Q-Pix:

pixel-based charge readout for kton scale LArTPC

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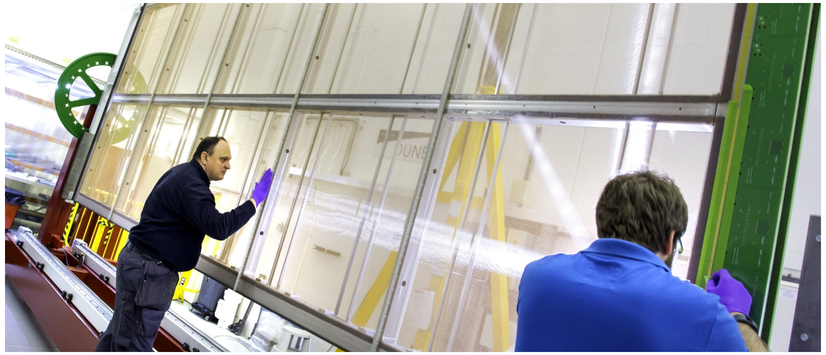
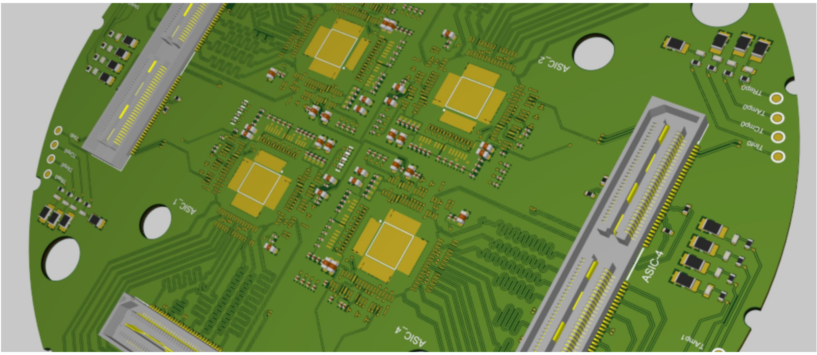
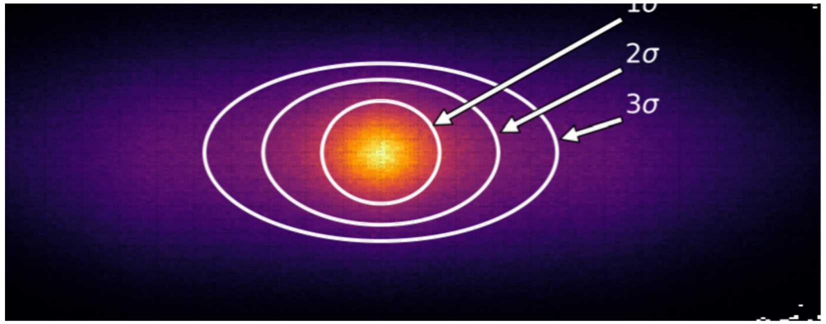
Harvard University / The University of Manchester

Contents





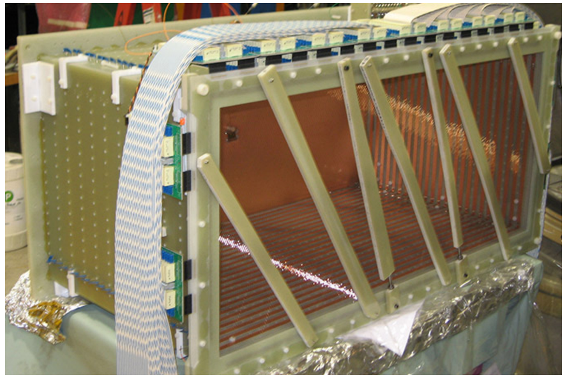


Liquid Argon Time Projection Chamber (LArTPC)

Traditional wire-based LArTPC has been used in many different experiments

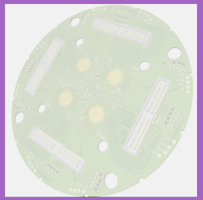
DUNE

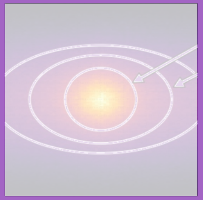
MicroBooNE



ArgoNeut

ICARUS





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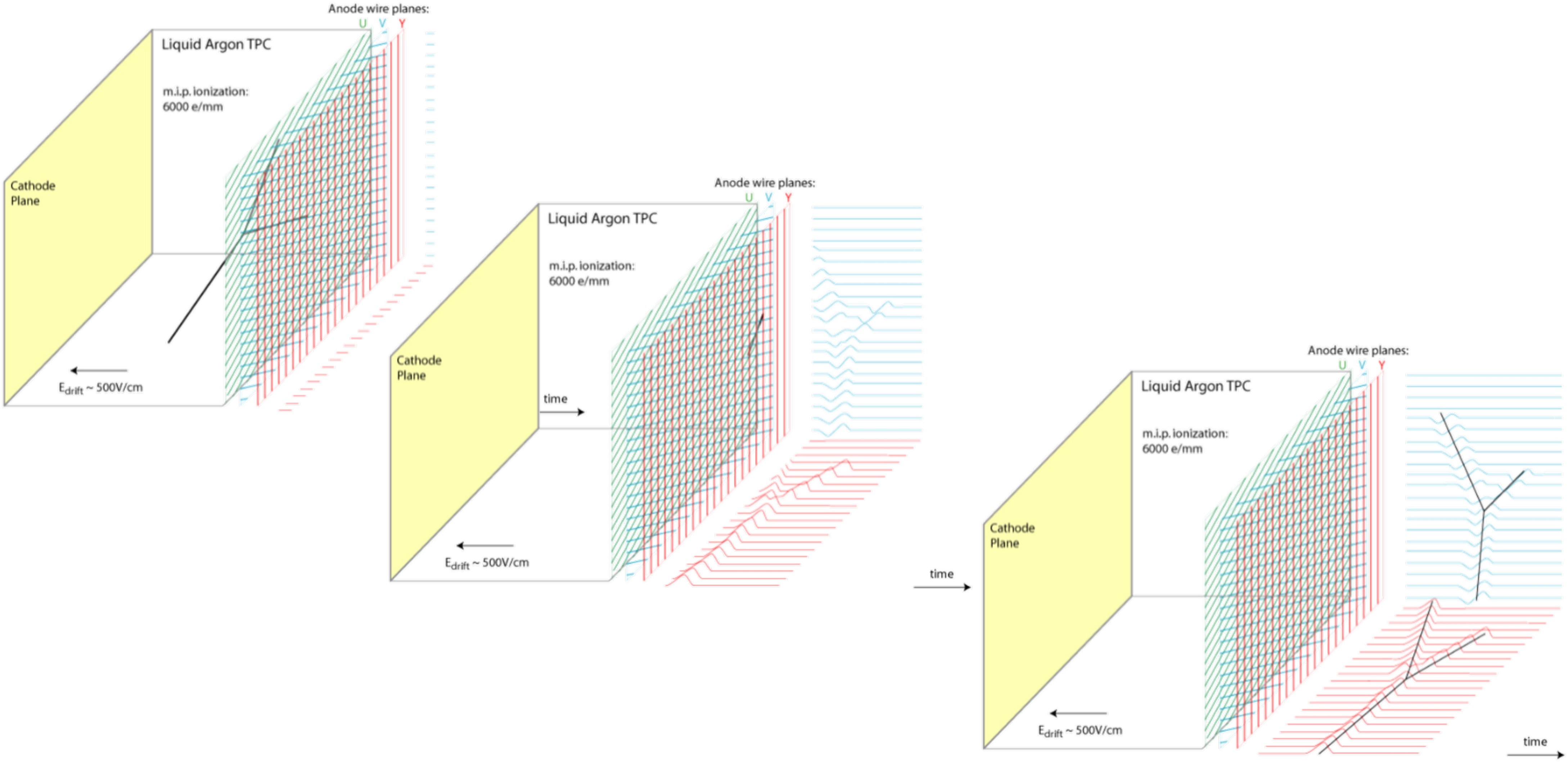

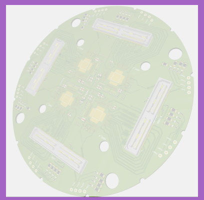

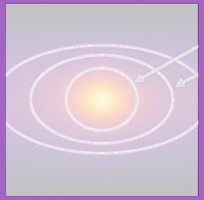

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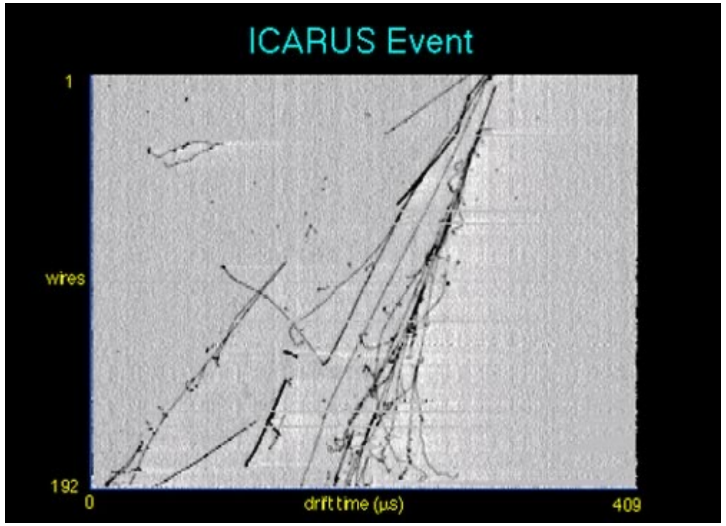
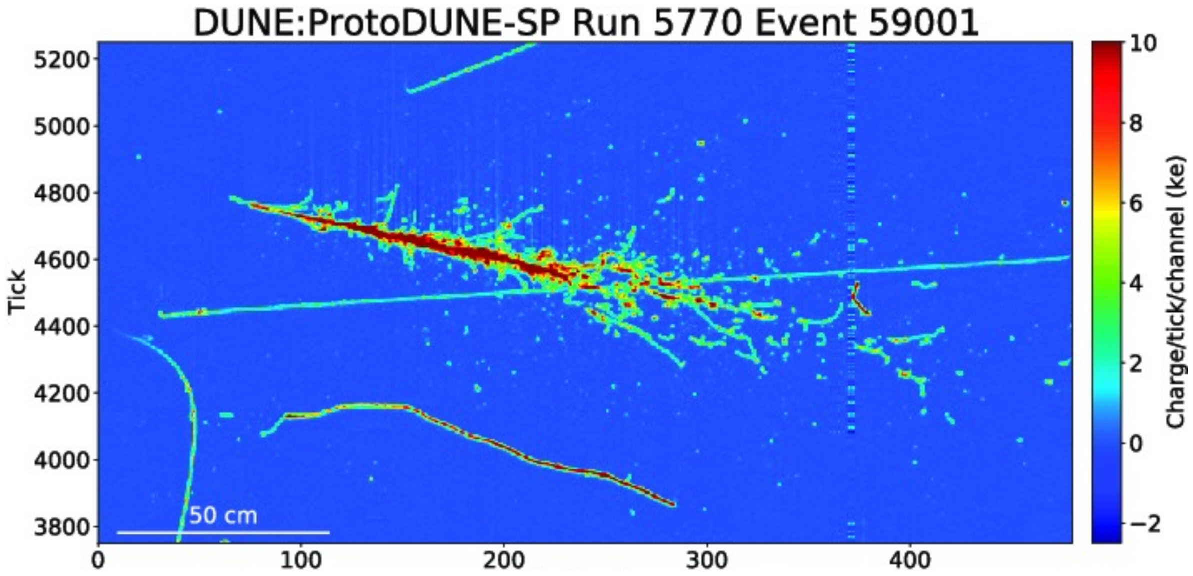
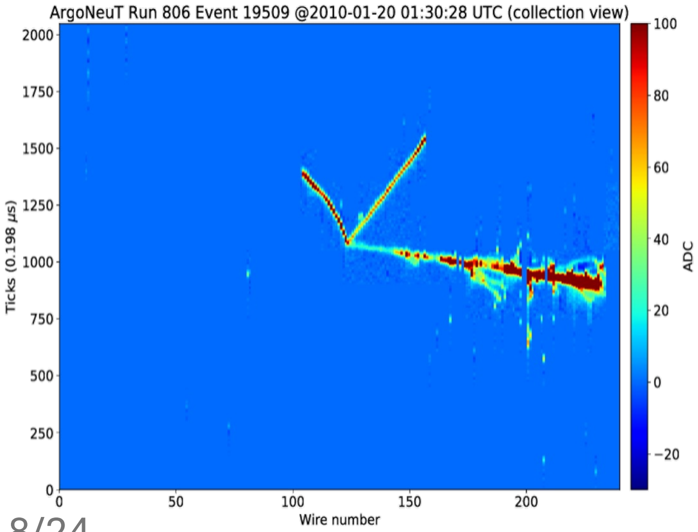
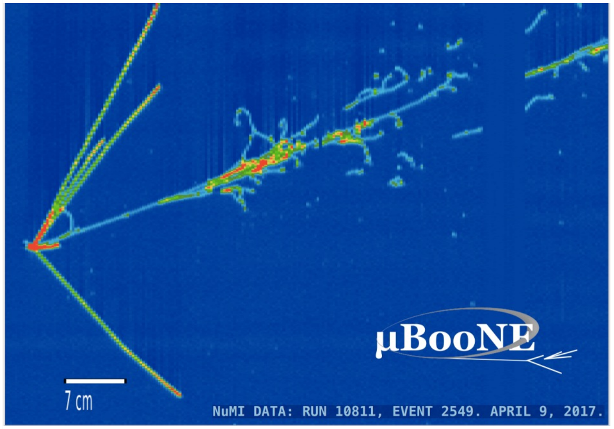
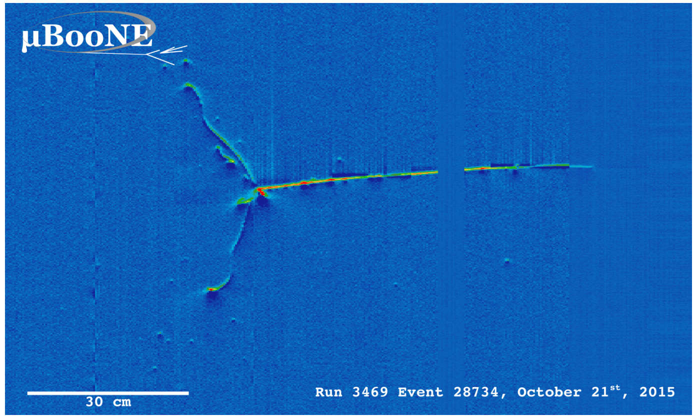
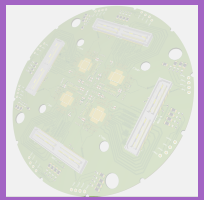


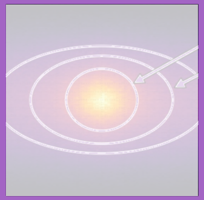






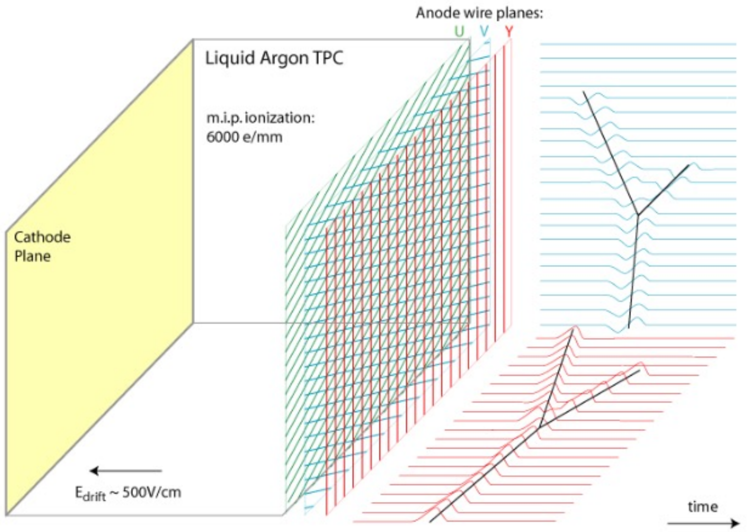




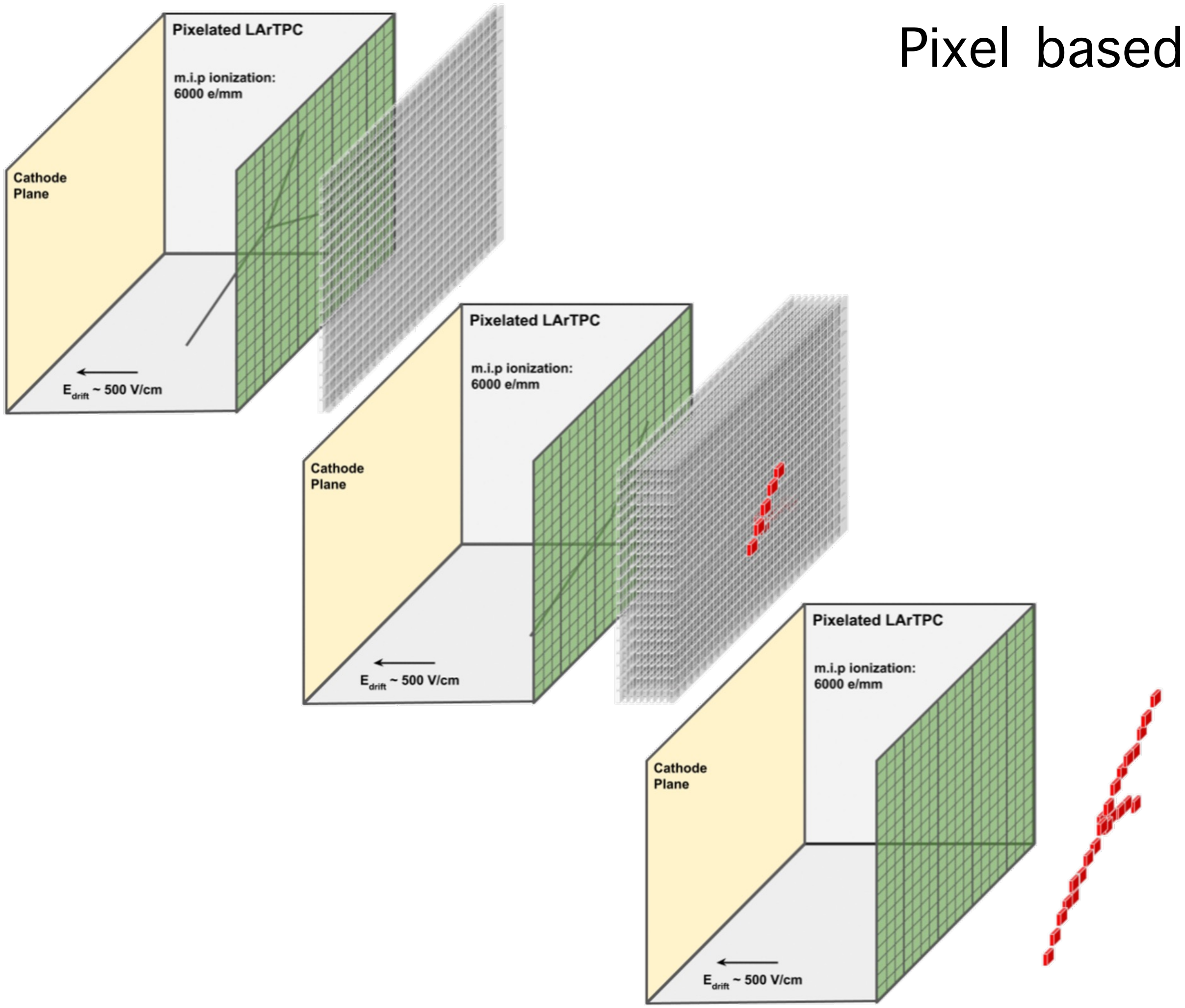


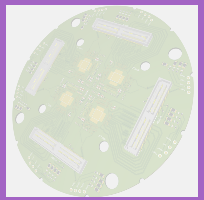
Wire vs Pixelated LArTPC

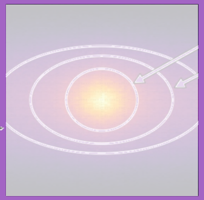
Wire based



Pixel based







Pixelated LArTPC – why not!

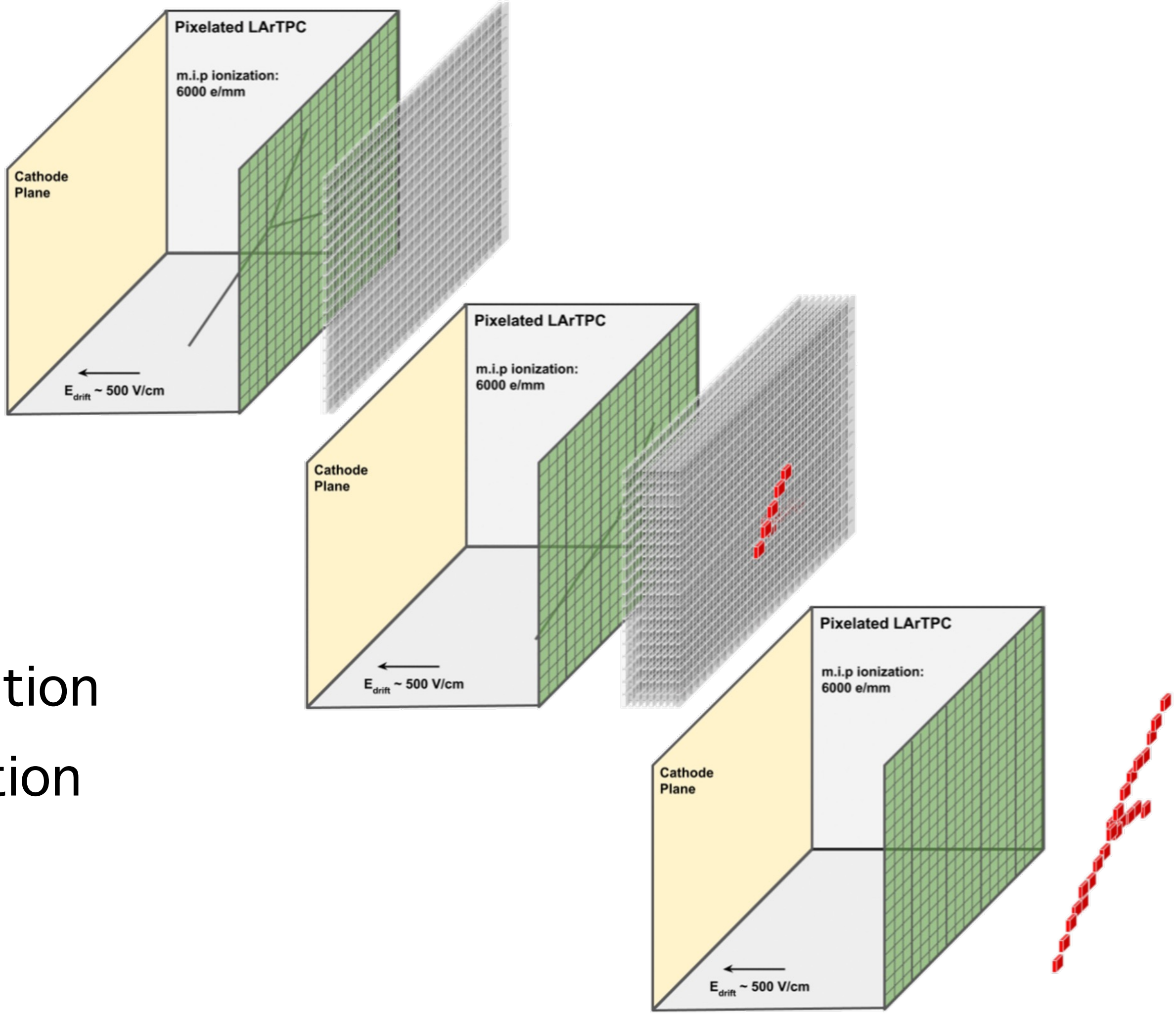
Challenge:

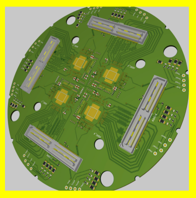
Great increase in

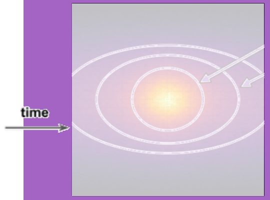
1) the number of channels
2) the amount of data

Solutions:

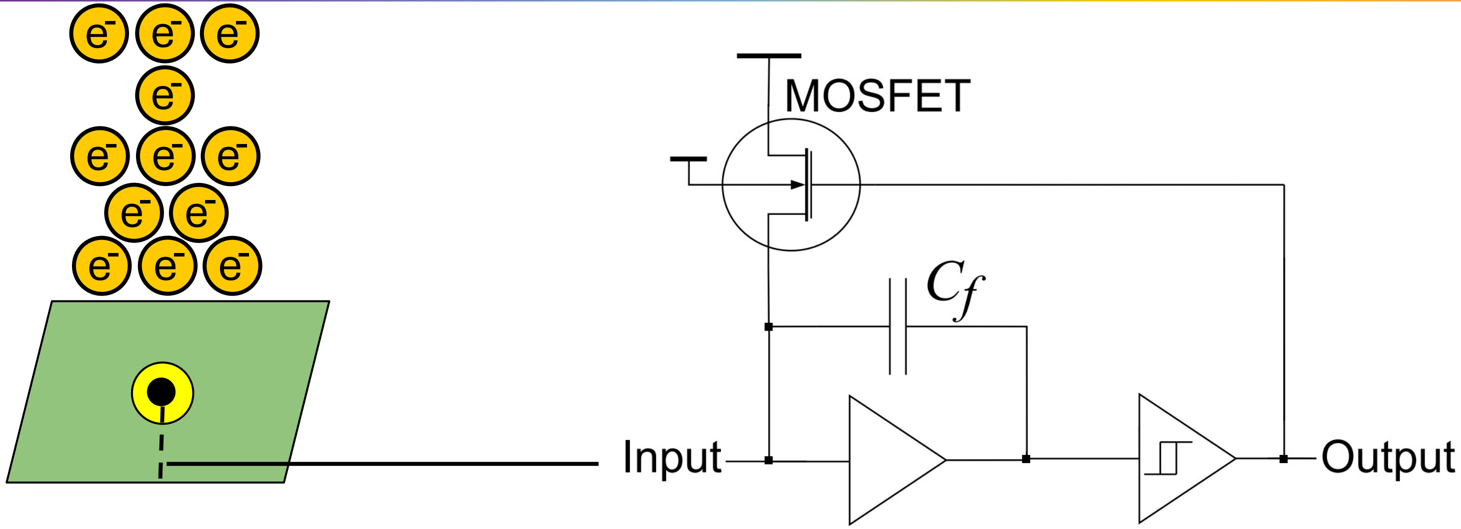
1) Electronic principle of least action
2) New way to quantize information

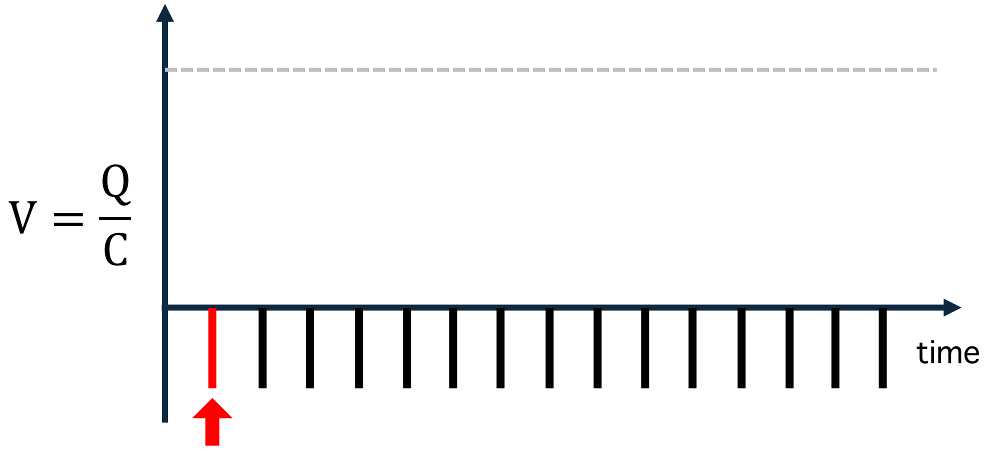


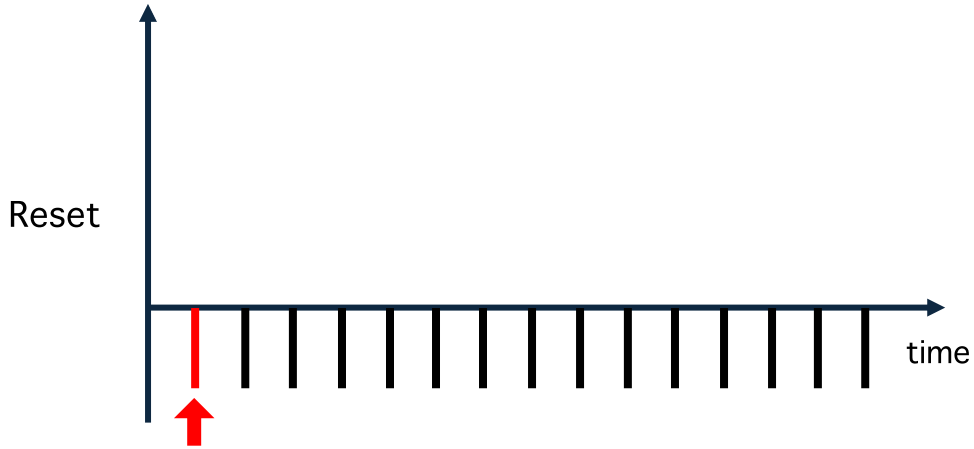


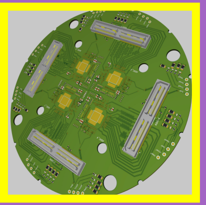


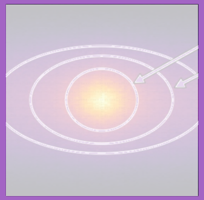
Toy Example











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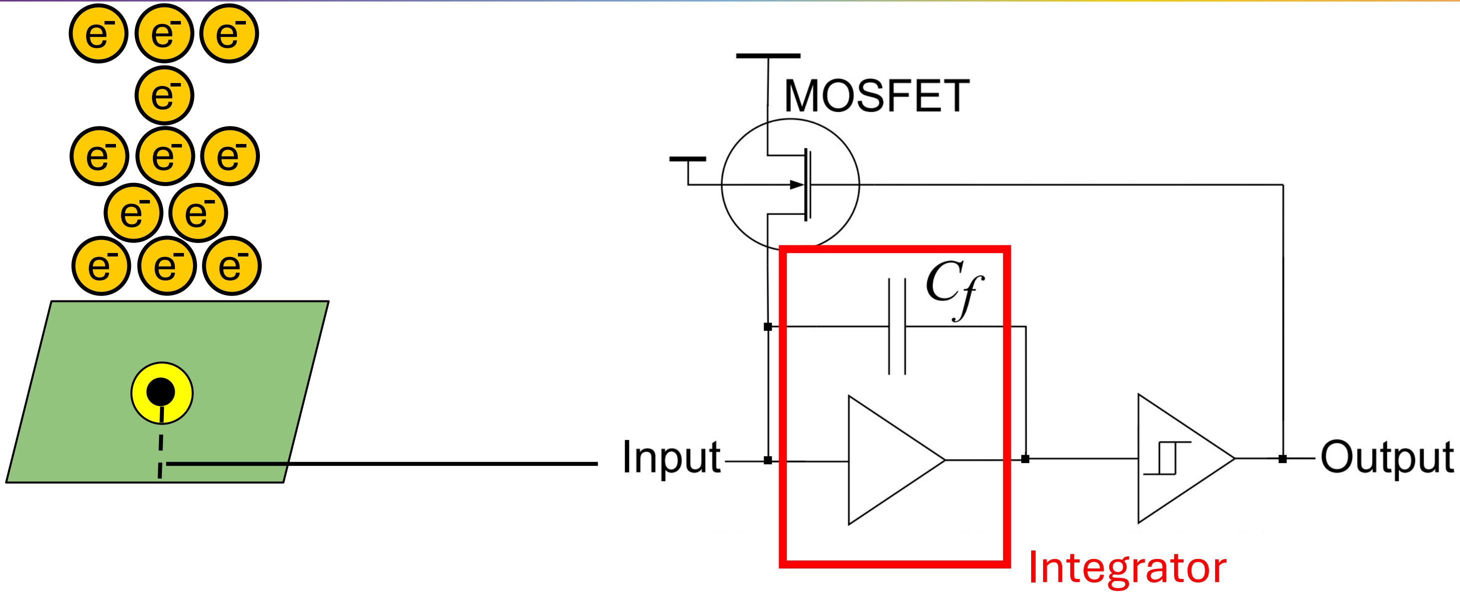

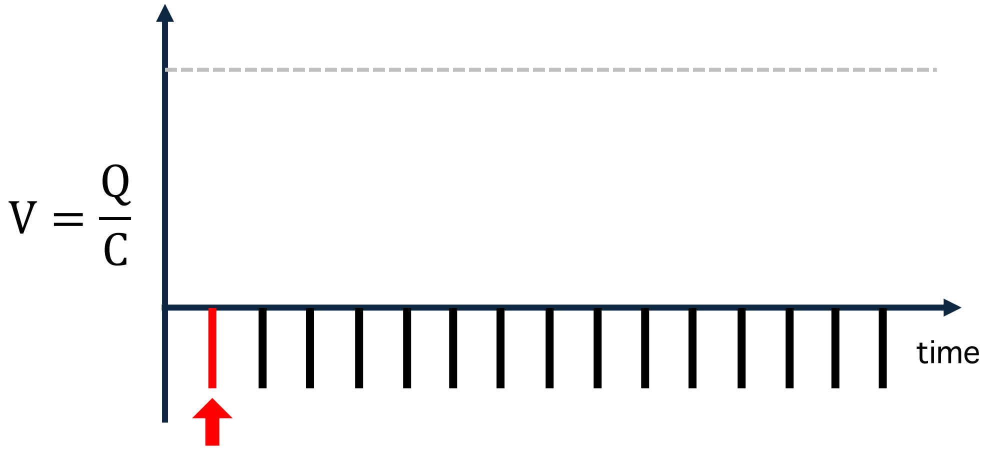

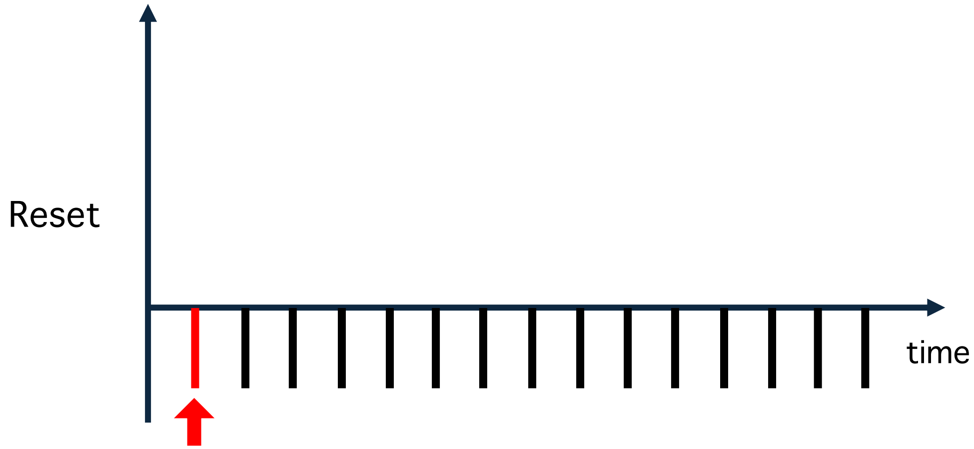

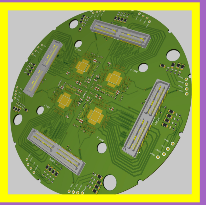

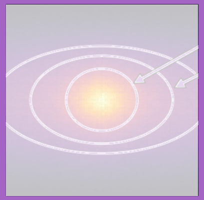

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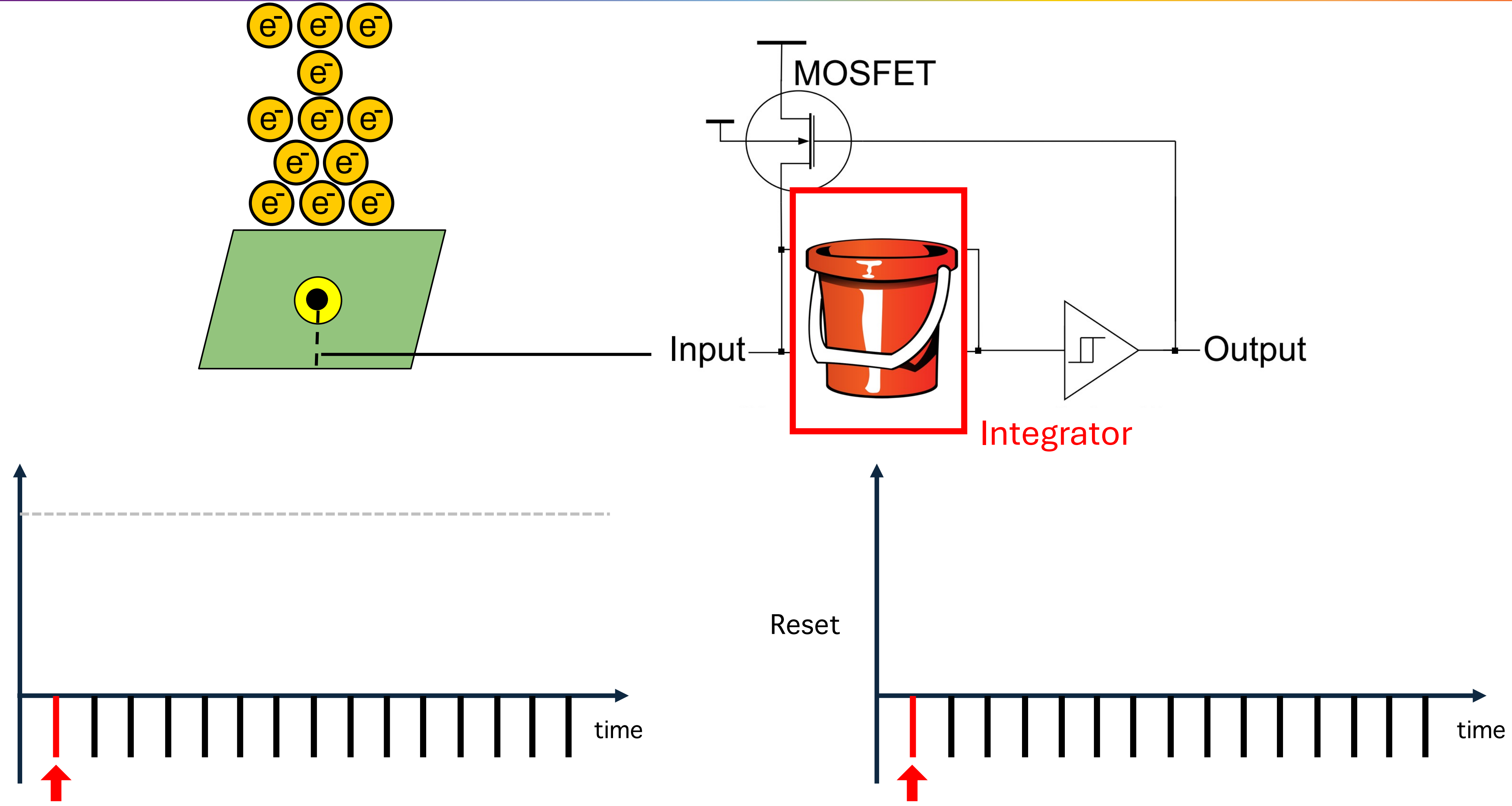
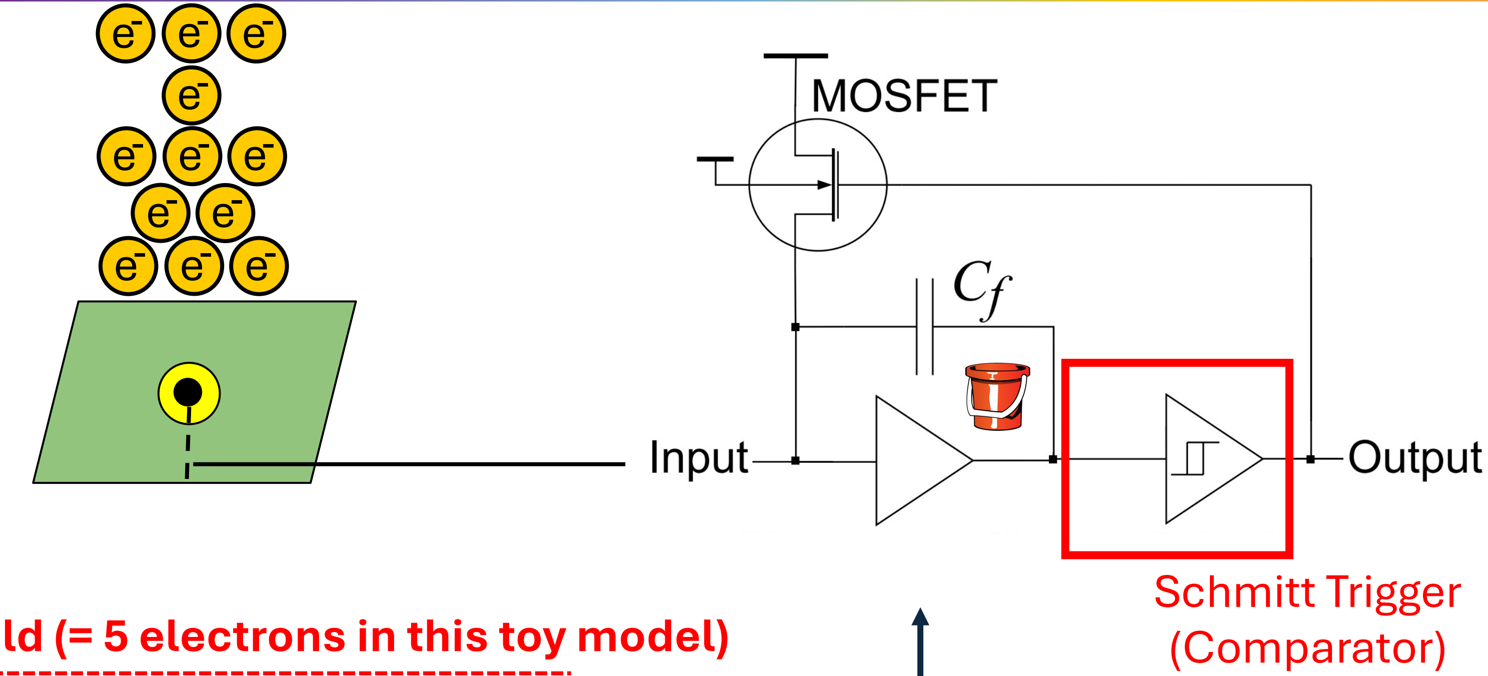
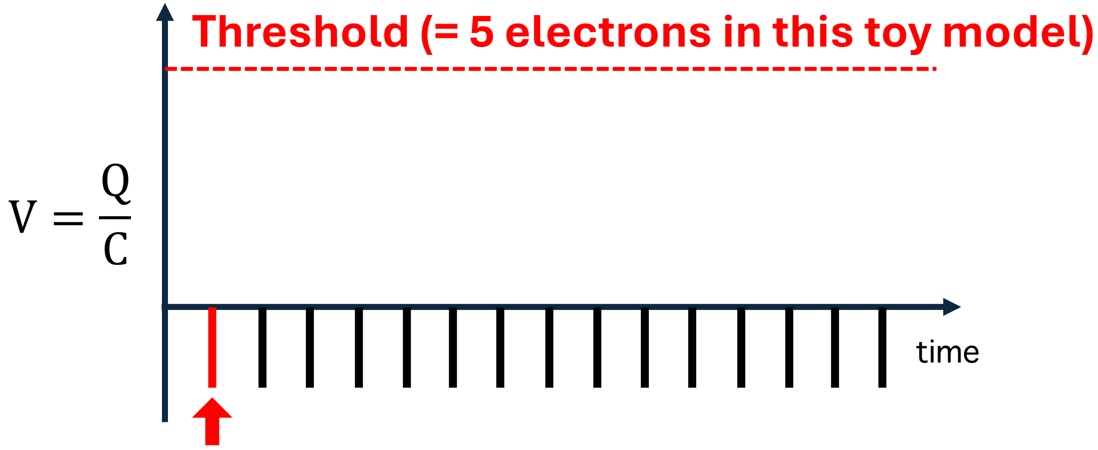


$$V = \frac{Q}{C}$$

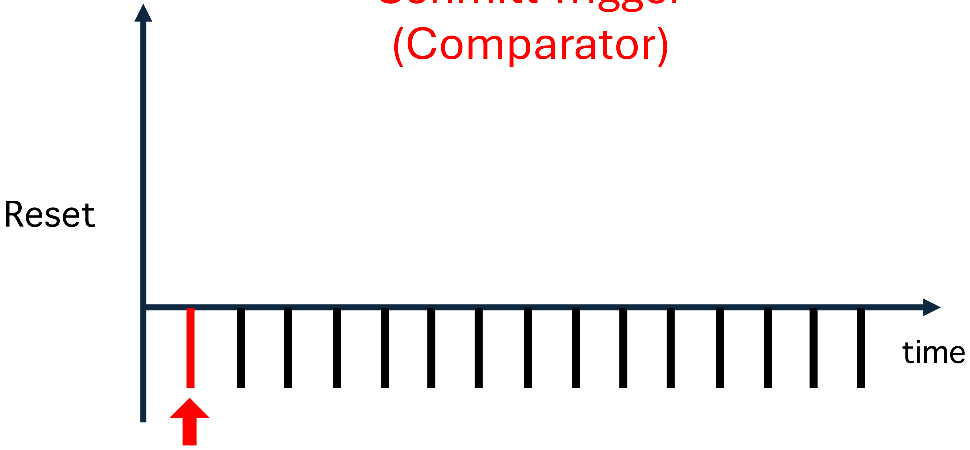
Toy Example



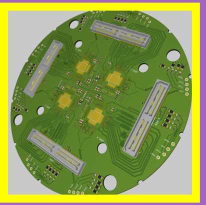
e⁻
MOSFET
C_f
Input
Output
Schmitt Trigger
(Comparator)

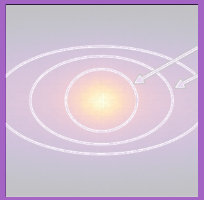


Threshold (= 5 electrons in this toy model)
$V = \frac{Q}{C}$
time



Reset
time





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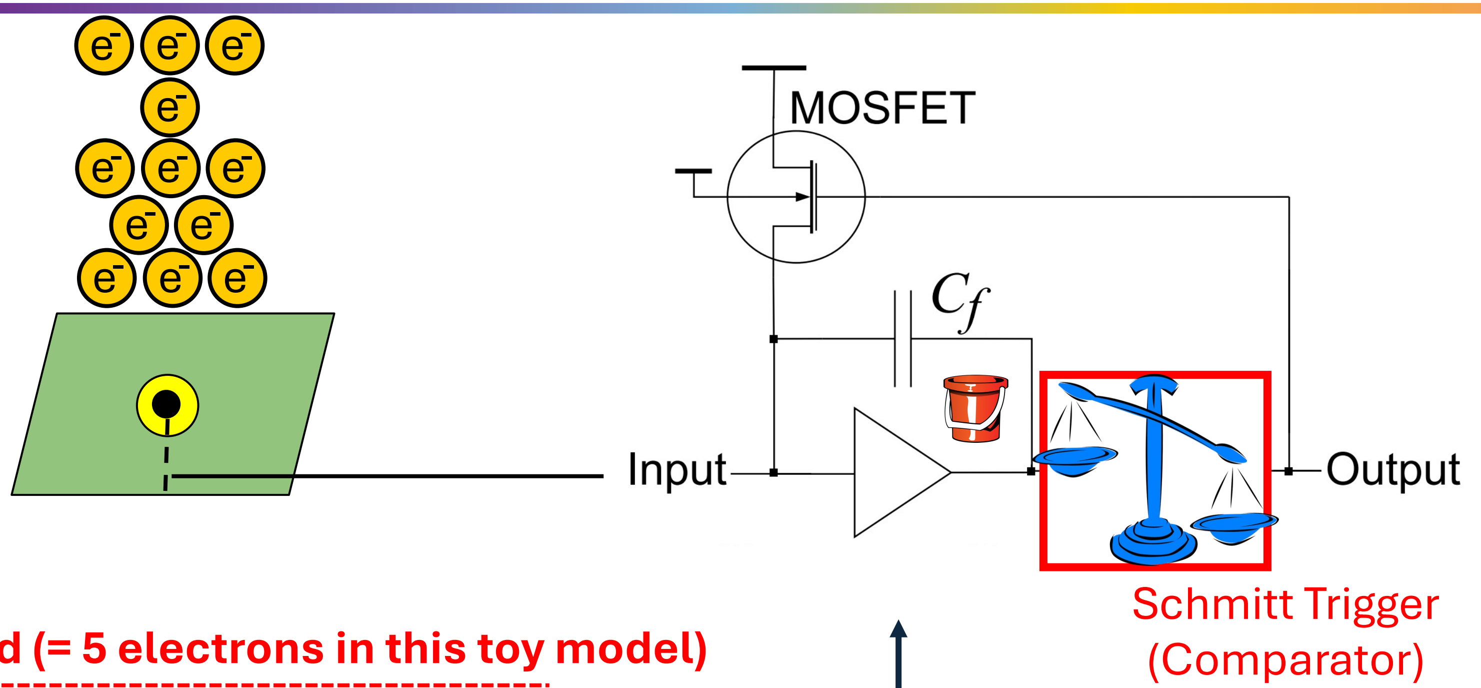

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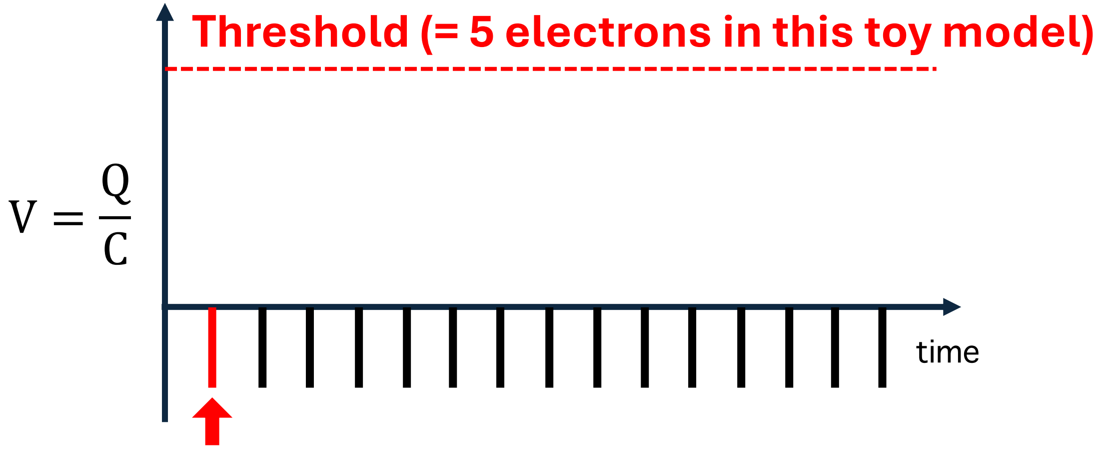

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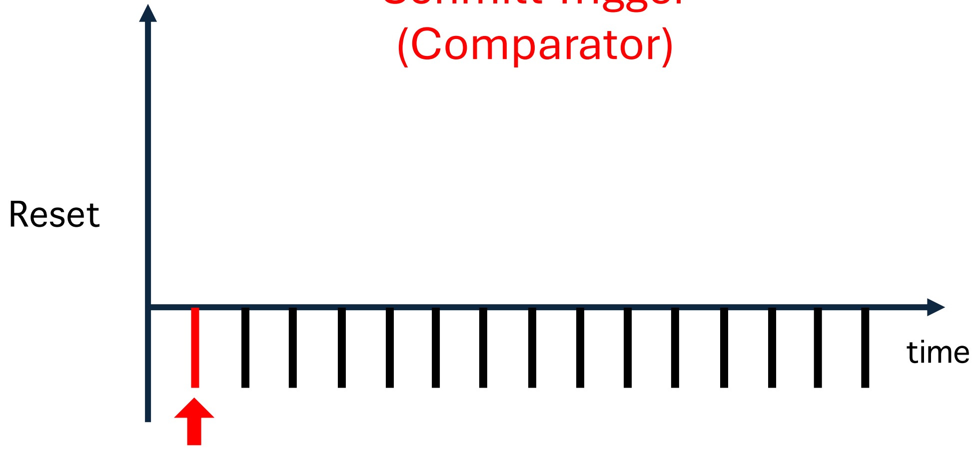

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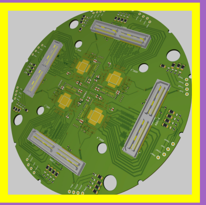

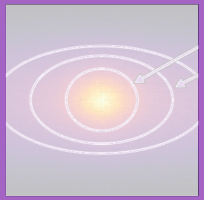

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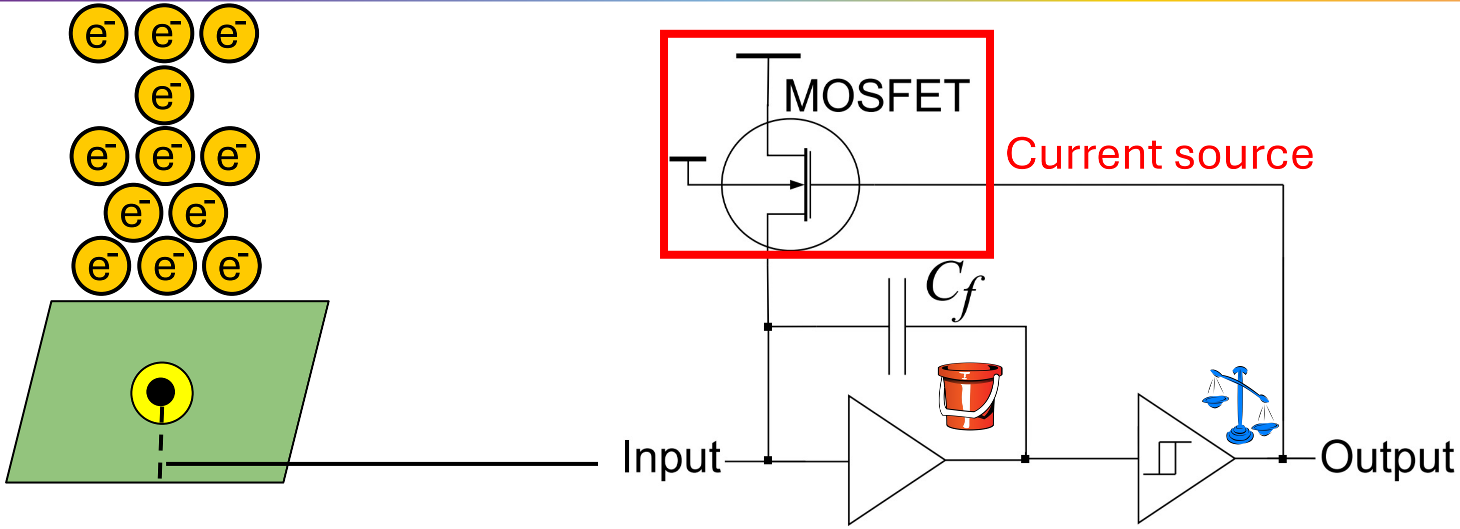

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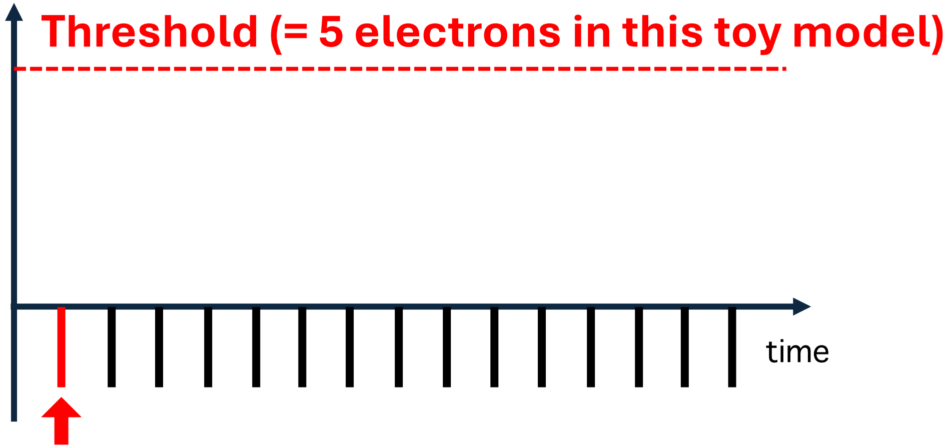
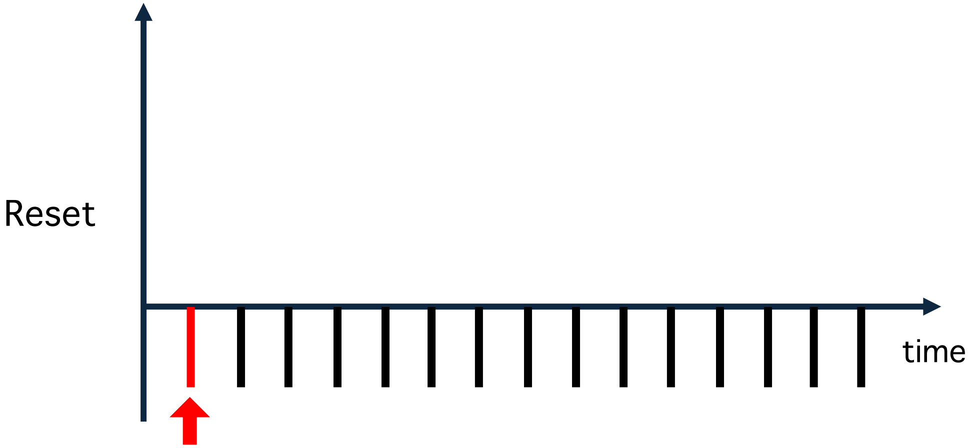
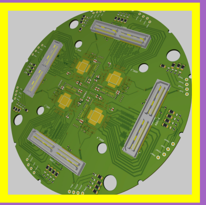
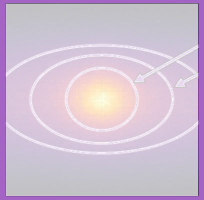




※ the reset happens for 5 electrons





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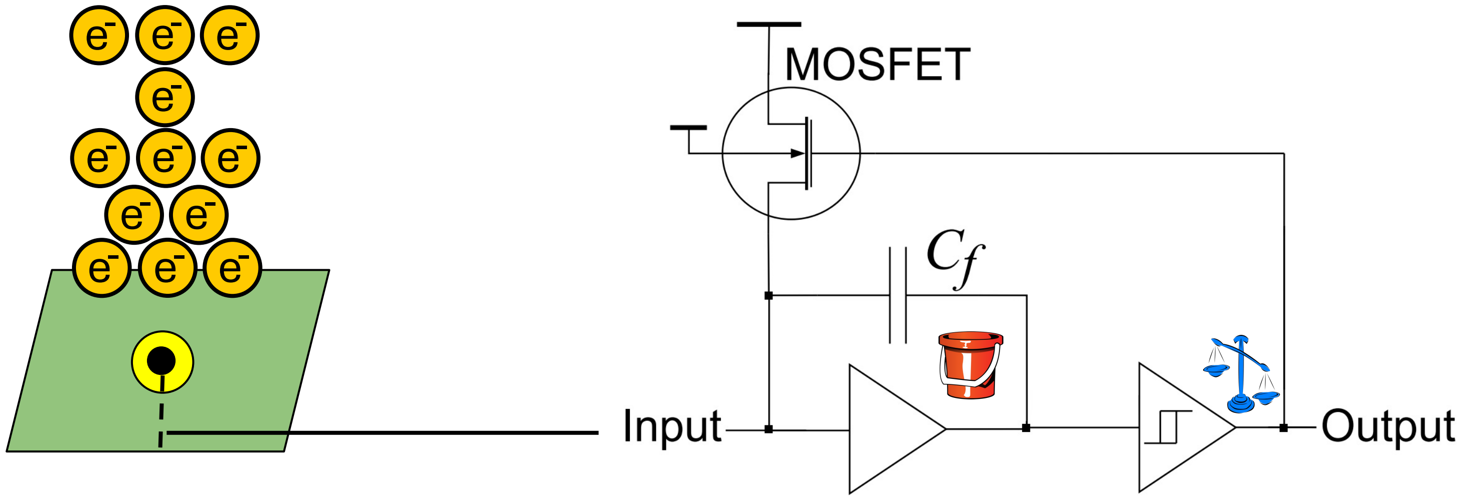

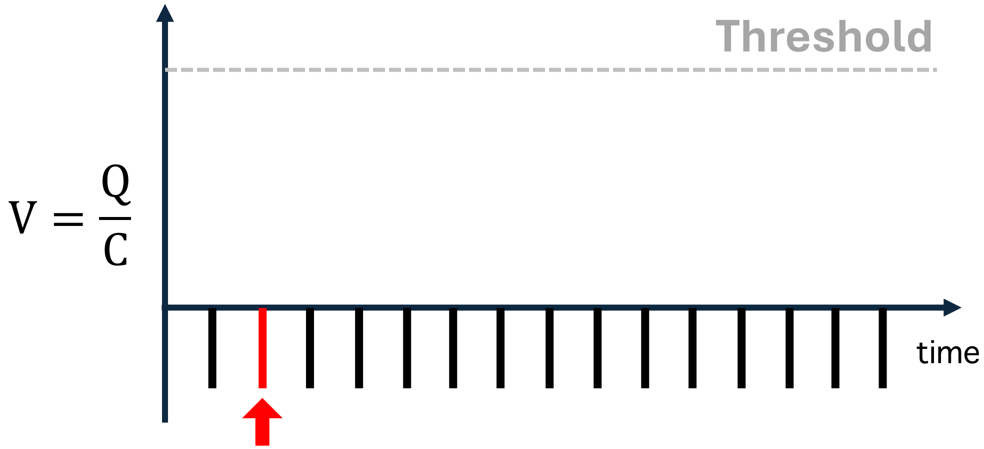

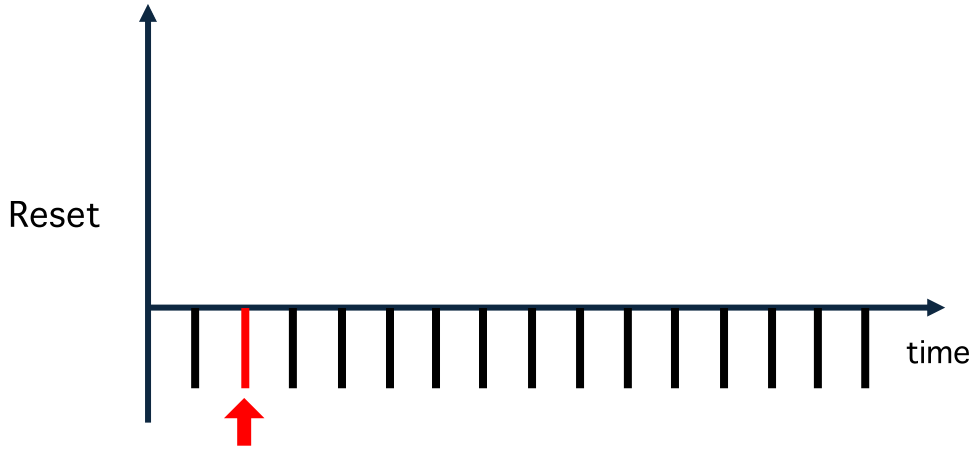

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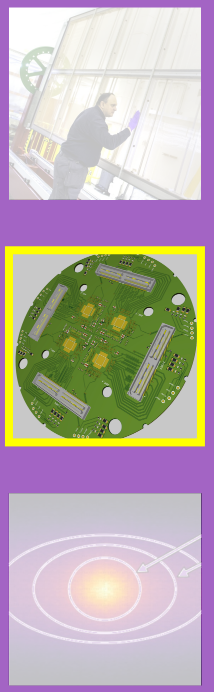

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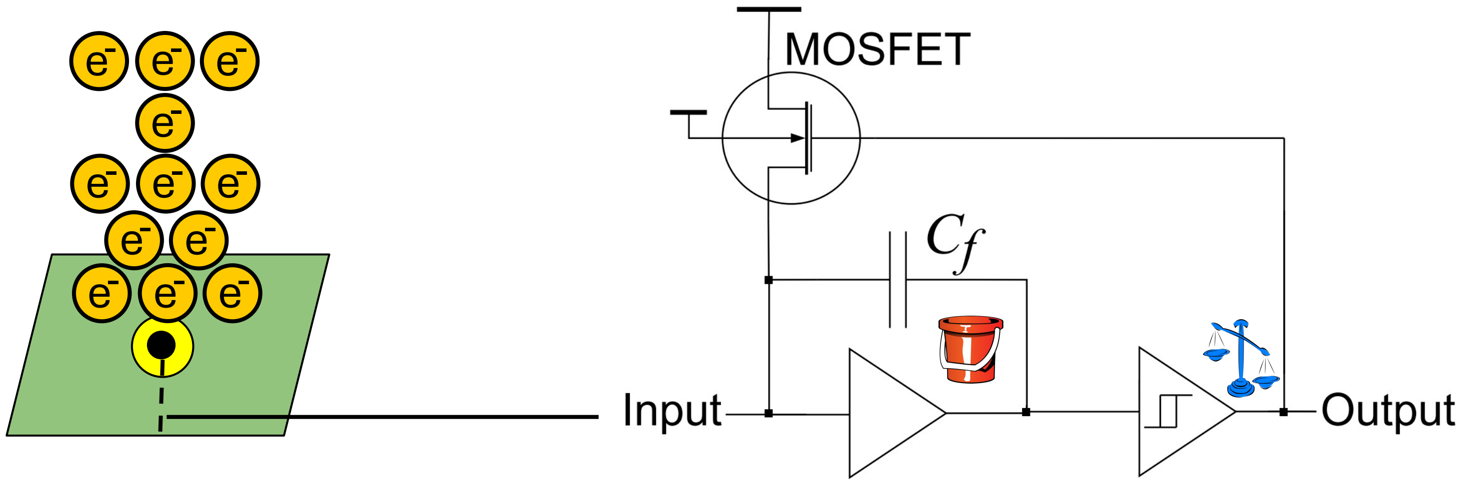

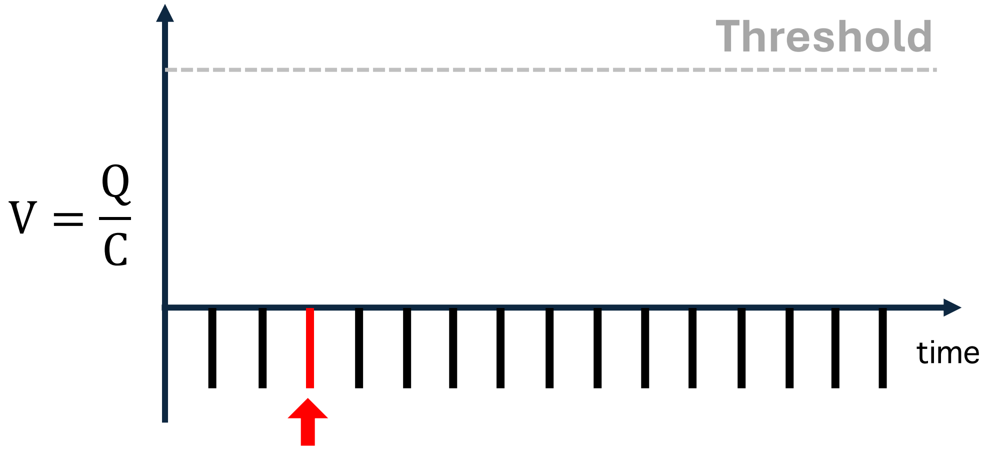

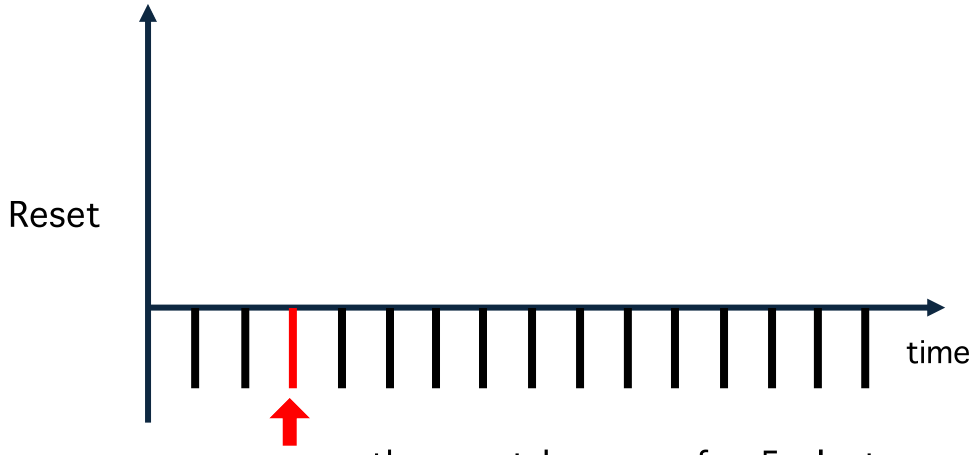

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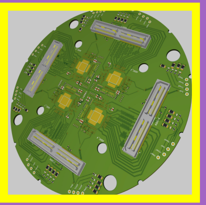

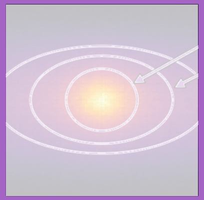

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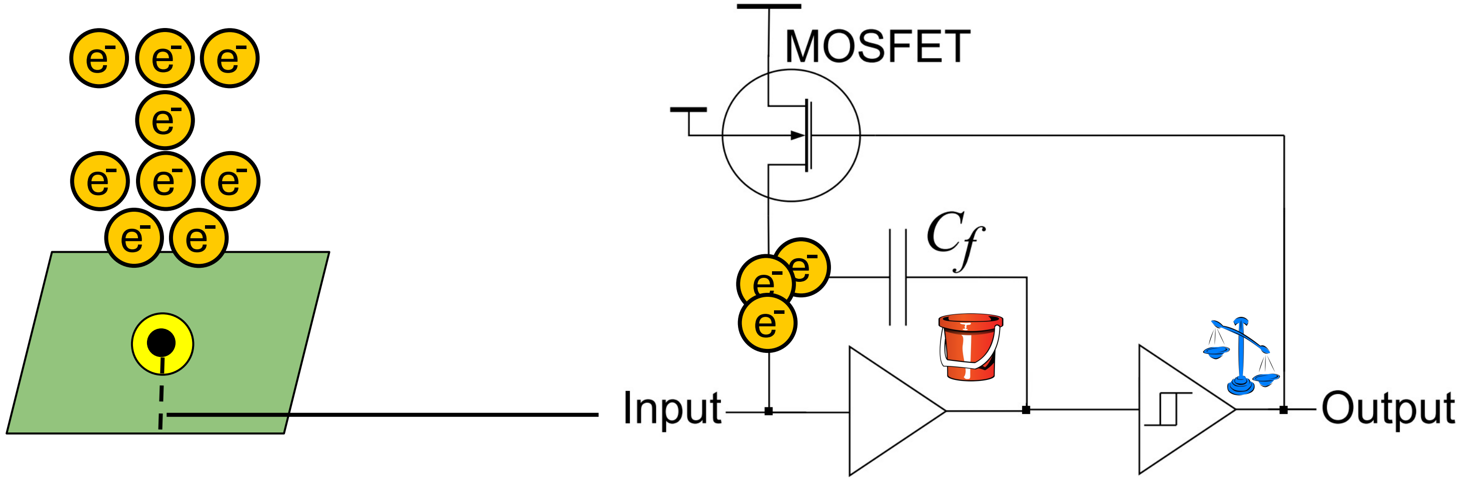

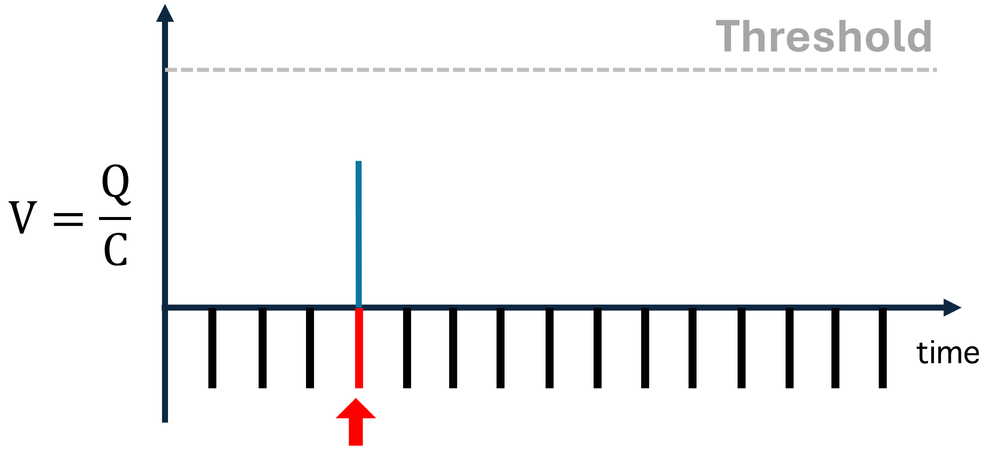

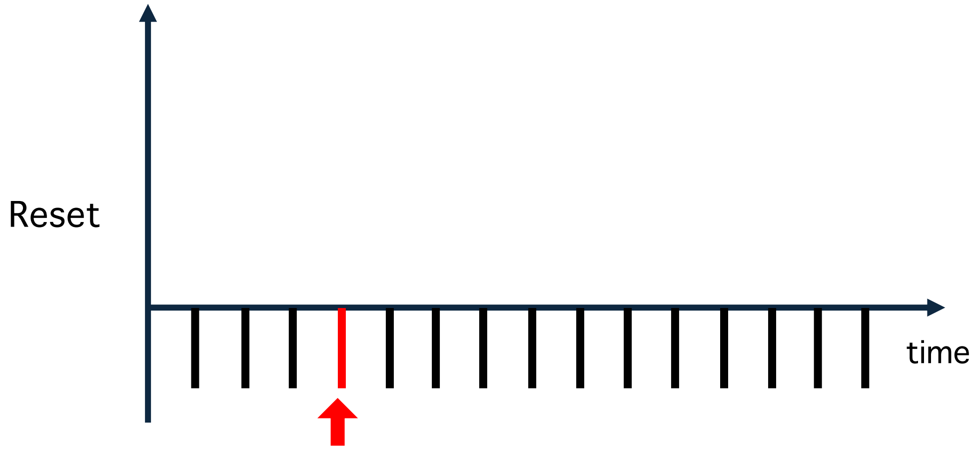

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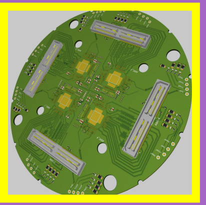

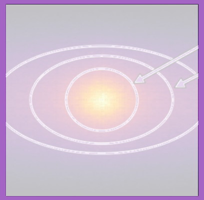

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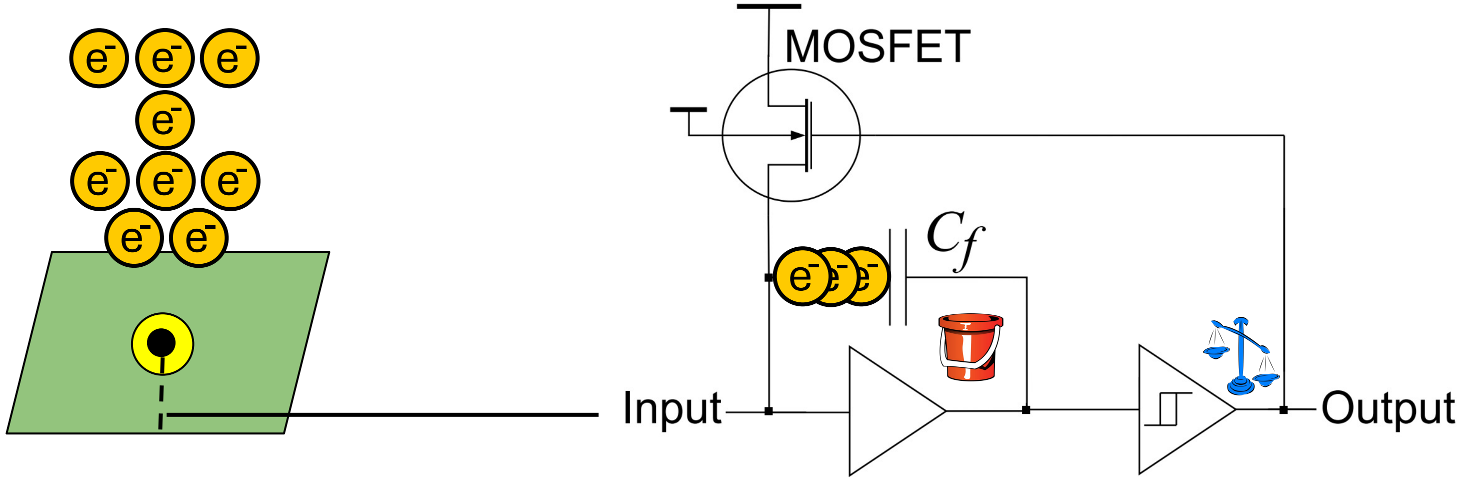

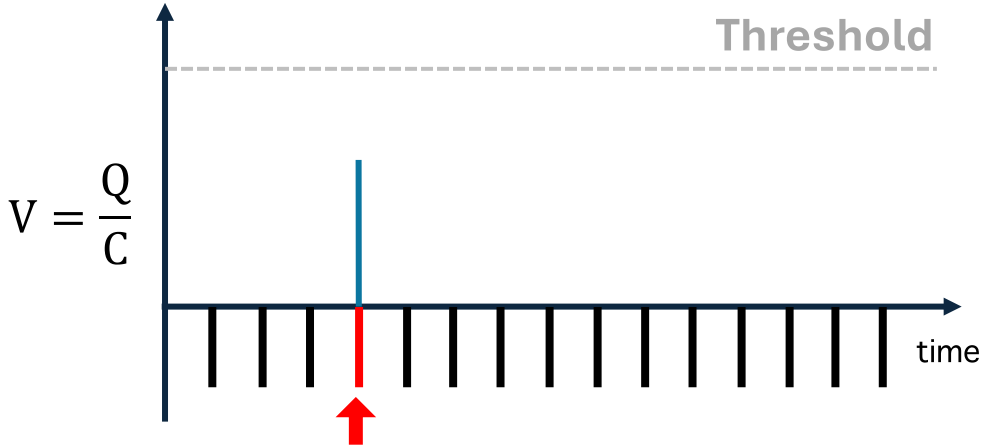

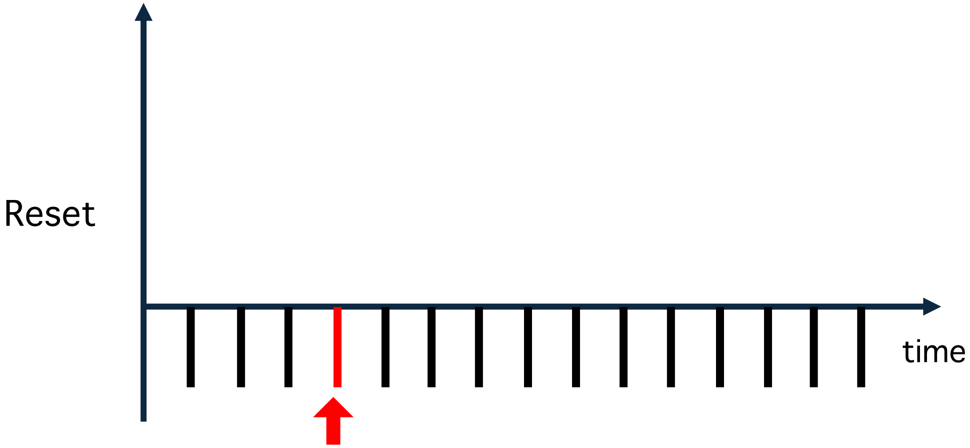

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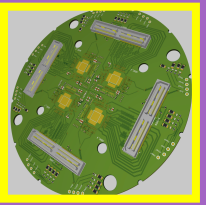

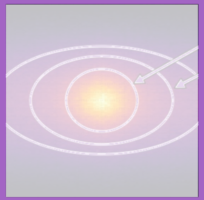

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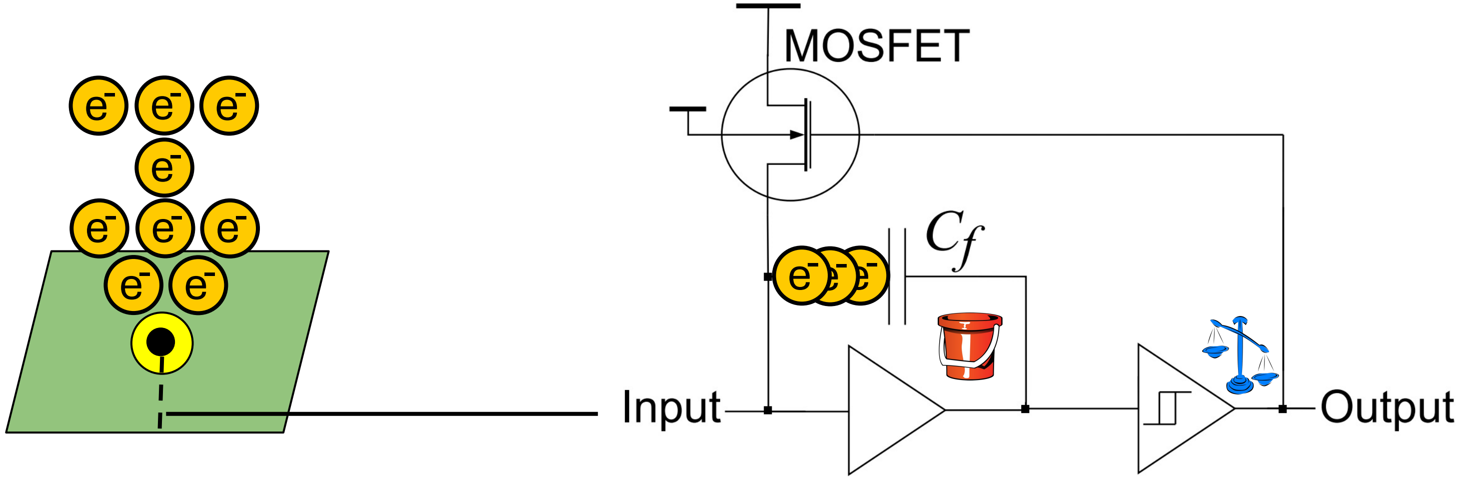

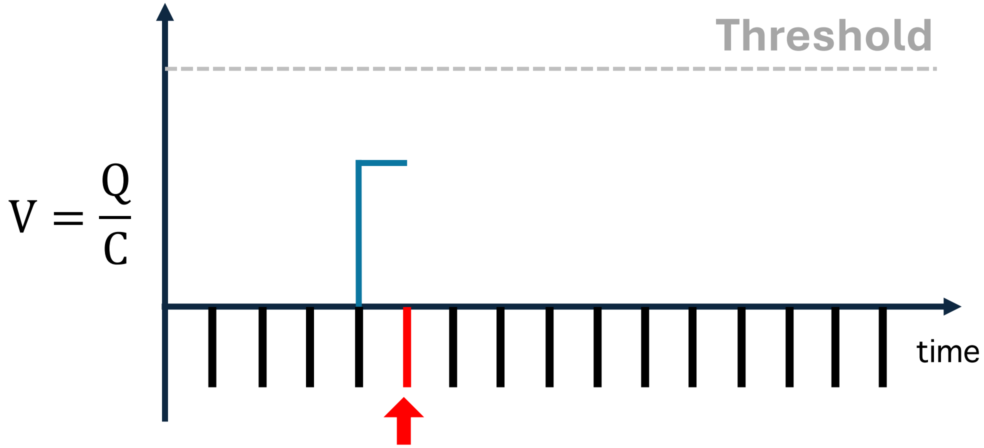

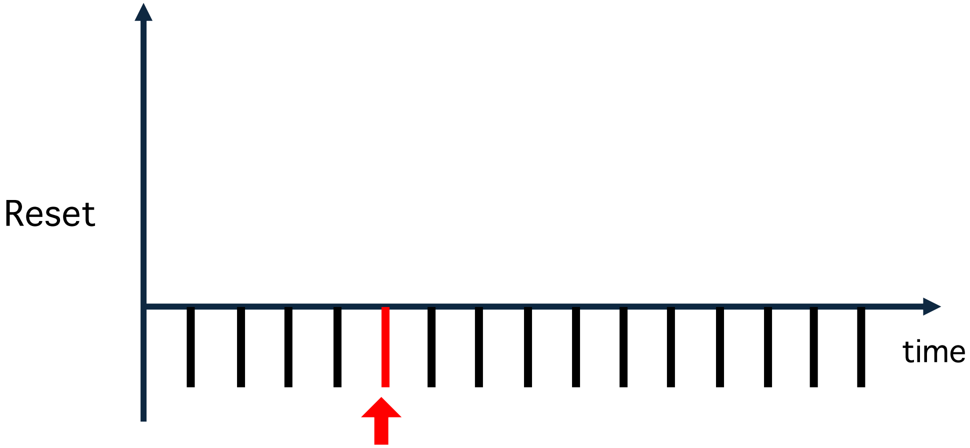

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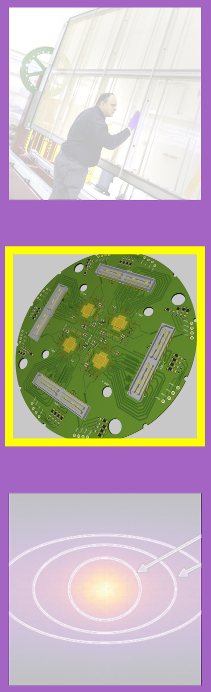
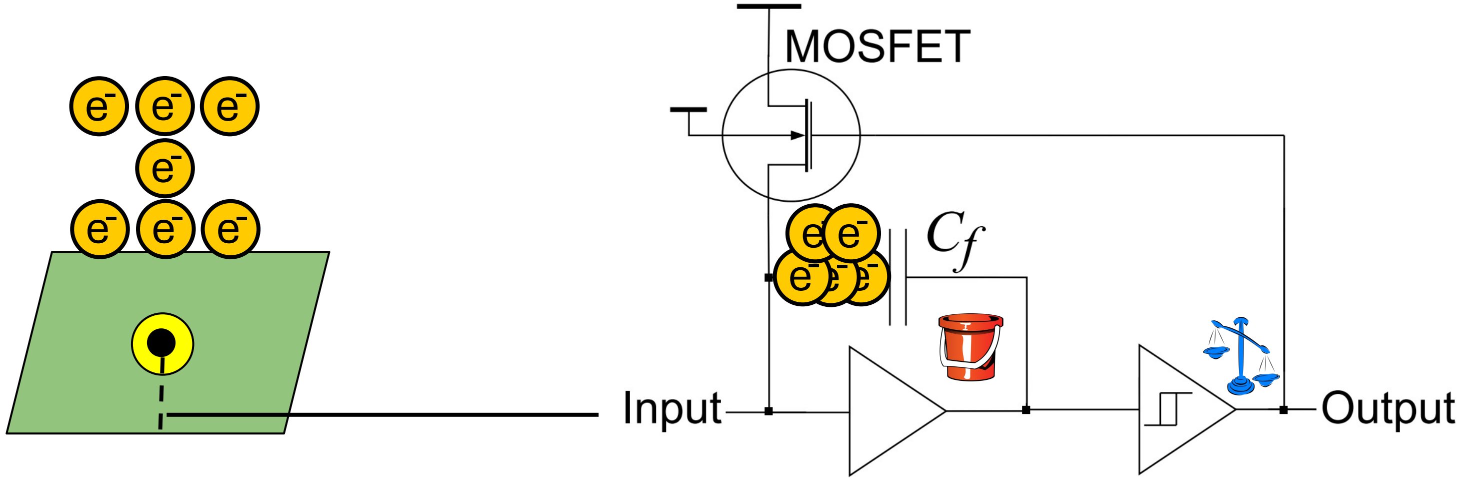
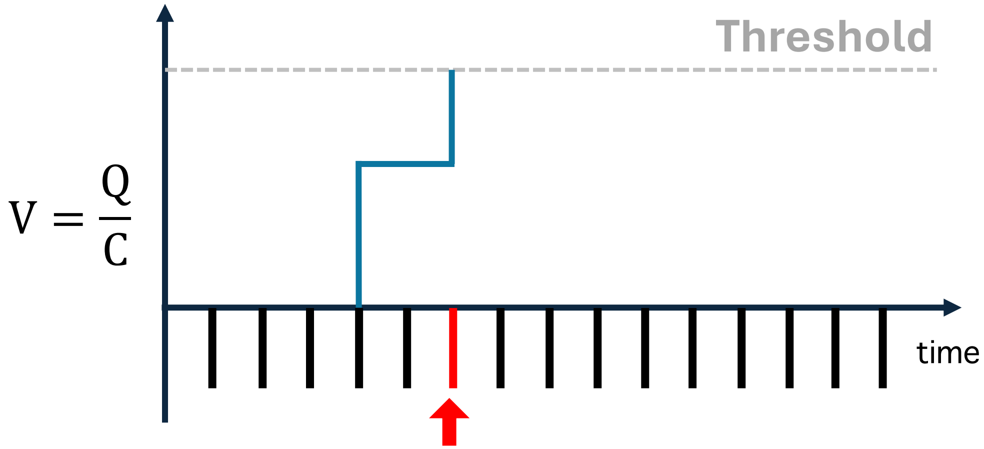
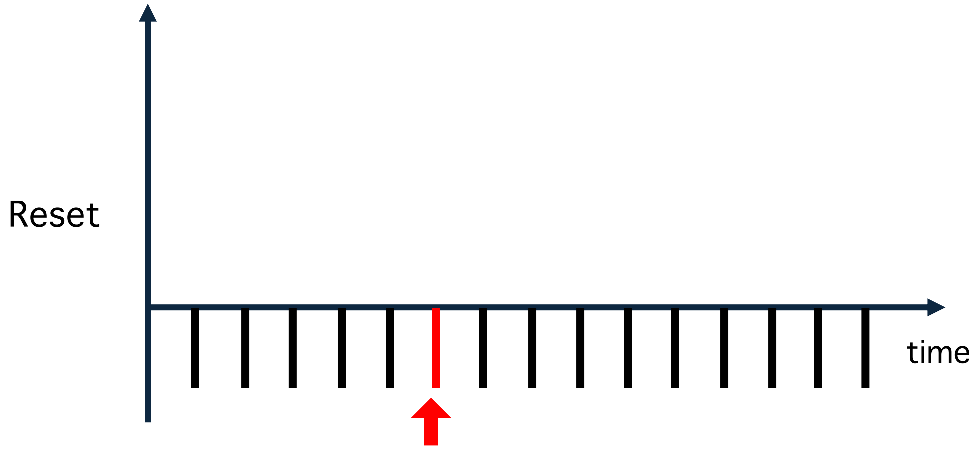


Toy Example

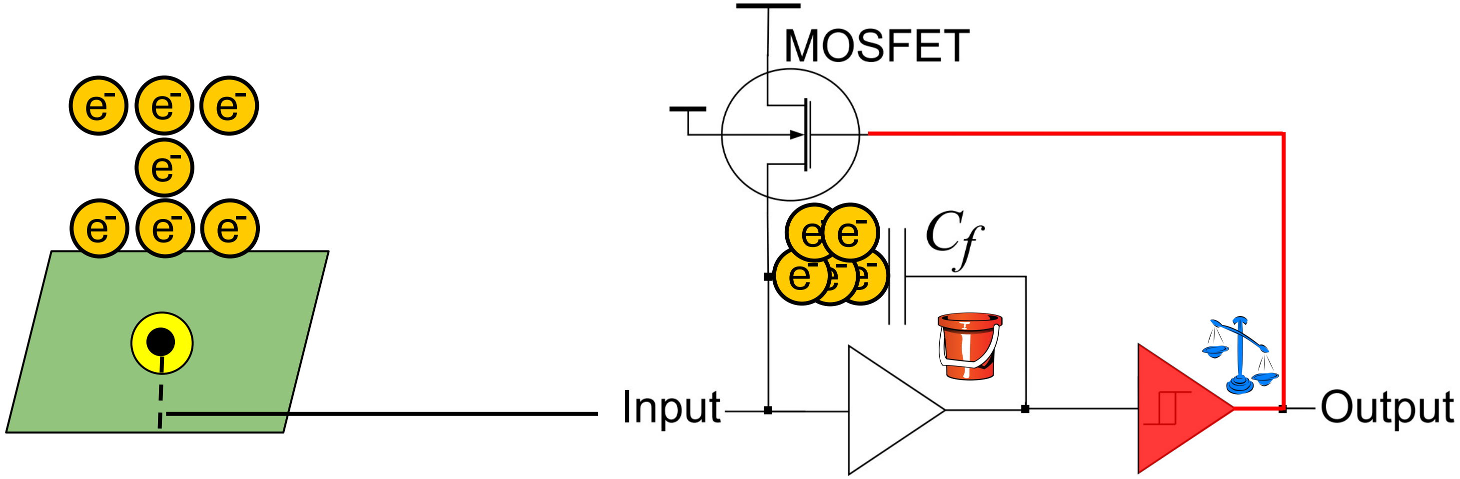


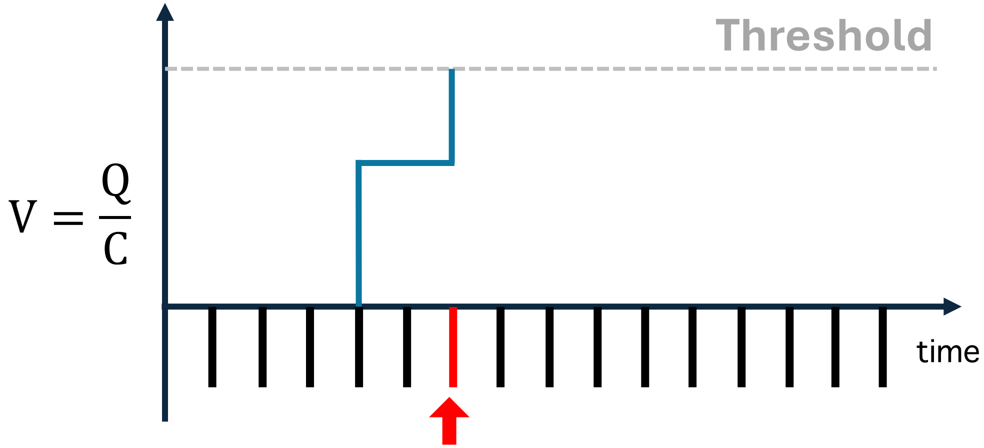


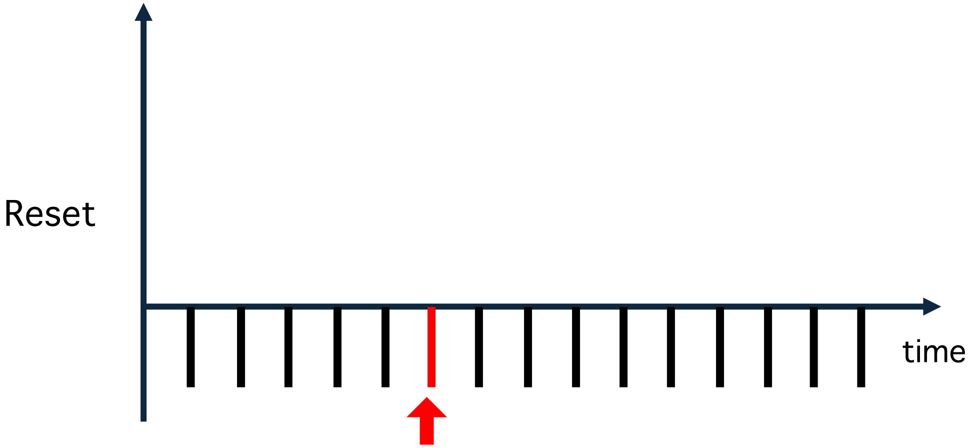


※ the reset happens for 5 electrons

Toy Example







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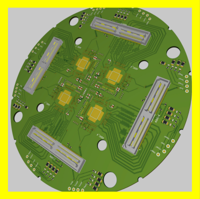
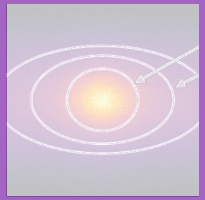
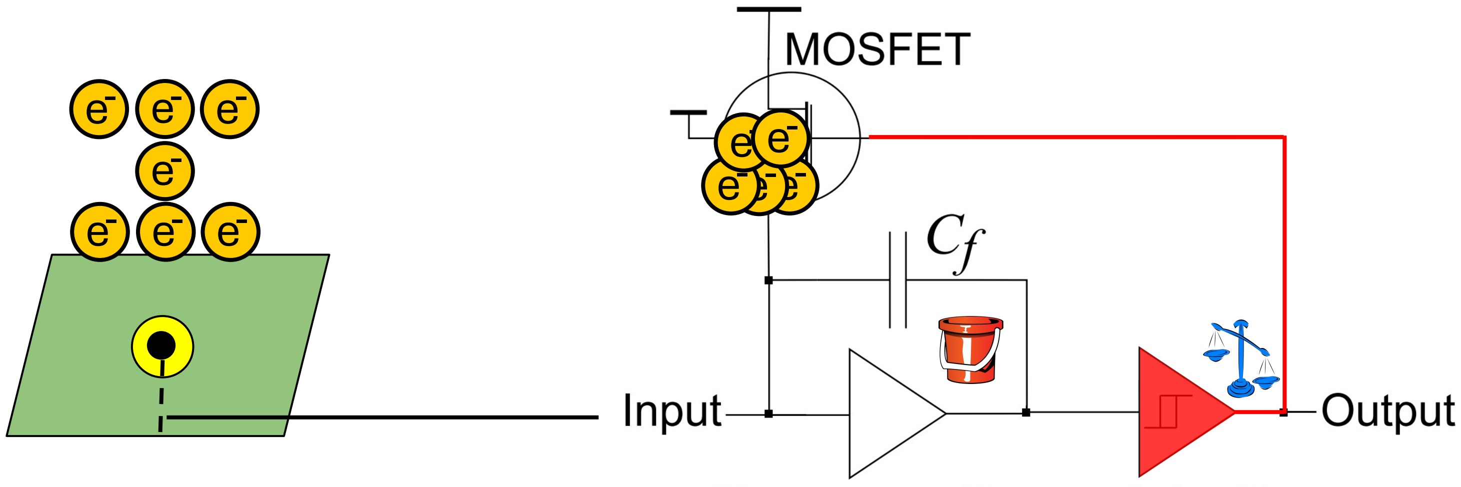
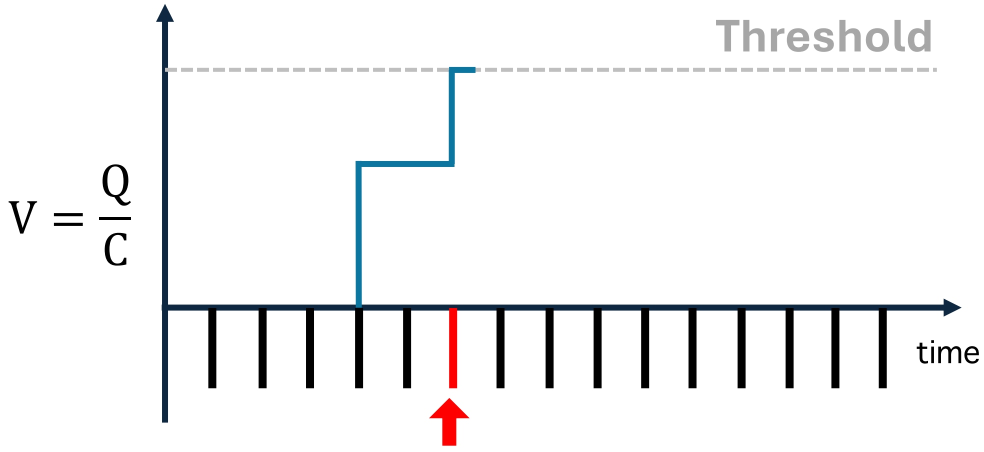
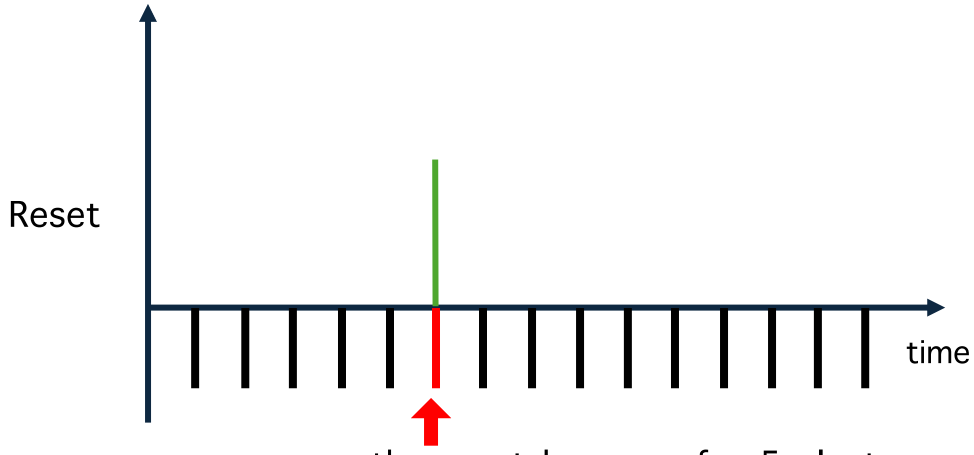




Toy Example







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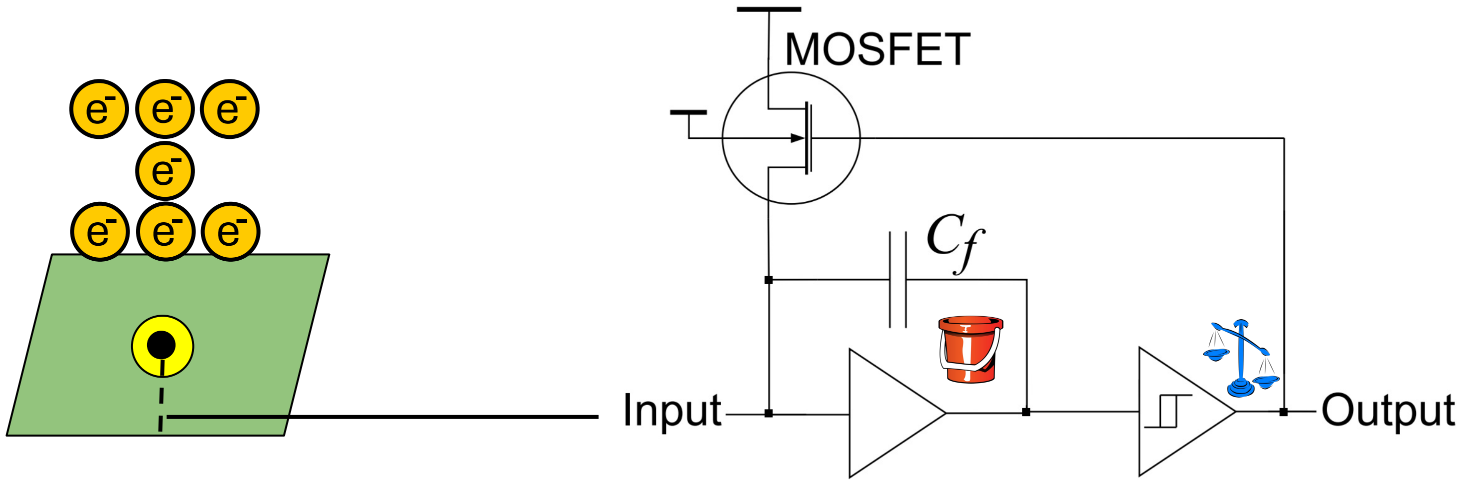
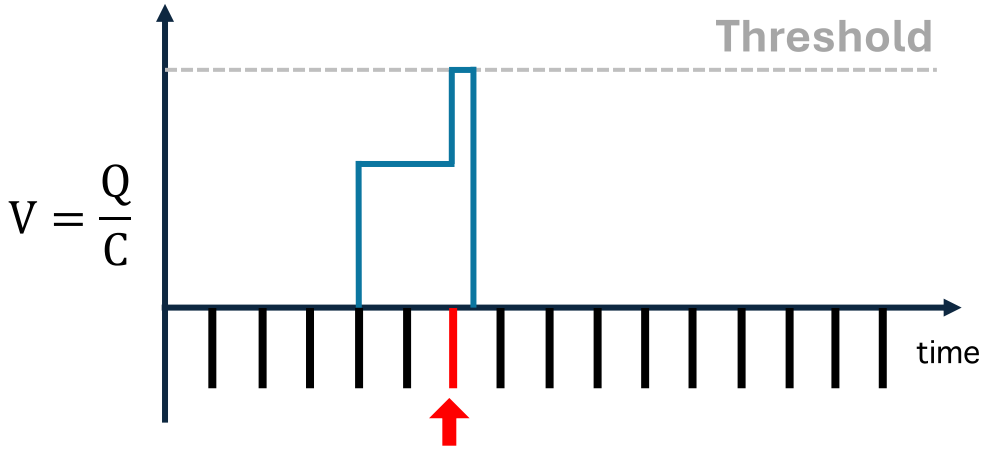
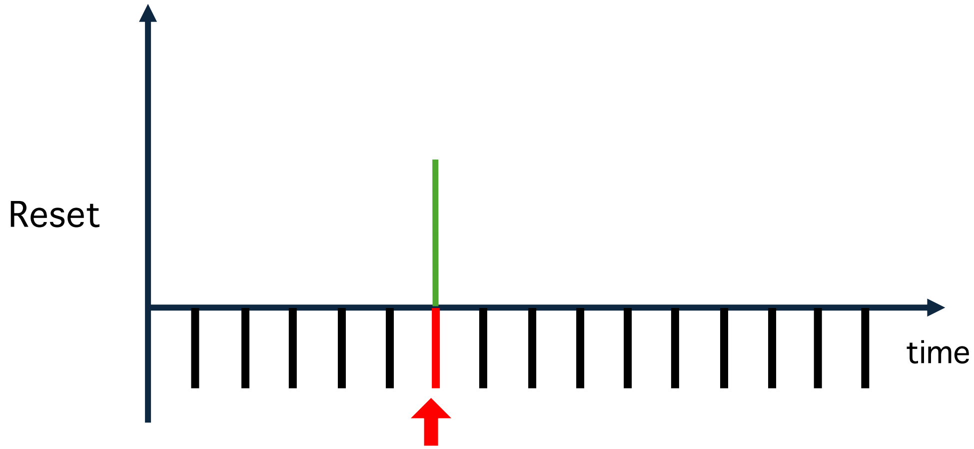
Toy Example

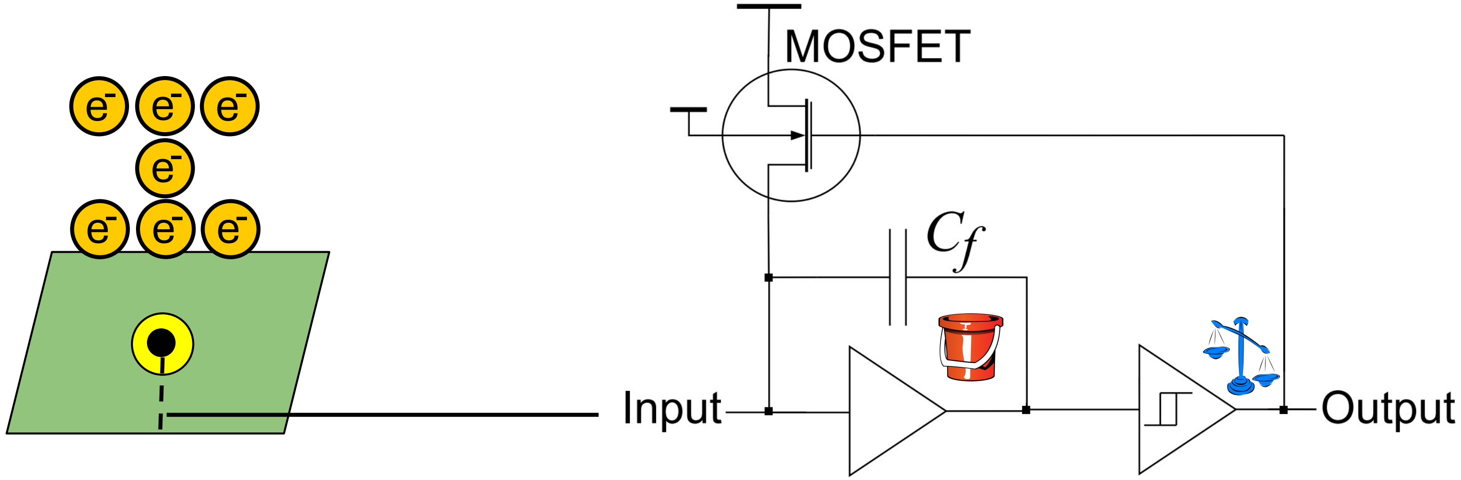


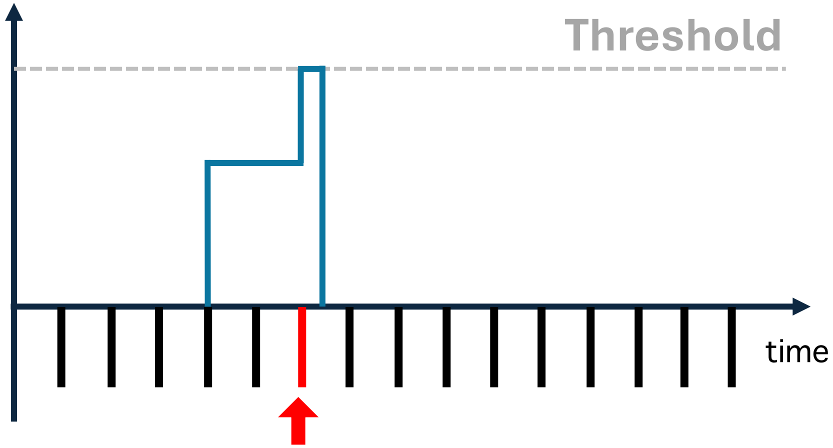




※ the reset happens for 5 electrons

Toy Example

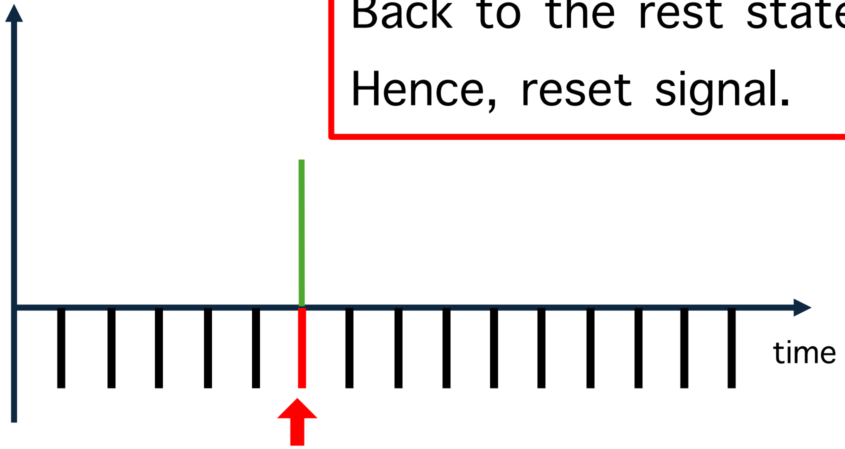




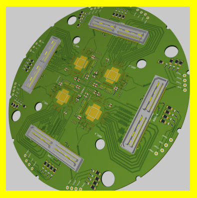
$V = \frac{Q}{C}$

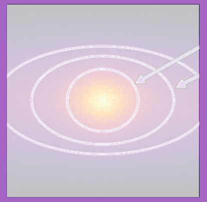
Back to the rest state!
Hence, reset signal.

Reset

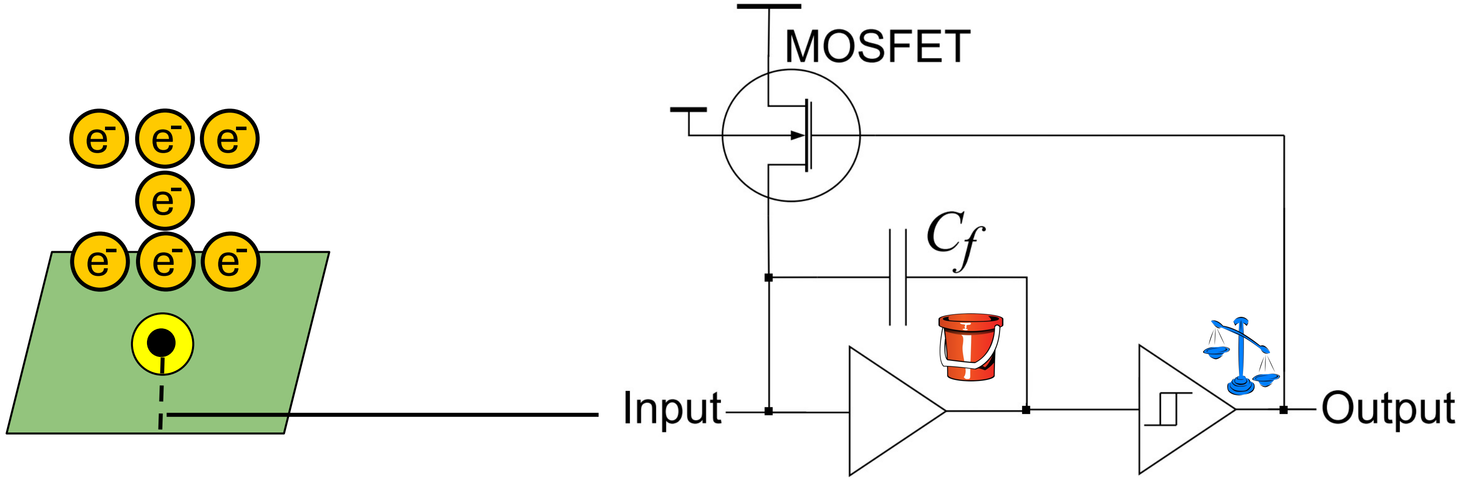


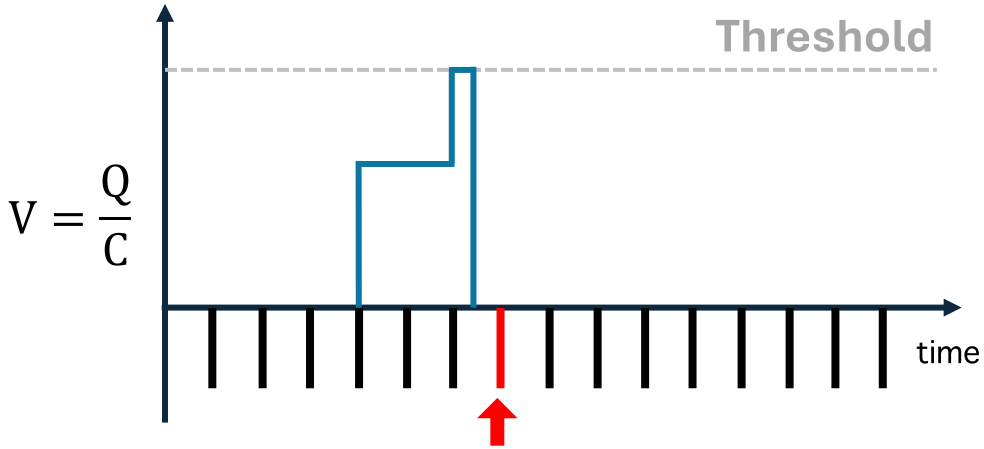
※ the reset happens for 5 electrons

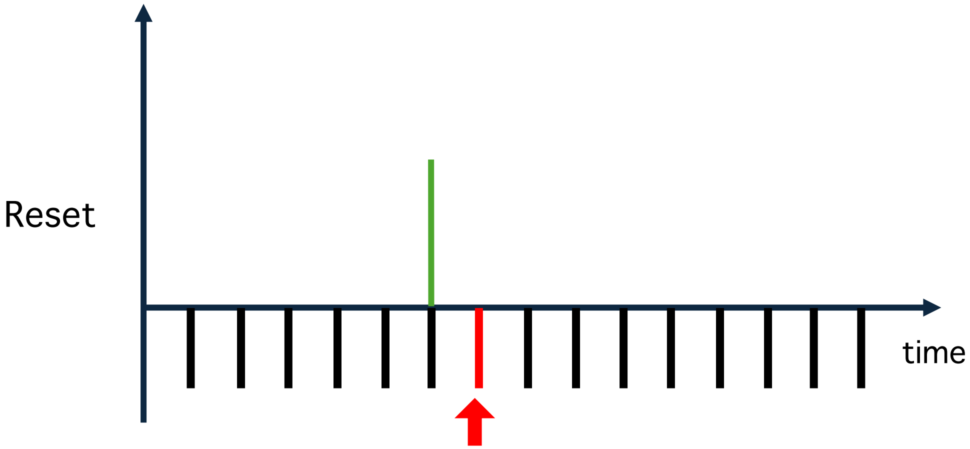




Toy Example







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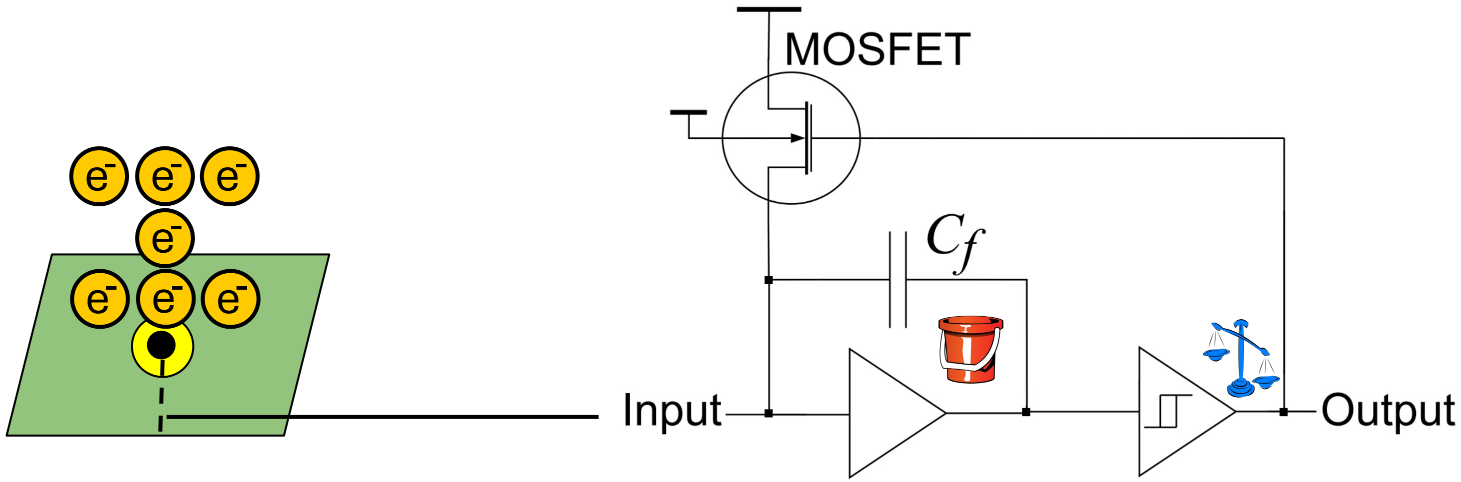
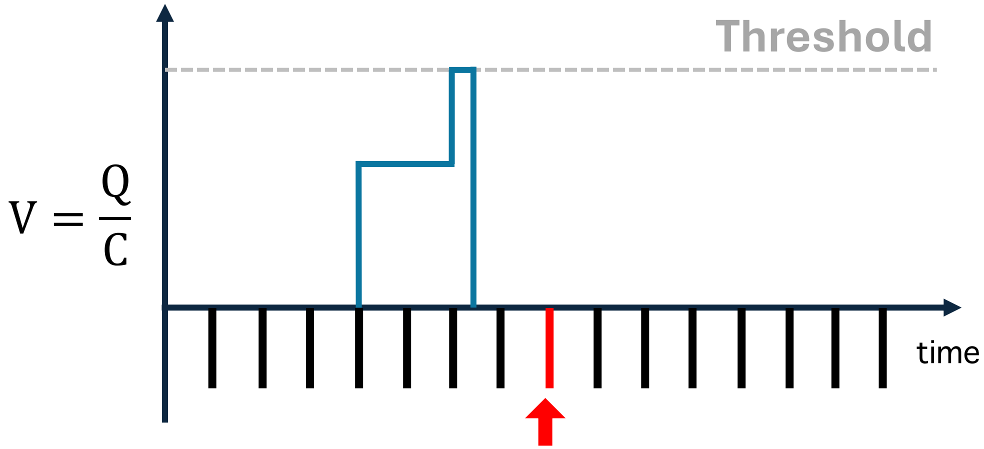
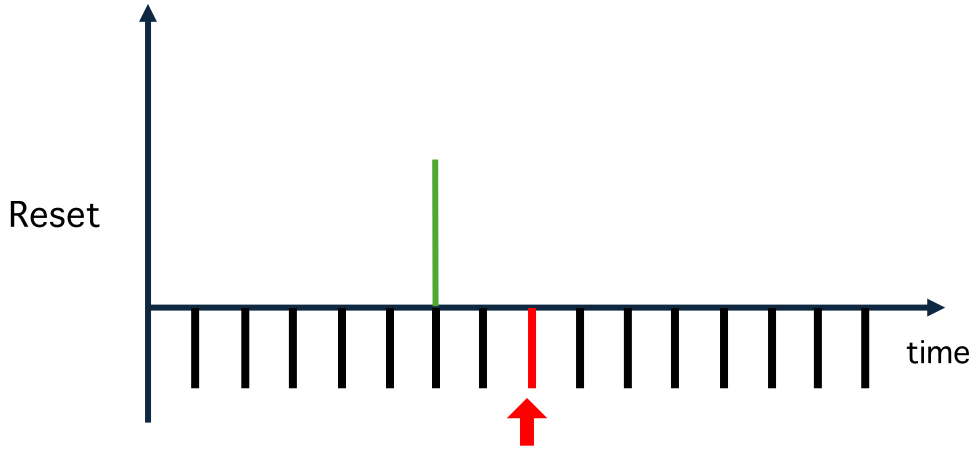
Toy Example







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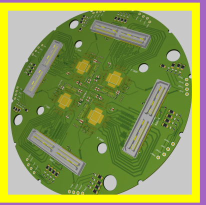
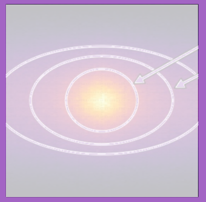
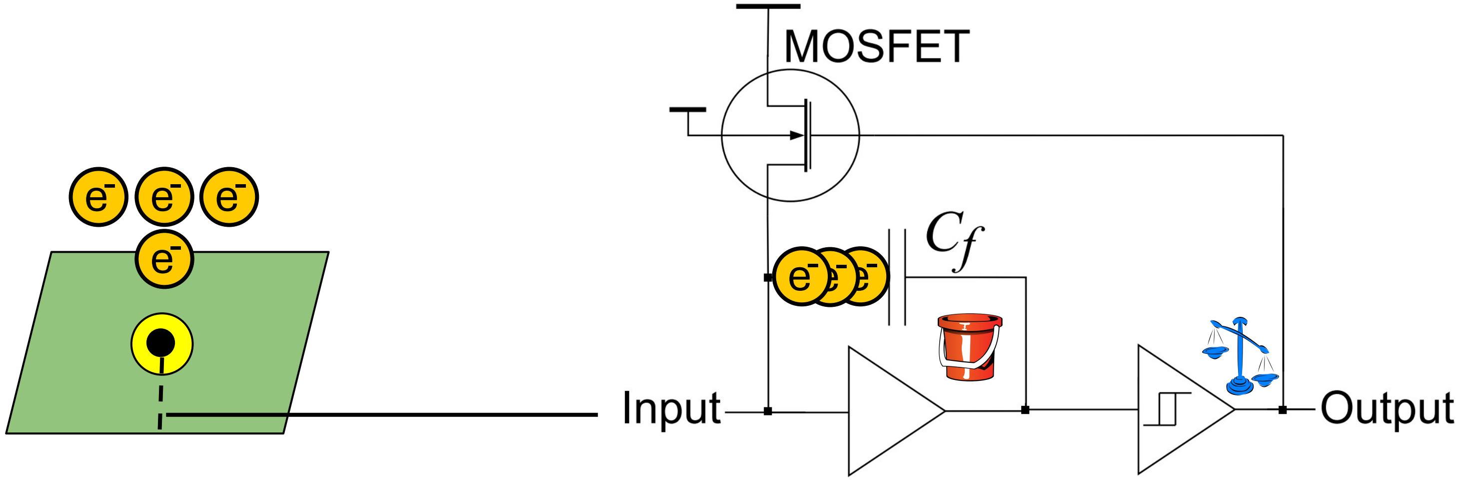
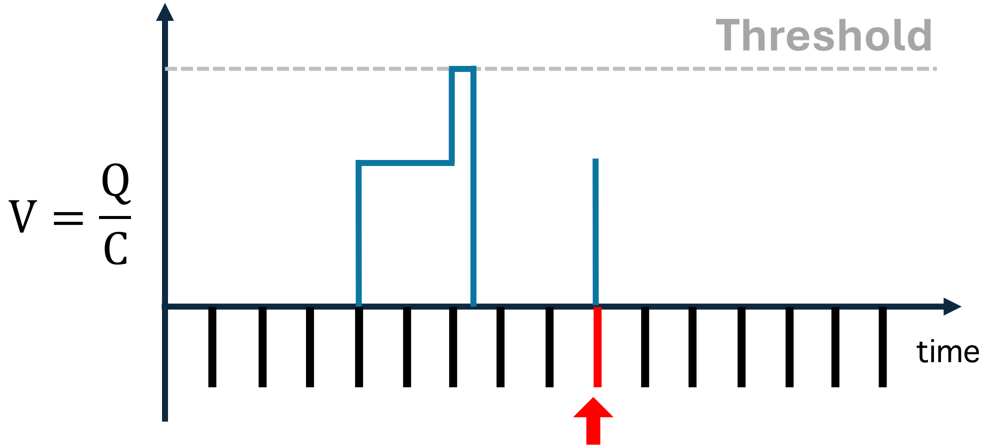
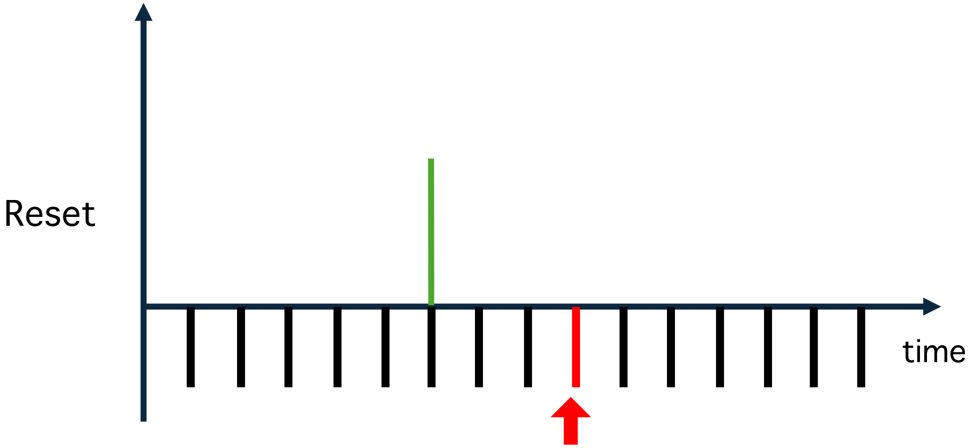




Toy Example







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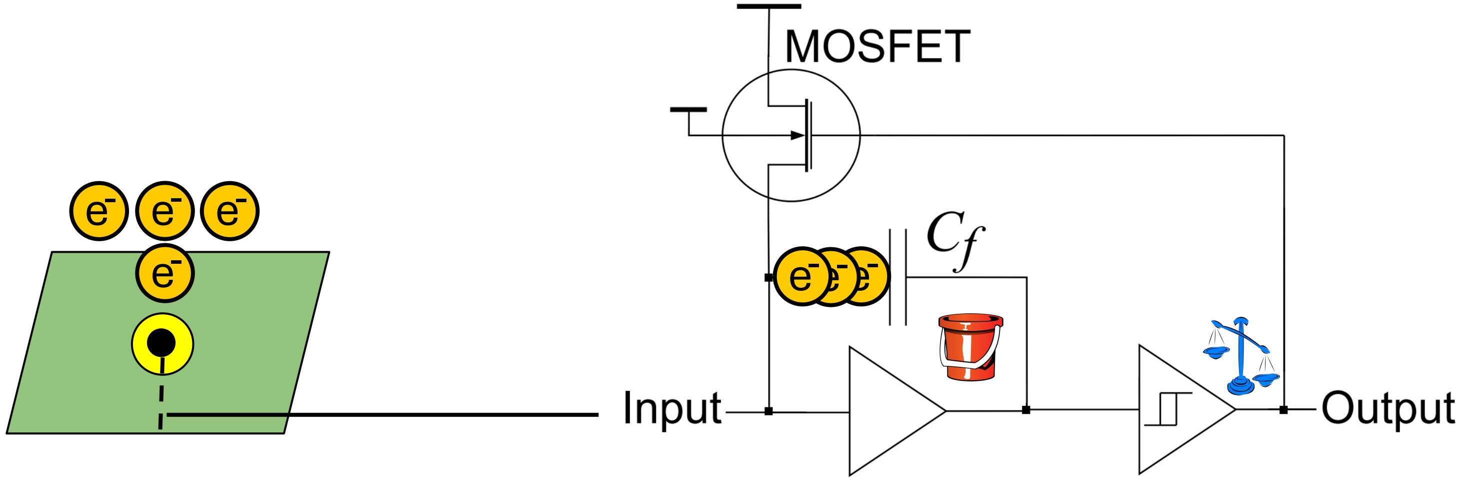
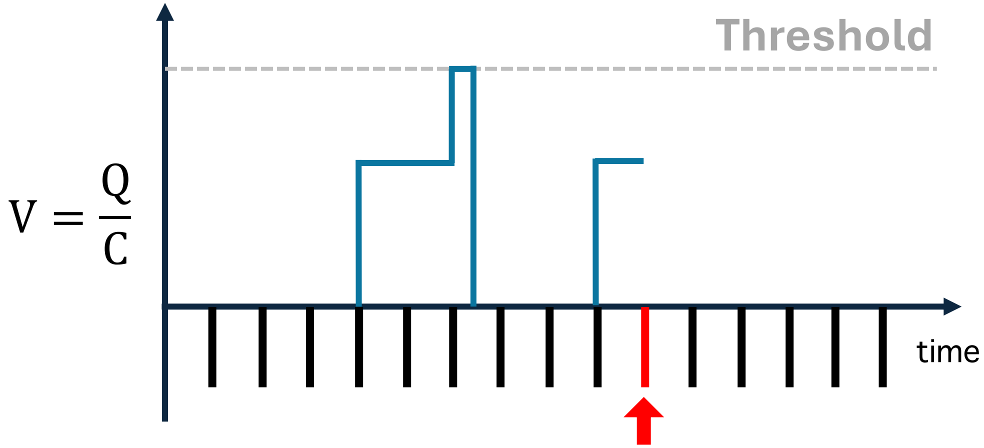
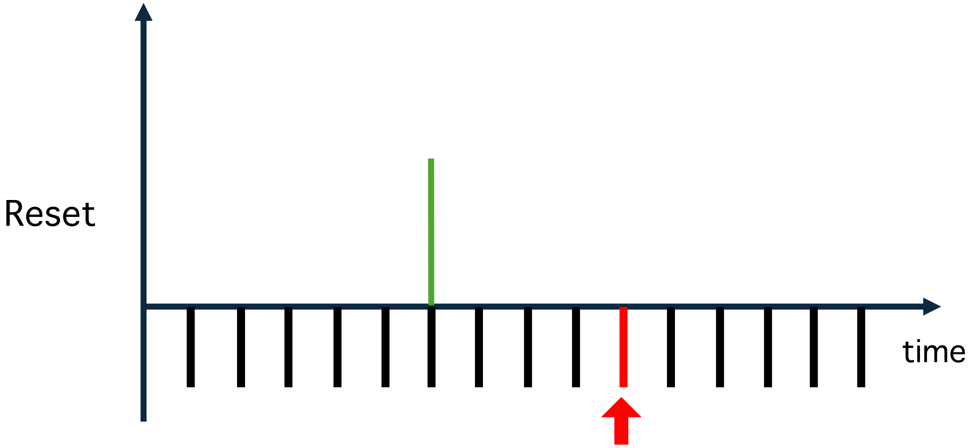
Toy Example







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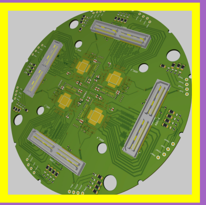
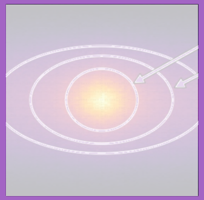
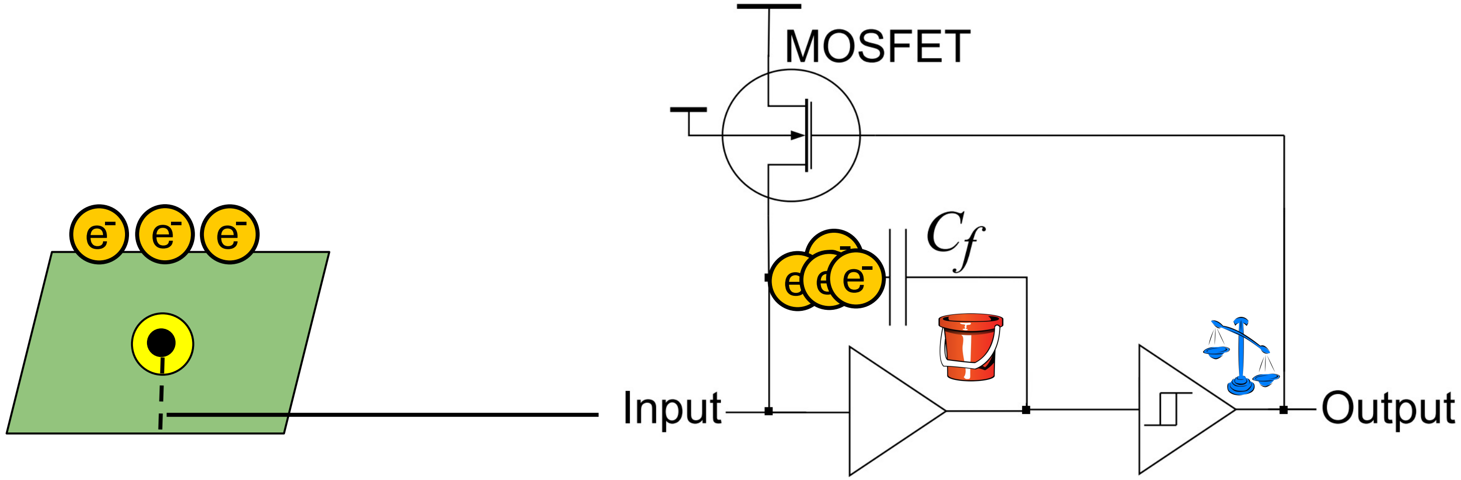
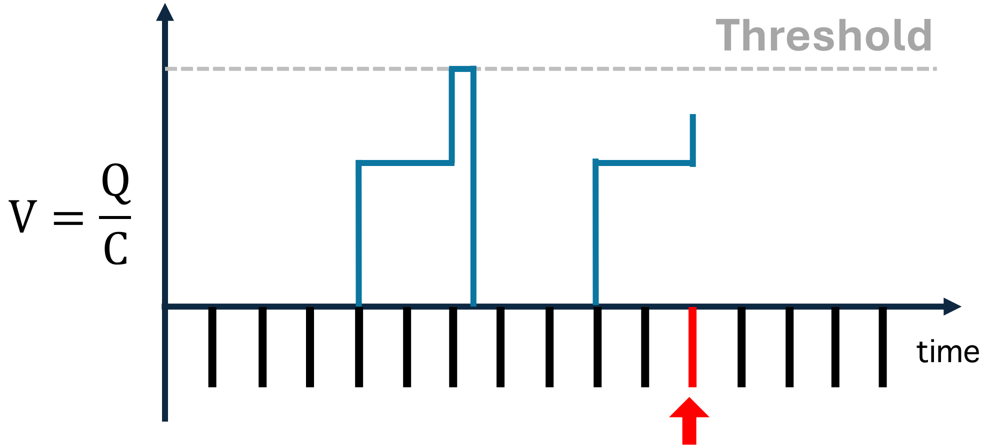
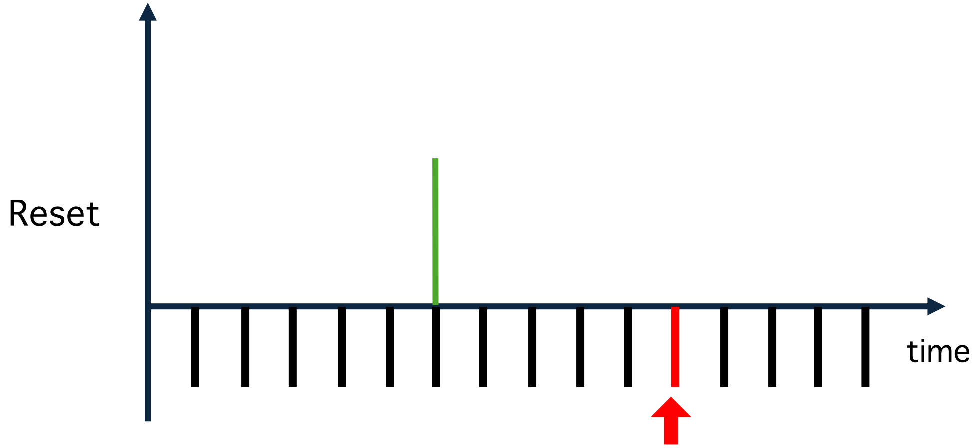




Toy Example







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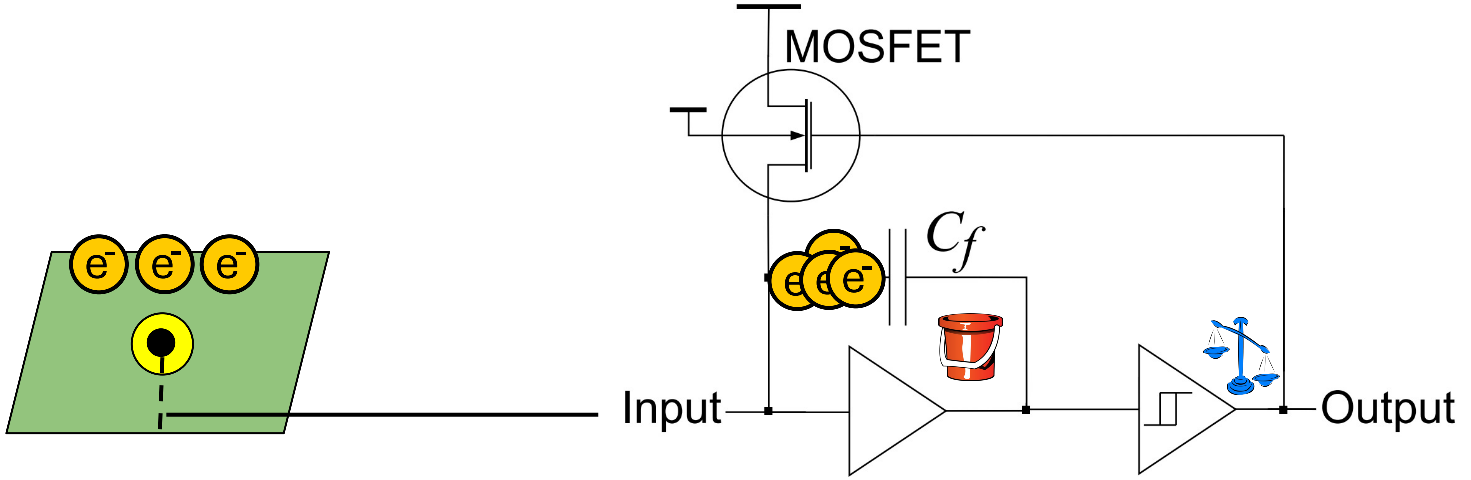

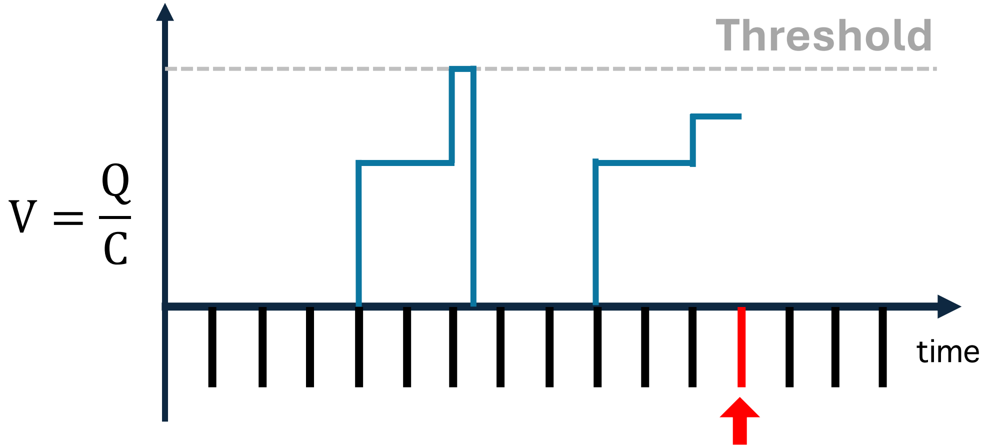

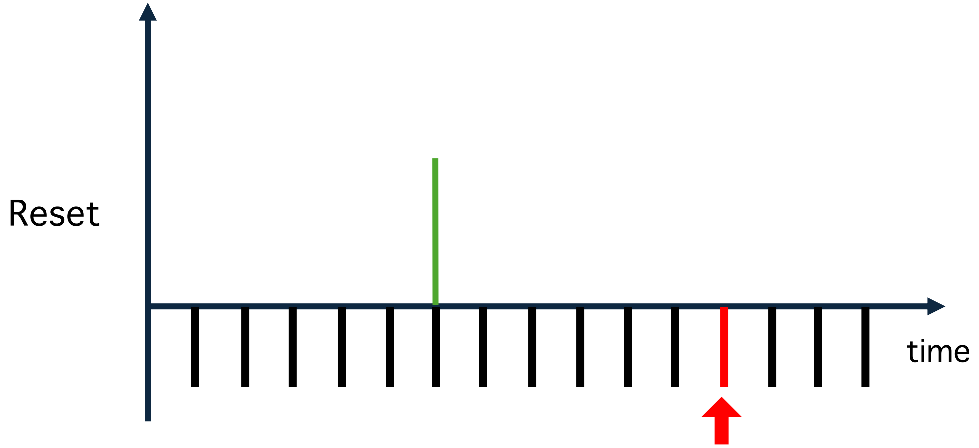

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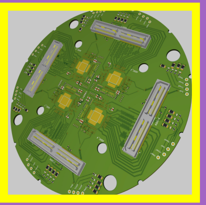
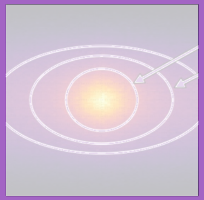
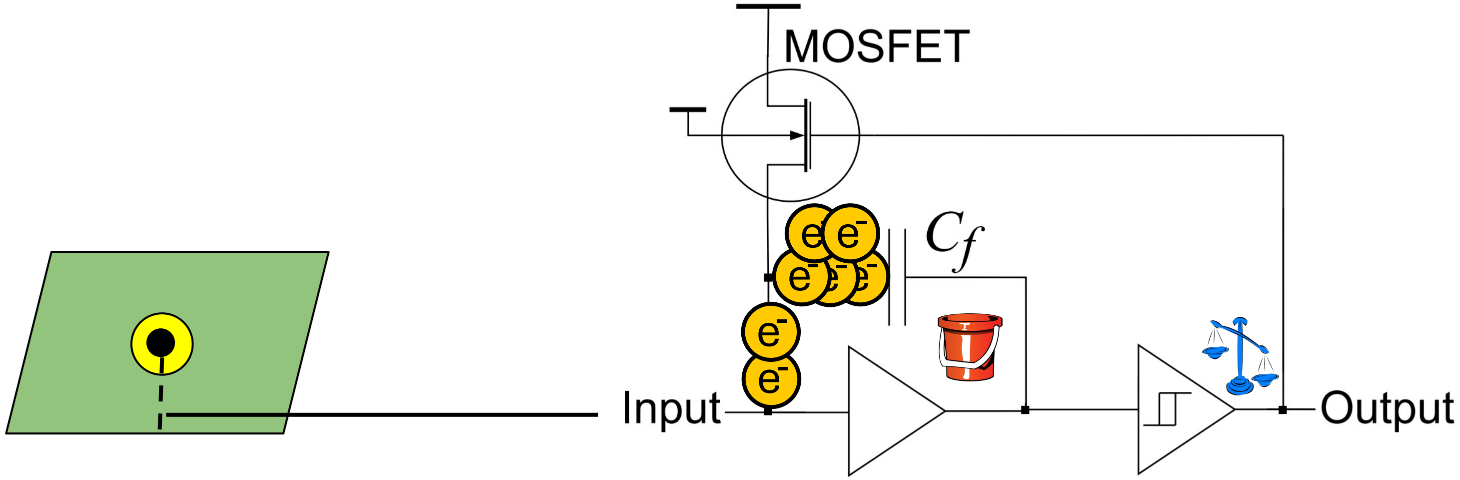
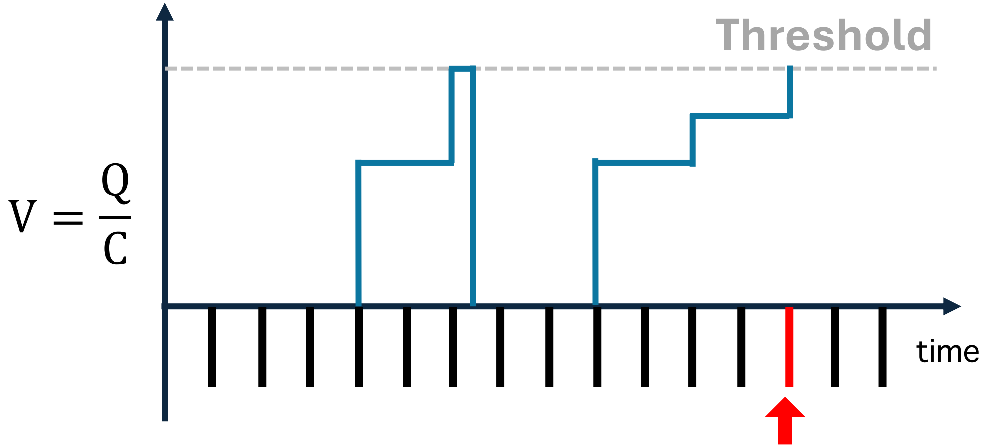
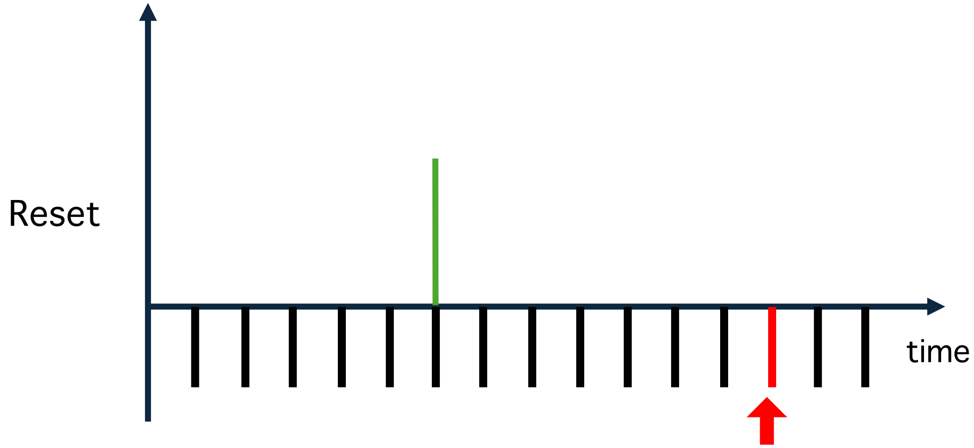




Toy Example







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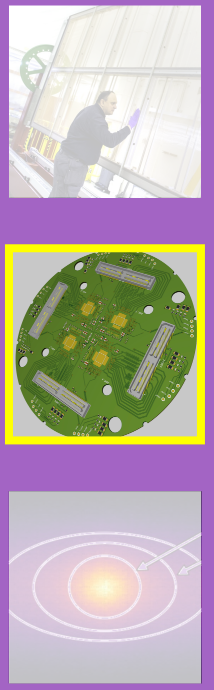
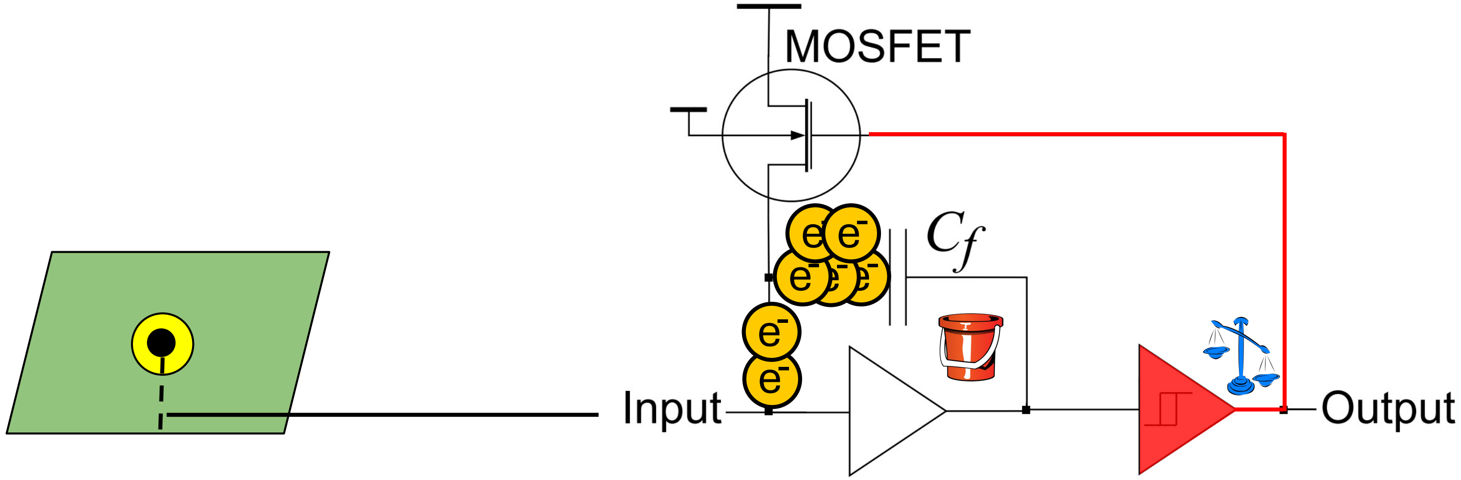
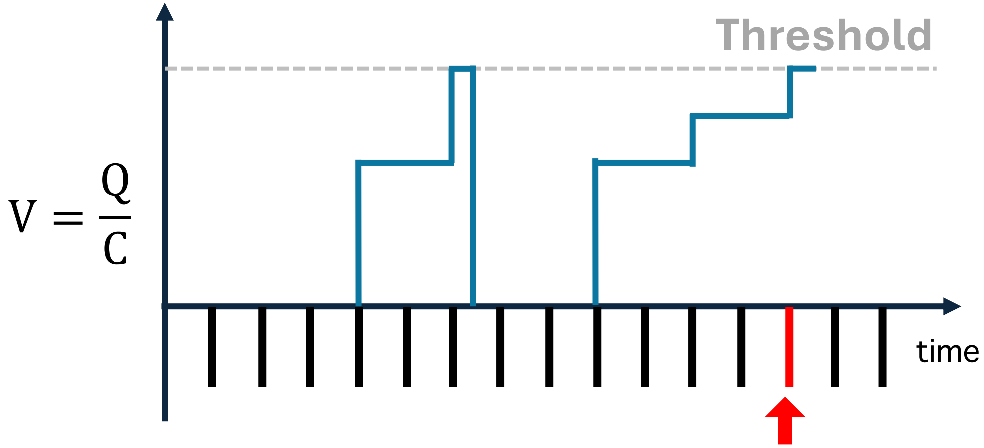
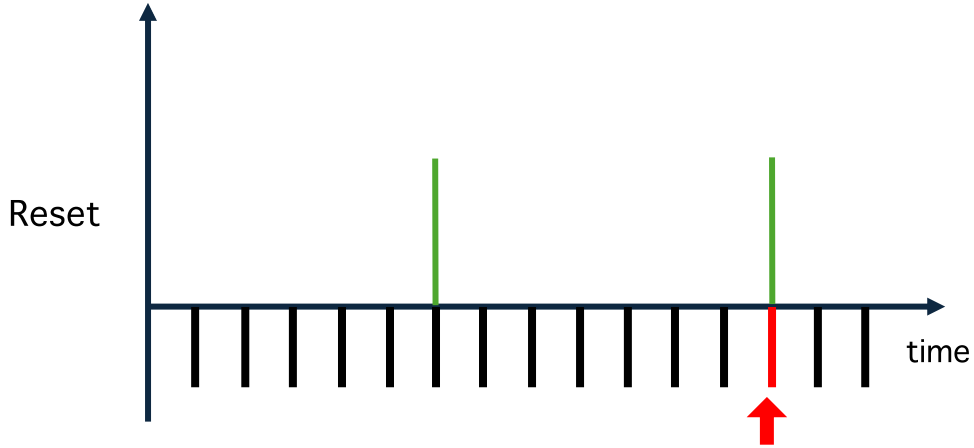


Toy Example







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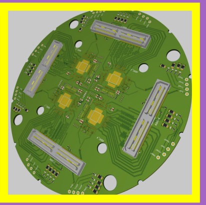
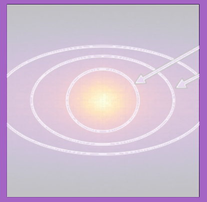
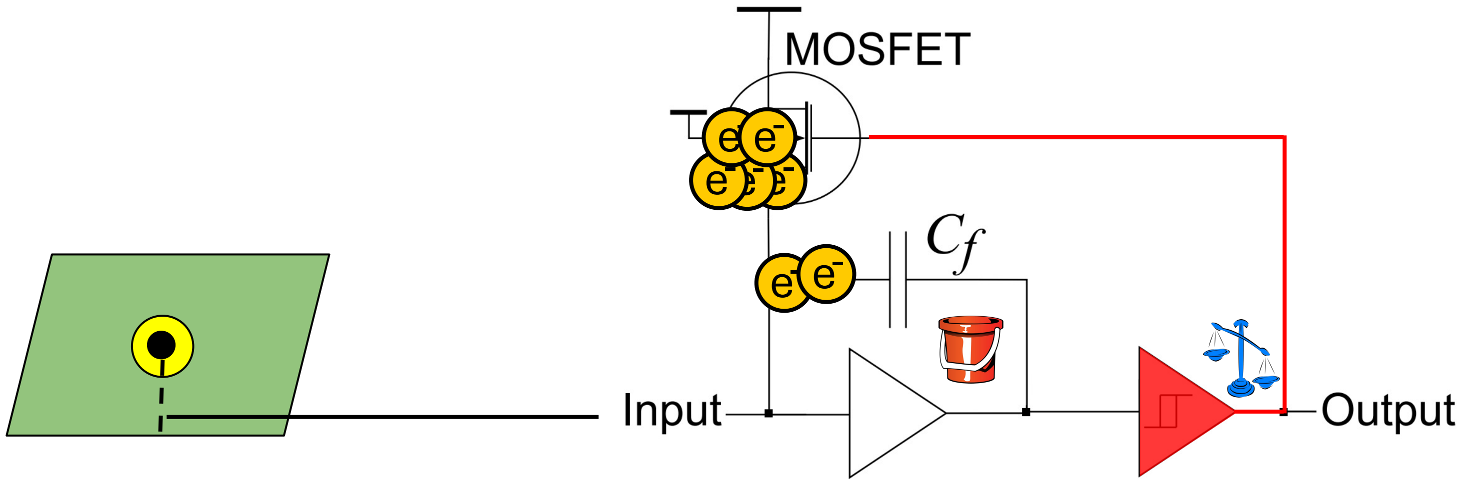
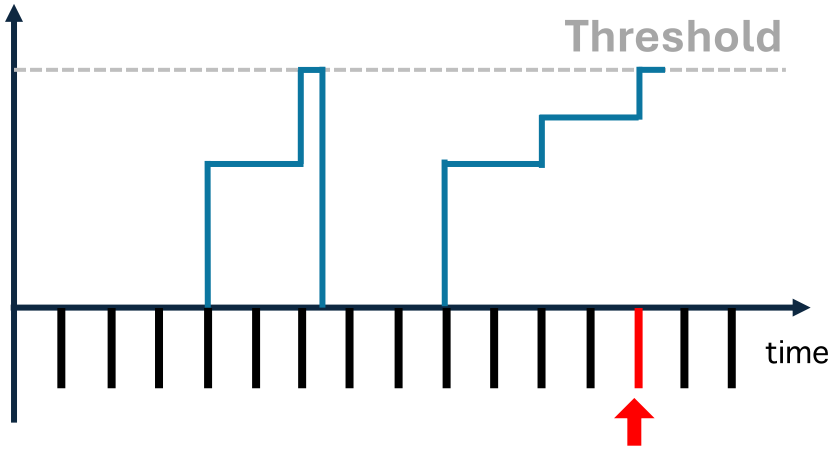


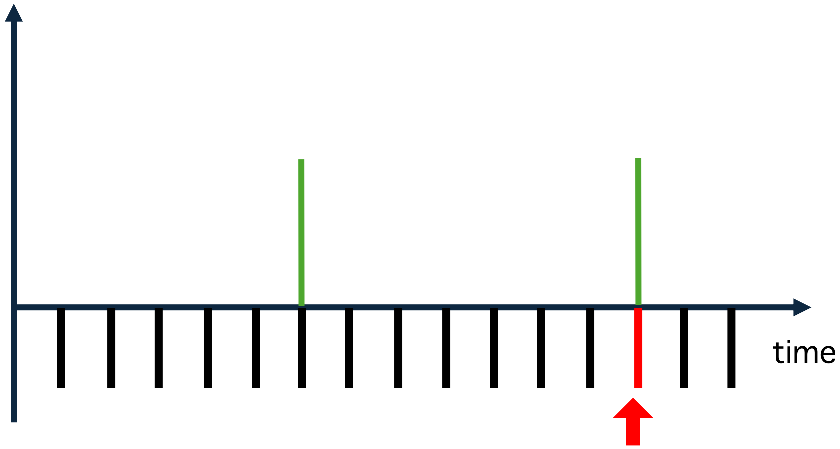


Toy Example





$$V = \frac{Q}{C}$$



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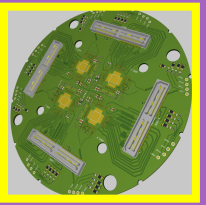
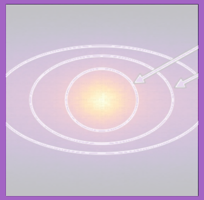
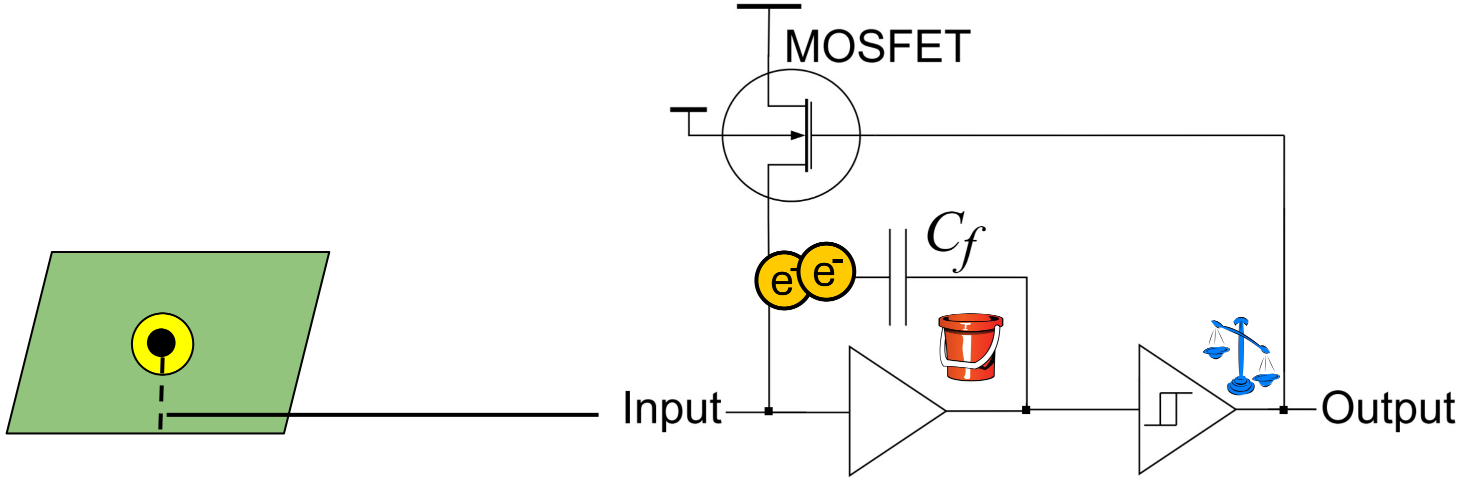
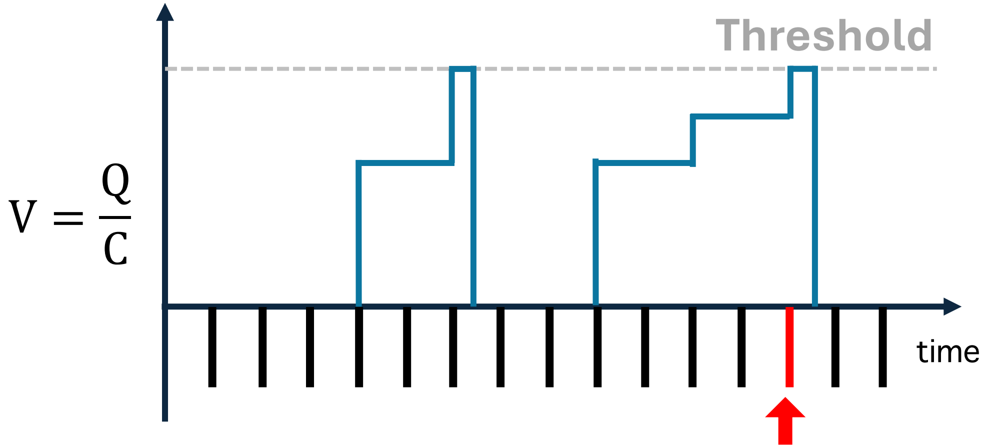
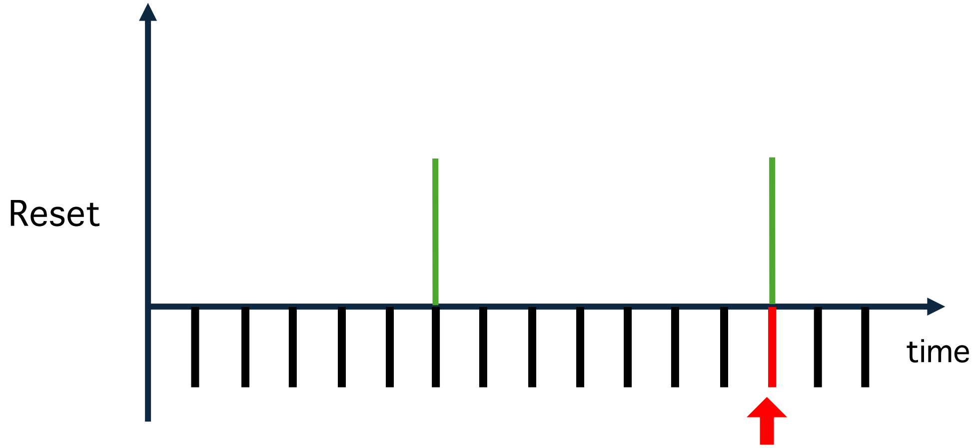




Toy Example







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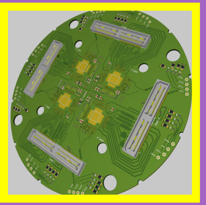
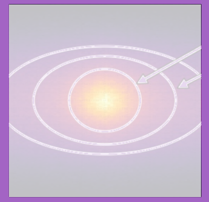
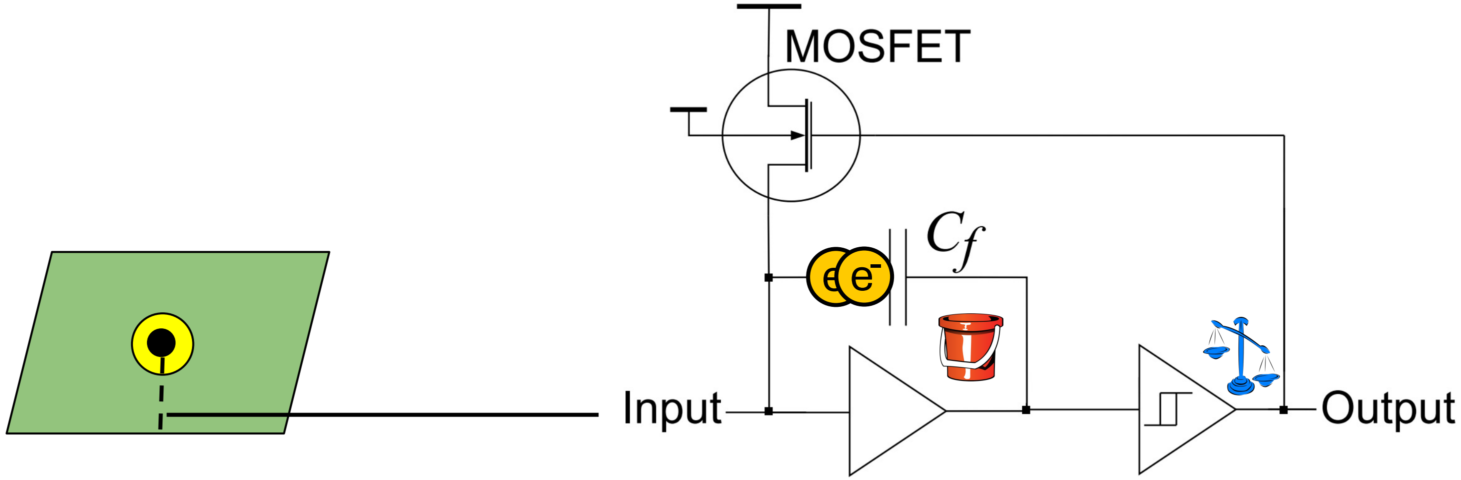
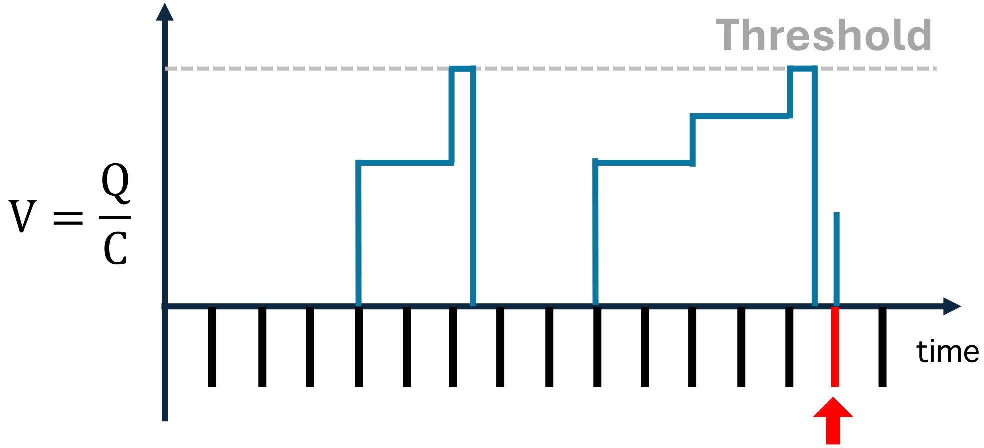
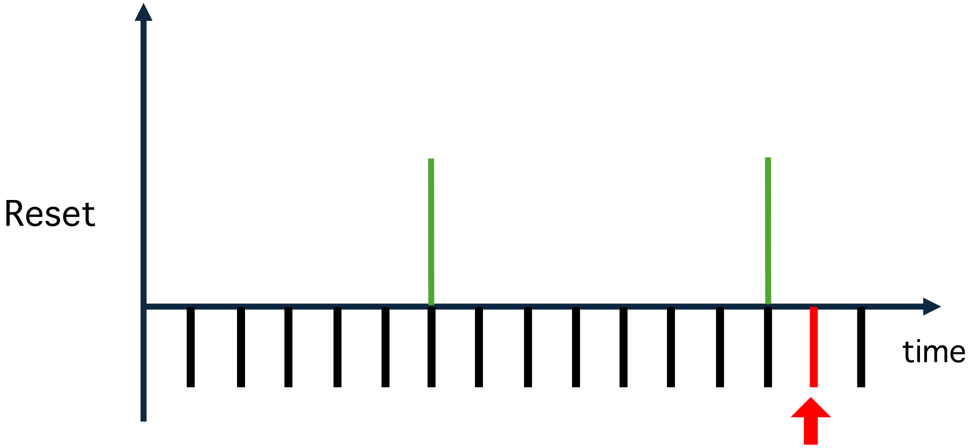




Toy Example







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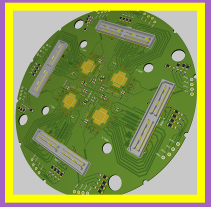
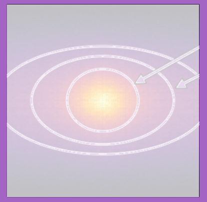


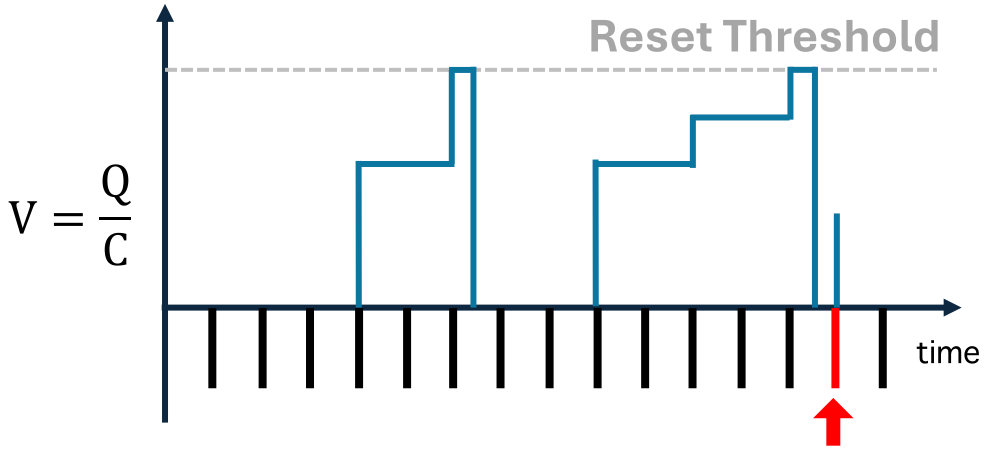


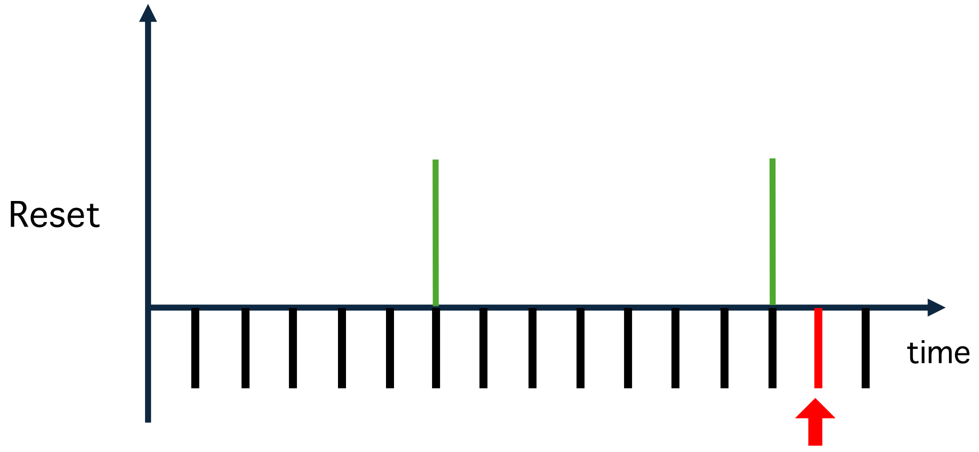
Toy Example

※ the reset happens for 5 electrons

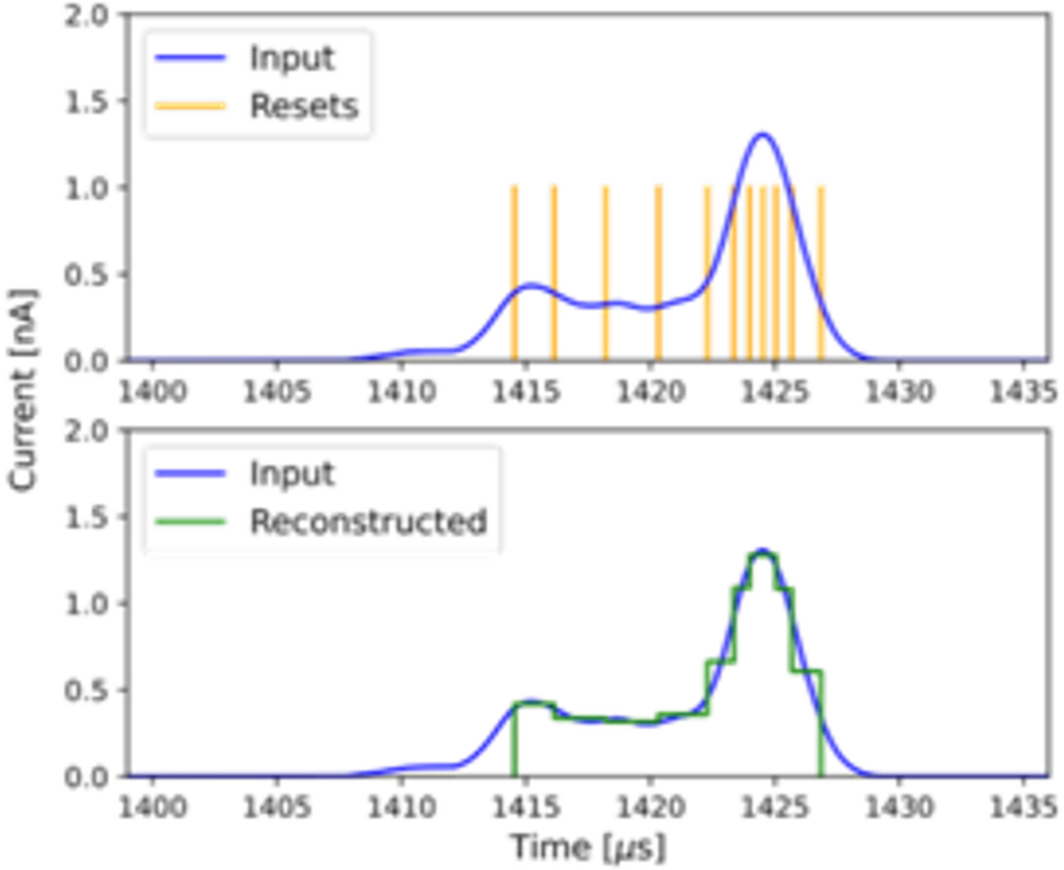
We are measuring
how long it took (Δt) to accumulate fixed amount of Q

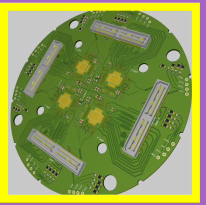
Instead of recording the entire waveform,
we only need to record time stamps!

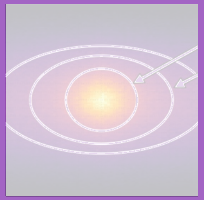




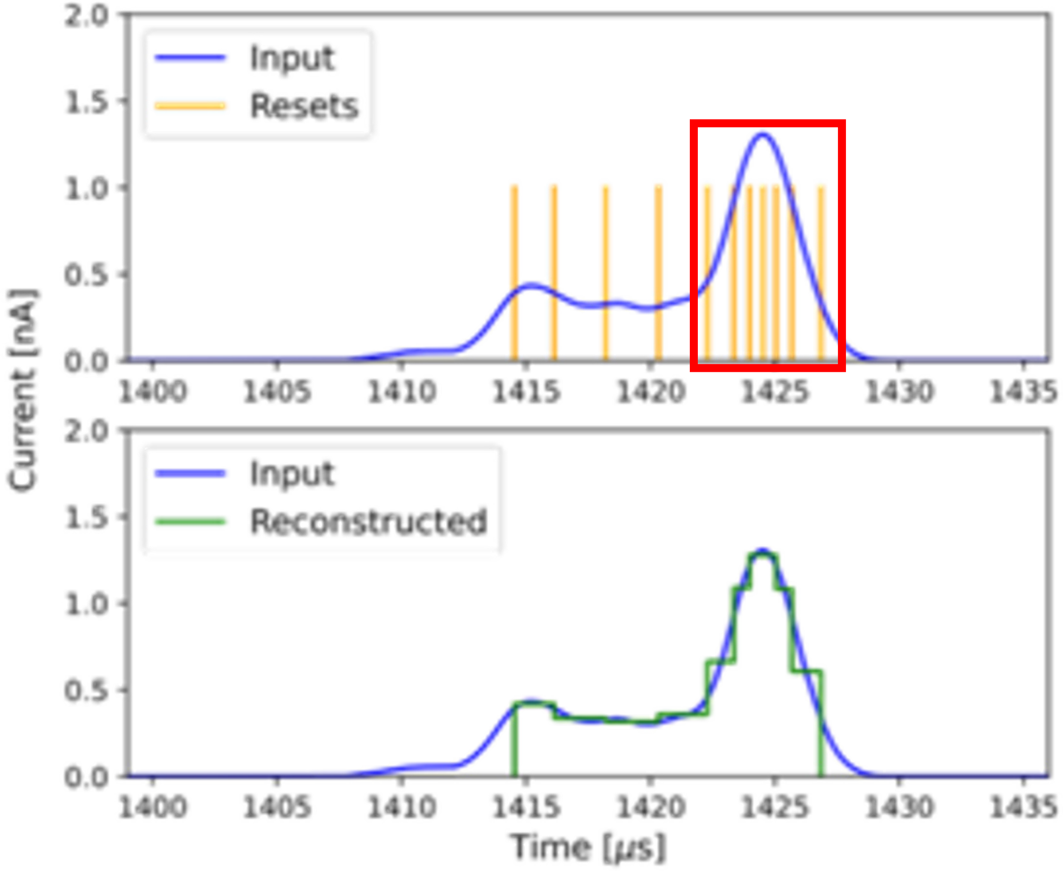
Solution : Pixelization







Solution : Pixelization



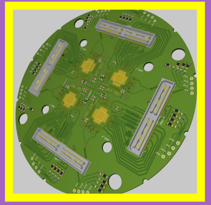
Denser area

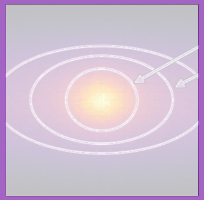
⇩

took pixel **less** time to get unit amount of charge

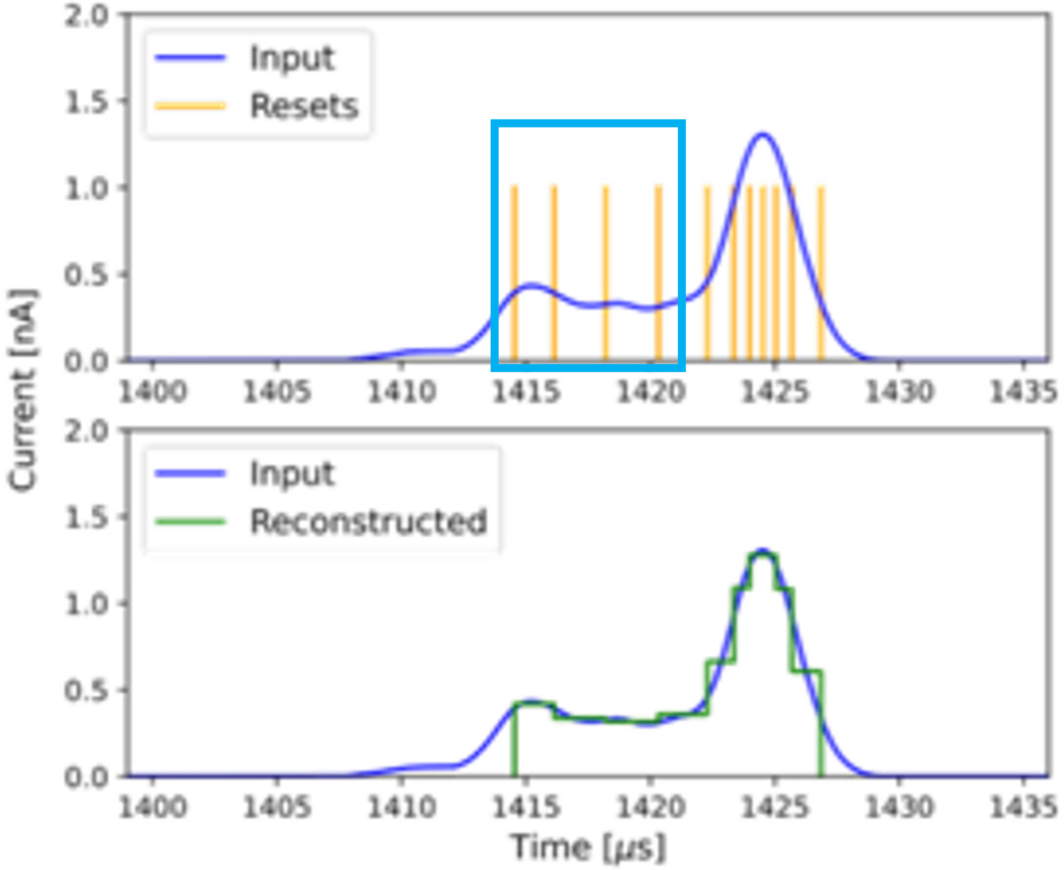
⇩

More current





Solution : Pixelization



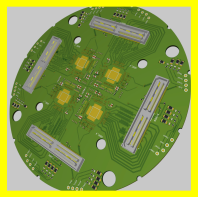
Sparse area

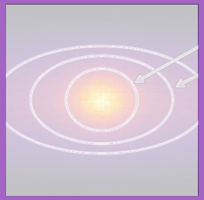
⇩

took pixel **more** time to get unit amount of charge

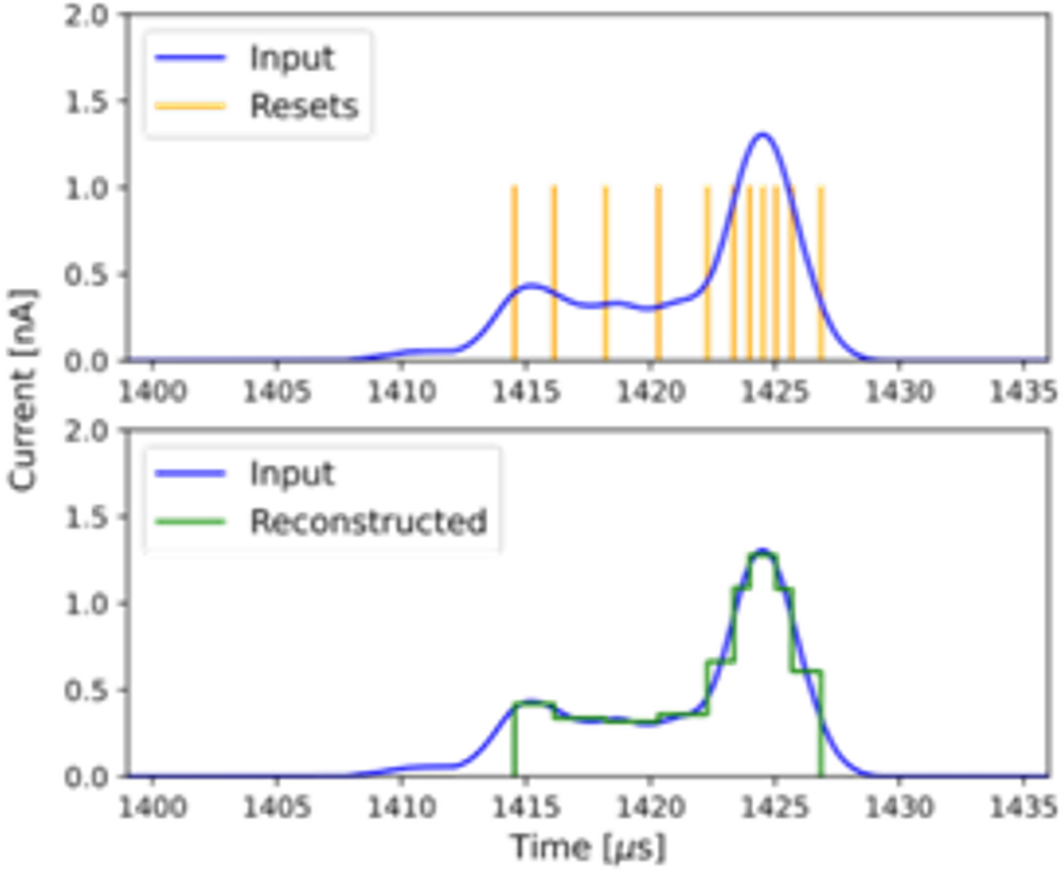
⇩

Less current





Solution : Pixelization



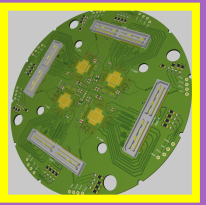
With this method,

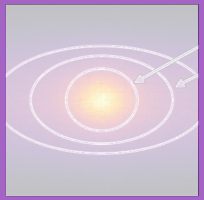
the data rate is **10^6 times less**

than the traditional wire readout!

⇩

Q-Pix can offer a direct solution to the data-rate problem that pixelized kton-scale LArTPC has





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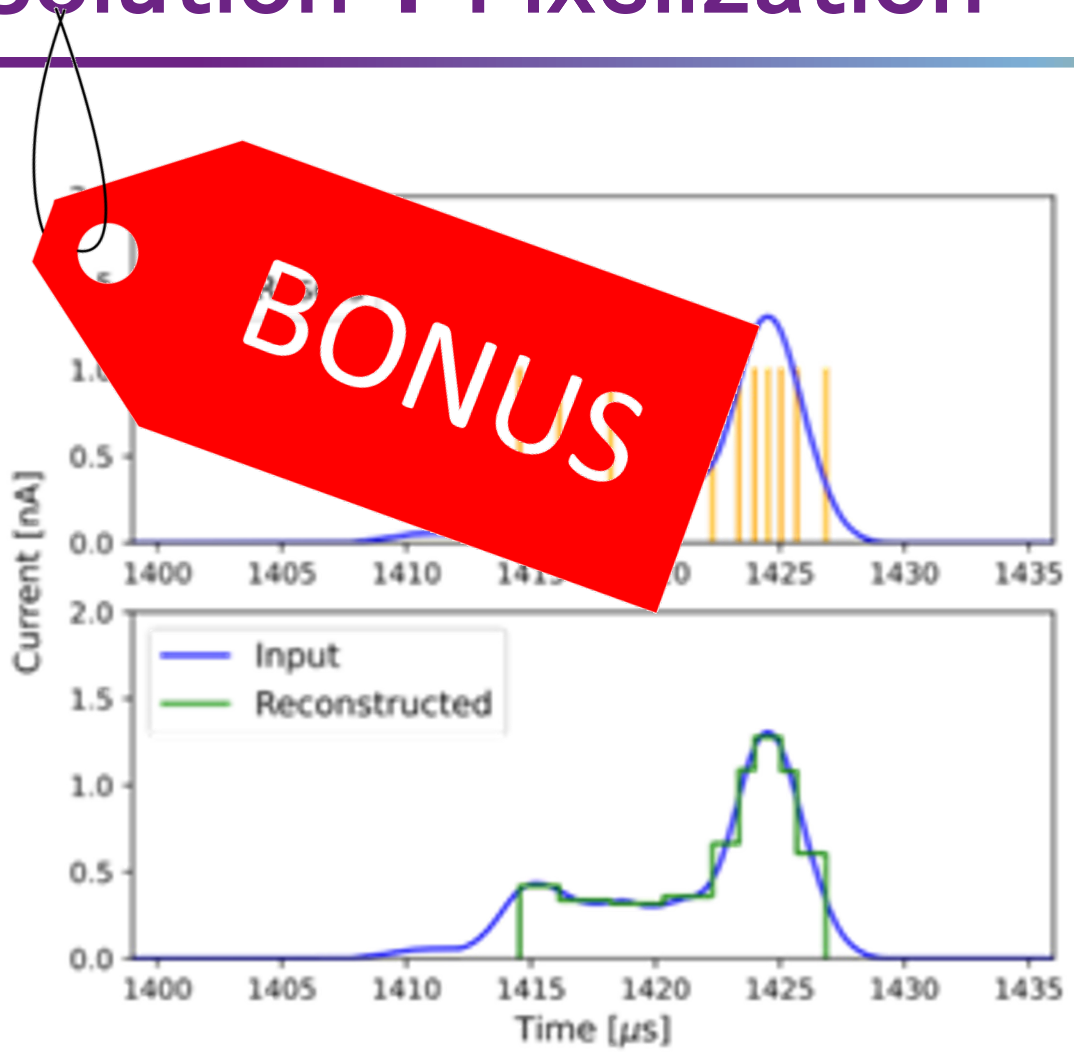
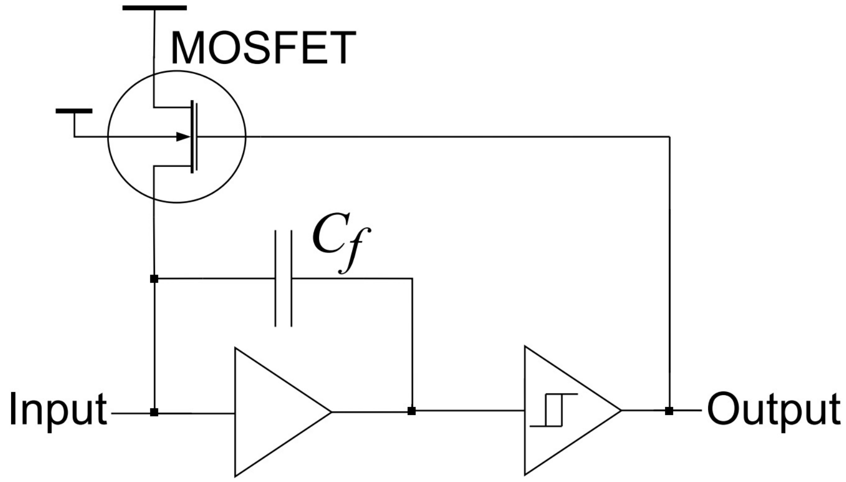
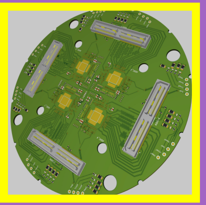
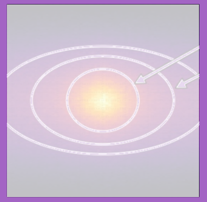




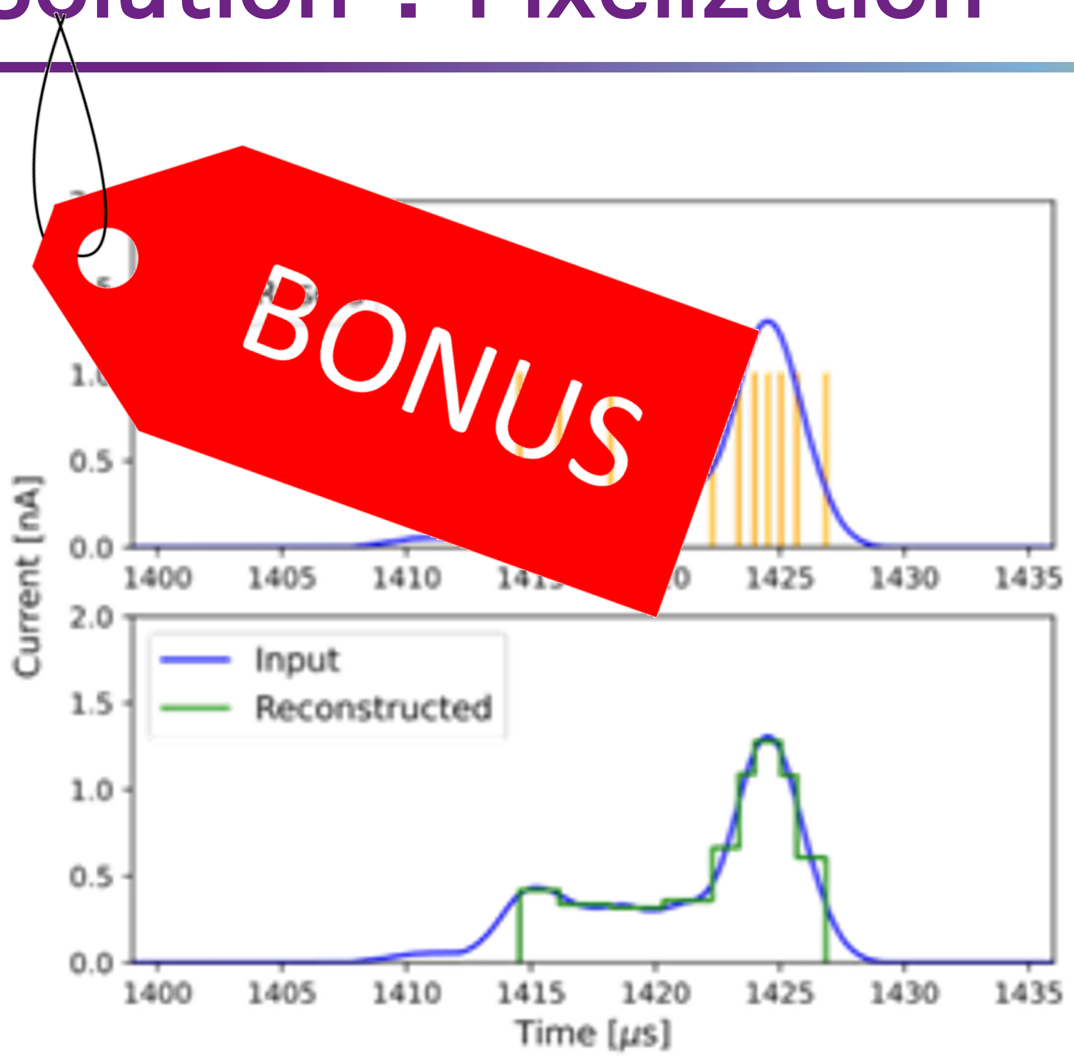
Moreover, this can be seen as

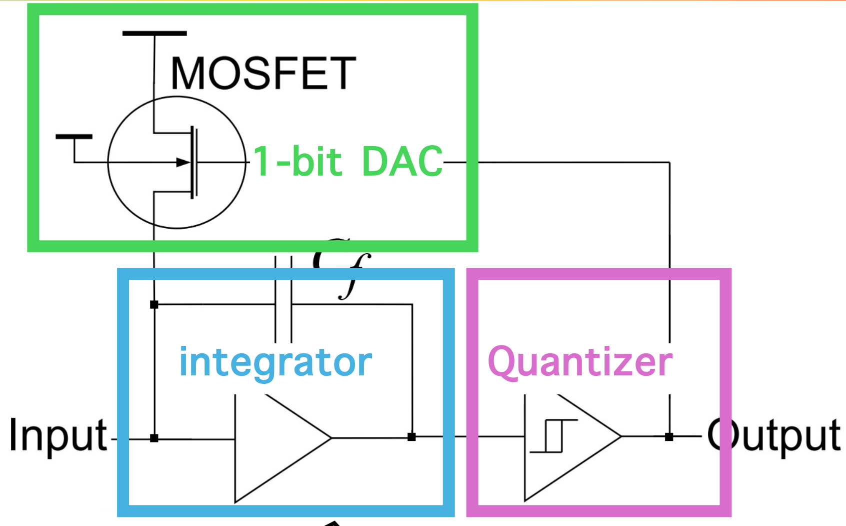
one-bit Sigma-Delta modulator





Solution : Pixelization

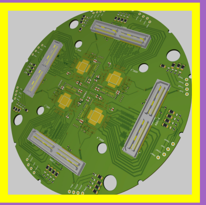


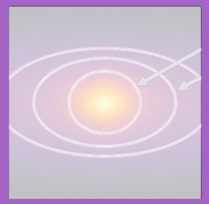


Moreover, this can be seen as

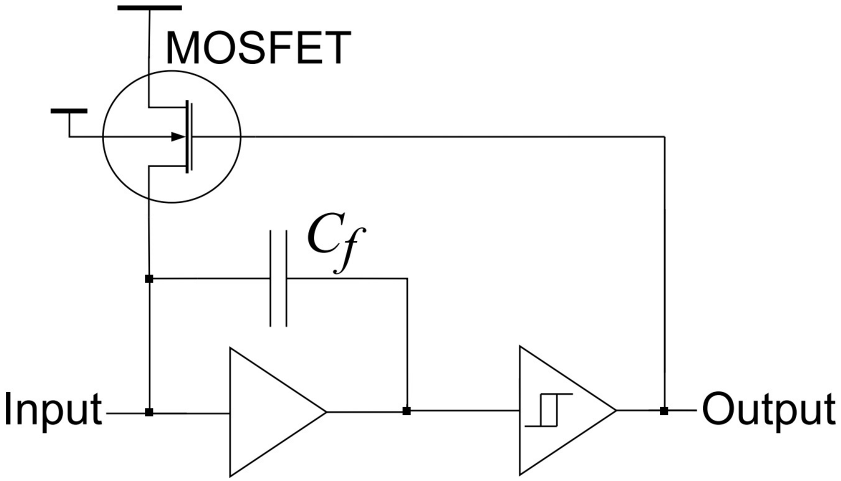
one-bit Sigma-Delta modulator

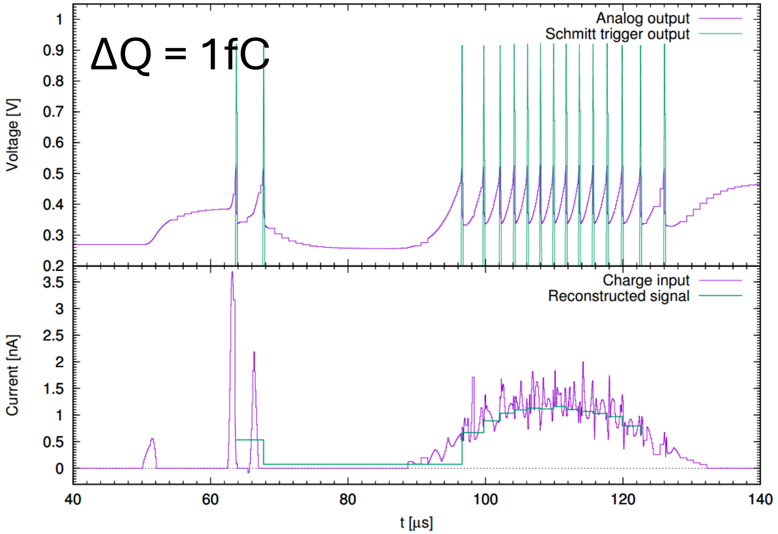
- Greater flexibility in data type
- More efficient data analysis
- More applications

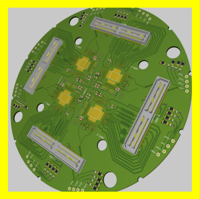


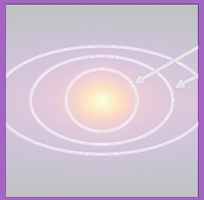


Demonstrating Q-Pix with commercial components









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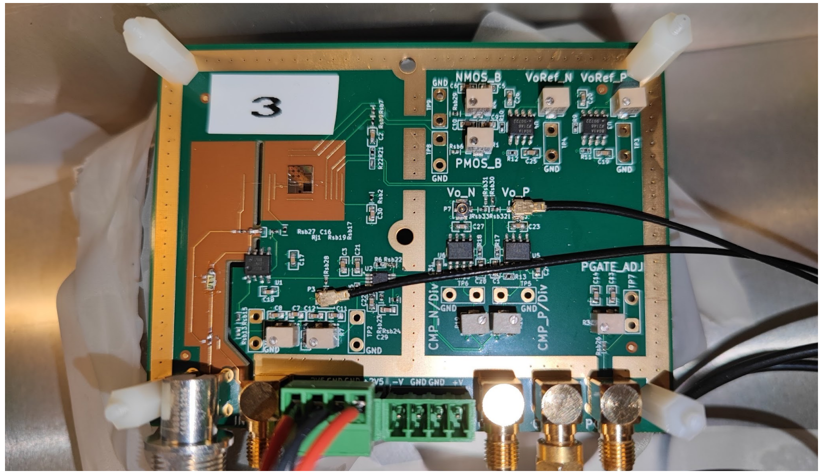

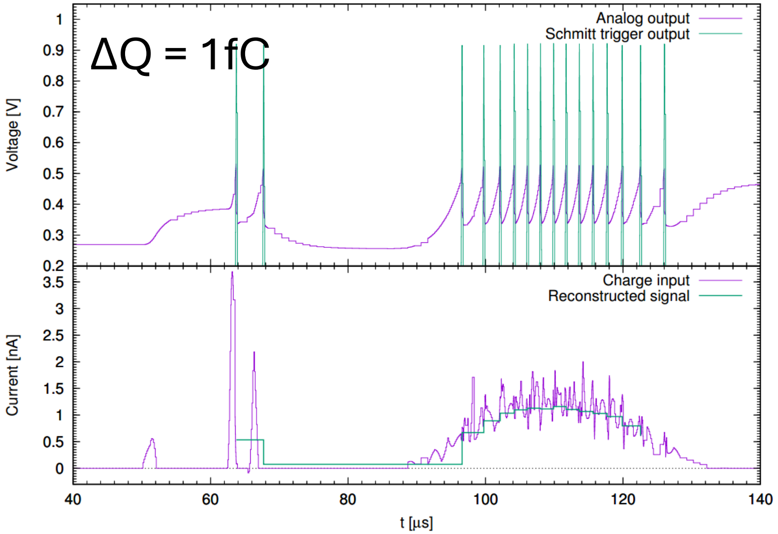

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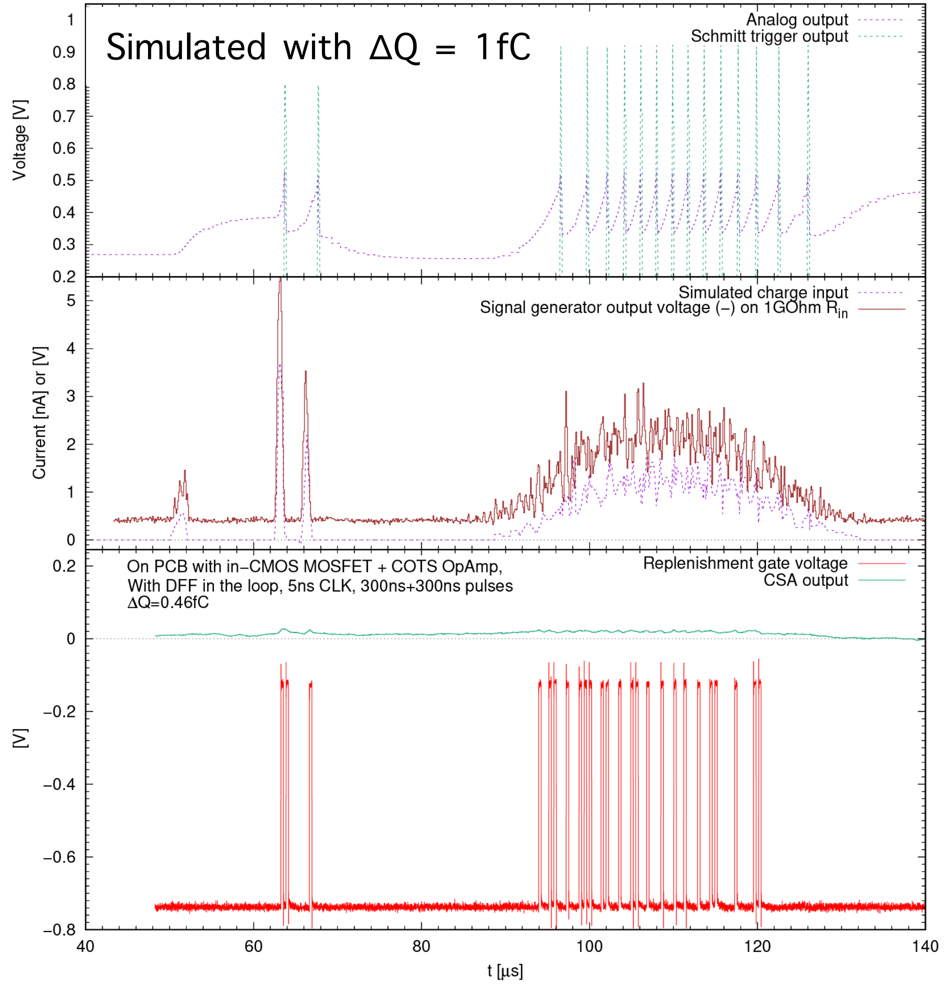

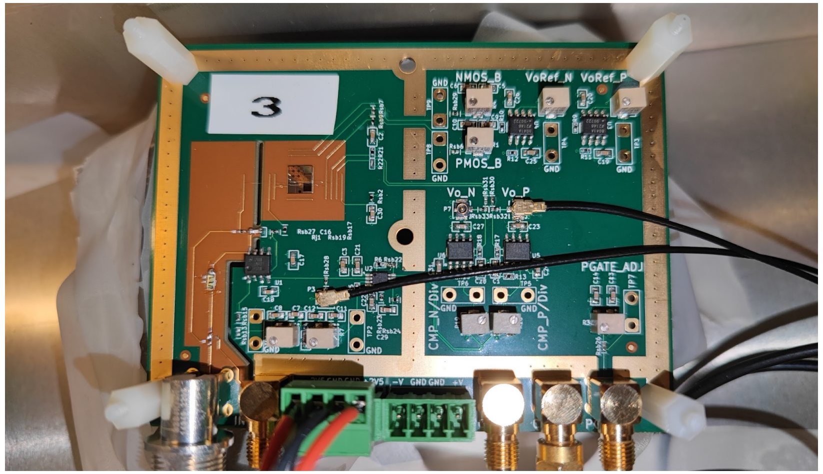

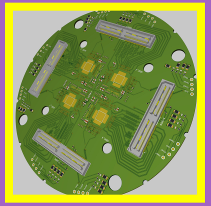

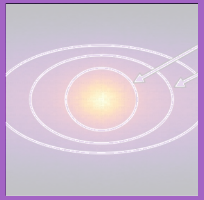

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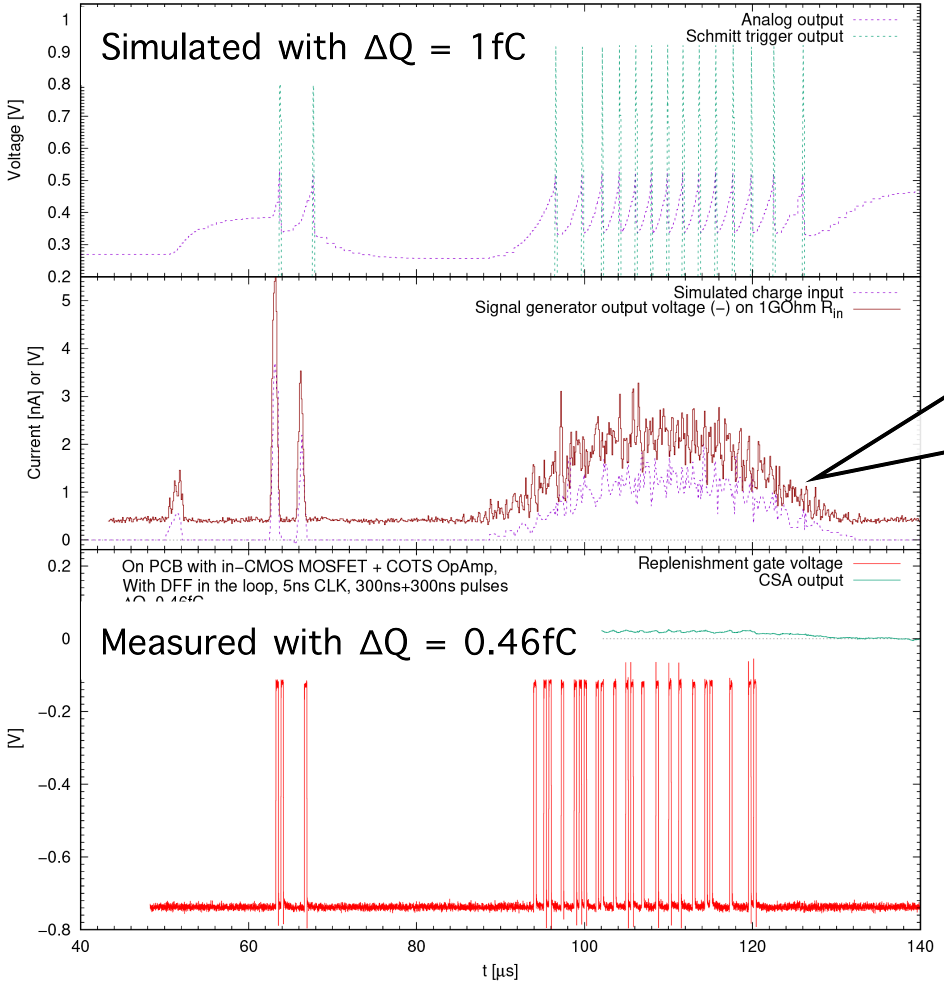
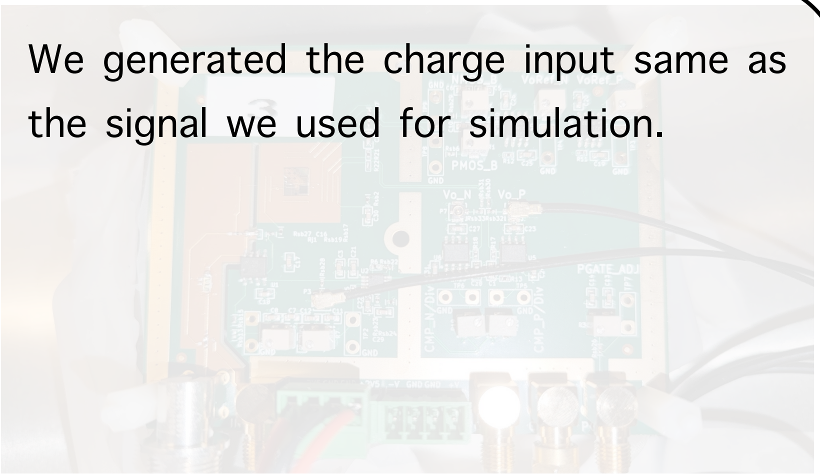
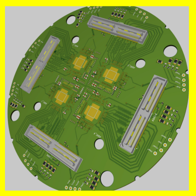
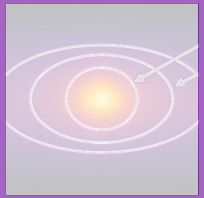


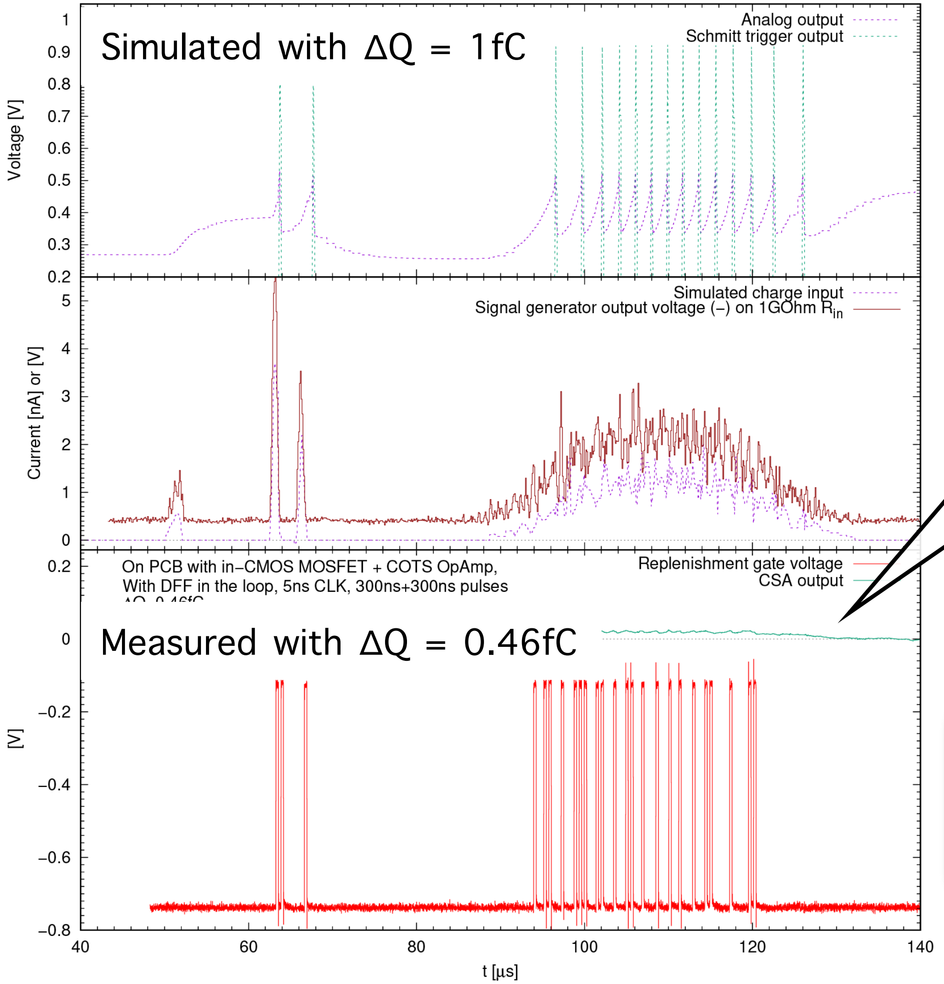


1. We generated the charge input same as the signal we used for simulation.





Demonstrating Q-Pix with commercial components

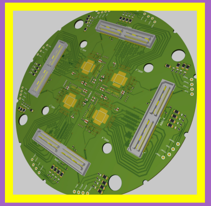


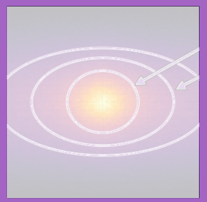
1. We generated the charge input same as the signal we used for simulation.
2. Fed it to the Q-Pix front-end made with commercial components
3. We could get better results than simulation!

0.46 fC enables **~50 KeV signal detection!**

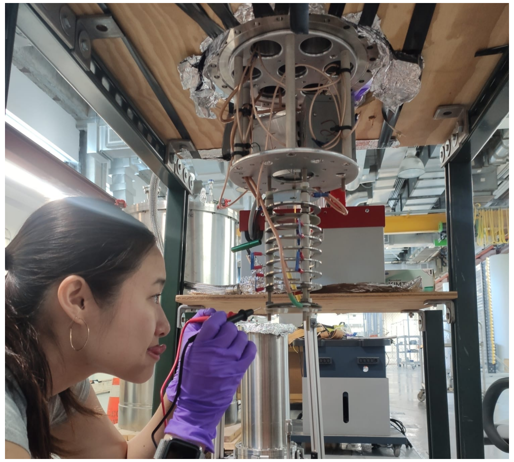
Demonstrating the Q-Pix front-end using discrete OpAmp and CMOS transistors

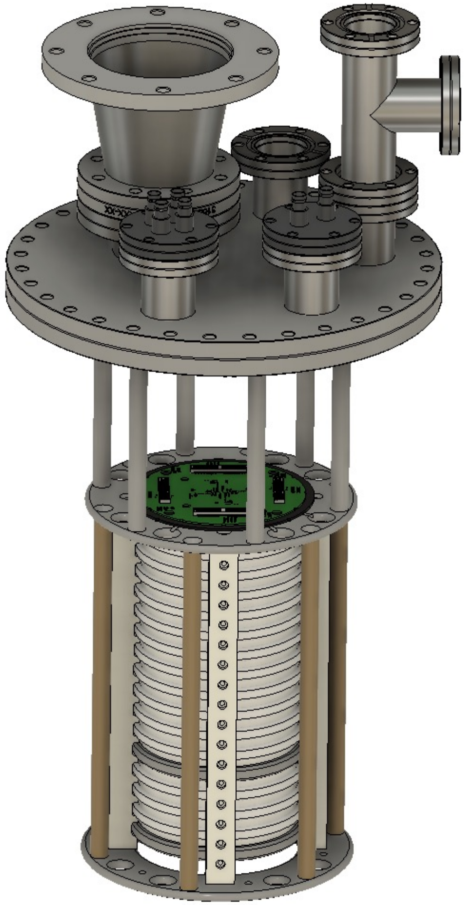
Peng Miao[a], Jonathan Asaadi[b], James B. R. Battat[d], Mikyung Han[a], Kevin Keefe[e,b], S. Kohani[e], Austin D. McDonald[b,c], David Nygren[b], Olivia Seidel[b], Yuan Mei[a,b,*]

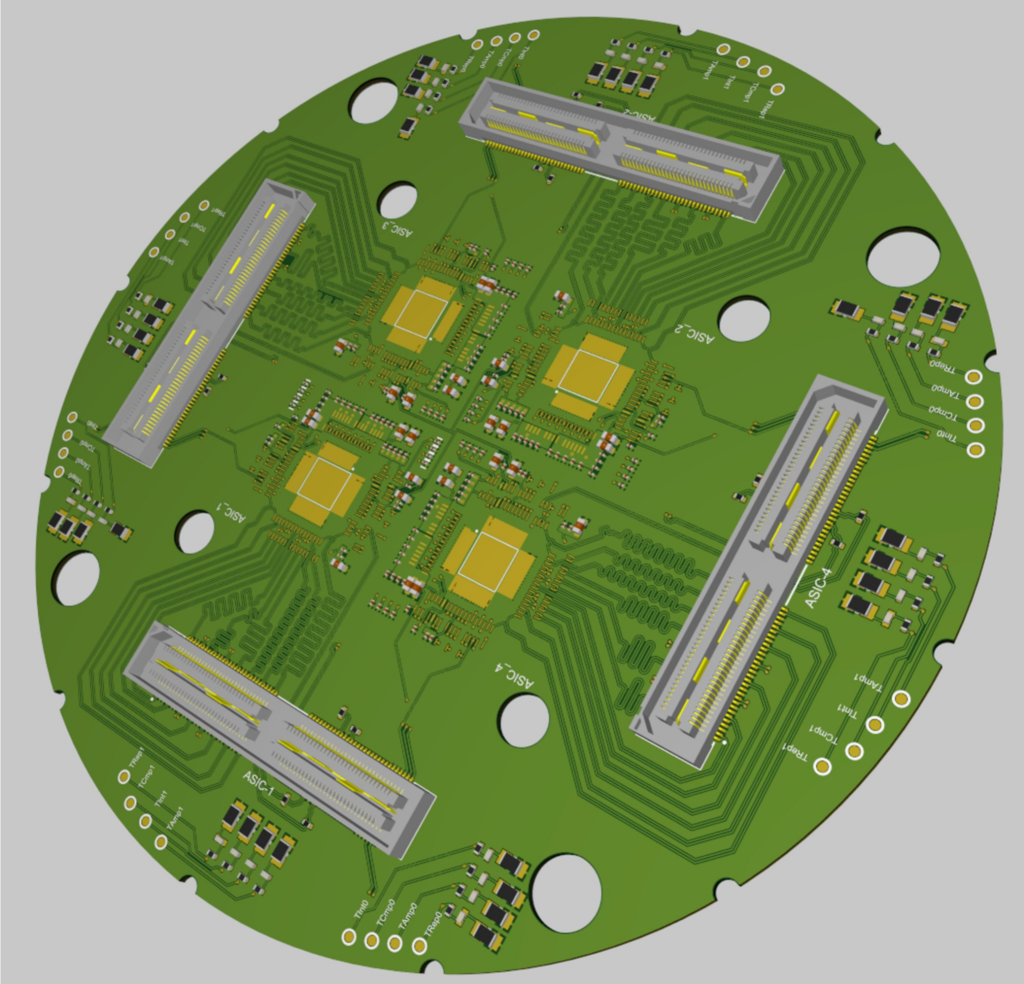




Future of Q-Pix

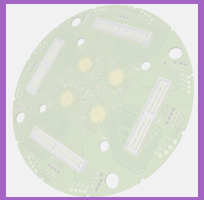


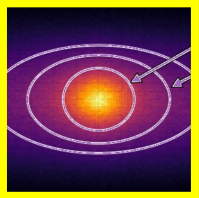




Physics with Q-Pix

'maximize the discovery potential of a kiloton scale LArTPC'





Physics with Q-Pix

‘maximize the discovery potential of a kiloton scale LArTPC’

Paper published from JINST in 2020

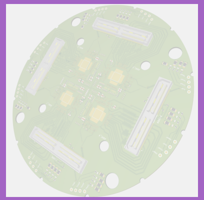
Enhancing beam event studies

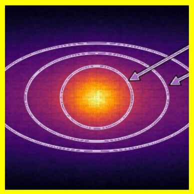
Paper published from PRD in 2022

Enhancing supernovae studies

Paper In preparation

Enabling solar studies





Physics with Q-Pix

‘maximize the discovery potential of a kiloton scale LArTPC’

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Enhancing beam event studies

～20GeV range : high energy neutrinos

Paper published from PRD in 2022

Enhancing supernovae studies

～40MeV range : low energy neutrinos

Paper In preparation

Enabling solar studies

～20MeV range : very low energy neutrinos

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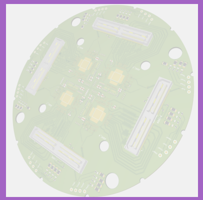
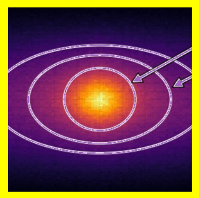




Beam physics (~20GeV)

Paper published from JINST in 2020

Enhancing beam event studies

Paper published from PRD in 2022

Enhancing supernovae studies

Paper In preparation

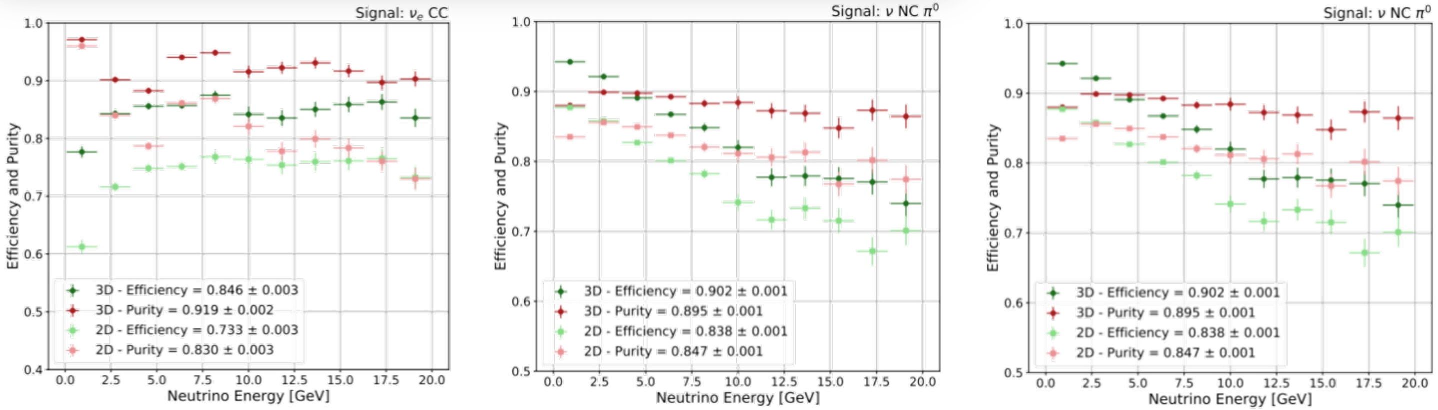
Enabling solar studies

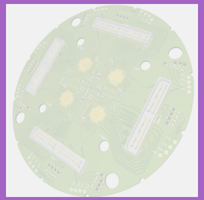
Enhancing neutrino event reconstruction with pixel-based 3D readout for liquid argon time projection chambers

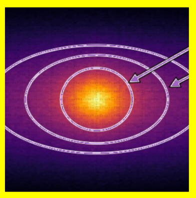
C. Adams,[a,1] M. Del Tutto,[b,c] J. Asaadi,[d] M. Bernstein,[c] E. Church,[e] R. Guenette,[c] J.M. Rojas,[c,2] H. Sullivan[d] and A. Tripathi[d]

[a] *Argonne National Laboratory, Lemont, IL, U.S.A.*
[b] *Fermi National Accelerator Laboratory, Batavia, IL, U.S.A.*
[c] *Harvard University, Cambridge, MA, U.S.A.*
[d] *University of Texas-Arlington, Arlington, TX, U.S.A.*
[e] *Pacific Northwest National Laboratory, Richland, WA, U.S.A.*

Category	Accuracy [%] 3D	Accuracy [%] 2D
Neutrino Interaction	**94**	91
Proton Multiplicity	**91**	87
Charge Pion Presence	**94**	91
Neutral Pion Presence	**95**	94







Beam physics (~20GeV)

Good tracking

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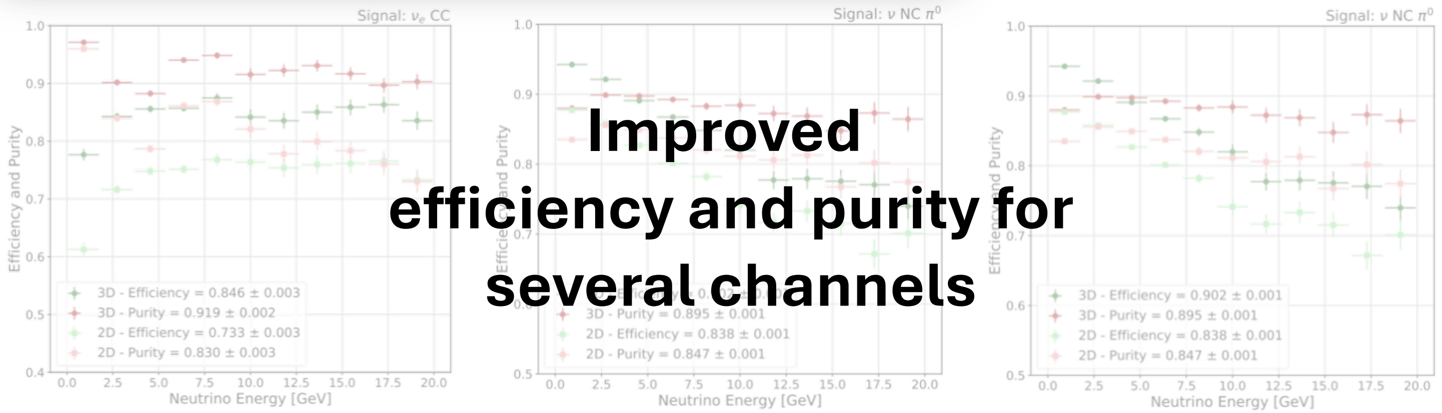
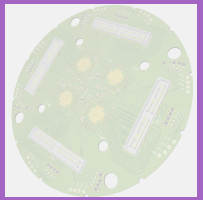
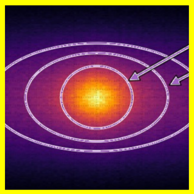
Enhanced event reconstruction accuracy



Improved efficiency and purity for several channels





Supernova physics (~40MeV)

Good tracking

Paper published from JINST in 2020

Enhancing beam event studies

Paper published from PRD in 2022

Enhancing supernovae studies

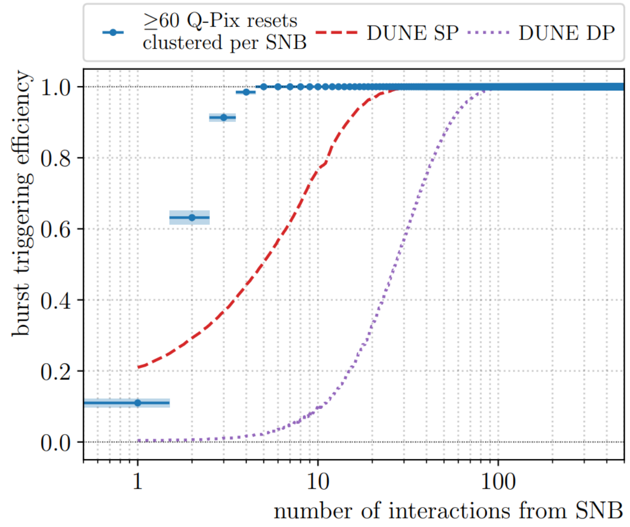
Paper In preparation

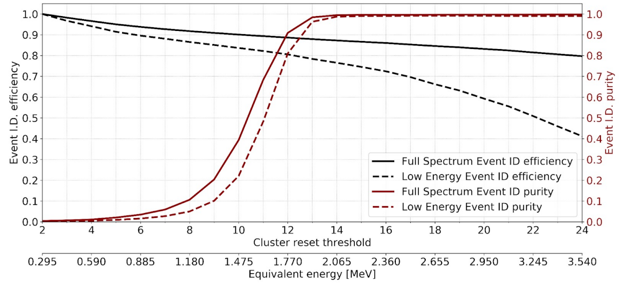
Enabling solar studies

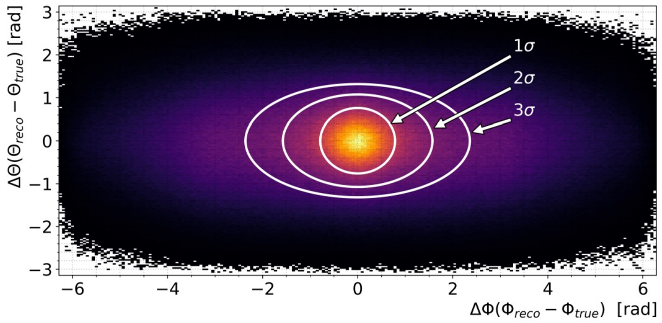
Enhanced low-energy supernova burst detection in large liquid argon time projection chambers enabled by Q-Pix

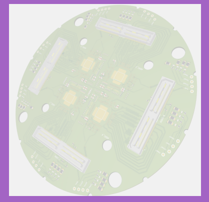
(The Q-Pix Collaboration)

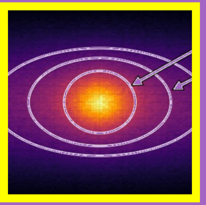
S. Kubota,[1] J. Ho,[1,*] A.D. McDonald,[2,1,†] N. Tata,[1] J. Asaadi,[2] R. Guenette,[3,1] J.B.R. Battat,[4] D. Braga,[5] M. Demarteau,[6] Z. Djurcic,[7] M. Febbraro,[6] E. Gramellini,[5] S. Kohani,[8] C. Mauger,[9] Y. Mei,[10] F.M. Newcomer,[9] K. Nishimura,[8] D. Nygren,[2] R. Van Berg,[9] G.S. Varner,[8] and K. Woodworth[5]











Supernova physics (~40MeV)

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Low energy threshold
Better energy resolution

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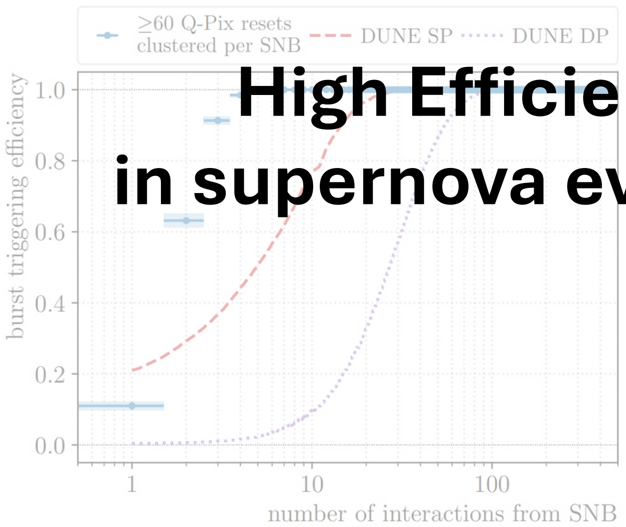
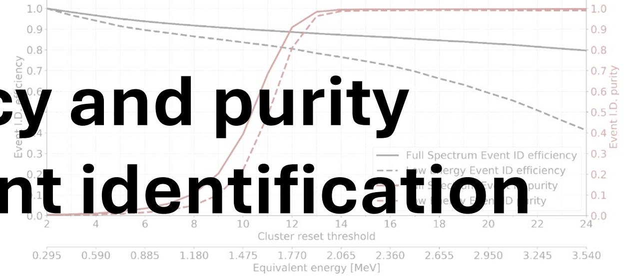
Paper published from JINST in 2020
Enhancing beam event studies

[Paper published from PRD in 2022]
Enhancing supernovae studies

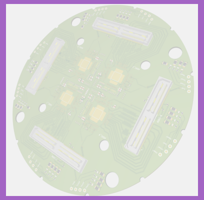
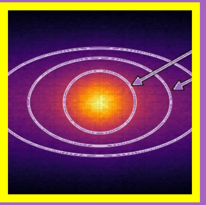
Paper In preparation
Enabling solar studies





High Efficiency and purity in supernova event identification

Supernova pointing

Solar physics (~20MeV)

Good tracking
Low energy threshold
Better energy resolution

Paper published from JINST in 2020

Enhancing beam event studies

Paper published from PRD in 2022

Enhancing supernovae studies

Paper In preparation

Enabling solar studies

Solar neutrinos offer many opportunities

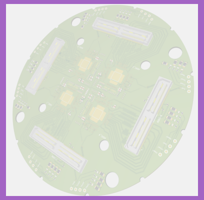
- Enhancement in oscillation parameters measurement
- Understanding solar models
- Potential discovery of hep neutrinos

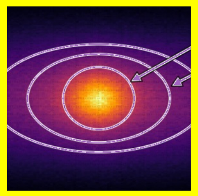
→ Requires :

better tracking

lower energy threshold

better energy resolution





Solar physics (~20MeV)

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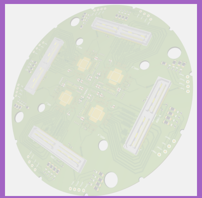
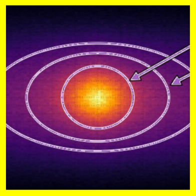
→ Requires :

better tracking
lower energy threshold
better energy resolution

Exactly the improvements Q-Pix can offer!

Solar physics (~20MeV)

Good tracking
Low energy threshold
Better energy resolution

Solar neutrinos offer many opportunities

- Enhancement in oscillation parameters measurement
- Understanding solar models
- Potential discovery of hep neutrinos

→ Requires :

better tracking
lower energy threshold
better energy resolution

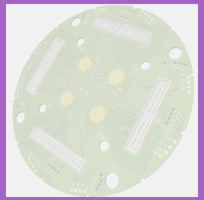
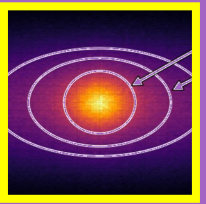
Exactly the improvements Q-Pix can offer!

Paper published from JINST in 2020
Enhancing beam event studies

Paper published from PRD in 2022
Enhancing supernovae studies

Paper In preparation
Enabling solar studies

→ <u>**Background control becomes crucial**</u> for continuous readouts
Paper on this is in preparation. Stay tuned!☺

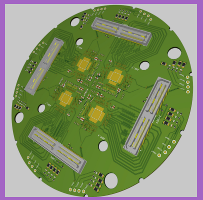




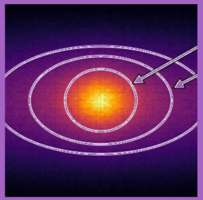
Summary

- Wire based LArTPC has been proven to be successful in many neutrino experiments but can be improved with pixel technology.

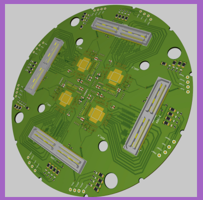
- Q-Pix is a new technology whose default state is ‘do-nothing’ – which is suited for large scale detector

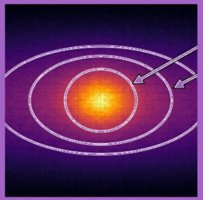
- Q-Pix offers [Good tracking / Low energy threshold / Better energy resolution] enabling low E neutrino studies.





Thank you!





Q-Pix consortium would like the thank the DOE and the Gordon & Betty Moore Foundation for their support

Links to papers

Q-Pix: Pixel-scale Signal Capture for Kiloton Liquid Argon TPC Detectors: Time-to-Charge Waveform Capture, Local Clocks, Dynamic Networks

⇨ Q-Pix general info

Demonstrating the Q-Pix front-end using discrete OpAmp and CMOS transistors

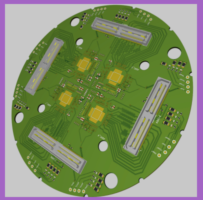
⇨ Q-Pix commercial components result

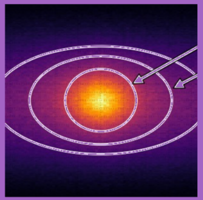
Enhancing Neutrino Event Reconstruction with Pixel-Based 3D Readout for Liquid Argon Time Projection Chambers

⇨ Beam physics with Q-Pix (High energy)

Enhanced low-energy supernova burst detection in large liquid argon time projection chambers enabled by Q-Pix

⇨ Supernovae neutrino physics with Q-Pix (Low energy)

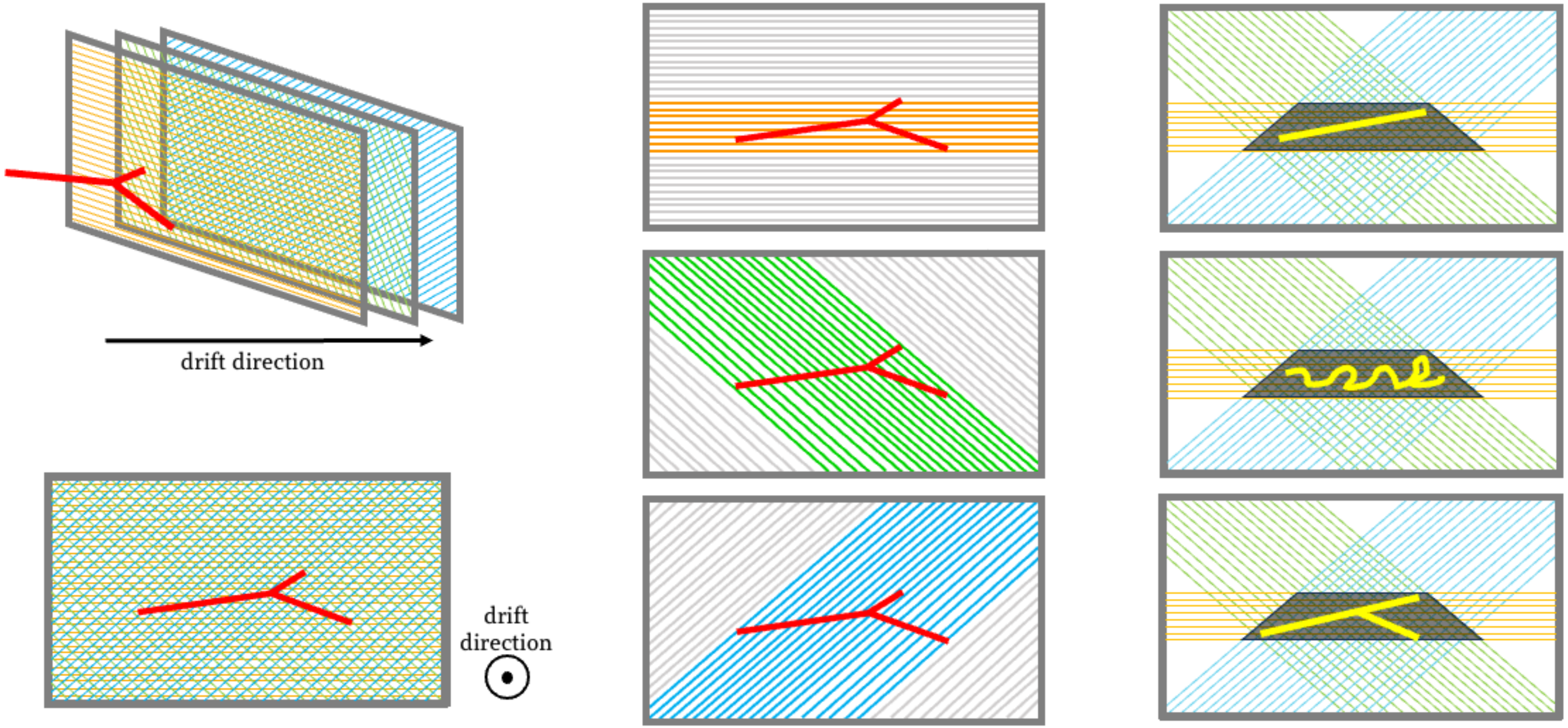


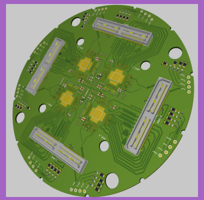


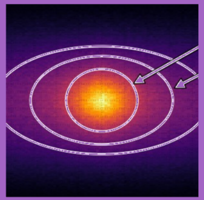
MANCHESTER 1824
The University of Manchester

Backup

Wire Ambiguity







Sigma Delta Modulator

Integrator

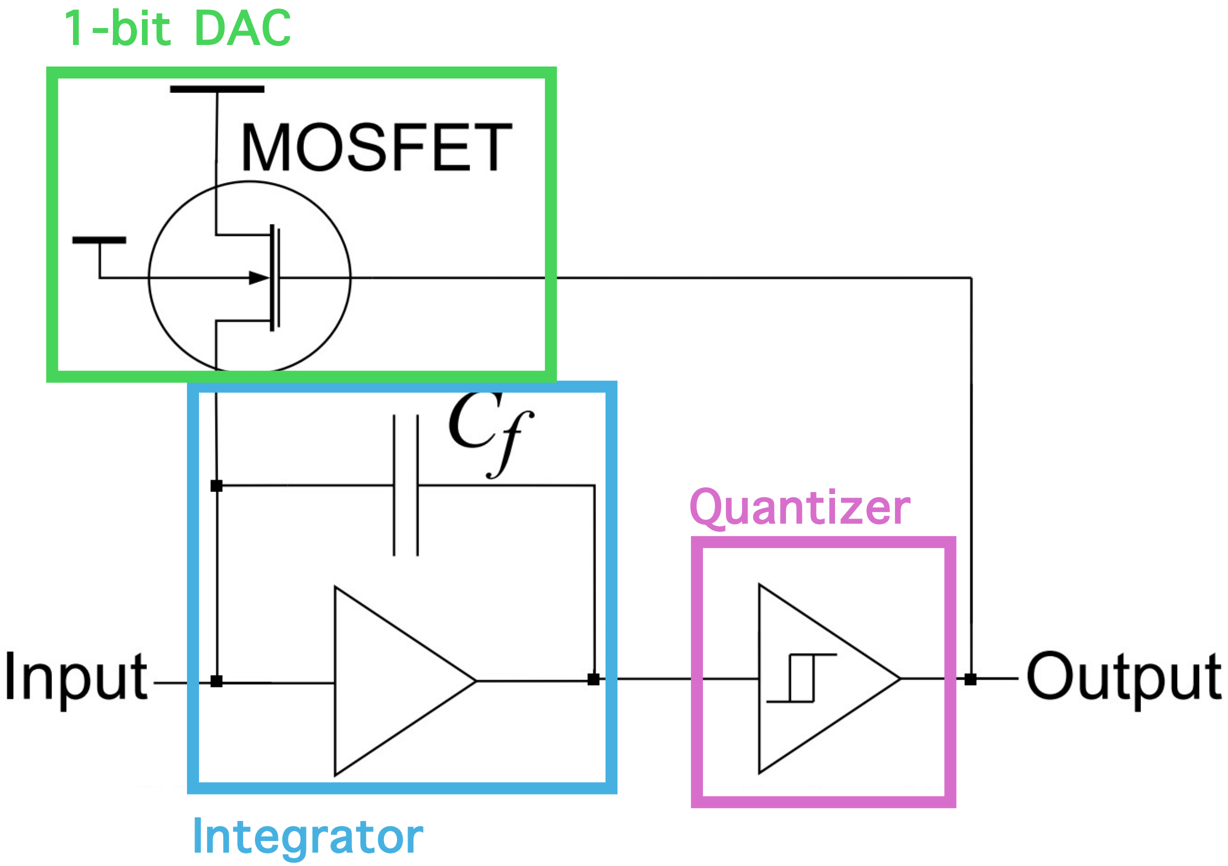
Sums the input signal over time, effectively smoothing it and making the system more tolerant to noise.

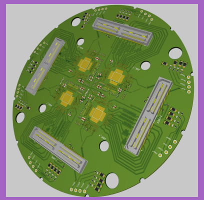
Quantizer

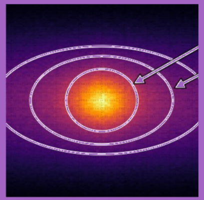
Converts the analog signal from the integrator into a digital signal. It is a single-bit quantizer with output of either 0 or 1.

DAC

Receives the output from the quantizer and converts the digital output back into an analog signal. It limits the amount of charge on Cf.



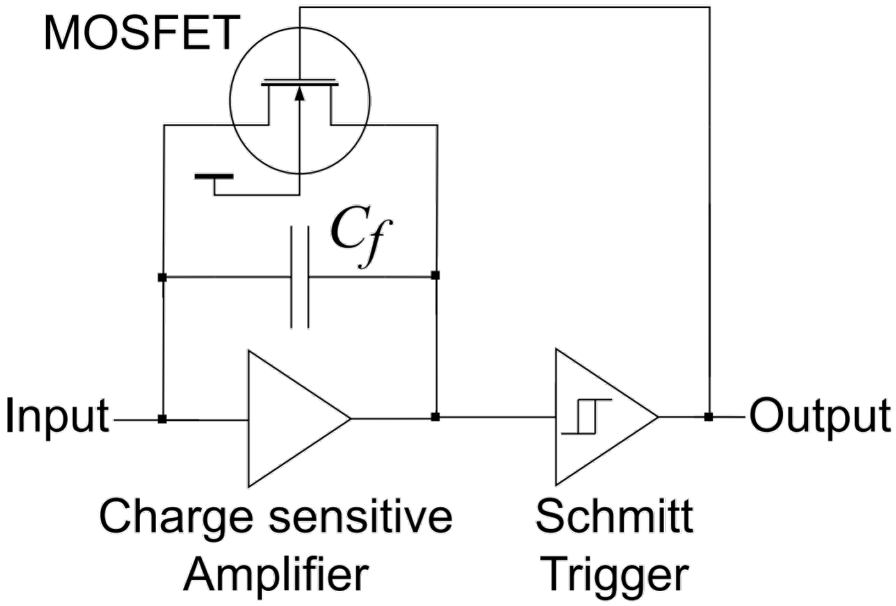




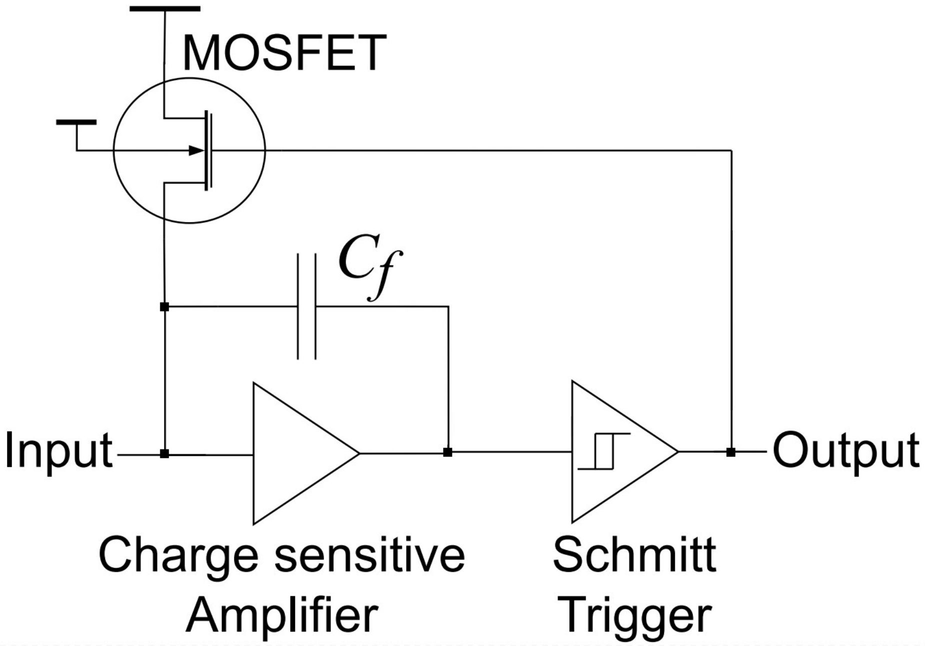
Paper published on this

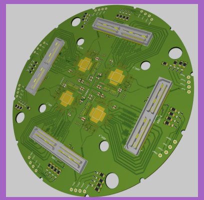
Reset vs Replenishment scheme

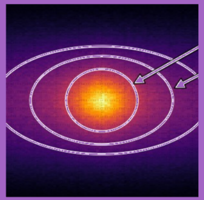
Reset



Replenishment

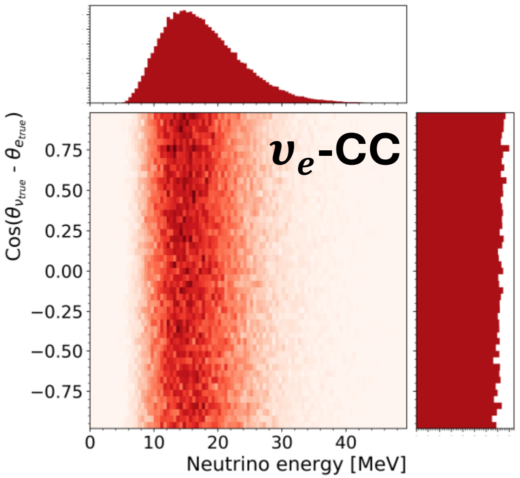


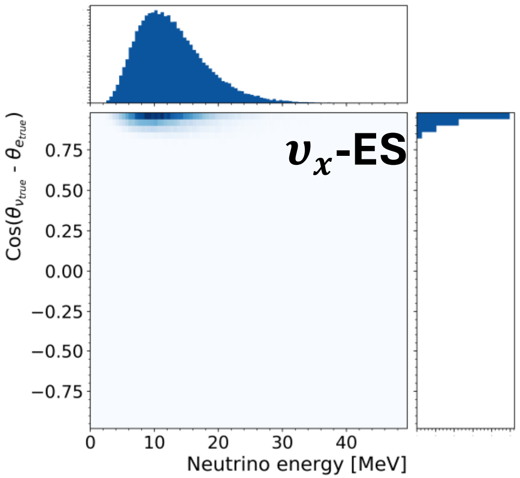


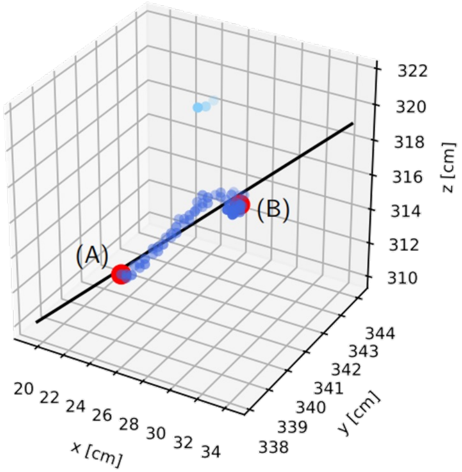


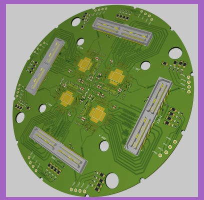
Supernova Pointing

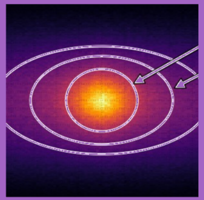
- The intrinsic 3D nature of the pixel data collected by Q-Pix allows us to get directional information from the identified supernova events.
- ~10% of all the events collected are neutrino-electron elastic scattering events (υ_x ES) and the rest are neutrino-charged current (υ_e-CC).
- υ_x-ES events preserve information about the direction of the neutrino
  - The direction of the neutrino tells us where in the sky the supernova burst occurs
  - This is a critical aspect of the identification of a SNB event for astronomers and particle physicists!







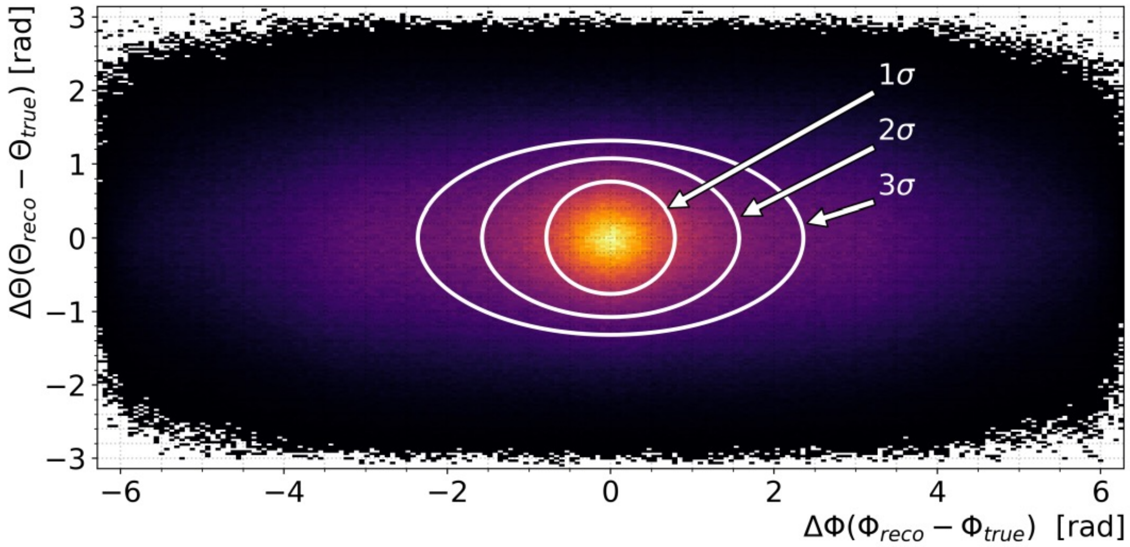


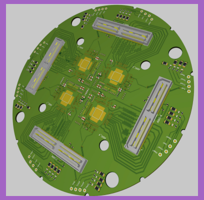


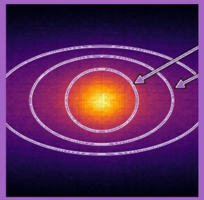
Supernova Pointing

- By reconstructing a direction vector for each neutrino interaction, we can come up with a hypothesis of where the SBN event occurred in the sky
  - We correctly identify the SBN direction within 20 degrees 80% of the time
  - The other 20% we have the direction wrong by 180 degrees

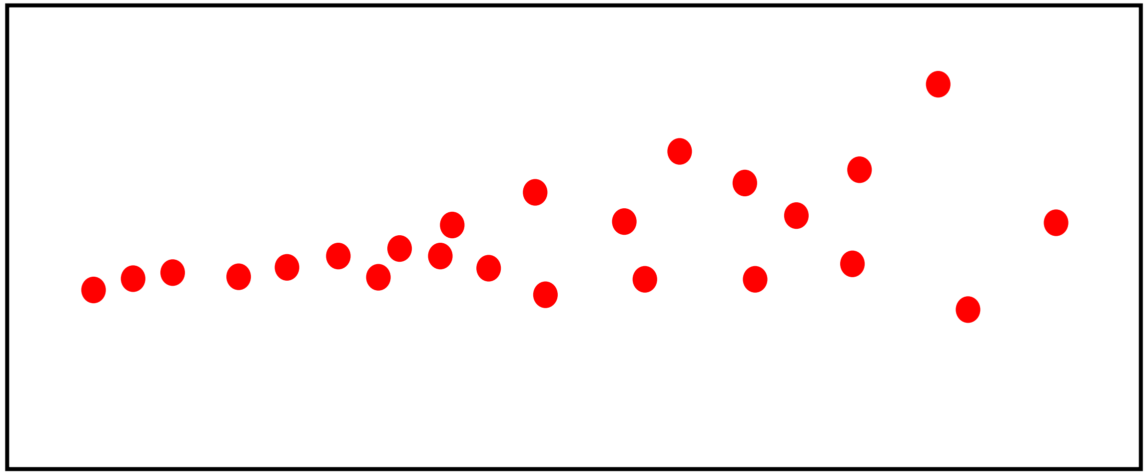
- Repeating this over 10,000 unique SBN events, we computed how confident we are with the direction with a 10 kTon Q-Pix module
  - 10 kpc supernova would be reconstructed within $\theta = 33^\circ$ and $\phi = 45^\circ$ at 1σ, and $\theta = 99^\circ$ and $\phi = 135^\circ$ at 3σ.

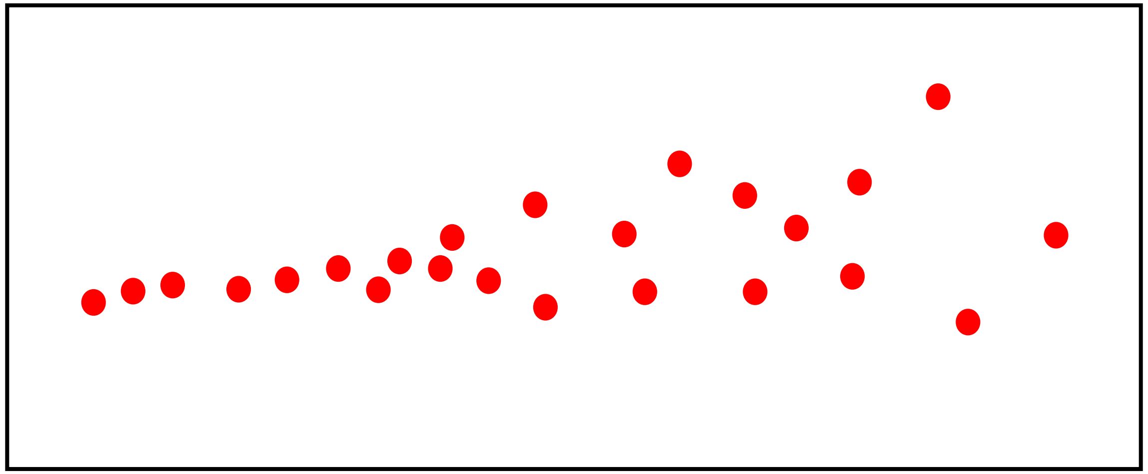


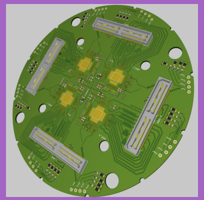


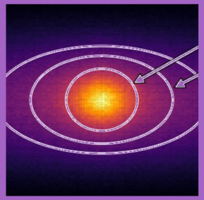


Directionality Flipping

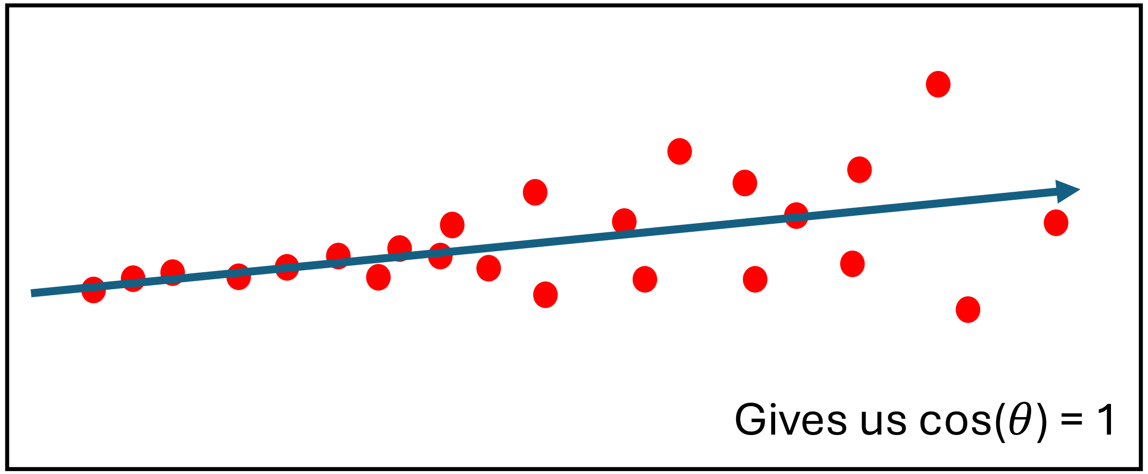


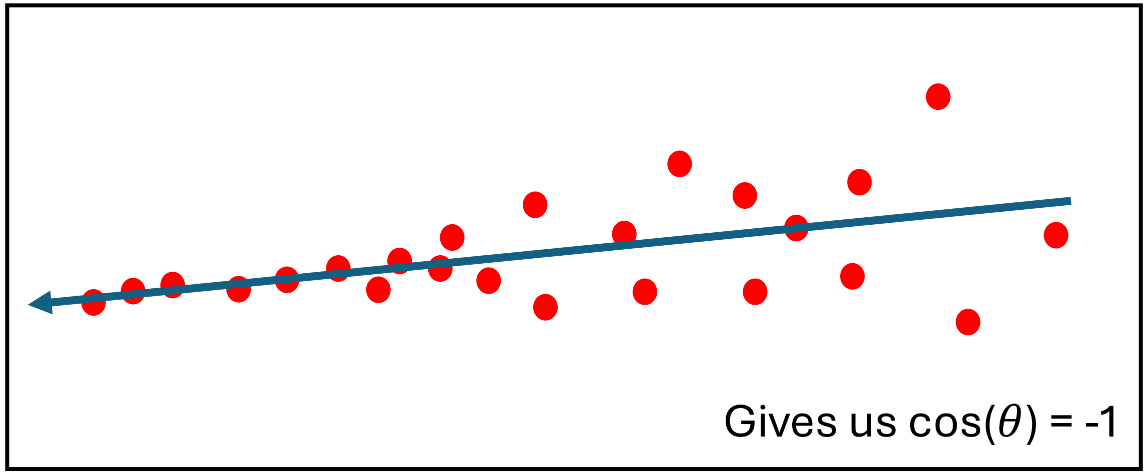


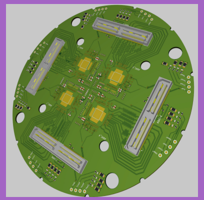


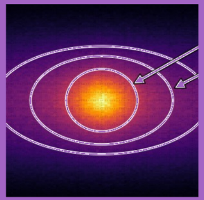


Directionality Flipping









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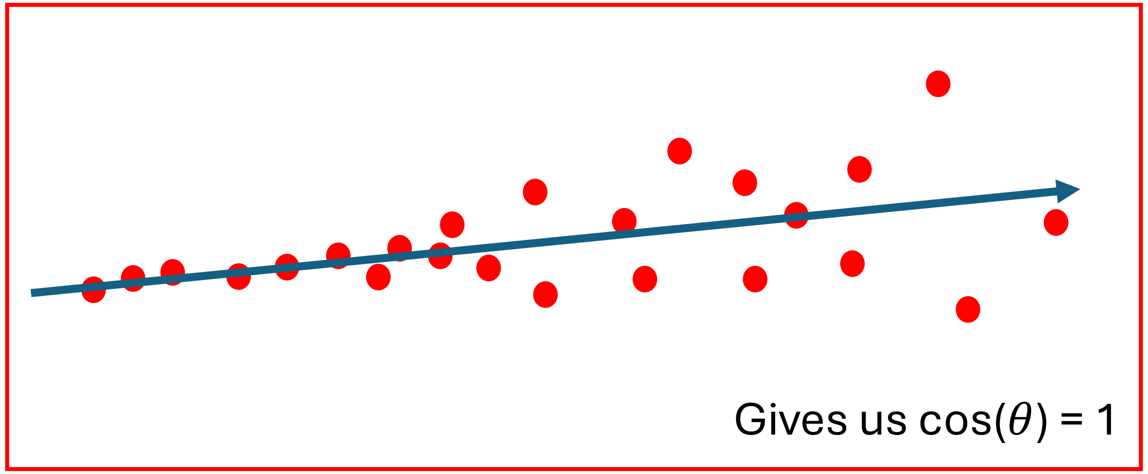
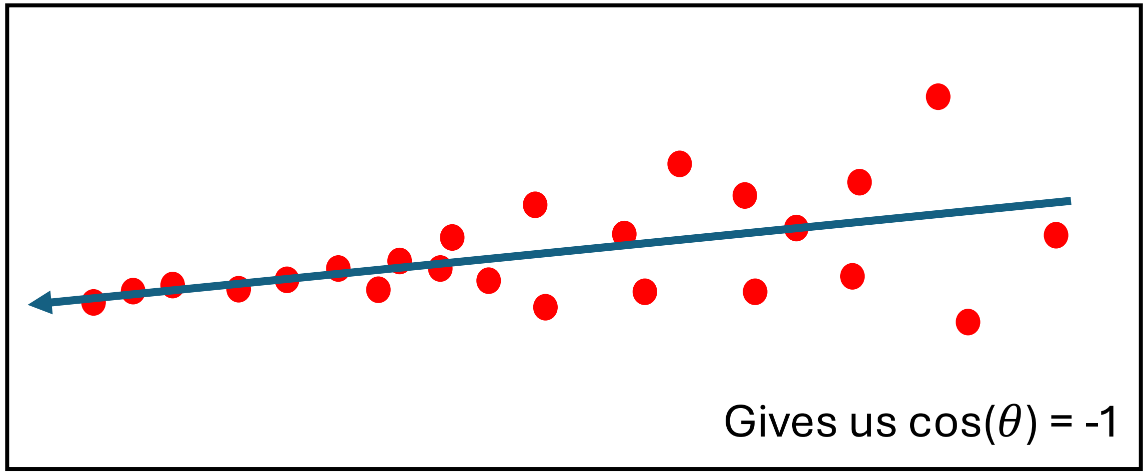
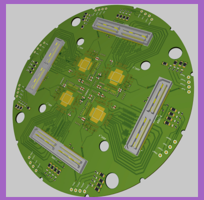


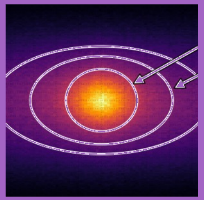


Electron scatters more often as it loses energy.

More scattered = less energy = end of the track

Less scattered = more energy = beginning of the track





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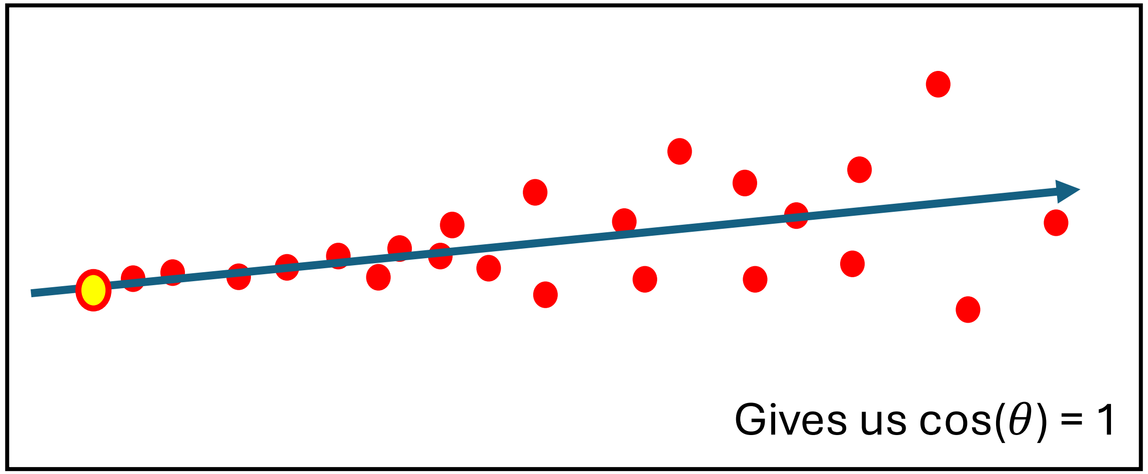
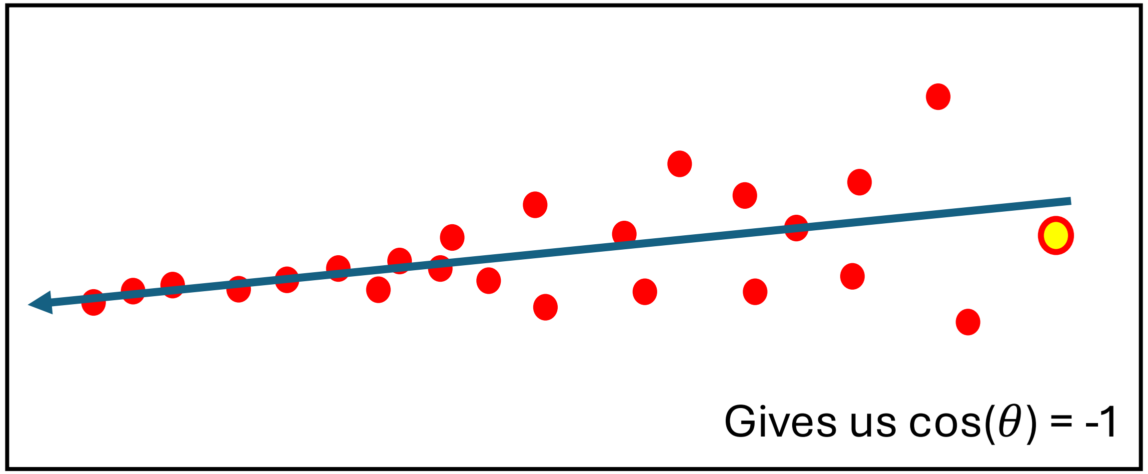
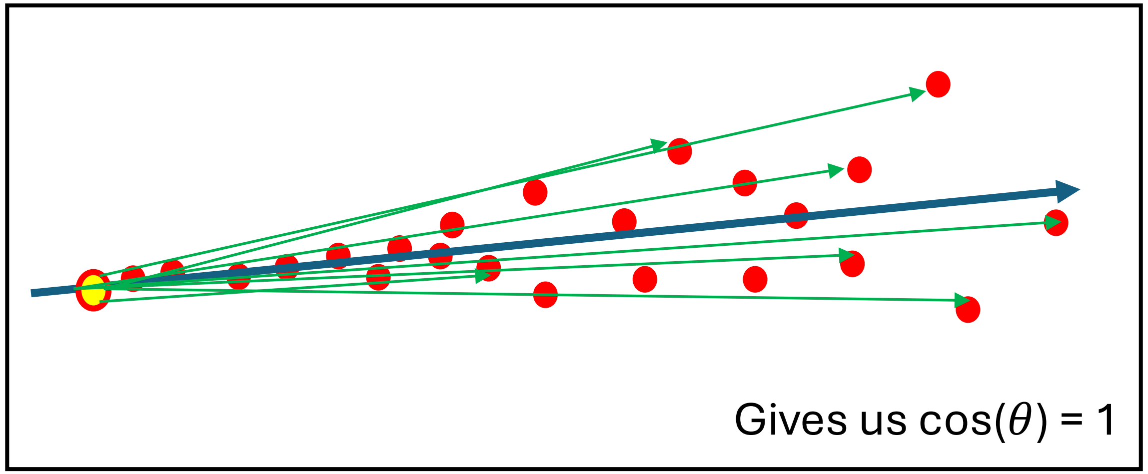
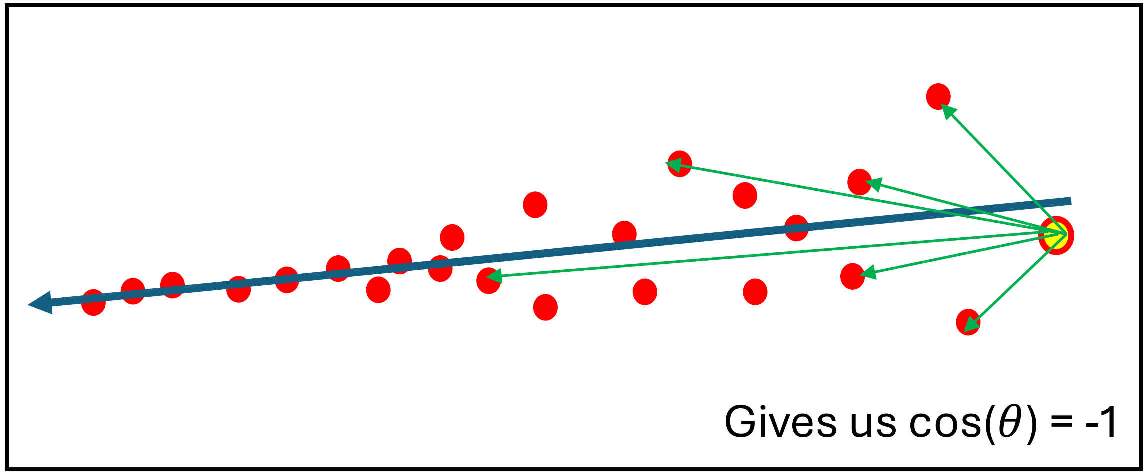




Pick data points at both ends

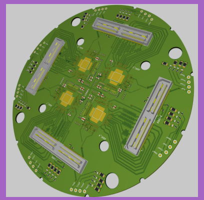
Directionality Flipping

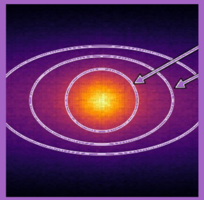




Pick data points at both ends

Draw line from two ends to every single data point





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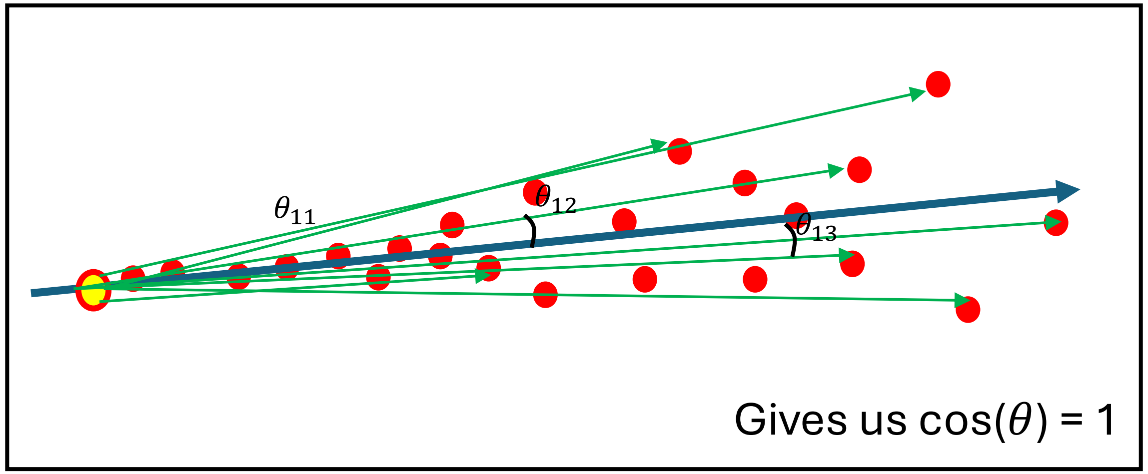
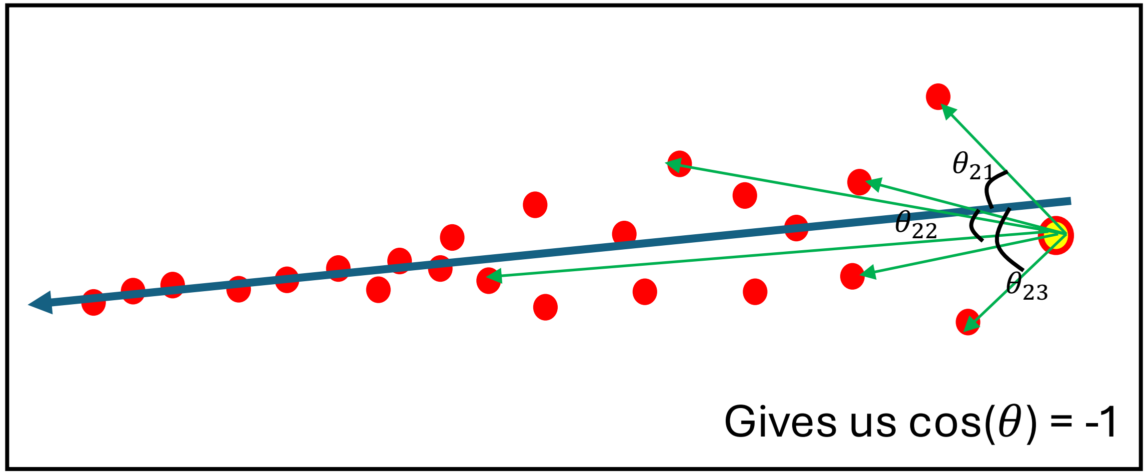


Gives us cos(θ) = 1

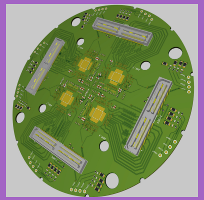


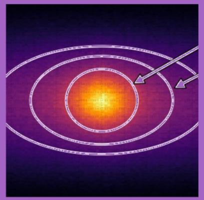
Gives us cos(θ) = -1

Pick data points at both ends

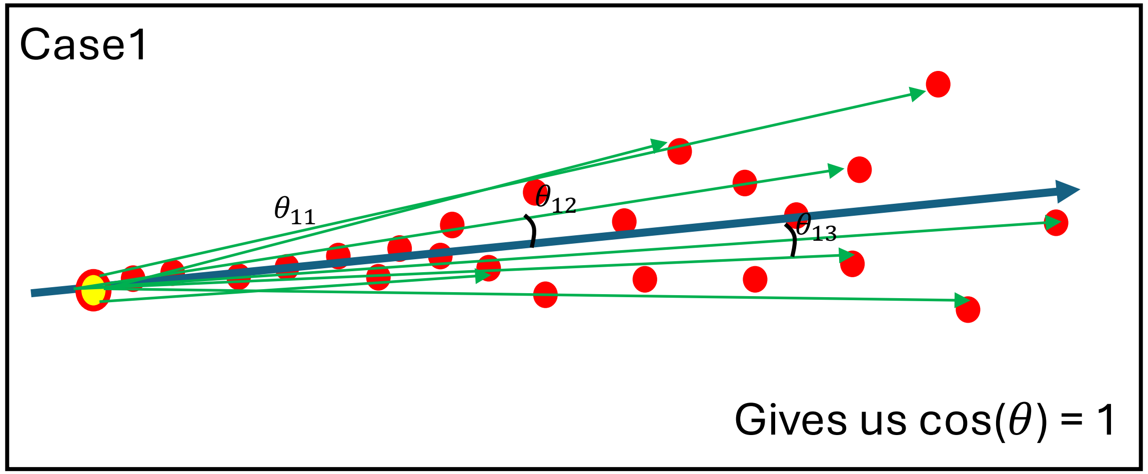
Draw line from two ends to every single data point

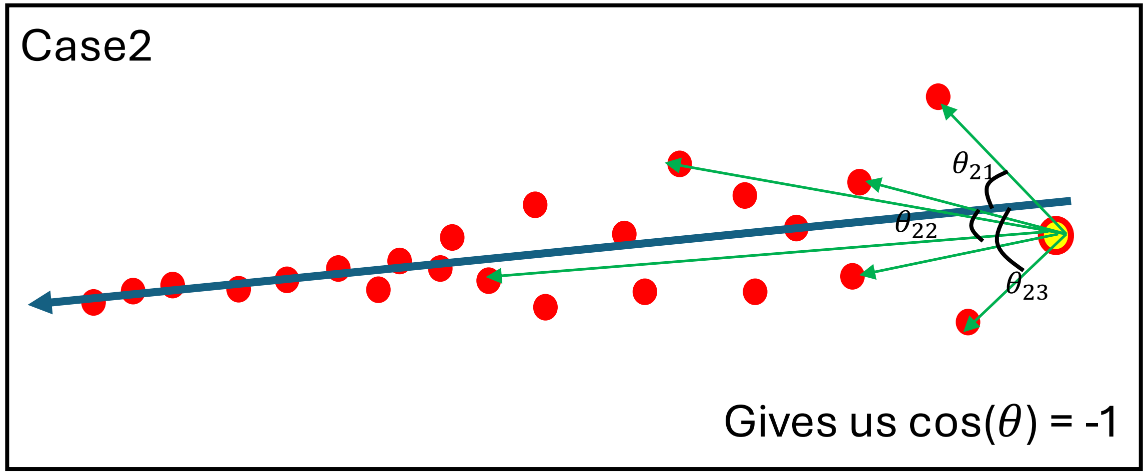
Calculate the cosine of angle between the lines and computed axis, and sum them





Directionality Flipping





Pick data points at both ends

Draw line from two ends to every single data point

Calculate the cosine of angle between the lines and computed axis, and sum them

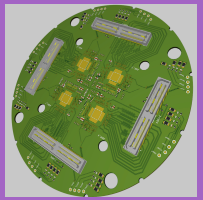
θ values are smaller $\rightarrow$ $\cos(\theta)$ values are bigger

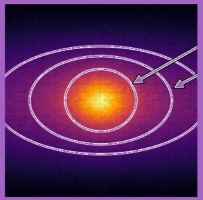
sum_cos_case1 = $\cos(\theta_{11}) + \cos(\theta_{12}) + \cos(\theta_{13}) \cdots$

θ values are bigger $\rightarrow$ $\cos(\theta)$ values are smaller

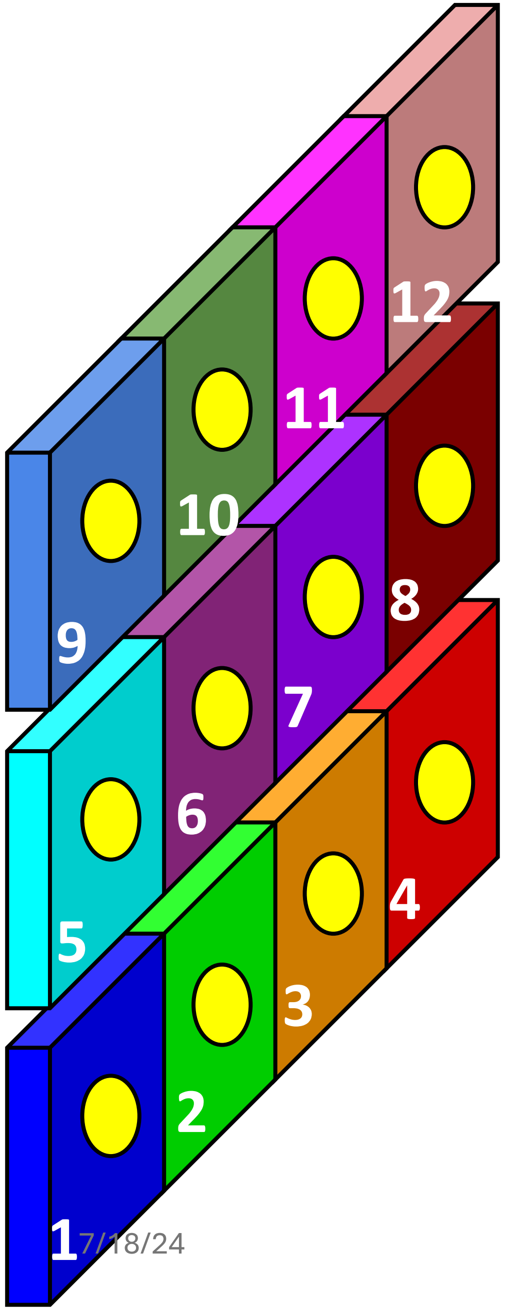
sum_cos_case2 = $\cos(\theta_{21}) + \cos(\theta_{22}) + \cos(\theta_{23}) \cdots$

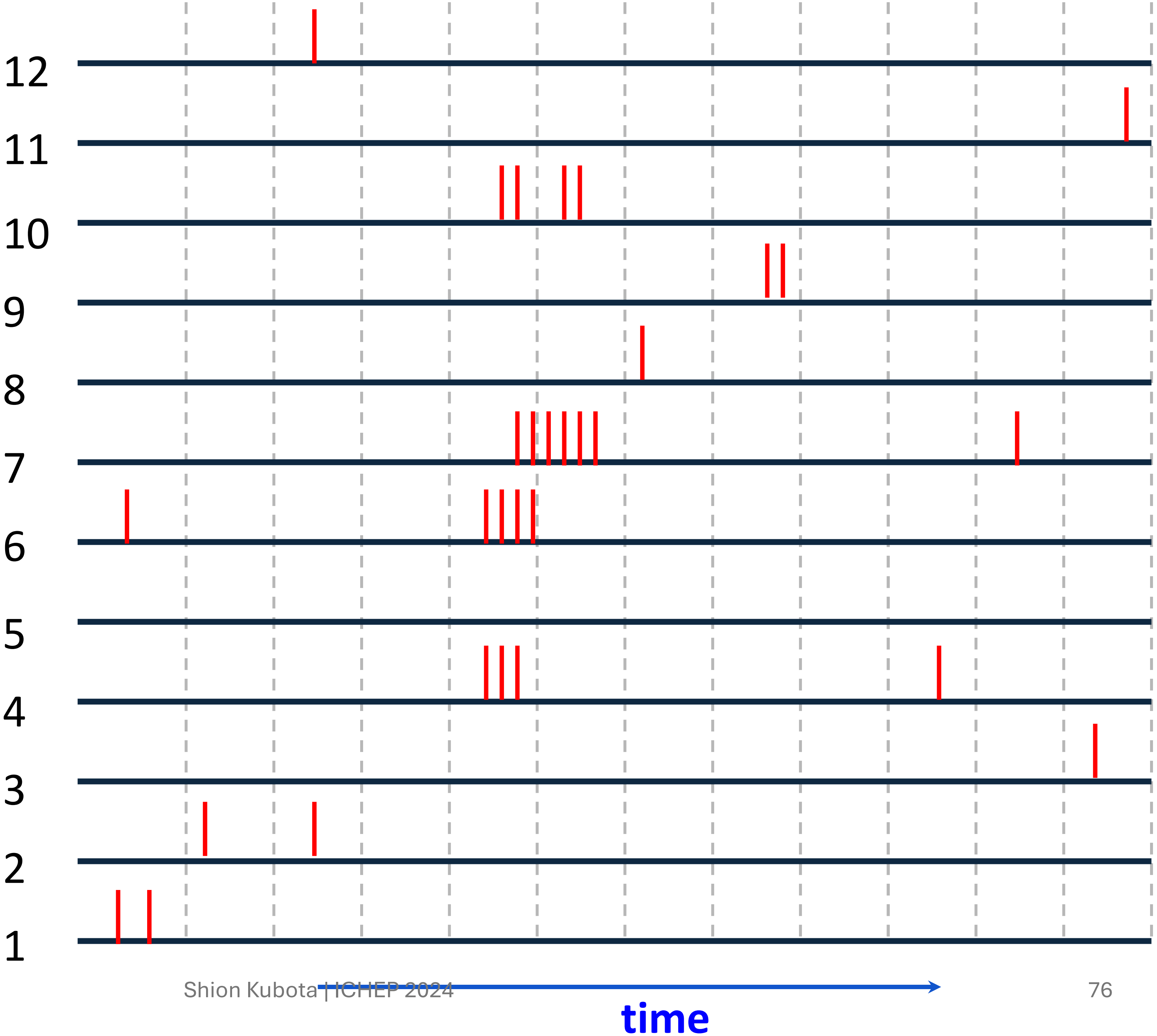
sum_cos_case1 > sum_cos_case2



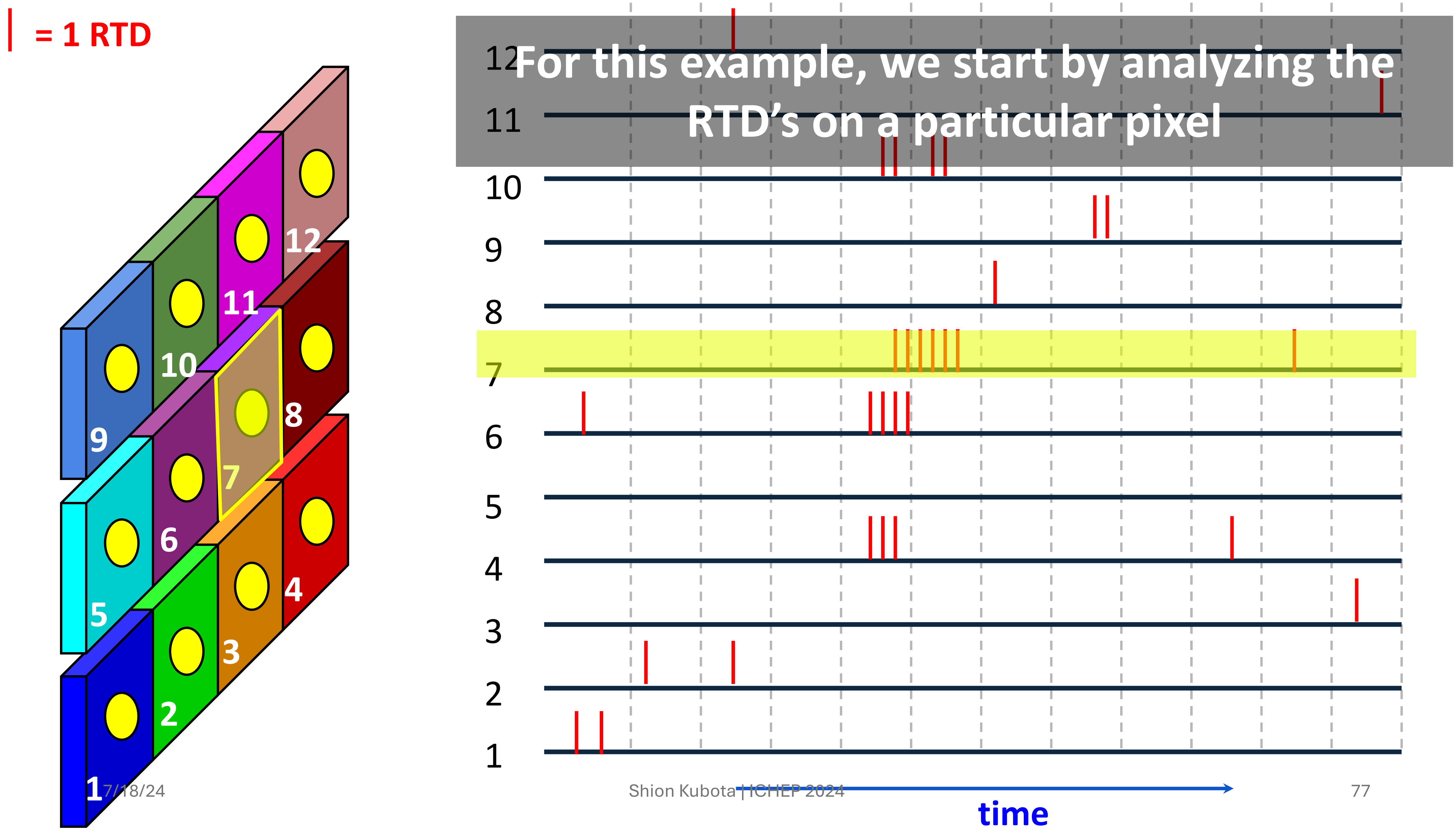


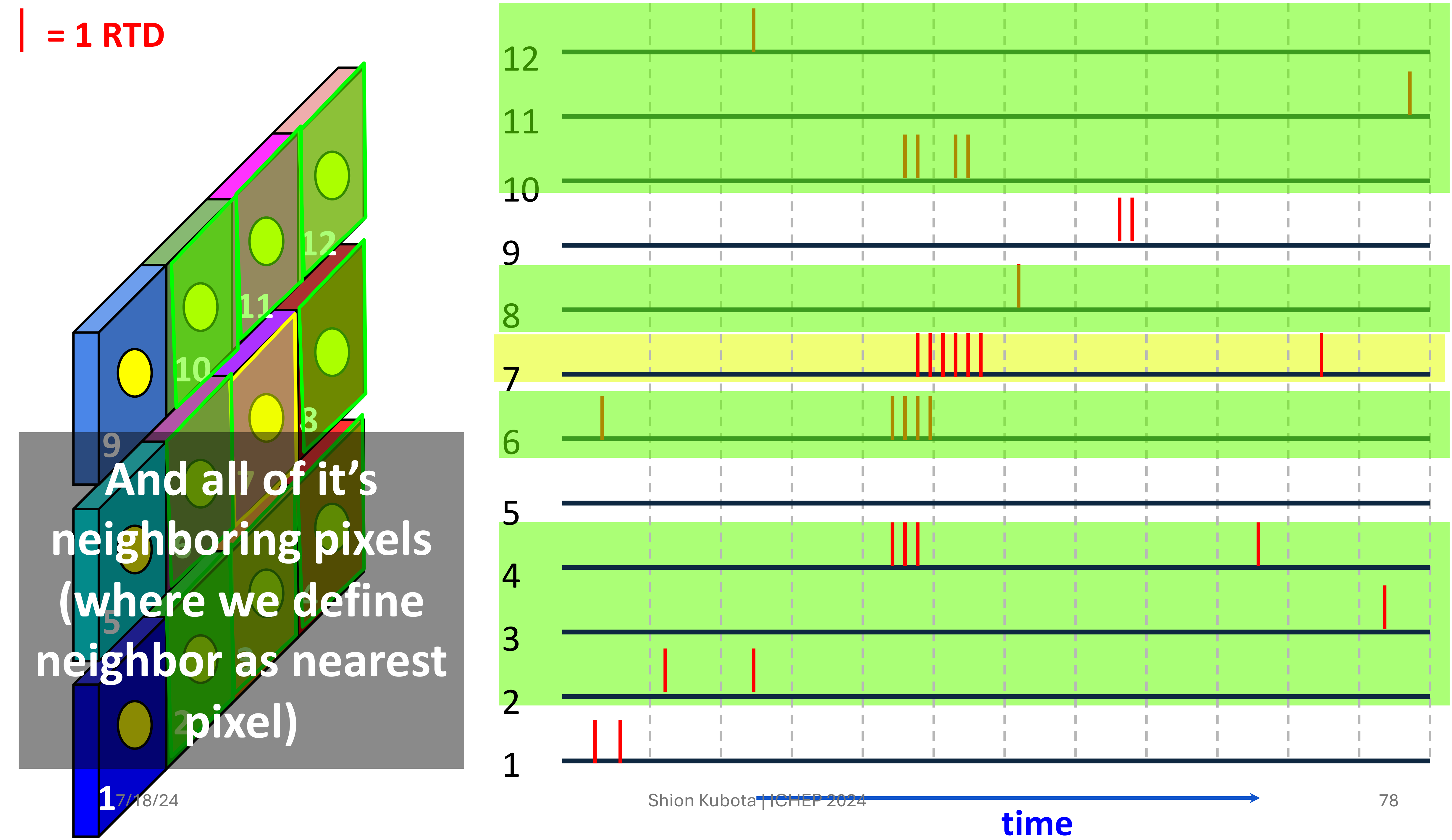
| = 1 RTD

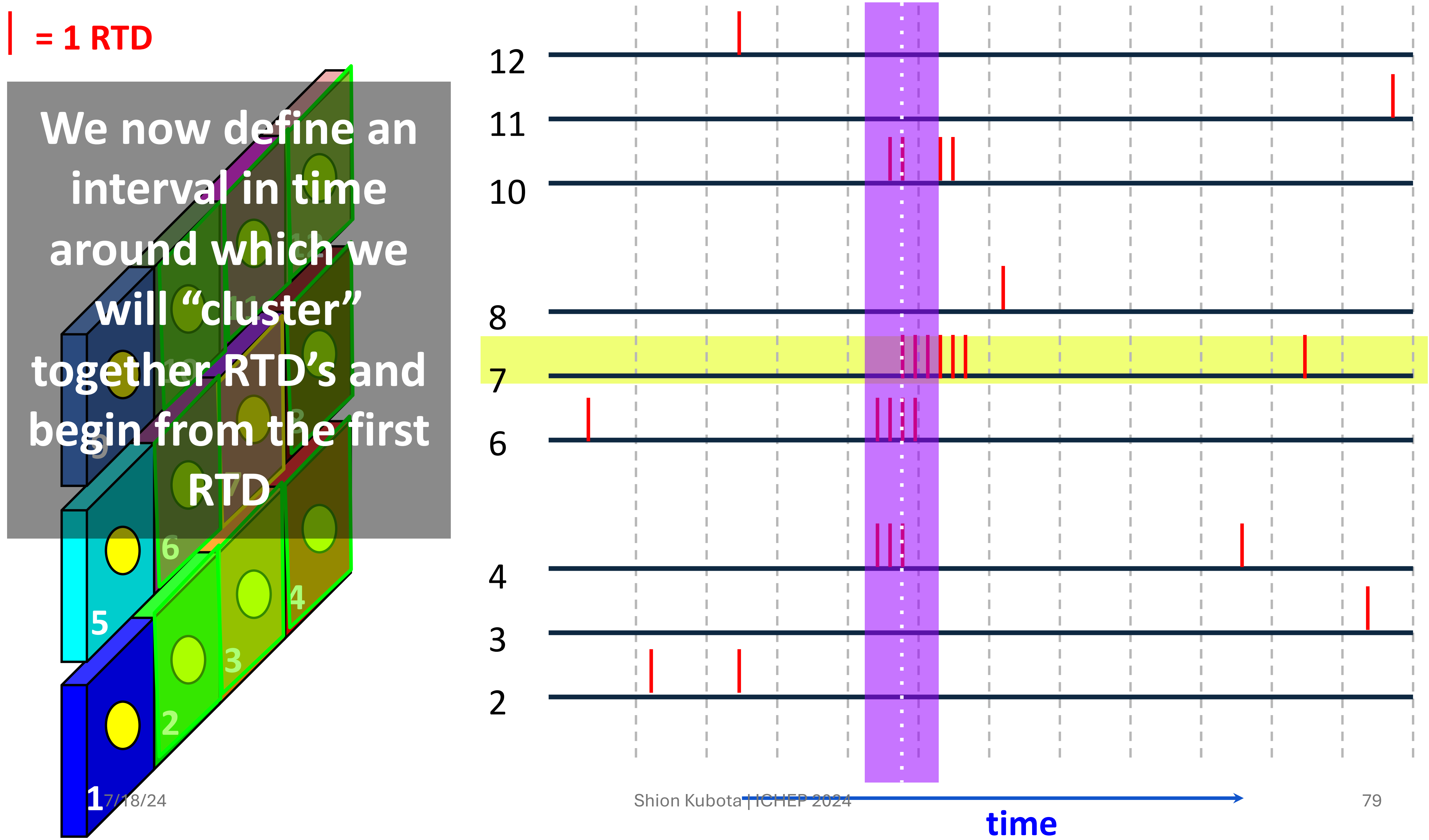




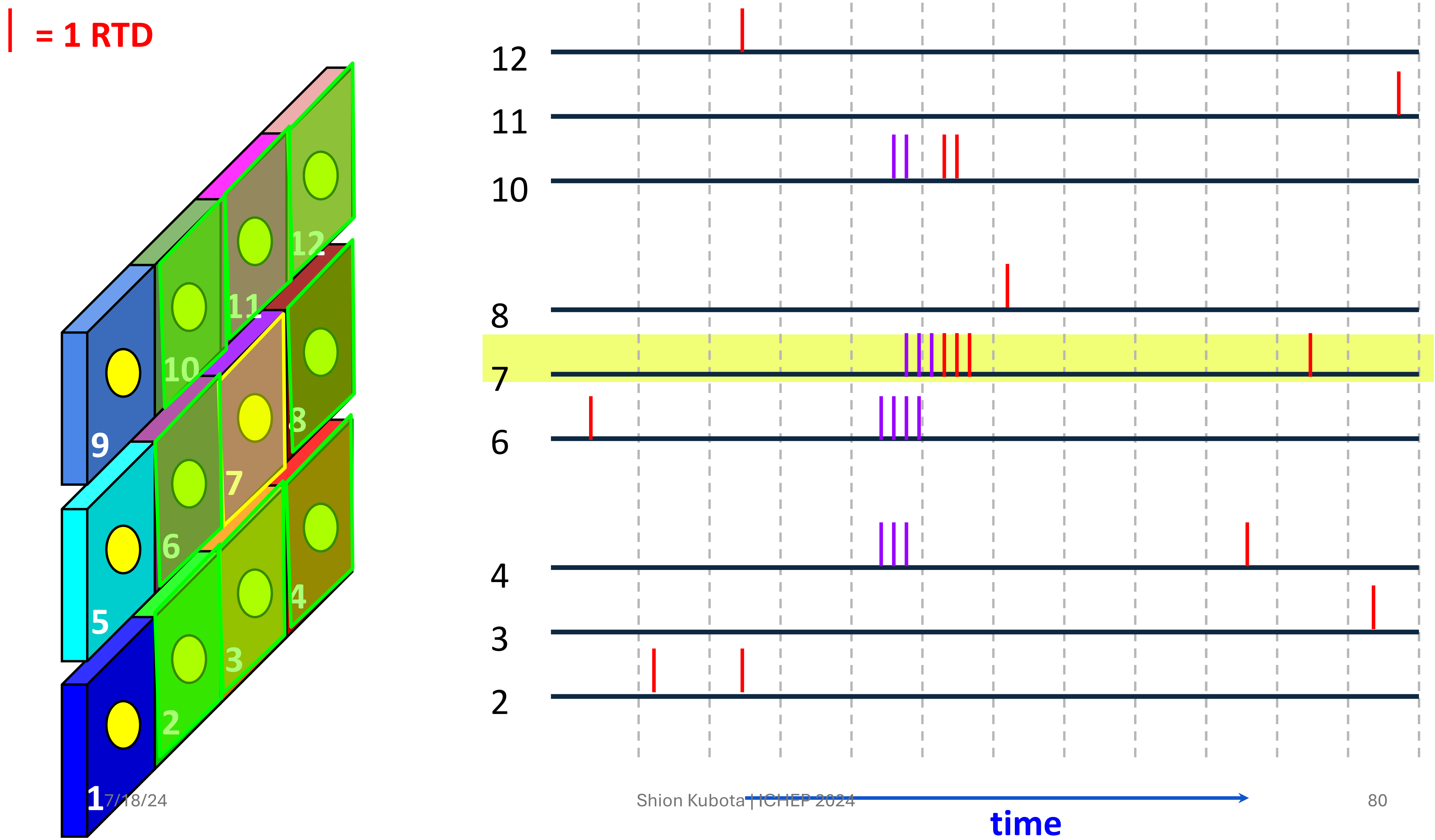
For this example, we start by analyzing the RTD's on a particular pixel




= 1 RTD
12
11
10
9
8
7
6
5
4
3
2
1
time
And all of it's neighboring pixels (where we define neighbor as nearest pixel)



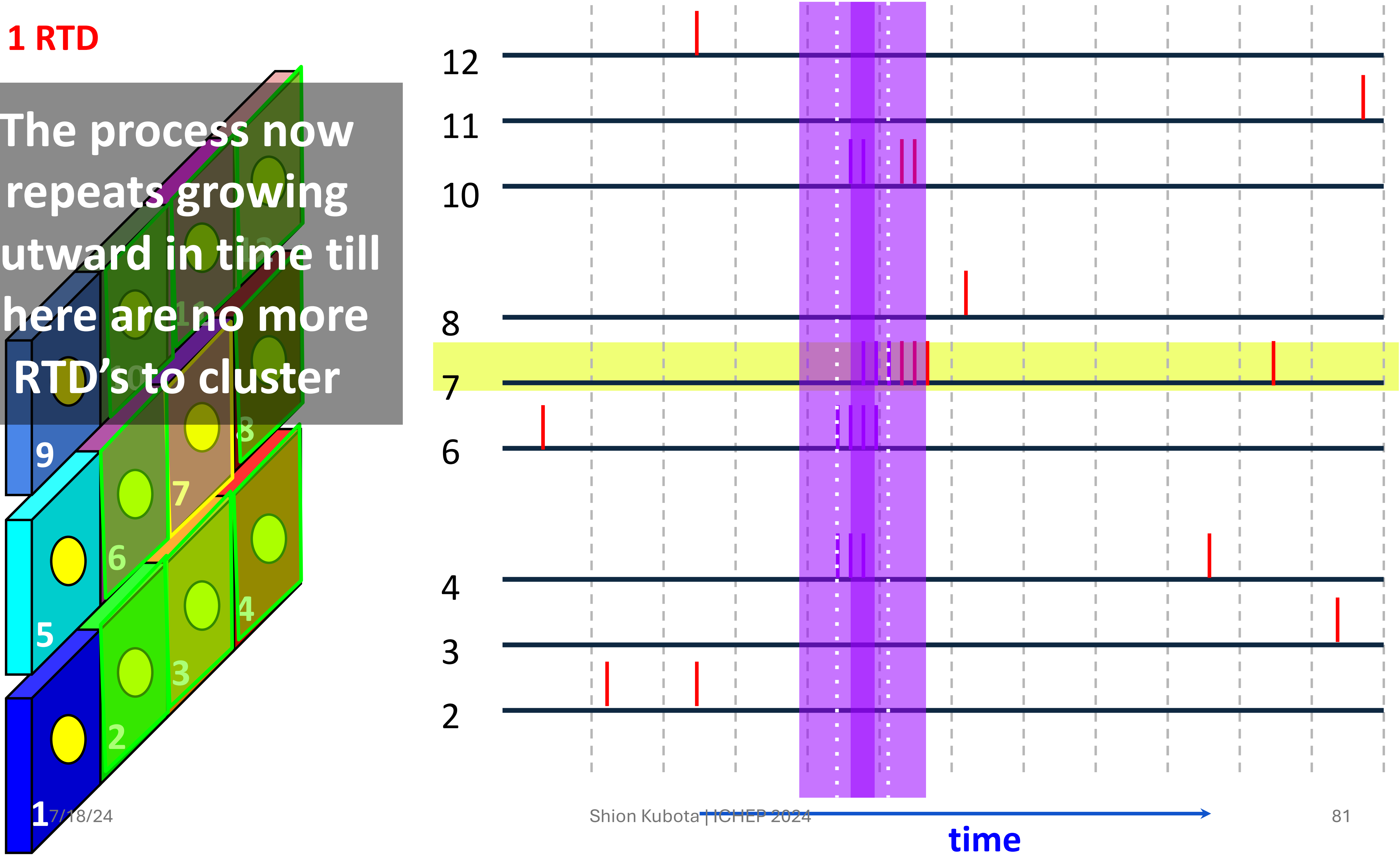
We now define an interval in time around which we will "cluster" together RTD's and begin from the first RTD

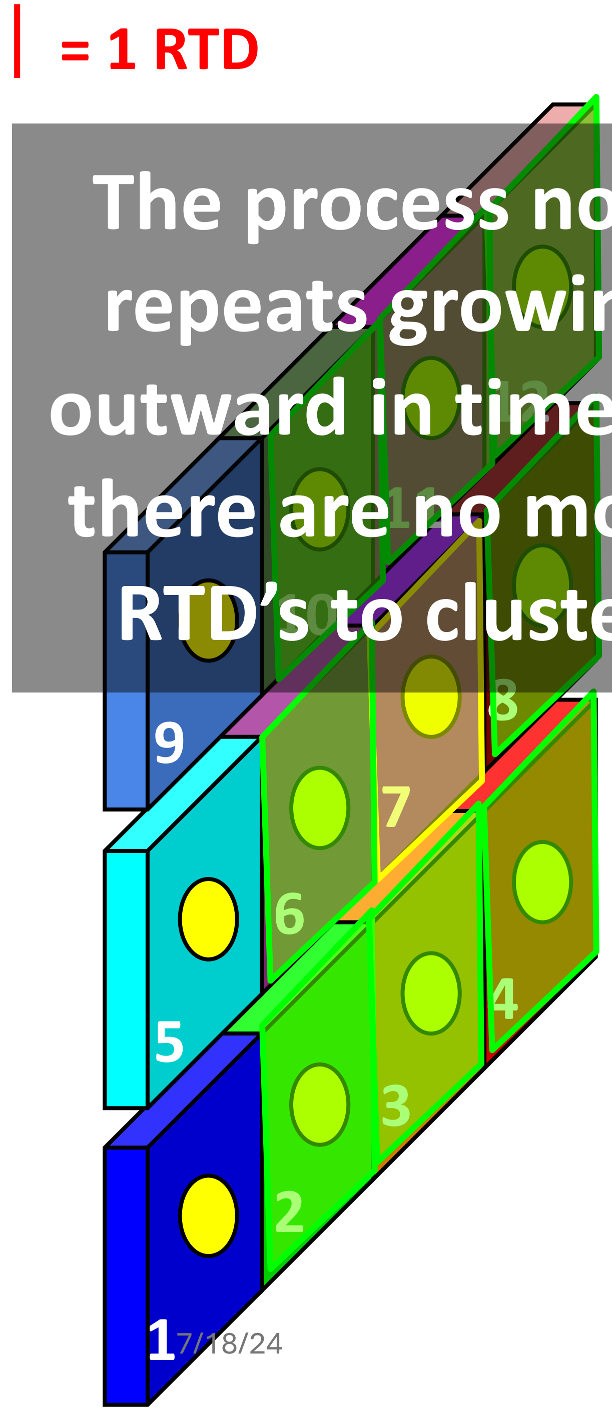


= 1 RTD
9
10
11
12
8
7
6
5
4
3
2
1
12
11
10
8
7
6
4
3
2
time

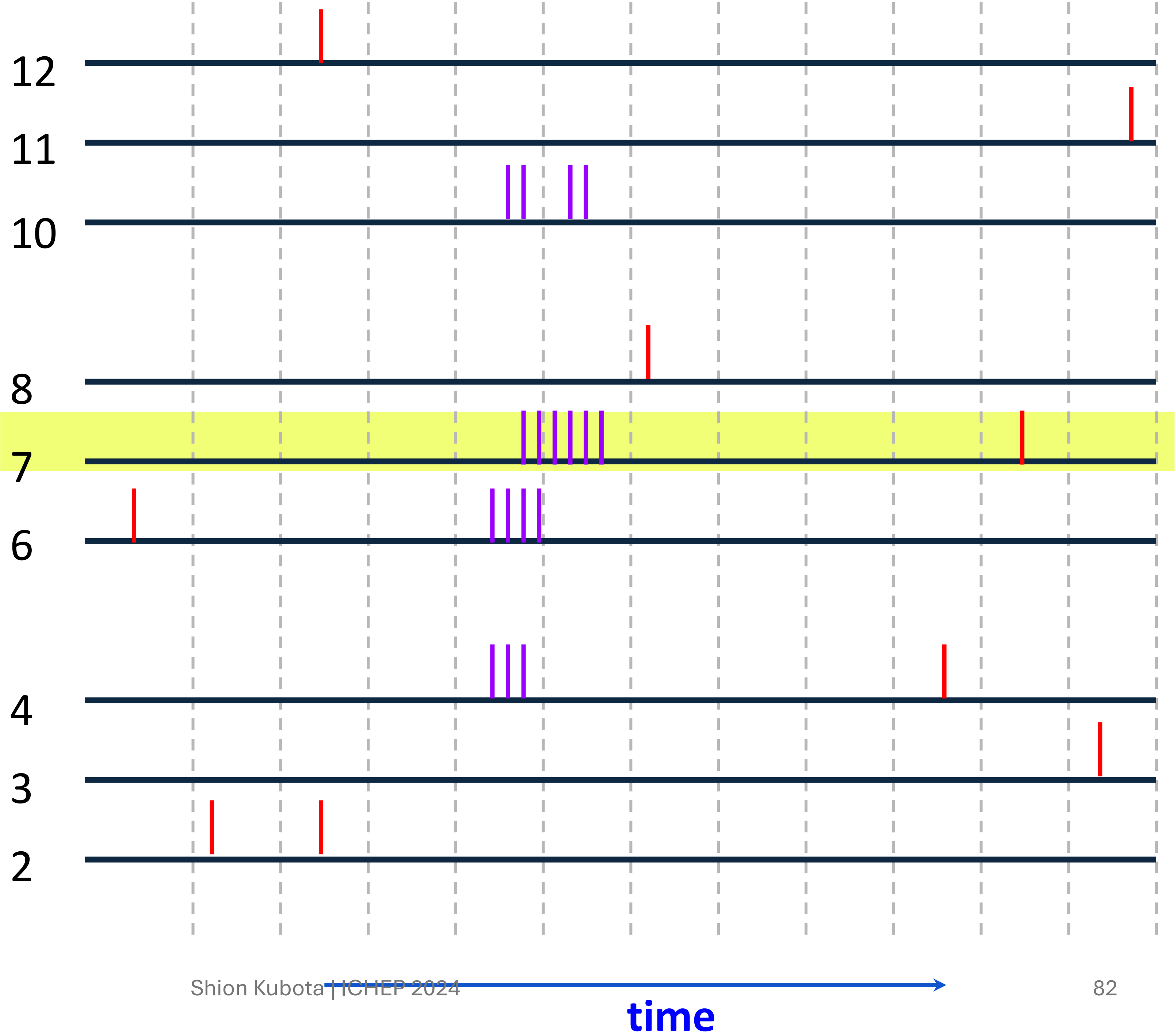
| = 1 RTD

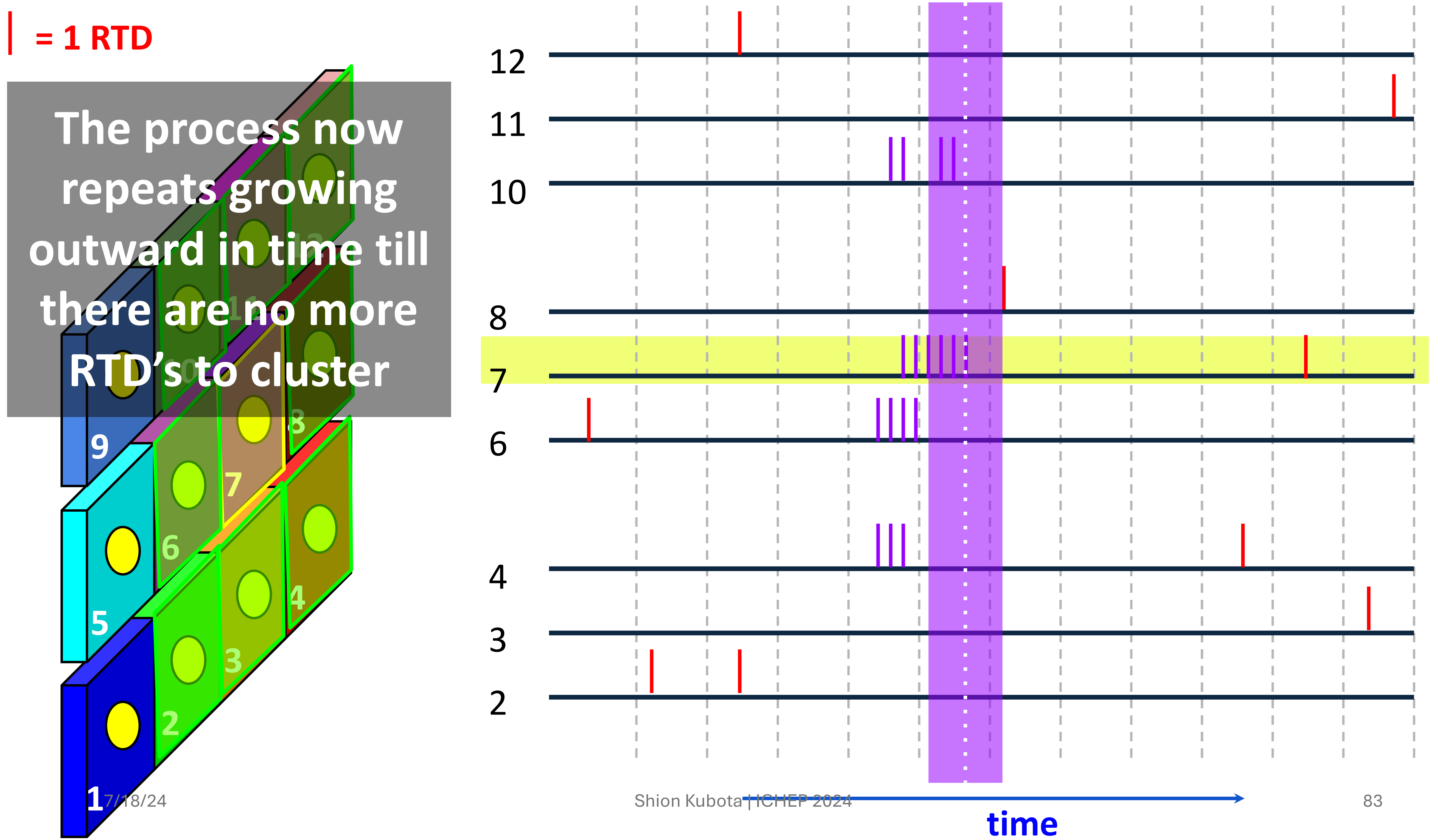
The process now repeats growing outward in time till there are no more RTD's to cluster

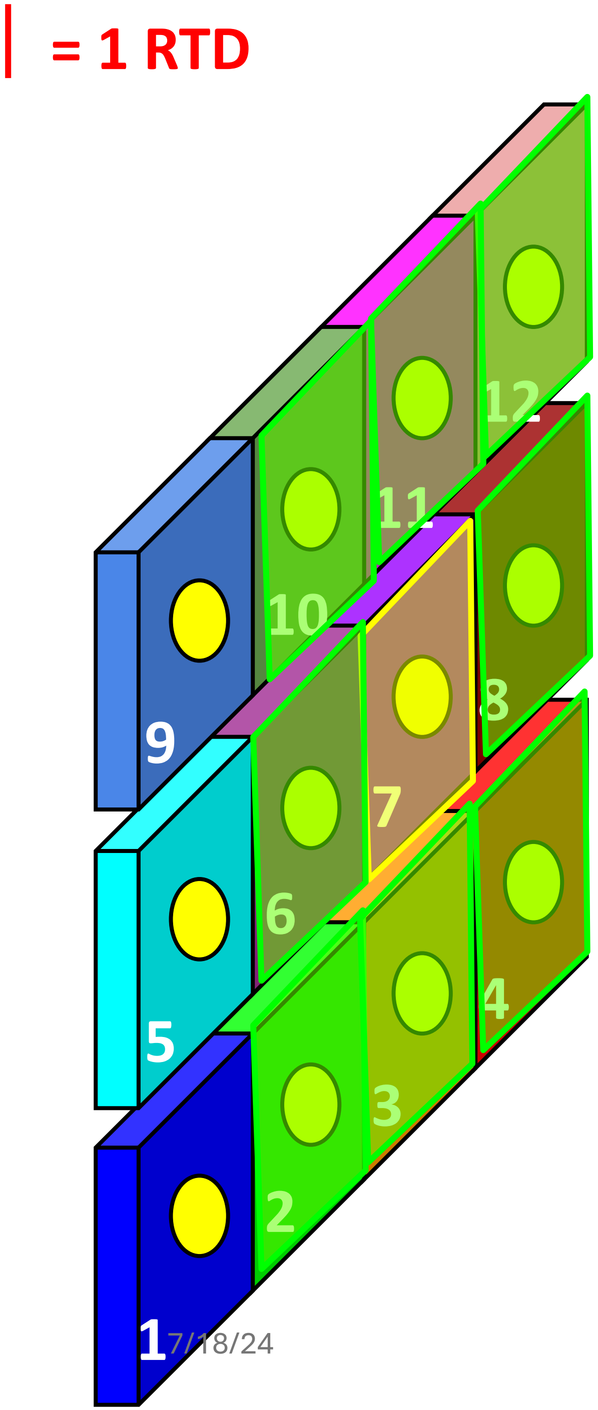


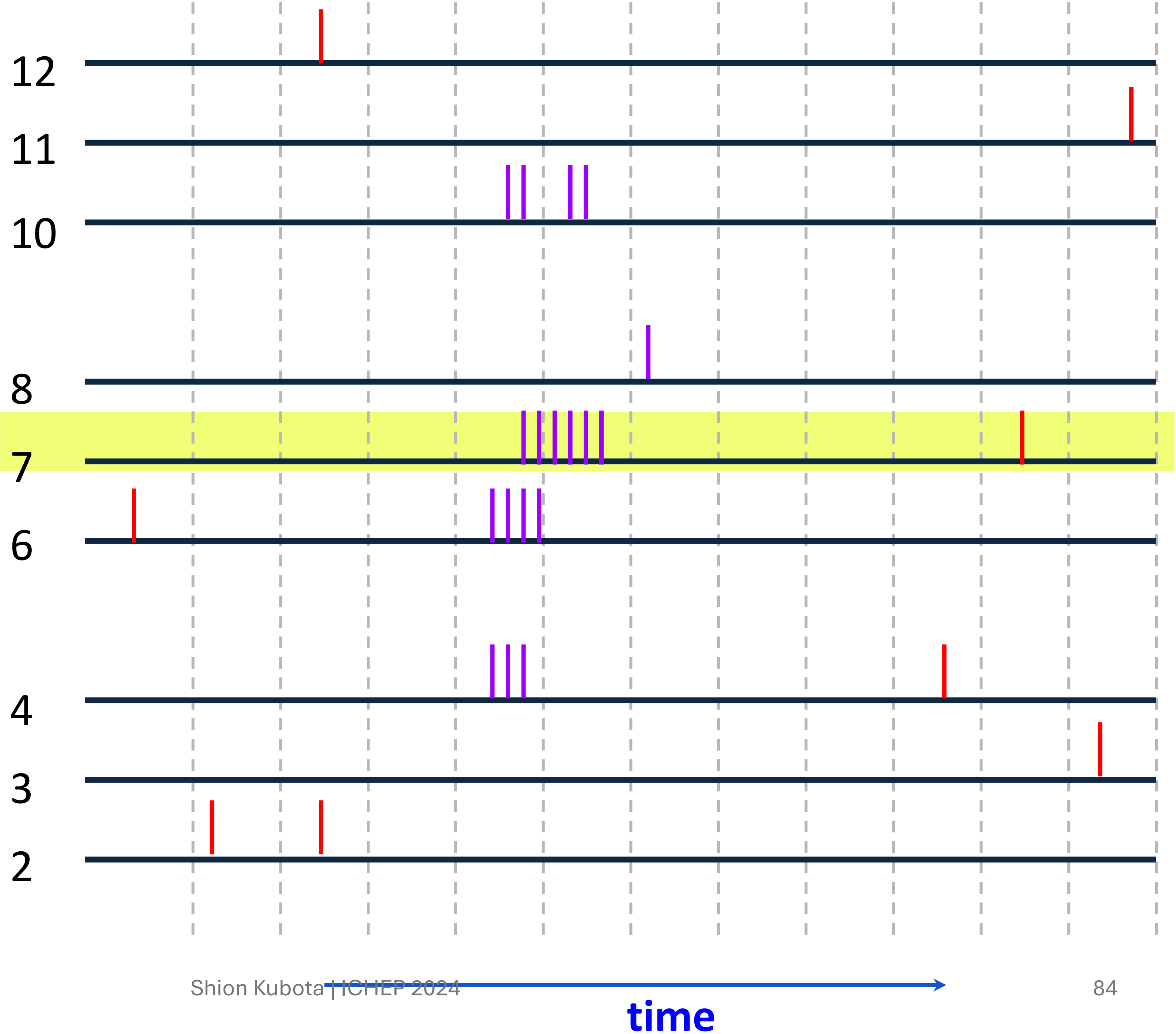


The process now repeats growing outward in time till there are no more RTD's to cluster



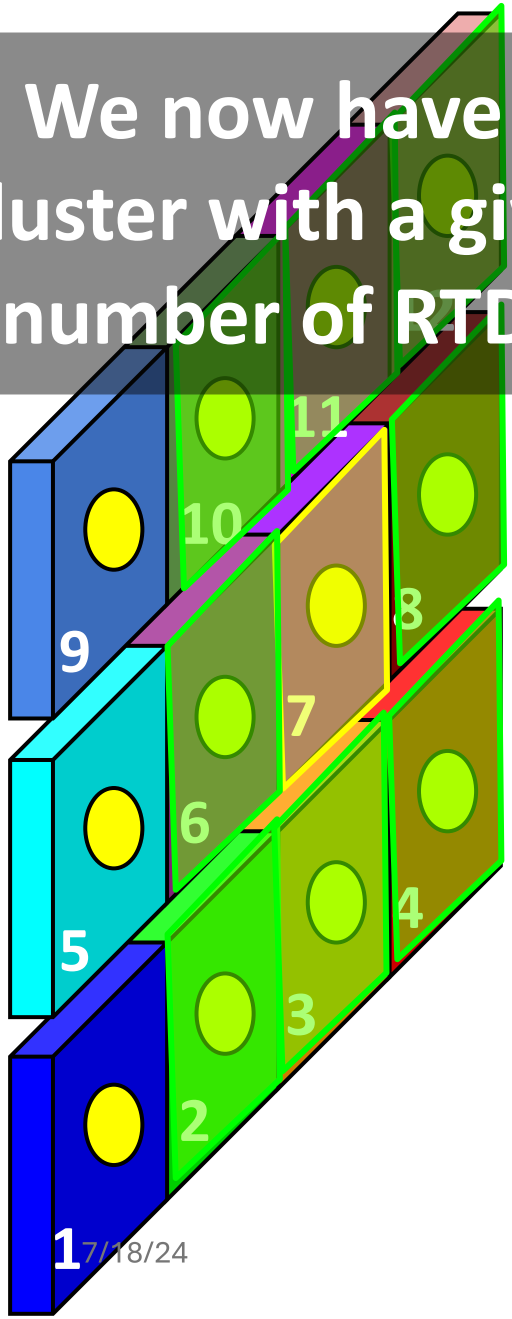

= 1 RTD
The process now repeats growing outward in time till there are no more RTD's to cluster
12
11
10
8
7
6
4
3
2
9
5
1
7
6
2
3
4
8
time

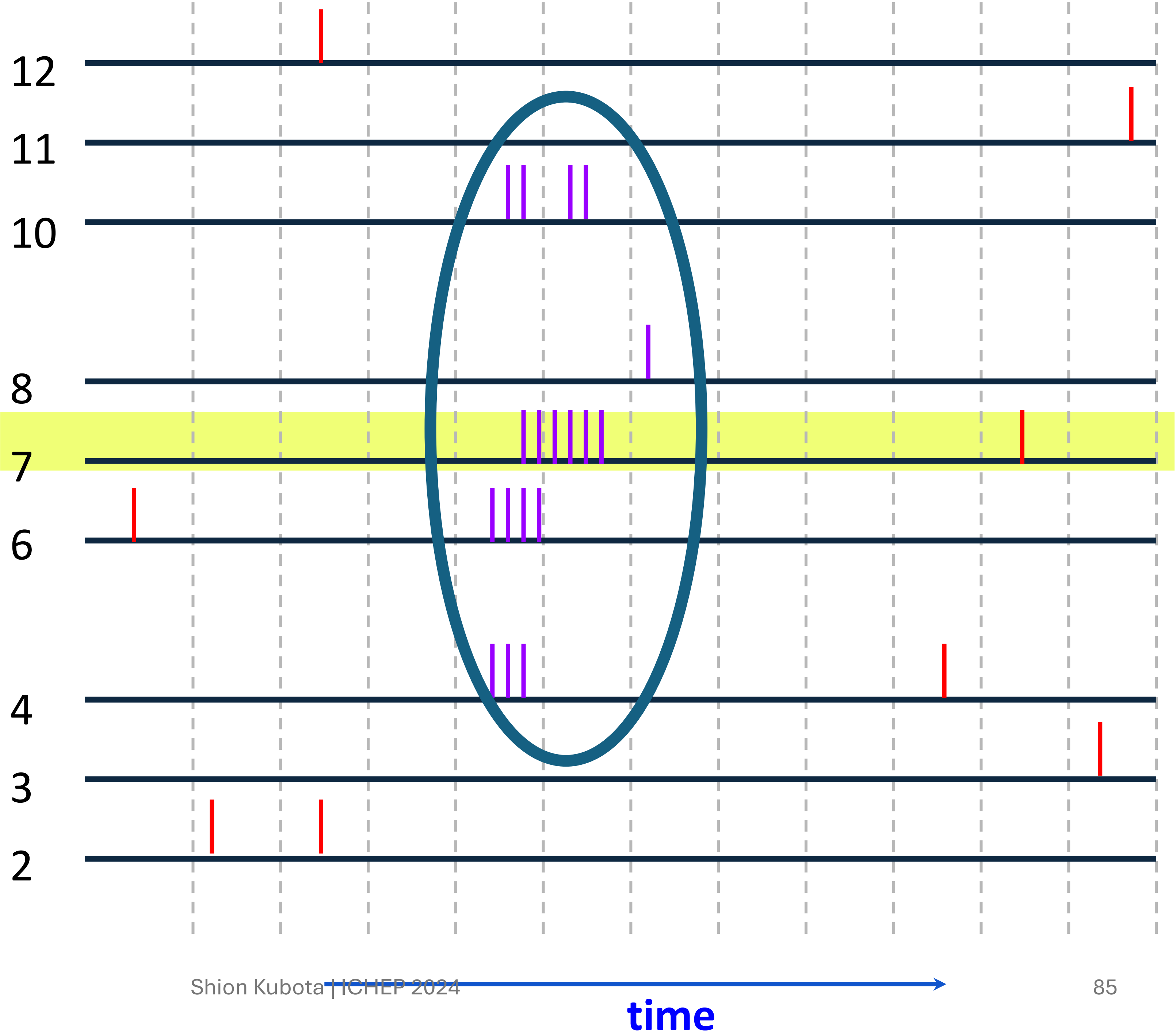

= 1 RTD
12
11
10
9
8
7
6
5
4
3
2
1


12
11
10
8
7
6
4
3
2
time

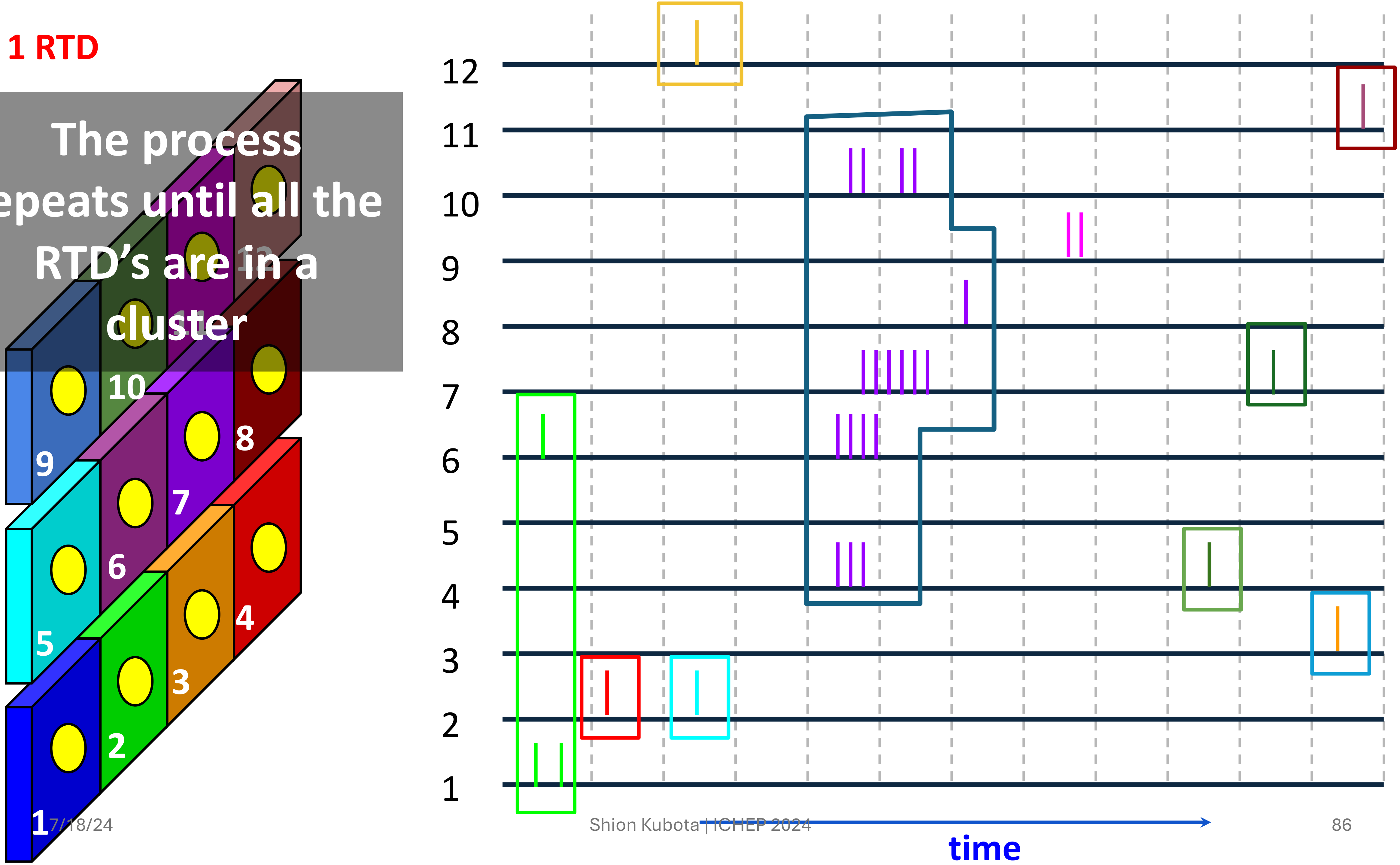
| = 1 RTD

We now have a cluster with a given number of RTD's





The process repeats until all the RTD's are in a cluster



Solar neutrino chains