Scalar Leptoquarks in Flavour Physics

Svjetlana Fajfer
Institute J. Stefan, Ljubljana and
Physics Department, University of Ljubljana, Slovenia

42nd International Conference on High Energy Physics
Prague, 17 -24 July 2024

Outline

Motivation

B meson puzzles $R_{D^{(*)}}$, $B \to K \nu\nu$

Experimental results
SM predictions

New Physics?

SMEFT

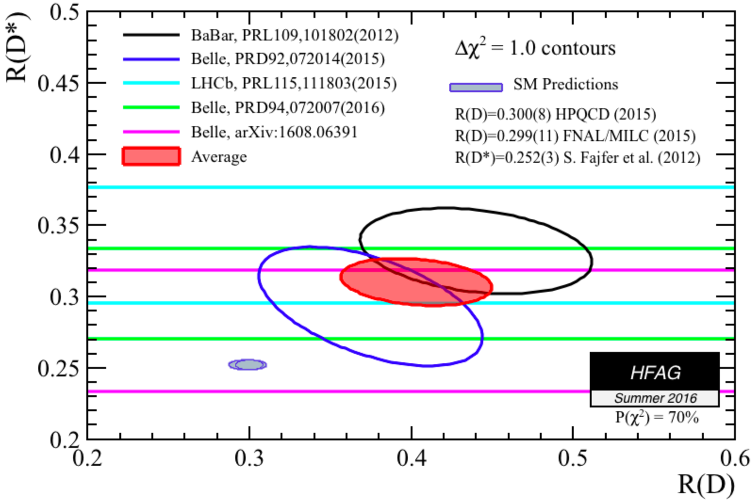
Low-energy and collider constraints

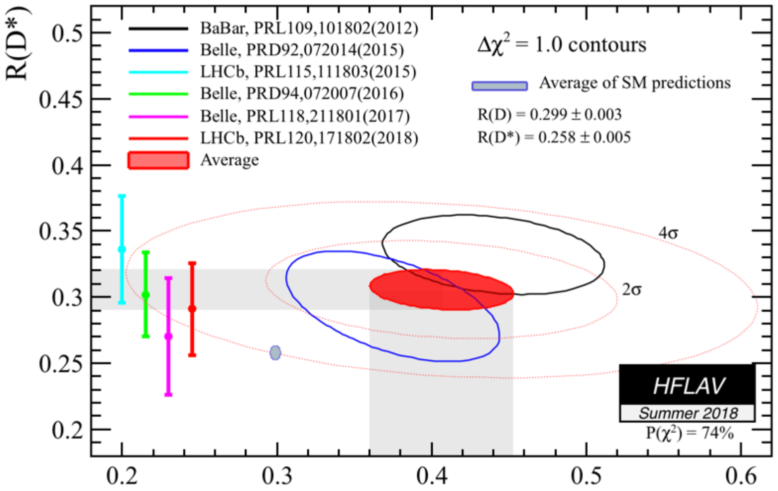
Leptoquarks

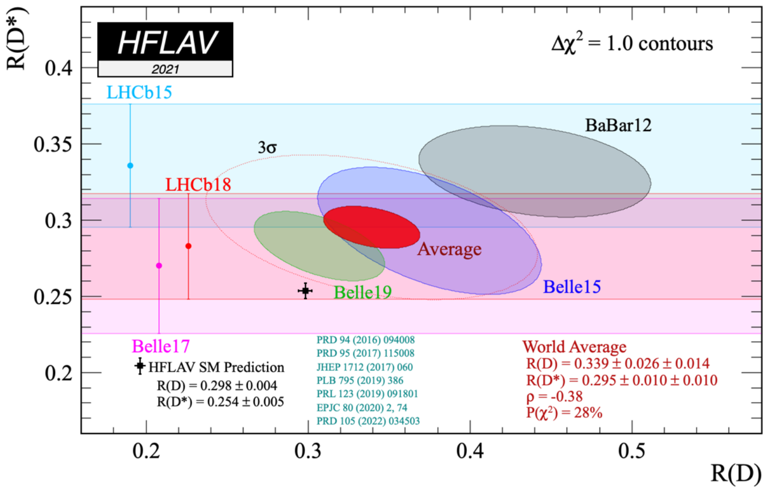
Testable Predictions

$R_{D^{(*)}}$ over the years

Started in 2012!







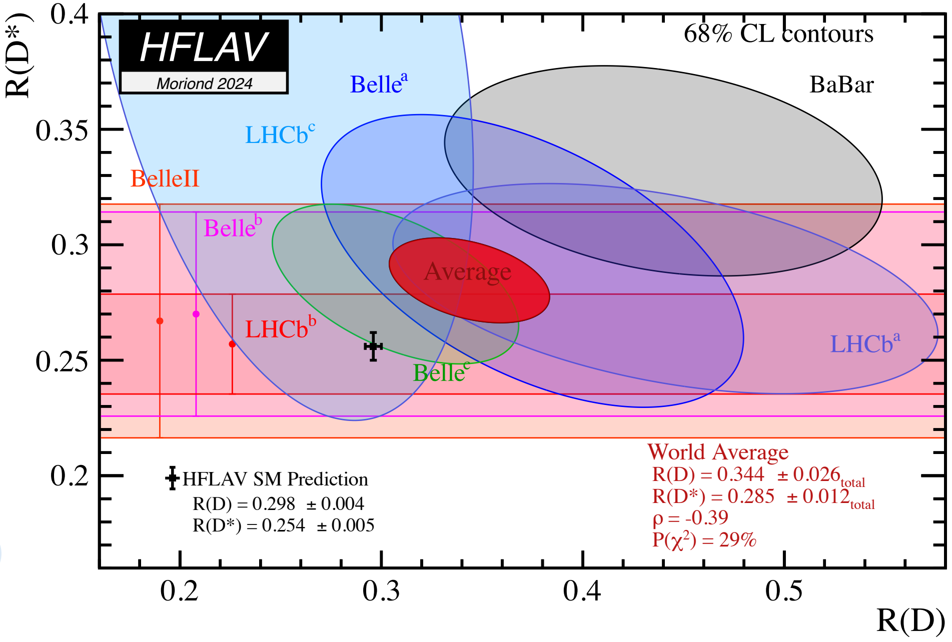
$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)} l\bar{\nu})}\bigg|_{l\in\{e,\mu\}}$$

- R_D^{exp} and $R_{D^*}^{exp}$: dominated by BaBar!
- In $R_{J/\psi}^{exp}$ and $R_{\Lambda c}^{exp}$ limited precision.

-Solution for the puzzle - New Physics (?!)

-Precise knowledge of form factors needed!

LHCb new results at Moriond 2024!



There are still some issues!

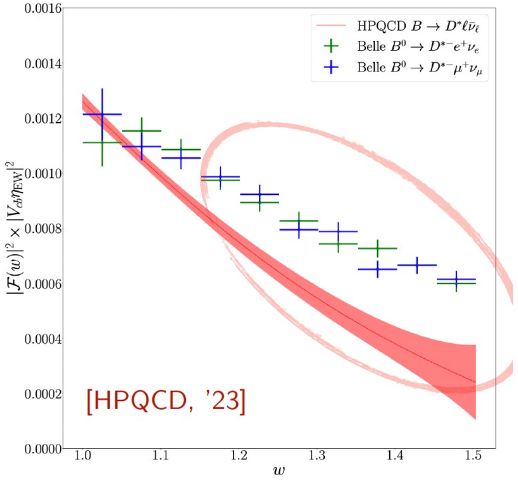
$$< D^{(*)}(p', (\epsilon))|\bar{c}\,\Gamma^{\mu}\, b|B(p) >= \sum_j K_j^{\mu} \mathcal{F}_j(q^2)$$

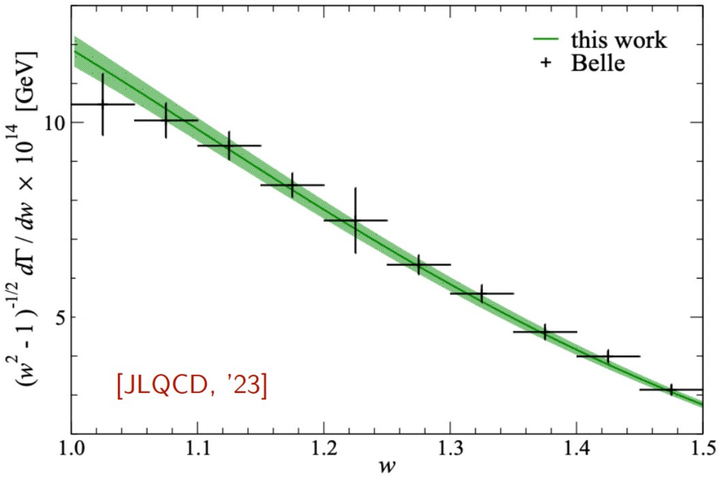
1) $B \to D$: one (two) form-factors with $f_0(0) = f_+(0)$ at $q^2 = 0$;
Lattice QCD at $q^2 \neq q^2_{max}$ for both form-factors.

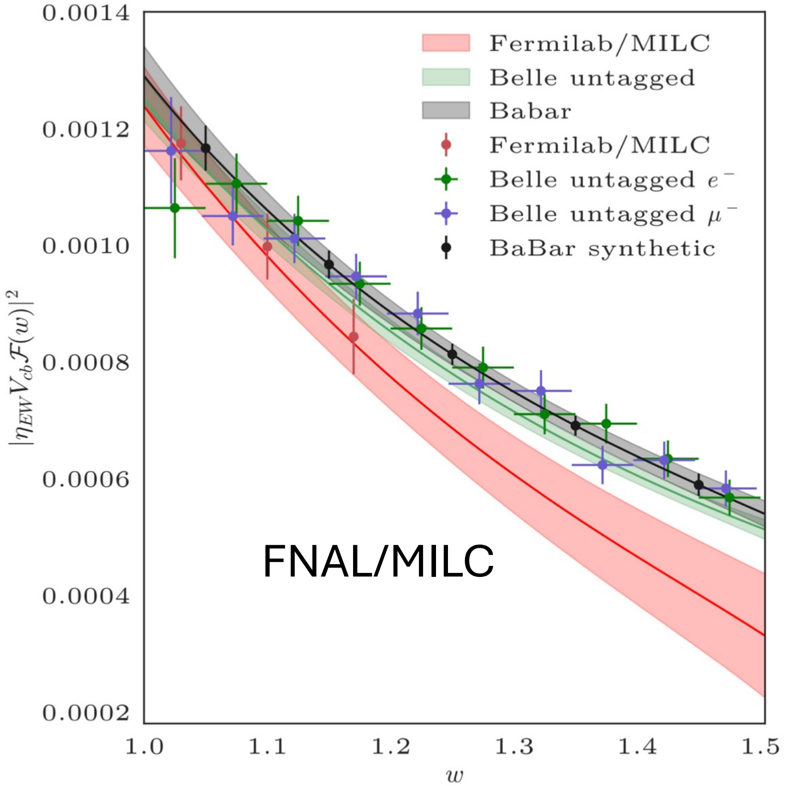
2) $B \to D^*$: three (four) form-factors;
First lattice results at $q^2 \neq q^2_{max}$! Tensions with $B \to D^* l\nu$ exp. data

$$R_D^{\rm exp} = 0.344(26)\,, \qquad R_{D*}^{\rm exp} = 0.285(12)$$

$$R_D^{\rm SM} = 0.293(8) \qquad R_{D*}^{\rm SM} = ?$$







$$w = \frac{m_B^2 + m_{D*}^2 - q^2}{2\, m_B\, m_{D*}}$$

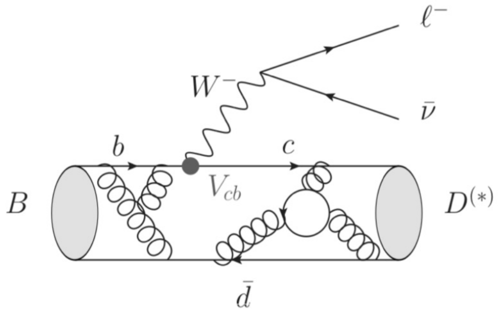
$$\langle D^*(k)|\bar{c}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho k^\sigma \frac{2V(q^2)}{m_B + m_{D*}} - i\varepsilon^*_\mu(m_B + m_{D*})A_1(q^2)$$
$$+i(p+k)_\mu(\varepsilon^*\cdot q)\frac{A_2(q^2)}{m_B + m_{D*}} + iq_\mu(\varepsilon^*\cdot q)\frac{2m_{D*}}{q^2}\left[A_3(q^2) - A_0(q^2)\right]$$

- Lattice QCD computations of these form factors :

(FERMILAB MILC, 2105.14019, HPQCD , 2304.03137, JLQCD, Y. Aoki et al., 2306.05657)

- consistent for the dominant form factor $A_1(q^2)$, but do not agree with the other form factors.

We used another approach to consider form factors is heavy quark effective theory (Caprini, Lellouch, Neubert -CLN), reducing the problem to four parameters and using all experimental information (HFLAV, in agreement with Belle II)



$$R^{\text{SM}}_{D*} = 0.247(2)$$

3σ smaller than the experimental avarege

$$R_{\nu\nu}^{K^{(*)}} = \mathcal{B}(B \to K^{(*)}\nu\bar{\nu})/\mathcal{B}(B \to K^{(*)}\nu\bar{\nu})^{\rm SM}$$

NP in b → s μμ: $RK_{(*)}$ anomaly is not an issue (only P_5’)!

$$\mathcal{L}_{\rm eff}^{b\to s\nu\nu} = \frac{4G_F\lambda_t}{\sqrt{2}}\frac{\alpha_{\rm em}}{2\pi}\sum_i C_L^{\rm SM}(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_{Li}\gamma^\mu\nu_{Li})$$

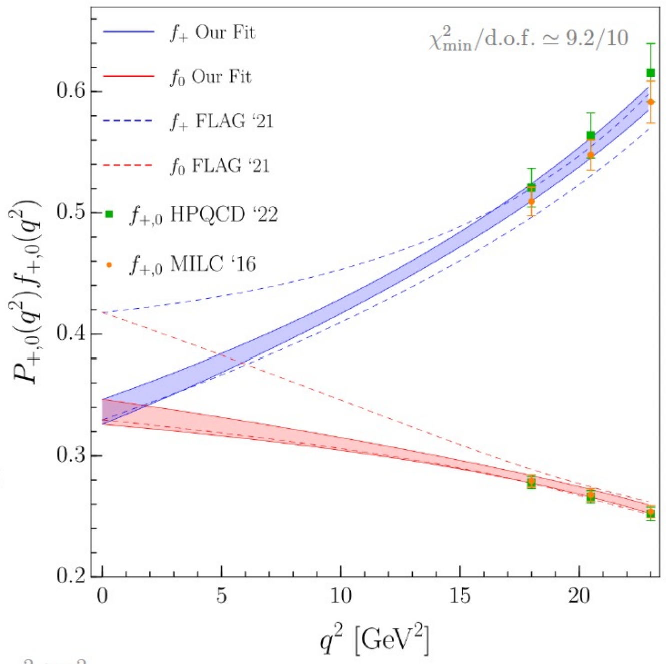
$\lambda_t = V_{tb}V_{ts}^*$

SM

$$C_L^{\rm SM} = -X_t/\sin^2\theta_W$$
$$= -6.32(7)$$

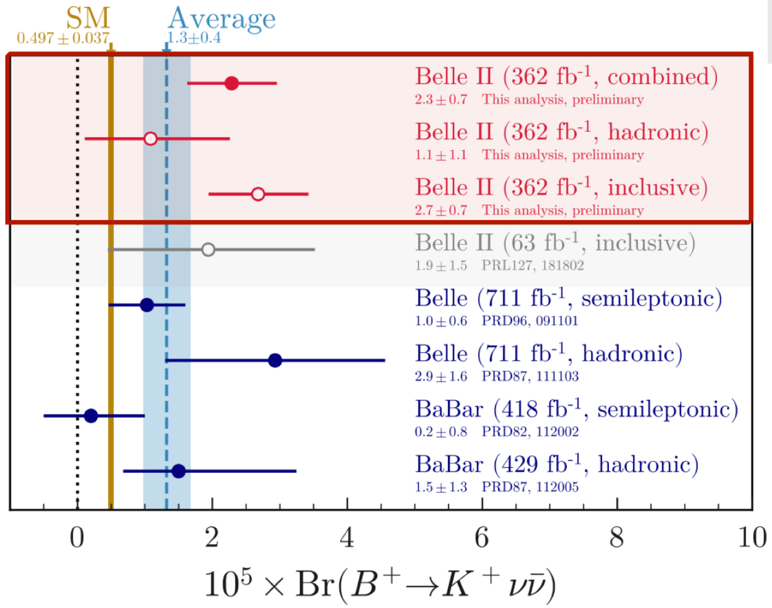
Buras et al.,1409.4557,
Altmannshofer et al., 0902.0160
Buras, 2209.03968
Bečirević et al, 2301.06990

$$\mathcal{B}(B^\pm \to K^\pm\nu\nu) = (4.44 \pm 0.30) \times 10^{-6}\,,$$
$$\mathcal{B}(B^\pm \to K^{\pm *}\nu\nu) = (9.8 \pm 1.4) \times 10^{-6}\,,$$



Pole factor:
$P_i(q^2) = 1 - q^2/M_i^2$

Bečirević et al, 2301.06990



Belle II, 2023 (2311.14647)

New Belle-II results

First Belle-II result

$$R_{\nu\nu}^{K} = 5.4 \pm 1.5$$

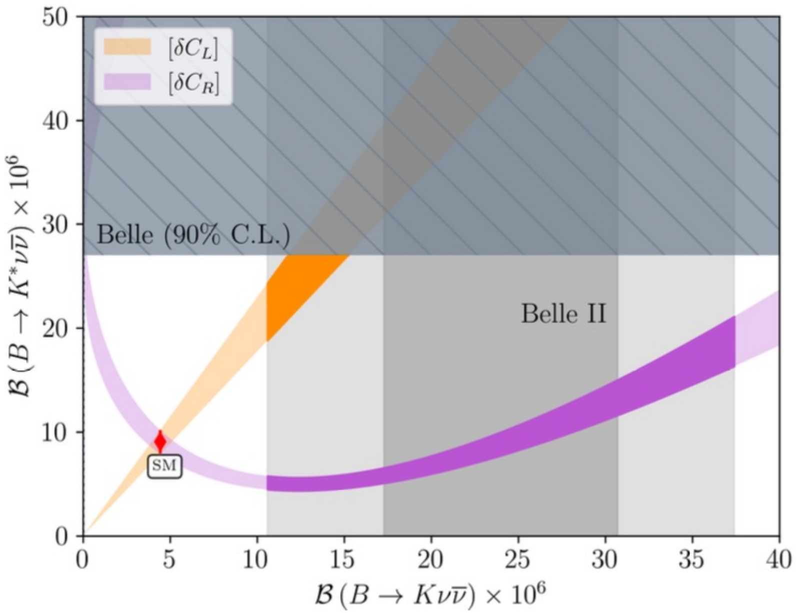
$$R_{\nu\nu}^{K^*} < 2.7 \quad (90\%\ {\rm C.L.})$$

NP in $b \to s \nu \bar{\nu}$

If SM neutrinos
the right-handed quark operator necessary!

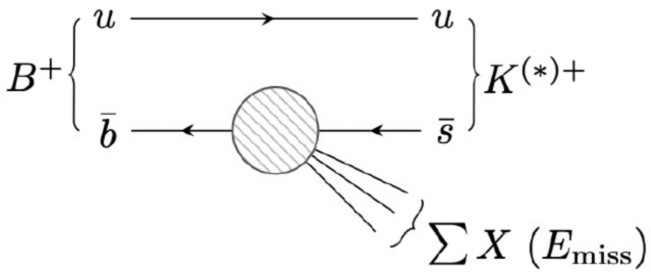
$$\mathcal{O}_L^{\nu_i \nu_j} = \frac{e^2}{(4\pi)^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_i \gamma^\mu (1-\gamma_5)\nu_j)\,,$$

$$\mathcal{O}_R^{\nu_i \nu_j} = \frac{e^2}{(4\pi)^2} (\bar{s}_R \gamma_\mu b_R)(\bar{\nu}_i \gamma^\mu (1-\gamma_5)\nu_j)\,,$$

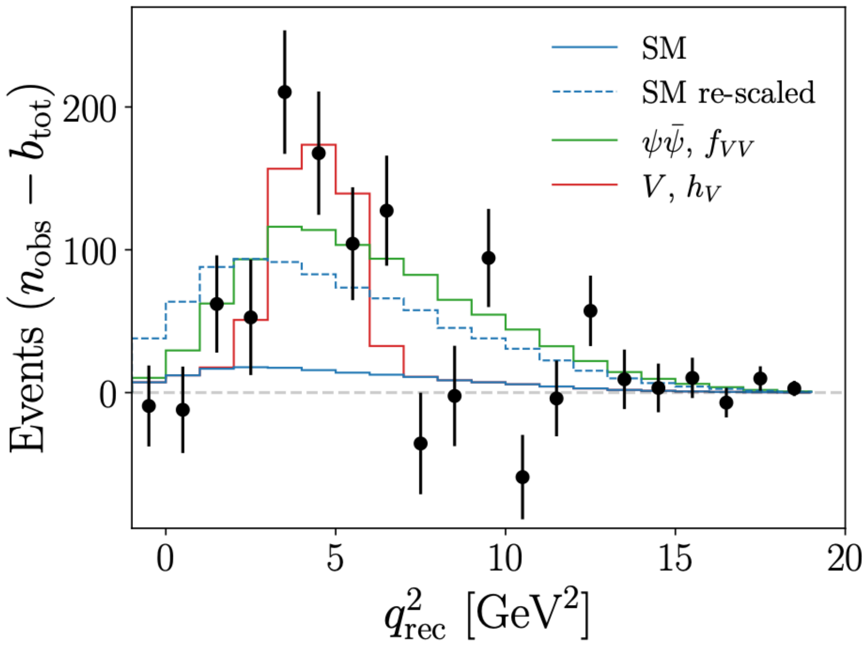


Allwicher et al, 2309.02246,
Bause et al., 2309.00075

Invisible sector?



B → KX
B → K $\nu_L \nu_R$
B→K $\chi\chi$
B→K$\phi\phi$
Bolton, SF, Kamenik,
Novoa-Brunet, 2403.13887

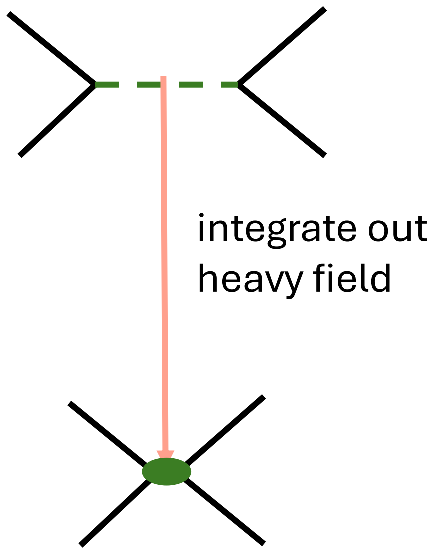


- two-body decay, best fit point (2.8 σ) for vector state
 $m_V \sim 2$ GeV

- for two inisible scalars or fermions m$\chi = 610$ MeV
Bolton et al. ’24

He et al., 2309.12741, Altmannshofer et al. 2311.1469, Alonso-Alvarez et al. 2310.13043, Bolton, SF, Kamenik, Novoa-Brunet 2403.13887,...

Approach: LQs → SMEFT→ LEFT



- $SU(3)_c \otimes U(1)_{em}$ H, W, Z, t are integrated out (1908.05295, Dekens&Stoffer)
- evolution (RGE) running of Wilson coefficents

$$\mathcal{L}_{\rm SMEFT} = \frac{1}{\Lambda^2}\sum_i C_i\, \mathcal{O}_i\,.$$

Warsaw basis, Grzadkowski et al, 1008.4884

SMEFT papers: Manohar et al., 1308.2627, 1309.0819, 1310,4838, 1312.2014

To analyze $R_{D^{(*)}}$ SMEFT operators

$$\mathcal{O}^{(1)}_{\substack{lequ\\prst}} = (\bar{l}^a_p e_r)\epsilon^{ab}(\bar{q}^b_s u_t)\,, \qquad \mathcal{O}^{(3)}_{\substack{lequ\\prst}} = (\bar{l}^a_p \sigma^{\mu\nu} e_r)\epsilon^{ab}(\bar{q}^b_s \sigma_{\mu\nu} u_t)\,,$$

$$\mathcal{O}^{(1)}_{\substack{lq\\prst}} = (\bar{l}^j_p \gamma^\mu l_r)(\bar{q}_s \gamma_\mu u_t)\,, \qquad \mathcal{O}^{(3)}_{\substack{lq\\prst}} = (\bar{l}_p \gamma^\mu \tau^I l_r)(\bar{q}_s \gamma_\mu \tau^I q_t)\,.$$

N_R-SMEFT operators
Del Aguila et al. 0806.0876
Fernando-Martinez et al., 2304.06772.

$$\mathcal{O}^{(1)}_{\substack{Nldq\\rst}} = (\bar{N}_R l^a_r)\epsilon^{ab}(\bar{d}_s q^b_t)\,, \qquad \mathcal{O}^{(3)}_{\substack{Nldq\\rst}} = (\bar{N}_R \sigma^{\mu\nu} l^a_r)\epsilon^{ab}(\bar{d}_s \sigma_{\mu\nu} q^b_t)\,.$$

$$g_{V_L}(m_b) = -\frac{v^2}{m^2_{\rm LQ}}\frac{V_{cs}}{V_{cb}} C^{(3)}_{\substack{lq\\\tau\tau sb}}(m_{\rm LQ}) - \frac{v^2}{m^2_{\rm LQ}} C^{(3)}_{\substack{lq\\\tau\tau bb}}(m_{\rm LQ})\,,$$

$$g_{S_L}(m_b) = -\frac{v^2}{2m^2_{\rm LQ}}\frac{1}{V_{cb}}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_t)}\right)^{12/23}\left(\frac{\alpha_s(m_t)}{\alpha_s(m_{\rm LQ})}\right)^{4/7} C^{(1)*}_{\substack{lequ\\\tau\tau bc}}(m_{\rm LQ}) = -0.56\, C^{(1)*}_{\substack{lequ\\\tau\tau bc}}(m_{\rm LQ})\,,$$

$$g_T(m_b) = -\frac{v^2}{2m^2_{\rm LQ}}\frac{1}{V_{cb}}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_t)}\right)^{-4/23}\left(\frac{\alpha_s(m_t)}{\alpha_s(m_{\rm LQ})}\right)^{-4/21} C^{(3)*}_{\substack{lequ\\\tau\tau bc}}(m_{\rm LQ}) = -0.28\, C^{(3)*}_{\substack{lequ\\\tau\tau bc}}(m_{\rm LQ})\,,$$

Bečirević, SF, Košnik, Pavičić, 2404.16772

Scalar and Vector Leptoquarks as NP meditaors

- Grand Unified Theories (GUTs), Jogesh Pati and Abdus Salam in 1974

- In 1997, the H1 and ZEUS collaborations at HERA an excess of events, production of leptoquarks (electron-proton system, H1 mass 200 GeV).

- In 21st century
  - a) LHC Searches
  - b) Flavour Anomalies
  - c) Neutrino Mass Models

$(SU(3), SU(2), U(1))$	Spin	Symbol	Type	F
$(\overline{\mathbf{3}}, \mathbf{3}, 1/3)$	0	S_3	$LL\,(S_1^L)$	-2
$(\mathbf{3}, \mathbf{2}, 7/6)$	0	R_2	$RL\,(S_{1/2}^L)$, $LR\,(S_{1/2}^R)$	0
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	$\tilde{R}_2$	$RL\,(\tilde{S}_{1/2}^L)$, $\overline{LR}\,(\tilde{S}_{1/2}^{\overline{L}})$	0
$(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$	0	$\tilde{S}_1$	$RR\,(\tilde{S}_0^R)$	-2
$(\overline{\mathbf{3}}, \mathbf{1}, 1/3)$	0	S_1	$LL\,(S_0^L)$, $RR\,(S_0^R)$, $\overline{RR}\,(S_0^{\overline{R}})$	-2
$(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$	0	$\bar{S}_1$	$\overline{RR}\,(\bar{S}_0^{\overline{R}})$	-2
$(\mathbf{3}, \mathbf{3}, 2/3)$	1	U_3	$LL\,(V_1^L)$	0
$(\overline{\mathbf{3}}, \mathbf{2}, 5/6)$	1	V_2	$RL\,(V_{1/2}^L)$, $LR\,(V_{1/2}^R)$	-2
$(\overline{\mathbf{3}}, \mathbf{2}, -1/6)$	1	$\tilde{V}_2$	$RL\,(\tilde{V}_{1/2}^L)$, $\overline{LR}\,(\tilde{V}_{1/2}^{\overline{R}})$	-2
$(\mathbf{3}, \mathbf{1}, 5/3)$	1	$\tilde{U}_1$	$RR\,(\tilde{V}_0^R)$	0
$(\mathbf{3}, \mathbf{1}, 2/3)$	1	U_1	$LL\,(V_0^L)$, $RR\,(V_0^R)$, $\overline{RR}\,(V_0^{\overline{R}})$	0
$(\mathbf{3}, \mathbf{1}, -1/3)$	1	$\bar{U}_1$	$\overline{RR}\,(\bar{V}_0^{\overline{R}})$	0

Scalar leptoquarks → Yukawa-like couplings $\bar{\ell}\, P_{L,R}\, q\, \Phi$

$Q = I_3 + Y$

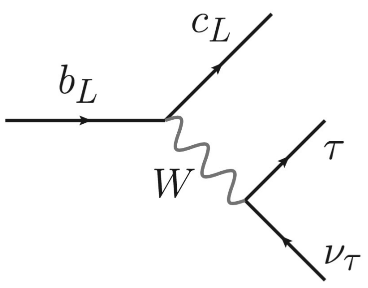
Vector leptoquarks → gauge bosons (in GUTs their masses at GUT scale) $\bar{\ell}\gamma_\mu\, P_{L,R}\, q\, V^\mu$

Dorsner, SF, Greljo, Kamenik, Košnik, 1603.04993

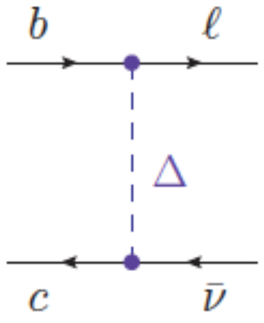
Scalar Leptoquarks in $R_{D^{(*)}}$

Goal of our study : whether or not any of the scalar leptoquarks, with a minimalistic set of Yukawa couplings, fits the current experimental world average of R_D and R_{D^*}

Scenarios in which the scalar leptoquark couples to τ and either to c or to b quark.



SM



LQs

$$\mathcal{L}_{b\to c\tau\nu} = -2\sqrt{2}G_F V_{cb}\Big[(1+g_{V_L})(\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_{\tau L}) + g_{V_R}(\bar{c}_R\gamma^\mu b_R)(\bar{\tau}_L\gamma_\mu\nu_{\tau L})$$
$$+ g_{S_L}(\bar{c}_R b_L)(\bar{\tau}_R\nu_{\tau L}) + g_T(\bar{c}_R\sigma^{\mu\nu}b_L)(\bar{\tau}_R\sigma_{\mu\nu}\nu_{\tau L}) +$$
$$+ \tilde{g}_{S_R}(\bar{c}_L b_R)(\bar{\tau}_L N_R) + \tilde{g}_T(\bar{c}_L\sigma^{\mu\nu}b_R)(\bar{\tau}_L\sigma_{\mu\nu}N_R)\Big] + \text{h.c.}$$

LQ →$(SU(3)_c, SU(2)_L, U(1)_Y)$

R_2 =(3,2,7/6)

$\tilde{R}_2$ = (3,2,1/6)

S_1= (3,1,1/3)

In ref. Angelescu et al., 2103.12504 all these could explain $R_{D^{(*)}}$

$$U_1 = (3,1,2/3) : g_V$$
$$R_2 = (3,2,7/6) : g_{S_L} = 4g_T$$
$$S_1 = (\bar{3},1,1/3) : g_{S_L} = -4g_T, g_V$$

Our update
Damir Bečirević, SF ,
Nejc Košnik, Lovre Pavičić, 2404.16772

$R_2 = (3, 2, 7/6)$

In $SU(2)_L$ R_2 is in a represention of dimension 2 (weak isospin 1/2).
There are two states $R_2^{5/3}$ and $R_2^{2/3}$

The minimal model: couplings to the third generations of leptons (ν_τ and τ only)

$$\mathcal{L}_{R_2} = y_R^{b\tau} V_{jb}^* (\overline{u}_j P_R \tau) R_2^{5/3} + y_R^{b\tau} (\overline{b} P_R \tau) R_2^{2/3} - y_L^{c\tau} (\overline{c} P_L \tau) R_2^{5/3} + y_L^{c\tau} (\overline{c} P_L \nu_\tau) R_2^{2/3} + \text{h.c.}$$

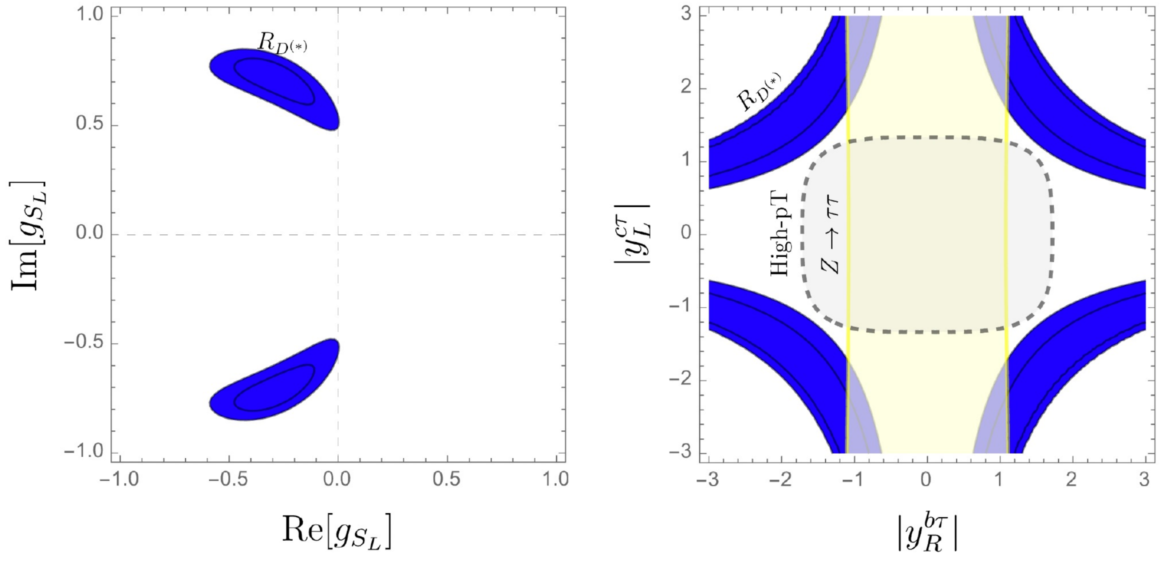
$$y_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_R^{b\tau} \end{pmatrix}, \qquad y_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_L^{c\tau} \\ 0 & 0 & 0 \end{pmatrix}$$

running from m_{R2} → mb

$$m_{R_2} = 1.5 \text{ TeV}$$

$$g_{S_L}(m_{R_2}) = 4 g_T(m_{R_2})$$

$$g_{S_L}(m_b) = 8.8 \times g_T(m_b)$$



One of the Yukawa couplings should be complex

$$g_{S_L}(m_b) = 0.60 \times \frac{1}{2} |y_R^{b\tau} y_L^{c\tau}| e^{i\varphi} .$$

Bečirević, SF, Košnik, Pavičić, 2404.16772

high p_T relevan- (Allwicher et al 2207.10756)

R_2 is out of game

$\widetilde{R}_2 = (3, 2, 1/6)$

$\widetilde{R}_2^{2/3}$ and $\widetilde{R}_2^{-1/3}$

- It can couple to non-SM right-handed neutrino N_R

$$\mathcal{L} = -\,\tilde{y}_L^{b\tau}(\bar{b}P_L\tau)\widetilde{R}_2^{2/3} + \tilde{y}_L^{b\tau}(\bar{b}P_L\nu)\widetilde{R}_2^{-1/3} +$$
$$+\,\tilde{y}_R^{sN}(\bar{s}P_RN_R)\widetilde{R}_2^{-1/3} + \tilde{y}_R^{sN}V_{js}(\bar{u}_jP_RN_R)\widetilde{R}_2^{2/3} + \text{h.c.}$$

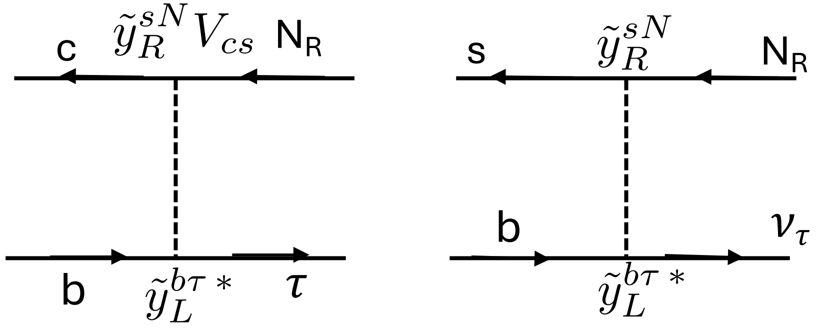
minimal set of couplings

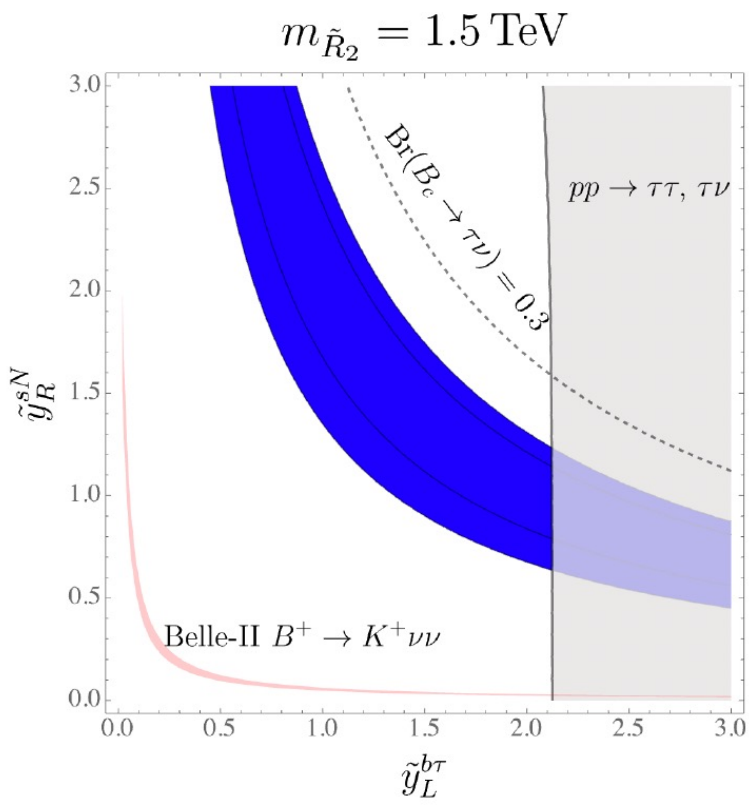
$$\tilde{y}_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \tilde{y}_L^{b\tau} \end{pmatrix}, \qquad \tilde{y}_R = \begin{pmatrix} 0 \\ \tilde{y}_R^{sN} \\ 0 \end{pmatrix},$$

In the branching ratio N_R cannot interfere with SM neutrino

$$\mathcal{B} \propto |\mathcal{A}_{\rm SM} + \mathcal{A}_{\rm NP}^{\nu_L}|^2 + |\mathcal{A}_{\rm NP}^{N_R}|^2$$

However, there is the tree diagram for $b \to s\nu\bar{N}_R$





To explain RD(*) and Belle II result for B→ $K\,\nu\,\nu$
this cannot be achieved with this LQ!
See also Rosauro-Alcaraz and Santos Leal ,2401.17440

$\tilde{R}_2$ is out of game!

$S_1 = (\overline{3}, 1, 1/3)$

Being a weak singlet S_1 has only one state with the electric charge 1/3.
It allows the the interactions with quark, lepton both being weak doublets, or weak singlets

$$\mathcal{L}_{S_1} = y_L^{b\tau}\, V_{ib}^* (\overline{u_i^C} P_L\, \tau)\, S_1 - y_L^{b\tau}\, (\overline{b^C} P_L\, \nu_\tau)\, S_1 + y_R^{c\tau}\, (\overline{c^C} P_R \tau)\, S_1 + \text{h.c.}$$

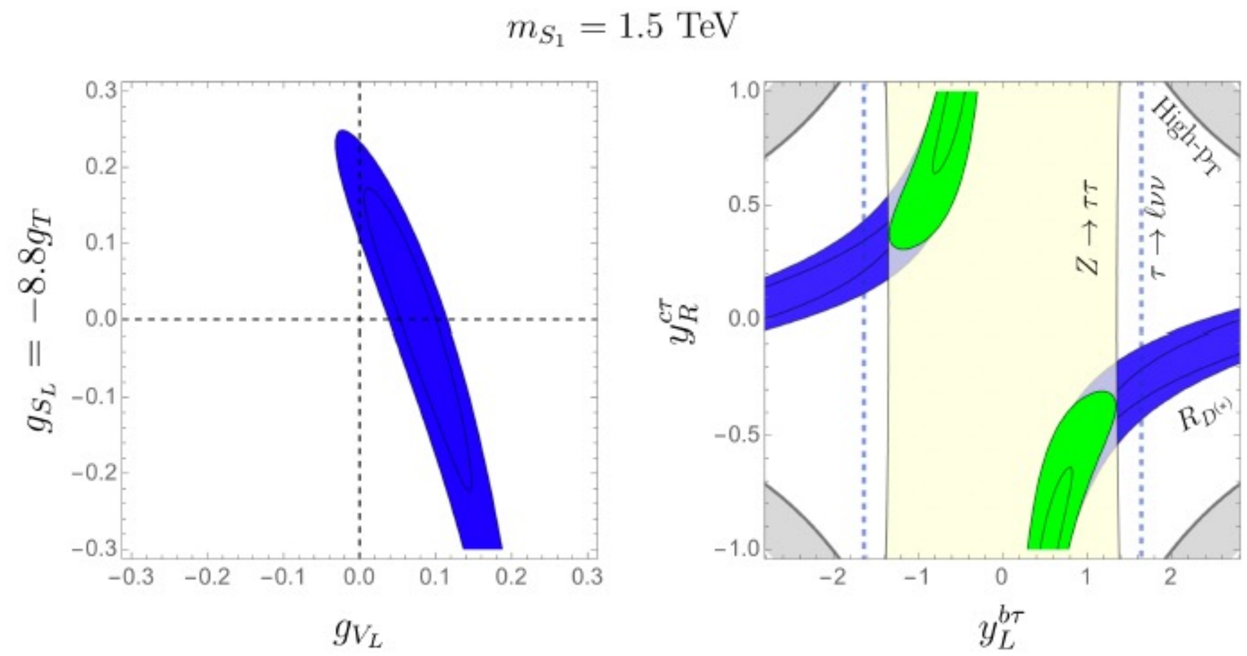
Minimal setting

$$y_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_L^{b\tau} \end{pmatrix}, \qquad y_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_R^{c\tau} \\ 0 & 0 & 0 \end{pmatrix}$$

$$g_{V_L} = \frac{v^2}{4V_{cb}} \frac{V_{cb}\, |y_L^{b\tau}|^2}{m_{S_1}^2}$$

$$g_{S_L}(m_{S_1}) = -\frac{v^2}{4V_{cb}} \frac{y_L^{b\tau}\, y_R^{c\tau *}}{m_{S_1}^2}$$

$$g_{S_L}(m_b) = -8.8 \times g_T(m_b)$$



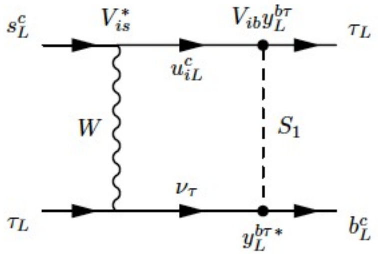
Damir Bečirević, SF, Nejc Košnik,
Lovre Pavičić, 2404.16772

S_1 - survival of the fittest!

Consequences

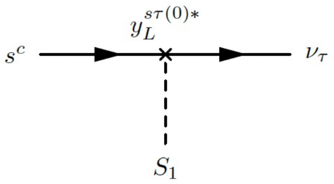
1) $$\frac{\mathcal{B}(B_c \to \tau\nu)^{S_1}}{\mathcal{B}(B_c \to \tau\nu)^{\rm SM}} \in [1.13, 1.48]\,, \qquad \mathcal{B}(B_c \to \tau\nu)^{\rm SM} = (2.24 \pm 0.07)\% \times \left(\frac{V_{cb}}{0.0417}\right)^2$$

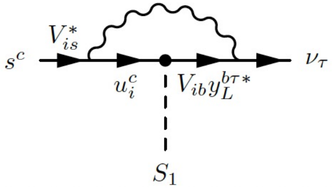
2) through the box or penguin diagrams involving one S_1 and one W-boson, a contribution to $b \to s\tau\tau$ or $b \to s\nu_\tau\nu_\tau$



$$\frac{\mathcal{B}(B_s \to \tau\tau)^{S_1}}{\mathcal{B}(B_s \to \tau\tau)^{\rm SM}} \in [0.73, 0.98]\,, \qquad \frac{\mathcal{B}(B \to K\tau\tau)^{S_1}}{\mathcal{B}(B \to K\tau\tau)^{\rm SM}} \in [0.73, 0.98]$$

3) $b \to s\nu_\tau\nu_\tau$





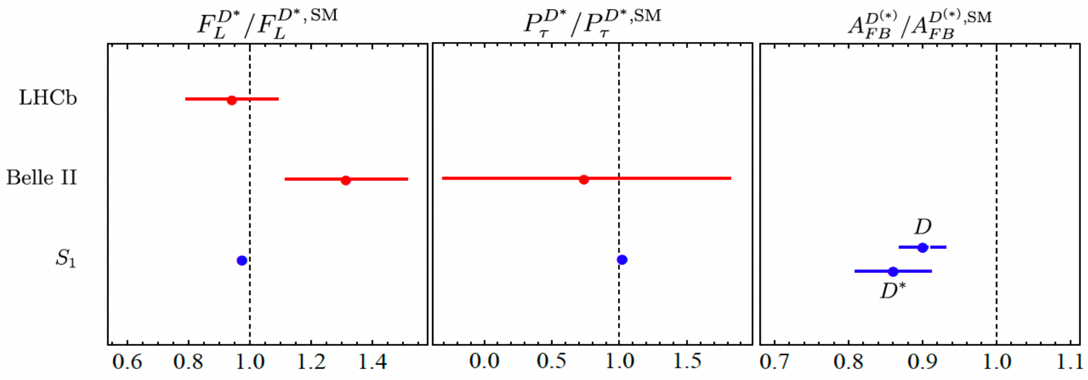
$$C_L^{S_1} = (-9.3 + 0.4\,i) \times 10^{-2}\,|y_L^{b\tau}|^2$$

(imaginary part comes from the fermions being on the mass shell in the loops)

$$\frac{\mathcal{B}(B \to K^{(*)}\nu\nu)^{S_1}}{\mathcal{B}(B \to K^{(*)}\nu\nu)^{\rm SM}} = \left|1 + \frac{\delta C_L^{S_1}}{3\,C_L^{\rm SM}}\right|^2 \in [1.001, 1.02] \quad (@2\sigma)$$

4) $V_{ub}|y_L^{b\tau}|^2$ in $\left\{\begin{array}{l}\mathcal{B}(B^- \to \tau\nu) \\ \mathcal{B}(B \to \pi\tau\nu)\end{array}\right\}$ only 3 % enhancement over the SM

5) for B →D(*)$\tau\nu$ the fraction of the decay rate to a longitudinally polarized D∗, the τ −lepton polarization asymmetry, and the forward-backward asymmetries

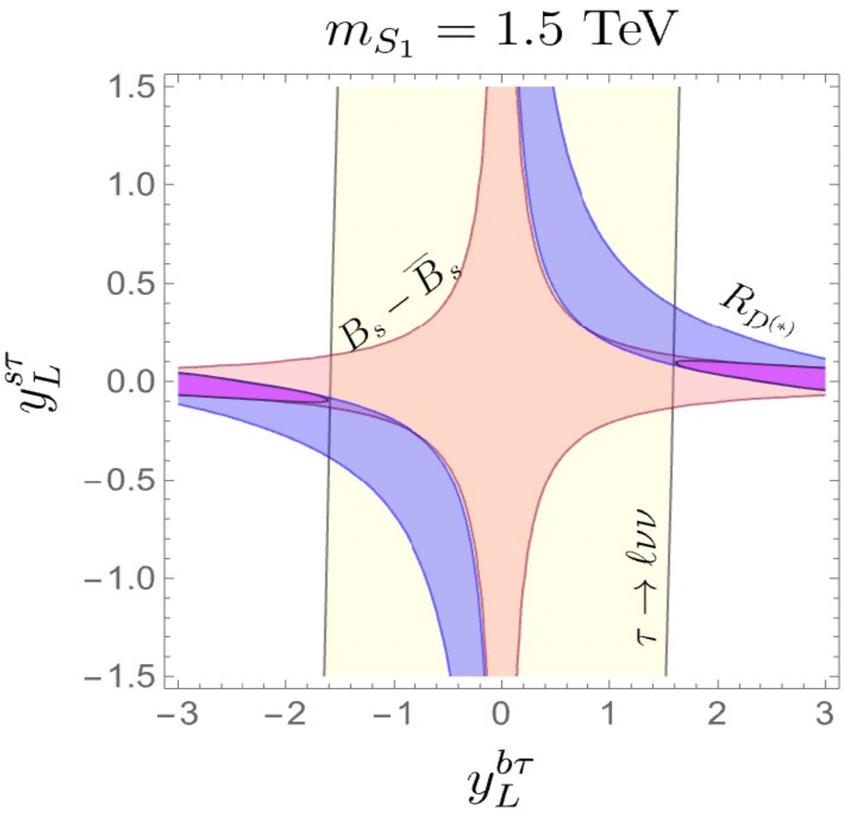


S$_1$ with V-A couplings only

$$g_{V_L} = -\frac{v^2}{m_{S_1}^2}\frac{V_{cs}}{V_{cb}} y_L^{s\tau} y_L^{b\tau}$$

$$y_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_L^{s\tau} \\ 0 & 0 & y_L^{b\tau} \end{pmatrix}, \qquad y_R = 0$$

No right-handed fermions ($y_R^{c\tau} = 0$)



S$_1$ (V-A) cannot survive!

Conclusions

Flavour puzzles persist

SMEFT a usefull tool for NP

LFU tests

Leptoquarks

S_1 leptoquark is the only one viable candidate to explain the $R_{D^{(*)}}$ puzzle!

LFU tests at LHC expected!

Děkuji!

Thanks!

Additional slides

B→ D^* form factors

CLN approach , hep-ph/9712417

$$\frac{1}{\sqrt{m_B m_{D*}}}\langle D^*(k)\,|\,\bar{c}\gamma_\mu(1-\gamma_5)b\,|\,B(p)\rangle = \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}v^\rho v'^\sigma h_V(w) - i\varepsilon^*_\mu(w+1)h_{A_1}(w)$$
$$+\, i(\varepsilon^*\cdot v)v_\mu h_{A_2}(w) + i(\varepsilon^*\cdot v)v'_\mu h_{A_3}(w)\,,$$

$$v_\mu = k_\mu/m_B$$

$$v'_\mu = k_\mu/m_{D*}$$

$$w = v\cdot v' = (m_B^2 + m_{D*}^2 - q^2)/(2m_B m_{D*}).$$

$$h_{A_1}(w) = h_{A_1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right],$$

$$z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$$

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)} = R_1(1) - 0.12(w-1) + 0.05(w-1)^2\,,$$

$$R_2(w) = \frac{h_{A_2}(w)}{h_{A_1}(w)} = R_2(1) + 0.11(w-1) - 0.06(w-1)^2\,,$$

$$A_1(q^2) = \frac{\sqrt{m_B m_{D*}}}{m_B + m_{D*}}(w+1)\;h_{A_1}(w)\,,$$

$$\frac{V(q^2)}{A_1(q^2)} = \left[1 - \frac{q^2}{(m_B + m_{D*})^2}\right]^{-1} R_1(w)\,,$$

$$\frac{A_2(q^2)}{A_1(q^2)} = \left[1 - \frac{q^2}{(m_B + m_{D*})^2}\right]^{-1} R_2(w)\,,$$

$$\rho^2 = 1.121(24), \quad R_1(1) = 1.269(26), \quad R_2(1) = 0.853(17),$$

HFLAV, 2206.07501, Belle II in agreement

$$R_0(1) = \frac{4m_B m_{D*}}{(m_B + m_{D*})^2}\frac{A_0(q^2_{\rm max})}{A_1(q^2_{\rm max})} = 1.087(26)\,,$$

$$R_0(w) = 1.09 - 0.16(w-1)$$

Peguin contribution to b → $s\nu\nu$ S_1 model

On shell renormalisation condition

$$y_L^{s\tau}(q^2 = m_{S_1}^2) = 0\,.$$

$$y_L^{s\tau *}(q^2) = \frac{g^2\, y_L^{b\tau *}}{32\pi^2} \sum_{i=u,c,t} \lambda_i \frac{m_{S_1}^2}{q^2}\, h\left(q^2/m_{S_1}^2, x_i, x_W\right)$$

$$\begin{aligned} h(z, x_q, x_W) = &-2(x_q - 1) z\, \mathrm{Li}_2\left(1 - \frac{x_W}{x_q - 1}\right) - 2x_q(z-1)\, \mathrm{Li}_2\left(1 - \frac{x_q}{x_W}\right) \\ &+ 2(x_q - z)\, \mathrm{Li}_2\left(1 - \frac{x_W}{x_q - z}\right) - z(x_q - 1)\log^2\left(\frac{x_W}{x_q - 1}\right) \\ &+ (x_q - z)\log^2\left(\frac{x_W}{x_q - z}\right)\,. \end{aligned}$$