Quarkonium dynamics in the quantum Brownian regime with non-abelian quantum master equations

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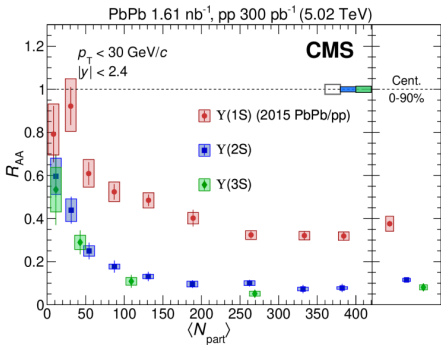
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Motivation & Outline

PbPb 1.61 nb⁻¹, pp 300 pb⁻¹ (5.02 TeV)

- Excited states are more suppressed $\rightarrow$ Sequential suppression.

- Quantum master equation provides us with in-QGP quarkonia survival probabilities $\rightarrow$ An input to compute R_{AA}.

- The dilute limit of this equation could be suitable for $b\bar{b}$ but not for $c\bar{c} \rightarrow$ Semiclassical equations are needed.

- Quantum vs. Semiclassical ❓

Outline:

1. Benchmark the quantum master equation with $b\bar{b}$ ✔

2. A comparative study: Quantum vs. Semiclassical for $c\bar{c} \rightarrow$ ✔ or ✗

Lindblad equation for in-QGP quarkonia

The master equation is derived from the von-Neumann equation of the total system

$$\frac{d\hat{\rho}_{tot}(t)}{dt} = -i\left[\hat{H}_{tot}, \hat{\rho}_{tot}(t)\right], \ \hat{\rho}_{tot} = |\Psi_{tot}\rangle\langle\Psi_{tot}| \tag{1}$$

with

$$\hat{H}_{tot} = \hat{H}_{Q\bar{Q}} \otimes \hat{I}_{QGP} + \hat{I}_{Q\bar{Q}} \otimes \bar{H}_{QGP} + g\hat{H}_{int} \tag{2}$$

Our system of interest is quarkonia $\rightarrow$ We can trace out the QGP degrees of freedom

$$\hat{\rho}_{Q\bar{Q}} = \text{Tr}_{QGP}(\hat{\rho}_{tot}) \tag{3}$$

and end up with an equation of the form [1-3]

$$\frac{d\hat{\rho}_{Q\bar{Q}}(t)}{dt} = \hat{\mathcal{L}}\left[\hat{\rho}_{Q\bar{Q}}(t)\right] = \sum_{i=0}^{4} \hat{\mathcal{L}}_i \hat{\rho}_{Q\bar{Q}} \tag{4}$$

Non-abelian quantum master equation

The Liouville superoperators $\mathcal{L}_i$ capture different aspects of the dynamics:

Unitary dynamics $\begin{cases} \mathcal{L}_0 : \text{Kinetic term} \\[2mm] \mathcal{L}_1 : \text{Static screening} \\ \quad \text{(real potential)} \end{cases}$

Non-unitary dynamics $\begin{cases} \mathcal{L}_2 : \text{Fluctuations and} \\ \text{heating (decoherence)} \\[2mm] \mathcal{L}_3 : \text{Dissipation} \\[2mm] \mathcal{L}_4 : \text{Preserve positivity} \\ \text{(sub-dominant)} \end{cases}$

The singlet and octet density matrices (ρ_s, ρ_o) evolve according to:

$$\frac{d}{dt} \begin{pmatrix} \rho_s \\ \rho_o \end{pmatrix} = \begin{pmatrix} \mathcal{L}_{ss} & \mathcal{L}_{so} \\ \mathcal{L}_{os} & \mathcal{L}_{oo} \end{pmatrix} \begin{pmatrix} \rho_s \\ \rho_o \end{pmatrix} \tag{5}$$

$\mathcal{L}_{so}$ & $\mathcal{L}_{os}$ describe singlet $\leftrightarrows$ octet transitions.

# Numerical resolution set-up for bottomonium system

The Lindblad equation was solved using 1D potentials that are based on a 3D potential inspired from Lattice results [5], with the following ingredients:

- Two different medium settings:
    1. Fixed temperature $T = 0.4$ GeV.
    2. Average temperature profile obtained from EPOS4 for the centrality class 0-10% with $|y| < 2.4$.
- A mixture of S and P states:

$$\psi(x) = e^{-\frac{x^2}{2\sigma^2}} \left(1 + a_{\text{odd}} \frac{x}{\sigma}\right)$$
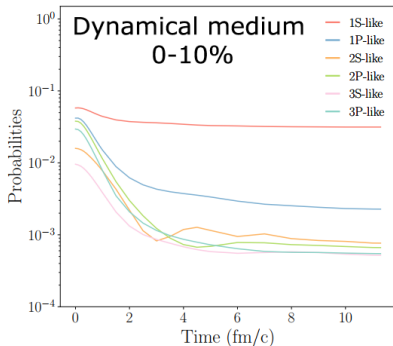
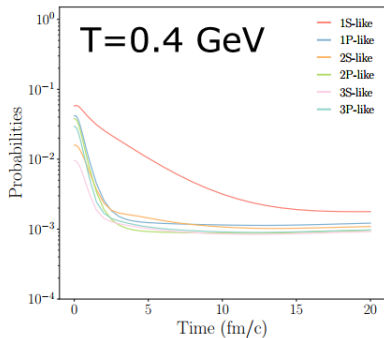$\sigma = 0.45$ fm, $a_{\text{odd}} = 3.5$

(see talk by P-B Gossiaux from HP2016)
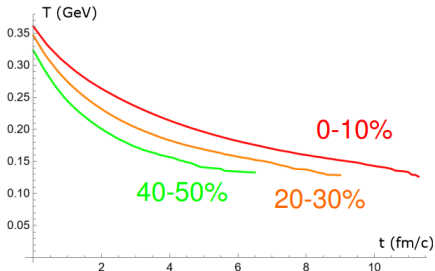
Preliminary study

Bottomonium dynamics

A comparison of the in-QGP bottomonia survival probabilities.

- The decreasing temperature in dynamical case enhances the survival probability.
- The excited state are more suppressed $\rightarrow$ Sequential suppression as observed in R_{AA}. ✔

Semiclassical approximation

- By implementing the semiclassical approximation (SCA) in the Lindblad equation, one can retrieve the semiclassical transport equations employed in various phenomenological models.

- This approximation consists in assuming that the system has a short quantum coherence length $y \equiv r - r'$ beyond which any superposition will be extremely suppressed.

$$\left\langle r' \left| \hat{\rho}_{Q\bar{Q}} \right| r \right\rangle \propto e^{-\frac{(r-r')^2}{\lambda_{th}^2}} \text{ with } \lambda_{th} \simeq \frac{1}{\sqrt{MT}} \tag{6}$$

- It not yet clear how to implement this approximation for the color dynamics $\rightarrow$ We switch to the Abelian limit.

Wigner equation in the semiclassical limit

An expansion, in QED limit, of the Lindblad equation in terms of coherence length followed by Wigner transform leads into the semiclassical Fokker-Planck equation [1]:

$$
\begin{aligned}
\frac{\partial W(r,\boldsymbol{p})}{\partial t} &= \left[-\frac{2\boldsymbol{p}.\boldsymbol{\nabla}_r}{M} - \boldsymbol{\nabla}_r V(r).\boldsymbol{\nabla}_{\boldsymbol{p}} + \frac{\eta(r)}{2}\boldsymbol{\nabla}_{\boldsymbol{p}}^2 \right. \\
&\quad \left. + \frac{\gamma(r)}{M}\boldsymbol{\nabla}_{\boldsymbol{p}}\boldsymbol{p} \right] W(r,\boldsymbol{p})
\end{aligned}
\tag{7}
$$

Steady state solution:

$$
W_{st}(r,\boldsymbol{p}) = N\exp\left[-\frac{1}{T}\left(\frac{\boldsymbol{p}^2}{M} + V(r) \right) \right]
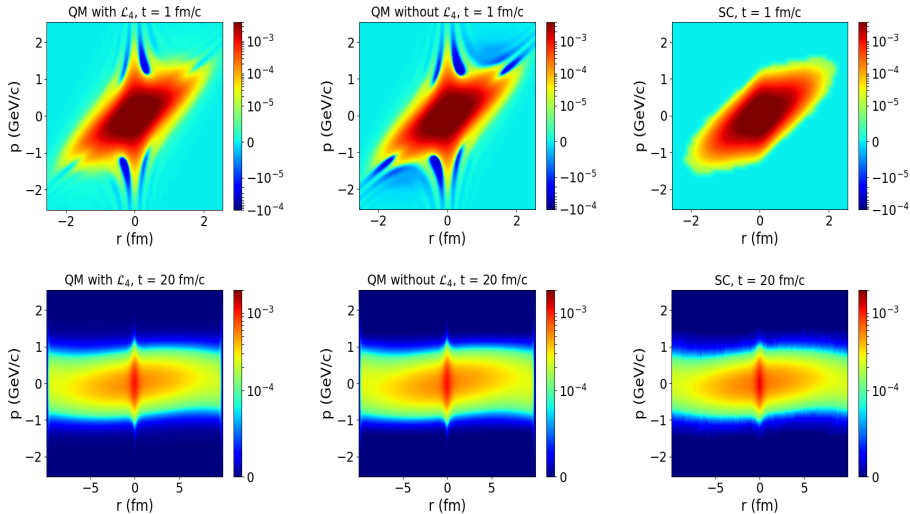\tag{8}
$$

Numerical resolution set-up for charmonium system

We adopt a 1S-like initial state and solve the two equations in 1D for a static QGP, using the same set of parameters [4].

$$\psi(r) = \left(\frac{2}{\pi\sigma^2}\right)^{\frac{1}{4}} e^{-\frac{r^2}{\sigma^2}} \text{ with } \sigma = 0.54 \text{ (fm)} \tag{9}$$
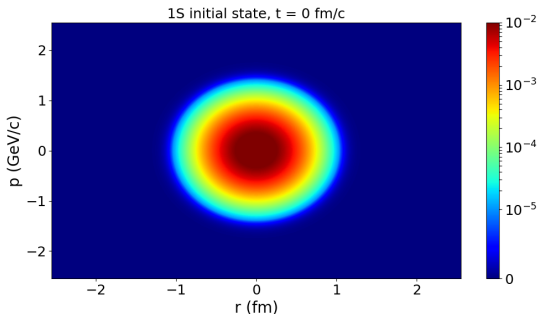
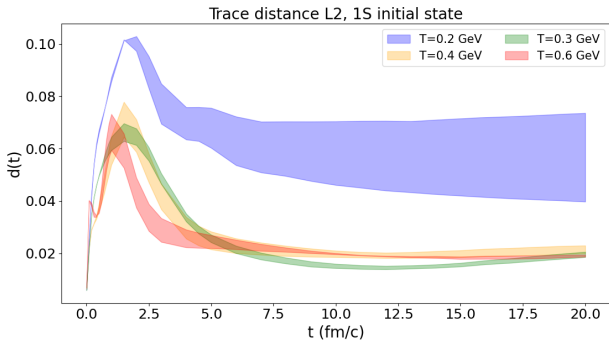# Quantum vs. semiclassical Wigner functions, $T = 0.3$ GeV

We observe a better agreement between the two descriptions at late times.

Trace distance between Wigner functions

A more quantitative comparison can be conducted by computing the trace distance

$$d(t) = \sqrt{2\pi\hbar \int drdp \left(W_{QM} - W_{SC}\right)^2} \rightarrow 0 \leq d(t) \leq 2$$

Trace distance L2, 1S initial state

- At least, $\sim 95\%$ of agreement between the two descriptions ! ✔
- Quantum decoherence is less efficient at low temperatures, hence, the quantum features survive longer $\Rightarrow$ The trace distance reaches higher values at lower temperatures and decreases at a slower rate.

1S-like state survival probability

A comparison of the quantum and semiclassical descriptions of the 1S-like state survival probability shows that the SCA performs remarkably well. ✔

1S state survival probabilities
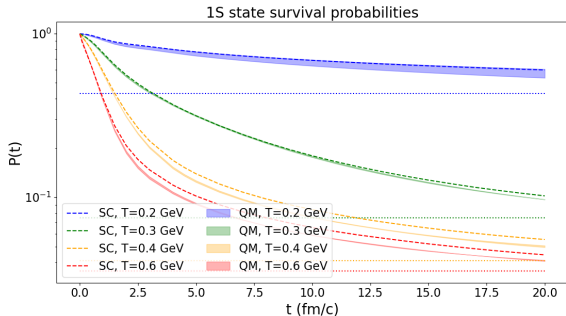
- The quantum overheating of quarkonium states results in a slight overdepletion of the population.
- Thermal equilibrium is not reached in any of the descriptions.

2S-like state generation probability

Some insights on the possible effect of interference terms and negativity of Wigner function can be inferred from the 2S-like state generation probability.

2S state generation probabilities
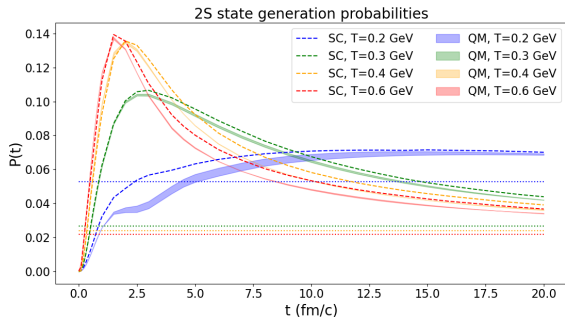
- A good agreement between the two descriptions is observed, especially, at high temperatures.
- The noticeable difference at early time for $T = 0.2$ GeV is due to the interference terms and their slow quenching by quantum decoherence.

Conclusions

- The Lindblad equation succeeds in producing the bottomonia sequential suppression observed in R_{AA}. As next step, we will compute this observable and have direct comparison with experimental data.

- The semiclassical description reproduces very well the results of the exact quantum description, especially, at high temperatures.

- The late time discrepancies are, mainly, due to the relaxation into different steady states. The steady sate of an open quantum system is still an active research topic !

- The quantum vs. semiclassical comparative study needs to be extended to the non-abelian case and to the low temperature regime.

Thank you for your attention !

References

[1] J-P.Blaizot et al. JHEP 2018.6 (2018): 1-57.

[2] S.Delorme et al. JHEP 06 (2024) 060.

[3] Y.Akamatsu. Prog. Part. Nucl. Phys. 123 (2022): 103932.

[4] R.Katz et al. Eur.Phys.J.A 58 (2022) 10, 198

Back up

Constraints on the master equation

The master equation, or dynamical map, governing time evolution of quarkonia density matrix must satisfy two requirements:

- Since probabilities could only be positive, the initial positive semidefinite density matrix must be evolved, i.e mapped, into another positive semidefinite matrix $\rightarrow$ The map must be positive.

$$\langle \phi | \hat{\rho}(t) | \phi \rangle \geq 0 \tag{10}$$

- Probabilities should always sum up to one , therefore, the trace of the initial normalized density matrix must be preserved during its time evolution $\rightarrow$ The map must be trace preserving.

$$\mathrm{Tr}(\hat{\rho}(t)) = \mathrm{Tr}(\hat{\rho}(0)) \tag{11}$$

In order to satisfy those requirements, a master equation must be in Lindblad form. To this end, some approximations are needed along the way of deriving a master equation!

Derivation of master equation

The main approximations usually adopted are

- QGP is assumed to be at thermal equilibrium all along the evolution time.
- The quarkonia-QGP total density matrix assumed to be initially and remain factorisable all along the time evolution $\rightarrow$ Born approximation.

$$\hat{\rho}_{\text{tot}}(t) \simeq \hat{\rho}_{Q\bar{Q}}(t) \otimes \hat{\rho}_{\text{QGP}}(0) \tag{12}$$

- The quarkonia state time evolution depends only on its current state, i.e the dynamical map is local in time $\rightarrow$ Markovian approximation ($\tau_R >> \tau_E$).

$$\frac{\mathrm{d}\hat{\rho}_{Q\bar{Q}}(t)}{\mathrm{d}t} = \mathcal{L}(t)\,\hat{\rho}_{Q\bar{Q}}(t) \tag{13}$$

Depending on whether we are in quantum optical or Brownian regime, additional approximations as gradient expansion or rotating wave approximation are needed to get complete positive trace preserving (CPTP) master equation !

Various regimes of quarkonium in QGP

- Several scales are involved in quarkonium-QGP system: $M, Mv, Mv^2, T, m_D, \Lambda_{QCD}$.

- The temperatures attained so far in heavy ion collisions satisfy $M \gg T$, and since $M \gg \Lambda_{QCD} \rightarrow$ Heavy quark mass is larger than thermal and quantum fluctuations $\rightarrow$ We can adopt a non relativistic description.

- Different hierarchies between the scales Mv, Mv^2, T, m_D leads to different regimes.

- In case of high QGP temperature, $T \gg E \sim Mv^2$ we are in the quantum Brownian motion regime and a gradient expansion is required to get a CPTP master equation.

- The appropriate basis of this regime is phase space variables.

- Effective field theories are a suitable starting point to derive a Lindblad equation in our case. The hierarchy between the soft scale $p \sim Mv$ and temperature T dictates the use of NRQCD or pNRQCD.

Quarkonium-QGP dynamics within NRQCD

$$\hat{H}_{\text{tot}} = \hat{H}_{Q\bar{Q}} \otimes \hat{I}_{QGP} + \hat{I}_{Q\bar{Q}} \otimes \hat{H}_{QGP} + g\hat{H}_{int}, \tag{14}$$

The system intrinsic Hamiltonian $\hat{H}_{Q\bar{Q}}$ corresponds to the free case

$$\hat{H}_{Q\bar{Q}} = \frac{\boldsymbol{p}_Q^2}{2M} + \frac{\boldsymbol{p}_{\bar{Q}}^2}{2M}, \tag{15}$$

while the QGP Hamiltonian corresponds to the light particle sector of NRQCD

$$\hat{H}_{QGP} = \hat{H}_{q+A}. \tag{16}$$

The interaction quarkonium-QGP is given by

$$\hat{H}_{int} = -g \int_{\boldsymbol{x}} n^a(\boldsymbol{x}) A_0^a(\boldsymbol{x}), \tag{17}$$

Non-abelian quantum master equations

$$
\begin{aligned}
\frac{d\hat{\rho}_{Q\bar{Q}}}{dt} &= -i\left[\hat{H}_{Q\bar{Q}} + \Delta\hat{H}_{Q\bar{Q}}, \hat{\rho}_{Q\bar{Q}}(t)\right] \\
&\quad + \int_{\boldsymbol{x},\boldsymbol{y}} W(\boldsymbol{x}-\boldsymbol{y})\left(\tilde{n}_{\boldsymbol{x}}^a \hat{\rho}_{Q\bar{Q}} \tilde{n}_{\boldsymbol{y}}^{a\dagger} - \frac{1}{2}\left\{\tilde{n}_{\boldsymbol{y}}^{a\dagger}\tilde{n}_{\boldsymbol{x}}^a, \hat{\rho}_{Q\bar{Q}}\right\}\right)
\end{aligned}
\tag{18}
$$

with

$$
\tilde{n}_{\boldsymbol{x}}^a = n_{\boldsymbol{x}}^a - \frac{i}{4T}\dot{n}_{\boldsymbol{x}}^a
\tag{19}
$$

The Lindblad map $\propto W(\boldsymbol{x}-\boldsymbol{y})$, it contains :

- The diffusive operator $\hat{\mathcal{L}}_2 \propto n_{\boldsymbol{x}}^a n_{\boldsymbol{y}}^a$ describes fluctuations, i.e the heating up of our system by QGP constituents kicks.
- The damping operator $\hat{\mathcal{L}}_3 \propto \dot{n}_{\boldsymbol{x}}^a n_{\boldsymbol{y}}^a$ and $\propto n_{\boldsymbol{x}}^a \dot{n}_{\boldsymbol{y}}^a$ describes dissipation.
- The operator $\hat{\mathcal{L}}_4 \propto \dot{n}_{\boldsymbol{x}}^a \dot{n}_{\boldsymbol{y}}^a$ is subdominant, but mandatory to preserve positivity.

One dimensional real potential for $c\bar{c}$
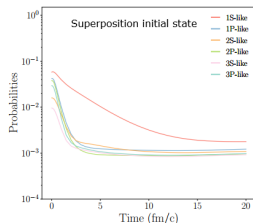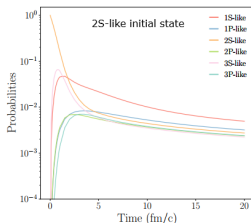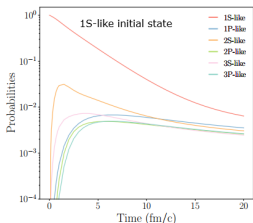

(a)
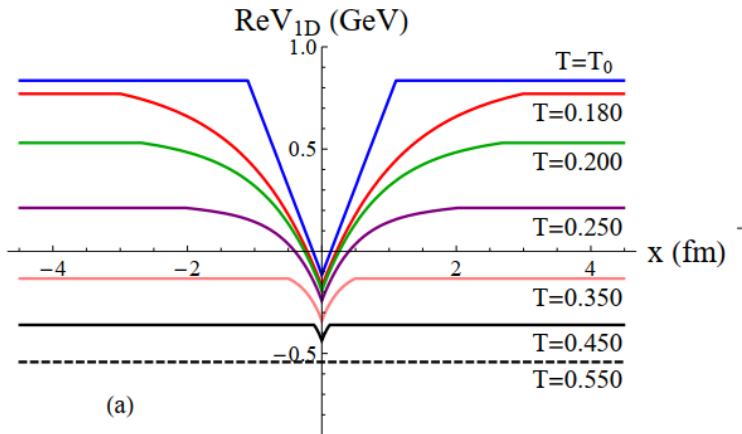
One dimensional imaginary potential for $c\bar{c}$
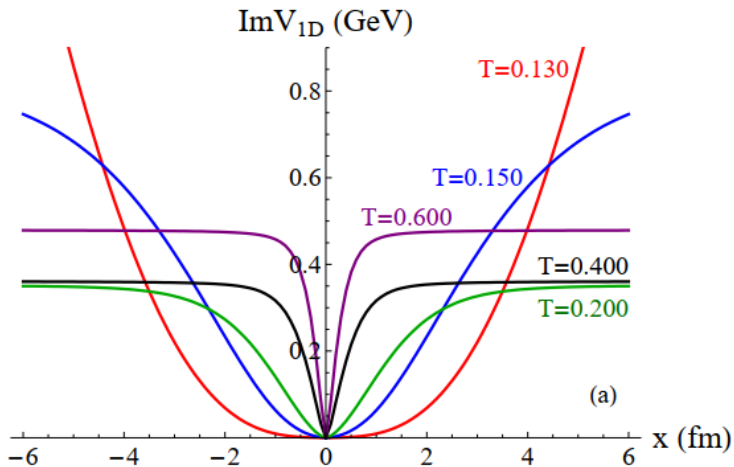
Different initial bottomonium states

We can test the effect of the initial state choice on late time survival probabilities.
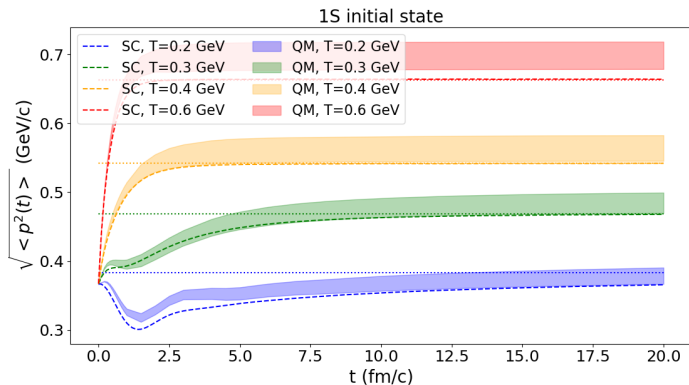
A key ingredient to solve a part of the puzzle !

A key concept in the classicalisation and thermalisation of our system is the energy exchange with medium and strength of momentum kicks it receives form it.

This could be reflected, to some extent, through root mean square momentum of our system $\sqrt{\langle p^2 \rangle}$.

Mean square momentum

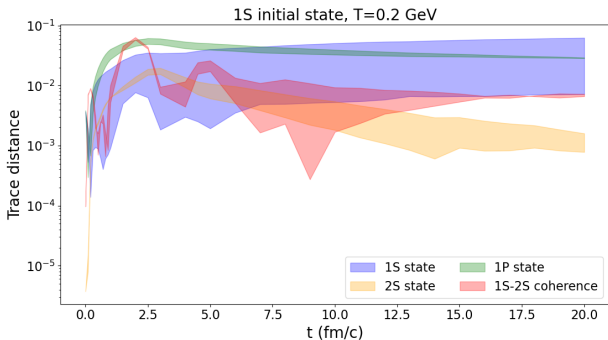At thermal equilibrium we have $\sqrt{\langle p^2 \rangle_{st}} = \sqrt{MT/2}$.

The quantum description overheats the quarkonium state $\rightarrow$ This induces more dissociation $\Rightarrow$ Lower survival probabilities.
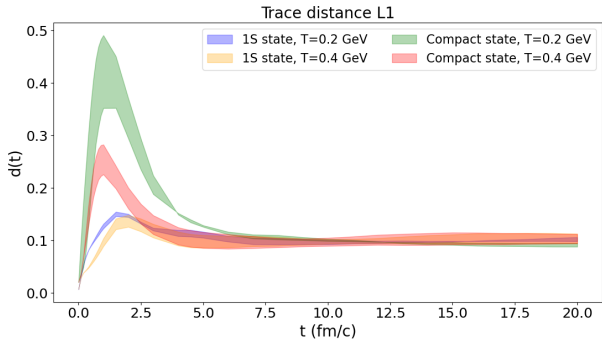
Trace distance in states basis

How about the observables sensitive to off diagonal elements of density matrix ? Could they dominate the trace distance between QM and SC descriptions?

$$\sqrt{\int drdp(W_{SC} - W_{QM})^2} \rightarrow \sqrt{\sum_{n,n\prime}(< n|(\rho_{SC} - \rho_{QM})|n\prime >)^2} \qquad (20)$$

1S-like initial state vs. compact initial state

Trace distance L1

Legend: 1S state, T=0.2 GeV; Compact state, T=0.2 GeV; 1S state, T=0.4 GeV; Compact state, T=0.4 GeV

x-axis: t (fm/c), y-axis: d(t)

Dangerous cross terms in Wigner function

The break down of SC description in case of 2S state at low T could be attributed to the "dangerous cross terms" in its Wigner function

J.E.Heller, J.Chem. Phys.65.4(1976):1289-1298

$$\Psi_{2S}\left(r\right) = \frac{1}{\sqrt{2}}\left(\psi_1\left(r\right) \pm \psi_2\left(r\right)\right) \tag{21}$$

corresponding to a Wigner function

$$W_{2S}\left(r,p\right) = \frac{1}{2}W_1\left(r,p\right) + \frac{1}{2}W_2\left(r,p\right) \pm W_{int}\left(r,p\right) \tag{22}$$

The interference term W_{int} results from cross terms as $\psi_1^*\psi_2$ and contains very oscillating terms

$$W_{int}\left(r,p\right) \propto \cos\left(\frac{rp}{\hbar c}\right), \sin\left(\frac{rp}{\hbar c}\right) \tag{23}$$

which spoil the SC expansion of unitary operators $\hat{\mathcal{L}}_{0,1}$ and necessitate all-orders resummation w.r.p to coherence length or $\leftrightarrow \hbar$.

Higher order quantum corrections

We define

$$\hat{H} = \frac{\mathbf{p}_Q^2}{2M} + \frac{\mathbf{p}_{\bar{Q}}^2}{2M} + \frac{1}{2} \int_{\mathbf{x}\mathbf{x}'} V\left(\mathbf{x} - \mathbf{x}'\right) \hat{n}_{\mathbf{x}}^a \hat{n}_{\mathbf{x}'}^a \tag{24}$$

So that the unitary evolution is given by

$$\frac{d\hat{\rho}_{Q\bar{Q}}(t)}{dt} = -\frac{i}{\hbar}\left[\hat{H}, \hat{\rho}_{Q\bar{Q}}(t)\right] \tag{25}$$

Higher order quantum corrections

$$
\begin{aligned}
\frac{\partial W(\mathbf{r},\mathbf{p})}{\partial t} &= -\frac{\partial H(\mathbf{r},\mathbf{p})}{\partial \mathbf{p}}\frac{\partial W(\mathbf{r},\mathbf{p})}{\partial \mathbf{r}} + \frac{\partial H(\mathbf{r},\mathbf{p})}{\partial \mathbf{r}}\frac{\partial W(\mathbf{r},\mathbf{p})}{\partial \mathbf{p}} - \frac{\hbar^2}{24}\frac{\partial^3 H(\mathbf{r},\mathbf{p})}{\partial \mathbf{r}^3}\frac{\partial^3 W(\mathbf{r},\mathbf{p})}{\partial \mathbf{p}^3} \\
&\quad + \frac{\hbar^4}{1920}\frac{\partial^5 H(\mathbf{r},\mathbf{p})}{\partial \mathbf{r}^5}\frac{\partial^5 W(\mathbf{r},\mathbf{p})}{\partial \mathbf{p}^5} + \mathcal{O}\left(\hbar^6\right) + \cdots \equiv L_Q W\left(\mathbf{r},\mathbf{p}\right)
\end{aligned}
\tag{26}
$$