Search for CPT and Lorentz Invariance Violation Effects in the Muon g-2 Experiment at Fermilab

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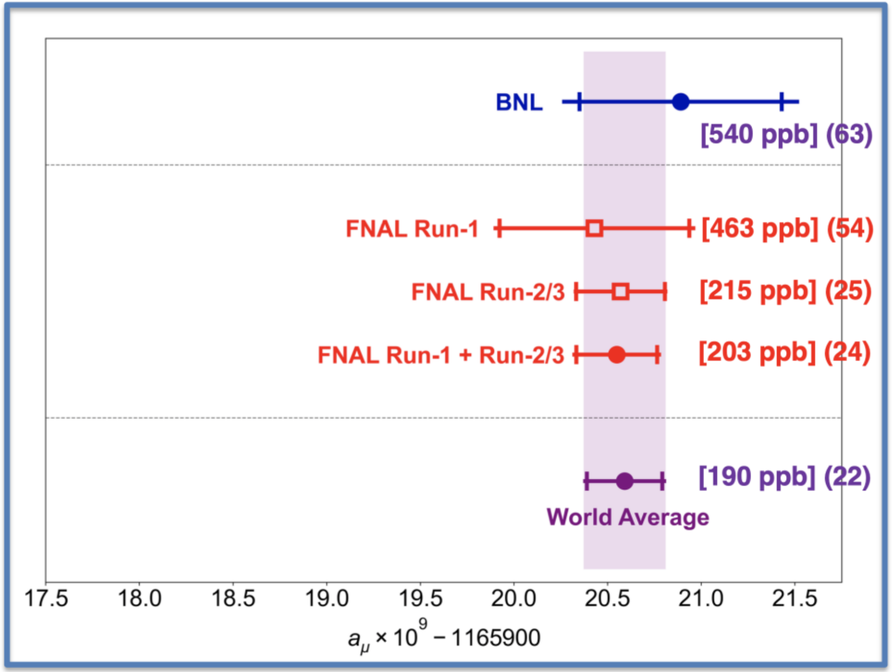
Introduction

Fermilab Muon $g-2$ Run 2/3 result

The world's most precise measurement yet of the anomalous magnetic moment of the muon.

a_μ(FNAL) = 116 592 055(24) x10^{-11} [203 ppb]



- FNAL combination: **203 ppb** uncertainty
- Both FNAL and BNL dominated by statistical error
- Combined world average **dominated by FNAL** values.

arXiv:2402.15410 [hep-ex]

a_μ(Exp) = 116 592 059(22) x 10^{-11} [190 ppb]

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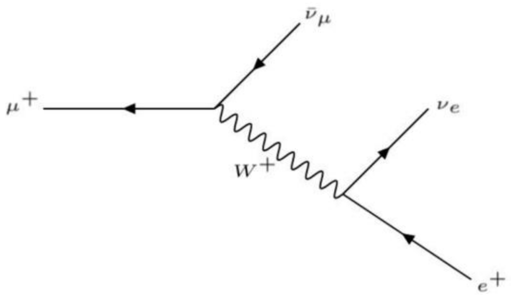
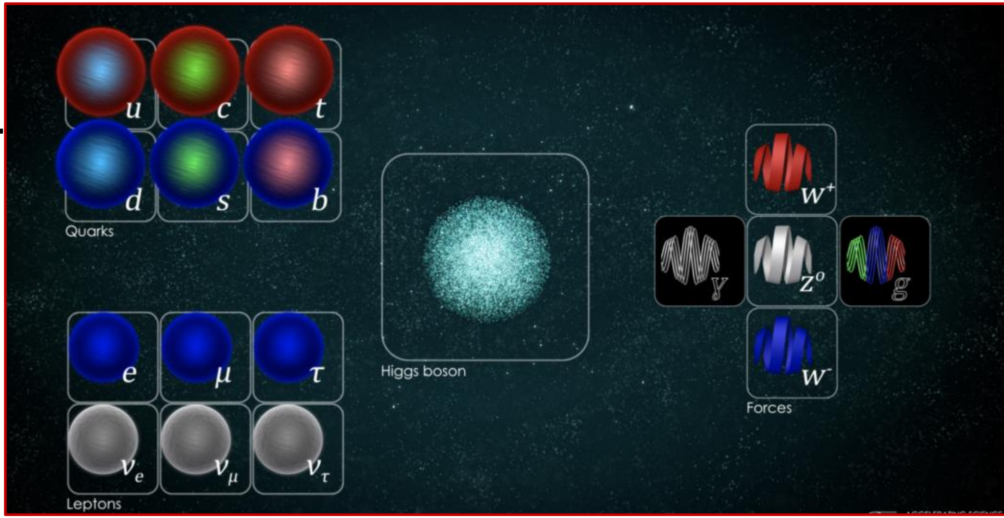
The theory behind the Fermilab Muon $g - 2$ experiment.

The Standard Model of Particle Physics (SM) and Muon

- Fundamental building blocks of SM.

- The muon is a similar cousin of the electron.
  - Same charge.
  - Same Spin.
  - ~ 200 times heavier than electron.
  - Rest lifetime $2.2\ \mu s$.



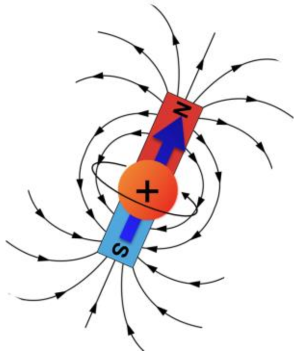


CERN:Infographics Gallery

Magnetic Moment

- A particle with charge and spin will act like a tiny bar magnet, with a magnetic moment:

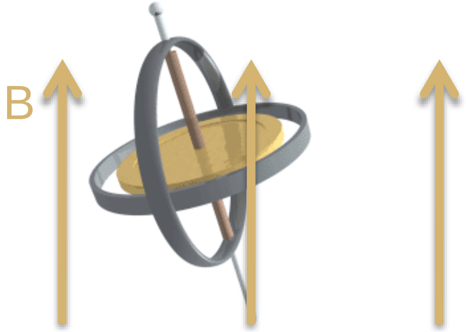
$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$



- If that particle is placed in a magnetic field, $\vec{B}$, the field will exert a torque on the particle, causing its spin axis to precess around the magnetic field direction. $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$

- Much like a spinning top in a gravitational field.

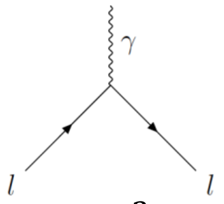
- The g factor affects how strong the interaction is, and thus how fast the spin precession is.



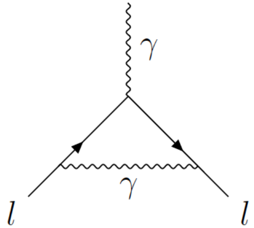
Magnetic Moment: g factor

- Classical physics: $g = 1$
- Relativistic quantum mechanics prediction for a point-like particle: $g = 2$ (Dirac, 1928).
- For electron, experimentally found to be $g_e = 2.00238(10)$.
- Schwinger theoretically calculated $g_e = 2.00232$
- We are interested in the difference between the g factor and 2.
- Define anomalous magnetic moment as

$$a = \frac{g-2}{2}$$



$g = 2$

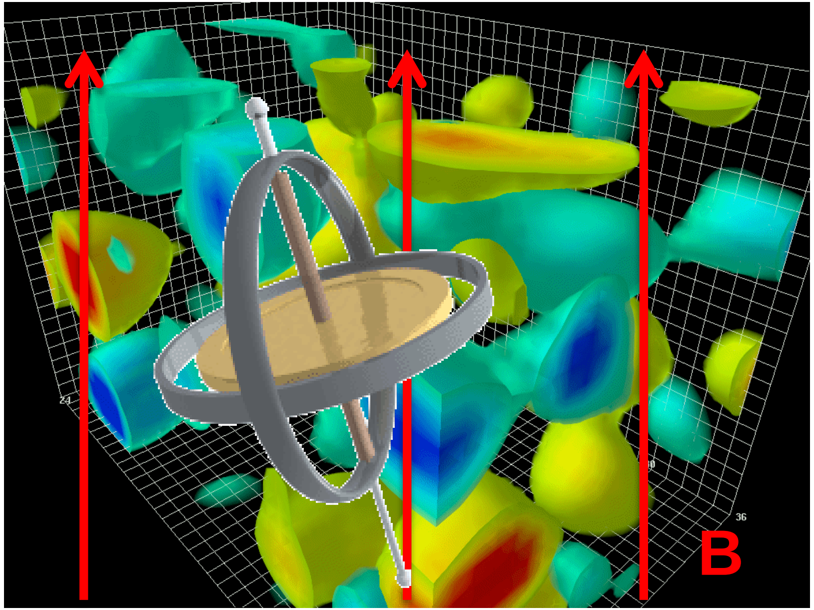


$g = 2.00232$

Anomalous Muon Magnetic Moment: a_μ

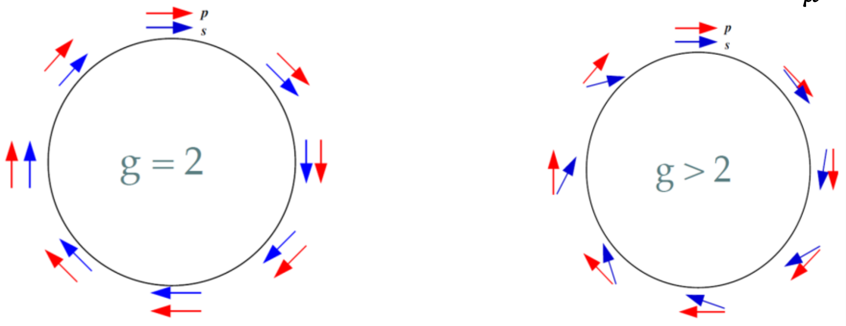
- But there's more to the story… Particles don't exist by themselves. They are constantly surrounded by a "quantum foam" of every other kind of "virtual" particle popping in and out of existence.

- They could all have an impact on the muon's interaction with the magnetic field.



Measurement of Anomalous Magnetic Moment of muon (a_μ)

- Inject a polarized muon beam into a magnetic storage ring.
- Spin precession frequency: $\boldsymbol{\omega_s} = \frac{g_\mu e\boldsymbol{B}}{2m_\mu c} + (1-\gamma)\frac{e\boldsymbol{B}}{m_\mu c\gamma}$
- Cyclotron frequency: $\boldsymbol{\omega_c} = \frac{e\boldsymbol{B}}{m_\mu c\gamma}$
- Anomalous precession frequency: $\boldsymbol{\omega_a} = \boldsymbol{\omega_s} - \boldsymbol{\omega_c} = a_\mu \frac{e}{m_\mu c}\boldsymbol{B}$ (Simplified),



- Spin and Momentum precess at the same rate: $g = 2, a_\mu = 0$.
- Spin advances with respect to the momentum: $g > 2, a_\mu > 0$.

Measurement of anomalous magnetic moment of muon (a_μ)

- Anomalous Magnetic Moment: a_μ

From literature

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

- $$\frac{\boldsymbol{\omega_a}}{\boldsymbol{\tilde{\omega}'_p}} = \frac{f_{\text{clock}}\, \omega_a^{meas} \left(1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd}\right)}{f_{\text{calib}} \left\langle \omega'_p(x,y,\phi) \times M(x,y,\phi) \right\rangle \left(1 + B_k + B_q\right)}$$

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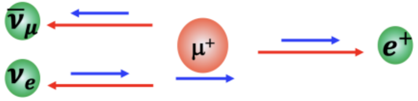
Measurement of precision frequency with positrons

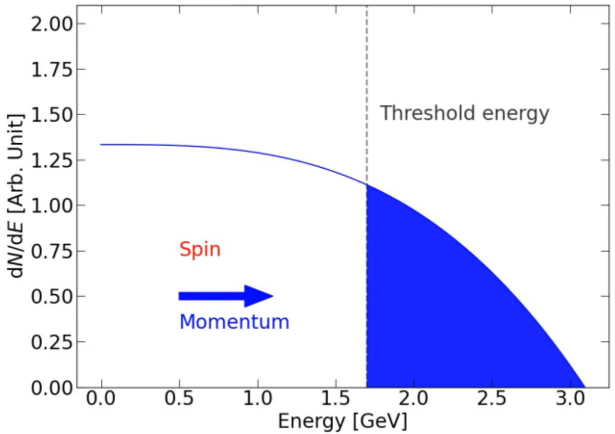
$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\, \omega_a^{meas} \left(1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd}\right)}{f_{\text{calib}} \left\langle \omega'_p(x,y,\phi) \times M(x,y,\phi) \right\rangle \left(1 + B_k + B_q\right)}$$

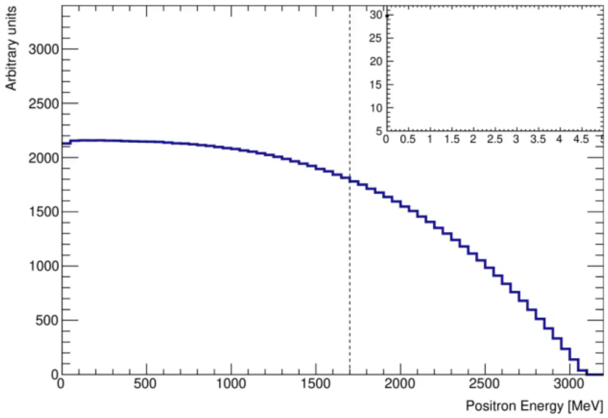
arXiv:2402.15410 [hep-ex]

- Due to parity violation, the highest energy decay positrons are preferentially emitted in the direction of the muon spin.

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$







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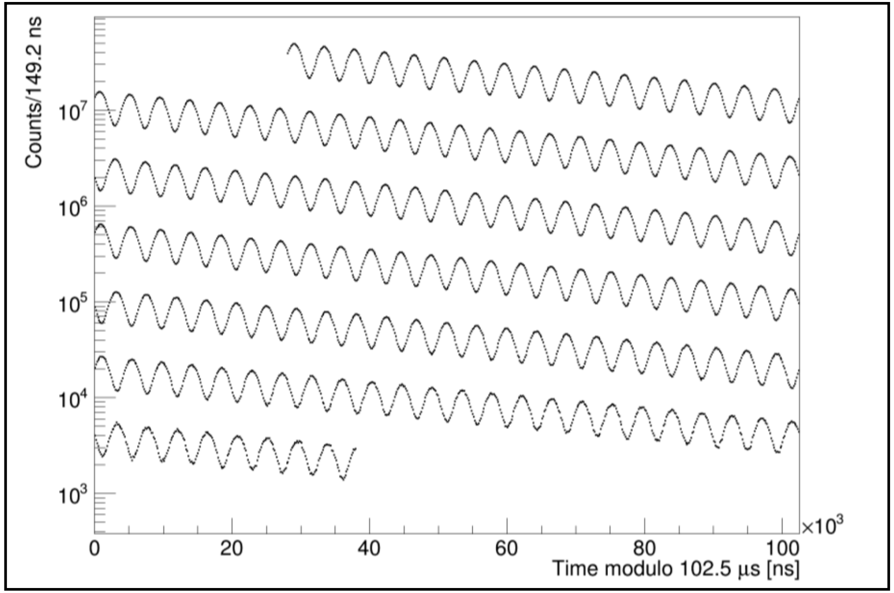
Measurement of precession frequency with positrons: Fitting the time spectrum

- Simple 5-parameter fit to extract ω_a^{meas}.

- This model captures exponential decay and $g-2$ oscillation.

- $N(t) = N_0 e^{-t/\tau_\mu}\,[1 + A\cos(\omega_a t + \phi_0)]$

The "wiggle plot"



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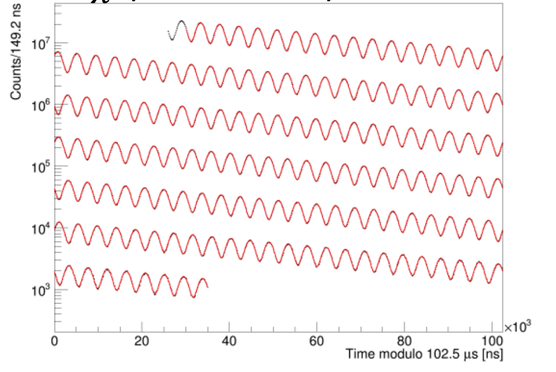
Measurement of precision frequency with positrons: Fitting the time spectrum

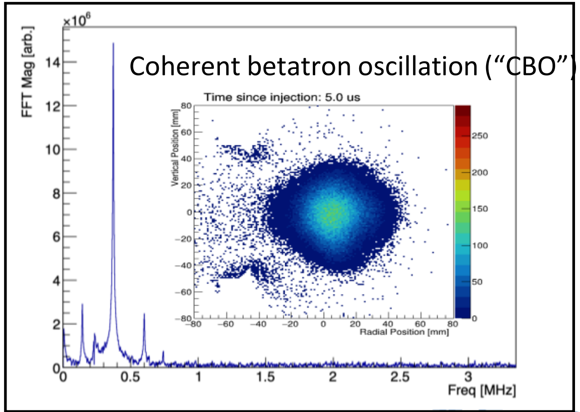
- A simple 5-parameter fit is not sufficient.
  - The most significant one is due to Coherent Betatron Oscillation (CBO)

- Each beam dynamic effect contributes an additional frequency component to the wiggle plot

- Need to account for beam oscillations that couples to detector acceptance, muons lost before decay to positron, and detector effects such as pileup, gain.

χ^2/ndf = 51530/4150





The anomalous precession frequency corrections

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\, \omega_a^{meas}\, (1+C_e+C_p+C_{ml}+C_{pa}+C_{dd})}{f_{\text{calib}}\, \langle \omega'_p(x,y,\phi) \times M(x,y,\phi) \rangle\, (1+B_k+B_q)}$$

- Electric field correction (C_e): Due to spread in injected muons momenta.
- Pitch correction (C_p): Due to vertical oscillation of muons.
- Muon loss correction (C_{ml}): It comes from the initial phase-momentum correlation in muons. As muons are lost in time, time-dependent change in phase is observed.
- Phase acceptance correction (C_{pa}) : It is caused by decay-position and energy dependence of the positron phase. Early-to-late beam motion modulation leads to a time-dependent phase.
- Differential decay correction (C_{dd}): It accounts for high-momentum muons having a longer lifetime.

Measurement of electric field corrections C_e

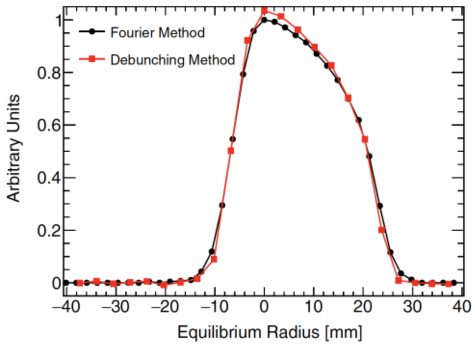
$$\frac{\boldsymbol{\omega_a}}{\boldsymbol{\tilde{\omega}'_p}} = \frac{f_{\text{clock}}\,\omega_a^{meas}\left(1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd}\right)}{f_{\text{calib}}\,\langle\,\omega'_p(x,y,\phi) \times M(x,y,\phi)\,\rangle\left(1 + B_k + B_q\right)}$$

- Muons off the magic momentum ($\gamma = 29.3$, p = 3.09 GeV/c) will produce a motional magnetic field contribution to ω_a.

$$C_e = 2n(1-n)\beta^2\frac{\langle x_e^2\rangle}{R_0^2}$$

- n is the weak focusing field index ($n = \frac{\partial E}{\partial x}\frac{r_0}{vB_0}$)

- x_e is radial equilibrium position which is proportional to the momentum offset.

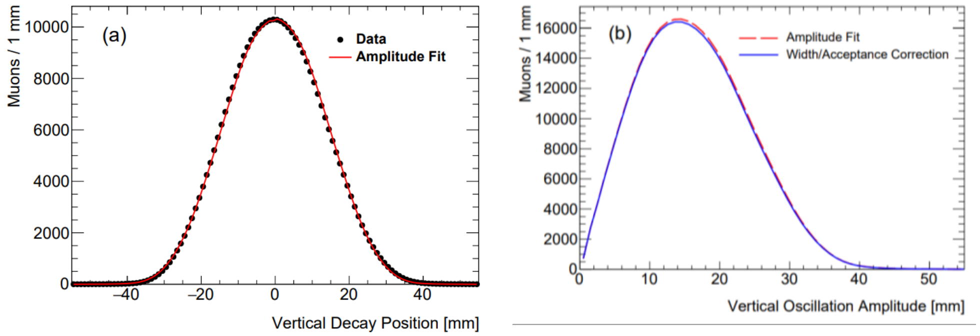


Measurement of pitch corrections C_p

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\, \omega_a^{meas} \left(1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd}\right)}{f_{\text{calib}} \left\langle \omega'_p(x,y,\phi) \times M(x,y,\phi) \right\rangle \left(1 + B_k + B_q\right)}$$

- The vertical motion of the muon causes the vertical spin precession.
- The horizontal precession (ω_a) is affected by coupled in-plane and out-of-plane precessions due to the vertical motion.
- Pitch correction estimated from vertical oscillation amplitude (A) distribution:

$$C_p = \frac{n}{2}\frac{\langle y^2 \rangle}{R_0^2} = \frac{n}{4}\frac{\langle A^2 \rangle}{R_0^2}$$



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The magnetic field corrections

$$\frac{\boldsymbol{\omega_a}}{\boldsymbol{\tilde{\omega}'_p}} = \frac{f_{\text{clock}}\, \omega_a^{meas}\left(1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd}\right)}{f_{\text{calib}}\, \langle\, \omega'_p(x,y,\phi) \times M(x,y,\phi)\, \rangle \left(1 + B_k + B_q\right)}$$

- Kicker transient (B_k): Magnetic field change caused by residual field after kicker pulse.
- Quad transient (B_q) : It is caused by vibration of ESQ plates, that perturbs the magnetic field.
- f_{calib} is calibration factor related to the magnetic field measurement.
- Muon weighted magnetic field: $\langle\, \omega'_p(x,y,\phi) \times M(x,y,\phi)\, \rangle$

Run 2/3 : Total Uncertainties (Statistical + Systematics)

Quantity	Correction [ppb]	Uncertainty [ppb]
ω_a^m (statistical)	–	201
ω_a^m (systematic)	–	25
C_e	451	32
C_p	170	10
C_{pa}	-27	13
C_{dd}	-15	17
C_{ml}	0	3
$f_{\text{calib}}\langle\omega'_p(\vec{r}) \times M(\vec{r})\rangle$	–	46
B_k	-21	13
B_q	-21	20
$\mu'_p(34.7°)/\mu_e$	–	11
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	70
Total external parameters	–	25
Totals	622	215

arXiv:2402.15410 [hep-ex]

- The total uncertainty is still dominated by statistical uncertainty.
- C_e, C_p, $\langle\, \omega'_p(x,y,\phi) \times M(x,y,\phi)\, \rangle$: important for CPT and Lorentz Invariance analysis.

Theoretical background of CPT and Lorentz Invariance Violation (CPTLV) search.

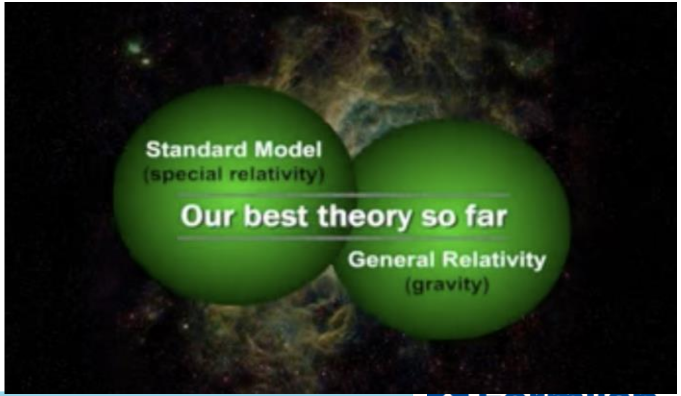
Toward a Unified Theory of Modern Physics

- Special Theory of Relativity: Physical theory of space and time in the absence of gravity.

- General Theory of Relativity: Incorporates gravity and describes it as the curvature of space and time.

- Modern theory consists of two parts:
    - Standard Model: It describes all fundamental physics except gravity. Special relativity is included in this part.
    - General Relativity: Describes gravity.

- Search for a Unified Theory: An attempt to describe all fundamental forces and the relationships between elementary particles in terms of a single theoretical framework.



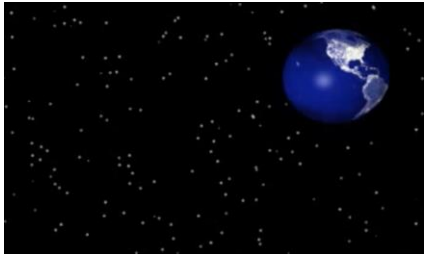
Background of Standard Model Extension (SME)

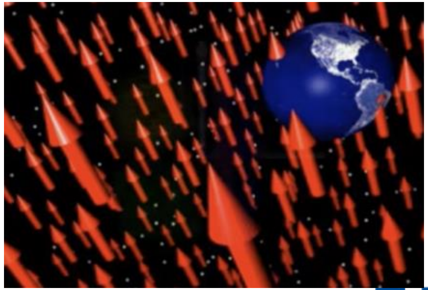
- Finding direct experimental evidence of Unified Theory is a big challenge.

- Indirect effects can be studied.
  - Minuscule deviation from laws of relativity.

- Deviation from laws of relativity:
  - If relativity is exact
    - ➢ Empty space-time looks the same in all directions.

  - If relativity is not exact
    - ➢ Empty space-time has inherent direction.



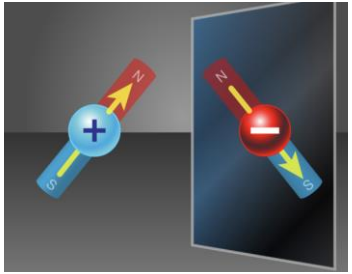


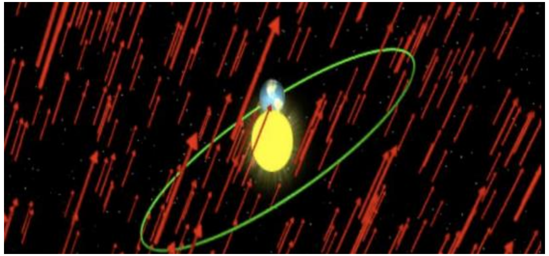
Courtesy: Alan Kostelecky et al.

Background of Standard-Model Extension (SME)

- Consequences of deviation from laws of relativity:
  - The length of measuring sticks, the ticking rate of clocks depend on direction.

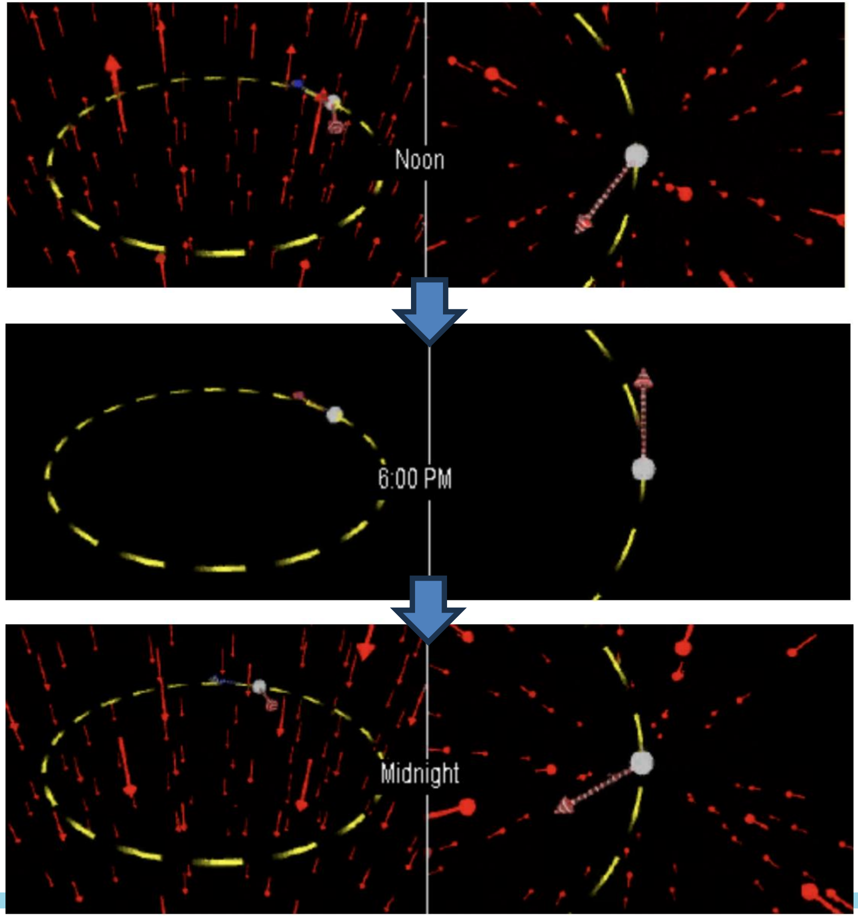
- Standard-Model Extension (SME): A theory extending the standard model in general relativity to include relativity violations.
  - It is a generalization of the usual Standard Model and General Relativity.
  - It has all the conventional desirable properties but allows for violations of Lorentz and CPT symmetry.





- SME predicts a huge variety of possible minuscule experimental signals in many different systems. Bluhm, Kostelecky,Lane: PRL 2000

Sidereal effects of the SME in the muon sector



The muon spin precession vector encounters different directions of the red arrows throughout the day in a cyclic fashion.

Courtesy: Bluhm, Kostelecky,Lane: PRL 2000

SME and CPTLV in Muon $g-2$

$$\mathcal{L} = -a_{\kappa\,AB}\,\bar{l}_A\gamma^\kappa l_B - b_{\kappa\,AB}\,\bar{l}_A\gamma_5\gamma^\kappa l_B$$
$$-\tfrac{1}{2}H_{\kappa\lambda\,AB}\,\bar{l}_A\sigma^{\kappa\lambda}l_B + \tfrac{1}{2}ic_{\kappa\lambda\,AB}\,\bar{l}_A\gamma^\kappa \overleftrightarrow{D}^\lambda\, l_B$$
$$+\tfrac{1}{2}id_{\kappa\lambda\,AB}\,\bar{l}_A\gamma_5\gamma^\kappa \overleftrightarrow{D}^\lambda\, l_B \quad .$$

$$b_\perp^{\mu^\pm} = \frac{\overset{\wedge}{\omega}{}_a^{\mu^\pm}}{2|sin\chi|}$$

arXiv:1407.7748

- Standard Model Extension (SME):
  Allows for CPT and Lorentz Invariance Violation(LV), quantitatively described by coefficients, experimentally determined/constrained.

- CPTLV effects with the signal at sidereal frequency:
  - In cartesian coordinate $c_{X\ or\ Y\ or\ antisymm\ XY\ pair}$
  - In spherical coordinate c_{kl1} (i.e. azimuthal index $m = 1$)

- CPTLV effects with the signal at sidereal frequency harmonics:
  - In cartesian coordinate $c_{symm\ XY\ pair}$
  - In spherical coordinate c_{klm} (i.e. azimuthal index $m > 1$)

SME and CPTLV in Muon $g-2$

$$A_m^{\pm} = \left| 4 \sum_{dnj} E_0^{d-3} G_{jm}(\chi) \left[\check{H}_{njm}^{(d)} \pm E_0 \, \check{g}_{njm}^{(d+1)} \right] \right|, \quad m \neq 0$$

arXiv:1407.7748

- A_m : Total amplitude of m^{th}harmonic. This term is energy dependent.
- E_0 : unperturbed muon energy.
- G_{jm} : Dimensionless factor. $G_{jm}(\chi) \equiv \sqrt{j(j+1)} \; {}_1Y_{j0}(\pi/2, 0) d_{0m}^{(j)}(-\chi)$
- $d \leq 4$: Minimal SME, $d > 4$: Nonminimal SME.
- Nonminimal terms produce effects that grow with energy.
- d : mass dimension of the SME coefficient. odd d : $m_{max} = d - 2$; even d: $m_{\max} = d - 3$.

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Previous results

- From BNL Muon $g-2$ Experiment: Phys.Rev.D90:076009,2014
  - $A_1^+ \leq 2.1 \text{ X } 10^{-24}$ GeV
  - $A_1^- \leq 4.0 \text{ X } 10^{-24}$ GeV
  - $\check{b}_\perp^{\mu^+} < 1.4 \text{ X } 10^{-24}$ GeV

- Sensitivity limit: The sensitivity to detect smaller amplitude signal, i.e. the upper bound of these SME parameters decreases with increase in statistics. With 4 fold increase in statistics in FNAL Muon $g-2$ experiment w.r.t. BNL, expected sensitivity limit ~ 0.5 ppm.

Description of analysis framework of CPT and Lorentz Invariance Violation search.

CPTLV Signals with Muon $g - 2$ experiment at Fermilab

- Sidereal Oscillation in $R_\mu = \omega_a / \tilde{\omega}'_p$

$$a_\mu = \frac{\omega_a}{\tilde{\omega}_{p'}} \frac{\mu_{p'}}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

  - ω_a is anomalous precession frequency of muon.
  - $\tilde{\omega}'_\mathrm{p}$ is proton Larmor precession frequency in water sample mapping the field and weighted by the muon distribution.

- Reason of measuring $R_\mu$instead of ω_a :
  - Variation of magnetic field affects ω_a , magnetic field used in $g - 2$ is not constant.

- One run in the Muon $g - 2$ experiment lasts $\sim$ 1 hour.

- Main ingredient of CPTLV signals: Run-by-run R_μ and timestamps for each run.

Analysis framework for run-by-run R_μ Extraction

Analyzer verification on the entire dataset

Extraction of Unix timestamp for each run

Extraction of ω_a and $\tilde{\omega}'_p$ for each run

Methods of ω_a extraction:
1. Threshold method.
2. Asymmetry-weighted method (more statistical power).

Extraction of $R_\mu = \frac{\omega_a}{\tilde{\omega}'_p}$ for each run

Extraction of oscillation amplitude from Run-by-Run R_μ: Using MPF

- Time domain extraction: Using Multi-Parameter fit (MPF)
- $R_\mu = C_0 + \frac{A}{\tilde{\omega}'_p}\sin(2\pi n f_s t + \varphi)$
  - C_0: Constant
  - A: oscillation amplitude
  - f_s: Sidereal frequency
  - n: Sidereal harmonic (1,2,3,4,5)

Extraction of oscillation amplitude from Run-by-Run R_μ: Using GLS

- Frequency domain extraction: Using Generalized Lomb-Scargle periodogram method.
- Spectral analysis technique for unequally spaced data. Scargle 1982
- Time domain model: $g(t) = \mathrm{a}\cos(2\pi f t) + b sin(2\pi f t) + c$
- Minimization of χ^2 at frequency f to obtain minimum $\chi^2 = \widetilde{\chi^2}$
- Lomb Power at f: $P_S(f) = \frac{[\widetilde{\chi_0^2} - \widetilde{\chi^2}(f)]}{\widetilde{\chi_0^2}}$, where $\widetilde{\chi_0^2}$ is $\widetilde{\chi^2}$ for $g(t) = c$. M. Zechmeister, M. Kürster

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Systematic Uncertainties

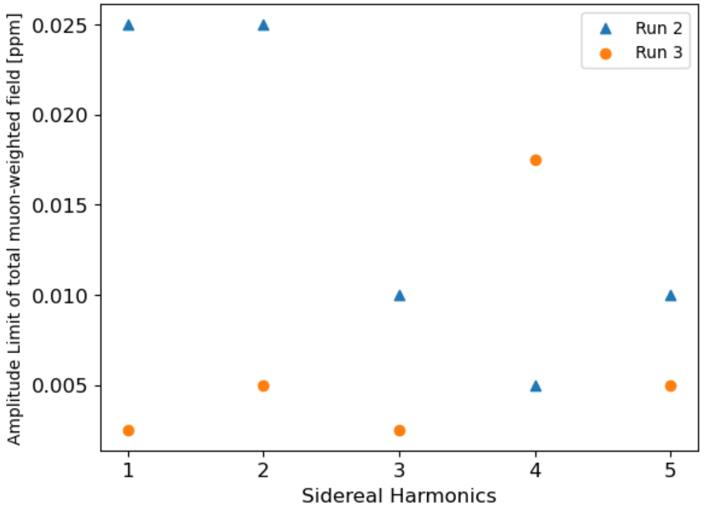
Magnetic Field related systematics

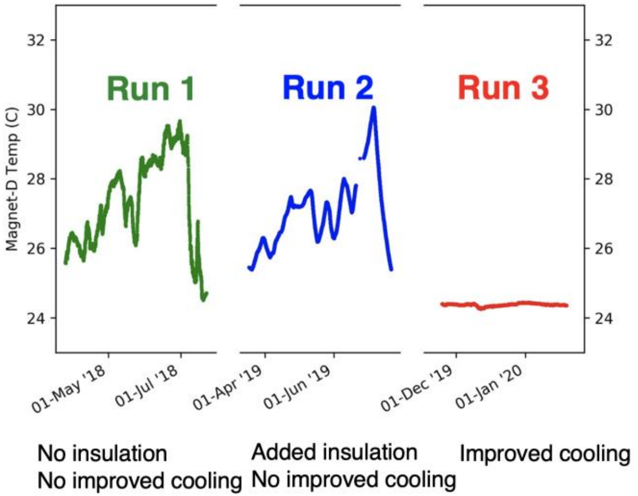
Detector gain calibration related systematics

Beam dynamics related systematics

Magnetic Field related systematics

- Search for any potential signal in $\widetilde{\omega_p'}$ at the sidereal frequency and its harmonics.
  - Set a limit on the oscillation amplitude of $\widetilde{\omega_p'}$ at the sidereal frequency and its harmonics.





- For most of the harmonics, magnetic field oscillation amplitude is larger in Run 2 than in Run 3. It is expected because of improved hall cooling in Run 3.
- The maximum oscillation amplitude is 0.025 ppm that is less than sensitivity limit 0.5 ppm

Conclusion

- From preliminary analysis with FNAL Muon $g-2$ Run 2 data using the Threshold method of ω_a extraction:
  - $A_1^+ \leq 2$ ppm => $A_1^+ \leq 2$ X 10^{-24} GeV at 95% C.L
  - $\check{b}_\perp^{\mu^+} < 1.27$ X 10^{-24} GeV at 95% C.L

Bhattacharya's PhD thesis: Run 2_Preliminary_Result

- A new analysis framework is built to improve the sensitivity limit. Asymmetry-weighted method of ω_a extraction will improve sensitivity.
- This framework will be applied to the combined Run 2, Run 3a, and Run 3b datasets. Due to enhanced statistics, further improvement in the sensitivity limit is expected.
- Future prospect: This paper is targeted for PRL publication. It will be the first paper on the CPTLV search with Fermilab Muon $g-2$ data.