# Thermal lattice QCD results from the FASTSUM collaboration

Gert Aarts[1], **Chris Allton**[1], Naeem Anwar[1], Ryan Bignell[6], Timothy Burns[1], Sergio Chaves García-Mascaraque[1], Simon Hands[2], Benjamin Jäger[3], Seyong Kim[4], Maria Paola Lombardo[5], Benjamin Page[1], Sinead Ryan[6], Jon-Ivar Skullerud[7], Antonio Smecca[1], Thomas Spriggs[1]

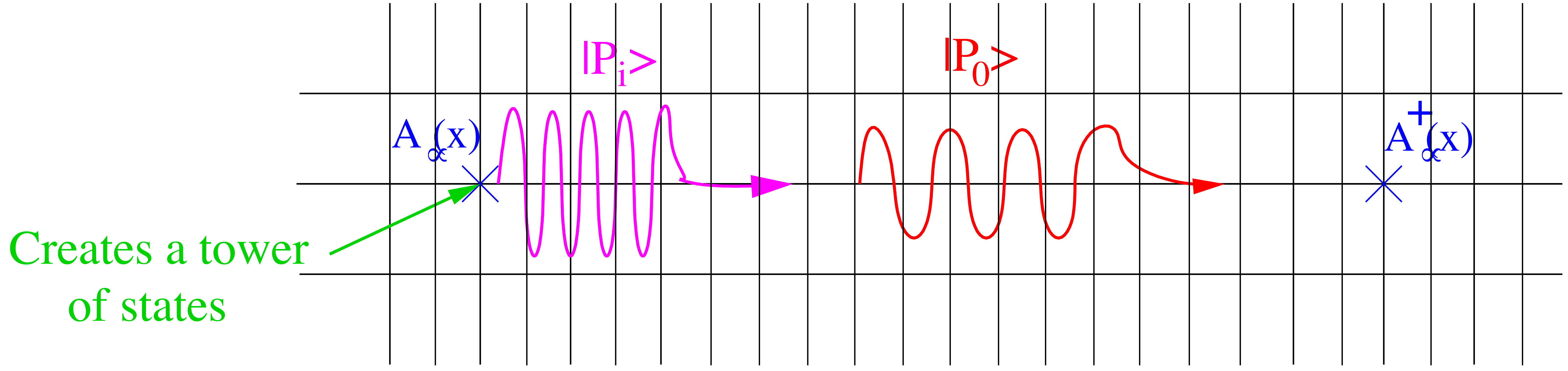
*(1) Swansea University, U.K.*
*(2) University of Liverpool, U.K.*
*(3) University of Southern Denmark, Denmark*
*(4) Sejoyng University, Korea*
*(5) INFN Firenze, Italy*
*(6) Trinity College, Dublin, Ireland*
*(7) National University of Ireland Maynooth, Ireland*

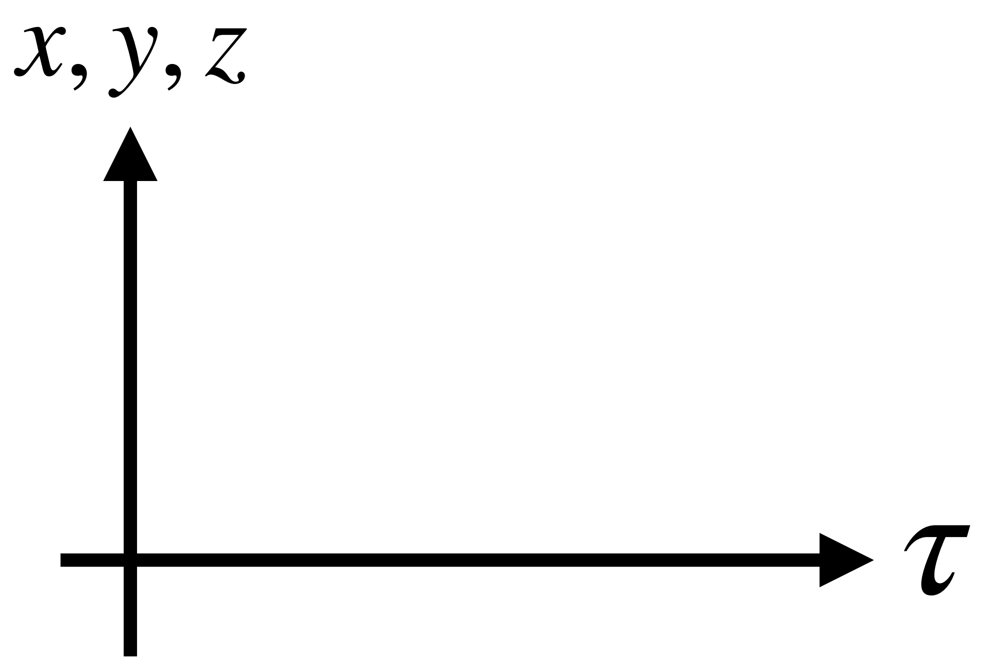
# Overview

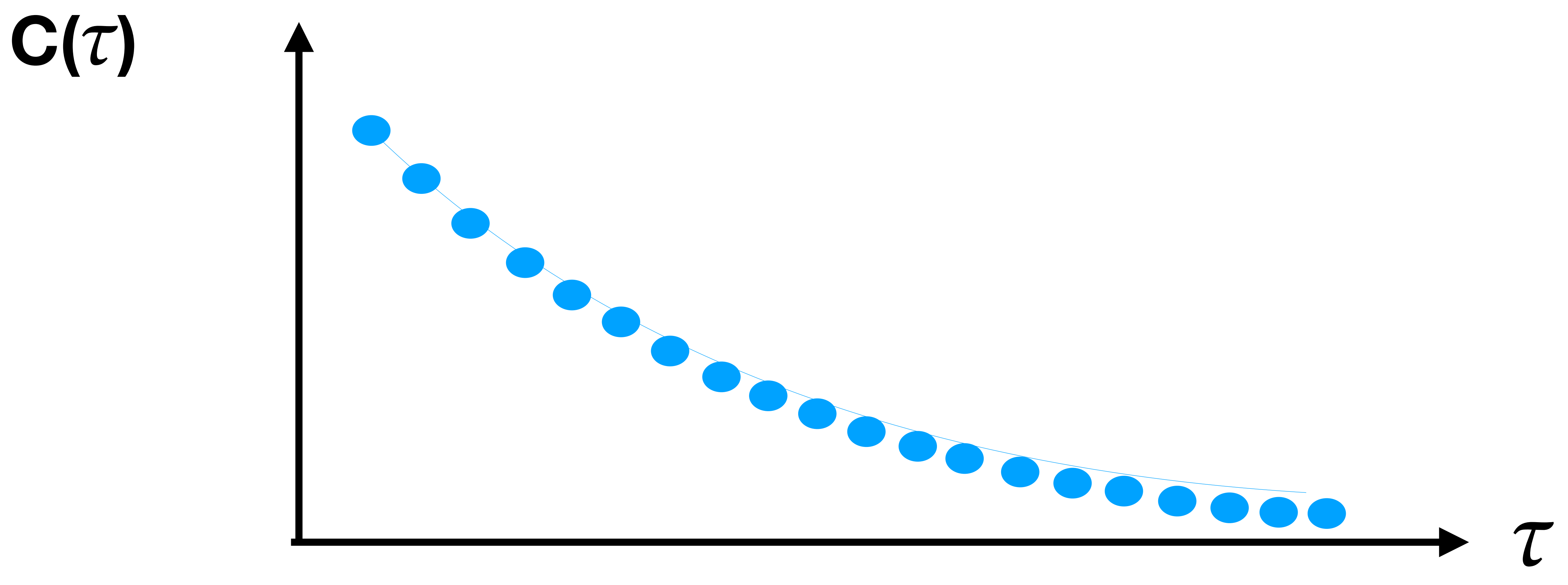
- FASTSUM approach
- Open Charm Mesons
- Charm Baryons
- Interquark potential in bottomonium
- Spectral Functions

# FASTSUM Approach:

## *Anisotropic* Lattice




**Spectral Quantities:**
- Bottomonium
- Charmed mesons
- Heavy Baryons
- Light Hadrons

**Interquark potential**

**Conductivity**

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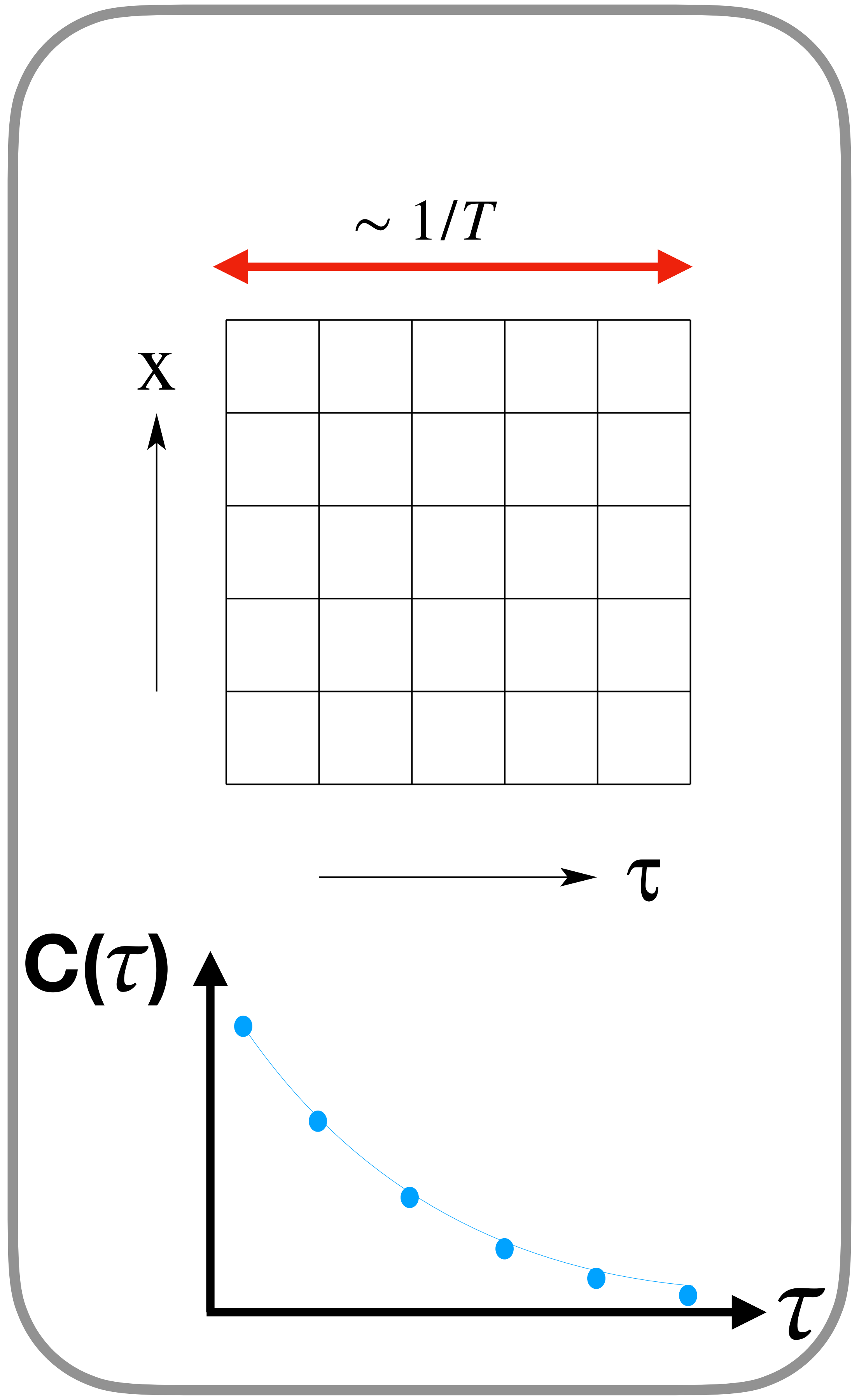
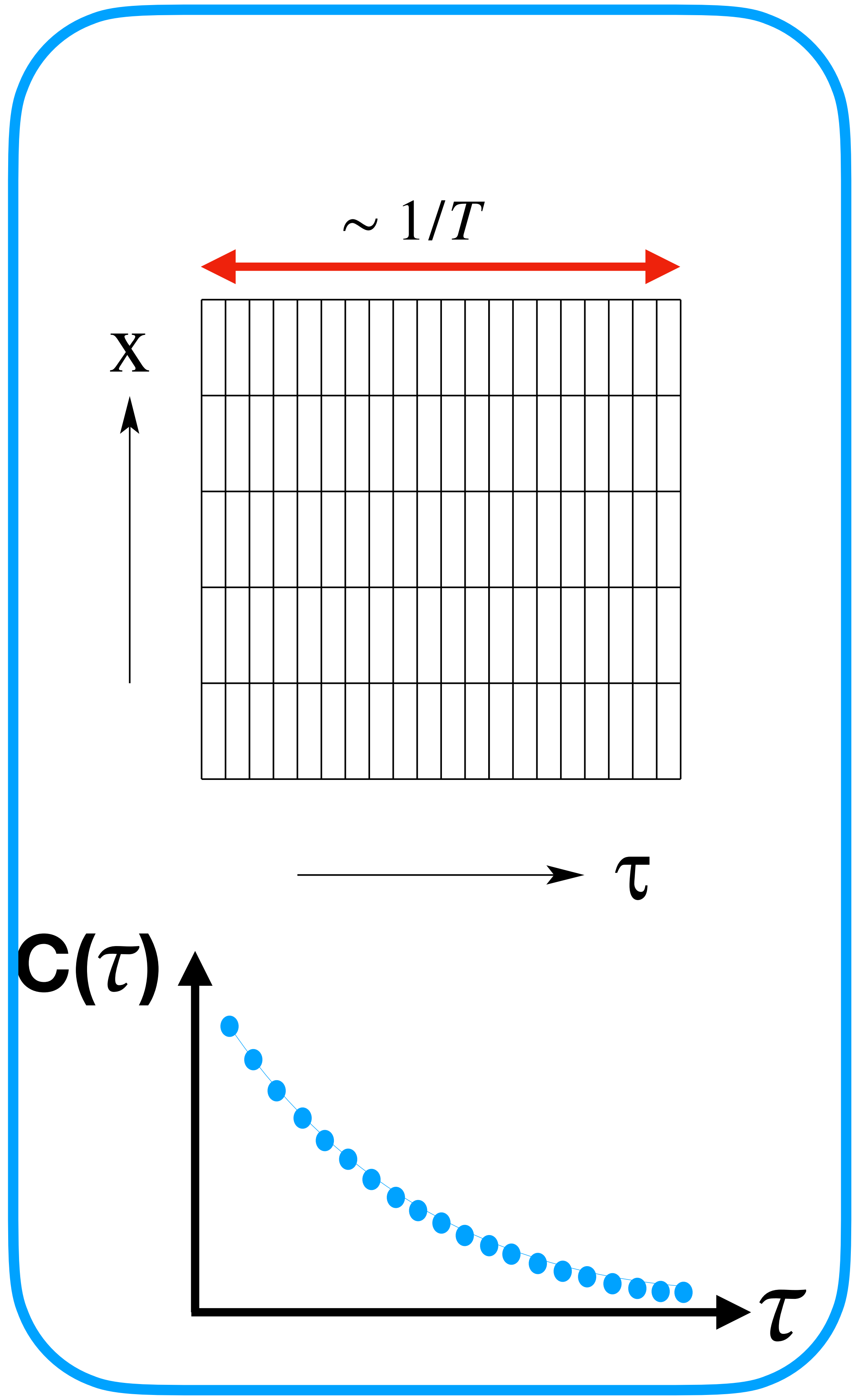


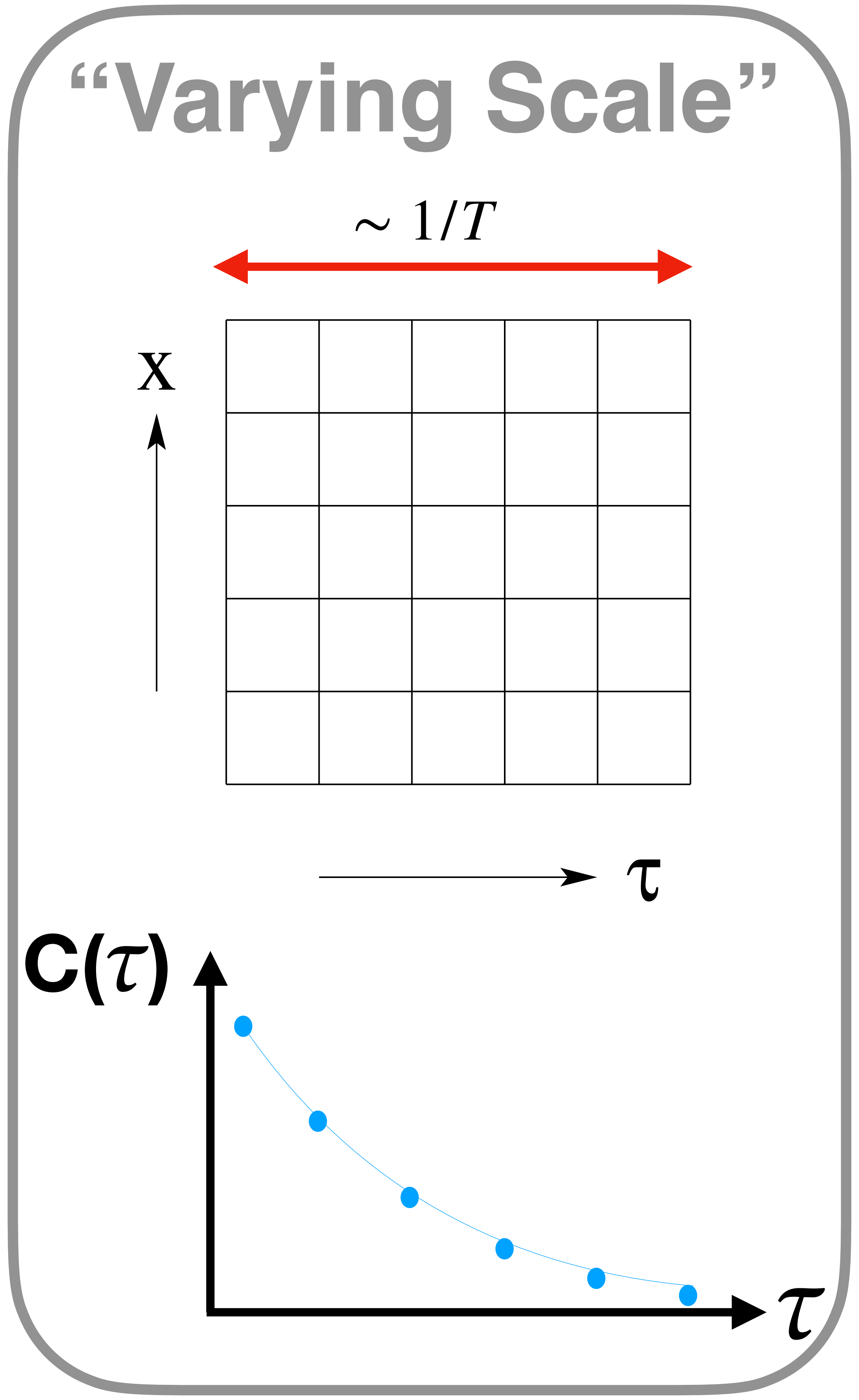




$$\sum_i \langle i \,|\, e^{-H L_\tau} \,|\, i \rangle = \sum_i \langle i \,|\, e^{-H/T} \,|\, i \rangle$$

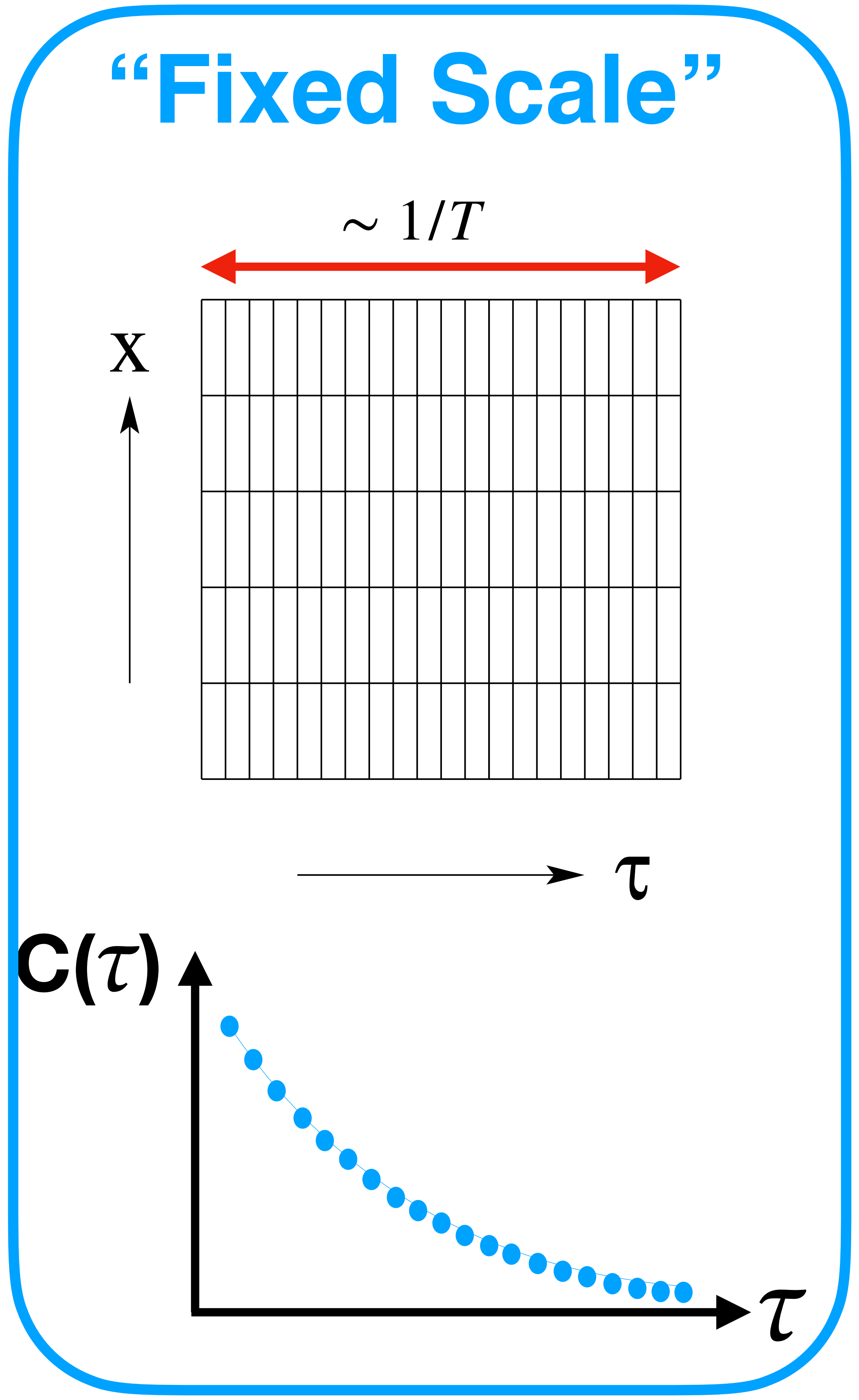
$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

# FASTSUM Approach:

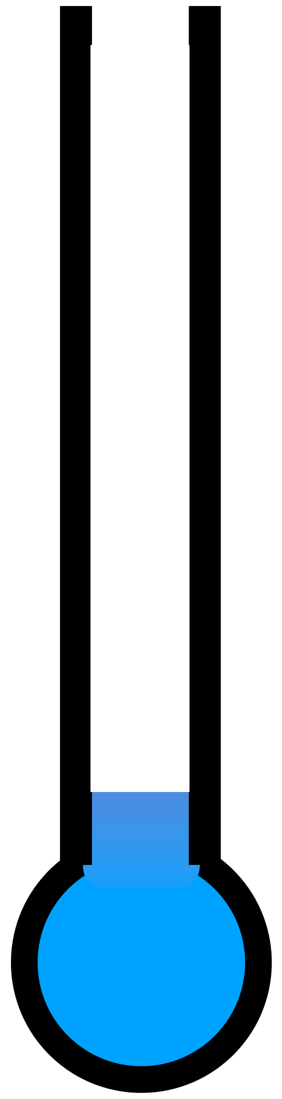
## *Anisotropic* Lattice




$a_\tau \to 0$




$N_\tau \to 0$



**Going hotter…**

$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

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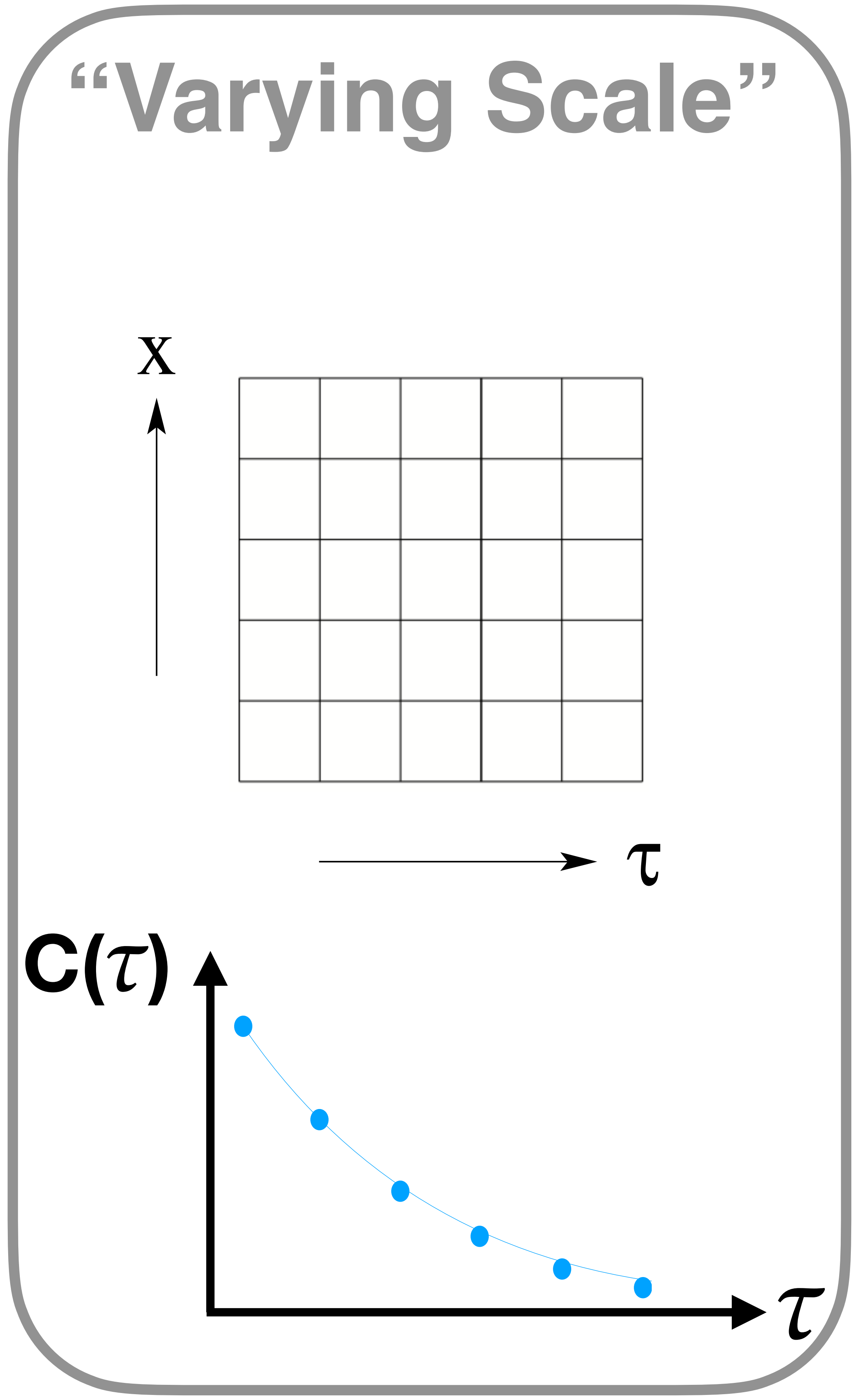
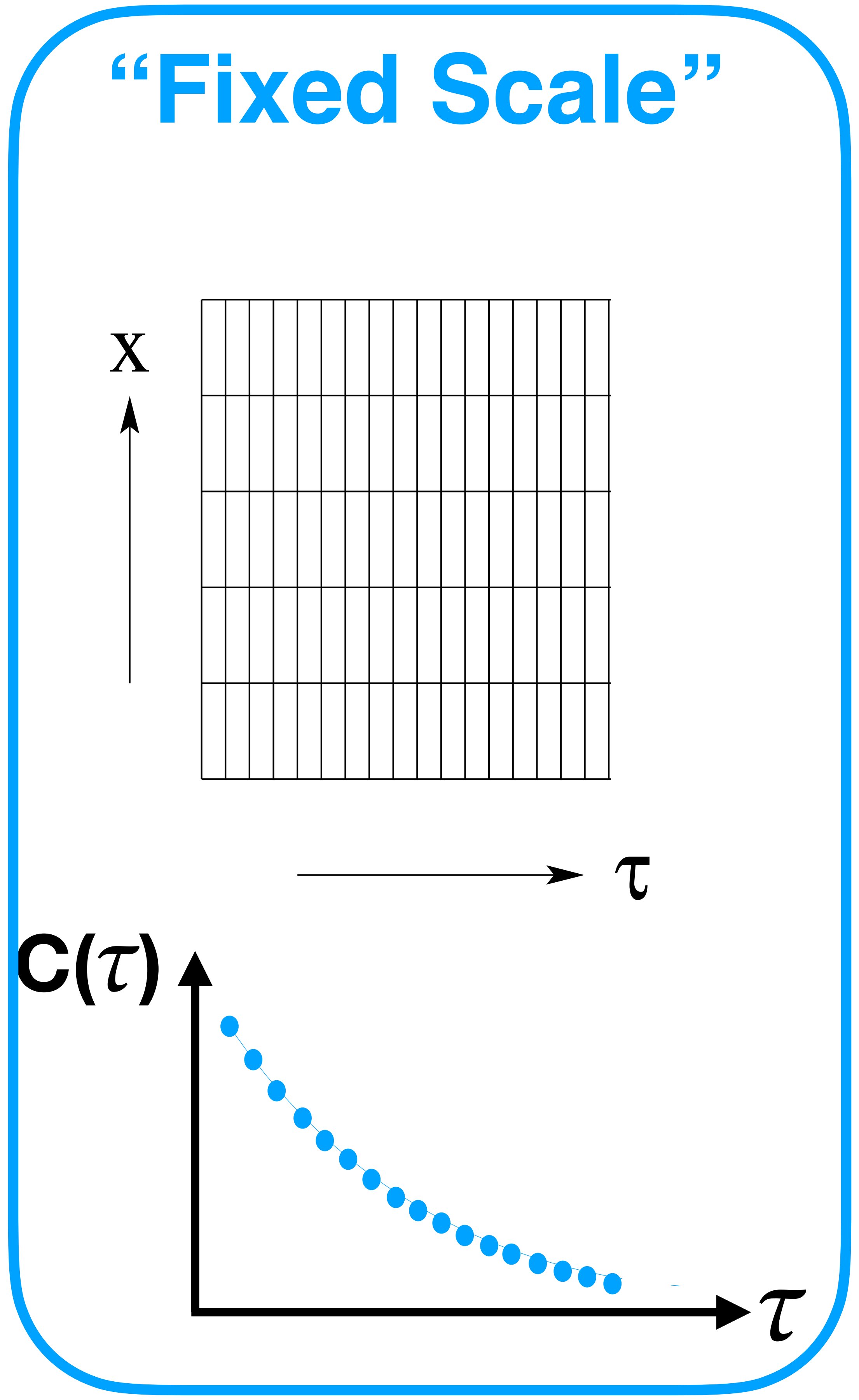
## *Anisotropic* Lattice




$$a_\tau \rightarrow 0$$

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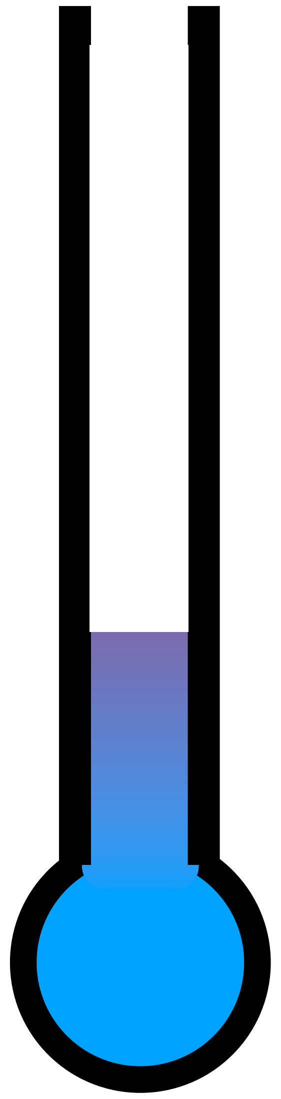

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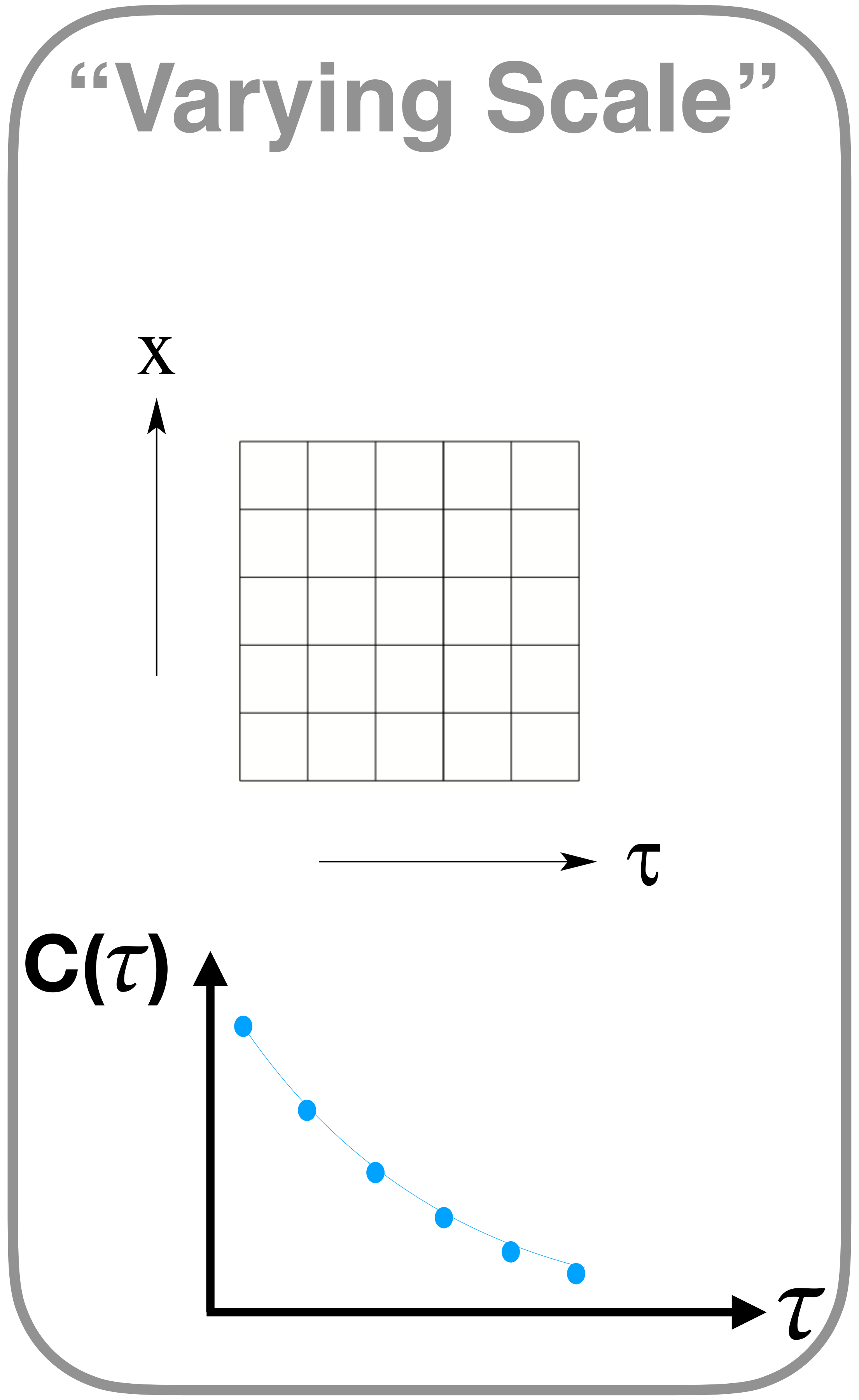
$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

# FASTSUM Approach:

*Anisotropic* Lattice

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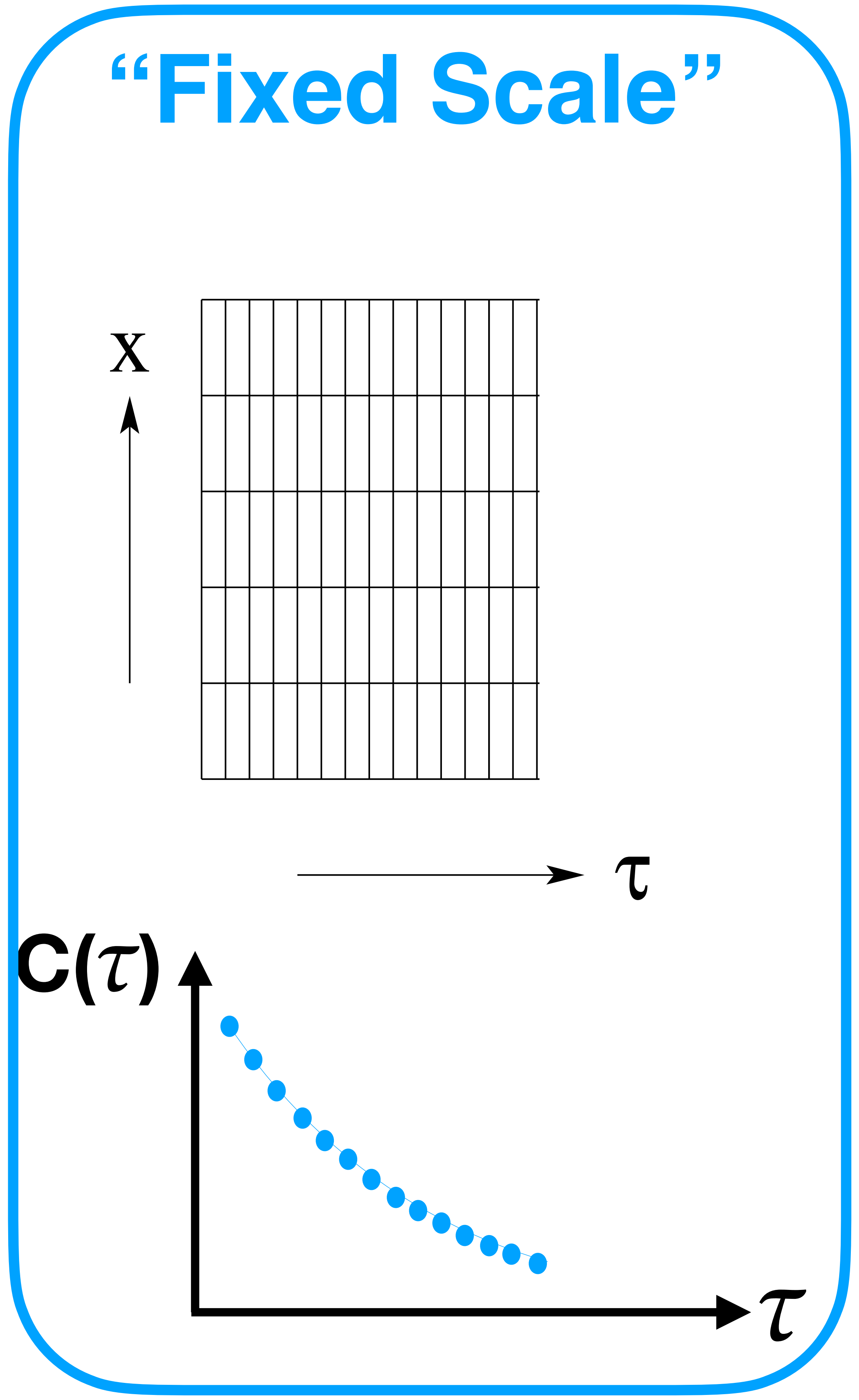


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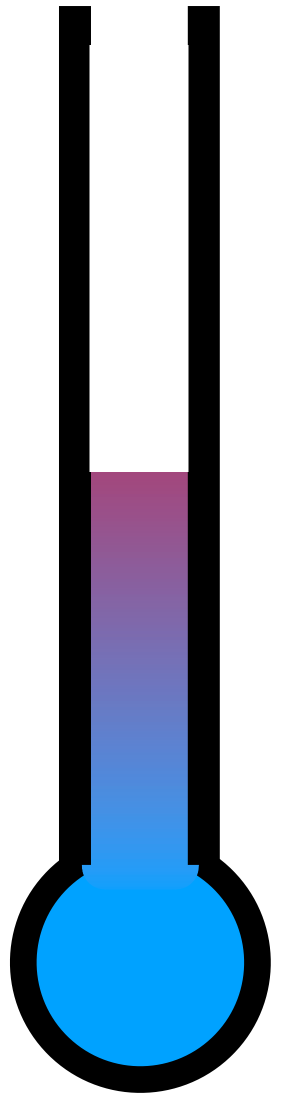

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$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

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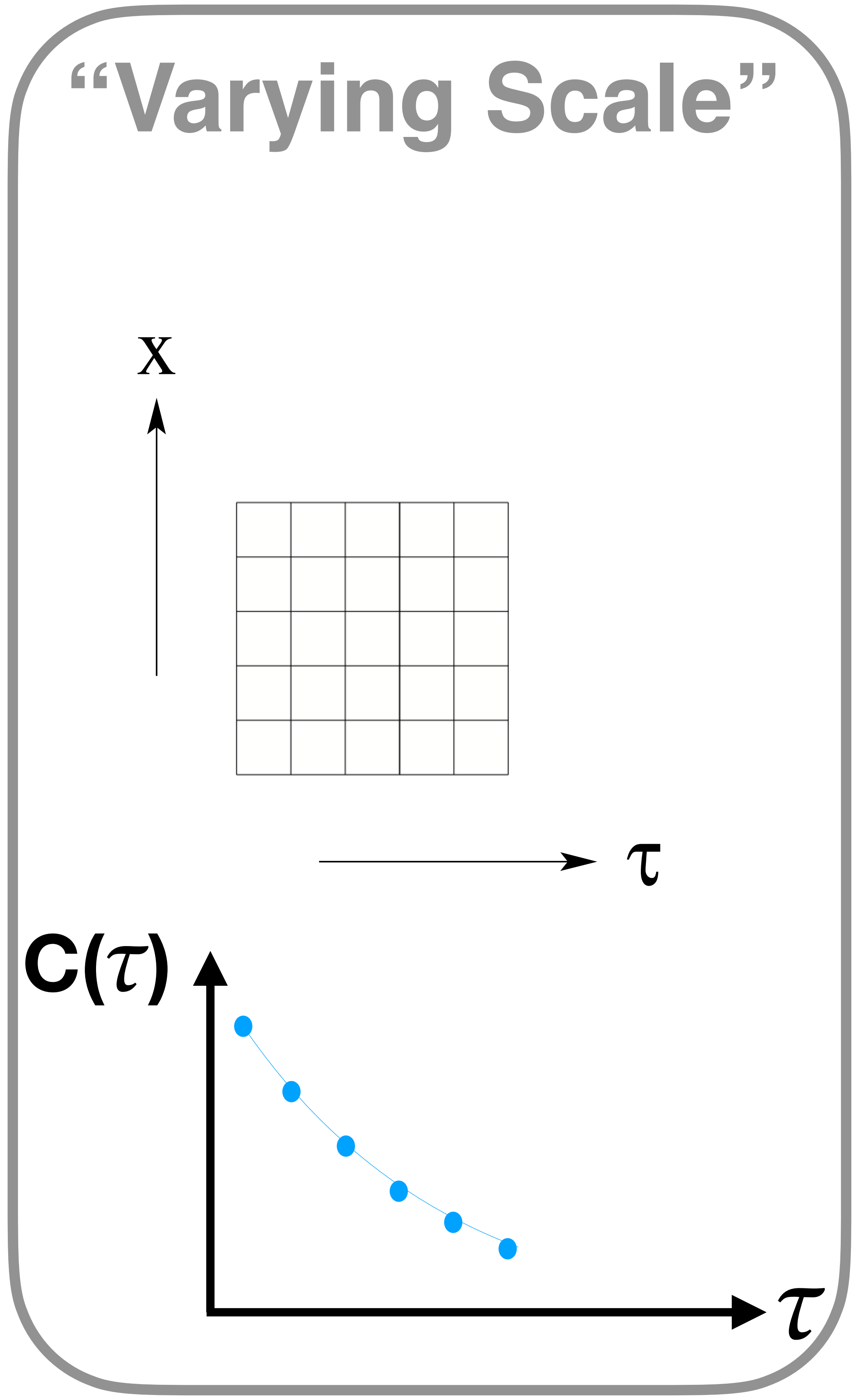


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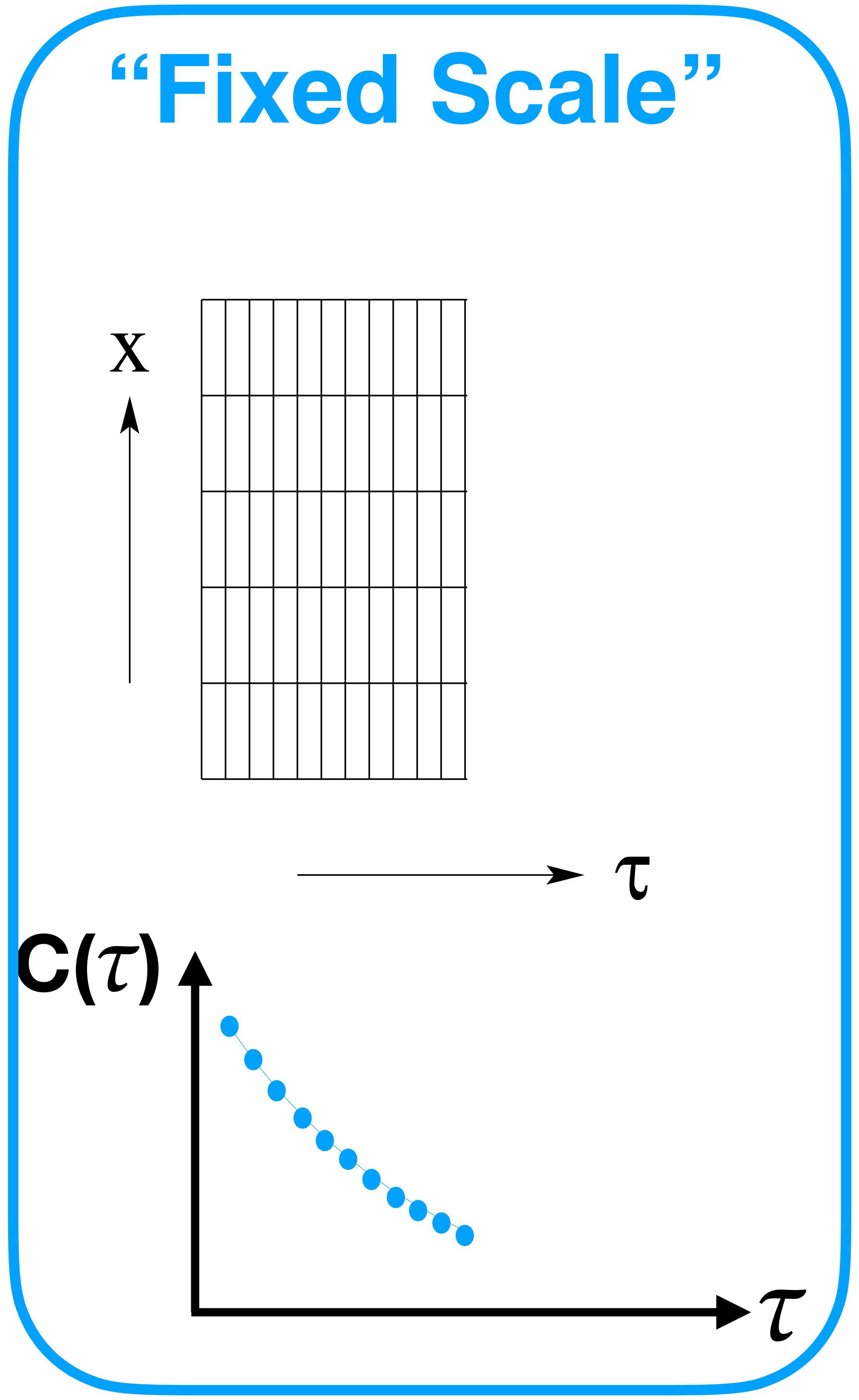


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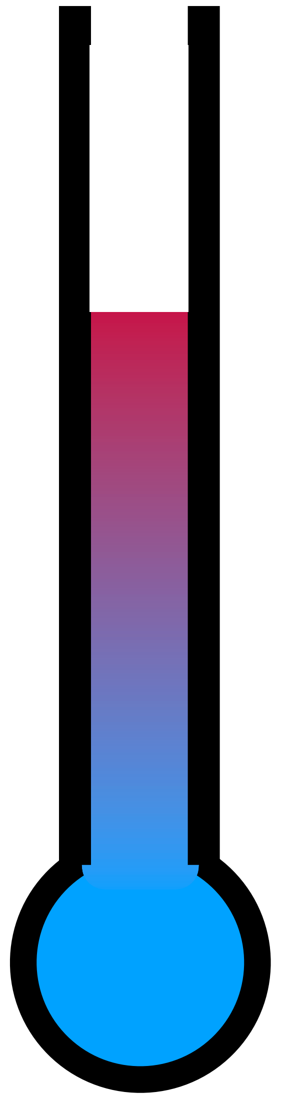

**Going hotter…**

$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

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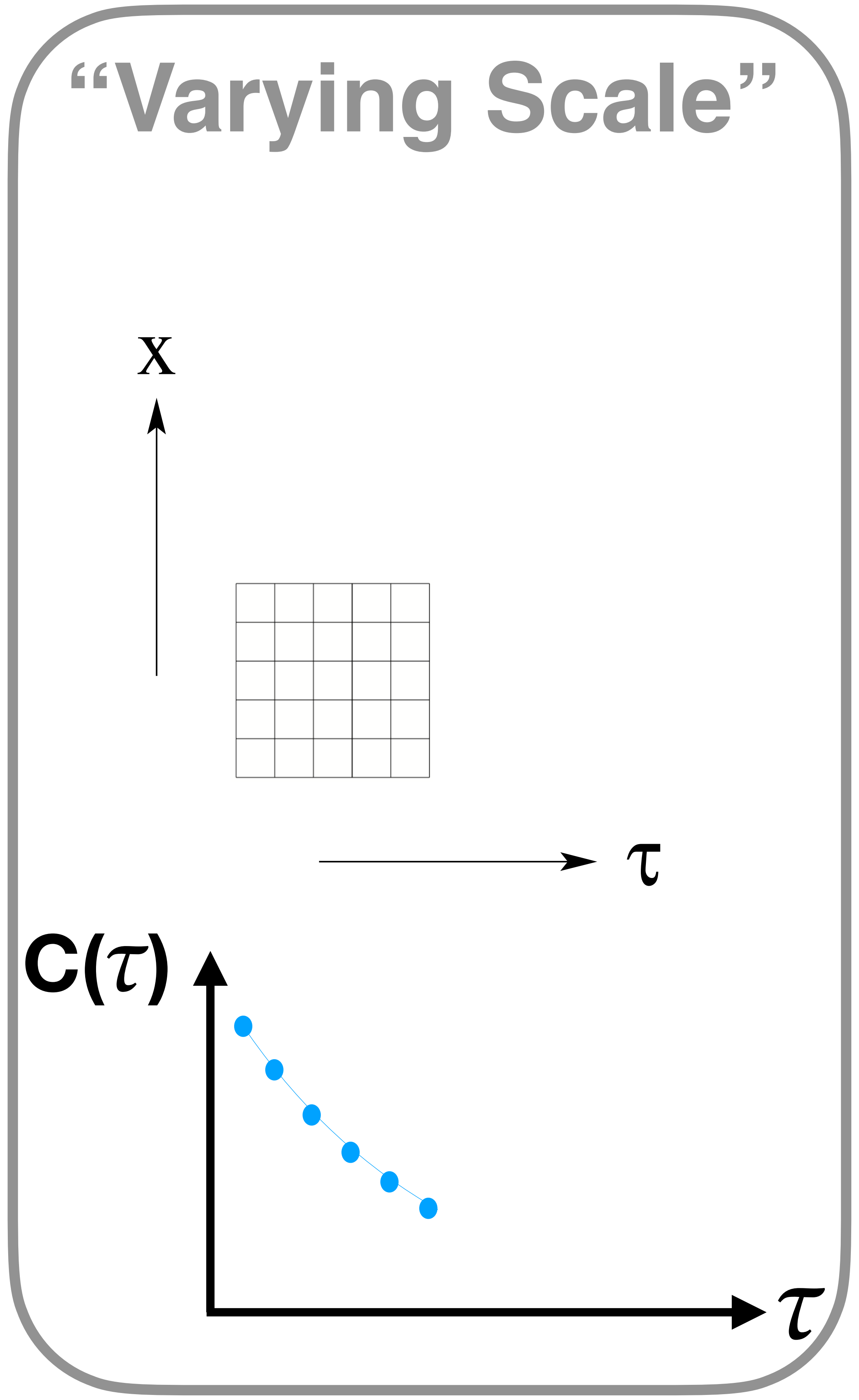


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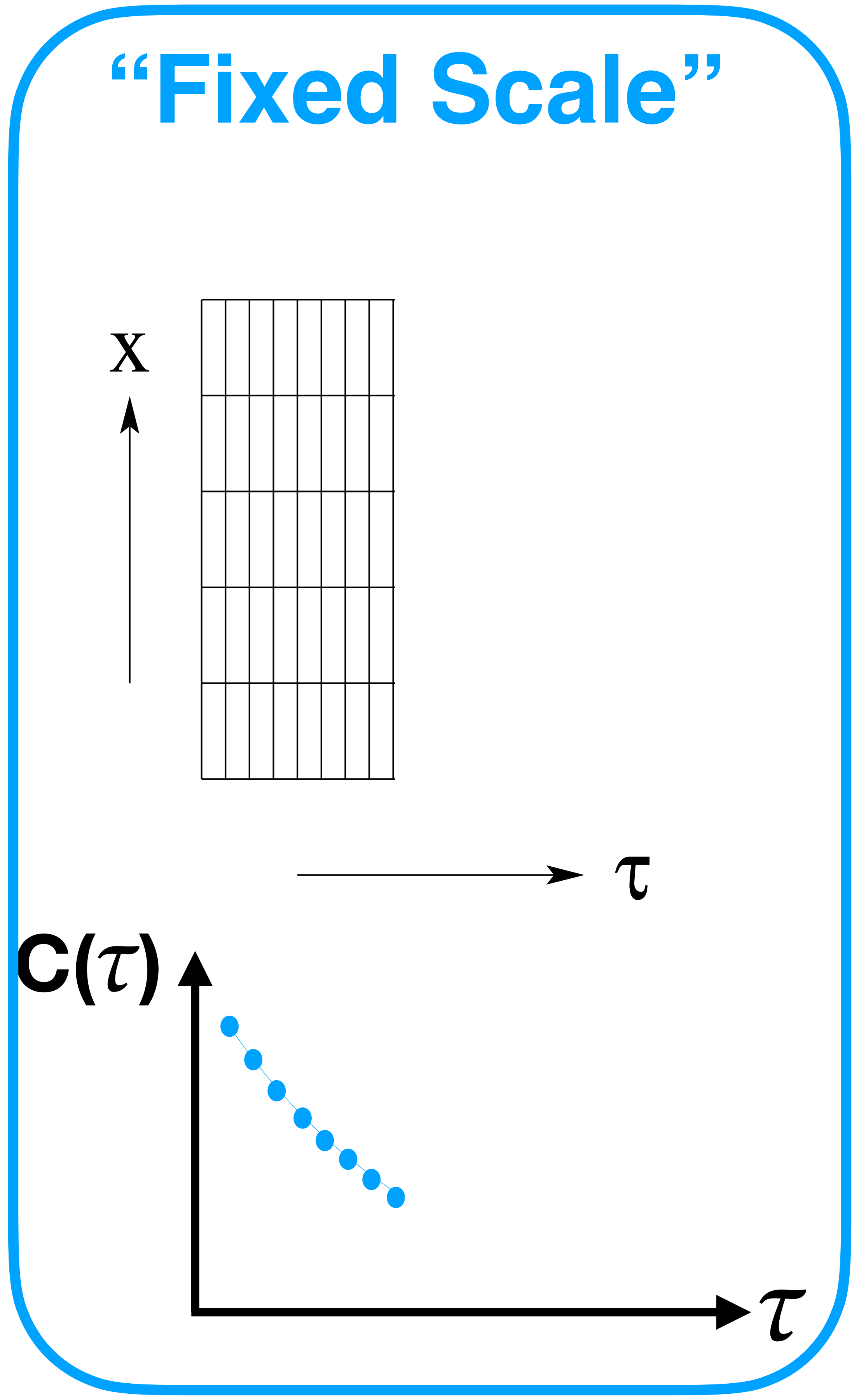


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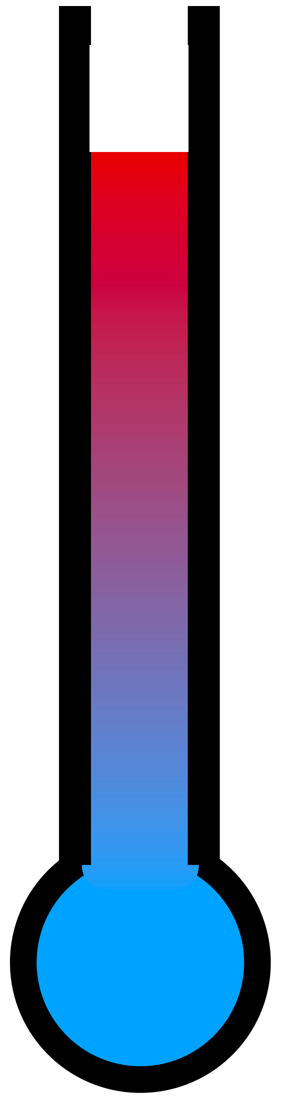

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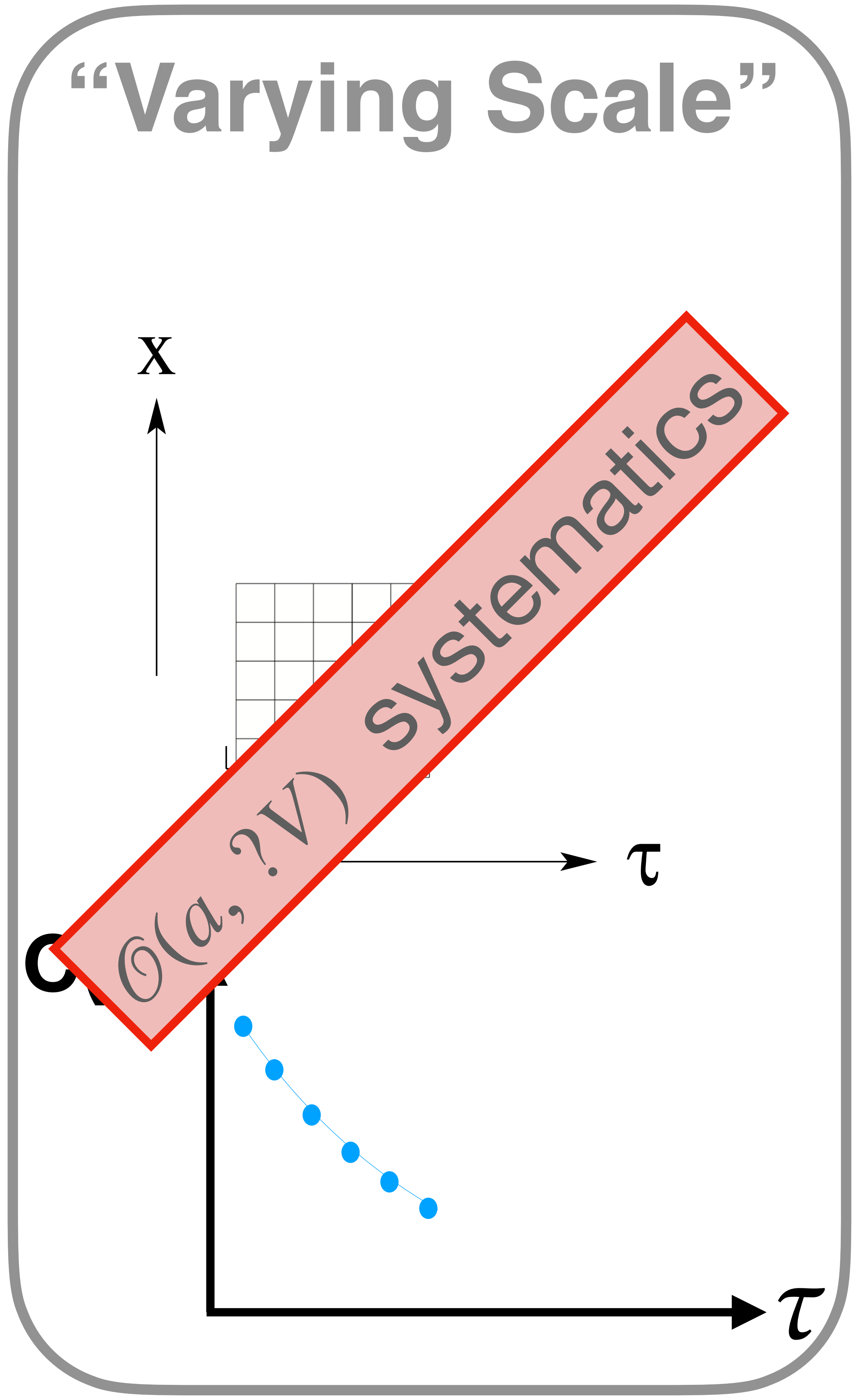
$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

# FASTSUM Approach:

## *Anisotropic* Lattice

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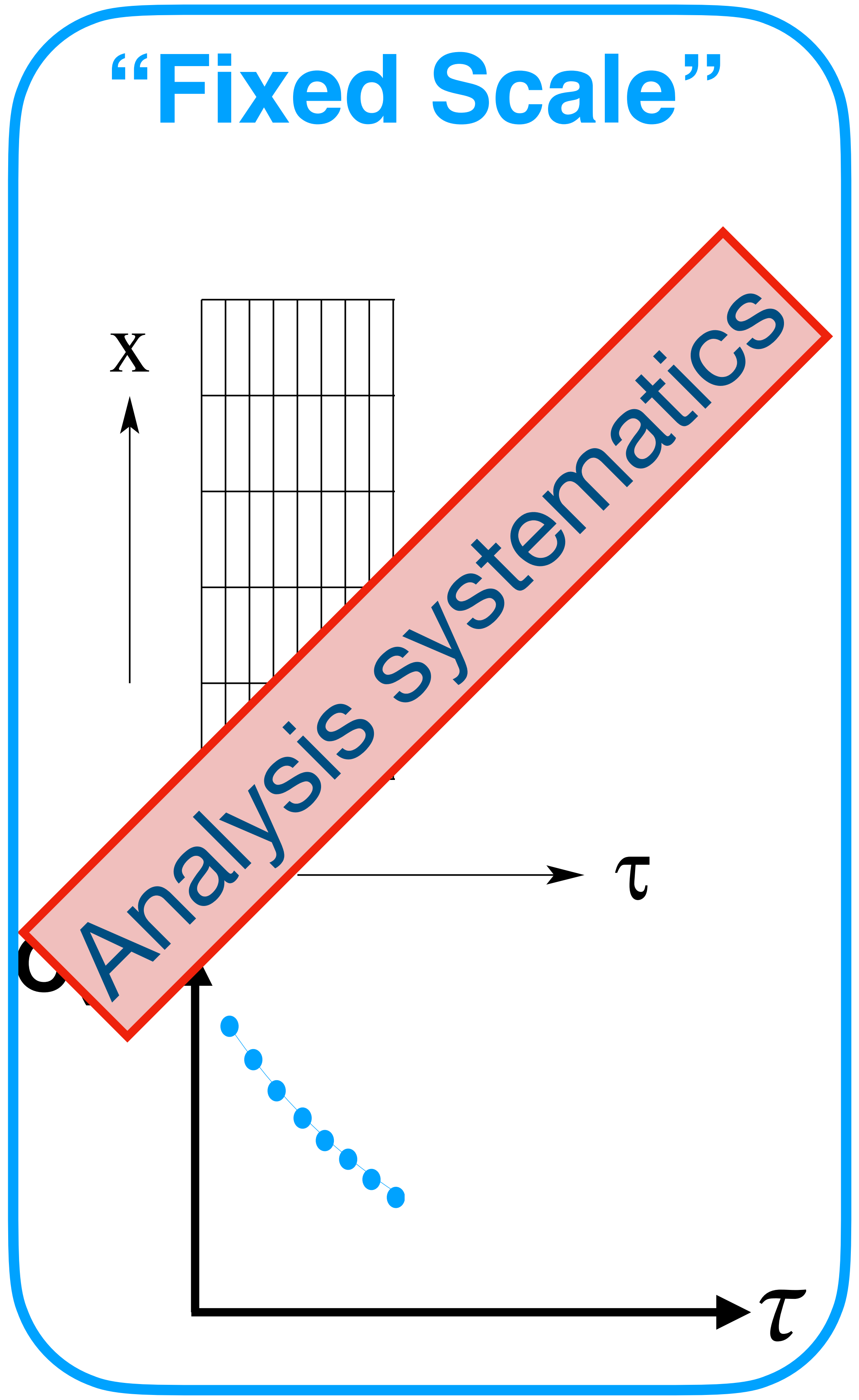


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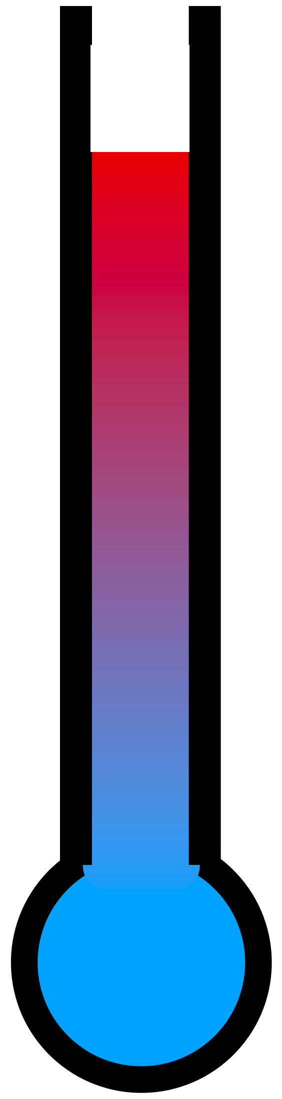
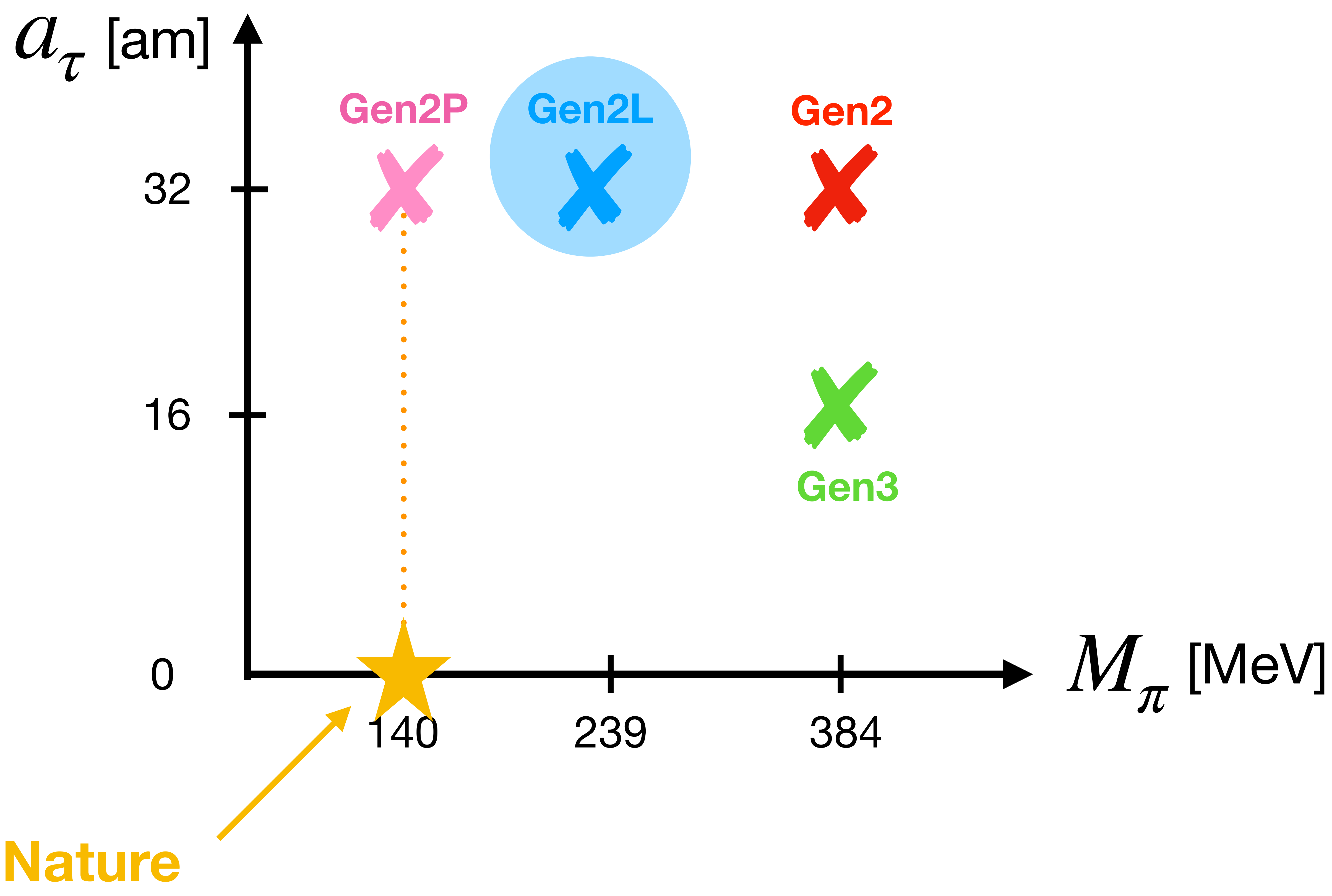
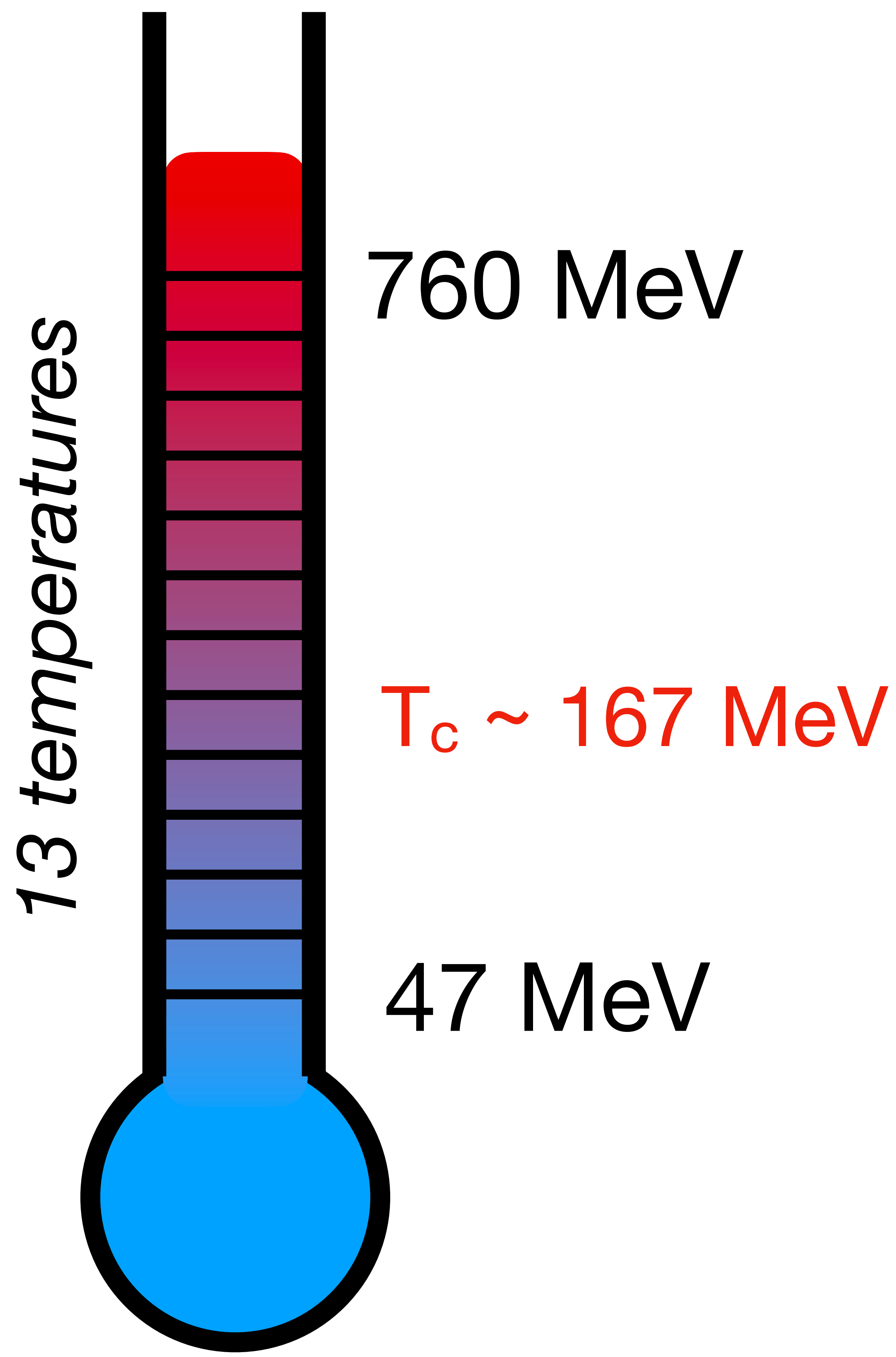


**Going hotter…**

$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

# FASTSUM Approach:
## Lattice Parameters




Generation 2L
(2+1) flavour
$a_s$ ~ 0.112 fm

**Gauge Action:**
Anisotropic,
Symanzik-improved

**Fermion Action:**
Wilson-clover,
tree-level tadpole,
stout-smeared links

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# Charmed Mesons: $D_{(s)}$ and $D^*_{(s)}$

Sergio Chaves

arXiv: 2209.14681

- Not studied at T ≠ 0 before (on lattice)

  (Open Charm)

- Confined phase:

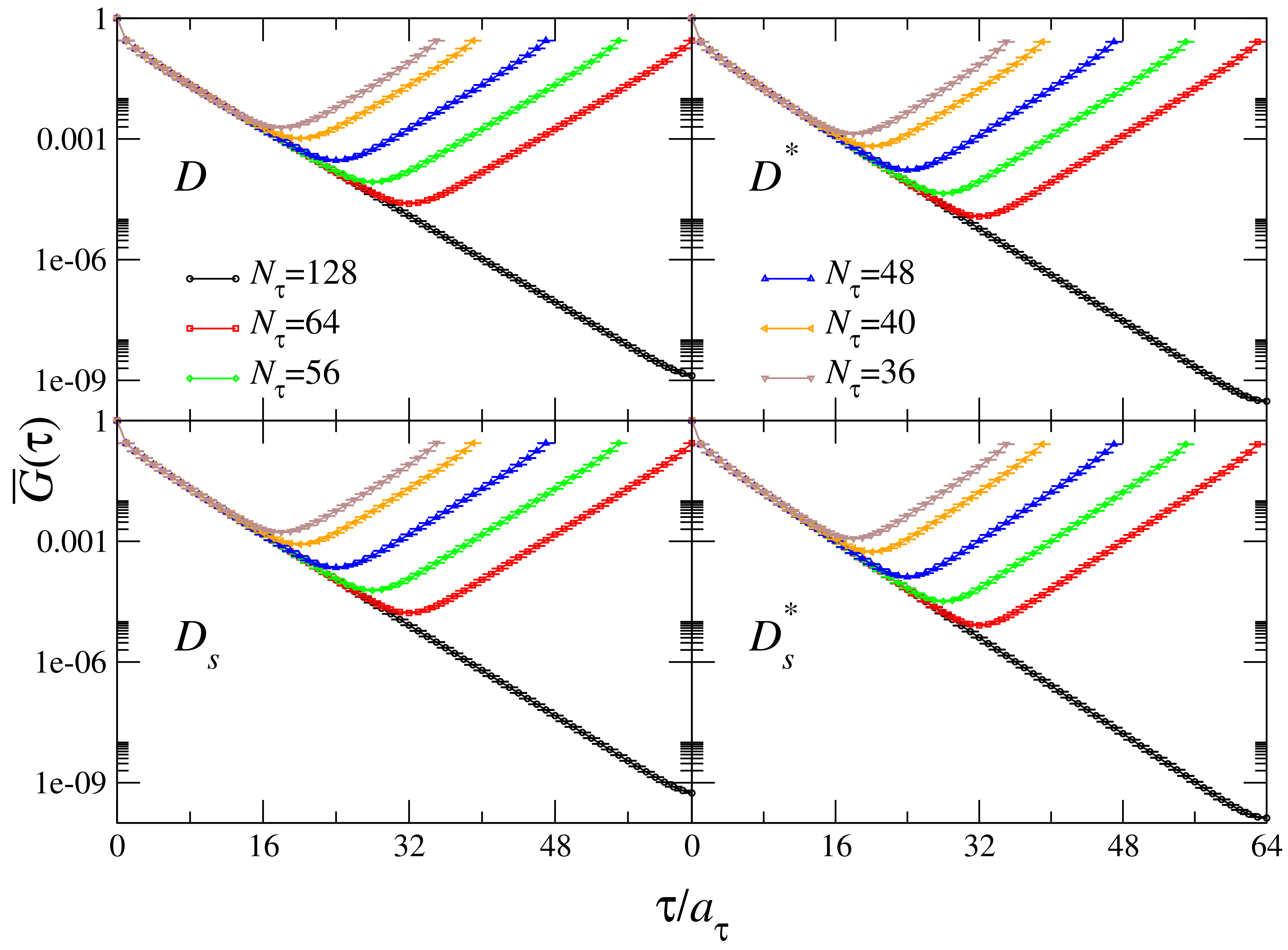
  $$G(\tau) \sim \cosh(-M(\tau - 1/2T))$$

- Periodic for all T:

  $$G(1/T - \tau) = G(\tau)$$

T=0

| | | $J^P$ | PDG [MeV] | $M$ [MeV] |
|---|---|---|---|---|
| $D$ | pseudoscalar | $0^-$ | 1869.65(5) | 1876(4) |
| $D^*$ | vector | $1^-$ | 2010.26(5) | 2001(4) |
| $D_0^*$ | scalar | $0^+$ | 2300(19) | 2222(10) |
| $D_1$ | axial-vector | $1^+$ | 2420.8(5) | 2325(43) |
| $D_s$ | pseudoscalar | $0^-$ | 1968.34(7) | 1972(5) |
| $D_s^*$ | vector | $1^-$ | 2112.2(4) | 2092(4) |
| $D_{s0}^*$ | scalar | $0^+$ | 2317.8(5) | 2115(29) |
| $D_{s1}$ | axial-vector | $1^+$ | 2459.5(6) | 2512(6) |



# Studying Thermal Effects

We use a 2-step procedure

Dominant behaviour is **ground state**
Use model for this
(confined phase):

Kernel Spectrum Spectrum Kernel

$$G_{\text{model}}(\tau; T, T_0) = Z\frac{\cosh[M(T_0)(\tau - 1/2T)]}{\sinh[M(T_0)/2T]}$$

Divide correlation f’n by model

$$R(\tau; T, T_0) = \frac{G(\tau; T)}{G_{\text{model}}(\tau; T, T_0)}$$

This is a constant as ($\tau \to \infty$)
if ground state has mass $M(T_0)$

Can now compare 2 temps
by taking ratio-of-ratios:

$$RoR(\tau; T, T_0) = \frac{R(\tau; T, T_0)}{R(\tau; T_0, T_0)}$$

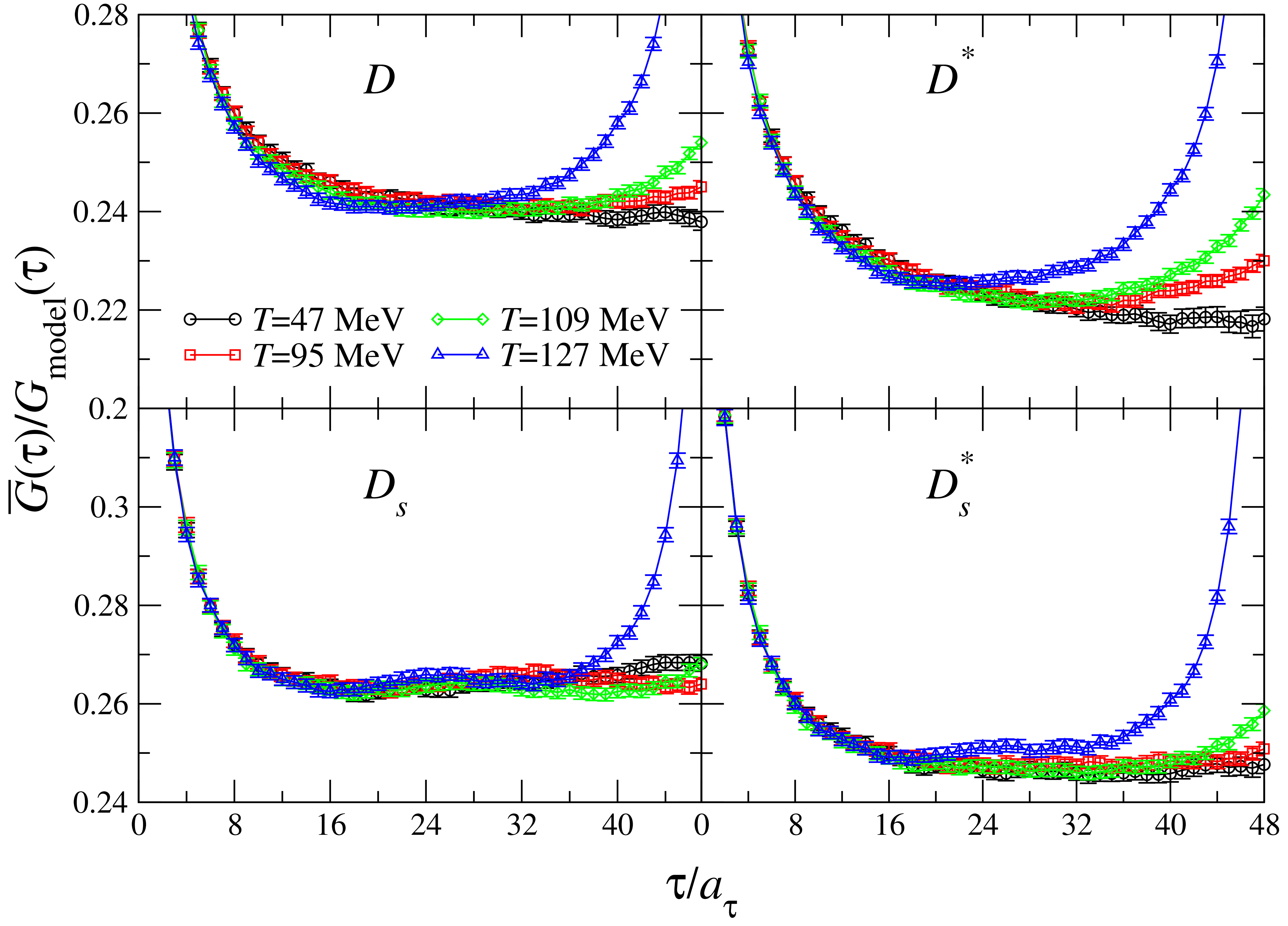
This is a unity (as $\tau \to \infty$) when $T$ and $T_0$
have same ground state mass $M(T_0)$

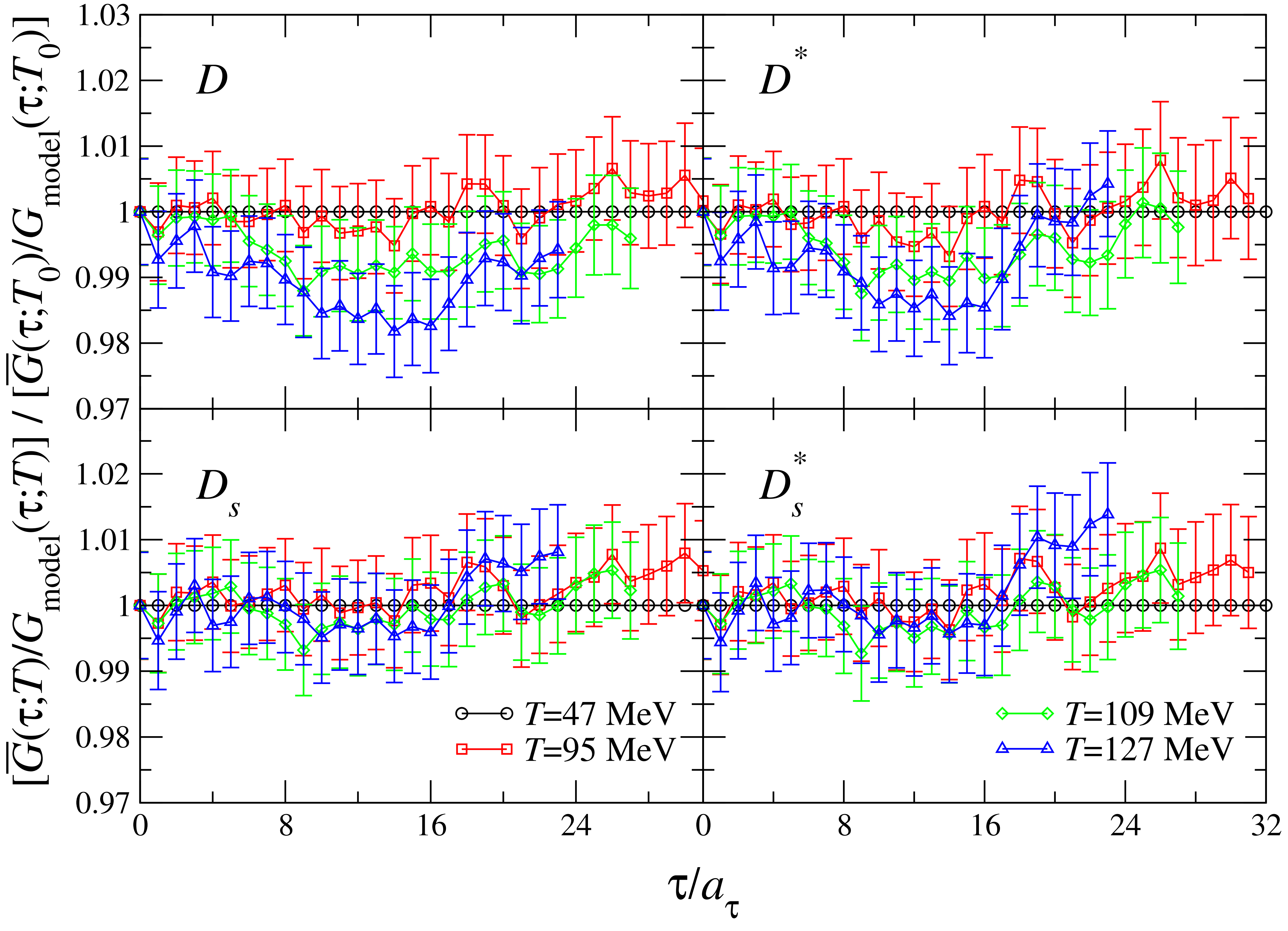
# $D_{(s)}$ and $D^*_{(s)}$ $T \leq 127$ MeV

$$R(\tau; T, T_0) = \frac{G(\tau; T)}{G_{\mathrm{model}}(\tau; T, T_0)}$$

$T_0 = 47$ MeV

$$RoR(\tau; T, T_0) = \frac{R(\tau; T, T_0)}{R(\tau; T_0, T_0)}$$



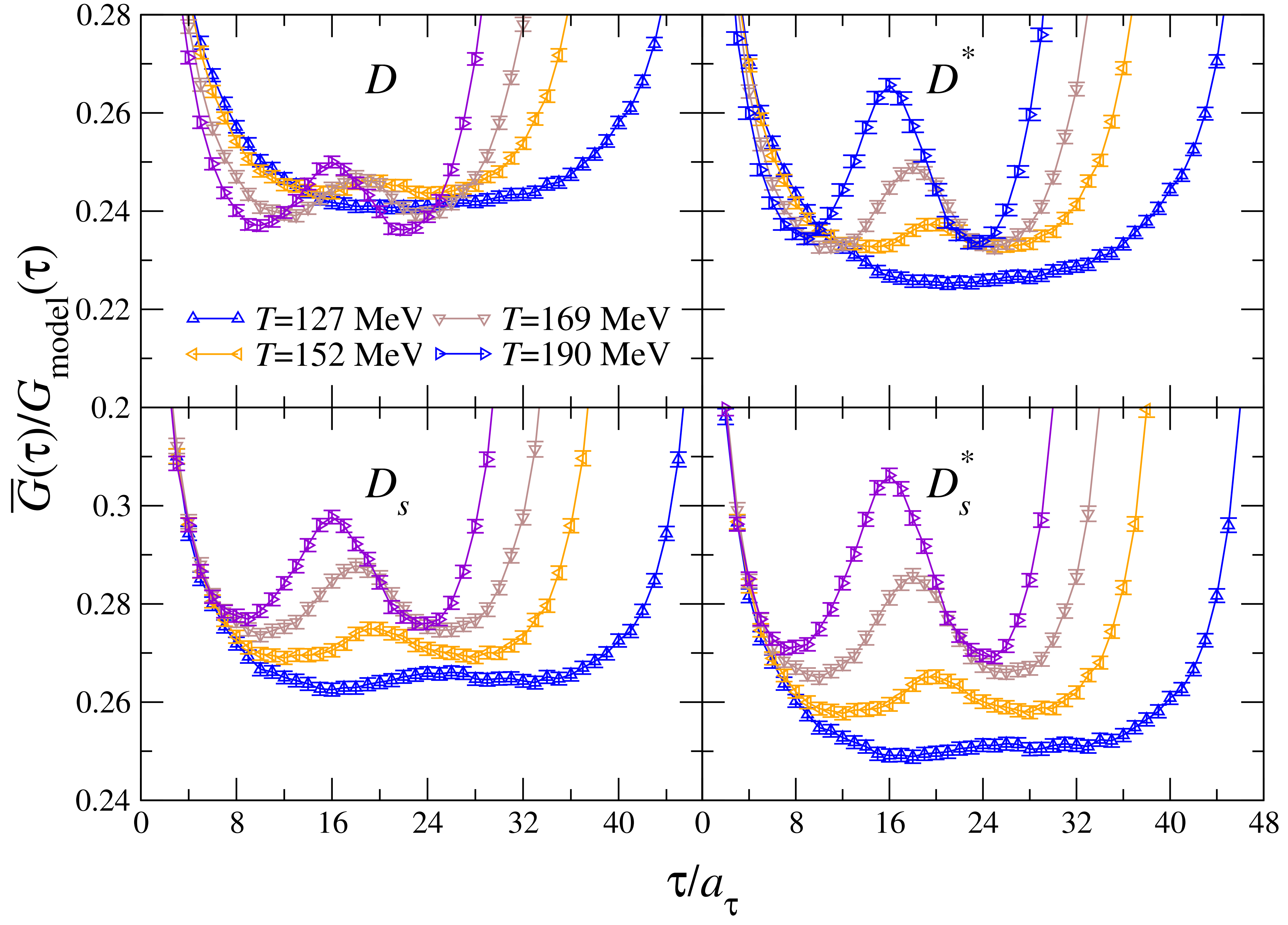



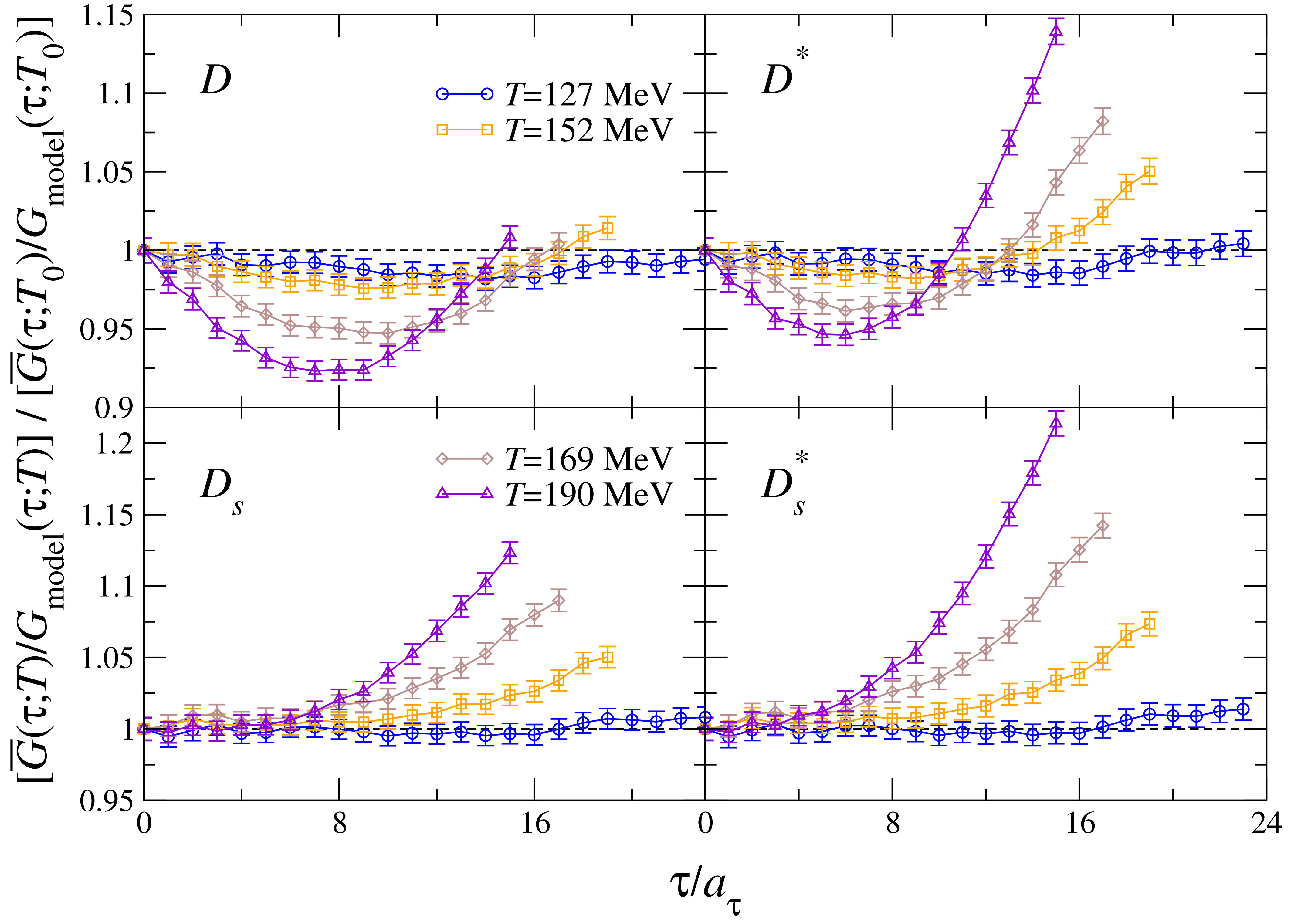

**No temperature dependence**

# $D_{(s)}$ and $D^*_{(s)}$   $127 \leq T \leq 190$ MeV

$$R(\tau; T, T_0) = \frac{G(\tau; T)}{G_{\text{model}}(\tau; T, T_0)}$$

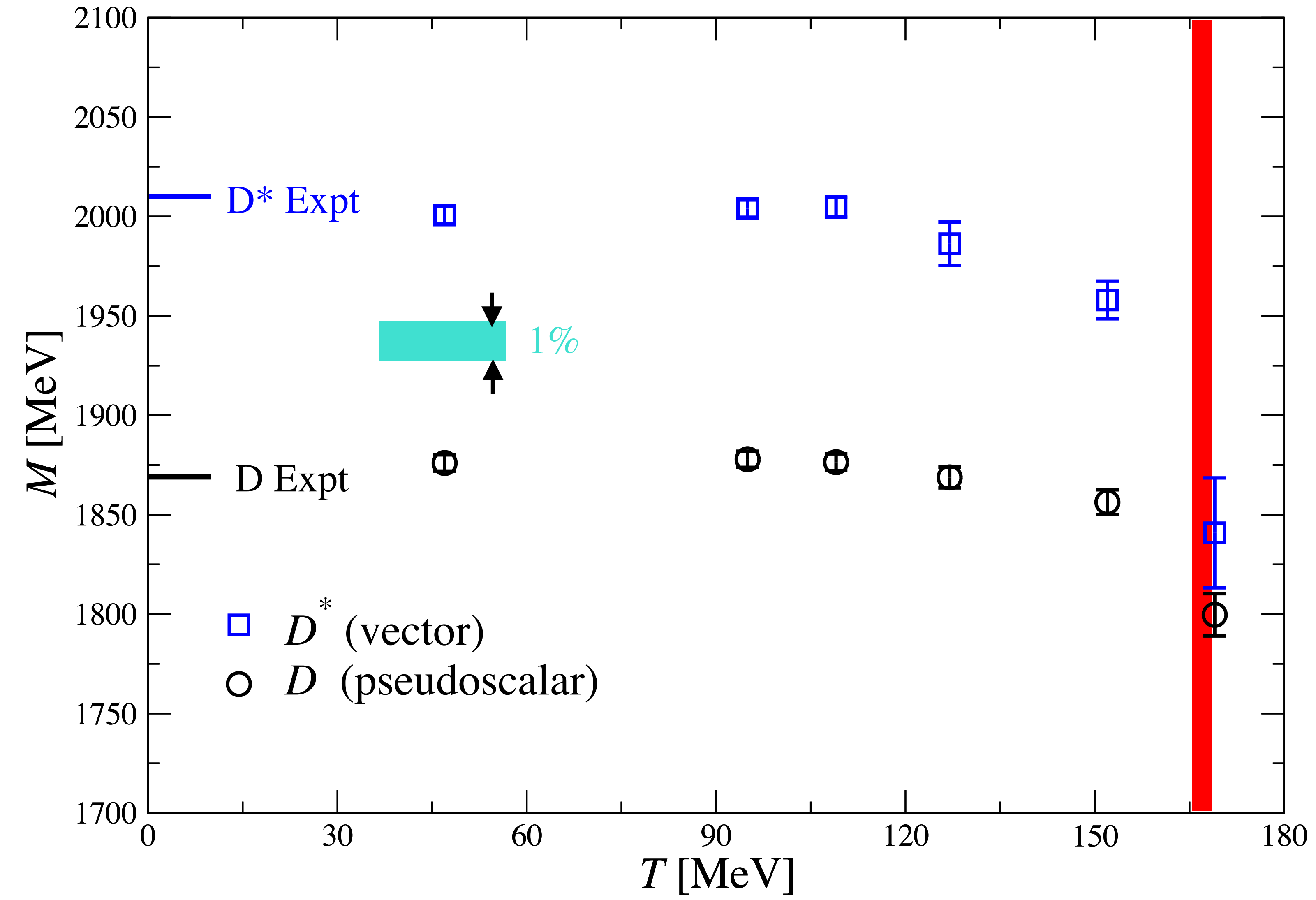
$$RoR(\tau; T, T_0) = \frac{R(\tau; T, T_0)}{R(\tau; T_0, T_0)}$$

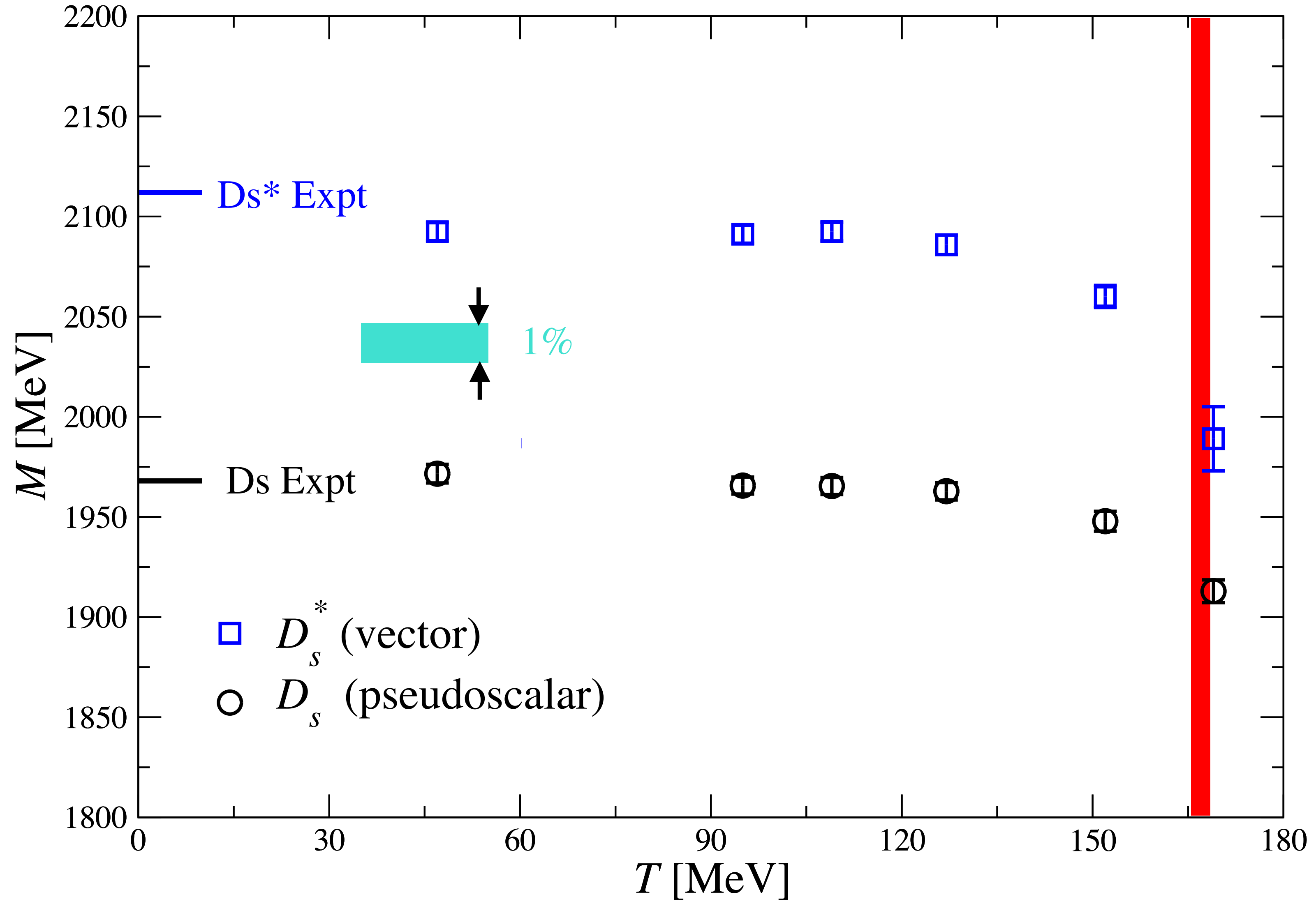



**Clear temperature dependence**

# $D_{(s)}$ and $D^*_{(s)}$ masses




- Ratio-of-ratio shows no temperature dependenceup to $T \sim 127$ MeV
- Temperature dependence clearly visible at $T \sim 152$ MeV
- Results for mass have 5MeV accuracy
- Scalar and axial vector channels have clear T effects (not shown)

| | $J^P$ | PDG | $T$[MeV]$=47$ | 95 | 109 | 127 | 152 | 169 |
|---|---|---|---|---|---|---|---|---|
| $D$ | $0^-$ | 1869.65(5) | 1876(4) | 1878(4) | 1876(4) | 1869(5) | 1856(6) | 1800(11) |
| $D^*$ | $1^-$ | 2010.26(5) | 2001(4) | 2004(4) | 2005(5) | 1986(11) | 1958(9) | 1841(28) |
| $D_s$ | $0^-$ | 1968.34(7) | 1972(5) | 1966(4) | 1965(4) | 1963(4) | 1948(5) | 1913(6) |
| $D_s^*$ | $1^-$ | 2112.2(4) | 2092(4) | 2091(5) | 2092(5) | 2086(5) | 2060(6) | 1989(16) |

# Overview

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- Open Charm Mesons
- **Charm Baryons**
- Interquark potential in bottomonium
- Spectral Functions

# Parity in the Baryonic Spectrum

Ryan Bignell

PRD 92 (2015) 014503 [arXiv:1502.03603]
JHEP 06 (2017) 034 [arXiv:1703.09246]
Phys.Rev. D99 (2019) no.7, 074503 [arXiv:1812.07393]
Eur.Phys.J.A 60 (2024) 3, 59 [arXiv: 2308.12207]

No parity doubling in (T=0) Nature:

+ve parity: $m_+ = m_N = 0.939$ GeV
-ve parity: $m_- = m_{N^*} = 1.535$ GeV

**Question:** What happens as T increases?

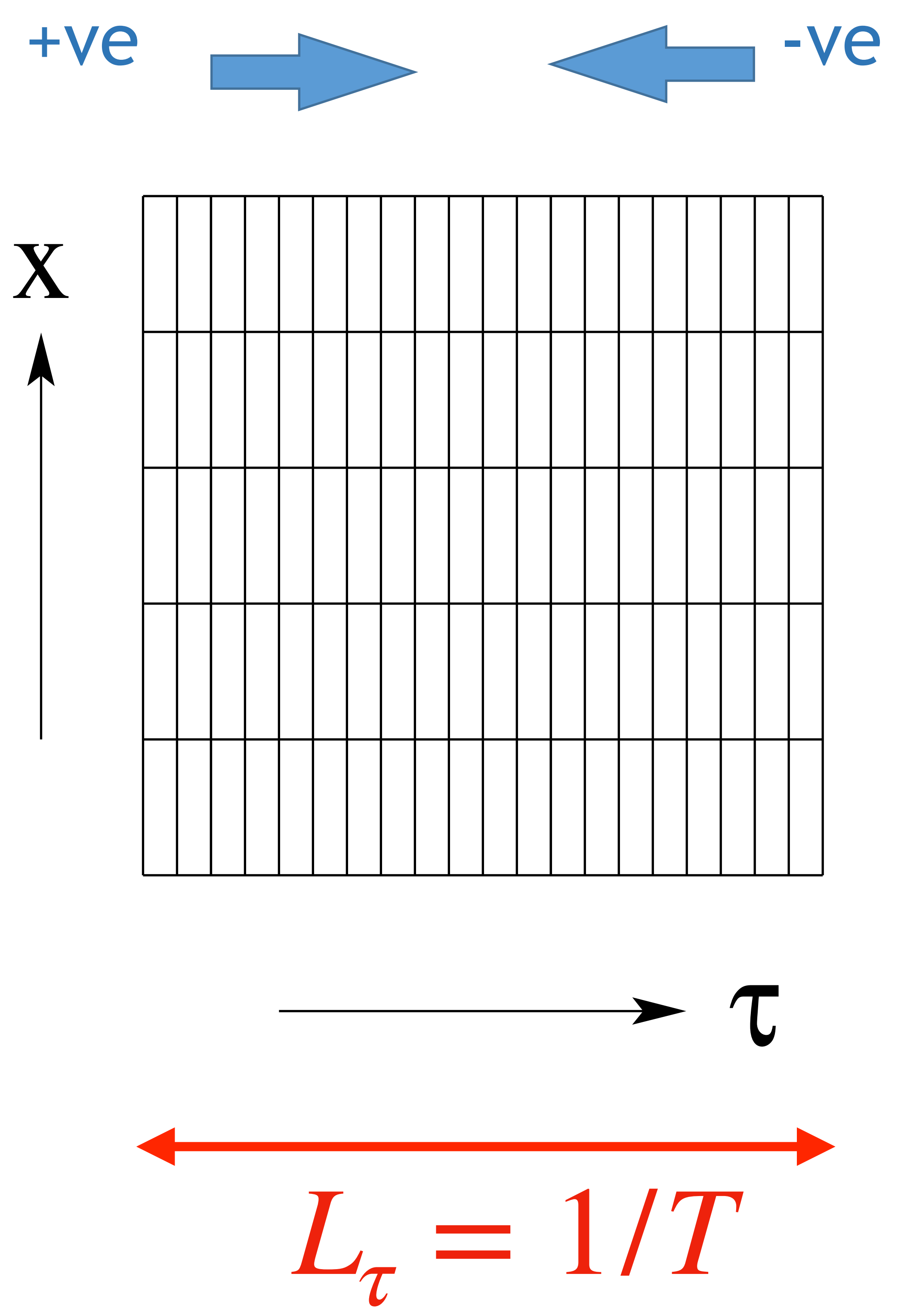
**Lattice:** Parity operation: $P\mathcal{O}(\tau,\vec{x})P^{-1} = \gamma_4\mathcal{O}(\tau,-\vec{x})$

- Use this to construct correlation f'ns

**Charge conjugation** (zero density): $G_\pm(\tau) = -G_\mp(1/T-\tau)$

**Chiral symmetry:** $G_+(\tau) = -G_-(\tau)$

$\Big\}$ $G_+(\tau) = G_+(1/T-\tau)$



# Results — "Reconstructed" Correlators

$$G(\tau; T) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K_F(\tau, \omega; T)\rho(\omega)$$ where the *fermonic* kernel is: $$K_F(\tau, \omega; T) = \frac{e^{-\omega T}}{1 + e^{-\omega/T}}$$

Following: H. T. Ding et al, Phys. Rev. D 86 (2012) 014509, [arXiv:1204.4945]

we write $1 + e^{-\omega m N_\tau} = (1 + e^{-\omega N_\tau}) \sum_{n=0}^{m-1} (-1)^n e^{-n\omega N_\tau}$ where $N_0 = m\, N_\tau$ and $m$ is odd

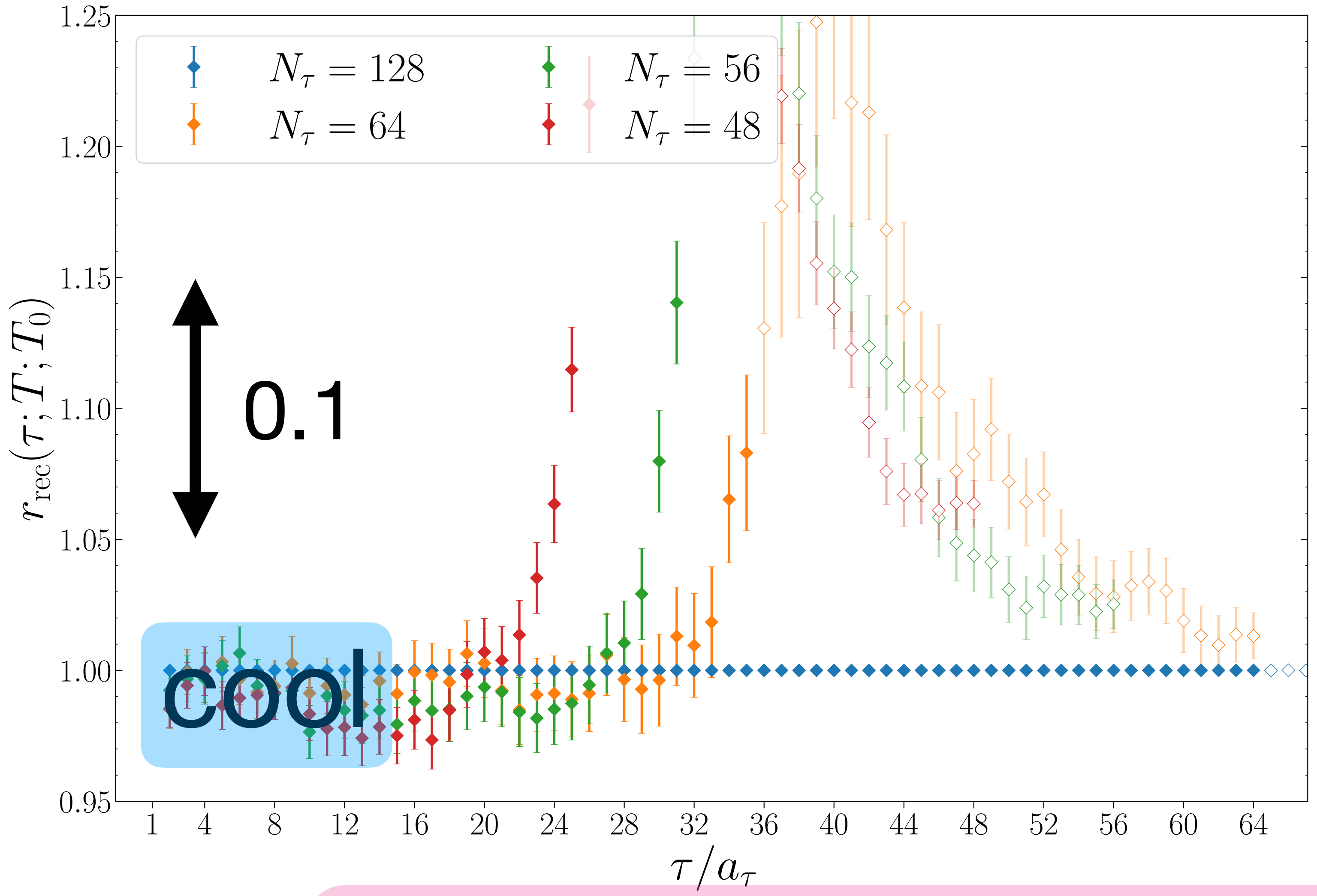
$$K_F(\tau, \omega; 1/N_\tau) = \frac{e^{-\omega\tau}}{1 + e^{-\omega N_\tau}} = \sum_{n=0}^{m-1} (-1)^n \frac{e^{-\omega(\tau + nN_\tau)}}{1 + e^{-\omega m N_\tau}} = \sum_{n=0}^{m-1} (-1)^n K_F(\tau + nN_\tau, \omega; 1/(mN_\tau))$$

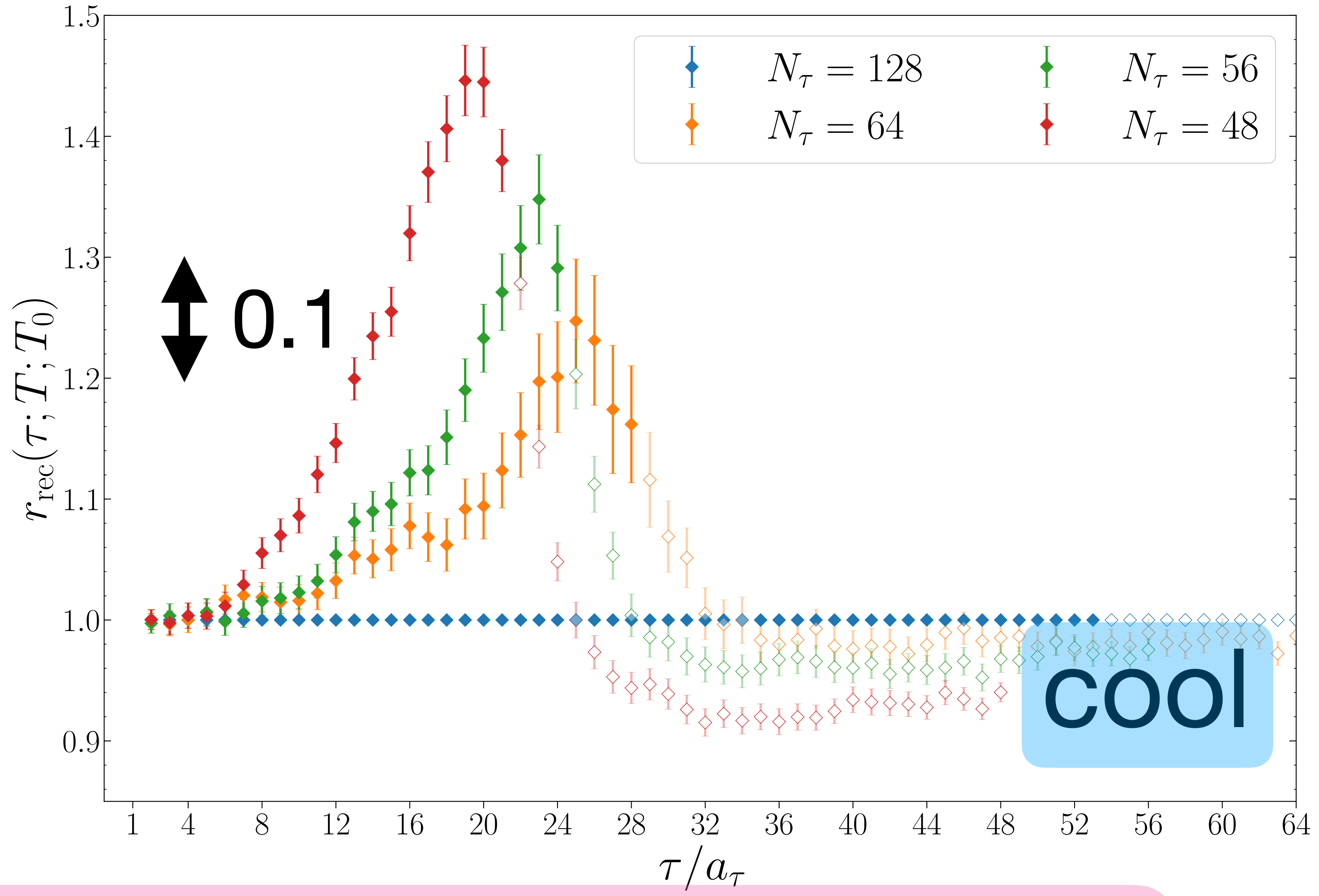
Suppose $\rho(\omega)$ was indept of $T$ : $$G_{\text{rec}}(\tau;\, 1/N_\tau;\, 1/N_0) = \sum_{n=0}^{m-1} (-1)^n G(\tau + nN_\tau;\, 1/N_0)$$

# Results - “Reconstructed” ratio: $G_{\text{rec}}/G$ $\Sigma_c\,(udc)$

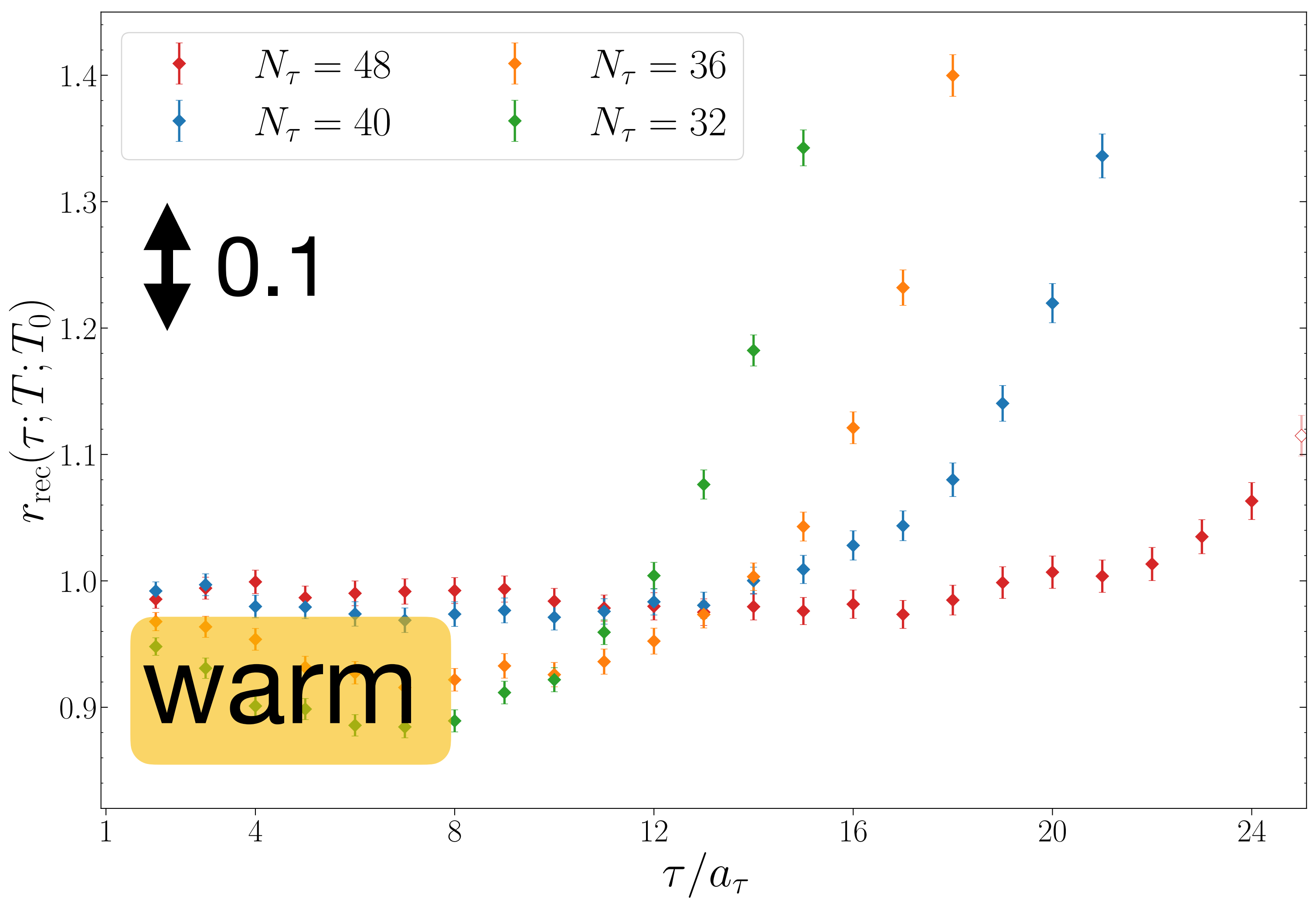
+ve parity

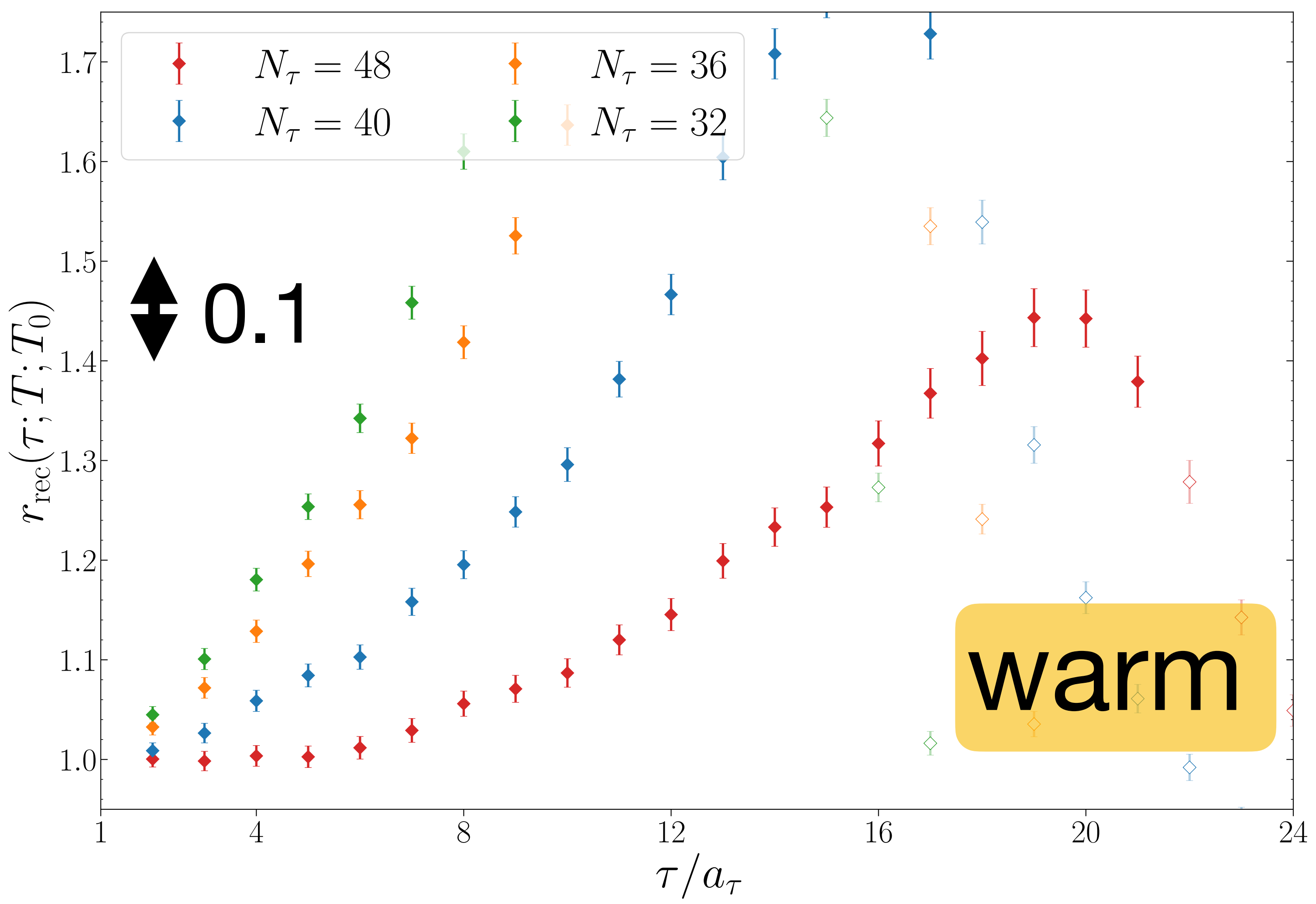
-ve parity



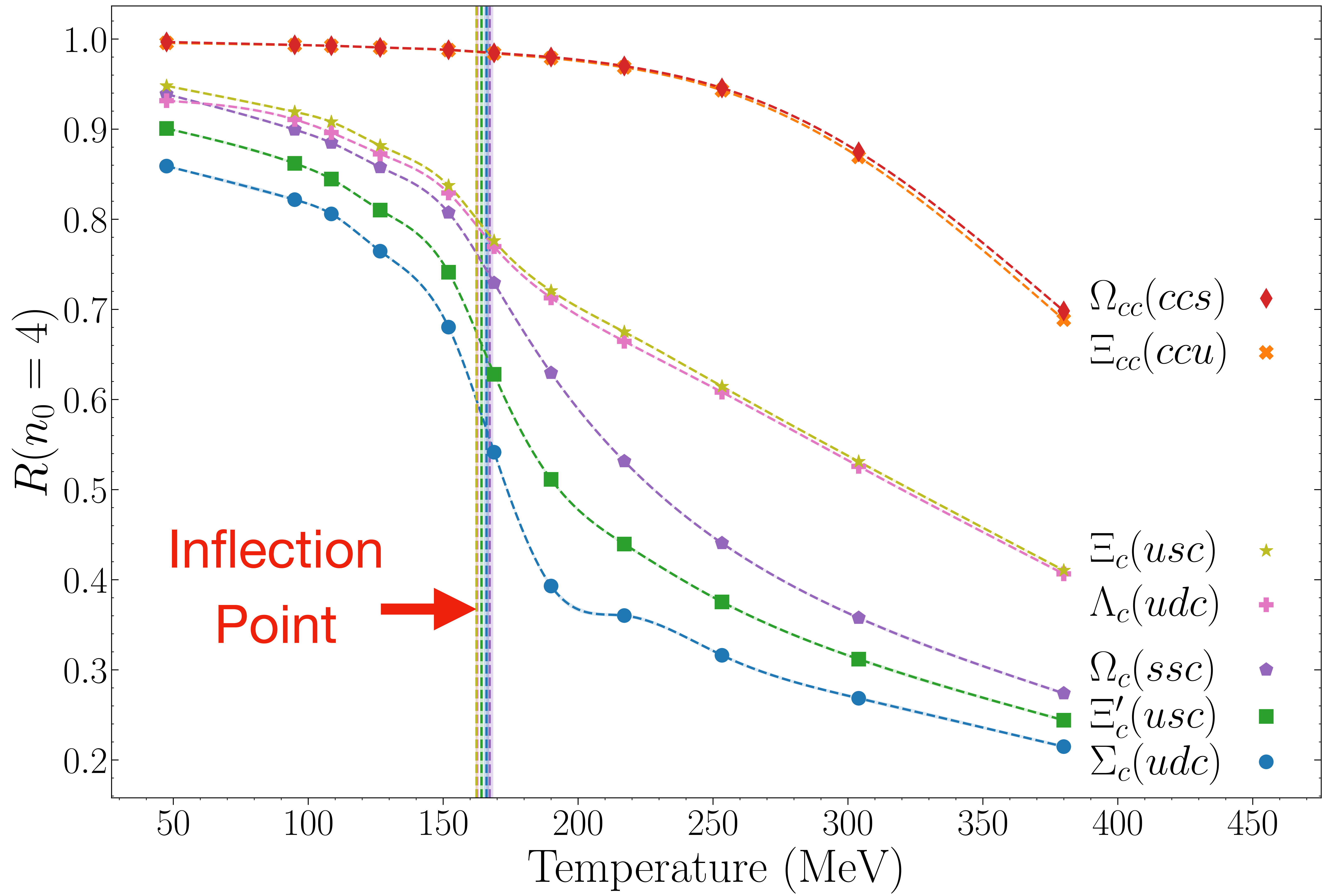




+ve parity sector less thermally sensitive than -ve parity



# Parity doubling in the correlators




$$R(\tau) = \frac{G_+(\tau) - G_+(1/T - \tau)}{G_+(\tau) + G_+(1/T - \tau)}$$

Parity doubling:

$G_+ = G_- \rightarrow R(\tau) \sim 0$

Parity max broken:

$G_+ \gg G_- \rightarrow R(\tau) \sim 1$

$$R = \frac{\sum_\tau R(\tau)/\sigma^2(\tau)}{\sum_\tau 1/\sigma^2(\tau)}$$

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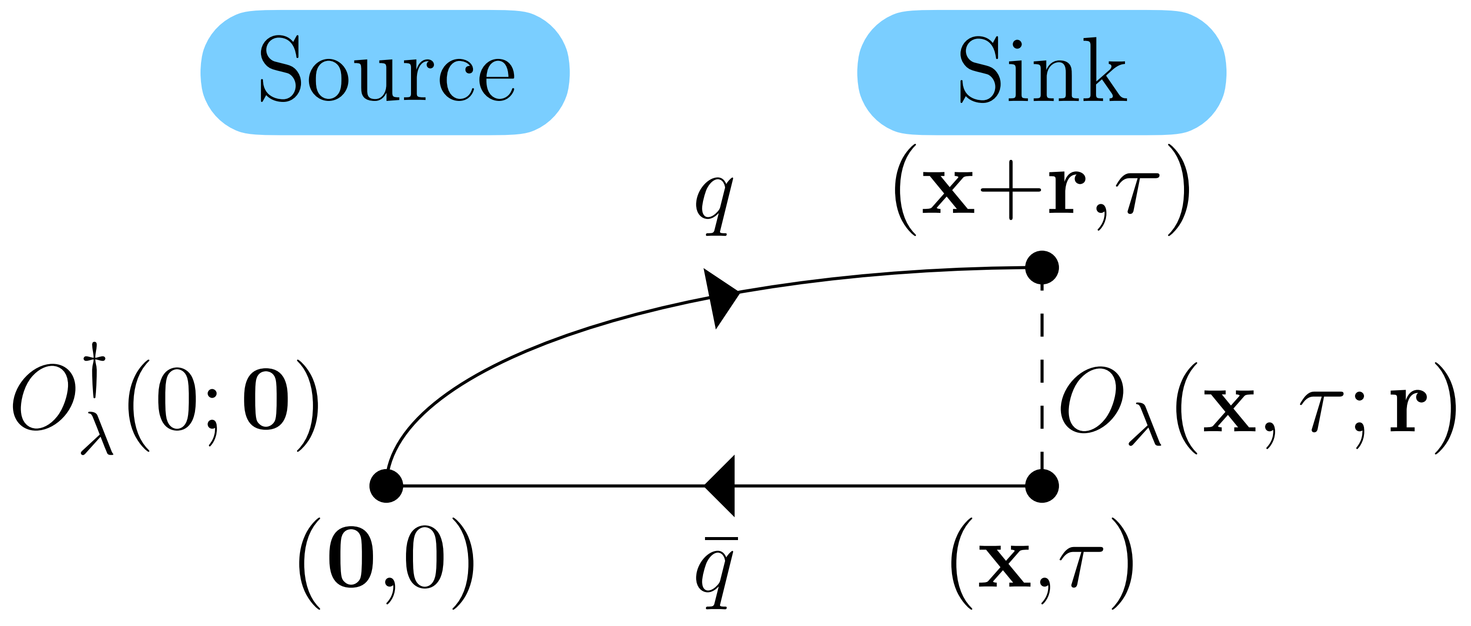
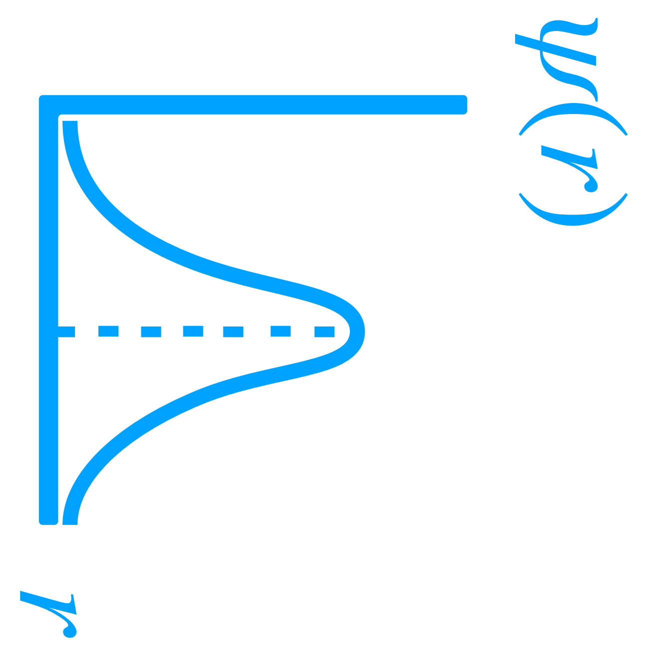
# Interquark Potential in a Meson

## HAL-QCD Method

Correlation F'n Considered, $C(\tau; r)$:




Schrödinger Equation: $H|\psi\rangle = E|\psi\rangle$

$$\left(-\frac{\nabla^2}{2\mu} + V(r)\right)\psi(r) \;=\; E\,\psi(r)$$

$$\left(-\frac{\nabla^2}{2\mu} + V(r)\right)C(\tau;r) \;=\; E\,C(\tau;r)$$

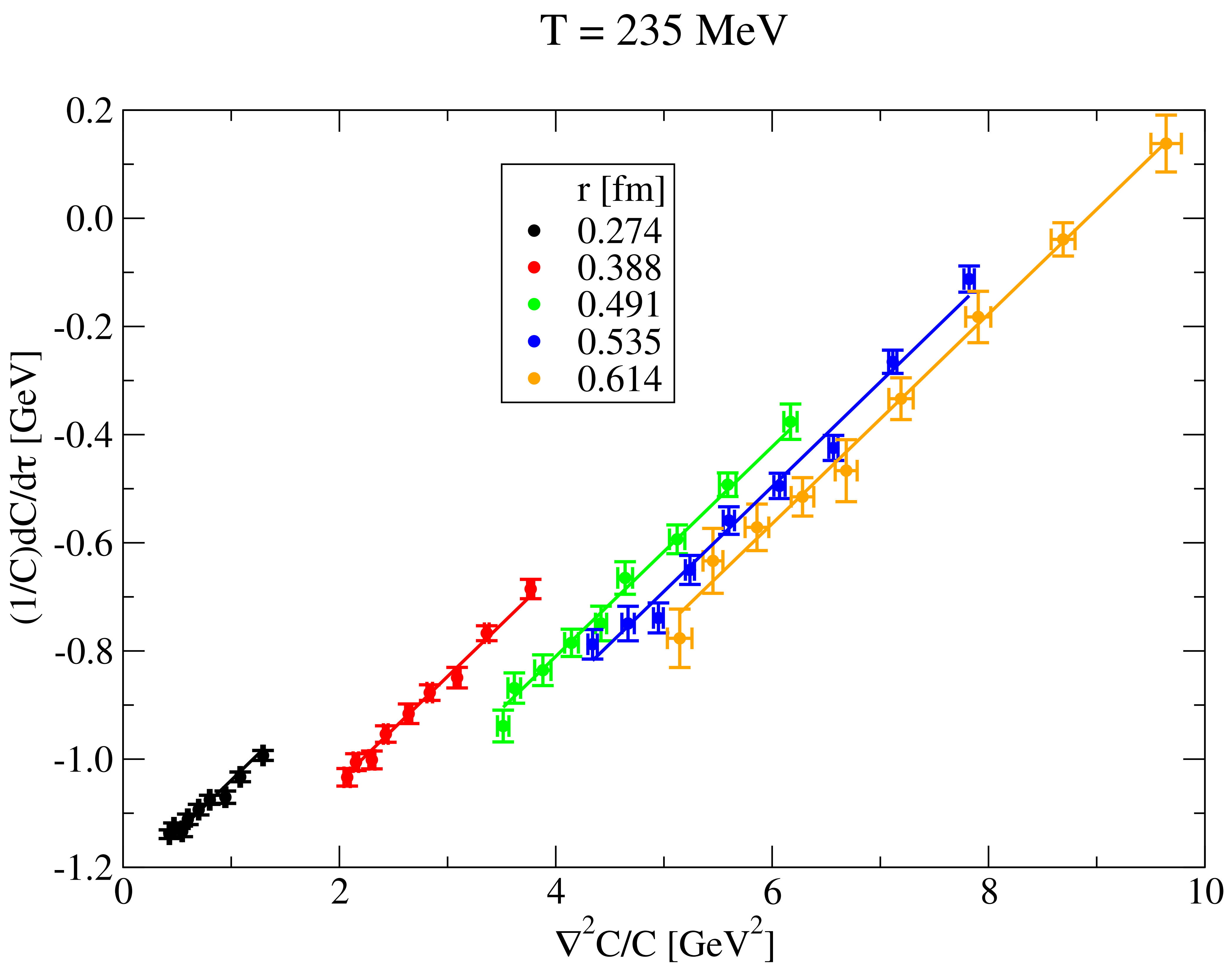
**Output** **Input**

# Linear Regression Method

Tim Burns

$$\left(-\frac{\nabla^2}{2\mu} + V(r)\right) C(\tau; r) = E\, C(\tau; r) \quad \longrightarrow \quad \frac{\partial C(\vec{r},t)}{C(\vec{r},t)} = \frac{1}{2\mu}\frac{\nabla_r^2 C(\vec{r},t)}{C(\vec{r},t)} - V(\vec{r})$$

i.e. it's linear: $y(\vec{r},t) = m(r)\, x(\vec{r},t) + c(r)$

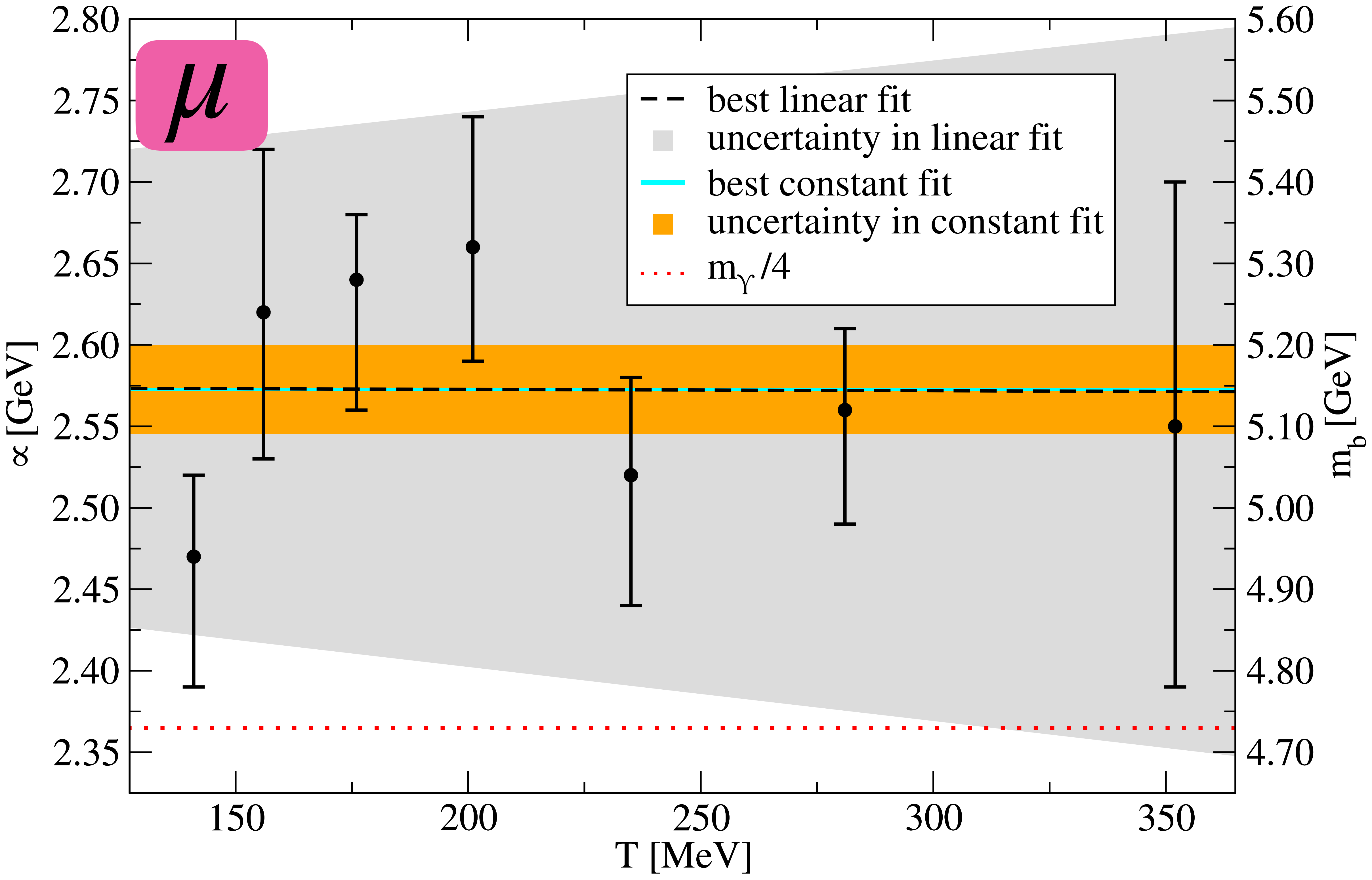


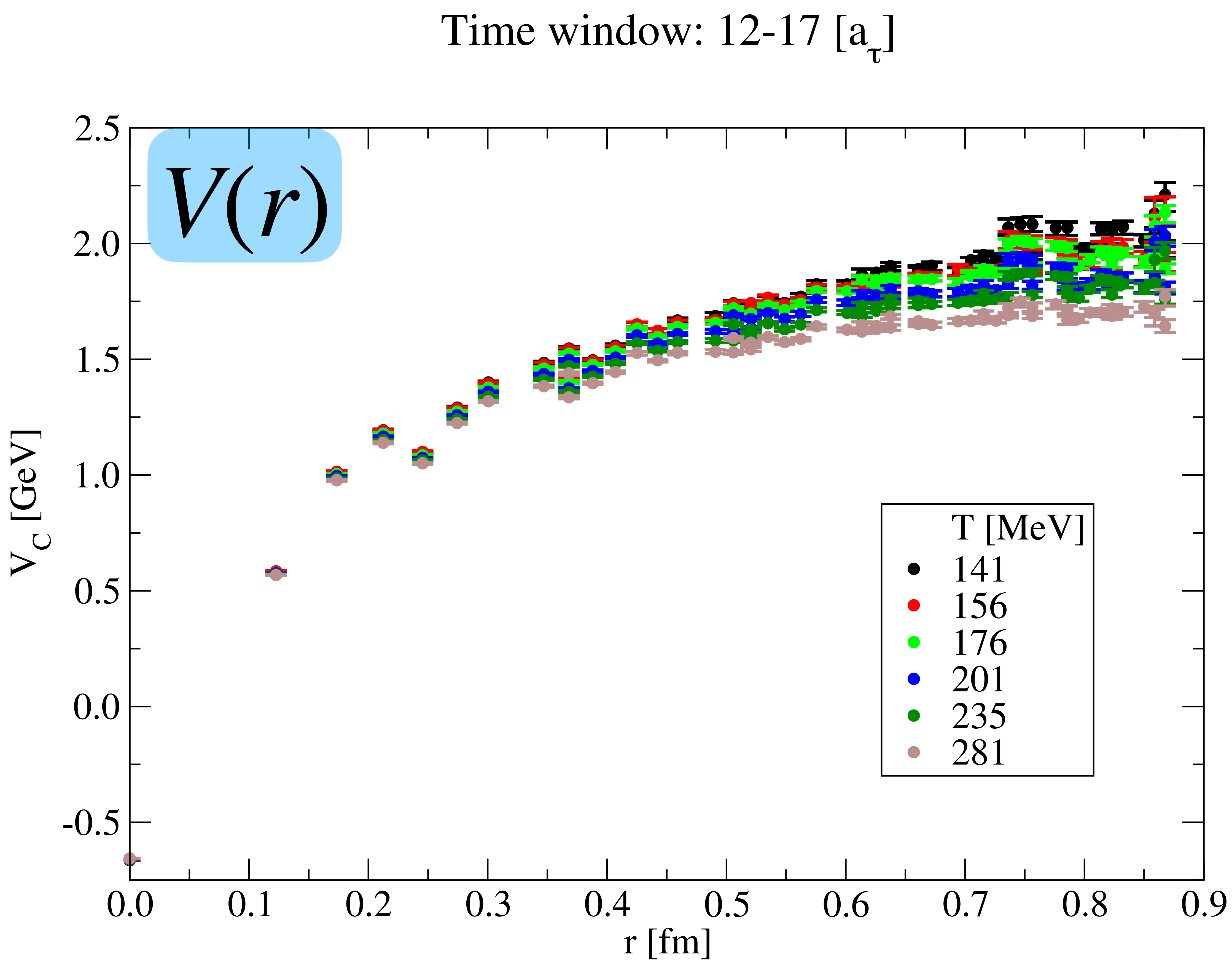
# Effective Mass & Potential in (NRQCD) Bottomonium

Tom Spriggs

*(Preliminary)*

$$\frac{\partial_t C(\vec{r},t)}{C(\vec{r},t)} = \frac{1}{2\mu}\,\frac{\nabla_r^2 C(\vec{r},t)}{C(\vec{r},t)} - V(\vec{r})$$



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# Studying Thermal Effects via Spectral Functions

Correlation Function's Spectral Representation:

$$G(\tau; T) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega; T)\ \rho(\omega; T)$$

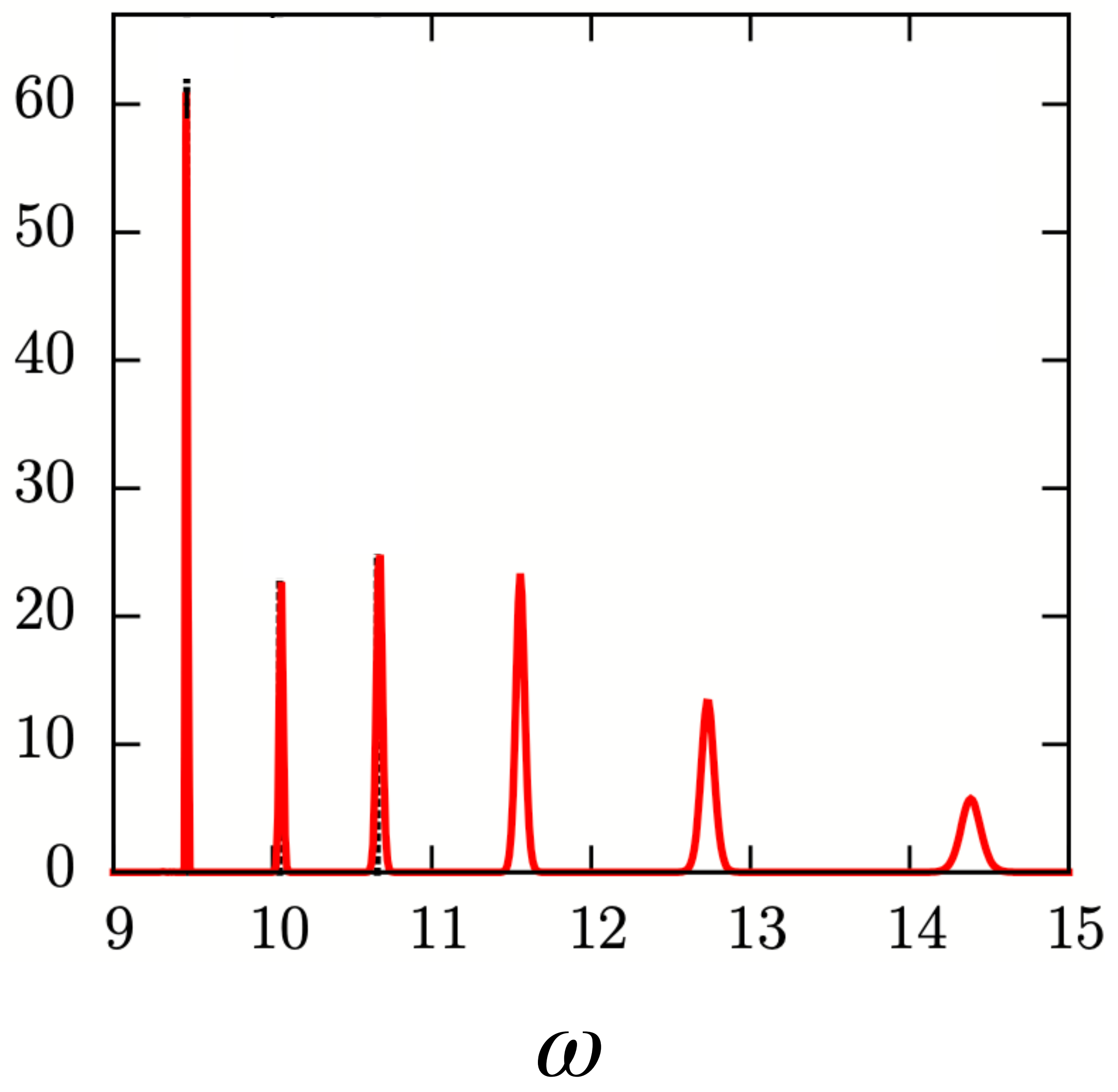
Two sources of Thermal Effects: *Kernel (Geometry / Periodicity)* *Spectral F'n (Physics)*

Kernel:

$$K(\tau, \omega; T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

Spectral F'n:

$\rho(\omega; T)$



# Many Approaches to Extract Spectral Information

1. Exponential (Conventional $\delta$ f'ns)
2. Gaussian Ground State (+ $\delta$ f'n excited)

} Maximum Likelihood

3. Moments of Correlation F'ns

Direct Method - "no" fit

4. Maximum Entropy Method
5. BR Method

} Bayesian Approaches

6. Kernel Ridge Regression

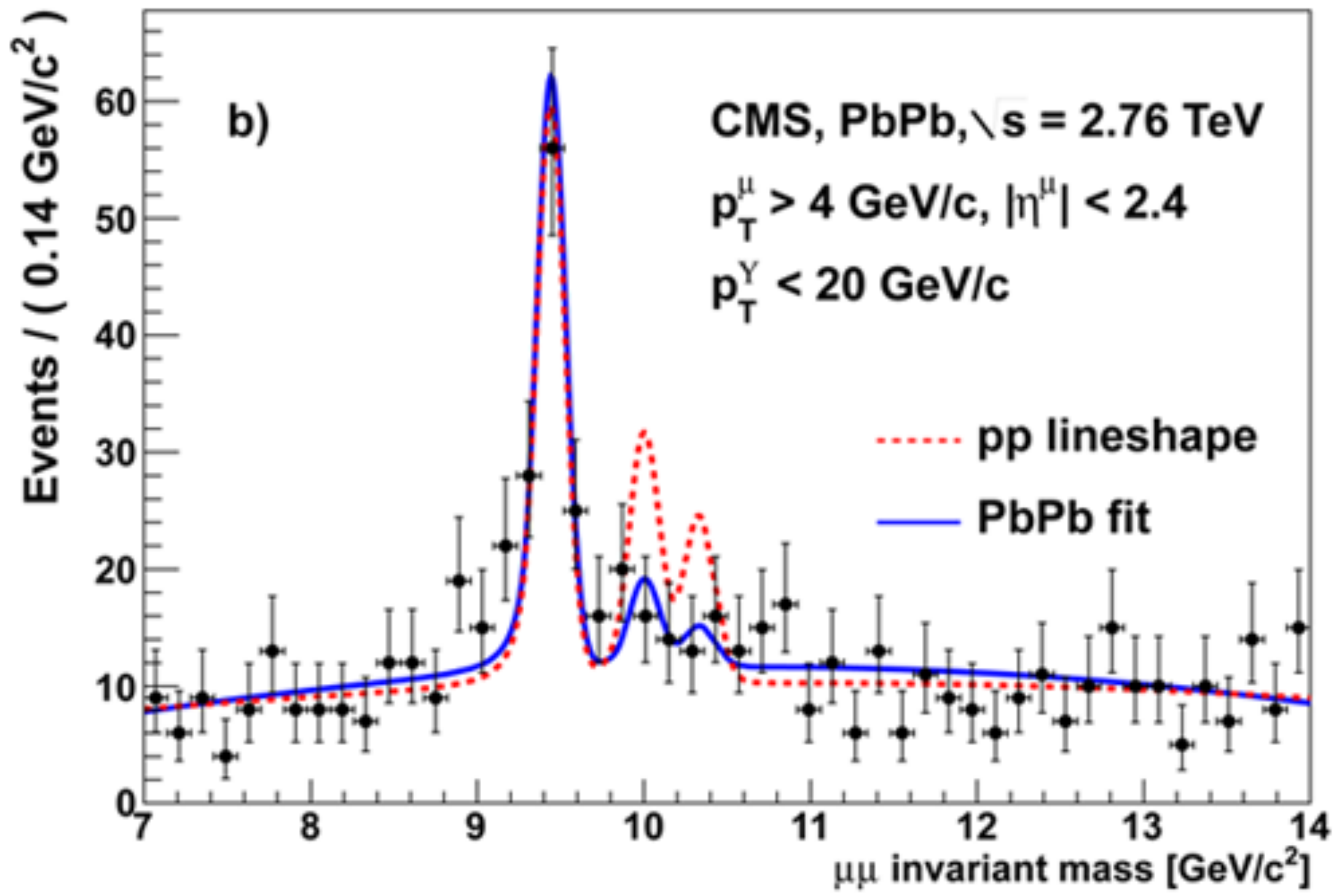
Machine Learning

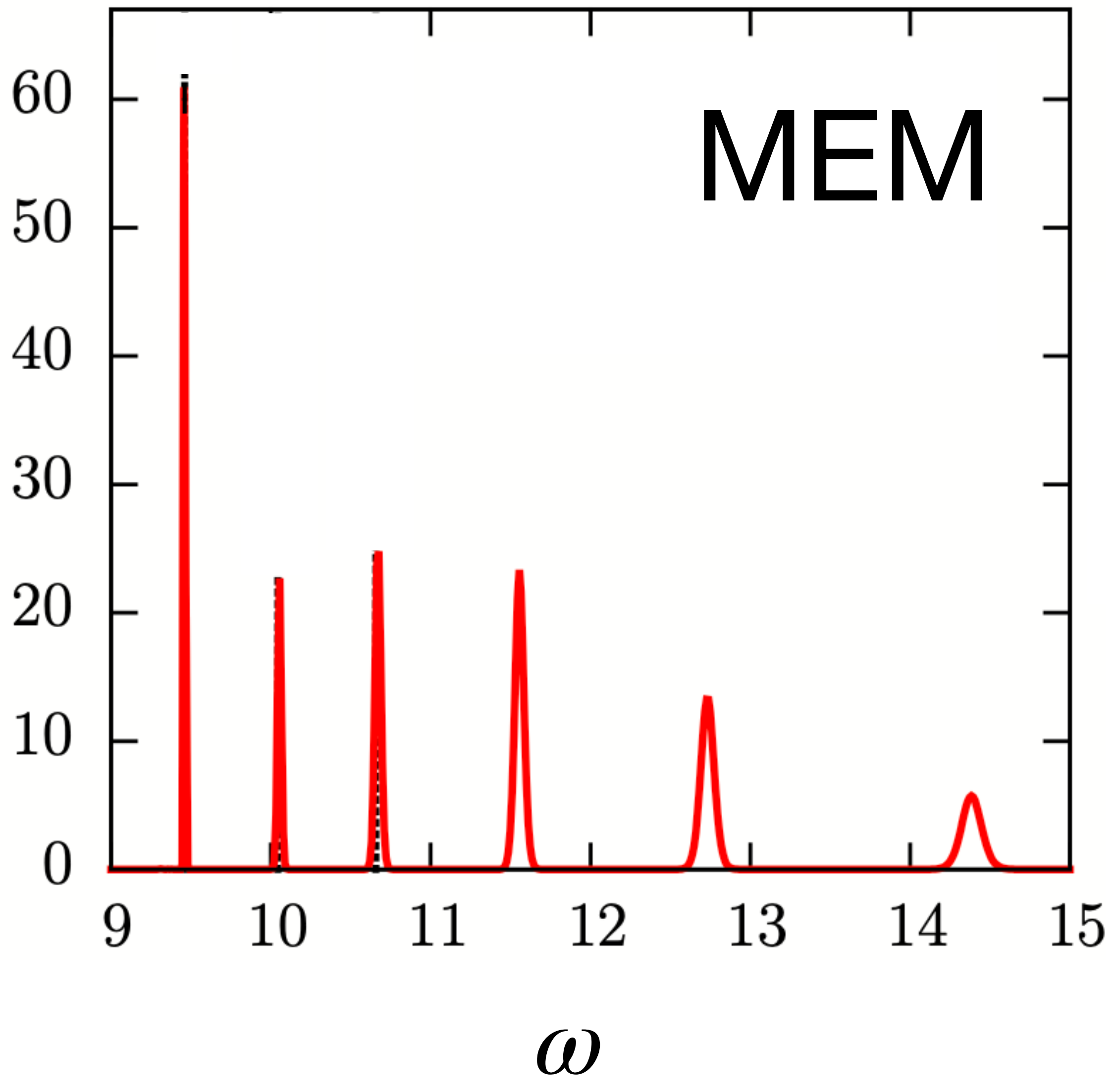
7. Backus Gilbert
8. HLT
9. HMR

Ben Page

Antonio Smecca

} from Geophysics



# Summary

**FASTSUM approach**

- *anisotropic*

**Open Charm Mesons**

- $D_{(s)}$ *and* $D^*_{(s)}$ *have no T dependency below 127 MeV*
- *Scalar and axial vector channels have strong thermal effects*

**Charm Baryons**

- *+ve parity less T dependent than -ve*
- *Signs of approx parity doubling*

**Interquark potential in bottomonium**

- *Thermal effects seen*

**Spectral Functions**

- *Work in progress!*

# Back-Up Slide

# Generation 2L

| $a_\tau$ [am] | $a_\tau^{-1}$ [GeV] | $\xi = a_s/a_\tau$ | $a_s$ [fm] | $m_\pi$ [MeV] | $T_{\rm pc}^{\bar{\psi}\psi}$ [MeV] |
|---|---|---|---|---|---|
| 32.46(7) | 6.079(13) | 3.453(6) | 0.1121(3) | 239(1) | 167(2)(1) |

| Generation 2L, $32^3 \times N_\tau$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $N_\tau$ | 128 | 64 | 56 | 48 | 40 | 36 | 32 | 28 | 24 | 20 |
| $T$ [MeV] | 47 | 95 | 109 | 127 | 152 | 169 | 190 | 217 | 253 | 304 |
| $N_{\rm cfg}$ | 1024 | 1041 | 1042 | 1123 | 1102 | 1119 | 1090 | 1031 | 1016 | 1030 |

$T_c \sim 167$ MeV

$a^{-1} = 6.079(13)$ GeV from HadSpec calculation of **Ω** baryon,

D. J. Wilson, et al., Phys. Rev. Lett. 123 (2019)