Model-independent Extraction of Form Factors and $|V_{cb}|$ in $\bar{B} \to D \ell^- \bar{\nu}$ with hadronic tagging at BABAR

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On behalf of the BABAR collaboration

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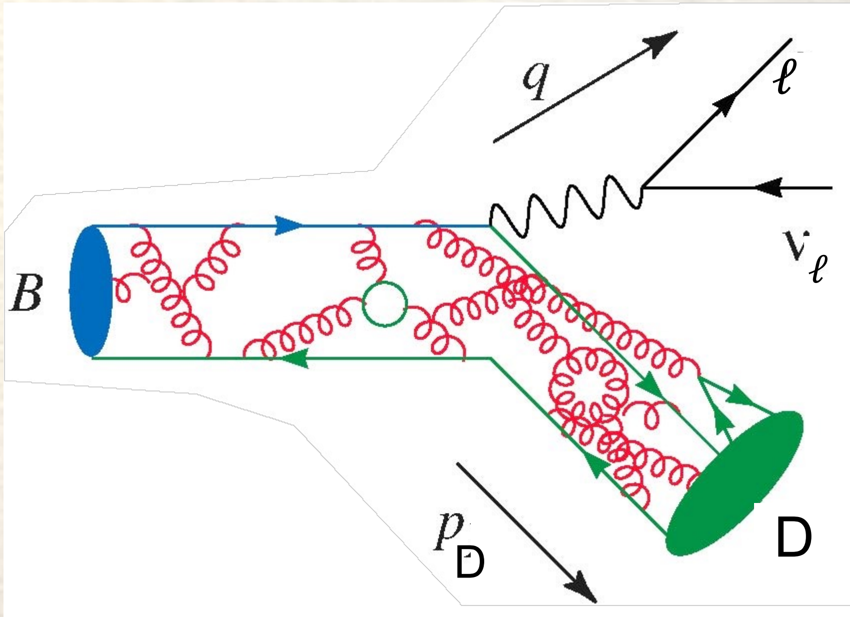
Introduction

- The decay $\bar{B}\to D\ell^-\bar{\nu}$ proceeds through a simple tree-level diagram and has been studied by many experiments
- The decay proceeds via the vector current
- The decay rate depends on the CKM element $|V_{cb}|$ and in the limit of neglecting the lepton mass on just one form factor $f_+(q^2)$
- Measurements of $|V_{cb}|$ from inclusive $b\to c\ell^-\bar{\nu}$ decay and exclusive $\bar{B}\to D^{(*)}\ell^-\bar{\nu}$ decays show a 3σ level disagreement



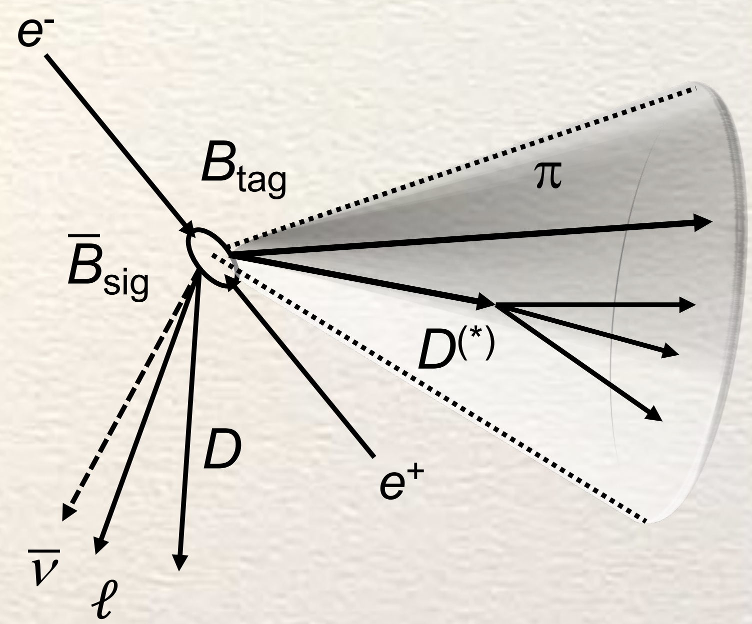
- Using the full data set, *BABAR* has performed a new study of $\bar{B}\to D\ell^-\bar{\nu}$ by analyzing the process $e^+e^-\to \Upsilon(4S)\to B_{tag}\bar{B}_{sig}$, where B_{tag} is reconstructed in B hadronic decays and $\bar{B}_{sig}$ represents the $\bar{B}\to D\ell^-\bar{\nu}$ signal mode
- Two different form factor parametrizations are employed, the model-independent Boyd-Grinstein-Lebed (BGL) expansion and the model-dependent Caprini-Lellouch-Neubert (CLN) expansion

Nucl.Phys. **B461**, 493 (1996)

Nucl.Phys. **B530**, 153 (1998)

Analysis Strategy

- Data sample consist of 471×10^6 $\Upsilon(4S)\rightarrow B\overline{B}$ events (426 fb^{-1}) NIM **A726**, 203 (2013)



- One B is tagged via a hadronic decay ($D^{(*)0}$, $D^{(*)+}$, $D_s^{(*)+}$, J/ψ) plus up to 5 charged charmless light mesons and 2 neutral mesons

- The reconstruction relies on 2 variables

$$m_{ES}=\sqrt{\tfrac{1}{4}s-\left|\vec{p}^{\,*}_{tag}\right|^2}$$
$$\Delta E=E^*_{tag}-\tfrac{1}{2}\sqrt{s}$$

where $\vec{p}^{\,*}_{tag}$ and E^*_{tag} are 3-momentum and energy of B_{tag} in the CM frame

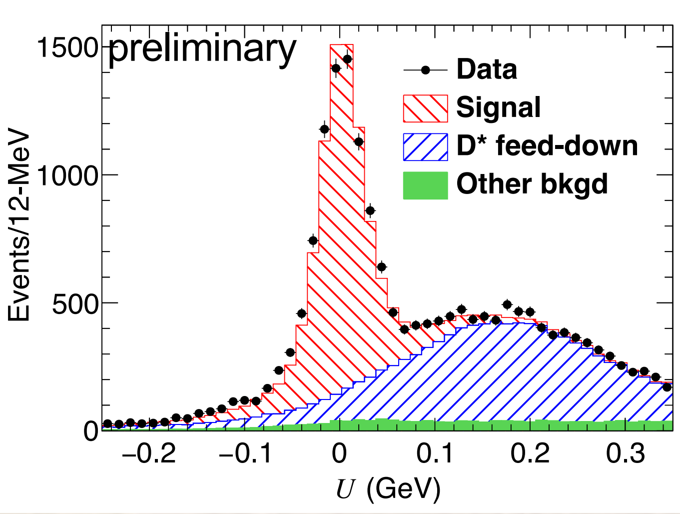
- Select events with m_{ES}>5.27 GeV/c^2 and $|\Delta E|$<72 MeV

- Select 10 modes on signal side: $D^0\rightarrow K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-\pi^+\pi^+\pi^-$, $D^+\rightarrow K^-\pi^+\pi^+$, $K^-\pi^+\pi^-\pi^0$ plus an e^- with p_e>200 MeV/c or a μ^- with p_μ> 300 MeV/c

- Analysis is similar to that of $\overline{B}\rightarrow D^*\ell^-\overline{\nu}$ PRL **123**, 091801 (2019)

Analysis Strategy cont.

- Determine missing momentum $p_{\bar{\nu}} \equiv p_{miss} = p_{e^+e^-} - p_{tag} - p_D - p_\ell$
- For a semileptonic decay with one missing neutrino this is fulfilled
- We use the discriminating variable $U = E^{**}_{miss} - \left|\vec{p}^{**}_{miss}\right|$
  (E^{**}_{miss} and $\vec{p}^{**}_{miss}$ are $\bar{\nu}$ energy and 3-momentum in $\bar{B}_{sig}$ rest frame)

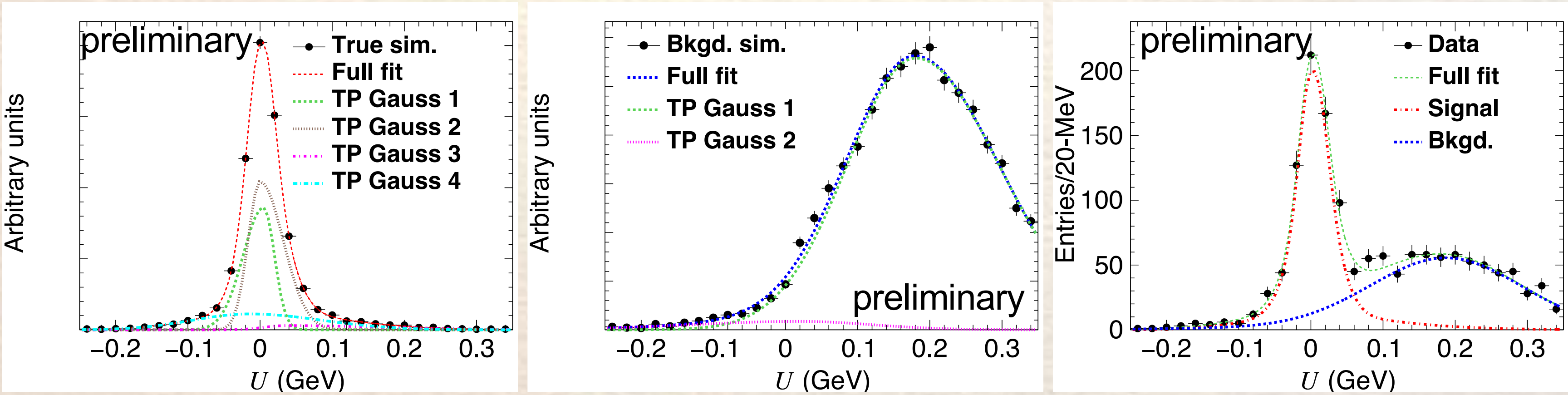


- We measure the extra energy in the calorimeter, require E_{Extra} ($\leq$ 80 MeV)
- We perform a kinematic fit of the entire event, constraining B_{tag}, B_{sig} and D mesons to their nominal masses, constrain B and D decay products to separate vertices
- In case of multiple candidates, we retain that with the lowest E_{Extra}
- A second kinematic fit with a U=0 constraint is done to improve the resolution in the variables q^2 and cos θ_ℓ (q is the momentum transfer to the $\ell^-\bar{\nu}$ system and θ_ℓ is the lepton helicity angle)

Signal-to-Background Separation

- We use a novel technique to separate signal from background since the background shape varies across phase space
- Primary background is from $\overline{B}\to D^{*}\ell^{-}\overline{\nu}$ with $D^{*}\to D\pi$ or $D^{*}\to D\gamma$

signal pdf | background pdf | Fit to data $K^{-}\pi^{+}e^{-}\overline{\nu}$ mode



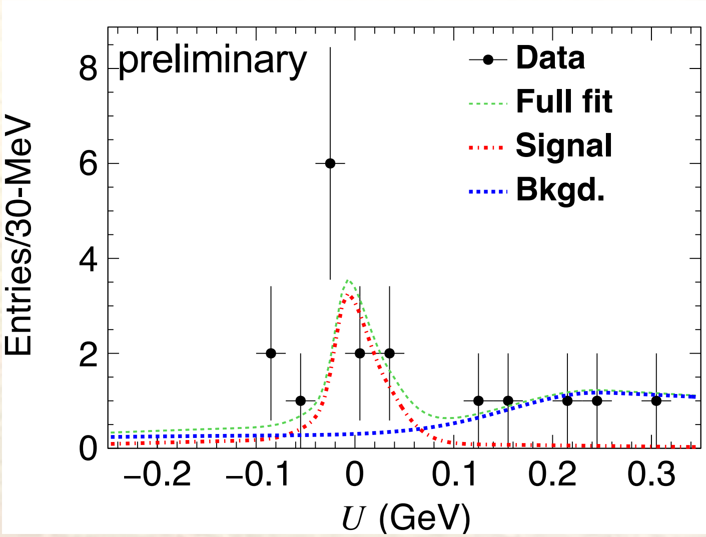
- Background from charmless B decays and $q\bar{q}$ continuum is small
- We define pdfs for signal (4 two-piece Gaussians) and background (2 two-piece Gaussians)
- We test the binned fit on the U distribution for the $K^{-}\pi^{+}e^{-}\overline{\nu}$ mode

Background Varies across Phase Space

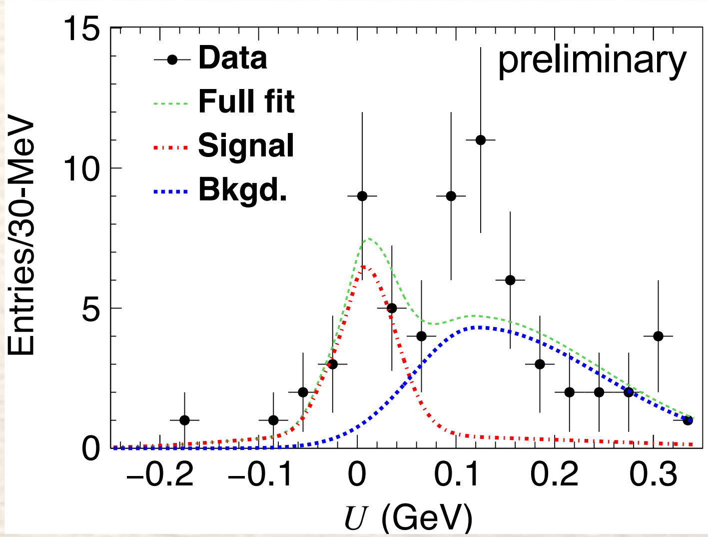
- We show that this method works in different regions of cos θ_ℓ and q^2

- Binned fits to data in $K^-\pi^+\pi^+e^-\bar{\nu}$ mode
- Fits describe data well

$|\cos\theta_\ell + 0.85| < 0.05$

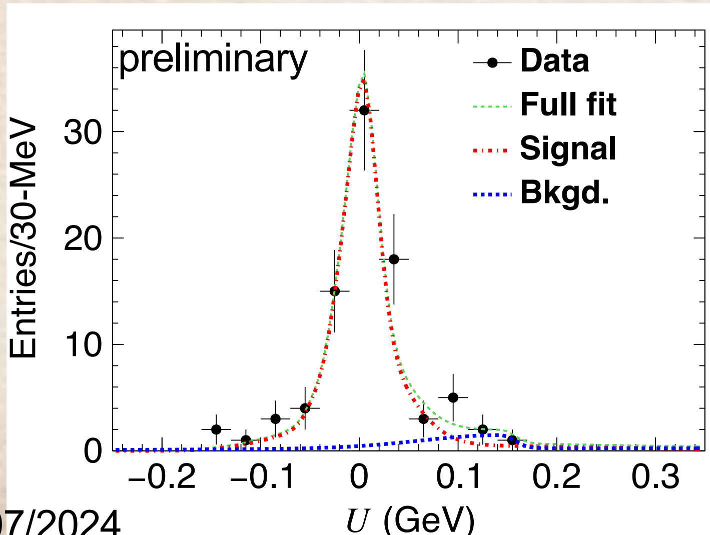


$|\cos\theta_\ell - 0.85| < 0.05$

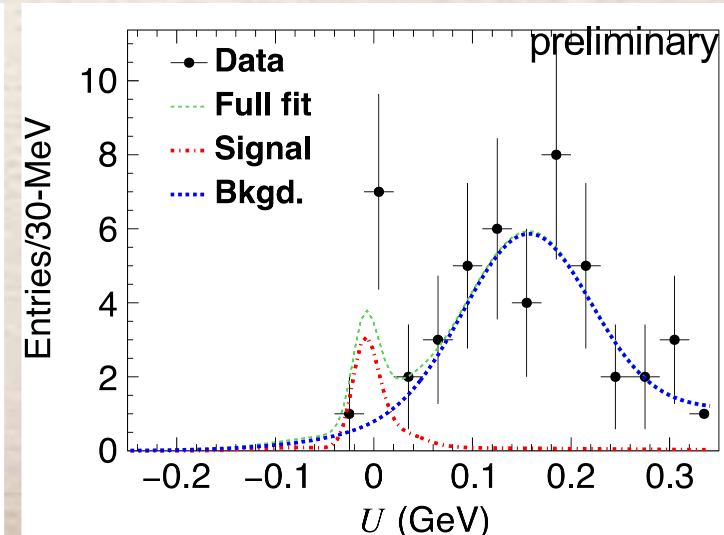


- Binned fits to data in $K^-\pi^+\pi^-\pi^+e^-\bar{\nu}$ mode
- Fits describe data well
- Distributions illustrate different background shapes

$|q^2 - 0.75| < 0.25$ GeV^2/c^2

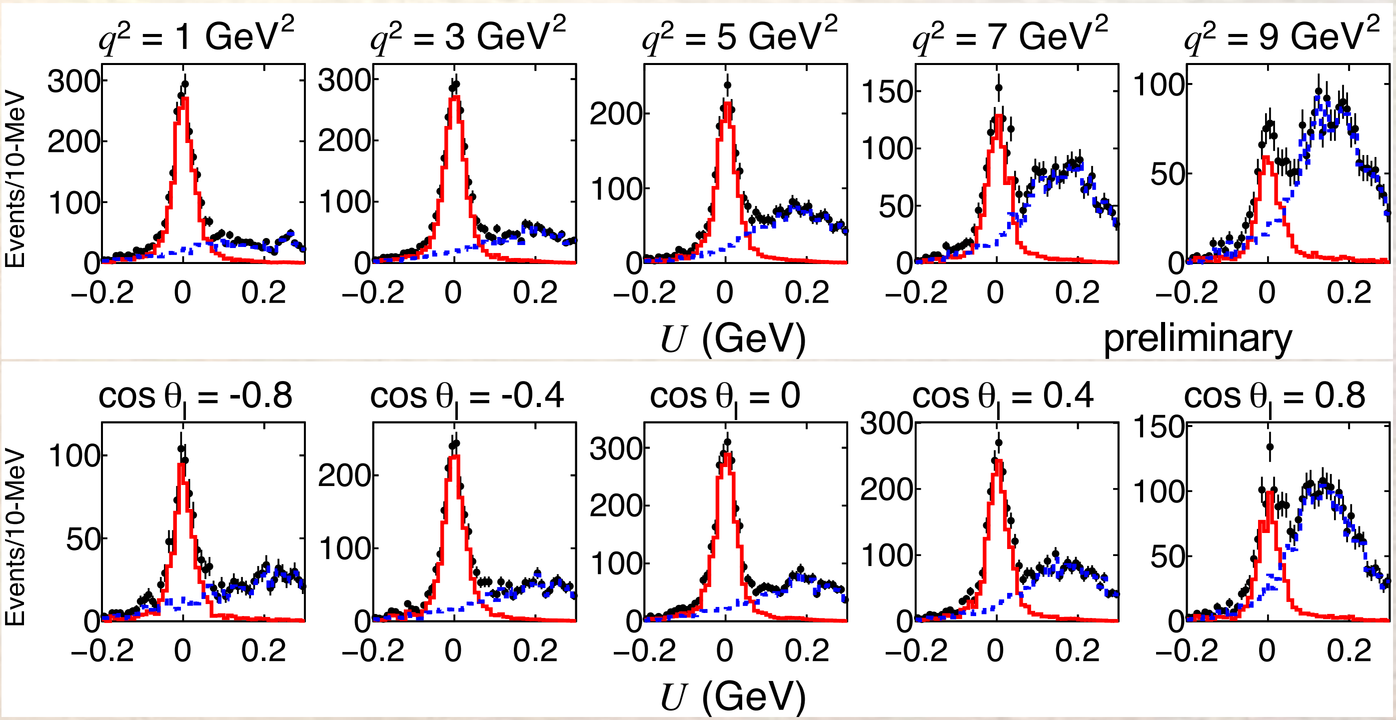


$|q^2 - 9.75| < 0.25$ GeV^2/c^2



Extraction of Signal Weight Factors

- We perform continuous U-variable fits in q^2 and cos θ_ℓ regions, selecting 50 events at a time that are closest to a selected event to determine signal and background components from which we determine signal weights for each event
- Signal weight $\mathcal{Q}_i = \dfrac{\mathcal{S}_i(U_i)}{\mathcal{S}_i(U_i)+\mathcal{B}_i(U_i)}$ and background weight $1-\mathcal{Q}_i = \dfrac{\mathcal{B}_i(U_i)}{\mathcal{S}_i(U_i)+\mathcal{B}_i(U_i)}$
- We observe 16,701 events in all ten modes
- To illustrate how well this procedure works, we show the U variable distributions for different q^2 and cos θ_ℓ regions, summing the $\mathcal{Q}_\mathrm{i}$ values of all 10 modes
- Red points result from signal weights $\mathcal{Q}_\mathrm{i}$ and blue points from background weights (1-$\mathcal{Q}_\mathrm{i}$)

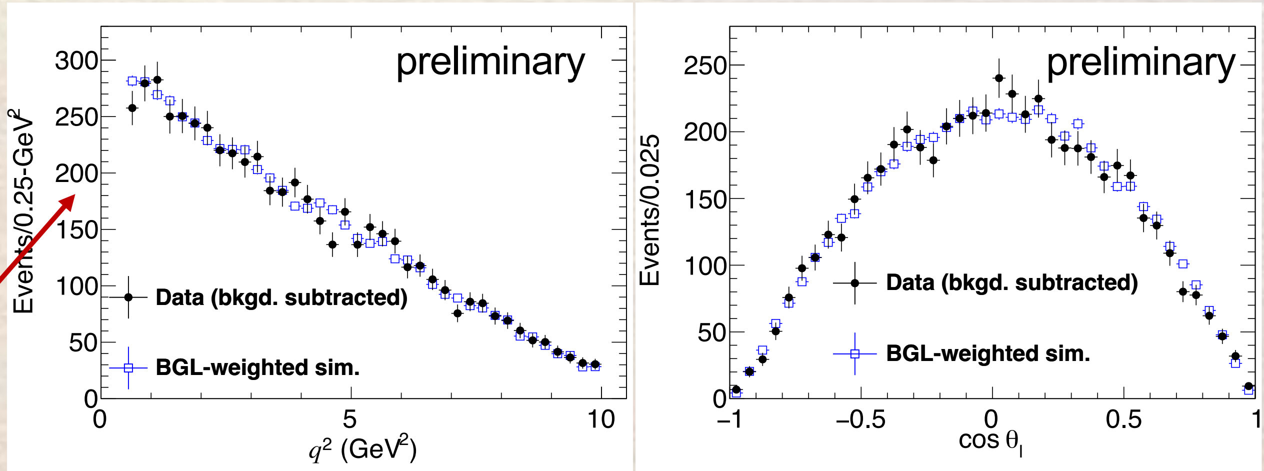


Unbinned Angular Fits

- We require $|U|<50$ MeV, $0.5 \leq q^2 \leq 10$ GeV$^2/c^2$ & $|\cos\theta_\ell|<0.97$ for the final sample
- We perform ML fits in the q^2-$\cos\theta_\ell$ plane using only signal weights Q_i
- We add two external constraints
  - To set normalization of the form factors, the $w\to1$ region calculations from lattice QCD are added as Gaussian constraints (6 $f_{0,+}(w)$ MILC data points) PRD **92**, 034506 (2015)
  - To access $|V_{cb}|$ the absolute q^2–differential decay rate data from Belle are also incorporated as Gaussian constraints (40 $d\Gamma/dw$ data points) PRD **93**, 032006 (2016)
- The total likelihood function is

$$\mathcal{L}(\vec{x})_{|tot} = -2\ln\mathcal{L}(\vec{x})_{|BABAR} + \chi^2(\vec{x})_{|Belle} + \chi^2(\vec{x})_{|FNAL/MILC}$$

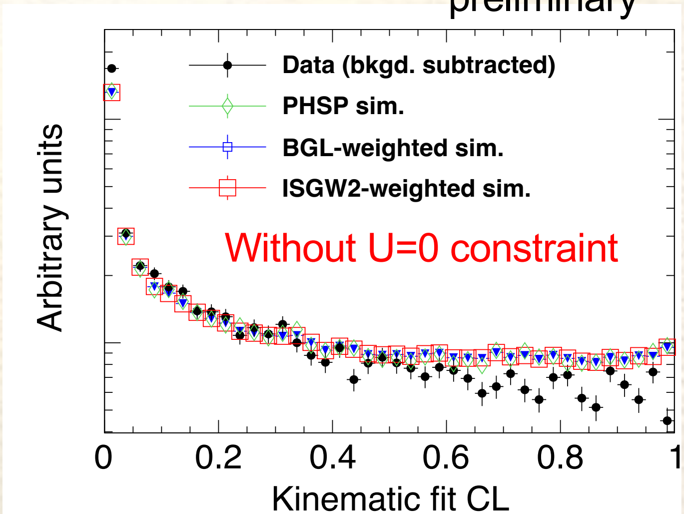
- We perform fits both with the BGL (N=2,3) and CLN forms
- 1d projections of the nominal fit in comparison with simulation using the BGL form

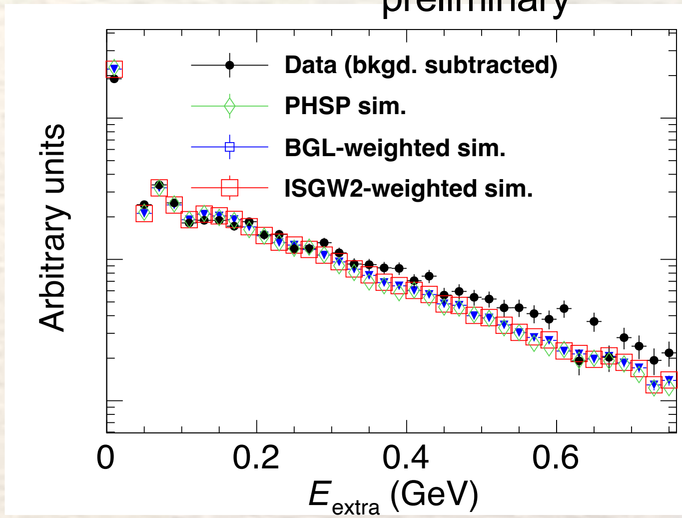


- The $\cos\theta_\ell$ distribution exhibits the $\sin^2\theta_\ell$ dependence expected in the SM ➡ this indicates that the $\bar{\nu}$ reconstruction works well

Cross Checks

- Besides the nominal fit, we perform 3 other fits with different background subtraction to study systematic uncertainties

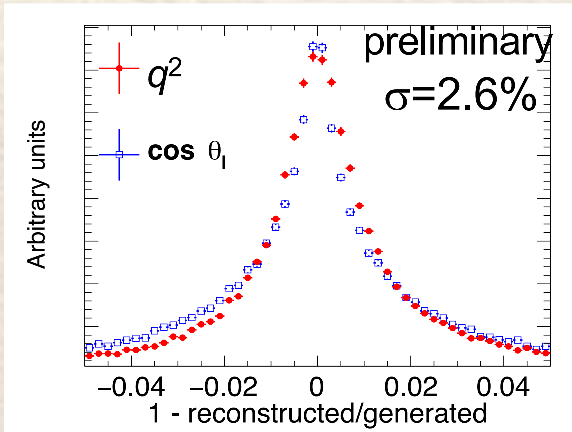


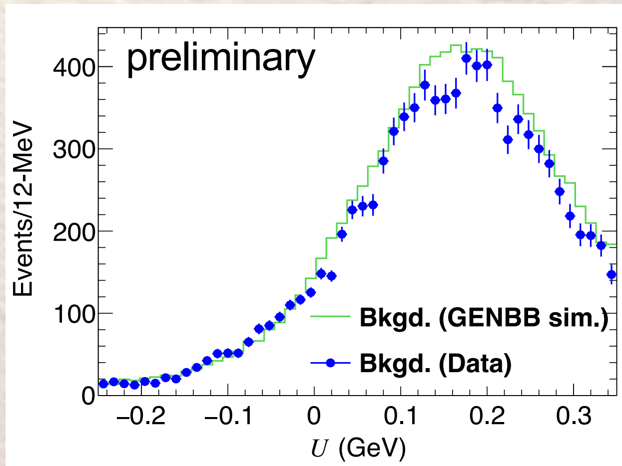


- We perform cross checks between background-subtracted data and efficiency-corrected simulations with BGL weighting and ISGW2 weighting for the confidence level of the fit and the E_{Extra} distribution

PRD **52**, 2783 (1995)

- The relative resolution of the deviation of the reconstructed-to-generated values for the q^2 and cos θ_ℓ distributions

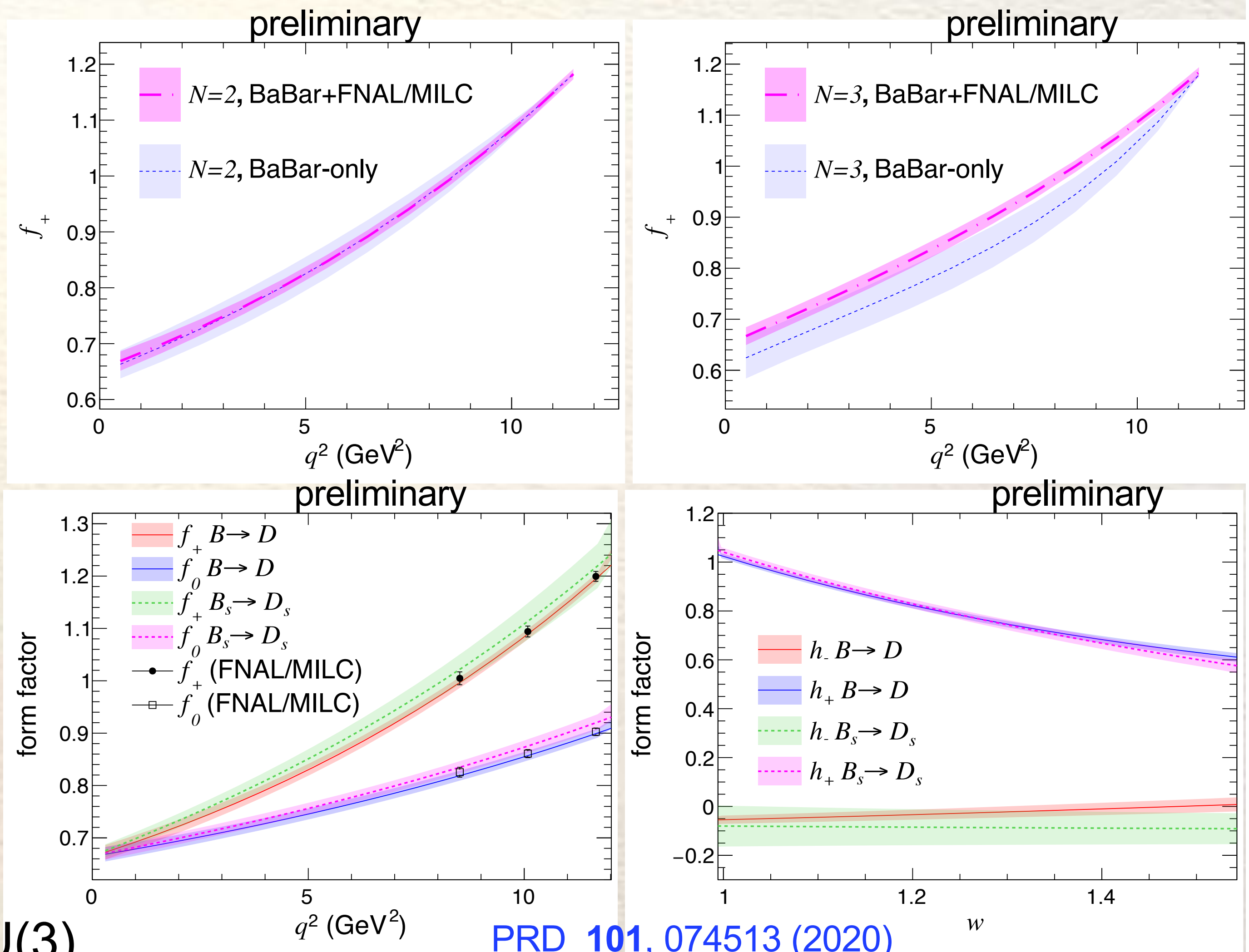




- Comparison of (1-$\mathcal{Q}$) weighted data and background simulation

Form Factor Results

- f^+ results for N=2 & N=3 for BABAR data only and BABAR+FNAL/MILC data
- Lattice points reduce errors
- The $B \to D$ form factors have improved precision and show good agreement with the new, full q^2 $B_s \to D_s$ calculation of the HPQCD Collaboration assuming flavor SU(3) symmetry
- Some slight tension exists for h_- in the HQET basis at maximum recoil point, $q^2 \to 0$, but otherwise the SU(3) flavor symmetry seems to hold ➔ SU(3) flavor symmetry breaking cannot be large
- This will be tested in $\bar{B} \to D^* \ell^- \bar{\nu}$ channel with a similar analysis

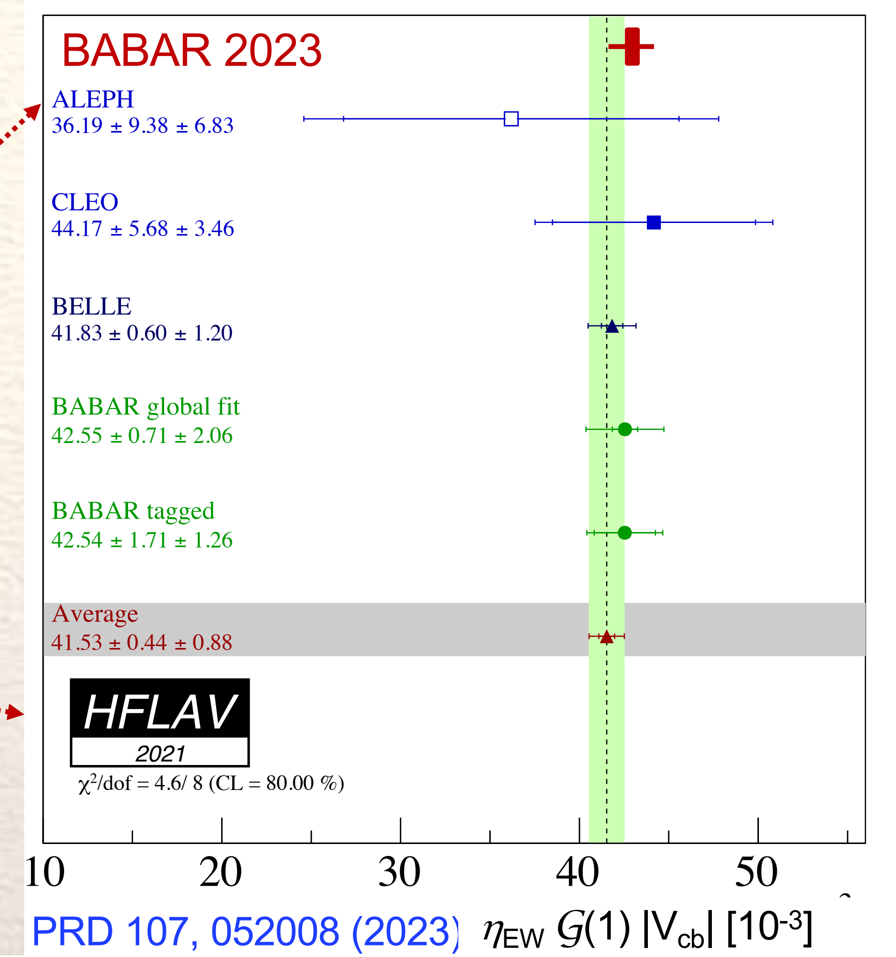


PRD **101**, 074513 (2020)

$|V_{cb}|$ Results from 2d Fit

- 2d fit to BABAR+Belle16+FNAL/MILC data: PRD **93**, 032006 (2016)
  - $|V_{cb}|^{BGL} = 0.04109 \pm 0.00116$ (preliminary)
  - $|V_{cb}|^{CLN} = 0.0409 \pm 0.00114$ (preliminary)
- Compute $|V_{cb}|\mathcal{G}(1)\eta_{EW}$ with $\mathcal{G}(1)=1.0530\pm0.0083$, $\eta_{EW}=1.0066\pm0.0050$
  - $\eta_{EW}\mathcal{G}(1)|V_{cb}| = 0.04355 \pm 0.00129$ (1.3 σ higher)
- Compared to the world average
  - $\eta_{EW}\mathcal{G}(1)|V_{cb}|_{WA} = 0.04153 \pm 0.00098$
- Good agreement with the $|V_{cb}|$ from inclusive analysis $|V_{cb}| = 0.04219 \pm 0.00078$
- Some tension with $|V_{cb}|$ from $\bar{B}\to D^*\ell^-\bar{\nu}$

  $|V_{cb}| = 0.03846 \pm 0.00040 \pm 0.00055$
- From HFLAV B^0 & B^+ branching fractions and Γ' from the fit we get $|V_{cb}|=\sqrt{\mathcal{B}/(\Gamma'\tau)}$



$\mathcal{B}$ measurement	$\lvert V_{cb}\rvert \times 10^3$
B^0: BABAR-10	$40.36 \pm 0.17 \pm 0.10 \pm 1.67$
B^+: BABAR-10	$38.98 \pm 0.15 \pm 0.09 \pm 1.30$
B^0: Belle-16	$42.01 \pm 0.18 \pm 0.10 \pm 1.06$
B^+: Belle-16	$41.60 \pm 0.17 \pm 0.10 \pm 1.07$

Conclusions

- We performed the first 2-dimensional unbinned angular analysis in the q^2- cos θ_ℓ plane for the $\bar{B}\rightarrow D\ell^-\bar{\nu}$ process
- We used a novel event-wise signal-to-background separation
- The lepton helicity follows a $\sin^2\theta_\ell$ distribution as expected in the SM; this is shown for the first time confirming that the ν reconstruction works well
- For the BGL form we measure $|V_{cb}|=0.04109\pm0.00116$, which is closer to the value measured in inclusive $b\rightarrow c\ell^-\bar{\nu}$ decays
- The $B\rightarrow D$ form factors are found to be consistent with the $B_s\rightarrow D_s$ form factors predicted by lattice calculations and expected by flavor SU(3) relations
- This *BABAR* analysis has been submitted to Physical Review D

Thank you for your attention

Backup Slides

$\bar{B}\to D\ell^-\bar{\nu}$ Decay Rate and Form Factors

- The amplitude for $\bar{B}\to D\ell^-\bar{\nu}$ comes from the vector interaction term

$$\langle D|\bar{c}\gamma_\mu b|\bar{B}\rangle_V = f_+(q^2)\left((p_B+p_D)_\mu - \frac{(p_B+p_D)\bullet q}{q^2}q_\mu\right) + f_0(q^2)\frac{(p_B+p_D)\bullet q}{q^2}q_\mu$$

- $q=p_B-p_D$ is the 4-momentum of the recoiling $(\ell\text{-}\bar{\nu})$ system
- $f_+(q^2)$ and $f_0(q^2)$ are the vector and scalar form factors
- In HQET the form factors are written in terms of B and D 4-velocities v and v'

$$\frac{\langle D|\bar{c}\gamma_\mu b|\bar{B}\rangle_V}{\sqrt{m_B m_D}} = h_+(w)(v+v')_\mu + h_-(w)(v-v')_\mu$$

where $w = v\bullet v' = \dfrac{m_B^2+m_D^2-q^2}{2m_B m_D}$

- The two form factors are related

$$f_+(q^2) = \frac{1}{2\sqrt{r}}\left((1+r)h_+(w) - (1-r)h_-(w)\right)$$

$$f_o(q^2) = \sqrt{r}\left(\frac{w+1}{1+r}h_+(w) - \frac{w-1}{1-r}h_-(w)\right)$$

where $r = \dfrac{m_D}{m_B}$ and $f_+(0) = f_0(0)$

$\bar{B}\to D\ell^{-}\bar{\nu}$ Decay Rate and Form Factors

- The differential $\bar{B}\to D\ell^{-}\bar{\nu}$ decay rate is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\mathrm{d}\cos\theta_\ell}=\frac{G_F^2\left|V_{cb}\right|^2\eta_{EW}^2}{32\pi^3}k^3\left|f_+(q^2)\right|^2\sin^2\theta_\ell$$

where $k=m_D\sqrt{w^2-1}$ ($|p_D|$ in B rest frame

- $f_+(q^2)$ is connected form factor G(w)

$$\mathcal{G}(w)=\frac{4r}{(1+r)^2}f_+(q^2)$$

The BGL Form

- In the model-independent BGL (Boyd, Grinstein, Lebed) form the form factors are expressed as $$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{N} a_n^i z^n$$ where i=0,+, $$z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}},$$

  $P_i(z)$: Blaschke factors that remove contributions of bound state $B_c^{(*)}$ poles,

  $\phi_i(z)$: non-perturbative outer functions,

  a_n^i: free parameters

  N: considered order of expansion

- Use following parameterizations
  - $P_i(z) = 1$
  - $$\phi_+(z) = \frac{1.1213(1+z)^2\sqrt{1-z}}{\left[(1+r)(1-z) + 2\sqrt{r}(1+z)\right]^5}$$ $$\phi_0(z) = \frac{0.5299(1+z)^2(1-z)^{3/2}}{\left[(1+r)(1-z) + 2\sqrt{r}(1+z)\right]^4}$$

- The coefficients a_n^i satisfy the normalization condition

$$\sum_n \left|a_n^i\right|^2 \le 1$$

The CLN Form

- In the model-dependent CLN (Caprini, Lellouch, Neubert) form the form factor is expressed as

$$\mathcal{G}(w)=\mathcal{G}(1)\left(1-8\rho_D^2 z(w)+(51\rho_D^2-10)z(w)^2-(252\rho_D^2-84)z(w)^3\right)$$

where QCD dispersion relations and HQET have been included, $G(1)$ is the normalization and ρ_{D} is the slope

- This form has been used in previous $\overline{B}\to D\ell\text{-}\overline{\nu}$ analyses

Binned Fits to *U* distribution

- The line shapes of signal and background in the *U* variable distribution are defined as

$$f_i(x;\mu_i,\sigma_{L,i},\sigma_{R,i},N_i) = N_i \begin{cases} \exp\dfrac{(x-\mu_i)^2}{2\sigma_{L,i}^2}, & \begin{matrix}\text{for x} \le \mu_i \\ \text{for x} > \mu_i\end{matrix} \\ \exp\dfrac{(x-\mu_i)^2}{2\sigma_{R,i}^2}, & \begin{matrix}\text{for x} \le \mu_i \\ \text{for x} > \mu_i\end{matrix} \end{cases}$$

- For signal we use 4 two-piece Gaussians (2 for the central peak and 2 for the tails on each side of *U*=0
  - $\sigma_{L,R,i}$ represent the widths of the two-piece Gaussians
  - α_i are relative fractions, $\alpha_0=1$
  - N_S is left unconstrained

$$\mathcal{S} = N_S\left(\sum_{i=0,1,2,3}\alpha_i \exp\frac{(x-\mu_i)^2}{2\sigma_{L,R,i}^2}\right)$$

- For background we use 2 two-piece Gaussians tails
  - $\alpha_0=1$

$$\mathcal{B} = N_B\left(\sum_{j=0,1}\alpha_j \exp\frac{(x-\mu_j)^2}{2\sigma_{L,R,j}^2}\right)$$

Binned Fits to *U* distribution cont.

- For fits to the data, normalizations of the signal and background components are always left unconstrained

- For the signal component, the shapes of the tails (μ_i, $\sigma_{L,R,i}$) for i=2,3 are fixed to values obtained from fit of truth-matched data

- Remaining 9 parameters ($\alpha_{1,2,3}, \mu_{0,1}, \sigma_{L,R,0,1}$) are allowed to vary between (1-κ, 1/(1-κ)× nominal value from truth-matched simulation fit (different κ values between 0, 5% and 30% were studied)

- For the background component, all seven parameters are allowed to vary between (1-κ, 1/(1-κ)× nominal value from non-truth-matched simulation (background) fit

Unbinned Fits to *U* distributions

- Measure closeness between i[th] and j[th] event in phase space

$$g_{ij}^2 = \sum_{k=1}^{n} \left[\frac{\phi_k^i - \phi_k^j}{r_k} \right]^2$$

- where $\vec{\phi}$ represents the n independent kinematic variables in phase space and $\vec{r}$ gives corresponding ranges for normalizations (r_{q^2} =10 GeV/c^2, $r_{\cos\,\theta}=2$ and n=2)
- In each q^2 and cos θ_ℓ bin an unbinned fit is performed in the *U* distribution to extract to the signal $\mathcal{S}_i(U_i)$ and background $\mathcal{B}_i(U_i)$ components for each event yielding a weight

$$\mathcal{Q}_i = \frac{\mathcal{S}_i(U_i)}{\mathcal{S}_i(U_i) + \mathcal{B}_i(U_i)}$$

- Now the total signal yield is $$y = \sum_i \mathcal{Q}_i$$
- Number of events in each q^2 and cos θ_ℓ bin is ≈50

Unbinned Fits to *U* distributions

- The pdf for detecting an event in the interval(ϕ, $\phi+\Delta\phi$) is

$$\mathcal{P}\left(\vec{x},\phi\right)=\frac{\frac{\mathrm{d}N(\vec{x},\phi)}{\mathrm{d}\phi}\eta(\phi)\Delta\phi}{\int\frac{\mathrm{d}N(\vec{x},\phi)}{\mathrm{d}\phi}\eta(\phi)d\phi}$$

- Where $\mathrm{d}N(\vec{x},\phi)/\mathrm{d}\phi$ is the rate term, $\eta(\phi)$ is the phase-space-dependent efficiency and $\vec{x}$ denotes the set of fit parameters

- The normalization integral constraint (pure signal) yields

$$\mathcal{N}(\vec{x})=\int\frac{\mathrm{d}N(\vec{x},\phi)}{\mathrm{d}\phi}\eta(\phi)d\phi=\bar{N}(\vec{x})=N_{data}$$

where $\bar{N}$ is equal to the measured yield

Likelihood function

- The non-extended likelihood function is

$$\mathcal{L}\left(\vec{x}\right) = -\prod_{i=1}^{N_{\text{data}}} \mathcal{P}\left(\vec{x},\phi_i\right)$$

- Taking the logarithm yields

$$-\ln\mathcal{L}\left(\vec{x}\right) = -\sum_{i=1}^{N_{\text{data}}} \mathcal{P}\left(\vec{x},\phi_i\right) \simeq N_{\text{data}}\ln\left[\mathcal{N}(\vec{x})\right] - \sum_{i=1}^{N_{\text{data}}} \ln\left[\frac{\mathrm{d}N}{\mathrm{d}\phi}\eta(\phi)\right]$$

- Using the approximation

$$\mathcal{N} = \int \frac{\mathrm{d}N}{\mathrm{d}\phi}\eta(\phi)\mathrm{d}\phi = \left(\int \mathrm{d}\phi\right)\left\langle \frac{\mathrm{d}N}{\mathrm{d}\phi}\eta(\phi)\right\rangle$$

where

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}\phi}\eta(\phi)\right\rangle = \sum_{i=1}^{N_{\text{sim}}^{\text{gen}}} \frac{\mathrm{d}N}{\mathrm{d}\phi}\frac{\eta(\phi)}{N_{\text{sim}}^{\text{gen}}} = \sum_{i=1}^{N_{\text{sim}}^{acc}} \frac{\mathrm{d}N}{\mathrm{d}\phi}\frac{1}{N_{\text{sim}}^{\text{gen}}}$$

- In the last step just accepted events are included, $\eta(\phi)$ is either 0 or 1

Likelihood function

- Ignoring term that are not variable in the fit yields

$$-\ln\mathcal{L}(\vec{x}) = N_{\text{data}} \times \ln\left[\sum_{i=1}^{N_{\text{sim}}^{\text{acc}}} \frac{dN}{d\phi}\right] - \sum_{i=1}^{N_{\text{data}}} \ln\left[\frac{dN}{d\phi}\right]$$

- Including the background subtraction procedure yield

$$-\ln\mathcal{L}(\vec{x}) = \left[\sum_{i=1}^{N_{\text{data}}} \mathcal{Q}_i\right] \times \ln\left[\sum_{i=1}^{N_{\text{sim}}^{\text{acc}}} \frac{dN}{d\phi}\right] - \sum_{i=1}^{N_{\text{data}}} \mathcal{Q}_i \ln\left[\frac{dN}{d\phi}\right]$$

- Since simulation includes model based form factor calculation (ISGW2 for $f_+(q^2)$), we need to include weight

$$\tilde{w}_i = 1 \Big/ \left[\frac{dN}{d\phi}\right]$$

yielding

$$-\ln\mathcal{L}(\vec{x}) = \left[\sum_{i=1}^{N_{\text{data}}} \mathcal{Q}_i\right] \times \ln\left[\sum_{i=1}^{N_{\text{sim}}^{\text{acc}}} \tilde{w}_i \frac{dN}{d\phi}\right] - \sum_{i=1}^{N_{\text{data}}} \mathcal{Q}_i \ln\left[\frac{dN}{d\phi}\right]$$

Fit Results

- Fit parameters for the BGL expansion with N=2

fit configuration	$a_0^{f+} \times 10$	a_1^{f+}	a_2^{f+}	a_1^{f0}	a_2^{f0}	$\|V_{cb}\| \times 10^3$	χ^2_{MILC}	χ^2_{Belle}
BABAR-1, Belle	0.126 ± 0.001	-0.096 ± 0.003	0.352 ± 0.052	-0.059 ± 0.003	0.155 ± 0.049	41.09 ± 1.16	1.15	24.50
BABAR-2, Belle	0.126 ± 0.001	-0.096 ± 0.003	0.352 ± 0.052	-0.059 ± 0.003	0.155 ± 0.049	41.12 ± 1.16	1.17	24.54
BABAR-3, Belle	0.126 ± 0.001	-0.096 ± 0.003	0.350 ± 0.052	-0.059 ± 0.003	0.153 ± 0.049	41.12 ± 1.16	1.18	24.55
BABAR-4, Belle	0.126 ± 0.001	-0.096 ± 0.003	0.352 ± 0.052	-0.059 ± 0.003	0.156 ± 0.049	41.05 ± 1.17	1.14	24.45
BABAR-1	0.126 ± 0.001	-0.097 ± 0.003	0.334 ± 0.063	-0.059 ± 0.003	0.133 ± 0.062	-	1.55	-

- Fit parameters for the BGL expansion with N=3

variable	value
$a_0^{f+} \times 10$	0.126 ± 0.001
a_1^{f+}	-0.098 ± 0.004
a_2^{f+}	0.626 ± 0.241
a_3^{f+}	-3.939 ± 3.194
a_1^{f0}	-0.061 ± 0.003
a_2^{f0}	0.435 ± 0.205
a_3^{f0}	-3.977 ± 2.840
$\|V_{cb}\| \times 10^3$	40.74 ± 1.18
$\chi^2_{\mathrm{FNAL/MILC}}$	0.001
χ^2_{Belle}	23.68

- Fit parameters for the CNL expansion

fit configuration	$\mathcal{G}(1)$	ρ_D^2	$\|V_{cb}\| \times 10^3$	$\chi^2_{\mathrm{FNAL/MILC}}$	χ^2_{Belle}
BABAR-1, Belle	1.056 ± 0.008	1.155 ± 0.023	40.90 ± 1.14	1.04	24.65
BABAR-2, Belle	1.056 ± 0.008	1.156 ± 0.023	40.92 ± 1.14	0.99	24.72
BABAR-3, Belle	1.056 ± 0.008	1.156 ± 0.023	40.92 ± 1.14	1.00	24.71
BABAR-4, Belle	1.056 ± 0.008	1.154 ± 0.023	40.87 ± 1.14	1.09	24.57
BABAR-1	1.053 ± 0.008	1.179 ± 0.027	–	0.53	–

- Reweighted $\overline{B} \to D\ell^-\overline{\nu}$ branching fraction

Measurement	$\mathcal{B}(\overline{B} \to D\ell^-\overline{\nu}_\ell) \times 10^2$	$\|V_{cb}\| \times 10^3$
BABAR-10 [14]	$\mathcal{B}_{B^0} = (2.15 \pm 0.11 \pm 0.14)$	40.02 ± 1.76
BABAR-10 [14]	$\mathcal{B}_{B^+} = (2.16 \pm 0.08 \pm 0.13)$	38.67 ± 1.41
Belle-16 [15]	$\mathcal{B}_{B^0} = (2.33 \pm 0.04 \pm 0.11)$	41.66 ± 1.22
Belle-16 [15]	$\mathcal{B}_{B^+} = (2.46 \pm 0.04 \pm 0.12)$	41.27 ± 1.23

Systematic Errors

- Add 3 fit configurations for determining systematics of background subtraction
  - BABAR-2, N_c=60, signal and background shapes locally fixed from simulation
  - BABAR-3, N_c=50, signal are allowed to vary by 5% from simulation
  - BABAR-3, N_c=50, tighter selection criteria (E_{Extra}< 0.6 GeV, $CL > 10^{-6}$)

- Compare resolutions of deviation of reconstructed-to-generated q^2 and $\cos\theta_\ell$ distributions included in the fit and not included in the fit ➡ σ=2.6% vs 3.4%

- We evaluate the effect of background subtraction

BGL $N=2$	value	CLN	value
$\lvert V_{cb}\rvert \times 10^3$	41.09 ± 1.16	$\lvert V_{cb}\rvert \times 10^3$	40.90 ± 1.14
$a_0^{f+} \times 10$	0.126 ± 0.001	$\mathcal{G}(1)$	1.056 ± 0.008
a_1^{f+}	-0.096 ± 0.003	ρ_D^2	1.155 ± 0.023
a_2^{f+}	0.352 ± 0.053		
$a_1^{f_0}$	-0.059 ± 0.003		
$a_2^{f_0}$	0.155 ± 0.049