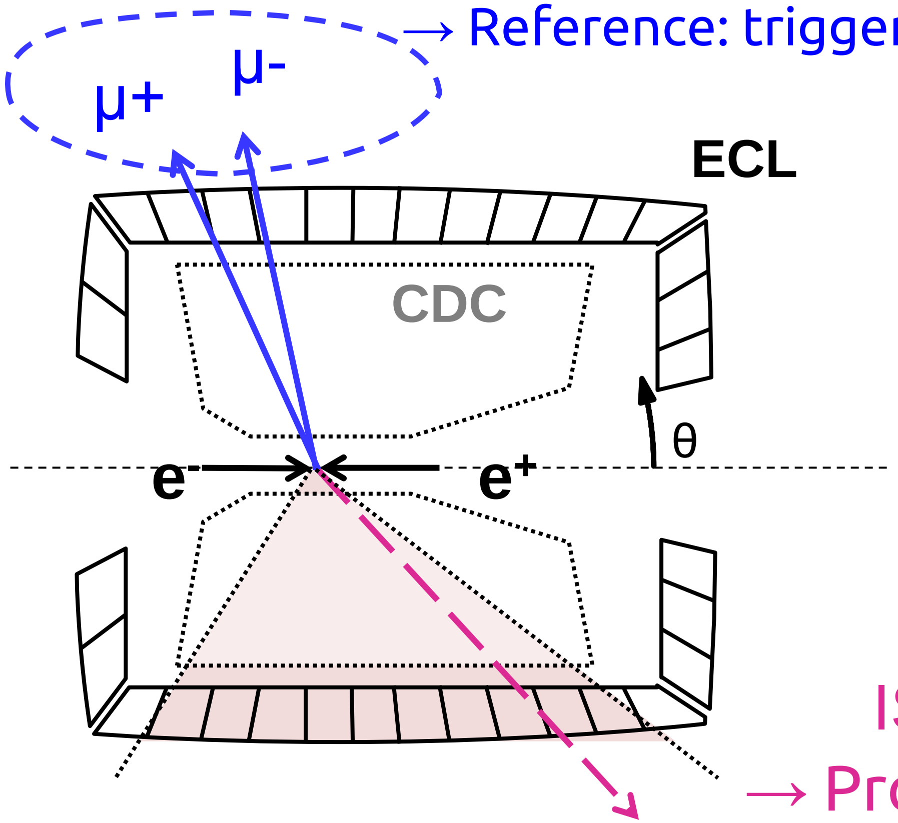
Mass resolution



Trigger efficiency

- ISR events are triggered by the calorimeter
- The efficiency can be measured by using the events triggered independently by the tracker
  - Efficiency for energetic ISR in barrel region: 99.9%
- The uncertainty related to trigger is small, 0.1%
- This also benefits other final-state measurements

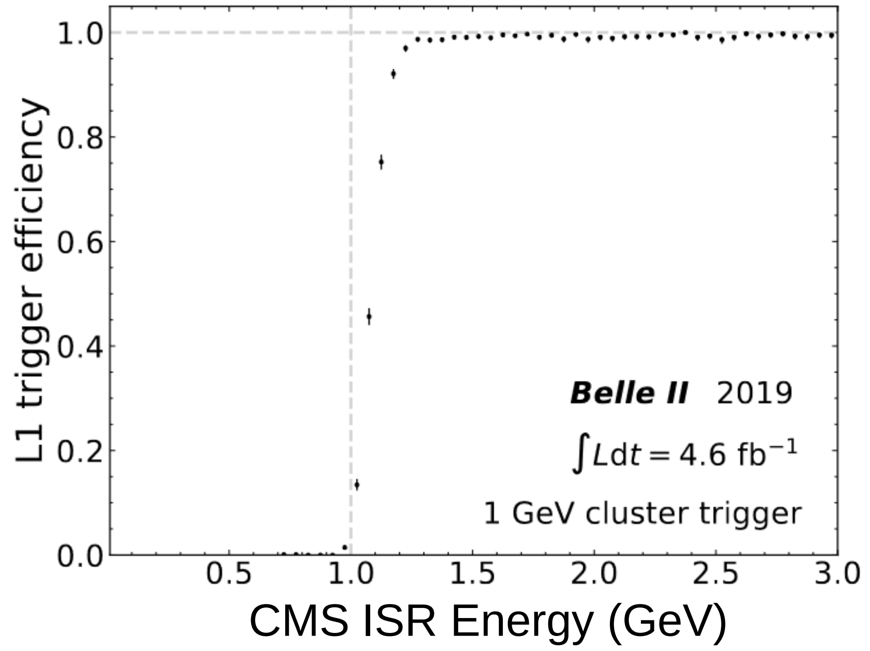
→ Reference: triggered by track trigger



ISR photon in barrel

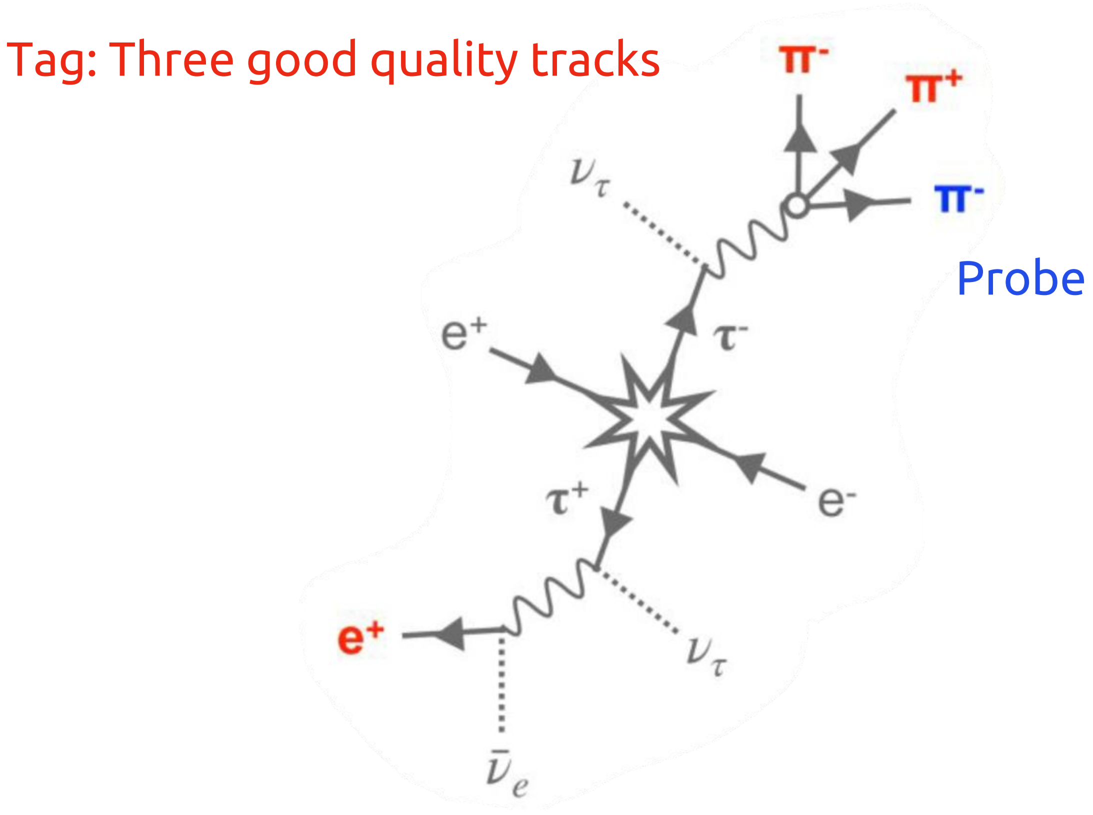
→ Probe: fire energy trigger

Belle II trigger efficiency measured by μμγ (data)

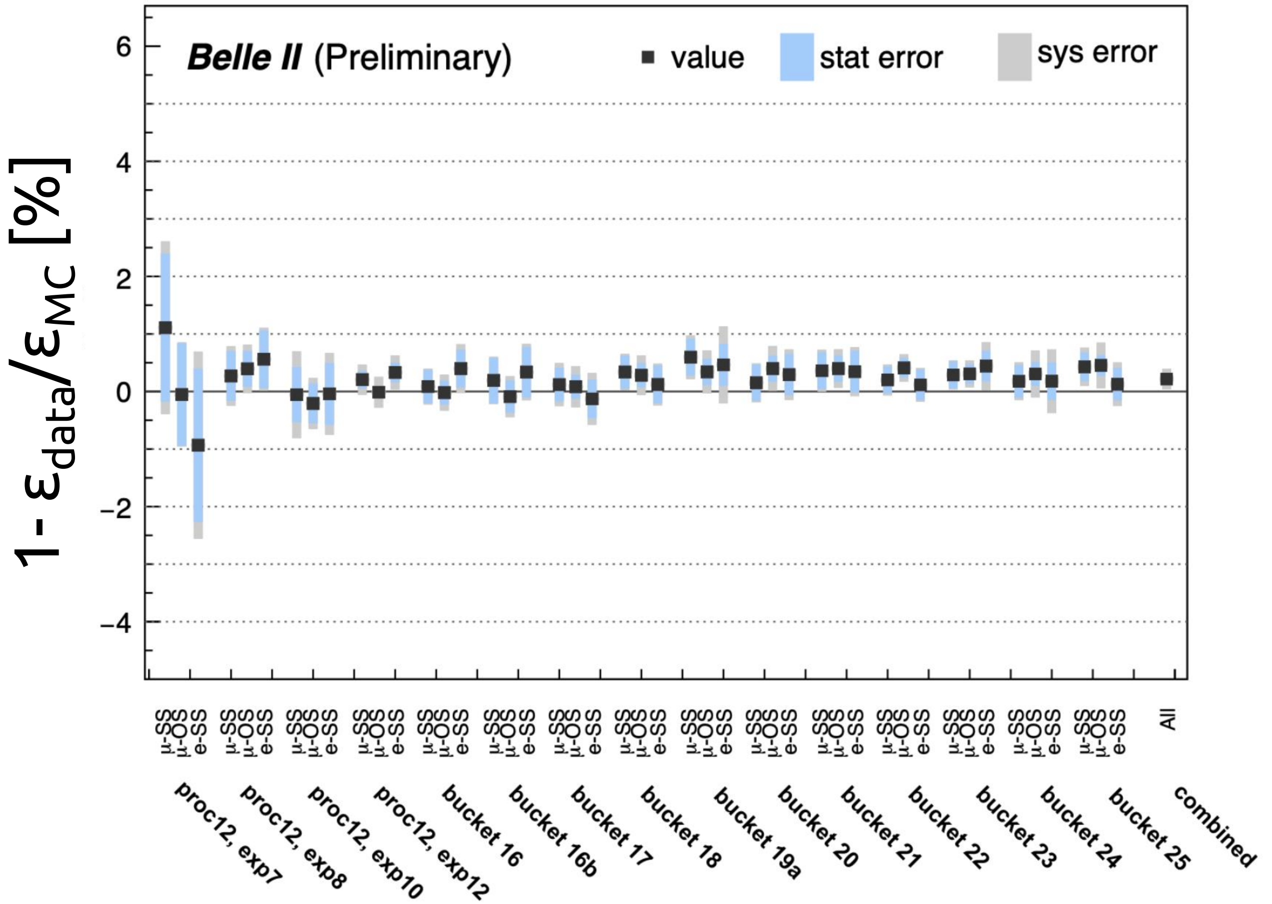


Tracking efficiency

- Tracking efficiency for pions is studied with the $e^+e^- \to \tau^+\tau^-$ process.
- Data-MC differences are confirmed to be small with 0.3% uncertainty per track.

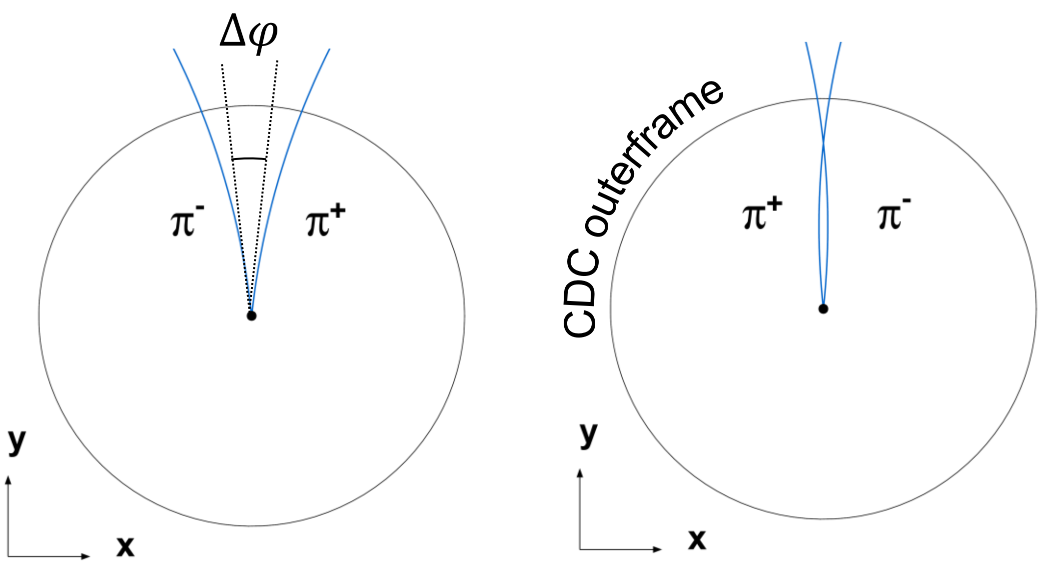


Data-MC discrepancy of tracking efficiency



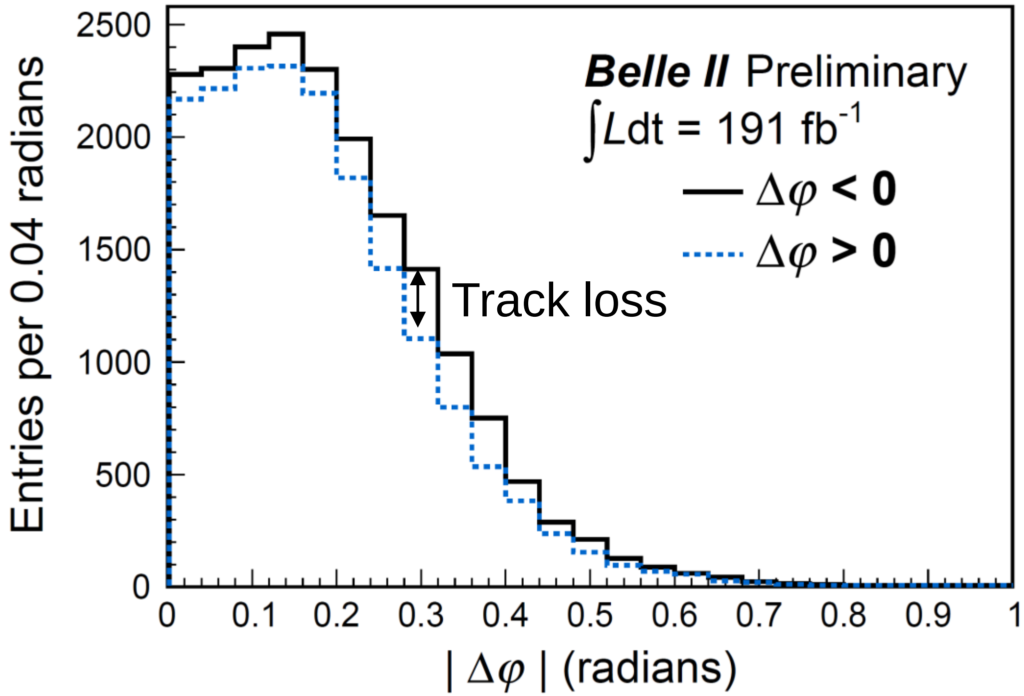
Tracking efficiency: Track loss

- Track loss due to shared hits on the drift chamber is confirmed using the $e^+e^- \to \pi^+\pi^-\pi^0\gamma$
- Define $\Delta\varphi \coloneqq \varphi(\pi^+) - \varphi(\pi^-)$
- The Inefficiency due to track loss is given by $f = \frac{N(\Delta\varphi<0) - N(\Delta\varphi>0)}{2N(\Delta\varphi<0)}$
  - The track loss is 5.0% in data and 4.0% in MC
- In total, the correction factor for tracking is (-1.4±0.8)%.
  - Dependency on no. of CDC hits and duplicated tracks are also studied.



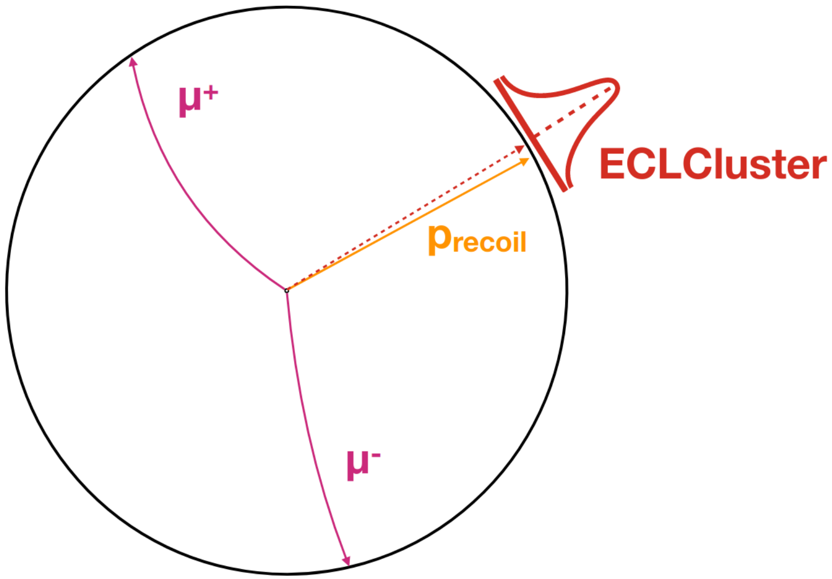
$\Delta\varphi < 0$ $\Delta\varphi > 0$

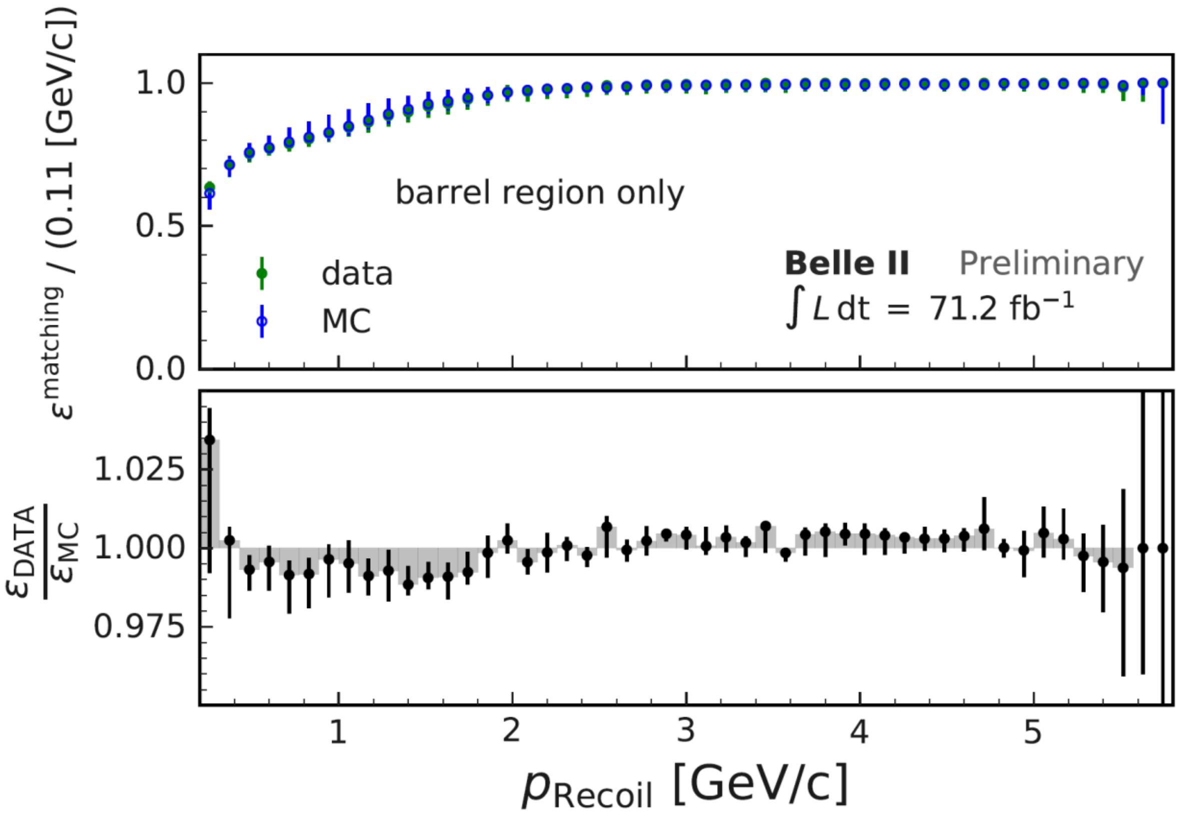
$\Delta\varphi$ distribution in data



ISR photon detection efficiency

- Photon detection efficiency is measured using $e^+e^- \to \mu^+\mu^-\gamma$ events
  - Taking a match between a ECL cluster and the missing momentum of dimuon system
- Efficiency is in good agreement with 0.7% systematic uncertainty



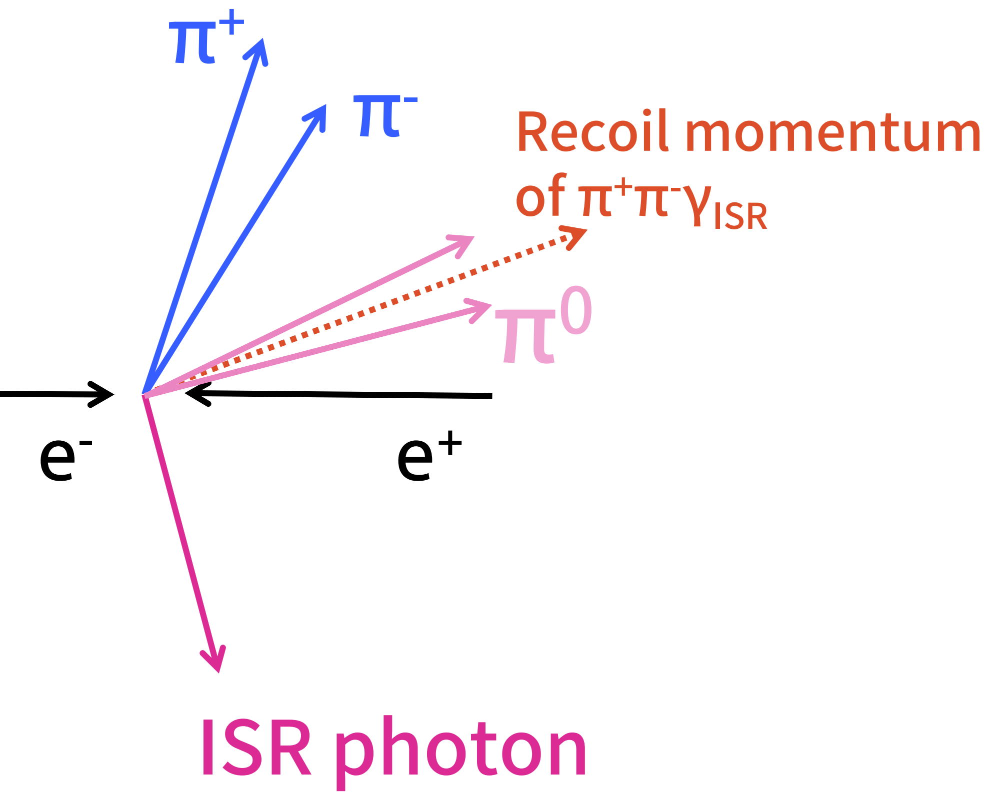


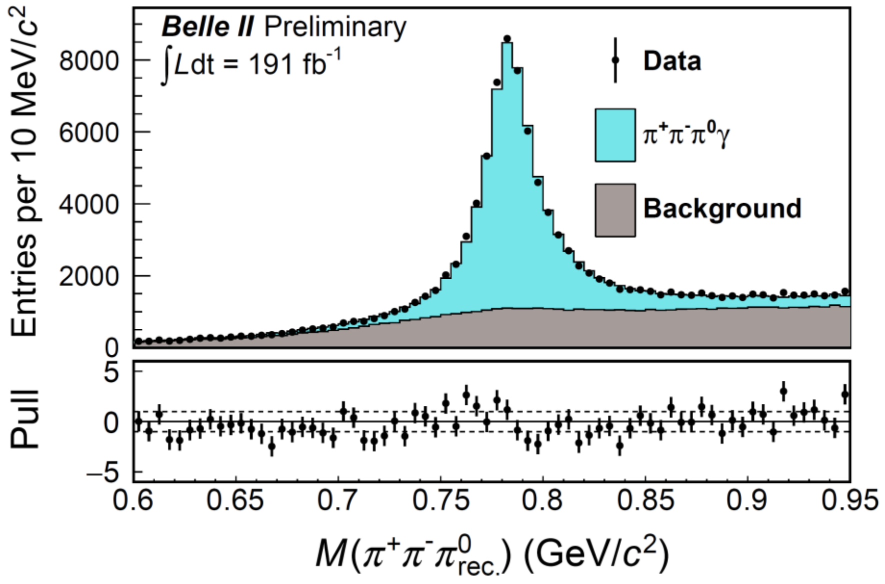
π^0 efficiency correction

- Accurate evaluation of π^0 efficiency in e^+e^- experiment is a challenging task.
  - Exclusive processes that include a π^0 are limited.
- Evaluate efficiency using the $e^+e^- \to \omega\gamma \to \pi^+\pi^-\pi^0\gamma$ events.

$$\varepsilon_{\pi^0} = \frac{N(\text{Full reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N(\text{Partial reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-)}$$

→ Count $\omega \to \pi^+\pi^-\pi^0$ decay without using π^0 information.





$$M^2(\pi^+\pi^-\pi^0_{\text{recoil}}) = (p_{\pi^+} + p_{\pi^-} + p_{\text{recoil}})^2$$

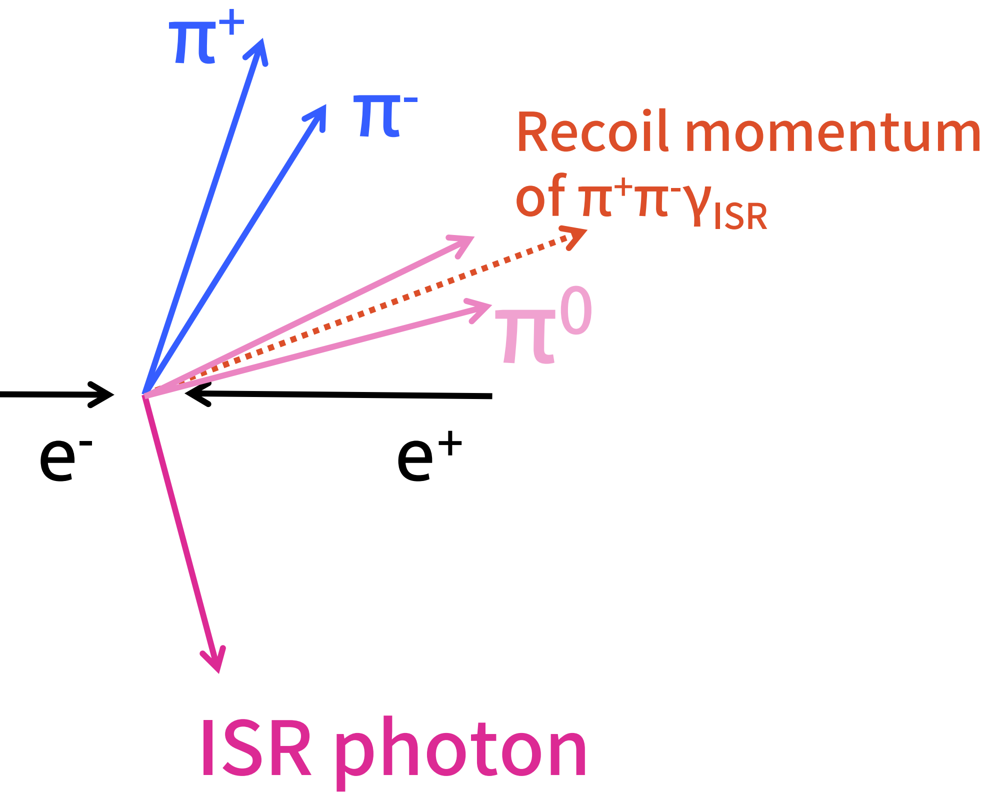
- π^0 momentum p_{recoil} is determined by kinematic fit to $\pi^+\pi^-\gamma$ with hypothesis that recoil mass equals π^0 mass

π^0 efficiency correction

- Accurate evaluation of π^0 efficiency in e^+e^- experiment is a challenging task.
  - Exclusive processes that include a π^0 are limited.
- Evaluate efficiency using the $e^+e^- \to \omega\gamma \to \pi^+\pi^-\pi^0\gamma$ events.

$$\varepsilon_{\pi^0} = \frac{N(\text{Full reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N(\text{Partial reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-)}$$

→ Count by reconstructing π^0 and fitting $M(\gamma\gamma)$



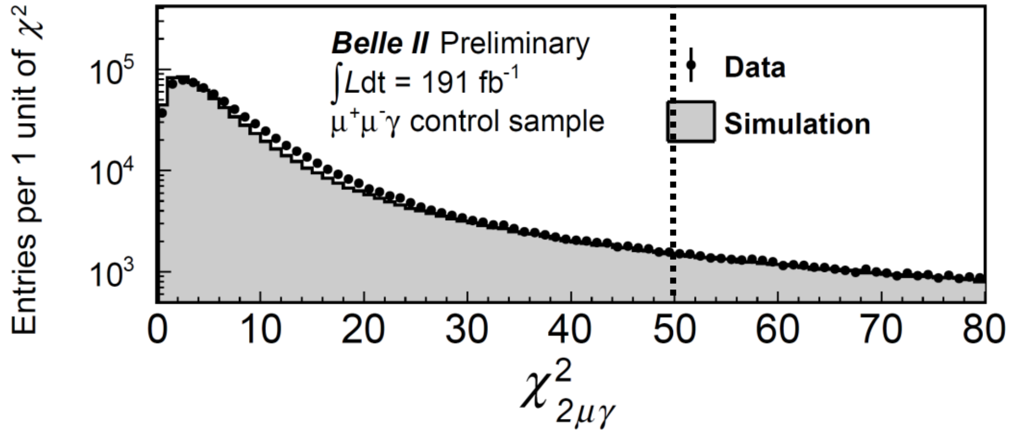
- ε_{π^0} are independently evaluated by the data and MC
  - Data/MC ratio = $0.986 \pm 0.006_{\text{stat}}$
- The systematic uncertainty related to π^0 is 1.0%
  - The uncertainty is evaluated by variations of the $M(\gamma\gamma)$ signal pdf, background pdfs, and selections

Background suppression efficiency

- Estimated by the ratio of signal yield before/after the criteria
- It is evaluated using ω and Φ, J/ψ resonances of good S/N
- In $M(3\pi) < 1.05$ GeV/c^2, efficiency is $(89.5 \pm 0.2)\%$ for data
  - Data-MC difference is $\varepsilon_{data}/\varepsilon_{MC} - 1 = (-1.90 \pm 0.20)\%$
- $M(3\pi) > 1.05$ GeV/c^2 : the number of J/ψ was obtained by $M(3\pi)$ fitting
  - Data-MC difference is $\varepsilon_{data}/\varepsilon_{MC} - 1 = (-1.78 \pm 1.85)\%$
  - Error is due to statistical errors in the sample

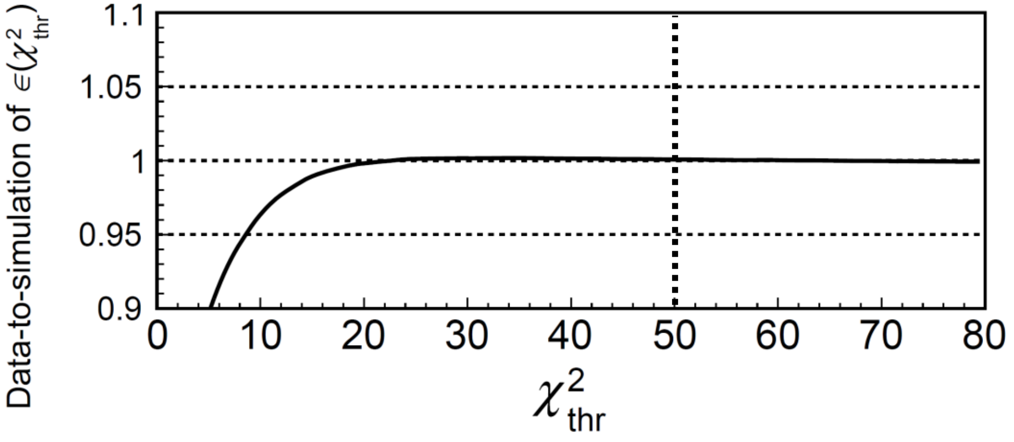
χ2 selection efficiency

- ISR and tracks χ^2-criteria efficiency is confirmed using $e^+e^- \to \mu^+\mu^-\gamma$ sample
- Confirm effects from differences in position, momentum, and energy of ISR and tracks
  - Agreement confirmed within ±0.6% uncertainty
- Dependence on multi-ISR photon calculations is discussed on the next page



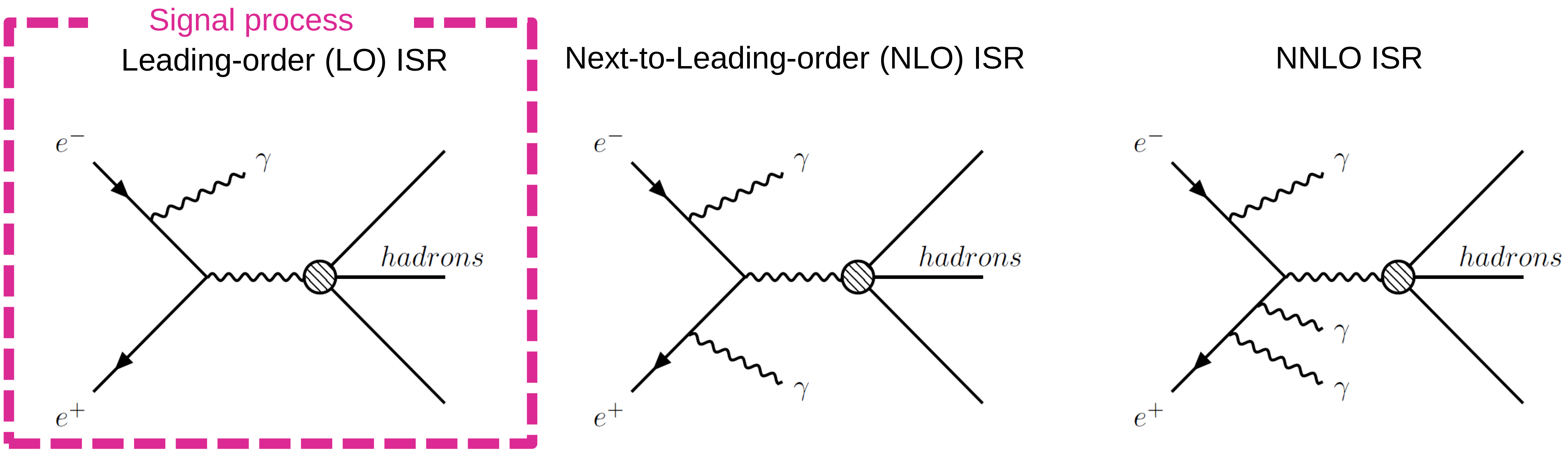
$$\varepsilon(\chi^2_{\mathrm{thr}}) = \frac{N(\chi^2 < \chi^2_{\mathrm{thr}})}{N_{\mathrm{all}}}$$

Data-MC ratio $\dfrac{\varepsilon_{\mathrm{data}}(\chi^2_{\mathrm{thr}})}{\varepsilon_{\mathrm{MC}}(\chi^2_{\mathrm{thr}})}$



Higher-order ISR effects

- Although a one-ISR photon emission process is set as the signal, in reality there are processes with multiple photon emissions.
- Two effects need to be considered from the existence of multiple photons:
  - A) Effective integrated luminosity L_{eff} (radiative correction): 0.5% unc.
  - B) χ2 selection efficiency due to ISR photon calculations in generator: 1.2% unc.



Higher-order ISR effects: A) radiative correction

- Using the LO ISR analytic function, ISR luminosity with the integrated luminosity $L_{\mathrm{int}} = 191\ \mathrm{fb}^{-1}$ is

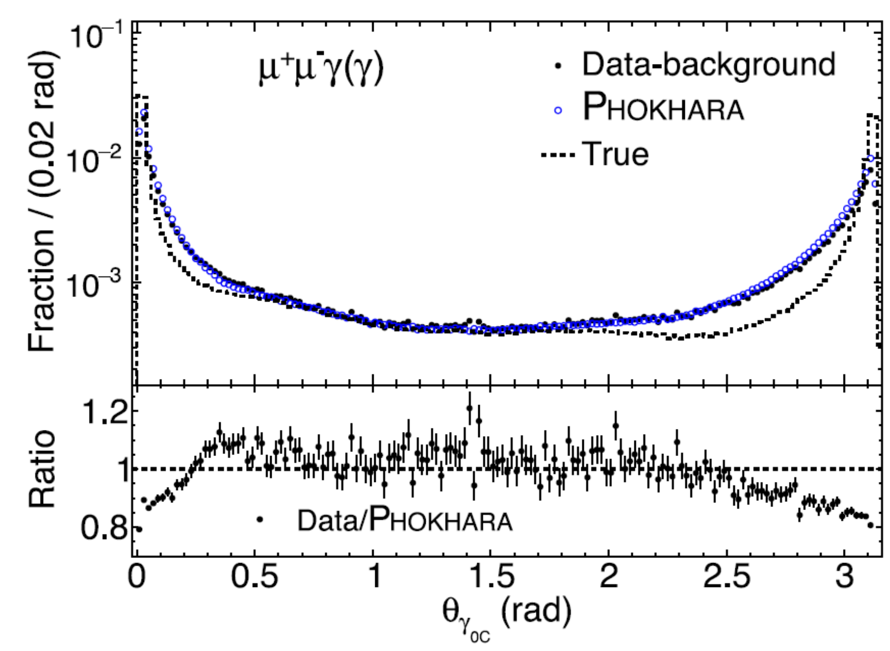
$$L_{\mathrm{eff}} = \frac{2\sqrt{s'}}{s}\frac{\alpha}{\pi}\left(\frac{s^2 + s'^2}{s(s-s')}\ln\frac{1+cos\theta}{1-\cos\theta} - \frac{s-s'}{s}\cos\theta\right) L_{\mathrm{int}}$$

- The ratio of the ISR emission probability including higher-order effects (LO+NLO+…) to LO is called radiative correction
- Higher order (LO+NLO) effects are calculated by MC generator, PHOKHARA, radiative correction is 1.008-1.013 depending on hadronic energy $\sqrt{s'}$

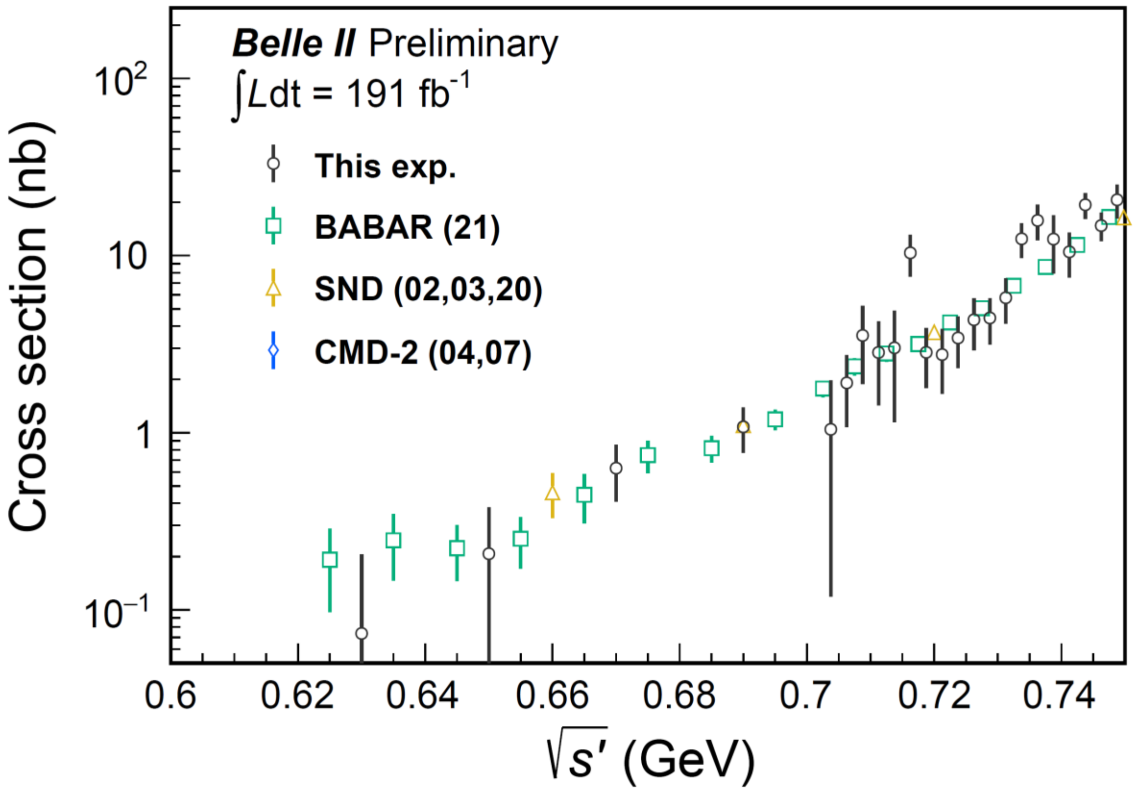
Higher-order ISR effects: B) χ2 efficiency

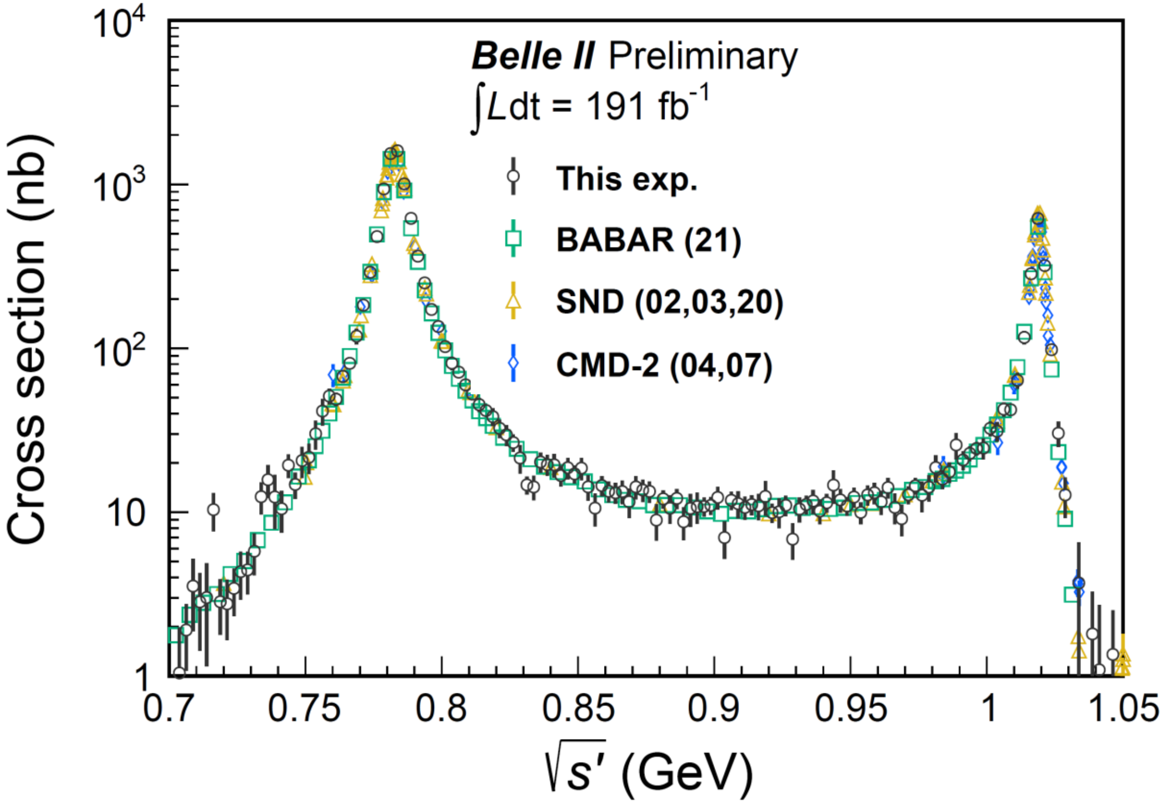
- 20% excess of the fraction of NLO (two ISR) events on the MC generator is reported by BABAR
  - This data-MC difference was also observed and verified by the Belle II data.
  - Signal efficiency changes because most NLO events are rejected by χ2 criteria.
  - Efficiency change in this case was evaluated on a simulation basis.
  - The χ2 efficiency is underestimated by (2.4±0.7)%.
- NNLO (three ISR) calculations not included in the generator, but (3.4±0.4)% observed by BABAR
  - Efficiency change due to NNLO is estimated to be 1.9% in the opposite direction of the NLO effect
- In conclusion,
  - No correction is assigned
  - 1.2% systematic uncertainty is accounted for as MC generator-derived error
    - 0.7%(NLO excess error) ⊕ 0.95% (half of NNLO effect) = 1.2%

PHYS. REV. D 108, L111103 (2023)



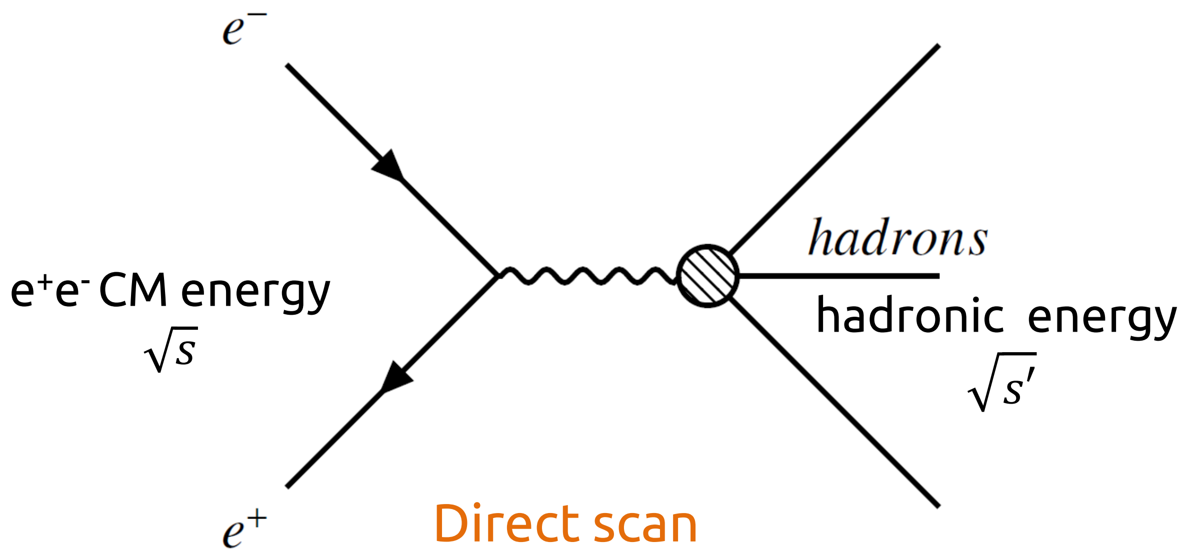
Result: cross section below 1.05 GeV





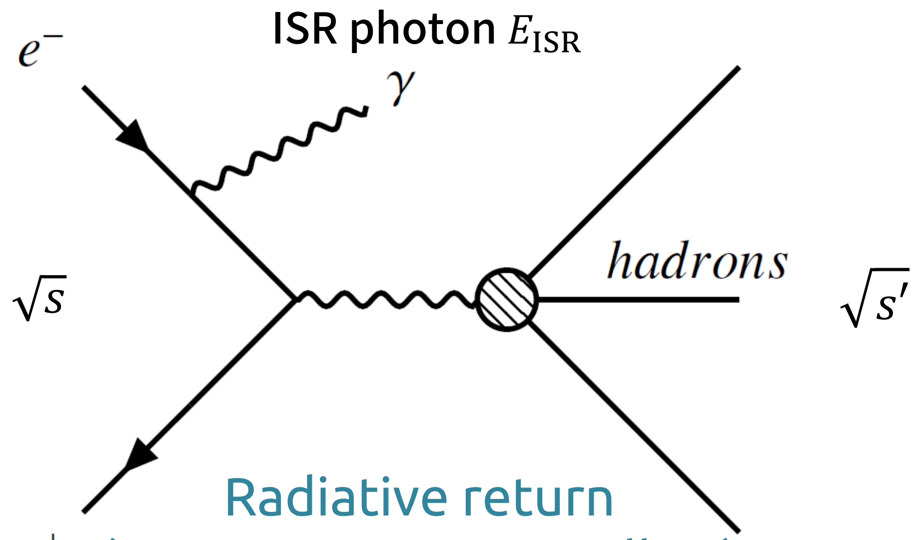
How to change the energies in e^+e^- experiment

- We want to scan the hadronic energy from √s ~ 0.5 to ~3 GeV.



Direct scan
(e.g. CMD, SND, KEDR)

$$s' = s$$



Radiative return
(e.g. BaBar, BES III, Belle II)

$$s' = s - \sqrt{s}\, E_{\mathrm{ISR}}$$

- **Larger dataset** at fixed energy
- Lower background
- Systematics depending on energy points
- Dynamic change of efficiency

- **Accessible to entire energies** with a single dataset
- Flat acceptance over a broad energy range
- Only 10% ISR emitted in detector acceptance
- Larger background from higher energies
- Energy resolution due to detector resolution

Major differences from BABAR 2021 measurement

- In quite a few respects, this analysis follows the BABAR method
- Systematic uncertainty is still nearly twice as large
  - NNLO generator is needed

	Belle II	BABAR (2021)
Dataset	191 fb^{-1}	469 fb^{-1}
Combinatorial γγ background	M(γγ) fit	Negligibly small(?)
ISR energy in kinematic fit	Used	Unused
Generator	PHOKHARA	AfkQed
Generator uncertainty	1.2%	-
Detection efficiency uncertainty	1.6%	1.1%
Integrated luminosity	0.6%	0.3%
Total systematic uncertainty for $a_\mu(3\pi)$	2.2%	1.3%