(2) Istituto Nazionale di Fisica Nucleare

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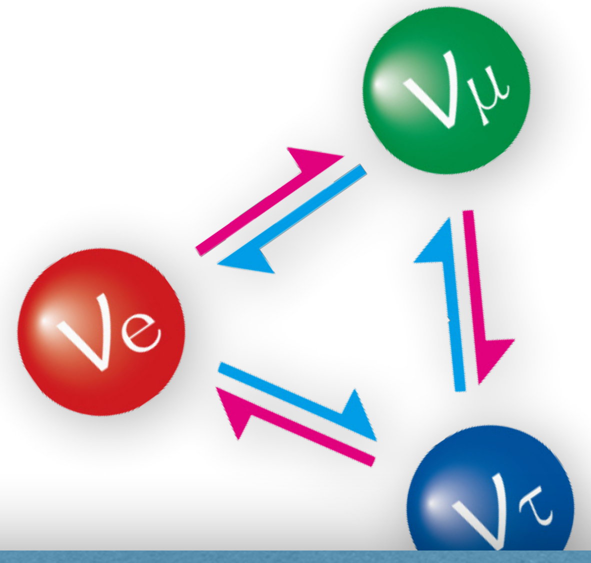


A minimalistic perspective on neutrino CP-violation and Leptogenesis: Modular Invariance

Matteo Parriciatu (1)(2)

Based on: JHEP05(2024)020, in collaboration with S. Marciano (1)(2) and D. Meloni (1)(2)

ICHEP 2024

July 19th, 2024 Prague

$3\,\nu$ Paradigm

$$\underbrace{|\nu_\alpha\rangle}_{\text{Interaction basis}} = \sum_{i=1}^{3} U^*_{\alpha i} \underbrace{|\nu_i\rangle}_{\text{Mass basis}}$$

$\theta_{12}\ \theta_{13}\ \theta_{23}$

δ_{CP} $\alpha_1\,,\alpha_2$

Majorana

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \times \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)$$

$c_{ij} \equiv \cos\theta_{ij}\ ,\ s_{ij} \equiv \sin\theta_{ij}$

Pontecorvo - Maki - Nakagawa - Sakata

$$P(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3}\sum_{j=1}^{3} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{ij}^2 L}{2E}\right)$$

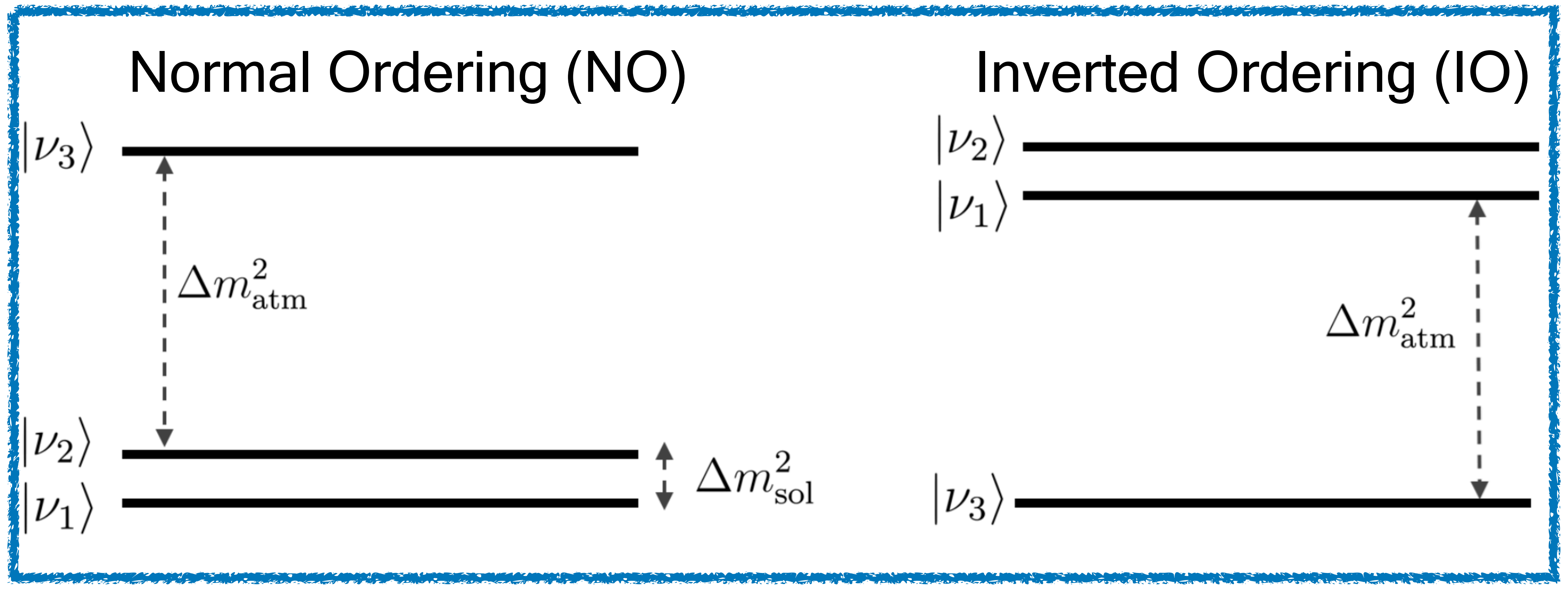
Sign determined from MSW

$$r \equiv \frac{\Delta m^2_{\text{sol}}}{|\Delta m^2_{\text{atm}}|} \approx 1/30$$

Undetermined sign

$|\Delta m^2_{\text{sol}}| \equiv |m_2^2 - m_1^2|$ $\quad$ $|\Delta m^2_{\text{atm}}| \equiv |m_3^2 - m_{1,2}^2|$

?

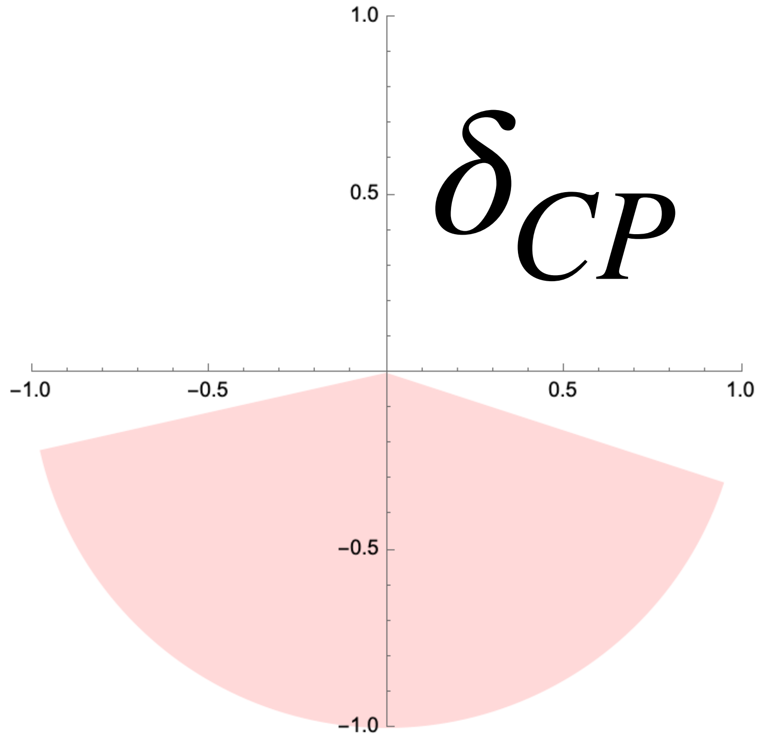


Best fit values

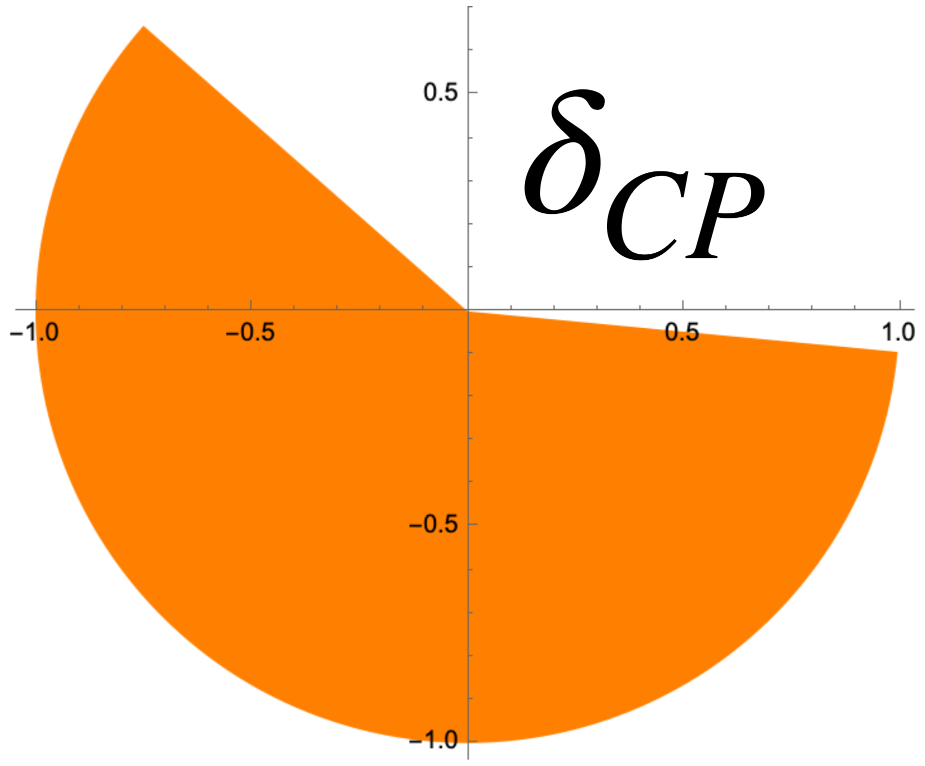
F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

$$\theta_{12} \approx 33.4^\circ \quad , \quad \theta_{13} \approx 8.5^\circ \quad , \quad \theta_{23} \approx 42.4^\circ \qquad (\mathbf{NO})$$
$$\theta_{12} \approx 33.4^\circ \quad , \quad \theta_{13} \approx 8.5^\circ \quad , \quad \theta_{23} \approx 48.9^\circ \qquad (\mathbf{IO})$$

Two big and one small angles



NO
IO



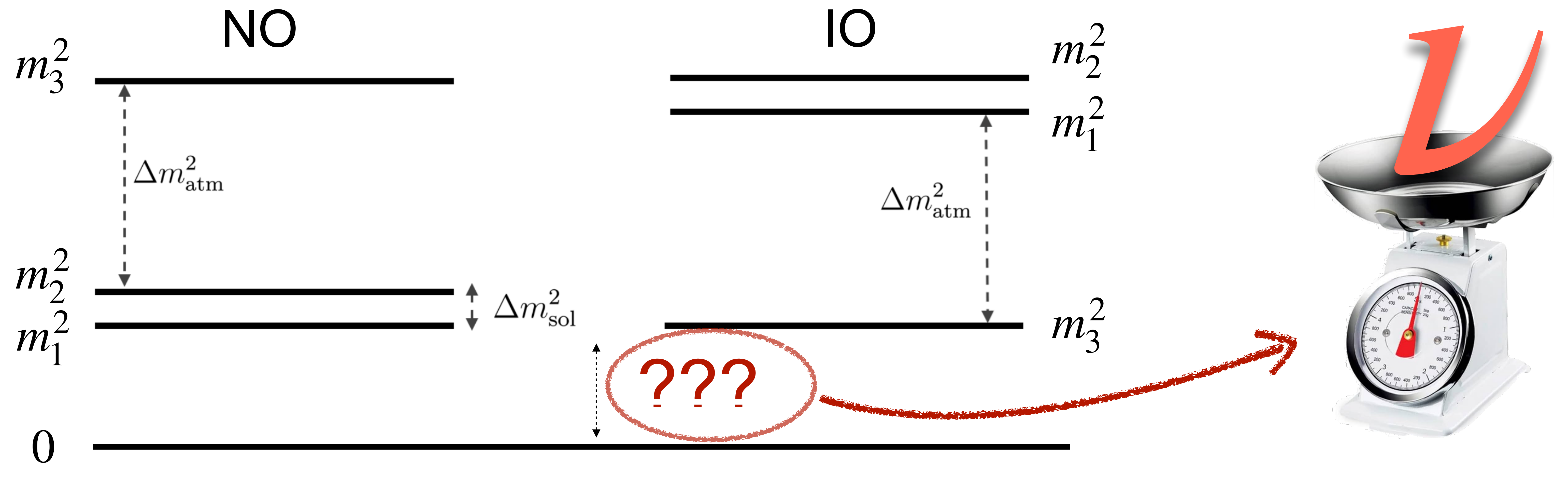
3σ ranges from
F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

$$J_{\mathrm{CP}} = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{\mathrm{CP}}$$

Violation if

$$\frac{\delta_{CP}}{\pi} \neq \{0,1\}$$

CP-conserving still allowed at 2σ (NO)



$$\sum_i^3 m_i < 0.115 \text{ eV} \qquad (95\% \text{ C.L.})$$

A. Shadab et al.

Phys. Rev. D **103** (Apr, 2021)

Cosmology

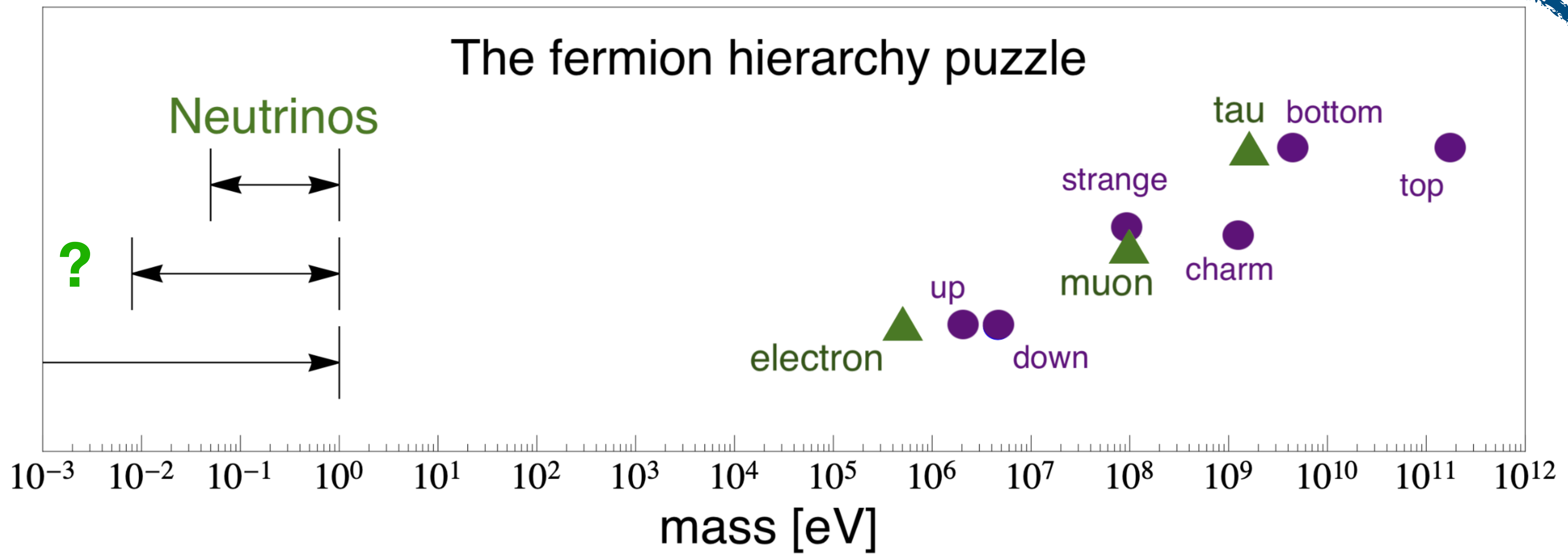
$$\sqrt{\sum_i m_i^2 |U_{ei}|^2} < 0.8 \text{ eV}$$

Nat. Phys. **18** (Feb, 2022)

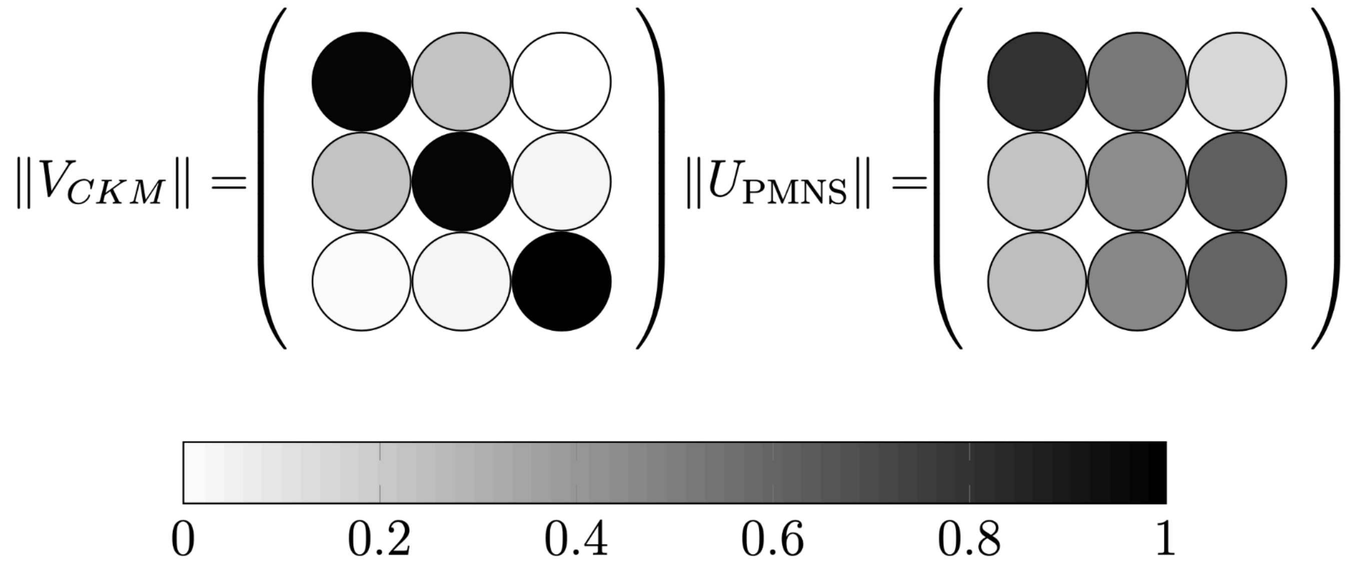
KATRIN

leptons

quarks

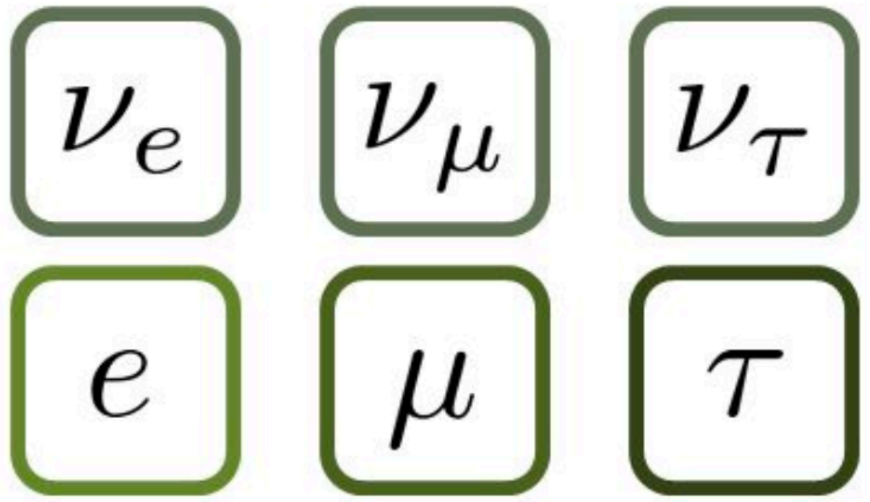


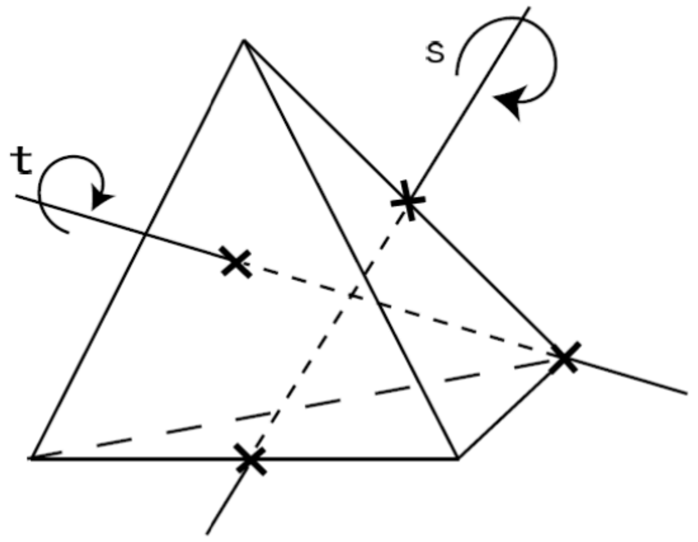
$$m_e/m_\mu \simeq \frac{1}{200} \qquad m_\mu/m_\tau \simeq \frac{1}{17}$$



D. Meloni [1709.02662v2]

Three “copies”, different masses

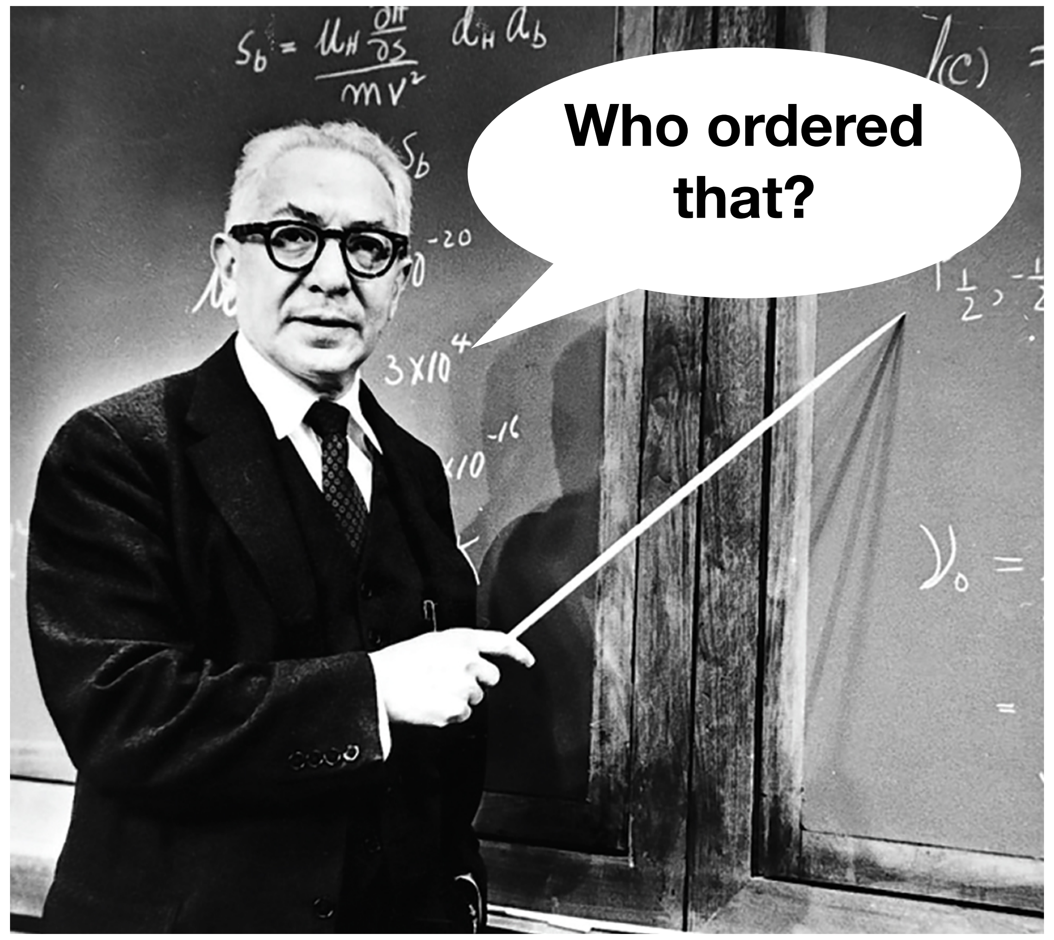




non-abelian discrete symmetries

$S_3 \quad A_4 \quad S_4 \quad A_5 \ldots\ldots$

$$\mathscr{W}_{Yukawa} \supset \frac{\alpha}{\Lambda} E^c (L\phi_i)_{\mathbf{1}} H_d$$



Isidor I. Rabi

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimaximal compatible with data until 2012

$$\theta_{13} = 0 \quad \theta_{12} \simeq 35^\circ \quad \theta_{23} = 45^\circ$$

Not zero

F. P. An *et al.*, “Observation of electron-antineutrino disappearance at daya bay,” *Phys. Rev. Lett.* **108** (Apr, 2012)

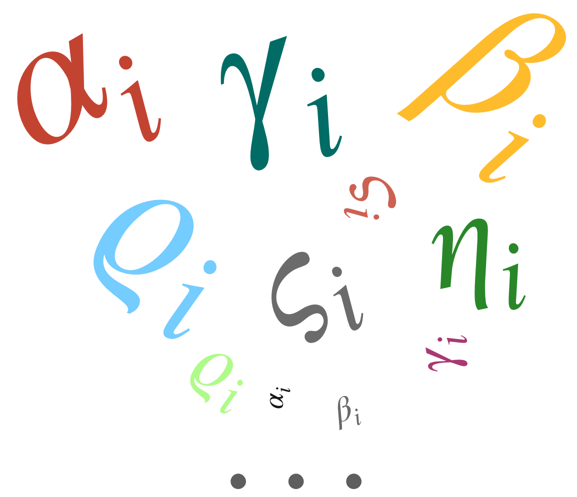
Shortcomings of the traditional approach

EFT with scalar "flavons" ϕ_i

❌ $U_{\text{PMNS}} = U^0_{\text{TBM}} + \ldots \text{corrections}$

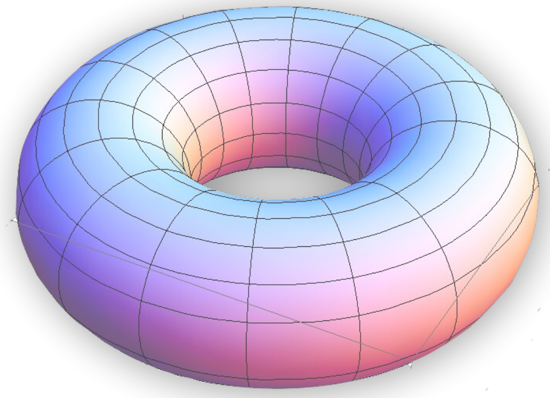
$\downarrow$ $\theta_{13} = 0$ — Automatic

$\downarrow$ $\theta_{13} \approx 8.5^\circ$? → $\delta\theta_{23}\ ,\ \delta\theta_{12} \approx 8.5^\circ$

❌ 

❌ $V(\phi_i) \to$ Mess!

F. Feruglio
[1706.08749]



$\tau \equiv$ modulus

$$(y_{jk}\bar{L}_j H \ell_{Rk} + \text{h.c.})$$

↓

Free structureless parameters
in the SM

→ **Modular Invariance**

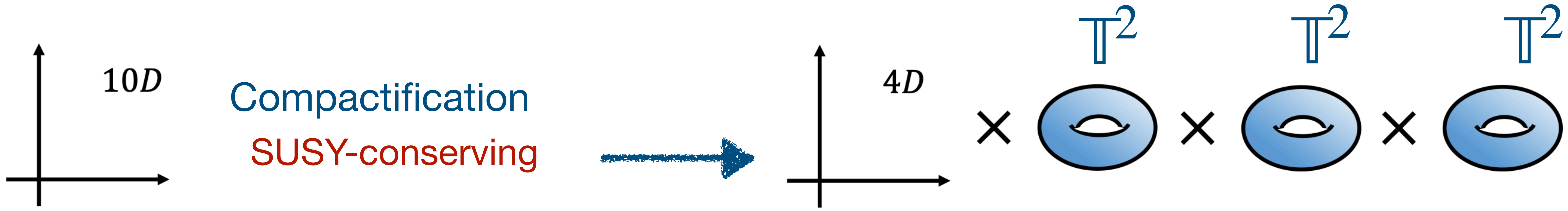
↓

$Y(\tau)$

Modular forms:
less free parameters

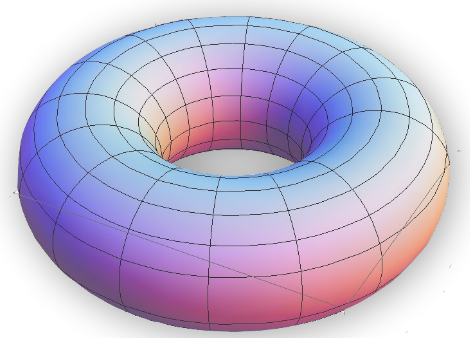
Predictivity ✓
User-friendly ✗

Modular invariance: a step forward?



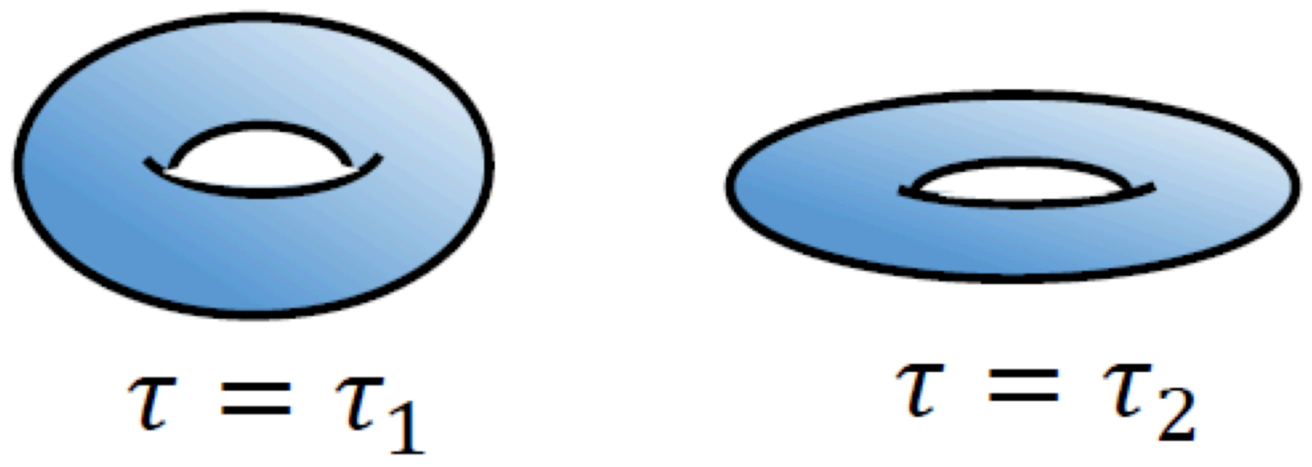
Discrete lattice Λ

$\omega_1, \omega_2 \in \mathbb{C}$

$\mathbb{C}/\Lambda \equiv$ 

$$\tau \equiv \frac{\omega_2}{\omega_1}$$

Modulus



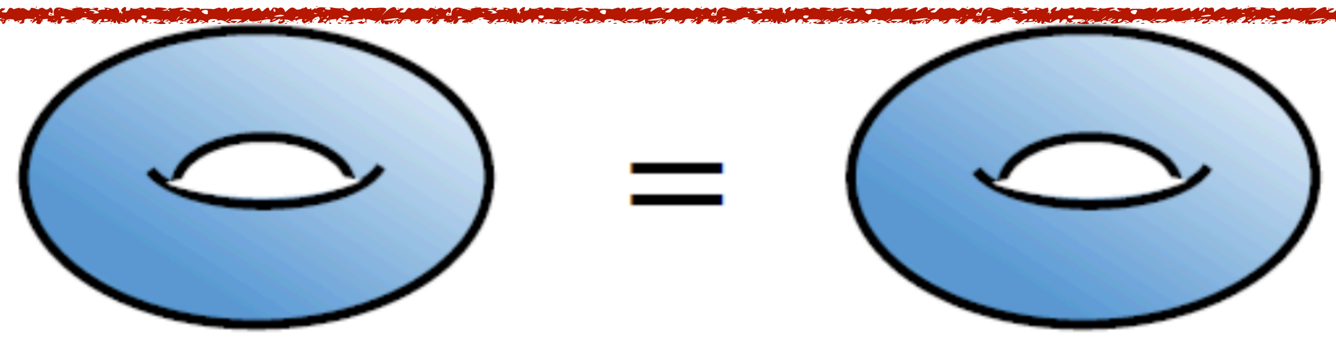
Change basis?

$$\begin{pmatrix}\omega_2' \\ \omega_1'\end{pmatrix} = \gamma \begin{pmatrix}\omega_2 \\ \omega_1\end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d\end{pmatrix}\begin{pmatrix}\omega_2 \\ \omega_1\end{pmatrix} \qquad a, b, c, d \in \mathbb{Z}$$

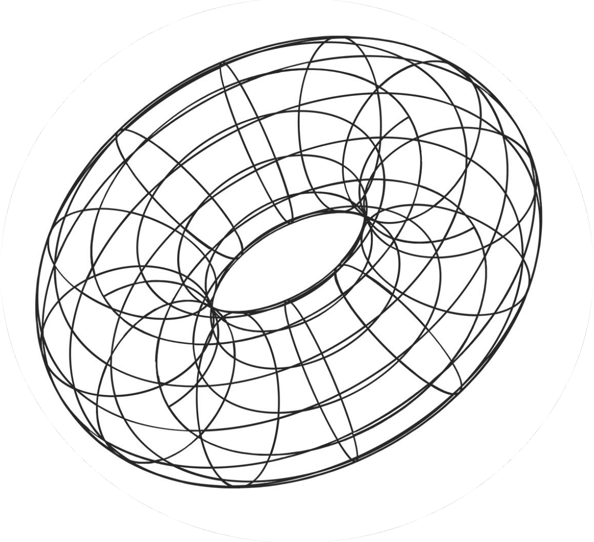
$\gamma \in$ SL$(2,\mathbb{Z})$ $\equiv$ Modular group $\equiv \Gamma$

$$\tau \equiv \frac{\omega_2}{\omega_1} \xrightarrow{\text{SL}(2,\mathbb{Z})} \tau' = \frac{a\,\omega_2 + b\,\omega_1}{c\,\omega_2 + d\,\omega_1} = \frac{a\tau + b}{c\tau + d}$$

$\tau \to \tau'$



τ = τ'



$$\int d^4x\, d^6y\, \mathcal{L}_{10D} \Longrightarrow \int d^4x\, \mathcal{L}_{EFT}$$

What is a modular form? $Y(\tau)$

- $$Y(\gamma(\tau)) = (c\tau + d)^k\, Y(\tau)$$

$c, d \in \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \gamma$

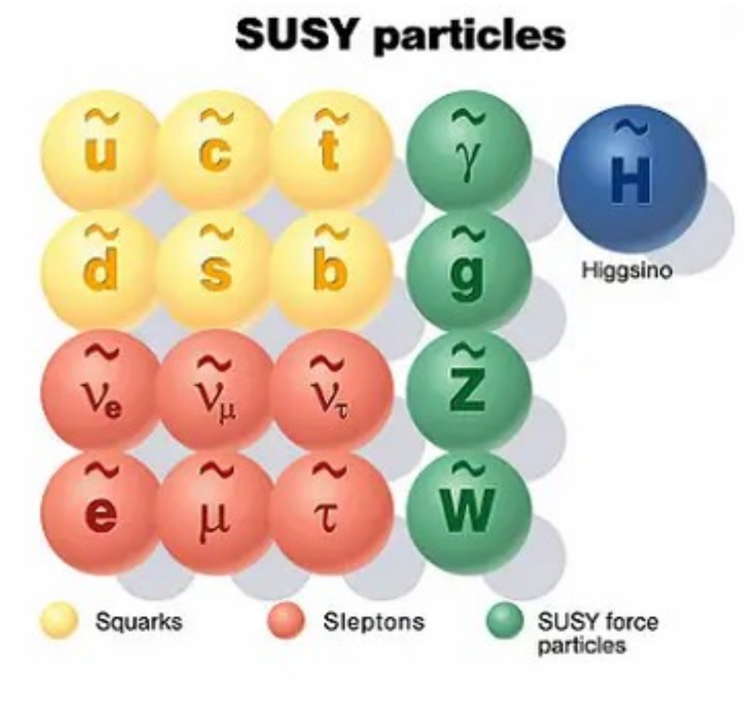
$a, b, c, d \in \mathbb{Z}$, $ad - bc = 1$

Holomorphic in: $\{\tau \in \mathbb{C} \,|\, \mathrm{Im}(\tau) > 0\}$

- "Weight" k>0

Very constraining!

Superfields transformations



$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} & \gamma \in \Gamma \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}$$

Usual matter fields ($\varphi^{(I)}$)

Unitary irrep. of $\Gamma_N \subset \Gamma$ ($\rho^{(I)}(\gamma)$)

$N = 1, 2, 3, \ldots$ "Level"

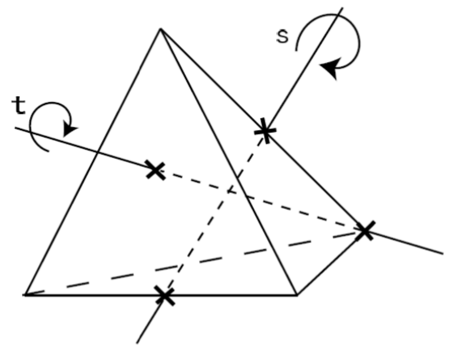
Finite modular group

$$\Gamma_N$$

for $N \leq 5$ isomorphic to

$N = 2$	$N = 3$	$N = 4$	$N = 5$
S_3	A_4	S_4	A_5

non-abelian discrete groups



- $\mathcal{W}(\Phi) = \sum (Y_{I_1 \dots I_n}(\tau)\, \varphi^{(I_1)} \dots \varphi^{(I_n)})_{\mathbf{1}}$

$\mathscr{W}(\Phi)$ modular invariant if:

$$\begin{cases} \rho \otimes \rho_{I_1} \otimes \rho_{I_2} \dots \otimes \rho_{I_n} \supset \mathbf{1} \\ k_Y = k_{I_1} + k_{I_2} + \dots + k_{I_n} \end{cases}$$

$\rho \otimes \rho_{I_1} \otimes \dots \otimes \rho_{I_n} \supset \mathbf{1}$ ⟶ Usual

$k_Y = k_{I_1} + \dots + k_{I_n}$ ⟶ Novelty

$$Y_{I_1 \dots I_n}(\tau) \to (c\tau + d)^{k_Y} \rho(\gamma) Y_{I_1 \dots I_n}(\tau)$$

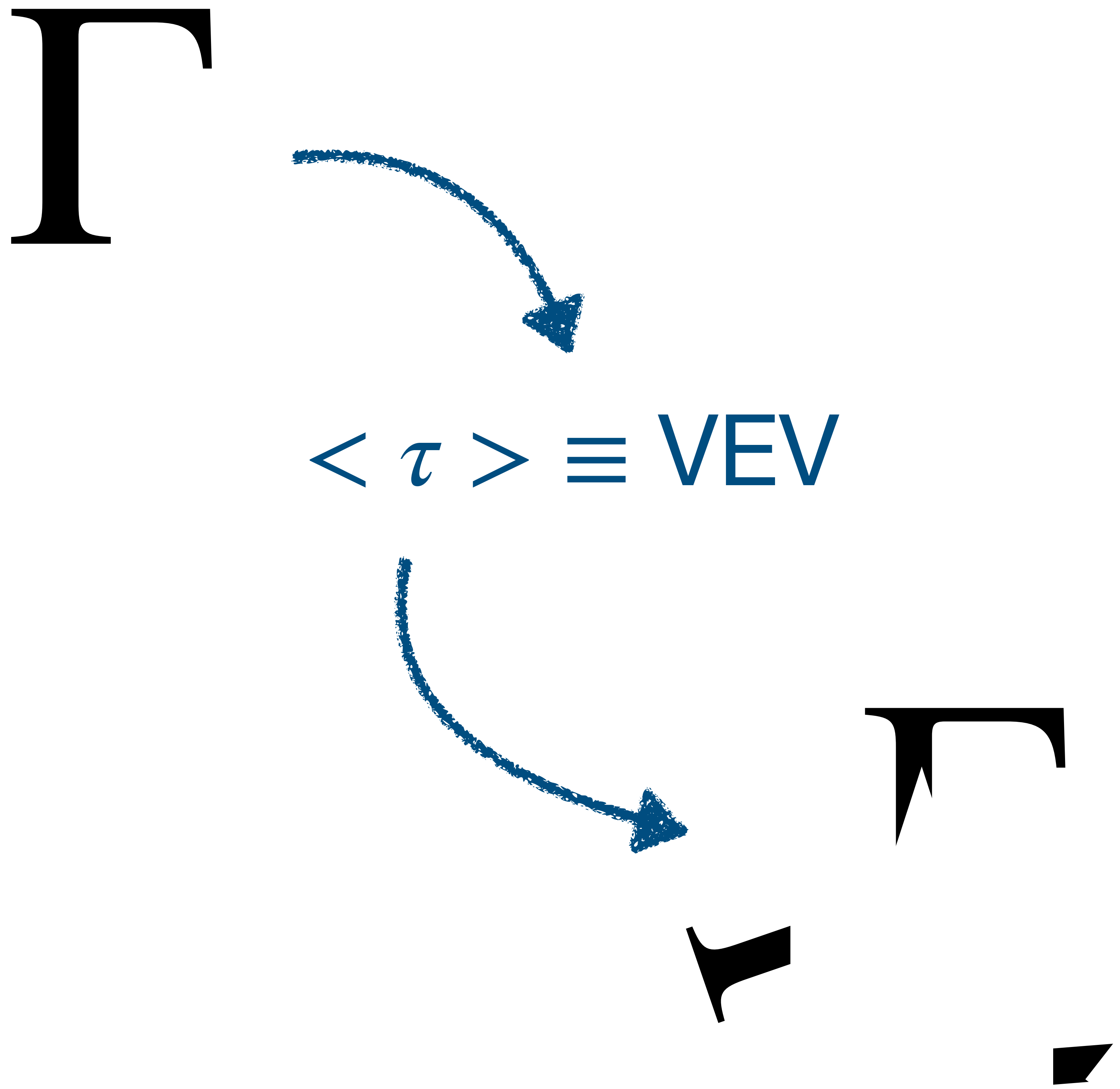
Yukawa: modular forms of weight k_Y

$$\varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)}$$

Superfields with modular charges $-k_I$

$$(c\tau + d)^{k_Y} (c\tau + d)^{-\sum k_{I_n}} = 1$$

Symmetry breaking



$Y(\tau)$

uniquely determined by τ

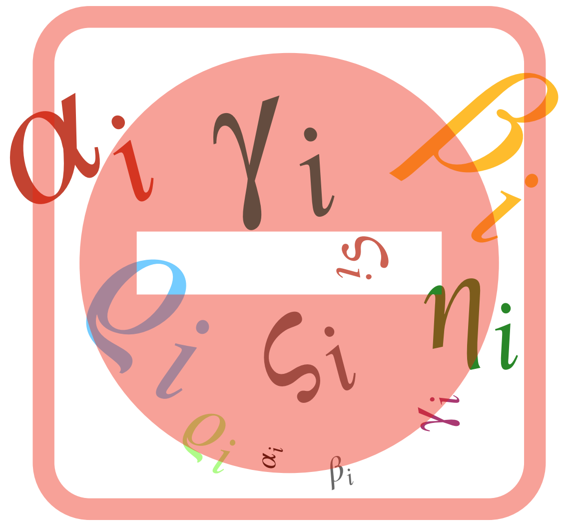
Fourier $\sim \sum_{n}^{\infty} a_n q^n$

$$q \equiv e^{2\pi i \tau}$$

Lepton mass matrices

$$\mathrm{M}_e \sim \sum_i \alpha_i \begin{pmatrix} f_{11}(\tau) & f_{12}(\tau) & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & f_{33}(\tau) \end{pmatrix}$$

$f_{ij} \equiv$ pre-determined functions of τ



$\alpha_i \equiv$ limited number of free parameters ✓

Impose CP symmetry on the model

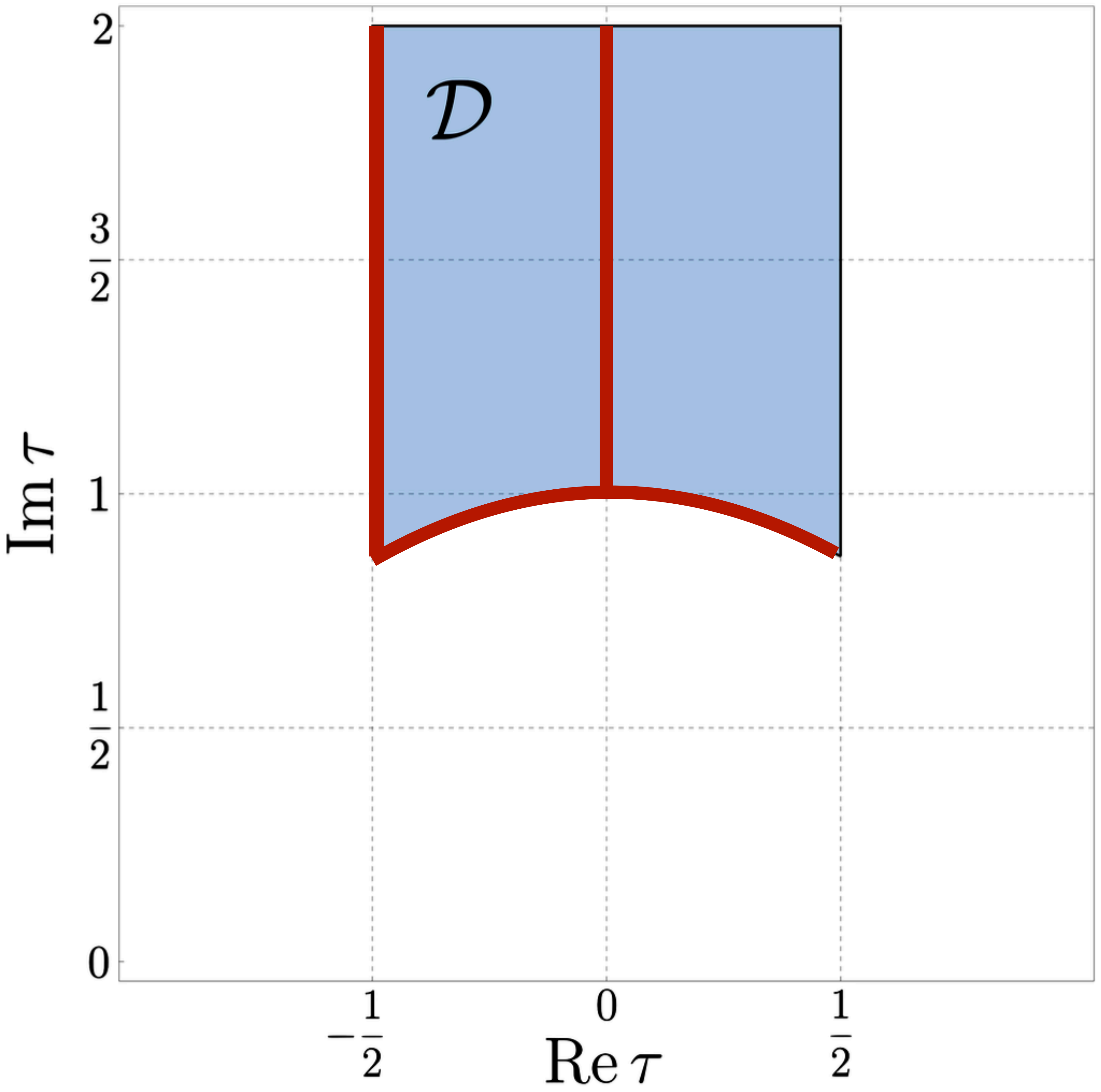
- gCP $\Rightarrow$ $\alpha_i \in \mathbb{R}$

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

Only source of CPV in the model is the VEV of τ

$$<\tau> = \mathrm{Re}\,\tau + i\,\mathrm{Im}\,\tau$$



- Every $\tau \notin \mathscr{D}$ can be mapped in $\tau' \in \mathscr{D}$ through Γ transformation
- CP conserving values

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

Only source of CPV in the model is the VEV of τ

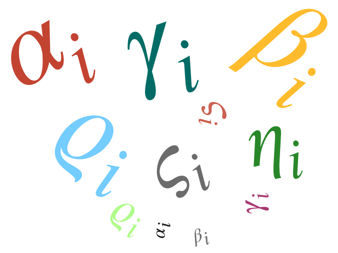
$$\mathcal{D} = \left\{ \tau \in \mathbb{C} \,:\, \mathrm{Im}\,\tau > 0\,,\ \ |\mathrm{Re}\,\tau| \leq \frac{1}{2}\,,\ \ |\tau| \geq 1 \right\}$$

Substantial number of successful models since 2018

$N = 2$

S_3	A_4	S_4	A_5	$\cdots$
✗	✓	✓	✓	

T. Kobayashi, K. Tanaka, T.H. Tatsuishi
Phys. Rev. D **98** (Jul, 2018)

- 

- Extra flavons
- Charged-leptons hierarchy by "hand"

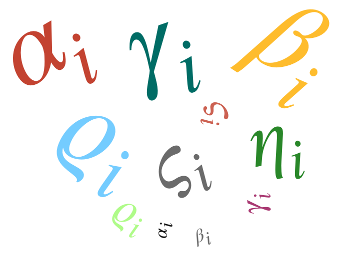
Substantial number of successful models since 2018

$N = 2$

S_3 ✓ A_4 ✓ S_4 ✓ A_5 ✓ $\cdots$

JHEP 09 (2023) 043

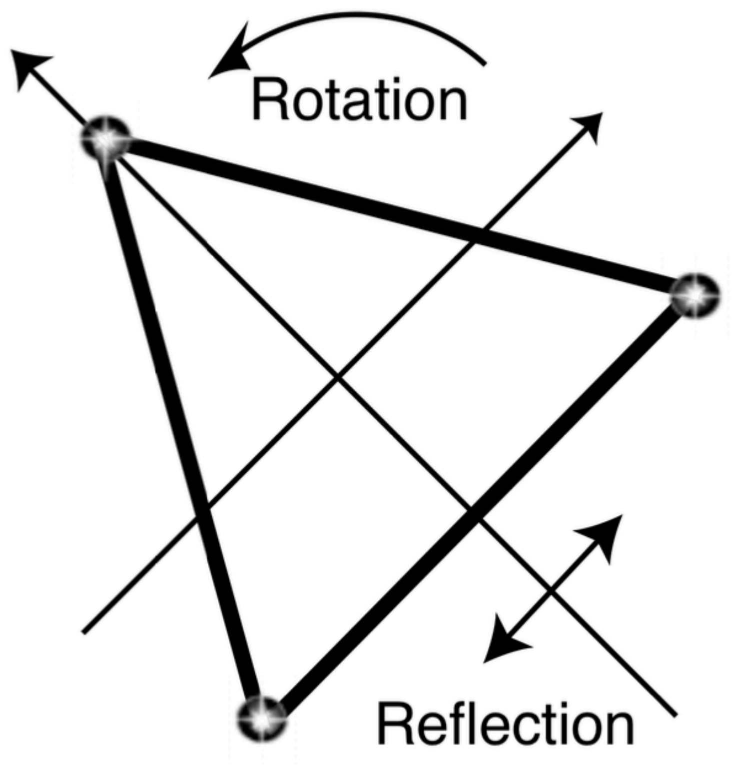
D. Meloni, M.Parriciatu

- Fewer 
- No flavons beside the modulus
- Charged-leptons hierarchy from symmetry

$N = 2$

S_3

Permutation group of 3 objects



- Smallest non-abelian discrete group ✓

- Three irreducible representations

$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$
singlet	pseudo-singlet	doublet

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \to \notin \mathbf{2}\ (S_3)$$

A basis for modular forms of weight k

- $\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)$ ⟶ $Y_1(\tau), Y_2(\tau) \sim \mathbf{2}$

Dedekind's eta function $(q \equiv e^{2\pi i \tau})$

$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}} \to (c\tau + d)^2 \rho(\gamma)_{\mathbf{2}} \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$$

Lowest weight: k=2

Very limited number! ✔

-

N	$d_k(\Gamma(N))$	Γ_N
2	$k/2+1$	S_3

$$\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1} \quad , \quad \mathbf{1}' \otimes \mathbf{2} = \mathbf{2} \quad , \quad \mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$$

Modular suppression

$$Y_1(\tau) = \frac{7}{100} + \frac{42}{25}q + \frac{42}{25}q^2 + \frac{168}{25}q^3 + \dots$$

$$Y_2(\tau) = \frac{14\sqrt{3}}{25}q^{1/2}(1 + 4q + 6q^2 + \dots)$$

Fourier expansion

$\longrightarrow$ $|Y_2(\tau)| \lesssim |Y_1(\tau)|$

$\mathrm{Im}\,\tau \gtrsim 1$

$$(q \equiv e^{2\pi i\,\tau})$$

Following guiding principles...

		E_1^c	E_2^c	E_3^c	D_ℓ	ℓ_3	$H_{d,u}$
	$SU(2)_L \times U(1)_Y$	$(\mathbf{1},+1)$	$(\mathbf{1},+1)$	$(\mathbf{1},+1)$	$(\mathbf{2},-1/2)$	$(\mathbf{2},-1/2)$	$(\mathbf{2},\mp 1/2)$
Irreps	$\Gamma_2 \cong S_3$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}$
Weights	k_I	4	0	-2	2	2	0

$$D_\ell \equiv \begin{pmatrix} \text{electron} \\ \text{muon} \end{pmatrix} \sim \mathbf{2}$$

$$\ell_3 \equiv \text{tau} \sim \mathbf{1}'$$

Following guiding principles...

$$\mathcal{W}_e^H = \alpha E_1^c H_d (D_\ell Y_{\mathbf{2}}^{(3)})_{\mathbf{1}} + \beta E_2^c H_d (D_\ell Y_{\mathbf{2}})_{\mathbf{1}'} + \gamma E_3^c H_d \ell_3 + \alpha_D E_1^c H_d \ell_3 Y_{\mathbf{1}'}^{(3)}$$

$$(m_\tau, m_\mu, m_e) \sim m_\tau(1, |Y_1|, |Y_1^3|)$$

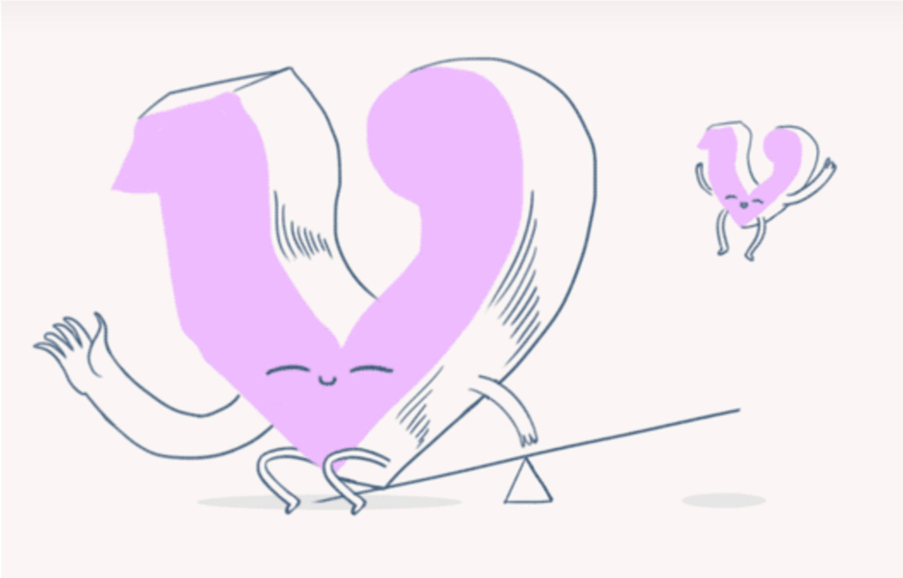
$$|Y_1| \sim \mathcal{O}(10^{-2})$$

$$M_\ell^\dagger = \begin{pmatrix} \alpha(Y_{\mathbf{2}}^{(3)})_1 & \alpha(Y_{\mathbf{2}}^{(3)})_2 & \alpha_D Y_{\mathbf{1}'}^{(3)} \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix} v_d$$

Modular invariance: texture zeros!

Charged-leptons masses reproduced with:

$$\frac{\beta}{\alpha} \sim \frac{\gamma}{\alpha} \sim \frac{\alpha_D}{\alpha} \approx \mathcal{O}(1)$$



Minimal seesaw scenario

2 RHN $\sim \mathbf{2}$ under S_3 weight 2

$$\begin{pmatrix} N_1^c \\ N_2^c \end{pmatrix} \sim \mathbf{2}$$

$$\mathcal{W}_\nu = g H_u N^c D_\ell Y_{\mathbf{2}}^{(2)} + g' H_u (N^c Y_{\mathbf{2}}^{(2)})_{\mathbf{1}'} \ell_3 + g'' H_u (N^c D_\ell)_{\mathbf{1}} Y_{\mathbf{1}}^{(2)} + \Lambda[(N^c N^c)_{\mathbf{2}} Y_{\mathbf{2}}^{(2)} + \lambda (N^c N^c)_{\mathbf{1}} Y_{\mathbf{1}}^{(2)}] \,,$$

Dirac mass term

Majorana mass term

$$M_D = g v_u \begin{pmatrix} -(Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & 2Y_1Y_2 & \frac{g'}{g}(2Y_1Y_2) \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & -\frac{g'}{g}(Y_2^2 - Y_1^2) \end{pmatrix}_{\mathrm{RL}}$$

$$m_\nu = -M_D^T \mathcal{M}_R^{-1} M_D \,.$$

$$\mathcal{M}_R = \Lambda \begin{pmatrix} -(Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) & 2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) \end{pmatrix}_{\mathrm{RR}}$$

$g''/g, \quad g'/g, \quad \lambda$

Free dimensionless parameters

- Fit VS 7 dimensionless observables

$$\left\{ \sin^2\theta_{23}, \sin^2\theta_{13}, \sin^2\theta_{12}, m_e/m_\mu, m_\mu/m_\tau, \frac{\Delta m^2_{atm}}{\Delta m^2_{sol}}, J_{CP} \right\}$$

$$\langle\tau\rangle = \mathrm{Re}\,\tau + i\,\mathrm{Im}\tau$$

Only source of CPV

$$q \equiv e^{2\pi i\tau}$$

Parameter	Best-fit value and 1σ range	
	NO	**IO**
$r \equiv \Delta m^2_{\rm sol}/\lvert\Delta m^2_{\rm atm}\rvert$	0.0295 ± 0.0008	0.0298 ± 0.0008
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.303^{+0.012}_{-0.011}$
$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.0223^{+0.00058}_{-0.00058}$
$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.569^{+0.016}_{-0.021}$
$J_{\rm CP}$	$-0.027^{+0.010}_{-0.010}$	$-0.032^{+0.007}_{-0.007}$
m_e/m_μ	0.0048 ± 0.0002	
m_μ/m_τ	0.0565 ± 0.0045	

NuFIT 5.2
I. Esteban et al. 2007.14792

J_{CP} ✓

$\sin^2\theta_{12}$ ✓

$\sin^2\theta_{13}$ ✓

$\sin^2\theta_{23}$ ✓

$r = \dfrac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}$ ✓

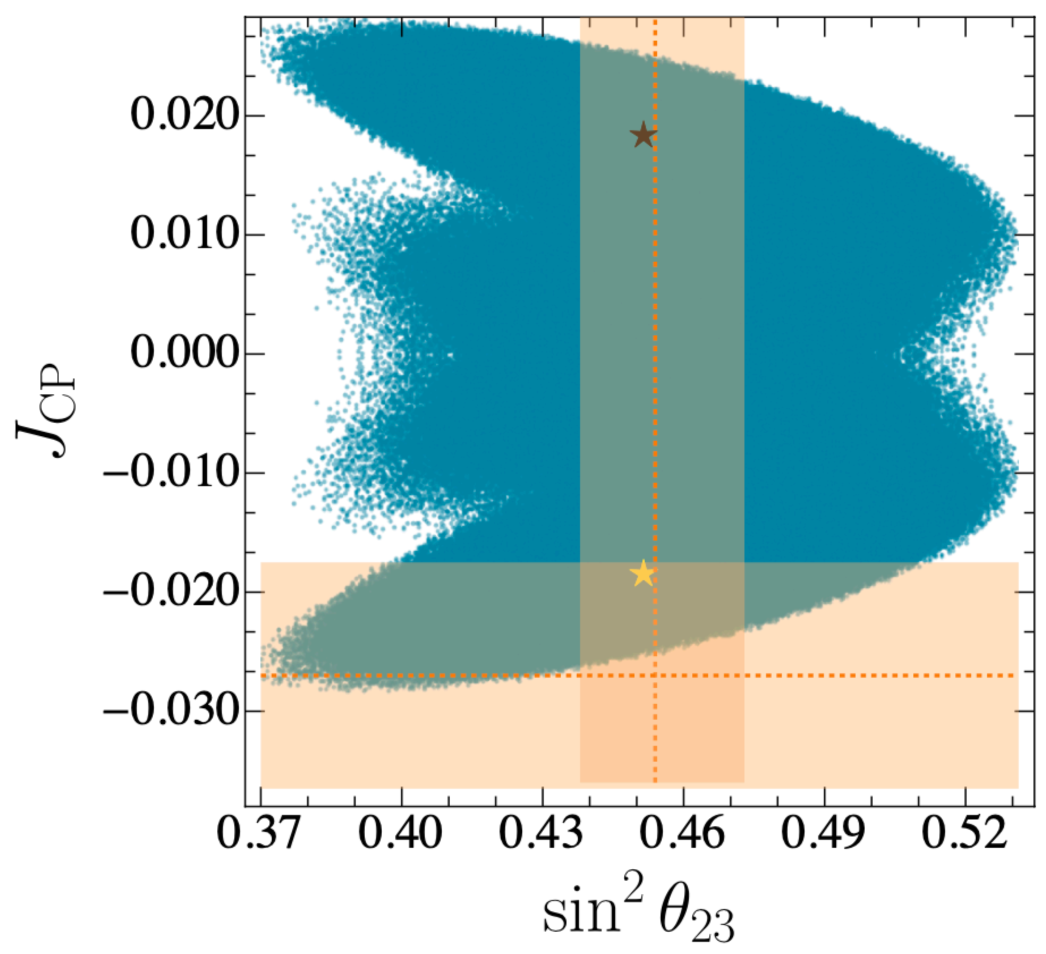
All reproduced within 1σ range

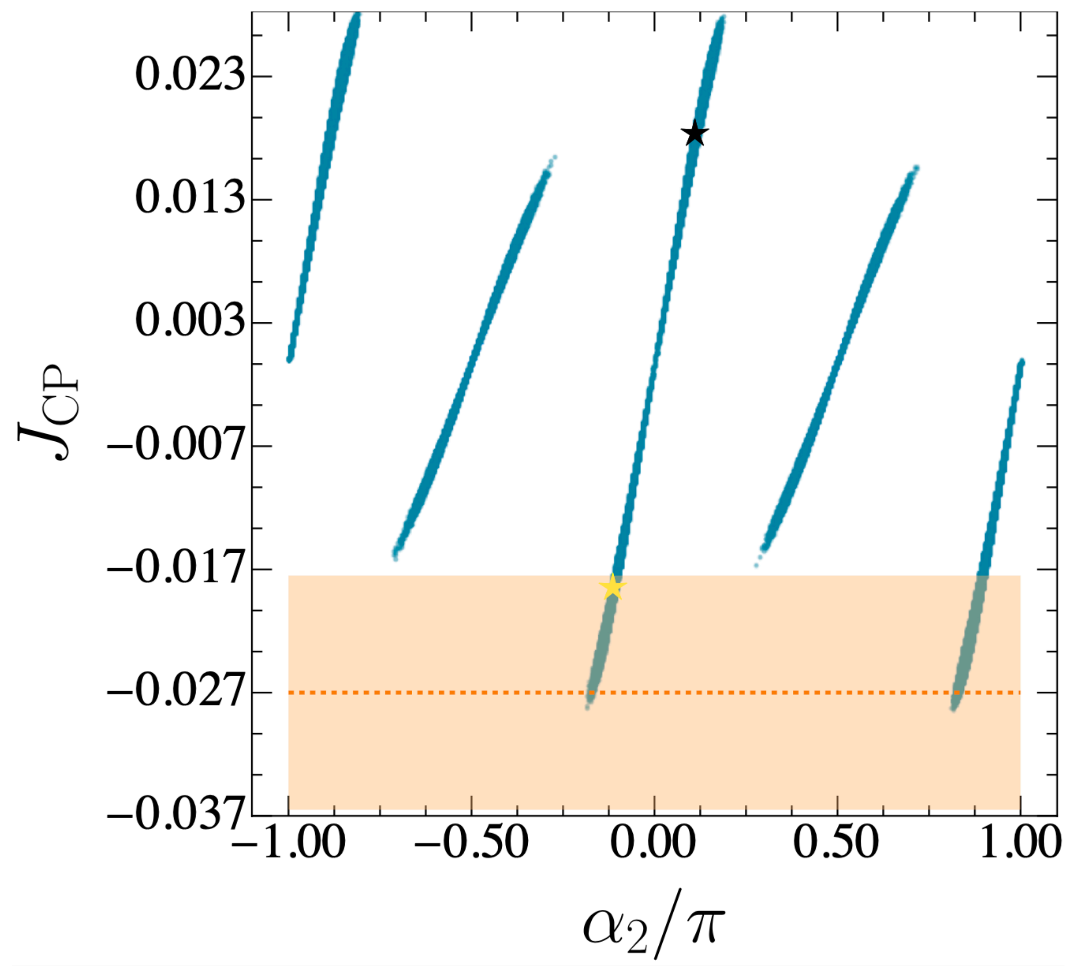
- One massless neutrino, Normal Ordering
- Excellent fit: $\chi^2 \sim 0.98$

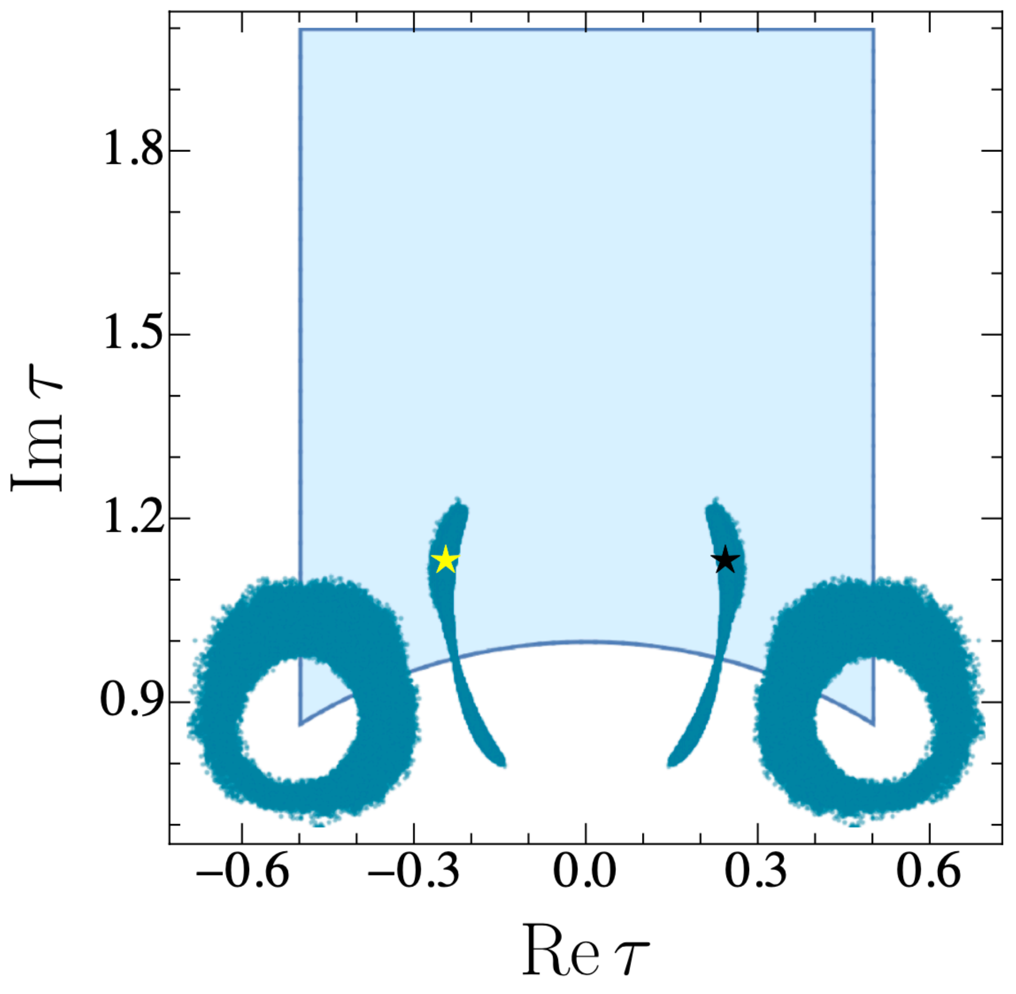
Ordering	NO
J_{CP}	$-0.018^{+0.002}_{-0.002}$
α_1/π	0
α_2/π	$\pm 0.112^{+0.792}_{-0.014}$
m_1 [meV]	0
m_2 [meV]	$8.620^{+0.095}_{-0.123}$
m_3 [meV]	$50.806^{+0.016}_{-0.021}$
$\sum_i m_i$ [eV]	$0.0594^{+0.0001}_{-0.0001}$
$\lvert m_{\beta\beta}\rvert$ [meV]	$3.61^{+0.09}_{-0.09}$
$m_\beta^{\rm eff}$ [meV]	$8.90^{+0.10}_{-0.09}$
$\chi^2_{\rm min}$	0.98

Neutrinoless double beta decay

Single beta decay (KATRIN)







★ Best fit

★ CP-symmetric solution $\delta_{CP} \to -\delta_{CP}$

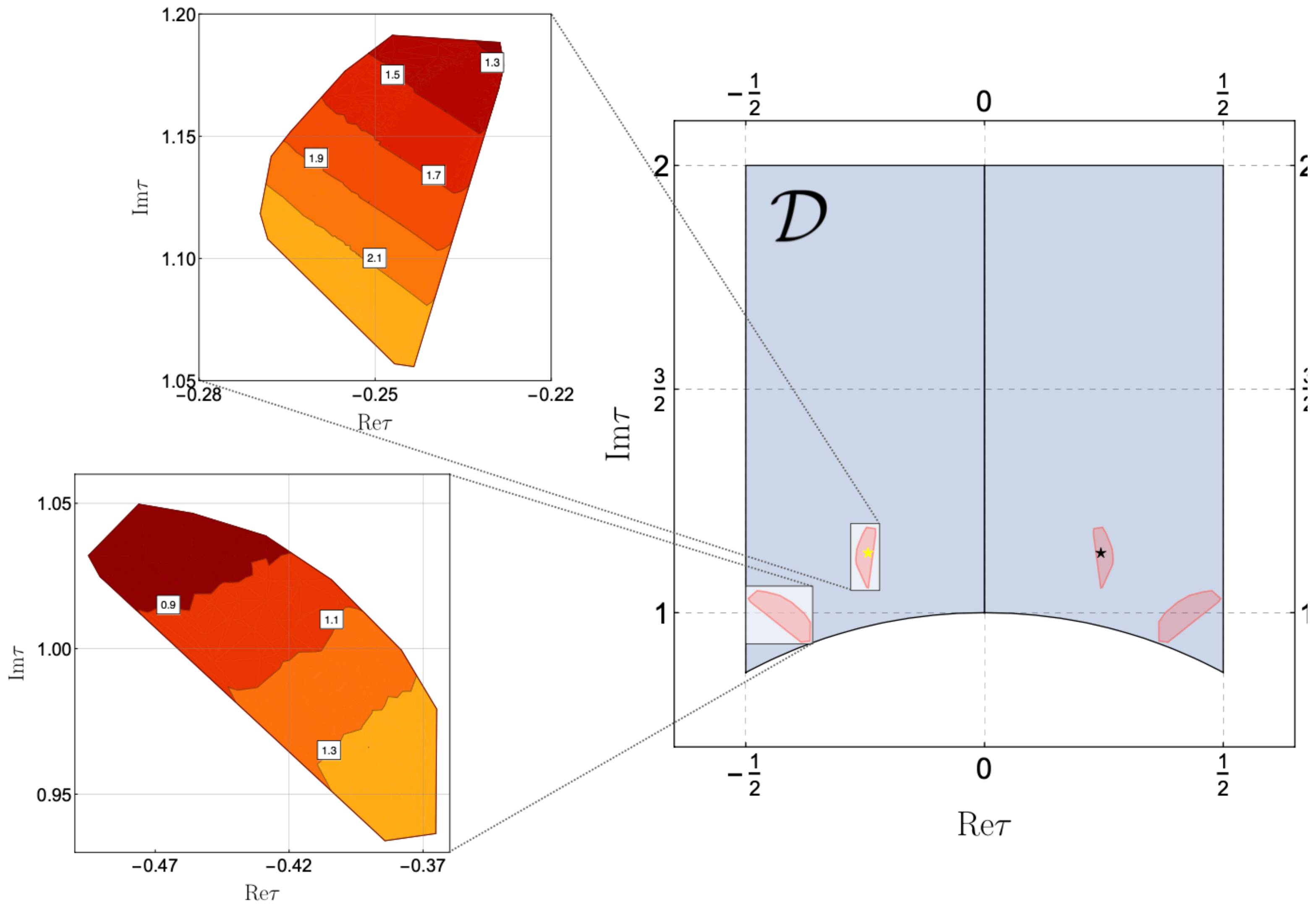
All points satisfy $\sqrt{\Delta\chi^2} < 5$

✔ Low-energy CP-violation

✔ Baryon Asymmetry through Leptogenesis

JHEP 05 (2024) 020

S. Marciano, D. Meloni, M.Parriciatu



N1-dominated decays of heavy neutrinos for specific regions of $\mathscr{D}$

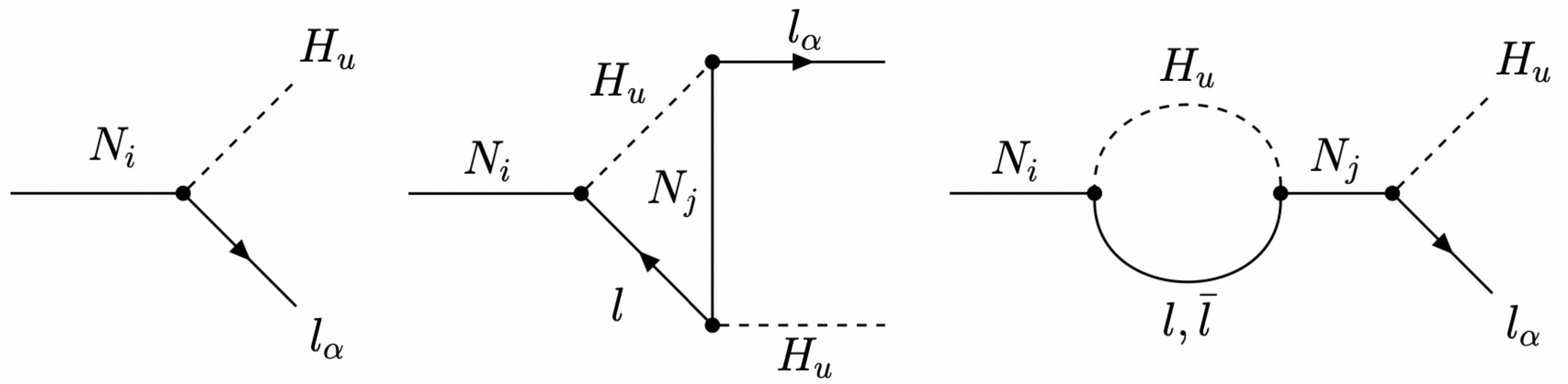
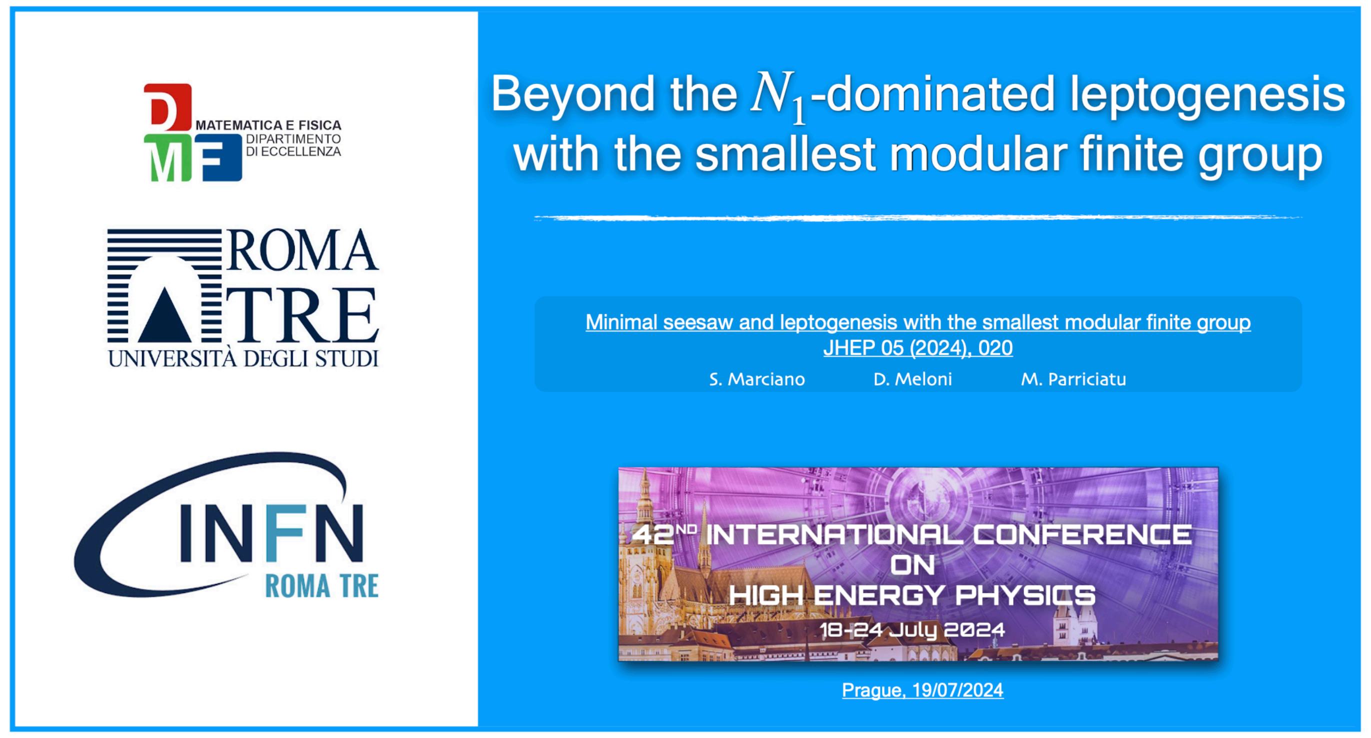


Figure 2: *CP-violating N_i decay.*

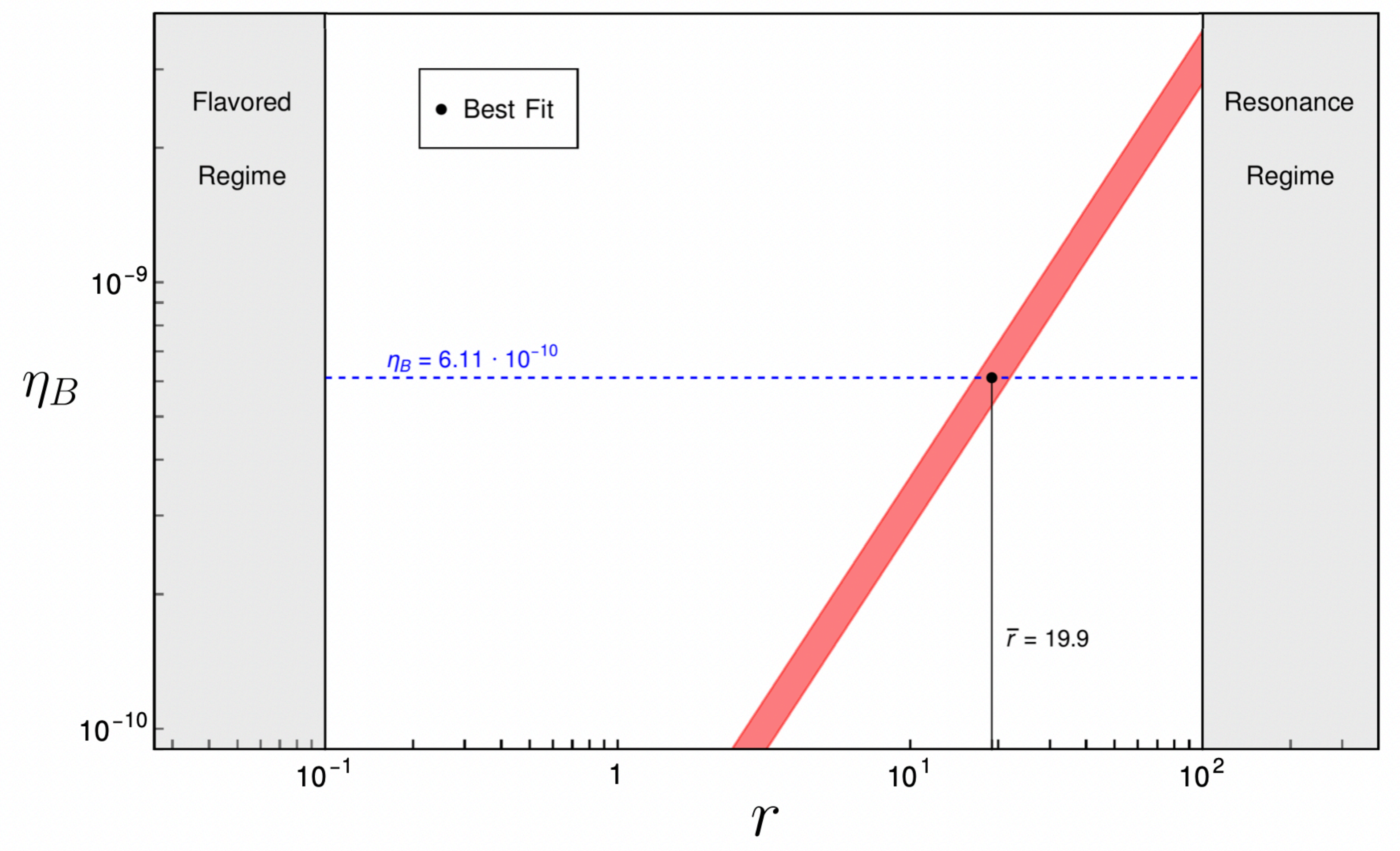
See S. Marciano's talk at 12:15 this morning



- ✔ Low-energy CP-violation
- ✔ Baryon Asymmetry through Leptogenesis

Not trivial!

Only source of CP-violation is Re τ



$\Lambda \to r\Lambda$ Majorana mass scale not completely fixed by low-energy

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S. Marciano, D. Meloni, M.Parriciatu

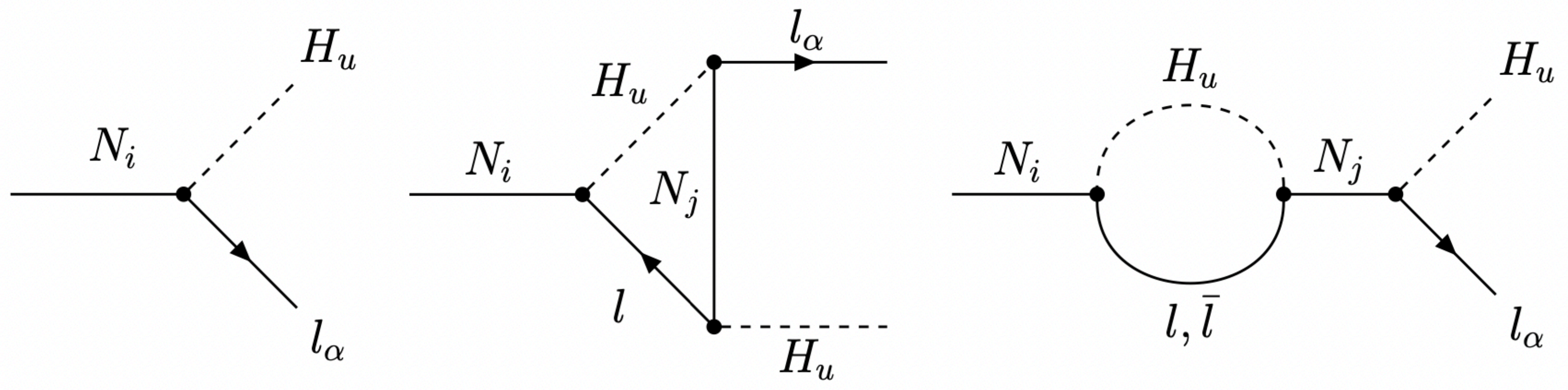
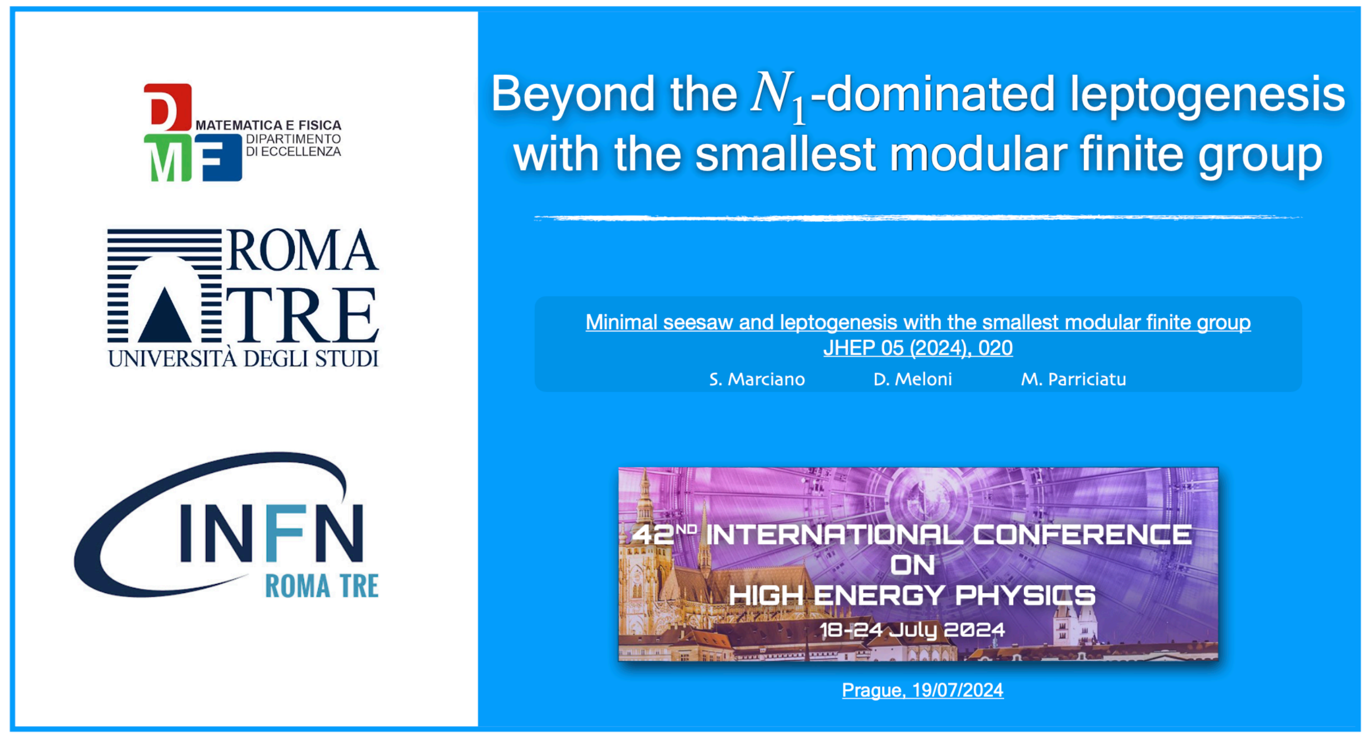


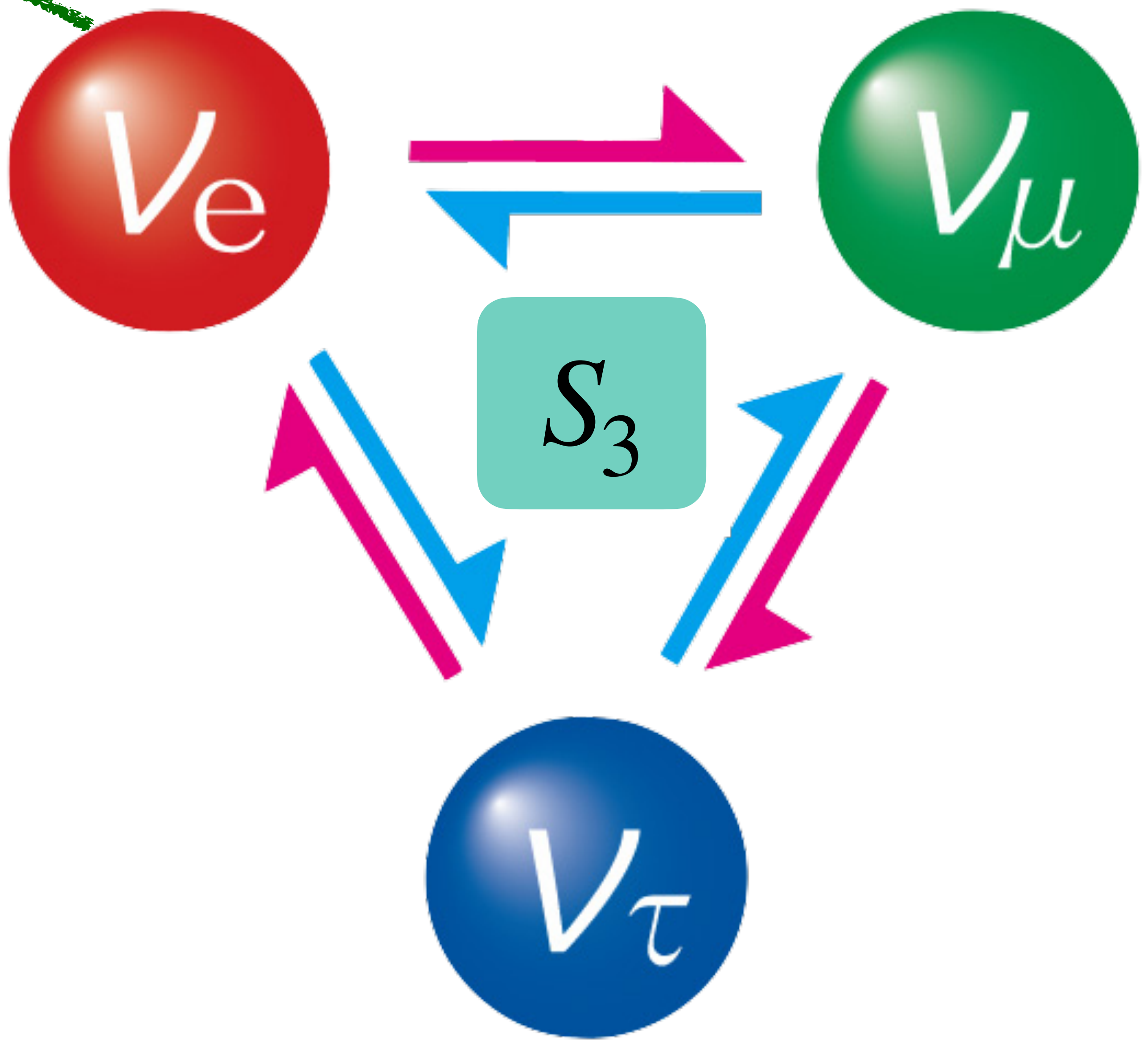
Figure 2: *CP-violating N_i decay.*

See S. Marciano's talk at 12:15 this morning



Quarks with S_3?

F.Feruglio, M.Parriciatu, A. Strumia, A. Titov 2406.01689



Simple model:
$\Gamma_2 \to$ few Yukawa forms

Low energy CP violation and Baryon Asymmetry using only one modulus

ICHEP 2024

Thank you for the attention

Roma Tre Neutrino Group

SCAN ME

BACKUP SLIDES

Backup slides: Clebsch-Gordan

- Clebsch-Gordan coefficients for S_3

$$\mathbf{2}\otimes\mathbf{2}=\mathbf{1}\otimes\mathbf{1}'\otimes\mathbf{2}\begin{cases}\mathbf{1} & \sim \quad \psi_1\varphi_1+\psi_2\varphi_2\\ \mathbf{1}' & \sim \quad \psi_1\varphi_2-\psi_2\varphi_1\\ \mathbf{2} & \sim \quad \begin{pmatrix}\psi_2\varphi_2-\psi_1\varphi_1\\ \psi_1\varphi_2+\psi_2\varphi_1\end{pmatrix}\end{cases}$$

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\mathbf{2}} \qquad \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}_{\mathbf{2}}$$

S_3 doublets

$$\mathbf{1}'\otimes\mathbf{1}'=\mathbf{1} \quad \sim \quad y_1y_2$$

$$\mathbf{1}'\otimes\mathbf{2}=\mathbf{2} \quad \sim \quad \begin{pmatrix}-y_1\psi_2\\ y_1\psi_1\end{pmatrix}$$

$y_1, y_2 \equiv$ pseudo-singlets $(\mathbf{1}')$

e.g. $(D_\ell Y_{\mathbf{2}}(\tau))_{\mathbf{1}'} = (D_\ell)_1(Y_{\mathbf{2}}(\tau))_2 - (D_\ell)_2(Y_{\mathbf{2}}(\tau))_1 \sim \mathbf{1}'$

The Modular symmetry approach

Modular-invariant SUSY action

$$\mathcal{S} = \int d^4x \int d^2\theta d^2\bar{\theta}\, K(\Phi, \bar{\Phi}) + \left[\int d^4x \int d^2\theta\, \mathcal{W}(\Phi) + \text{h.c.}\right]$$

$\sigma \equiv \Lambda_\tau \tau$

Kähler potential

- Gives the kinetic terms after the modulus acquires a VEV
- A minimalistic form is chosen

Superpotential

- Holomorphic function of superfields
- Encodes the Higgs Yukawa interactions

The superfields transform as:

$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}, \quad \text{with} \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_N$$

Action is invariant if, under Γ_N:

$$\begin{cases} \mathcal{W}(\Phi) \to \mathcal{W}(\Phi) \\ K(\Phi, \bar{\Phi}) \to \underbrace{K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi})}_{\text{Kähler transformation}} \end{cases}$$

- $\theta, \bar{\theta}$ Grassmann spinor coordinates
- $\Phi = (\tau, \varphi)$ Chiral superfields
- φ Usual matter supermultiplets
- $\rho(\gamma)$ Unitary representation of Γ_N

The Kähler potential...

- Minimalist choice for the Kähler

$$K(\Phi,\bar{\Phi}) = -h\Lambda_\tau^2 \log(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\varphi^{(I)}|^2$$

$\Lambda_\tau \equiv$ dimensions of mass

Satisfies $K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + f(\Phi) + f(\bar{\Phi})$ under Γ_N

$h \equiv$ positive constant

- In a bottom-up approach, this is just a choice

- Corrections of the Kähler potential can spoil the predictivity of the model

M.-C. Chen, S. Ramos-Sánchez, and M. Ratz, "A note on the predictions of models with modular flavor symmetries," *Physics Letters B* **801** (Feb, 2020) 135153.

- This question is an open one

The Modular symmetry approach

The group generators

- Finite modular group can be defined: $\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}) \middle| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

subgroups of Γ N=1,2,3...called "level"

$$\overline{\Gamma} \equiv \Gamma/\{\pm \mathbb{I}\}$$
$$\overline{\Gamma}(N) \equiv \Gamma(N)/\{\pm \mathbb{I}\}$$

- Generators S and T of the modular group Γ_N

$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau + 1 \qquad S^2 = T^N = (ST)^3 = \mathbb{I}$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

- S_3 Generators S and T satisfy:

$$S^2 = T^2 = (ST)^3 = \mathbb{I}$$

$$\rho(S) = \frac{1}{2} \begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I},$$

The Modular S_3 model: lowest weights

Level 2 modular forms of lowest weight (2) constructed from Dedekind's Eta "seed function"

- $\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \quad , \quad q \equiv e^{2\pi i \tau}$

$$\{\eta(\tau/2), \eta\left(\frac{\tau+1}{2}\right), \eta(2\tau)\}$$

Closed set under the modular group

$$\alpha + \beta + \gamma = 0$$

$$Y(\alpha, \beta, \gamma|\tau) = \frac{d}{d\tau}[\alpha \log \eta(\tau/2) + \beta \log \eta((\tau+1)/2) + \gamma \log \eta(2\tau)]$$

This fixes the constants

$$\begin{cases} Y_1(\tau) = \frac{C}{2}\left[\frac{\eta'(\tau/2)}{\eta(\tau/2)} + \frac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)} - \frac{8\eta'(2\tau)}{\eta(2\tau)}\right] \\ Y_2(\tau) = \frac{C}{2}\sqrt{3}\left[\frac{\eta'(\tau/2)}{\eta(\tau/2)} - \frac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)}\right] \end{cases}$$

"C" arbitrary

Impose transformation properties under S_3 generators

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I},$$

- $\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}} \to (c\tau + d)^2 \rho(\gamma)_{\mathbf{2}} \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$

The Modular S_3 model: the normalisation

Level 2 modular forms of lowest weight (2) constructed from Dedekind's Eta "seed function"

- $\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \quad , \quad q \equiv e^{2\pi i \tau}$ $\longrightarrow$ $\{\eta(\tau/2), \eta\left(\frac{\tau+1}{2}\right), \eta(2\tau)\}$

Closed set under the modular group

"C" arbitrary

$$\begin{cases} Y_1(\tau) = \frac{C}{2}\left[\frac{\eta'(\tau/2)}{\eta(\tau/2)} + \frac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)} - \frac{8\eta'(2\tau)}{\eta(2\tau)}\right] \\ Y_2(\tau) = \frac{C}{2}\sqrt{3}\left[\frac{\eta'(\tau/2)}{\eta(\tau/2)} - \frac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)}\right] \end{cases}$$

Impose CP symmetry on the model

- Superpotential parameters must be real: less free parameters

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

$$Y(\tau) \xrightarrow{\text{CP}} Y(-\tau^*) = Y^*(\tau)$$

- In our case, this is true if C is purely imaginary

$$C = \frac{7i}{25\pi}$$

The choice made in this work

Only source of CPV is the VEV of modulus τ

Backup slides

Numerical procedure

- Define a "figure of merit", i.e. chi-square for every set of parameters $l(p_i) \equiv \sqrt{\chi^2(p_i)}$

$$\chi^2(p_i) = \sum_{j=1}^{6} \left(\frac{q_j(p_i) - q_j^{\text{b-f}}}{\sigma_j} \right)^2$$

$$p_i = \{\tau, \beta/\alpha, \gamma/\alpha, ..., g'/g, g_p/g, ...\}$$

$$q_j = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, m_e/m_\mu, m_\mu/m_\tau, r\}$$

- Define a "potential" with a given temperature T and a threshold

$$V(p_i) = \begin{cases} l(p_i) & , \quad l(p_i) \le l_{\max} \\ +\infty & , \quad \text{otherwise} \end{cases}$$

P. P. Novichkov, J. T. Penedo, S. T. Petcov, and A. V. Titov, "Modular S_4 models of lepton masses and mixing," (2019)

- At iteration "t", generate a new point from a Gaussian centred on the previous one → Accept the new point with a probability given by:

$$P_\alpha = \min[1, \exp(V(p_i^{(t)}) - V(p_i'))/T]$$

Backup slides

A measure of fine-tuning:
Altarelli-Blankenburg

$$\text{Fine-tuning} = \frac{\sum_i \left|\frac{\text{par}_i}{\delta\text{par}_i}\right|}{\sum_i \left|\frac{\text{obs}_i}{\sigma_i}\right|}$$

- δpar_i The shift of the parameter from the point of minimum, which increases the chi-square by one unit, while keeping all other parameters fixed.
- σ_i Experimental errors for each observable as extracted from NuFIT

Non-standard interactions

tests of modulus couplings

G-J. Ding, FF, 2003.13448

non standard neutrino interactions

$$\mathcal{L} = i \sum_{f=e,e^c,\nu} \bar{f}\,\bar{\sigma}^\mu \partial_\mu f + \frac{1}{2}\partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha - \frac{1}{2} M_\alpha^2 \varphi_\alpha^2$$
$$- (m_e + \mathcal{Z}_\alpha^e \varphi_\alpha) e^c e - \frac{1}{2}\nu(m_\nu + \mathcal{Z}_\alpha^\nu \varphi_\alpha)\nu + h.c. + \ldots .$$

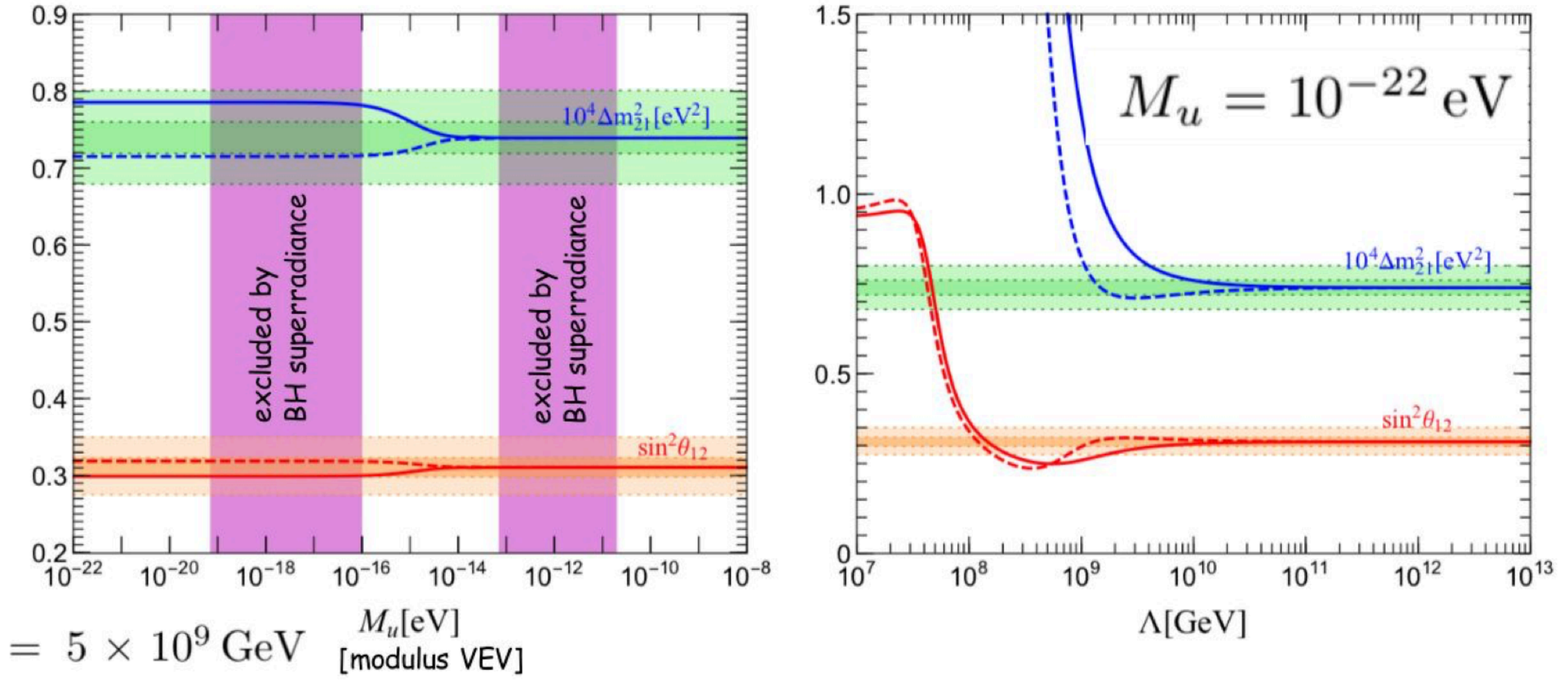
$$\tau = \langle\tau\rangle + \frac{\varphi_u + i\,\varphi_v}{\sqrt{2}}$$

in medium with non-zero electron number density

small, unless the modulus is very light

$$\delta m_\nu(0) = -n_e^0 \frac{\mathrm{Re}(\mathcal{Z}^e)\mathcal{Z}^\nu}{M^2(R)},$$

in the sun:



$\Lambda = 5 \times 10^9$ GeV

from Feruglio's slides at Mod. Symmetry Bethe Workshop

Bottom-up problems

Kähler potential

Normalisation of the modular forms

Modulus stabilization

SUSY-dependent theory