

# Revisiting experimental mass limits on HECOs using Dyson-Schwinger resummation

Emanuela Musumeci (IFIC, Valencia)

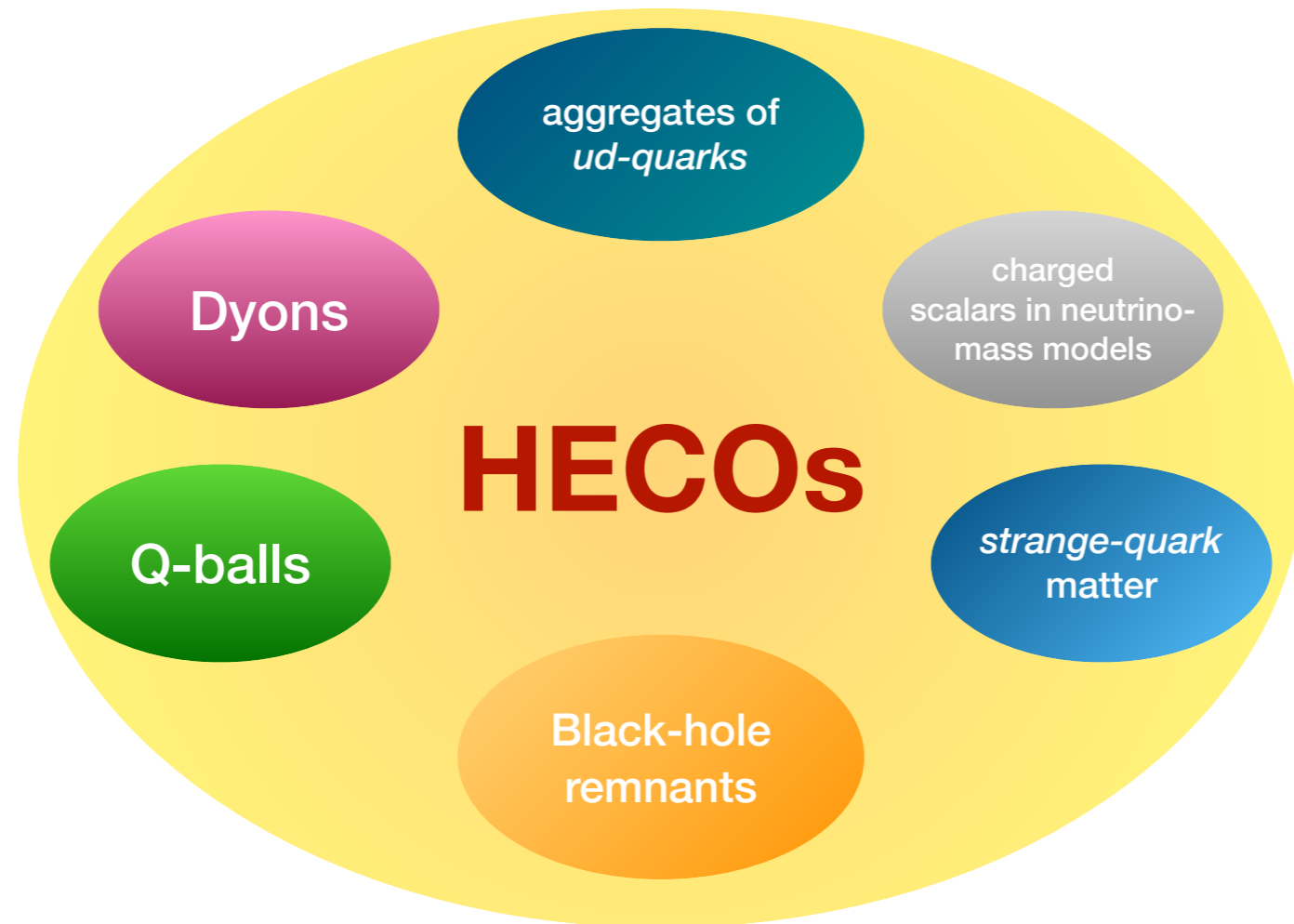
in collaboration with  
J.Alexandre , N.E. Mavromatos, V. A. Mitsou

Based on [Phys. Rev. D 109, 036026](#)

[PoS LHCP2023 \(2024\) 261](#)

# High Electric Charge Objects

Predicted by various theories Beyond the Standard Model

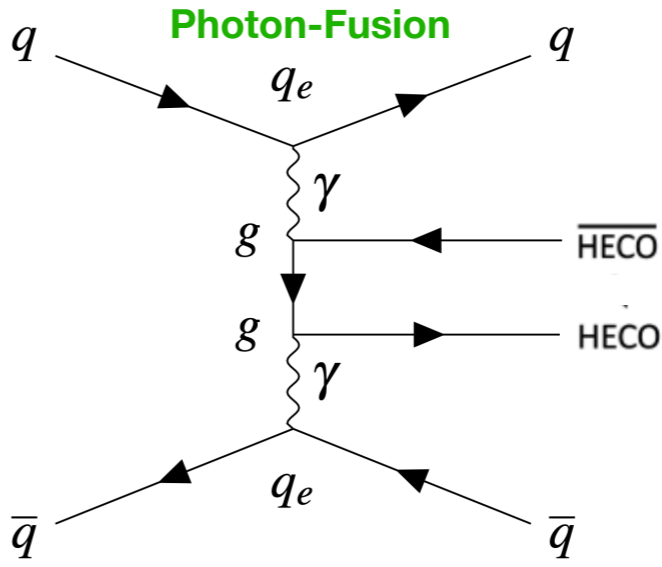
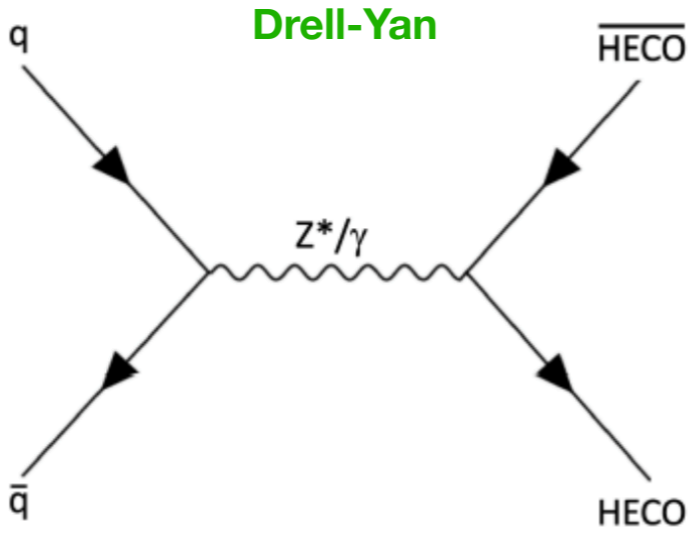


- ❖  $Q = ne, n \in \mathbb{Z}$
- ❖ High Ionisation
- ❖ Mass and Spin *free parameters*

# High Electric Charge Objects

SPIN 1/2

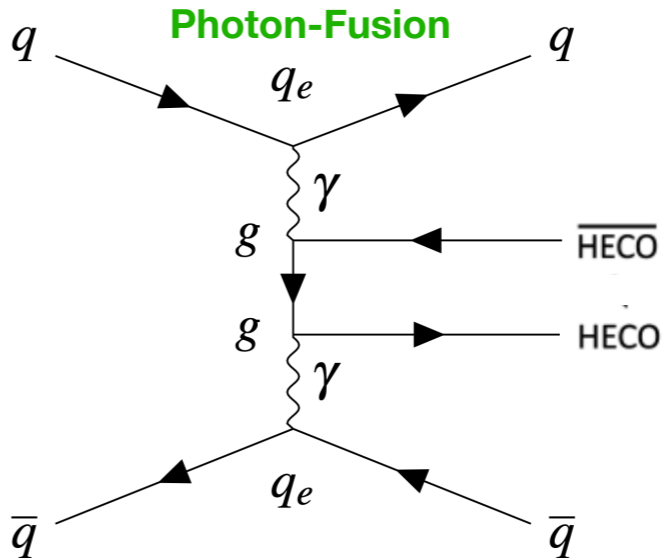
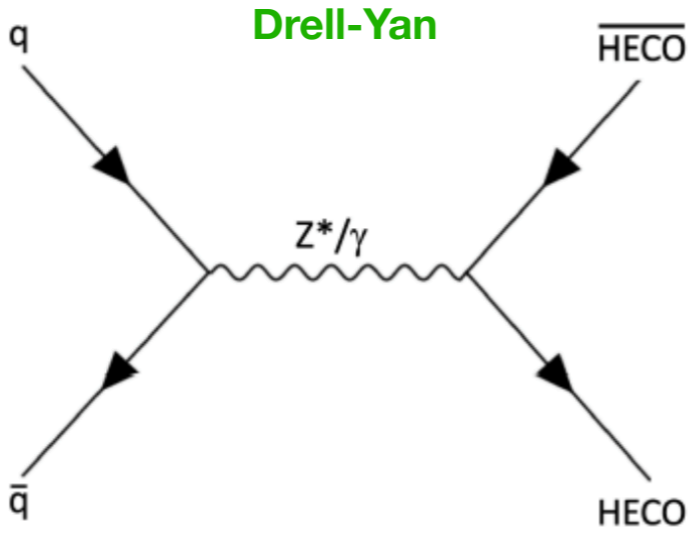
## Production mechanisms at colliders



# High Electric Charge Objects

## SPIN 1/2

### Production mechanisms at colliders



### Recent searches at the LHC

**ATLAS**  
[PRL 124, \(2020\) 3, 031802](#)  
[JHEP 11 \(2023\) 112](#)

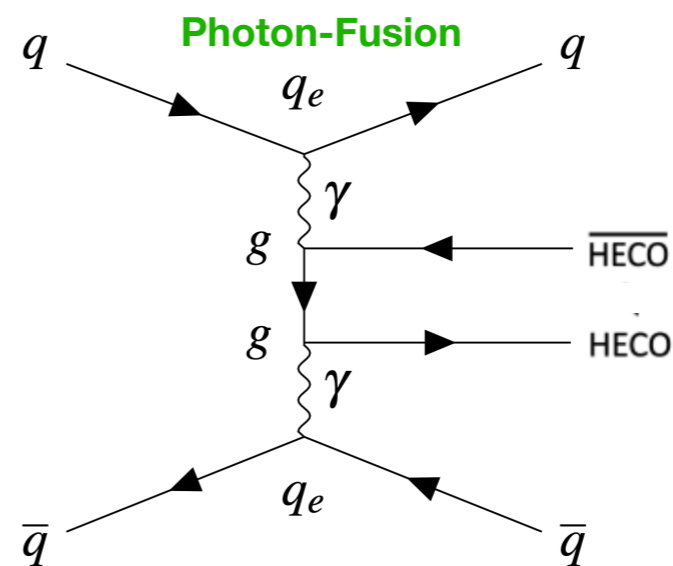
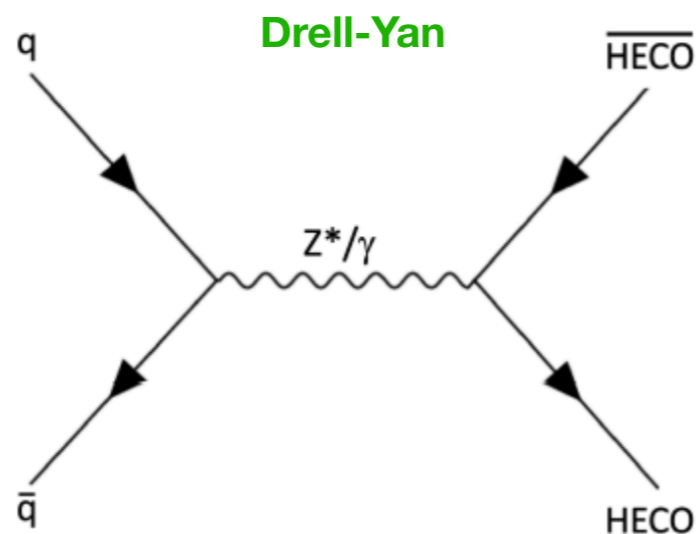
**MoEDAL**  
[EPJ C82,694 \(2022\)](#)  
[2311.06509](#)

See [A.Upreti's talk](#)

# High Electric Charge Objects

**SPIN 1/2**

## Production mechanisms at colliders



Recent searches at the LHC

**! At tree level**

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**MoEDAL**  
[EPJ C82,694 \(2022\)](#)  
[2311.06509](#)

See A.Upreti's talk



Large coupling  $g$

Perturbation theory breaks down

**Resummation needed!**

or, in general, some non-perturbative (e.g. Lattice) treatment

# RESUMMATION

## Electromagnetic Interactions - QED-Like Lagrangian

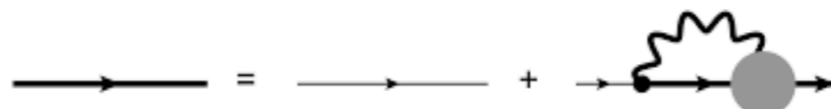
$$\mathcal{L}_{\text{bare}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left( i\not{\partial} + g\cancel{A} - m \right) \psi$$

Photon  
 ↓  
 HECO fermion    HECO charge  $g = ne$     Bare mass

$$n \geq 11$$

Fermion dressed propagator

$$G = i \frac{Z\not{p} + M}{Z^2 p^2 - M^2}$$



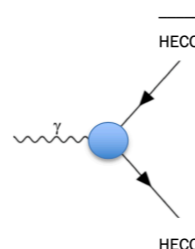
Photon dressed propagator

$$\Delta_{\mu\nu} = \frac{-i}{(1 + \omega)q^2} \left( \eta_{\mu\nu} + \frac{1 + \omega - \lambda}{\lambda} \frac{q_\mu q_\nu}{q^2} \right)$$



HECO- $\gamma$  dressed vertex

$$* \Gamma_\mu = gZ\gamma_\mu$$



Z,  $\omega$ , M quantum corrections

\* This is due to the fact that the basic effect of the resummation is equivalent to a gauge-fixed free QED Lagrangian in which the standard Ward identity is not valid, leading to the above result, assuming naturally that the coupling renormalisation function is of the same order as the resummation. (see preprint version of Alexandre et al., e-Print:2310.17452 [hep-ph] (*Phys.Rev.D* 109 (2024) 3, 3))

# RESUMMATION

## Electromagnetic Interactions - QED-Like Lagrangian

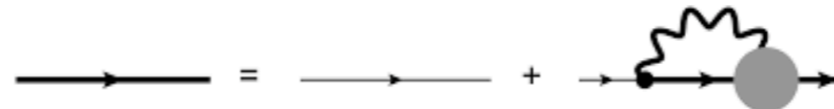
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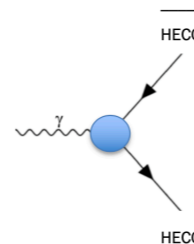
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HECO- $\gamma$  dressed vertex

$$\Gamma_\mu = gZ\gamma_\mu$$



$Z, \omega, M$  quantum corrections

➤ The aim is to solve Dyson-Schwinger equations for HECO and  $\gamma$  self-energies without assuming  $g^2 \ll 1$

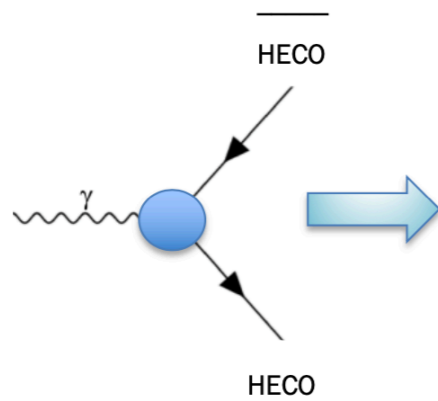
$$\begin{aligned} Z &= 1 + \frac{g^2}{8\pi^2\lambda} \ln\left(\frac{Zk}{M}\right) \\ Z\omega &= \frac{g^2}{6\pi^2} \ln\left(\frac{Zk}{M}\right) \\ Z\left(1 - \frac{m}{M}\right) &= \frac{g^2}{8\pi^2\lambda} \frac{1+3\lambda+\omega}{1+\omega} \ln\left(\frac{Zk}{M}\right) \end{aligned}$$

**UV Fixed-point solution**

$$\lim_{k \rightarrow \infty} (Z, \omega, \tilde{M}) = (Z^*, \omega^*, \tilde{M}^*)$$

such that  $\lim_{k \rightarrow \infty} \frac{kZ}{M} = \text{finite}$

# RESUMMATION



Lagrangian based on the running effective parameters

$$\mathcal{L}_{eff} = \frac{1}{2} A_\mu \left( (1 + \omega) \eta^{\mu\nu} \square - \omega \partial^\mu \partial^\nu \right) A_\nu + \bar{\psi} \left( Z i \not{\partial} + Z g \not{A} - M \right) \psi$$

$$A_\mu \rightarrow A_\mu / \sqrt{1 + \omega} \quad \text{and} \quad \psi \rightarrow \psi / \sqrt{Z} \quad \text{rescaling}$$

$$\mathcal{L}_{eff} \rightarrow \frac{1}{2} A_\mu \left( \eta^{\mu\nu} \square - \frac{\omega}{1 + \omega} \partial^\mu \partial^\nu \right) A_\nu + \bar{\psi} \left( i \not{\partial} + \frac{g}{\sqrt{1 + \omega}} \not{A} - \frac{M}{Z} \right) \psi$$

$$\text{Shifted covariant gauge parameter } \lambda_{eff} = \frac{1}{1 + \omega}$$

**Running coupling:**  $\alpha(k) = \frac{g^2 / (4\pi)}{1 + \omega(k)}$  with  $g = n e, \omega(k) > 0$

**Effective HECO Mass:**  $\mathcal{M}(k) = \frac{M(k)}{Z(k)} = k \exp \left( -\frac{2\pi}{\alpha(k)} (Z(k) - 1) \right)$

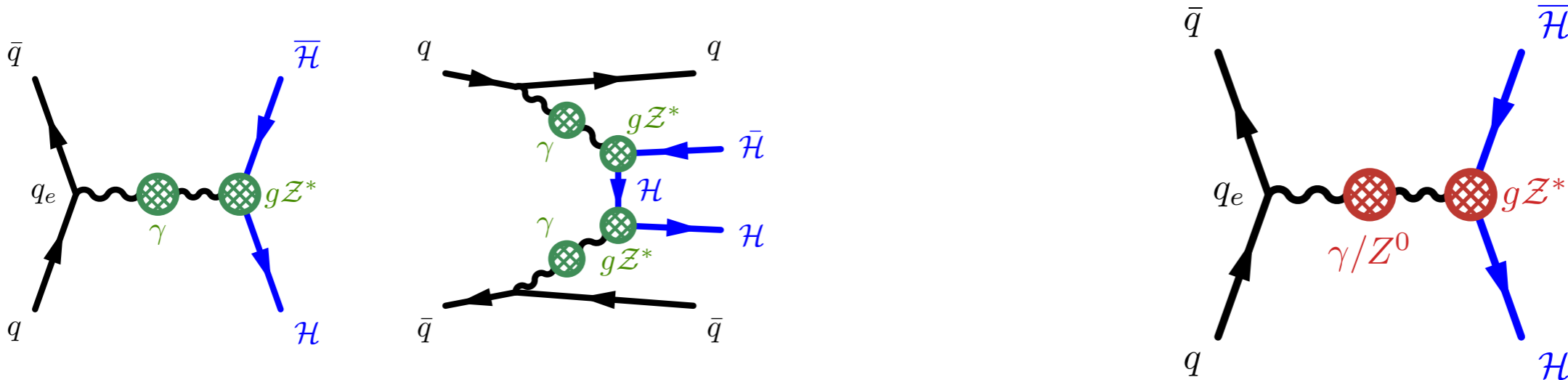
@ UV fixed point  $(Z^*, \omega^*, M^*)$

$$\lim_{k \rightarrow \Lambda} M(k) \equiv M(\Lambda) = \Lambda \exp \left( -\frac{2\pi}{\alpha^*} (Z^* - 1) \right), \quad \Lambda \gg m$$



# RESUMMATION

## Feynman Rules



### $\gamma$ -only exchange

Running mass:  $\mathcal{M}(\Lambda) = \Lambda \exp\left(-\frac{2\pi}{\hat{\alpha}^*}(Z^* - 1)\right)$

HECO-fermion propagator:  $G^{\text{eff}} = i \frac{\not{p} + \mathcal{M}(\Lambda)}{p^2 - \mathcal{M}(\Lambda)^2}$

Photon propagator:  $\Delta_{\mu\nu}^{\text{eff}} = \frac{-i}{q^2} \left( \eta_{\mu\nu} + i \omega^* \frac{q_\mu q_\nu}{q^2} \right)$

Photon-HECO vertex:  $\Gamma_\mu^{\text{eff}} = g Z^* \gamma_\mu$

with  $\hat{\alpha}^*$  is the rescaled electric coupling  $\hat{\alpha}^* = \frac{g^2/4\pi}{1 + \hat{\omega}^*}$ ,  $Z^* = 1.477$

the wavefunction renormalization and  $\hat{\omega}^* = \frac{4}{3} \left(1 - \frac{1}{Z^*}\right) \simeq 0.431$

### $Z^0$ inclusion

SU(2) singlet

Same procedure as for photon with the replacement:

$g^2 \rightarrow \hat{g}^2 \equiv g^2 + 3g'^2/4$  where  $g'$  is the  $Z_0$ -HECO coupling

$\mathcal{M}(\Lambda) = \Lambda \exp\left(-\frac{2\pi}{\hat{\alpha}^*}(\hat{Z}^* - 1)\right)$

$\hat{Z}^* = \hat{Z}_+ = \frac{2}{9}(3 + \eta) \left(1 + \sqrt{1 - \frac{9\eta}{(3+\eta)^2}}\right)$

$\hat{\omega}^* = \frac{4}{3}\eta \left(1 - \frac{1}{\hat{Z}^*}\right)$  with  $\eta \equiv g^2/\hat{g}^2 < 1$ .

# UNIVERSAL FEYNRULES OUTPUT

## SPIN 1/2

- Two UFO models created:
- i)  $\gamma$ -HECO only interaction
  - ii) Including  $Z^0$  boson

Tested on Madgraph!

Available on the [FeynRules model database](#) > [Simple extensions of the SM](#)

<a href="#">General 2HDM</a>	The most general 2HDM, including all flavor violation and mixing terms.	C. Duhr, M. Herquet	Availa
<a href="#">Heavy Scalar Effective Model</a>	A model with one heavy scalar with effective couplings to the vector bosons.	Y. Wu, Y. Xu, X. Chen	Availa
<a href="#">Heavy Neutrino</a>	The SM with three heavy Majorana neutrinos that couple to SM fields through mixing with active neutrinos.	R. Ruiz	Availa
<a href="#">Heavy Neutral Leptons</a>	The SM with heavy neutrinos interacting with mesons.	P. Coloma, E. Fernández-Martínez, M. González-López, J. Hernández-García	Availa
<a href="#">Hidden Abelian Higgs Model</a>	A $Z'$ model where the $Z'$ interacts with the SM through mixings, leading to very small non-SM like $Z'$ couplings.	D. Curtin	Availa
<a href="#">HECO</a>	High Electric Charge Objects (HECOs) pair production by including resummation effects	E. Musumeci	Availa
<a href="#">Hill Model</a>	A model with an unusual extension of the SM Higgs sector.	P. de Aquino, C. Durr	Availa
<a href="#">Inert Doublet Model</a>	A model with an additional complex scalar SU(2) <sub>L</sub> doublet and an unbroken Z <sub>2</sub> symmetry under which all SM particles are even while the extra doublet is odd.	A. Goudelis, B. Herrmann, O. Stal	Availa
<a href="#">Flavor-violating KK gluon</a>	A Kaluza-Klein Gluon Model with FCNC Decay to a Single Top Quark	E. Drueke, R. Schwienhorst, N. Vignaroli, J. Nutter, D. Walker, J.-H. Yu, R. S. Chivukula, E. Simmons	Availa
<a href="#">Simplified Freeze-in models</a>	LHC-friendly minimal freeze-in models with a charged parent	A. Goudelis	Availa
<a href="#">Leptoquarks + dark matter</a>	Minimal model including dark matter and leptoquarks	B. Fuks	Availa
<a href="#">Minimal Zp models</a>	The minimal Z' extension of the SM.	L. Basso	Availa
<a href="#">Minimal Dilaton Model</a>	Minimal Dilaton Model	J. Cao, X. Hao, Z. Han, L. Shang, Y. Zhang	Availa

Two new input parameters:

- i) Multiplicity of the electric charge  $n$  ( $Q = n e$ )
- ii) Cut-off  $\Lambda$

# UNIVERSAL FEYNRULES OUTPUT

## SPIN 1/2

Two new input parameters:

- i) Multiplicity of the electric charge  $n$  ( $Q = n e$ )
- ii) Cut-off  $\Lambda$

### Commands for MadGraph5\_aMC@NLO

#### 1. $\gamma$ -only exchange

- Drell-Yan

```
import model heco_spinhalf_photononly
generate p p > heco heco~
output DY_HECO
```

- Photon-Fusion

```
import model heco_spinhalf_photononly
generate a a > heco heco~
output PF_HECO
```

#### 2. $\gamma/Z0$ exchange

- Drell-Yan

```
import model heco_spinhalf_withZ0
generate p p > heco heco~
output DY_HECO_Z0
```

### Set run card and parameters

```
launch name_of_theoutput ### replace name_of_theoutput accordingly, i.e. DY_HECO_Z0
set ebeam1 6500 ### beam 1 energy in GeV
set ebeam2 6500 ### beam 2 energy in GeV
set lpp1 1 ### beam 1 type (1=proton)
set lpp2 1 ### beam 2 type (1=proton)
set pdlabel lhpdf ### set the pdlabel argument
set lhaid 82000 ### set the pdf set
set nevents 10000 ### number of events

set n 20 ### set the multiplicity of the charge
set lambda 1000 ### set the cutoff energy scale in GeV
```

# VALIDATION OF UFO MODELS

## SPIN 1/2

- ✓ Calculation of the analytical cross section (Wolfram Mathematica)
- ✓ Comparison results from Madgraph with those obtained from Mathematica

### DRELL-YAN

$Q (e)$	DY with resummation $u\bar{u} \rightarrow \mathcal{H}\bar{\mathcal{H}}$ @ $\sqrt{s} = 13$ TeV, $\Lambda = 2$ TeV						NO PDF
	$\gamma$ -only exchange			$\gamma/Z^0$ exchange			
	$\sigma_{\text{MADGRAPH}}$ (pb)	$\sigma_{\text{MATHEMATICA}}$ (pb)	UFO/Theory	$\sigma_{\text{MADGRAPH}}$ (pb)	$\sigma_{\text{MATHEMATICA}}$ (pb)	UFO/Theory	
20	0.0762	0.0758	1.005	0.0659	0.0655	1.006	
40	0.3048	0.3027	1.007	0.2632	0.2616	1.006	
60	0.6837	0.6807	1.004	0.5919	0.5886	1.005	
80	1.2151	1.2097	1.004	1.0508	1.0462	1.004	
100	1.8976	1.8898	1.004	1.6460	1.6345	1.007	
120	2.7372	2.7210	1.006	2.3682	2.3537	1.006	
140	3.7261	3.7032	1.006	3.2251	3.2035	1.006	
160	4.8712	4.8366	1.007	4.2083	4.1843	1.005	
180	6.1548	6.1210	1.006	5.3224	5.2955	1.005	
200	7.5927	7.5568	1.004	6.5758	6.5379	1.006	

### PHOTON FUSION

$Q (e)$	PF with resummation $\gamma\gamma \rightarrow \mathcal{H}\bar{\mathcal{H}}$ @ $\sqrt{s} = 13$ TeV, $\Lambda = 2$ TeV			NO PDF
	$\sigma_{\text{MADGRAPH}}$ (pb)	$\sigma_{\text{MATHEMATICA}}$ (pb)	UFO/Theory	
20	$7.438 \times 10^3$	$7.398 \times 10^3$	1.005	
40	$7.732 \times 10^4$	$7.692 \times 10^4$	1.004	
60	$3.539 \times 10^5$	$3.528 \times 10^5$	1.003	
80	$1.082 \times 10^6$	$1.076 \times 10^6$	1.006	
100	$2.596 \times 10^6$	$2.580 \times 10^9$	1.006	
120	$5.327 \times 10^6$	$5.300 \times 10^6$	1.005	
140	$9.814 \times 10^6$	$9.761 \times 10^6$	1.005	
160	$1.668 \times 10^7$	$1.659 \times 10^7$	1.005	
180	$2.663 \times 10^7$	$2.651 \times 10^7$	1.004	
200	$4.052 \times 10^7$	$4.033 \times 10^7$	1.005	

# IMPACT ON PRODUCTION CROSS SECTIONS

## DRELL-YAN

DY $pp \rightarrow \gamma \rightarrow \mathcal{H}\overline{\mathcal{H}}$ @ $\sqrt{s} = 13$ TeV, $\Lambda = 2$ TeV			
$Q$ (e)	$\sigma_{\text{tree-level}}$ (fb)	$\sigma_{\text{resum}}$ (fb)	NNPDF23 $M$ (TeV)
20	$7.75 \times 10^2$	$1.692 \times 10^3$	0.507
60	4.959	10.79	1.717
100	5.949	12.95	1.893
140	9.134	19.86	1.945
180	13.67	29.71	1.966
220	19.31	42.08	1.977

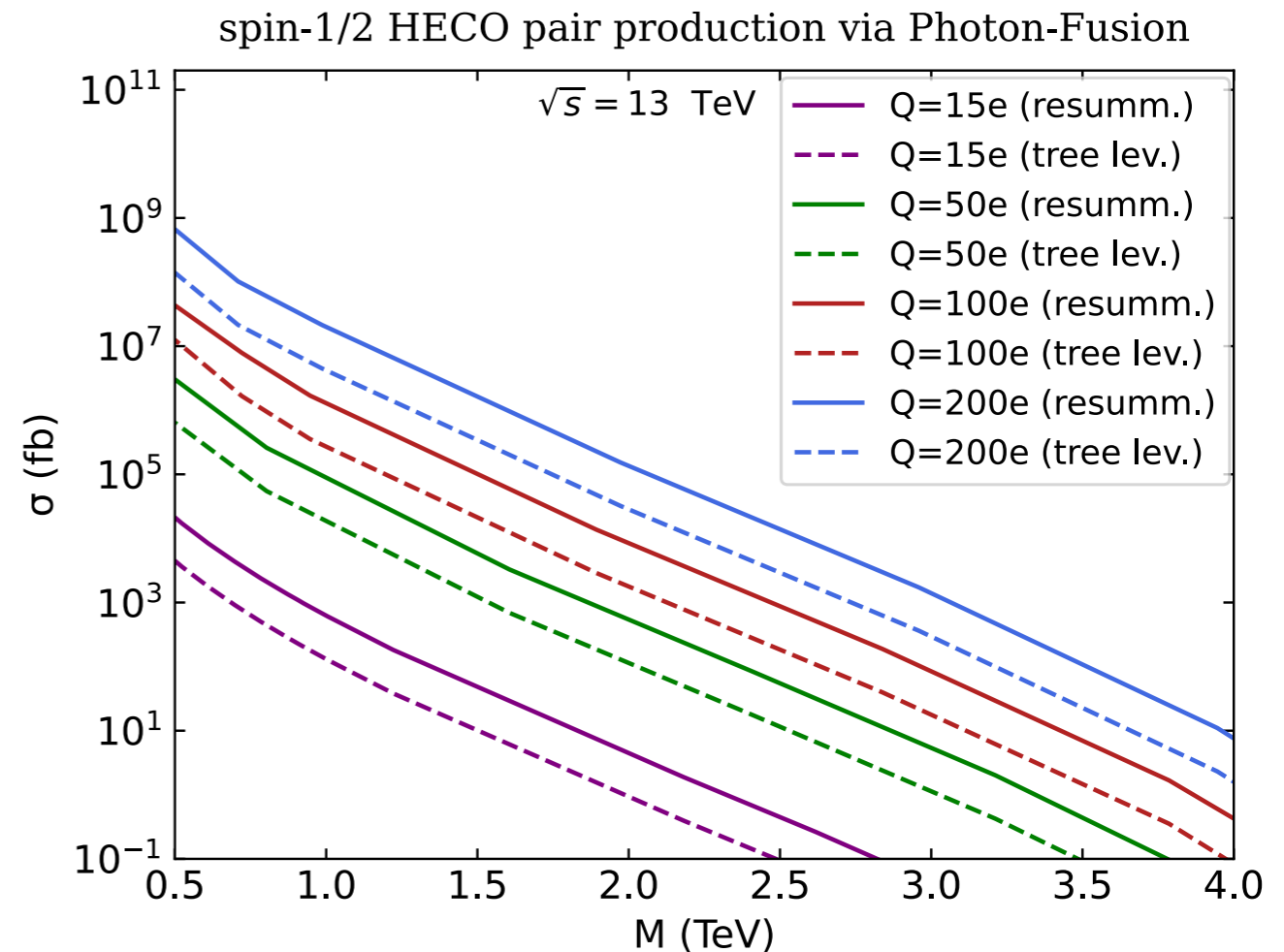
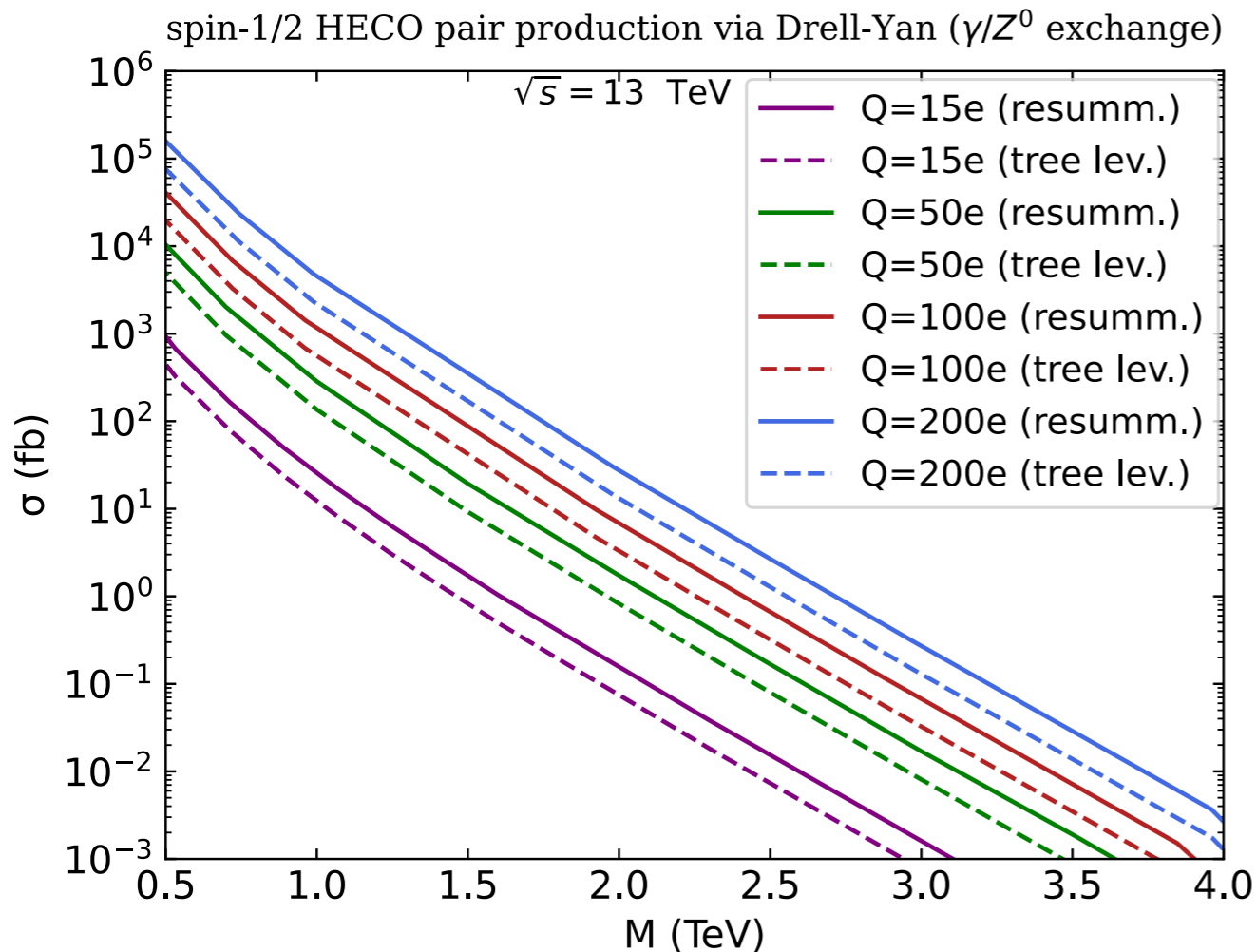
DY $pp \rightarrow \gamma/Z^0 \rightarrow \mathcal{H}\overline{\mathcal{H}}$ @ $\sqrt{s} = 13$ TeV, $\Lambda = 2$ TeV			
$Q$ (e)	$\sigma_{\text{tree-level}}$ (fb)	$\sigma_{\text{resum}}$ (fb)	$M$ (TeV)
20	$1.014 \times 10^2$	$2.118 \times 10^2$	0.798
60	3.367	6.527	1.780
100	4.722	9.835	1.924
140	7.752	16.25	1.961
180	11.95	24.92	1.976
220	17.22	35.94	1.984

## PHOTON FUSION

PF $pp \rightarrow \mathcal{H}\overline{\mathcal{H}}$ @ $\sqrt{s} = 13$ TeV, $\Lambda = 2$ TeV			
$Q$ (e)	$\sigma_{\text{tree-level}}$ (fb)	$\sigma_{\text{resum}}$ (fb)	LUXqed17 $M$ (TeV)
20	$1.321 \times 10^4$	$6.271 \times 10^4$	0.507
60	$8.466 \times 10^2$	$4.025 \times 10^3$	1.717
100	$2.895 \times 10^3$	$1.372 \times 10^4$	1.893
140	$8.753 \times 10^3$	$4.170 \times 10^4$	1.945
180	$2.175 \times 10^4$	$1.030 \times 10^5$	1.966
220	$4.612 \times 10^4$	$2.184 \times 10^5$	1.977

# IMPACT ON PRODUCTION CROSS SECTIONS

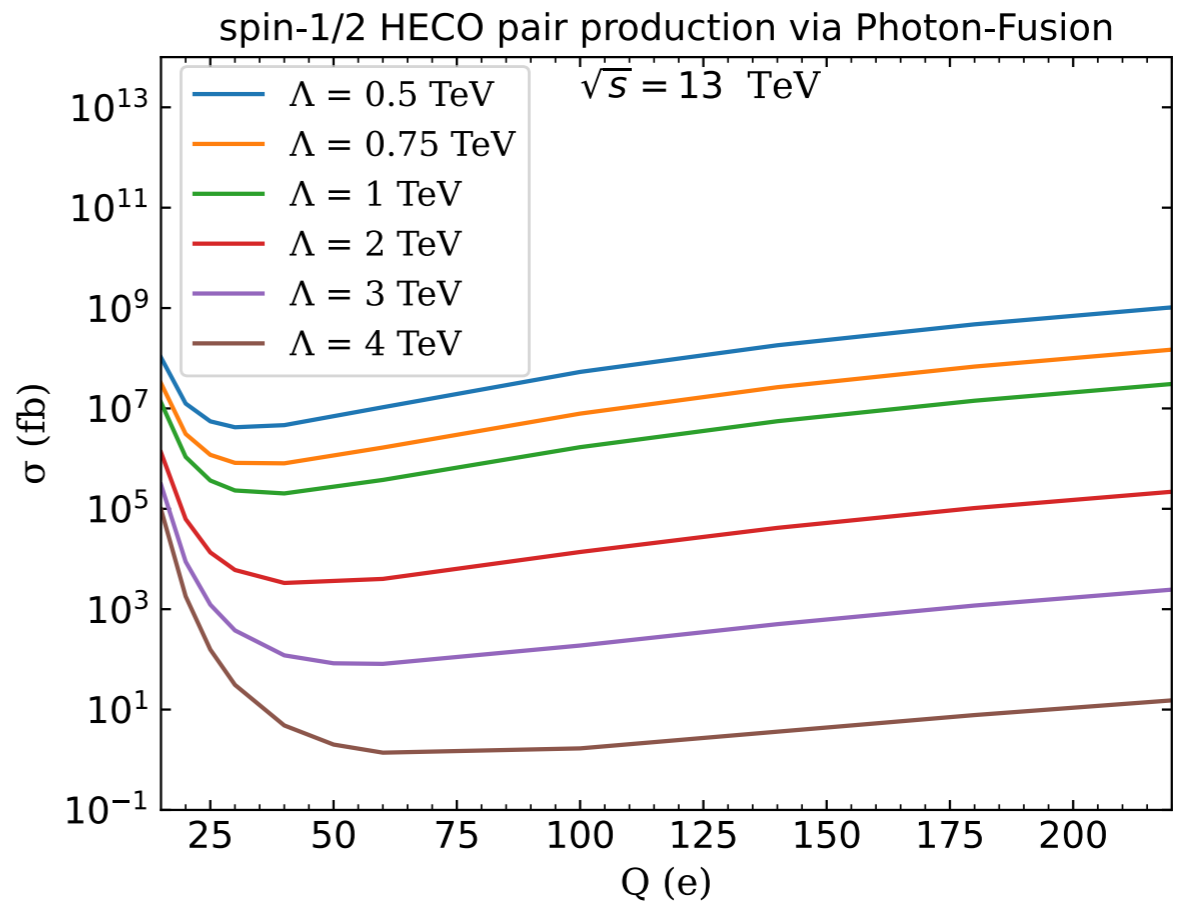
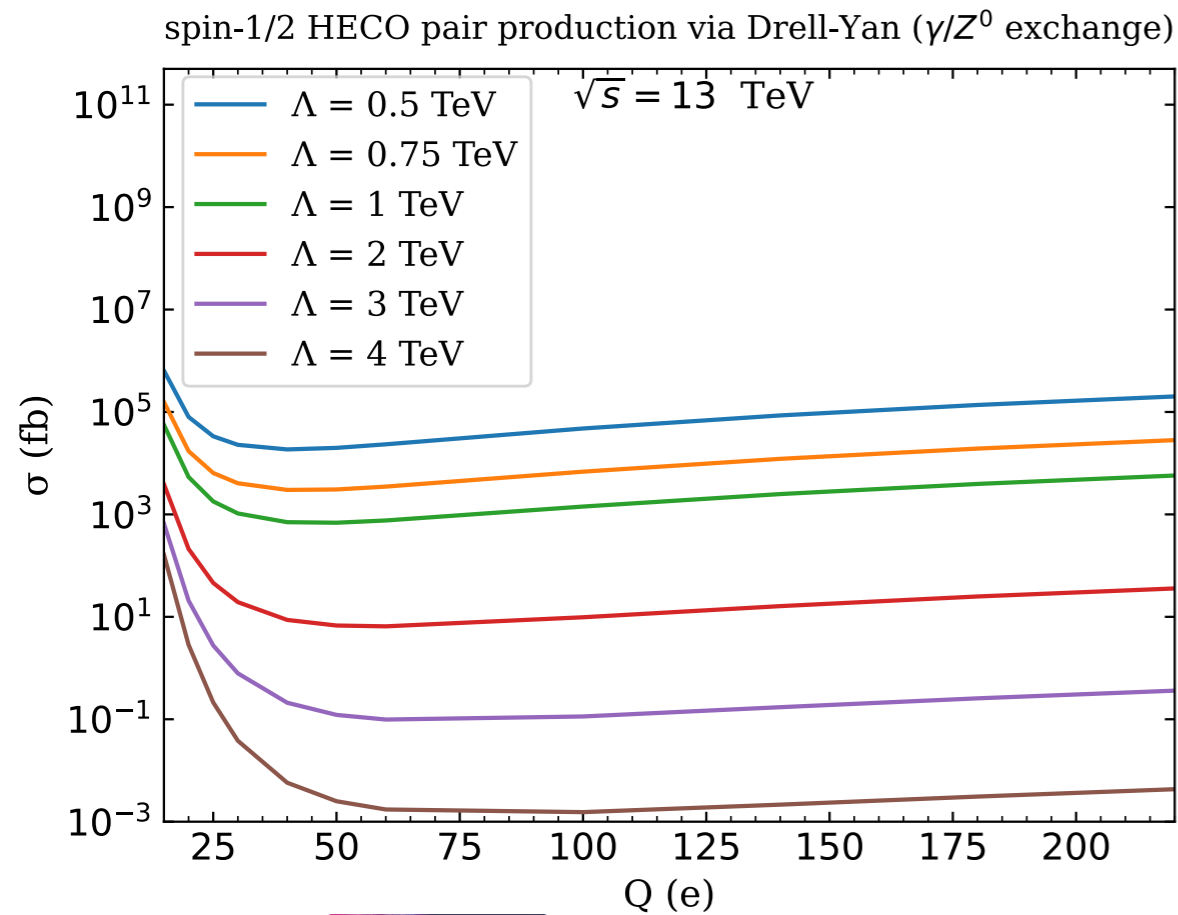
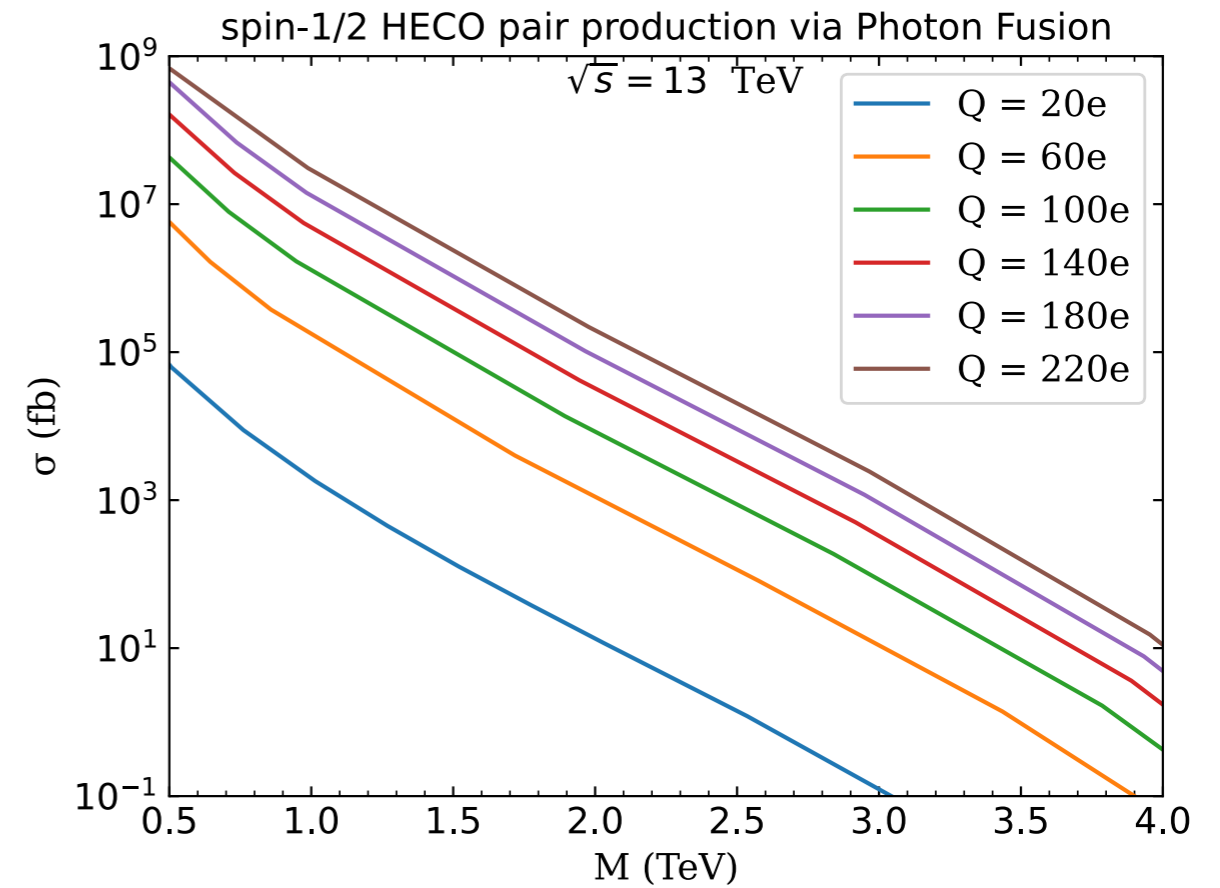
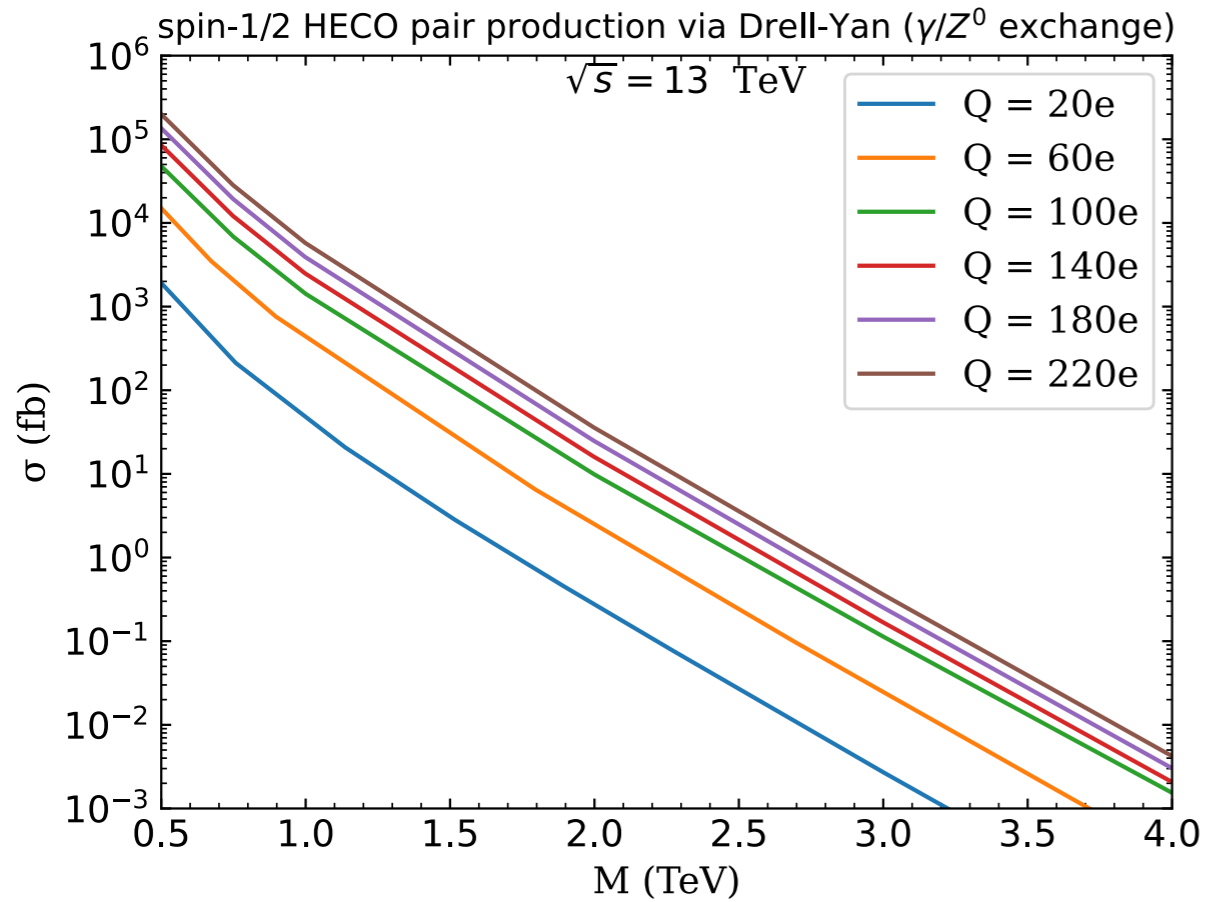
## SPIN-1/2



*After resummation...*

- the production via Drell-Yan cross section increases by a factor of  $\sim 1.66$
- the production via Photon-Fusion cross section increases by a factor of  $\sim 2.76$

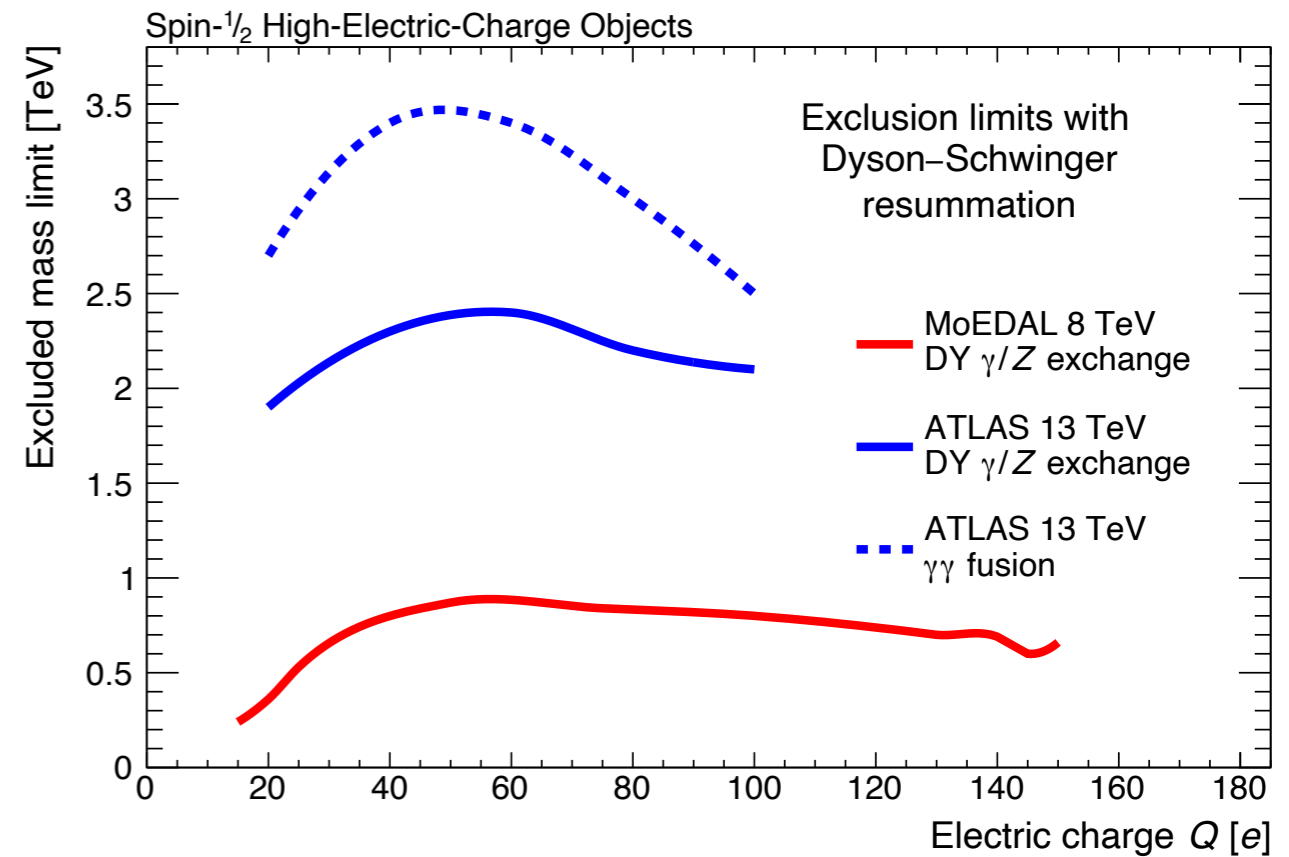
# IMPACT ON PRODUCTION CROSS SECTIONS



# NEW MASS LIMITS

Experimental lower limits at 95% CL on spin-1/2 HECO mass (TeV)

Experiment/ energy	$Q$ (e)	DY $\gamma$ exchange		DY $\gamma/Z^0$ exchange		$\gamma\gamma$ fusion	
		LO	DS	LO	DS	LO	DS
MoEDAL [14] $\sqrt{s} = 8$ TeV	15	0.18	0.24	0.17	0.24	–	–
	20	0.28	0.36	0.31	0.36	–	–
	25	0.44	0.55	0.44	0.53	–	–
	50	0.78	0.88	0.78	0.87	–	–
	75	0.78	0.88	0.78	0.84	–	–
	100	0.73	0.84	0.71	0.80	–	–
	125	0.66	0.75	0.64	0.72	–	–
	130	0.64	0.74	0.62	0.70	–	–
	140	0.58	0.68	0.62	0.69	–	–
	145	0.52	0.66	0.51	0.60	–	–
ATLAS [11] $\sqrt{s} = 8$ TeV	10	0.78	0.78 <sup>a</sup>	–	–	–	–
	20	1.05	1.14	–	–	–	–
	40	1.16	1.25	–	–	–	–
	60	1.07	1.15	–	–	–	–
ATLAS [12, 13] $\sqrt{s} = 13$ TeV	20	1.83	2.02	1.8	1.9	2.5	2.7
	40	2.05	2.22	2.2	2.3	3.1	3.4
	60	2.00	2.18	2.2	2.4	3.1	3.4
	80	1.86	2.02	2.1	2.2	3.0	3.0 <sup>b</sup>
	100	1.65	1.80	1.9	2.1	2.5	2.5 <sup>b</sup>



The increase in the mass limits spans a wide range of values up to 30%!!

**NOTE:** The latest [MoEDAL results](#) @ 13 TeV are not included here!



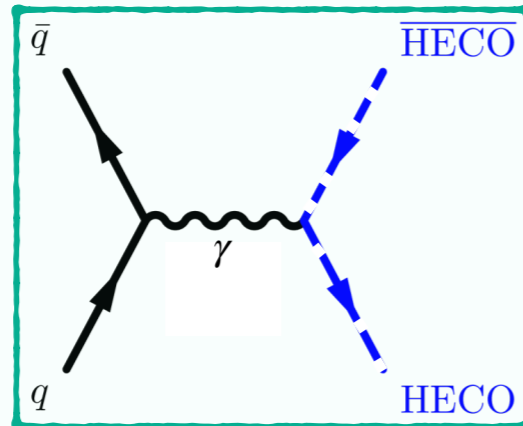
# High Electric Charge Objects

WORK IN PROGRESS

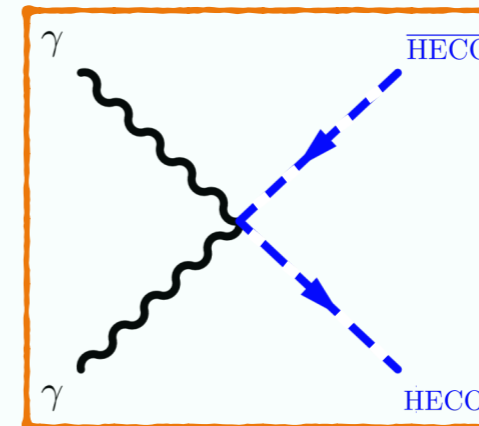
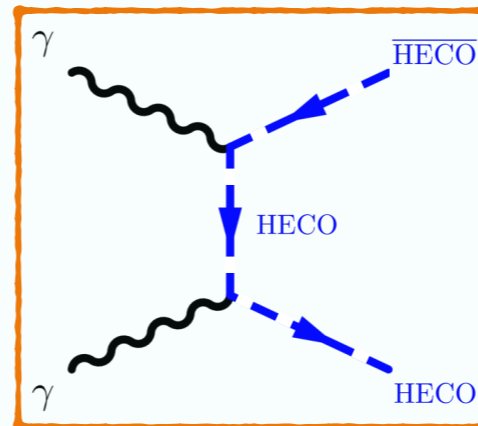
SPIN-0

## Production mechanisms at colliders

Drell-Yan



Photon-Fusion



## Feynman Rules @ UV fixed-point

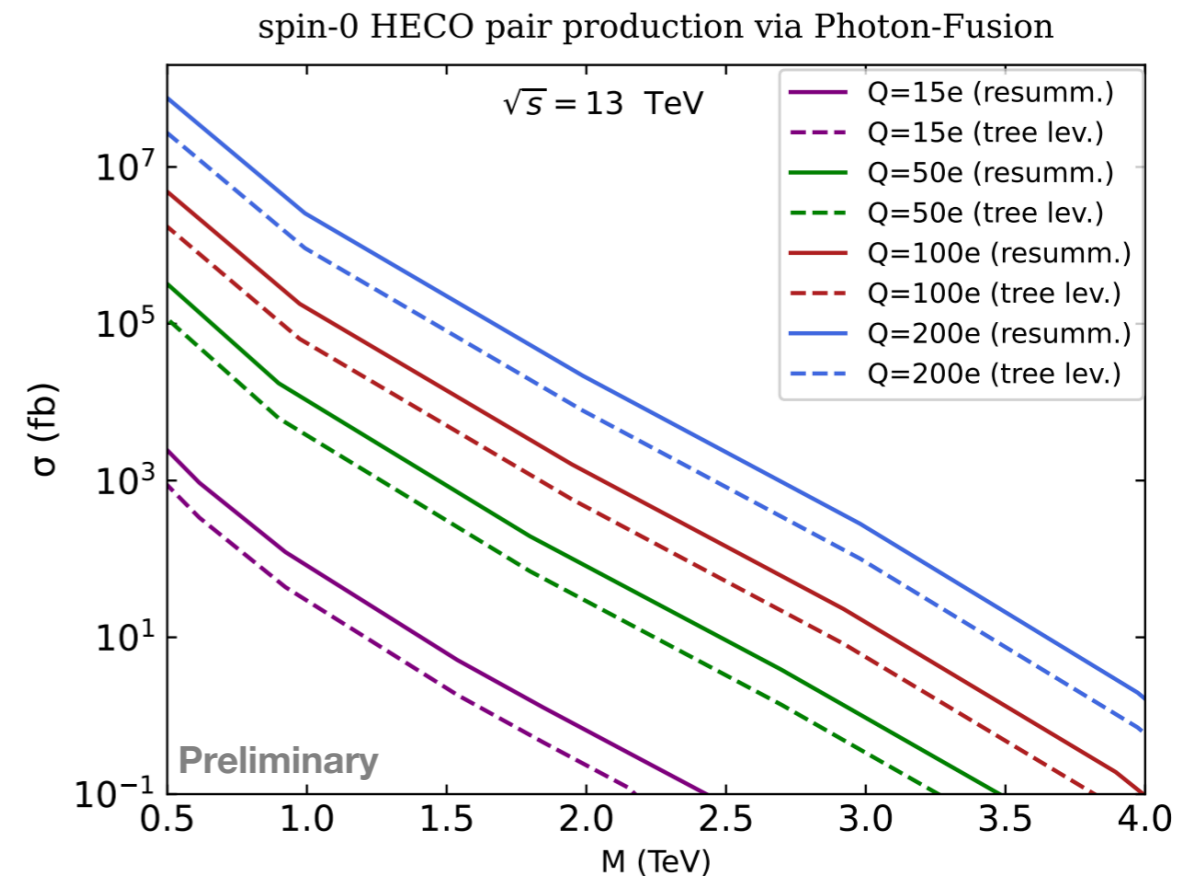
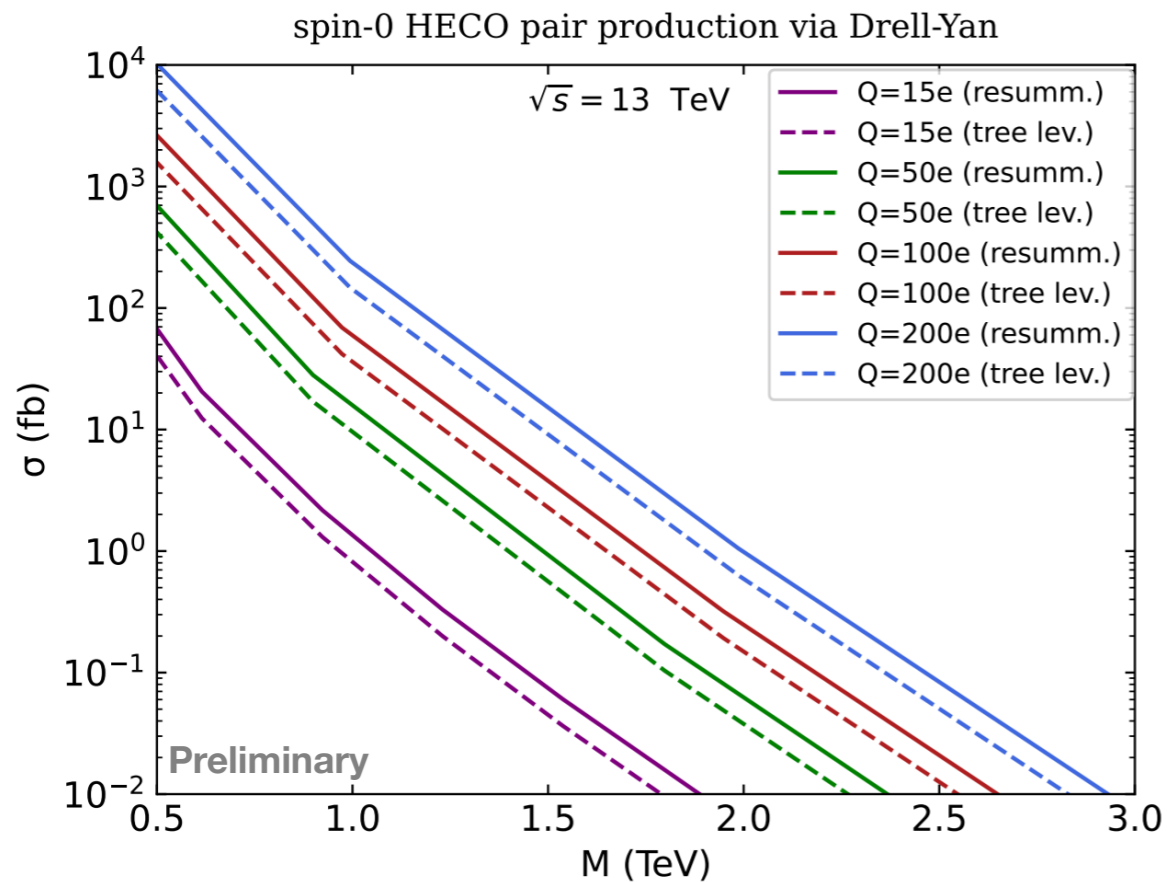
- ❖ **Scalar- $\gamma$  vertex:**  $-i\tilde{g}Z^* \simeq -ig\sqrt{1 + \frac{8g^2}{h}}, \left(\frac{g^2}{h} \lesssim 0.0825\right)$
- ❖ **Self-interaction vertex:**  $-i\frac{H^*}{Z^{*4}} \simeq -i\frac{h}{4}\left(1 + \frac{8g^2}{h}\right)^{-2}$
- ❖ **Running mass:**  $\tilde{M} \simeq \Lambda \exp\left(-\frac{32\pi^2}{h}\right) \lesssim \Lambda \exp\left(-\frac{2.64\pi^2}{n^2 e^2}\right)$
- ❖ **Gauge Boson propagator:**  $\frac{i}{p^2 - i\epsilon}\left(-\eta_{\mu\nu} + \frac{4g^2}{3h}\right), \epsilon \rightarrow 0^+$
- ❖ **Charged scalar propagator:**  $\frac{i}{p^2 - \tilde{M}^2 + i\epsilon}, \epsilon \rightarrow 0^+$

# High Electric Charge Objects

**WORK IN PROGRESS**

**SPIN-0**

- ✓ Implementation of the UFO model
- ✓ Calculation of the analytical cross section (Wolfram Mathematica)
- ✓ Comparison Madgraph vs Mathematica
- ✓ Studying resummation effects on cross-sections



After resummation...

- the production via Drell-Yan cross section improves by a factor of  $\sim 2.1$
- the production via Photon-Fusion cross section improves by a factor of  $\sim 4.75$

## CONCLUSIONS

- ❖ Dyson–Schwinger resummation performed
- ❖ UFO models available for experimental HECOs searches
- ❖ Reliable results compared to those obtained at tree-level
- ❖ Ongoing project:
  - ➔ resummation for *spin-0* HECOs (Alexandre, Mavromatos, Mitsou, Musumeci)
- ❖ Future project:
  - ➔ resum. for Magnetic Monopoles (based on Alexandre & Mavromatos *Phys.Rev.D 100(2019),9*)



Thank you  
for your  
attention! :)

# RESUMMATION

## Dyson-Schwinger Resummation

$$Z = 1 + \frac{g^2}{8\pi^2\lambda} \frac{1}{\epsilon} \left( \frac{Zk}{M} \right)^\epsilon + \text{finite}$$

$k$  is an energy scale

$$\omega = \frac{g^2}{6\pi^2 Z} \frac{1}{\epsilon} \left( \frac{Zk}{M} \right)^\epsilon + \text{finite}$$

$g$  and  $m$  are independent of  $k$

$$1 - \frac{m}{M} = \frac{g^2}{8\pi^2\lambda Z} \frac{1 + 3\lambda + \omega}{1 + \omega} \frac{1}{\epsilon} \left( \frac{Zk}{M} \right)^\epsilon + \text{finite}$$

The aim is to solve the full set of equations without assuming  $g^2 \ll 1$

Boundary conditions:

$$(Z, \omega, M) = (1, 0, m) \quad \text{when} \quad k = m \quad \Longrightarrow$$

$$\begin{aligned} Z &= 1 + \frac{g^2}{8\pi^2\lambda} \ln \left( \frac{Zk}{M} \right) \\ Z\omega &= \frac{g^2}{6\pi^2} \ln \left( \frac{Zk}{M} \right) \\ Z \left( 1 - \frac{m}{M} \right) &= \frac{g^2}{8\pi^2\lambda} \frac{1 + 3\lambda + \omega}{1 + \omega} \ln \left( \frac{Zk}{M} \right) \end{aligned}$$

Usual one-loop results for  $g^2 \ll 1$

Feynman gauge  $\lambda=1$

$$\lim_{k \rightarrow \infty} \frac{kZ}{M} = \text{finite}$$

$$\begin{aligned} Z &= 1 + \frac{g^2}{8\pi^2} \ln \left( \frac{Zk}{M} \right) \\ \omega &= \frac{4}{3} \left( 1 - \frac{1}{Z} \right) \\ 1 - \frac{m}{M} &= 4 \frac{4Z - 1}{7Z - 4} \left( 1 - \frac{1}{Z} \right) \end{aligned}$$

$$\lim_{k \rightarrow \infty} (Z, \omega, \tilde{M}) = (Z^*, \omega^*, \tilde{M}^*)$$

**UV Fixed-point**

$$\frac{k}{m} = \frac{(7Z - 4)}{9(Z_+ - Z)(Z - Z_-)} \exp \left( \frac{8\pi^2}{g^2} (Z - 1) \right)$$

$$Z_{\pm} = \frac{8}{9} \left( 1 \pm \frac{\sqrt{7}}{4} \right) \simeq \begin{cases} 1.477 = Z^* \\ 0.301 \end{cases}$$

$$\omega \rightarrow \omega^* = \frac{4}{3} \left( 1 - \frac{1}{Z^*} \right) \simeq 0.431,$$

$$\tilde{M} \equiv \frac{M}{k} \rightarrow Z^* \exp \left( -\frac{8\pi^2}{g^2} (Z^* - 1) \right)$$

# $Z^0$ boson inclusion

## SPIN-1/2

$$\mathcal{L}_{\text{int}} = -\frac{e}{\sin(2\theta_W)} \bar{H} \gamma^\mu Z_\mu^0 \left( c_L \frac{1}{2} (1 - \gamma^5) + c_R \frac{1}{2} (1 + \gamma^5) \right) H = -\frac{e}{\sin(2\theta_W)} \bar{H} \gamma^\mu Z_\mu^0 \left[ \frac{1}{2} (c_L + c_R) - \frac{1}{2} (c_L - c_R) \gamma^5 \right] H$$

with

$$c_L = t^3 - |n| \sin^2 \theta_W \quad c_R = -|n| \sin^2 \theta_W$$

SU(2) singlet

$$\mathcal{L}_{\text{int}} = \frac{1}{2} e |n| \tan \theta_W \bar{H} \gamma^\mu Z_\mu^0 H$$

In the Feynman gauge, in the high energy limit, the  $Z^0$  boson behaves as a second photon

$$\Delta_{\mu\nu}^{Z^0} = -\frac{i}{p^2 - M_Z^2 + i\epsilon} \left( \eta_{\mu\nu} + \frac{\omega_{Z^0}}{1 + \omega_{Z^0}} \frac{p_\mu p_\nu}{M_Z^2} \right), \quad \epsilon \rightarrow 0^+$$

In the unitary gauge, for  $M_Z \neq 0$ , leads to the standard massive (Proca)  $Z^0$  boson bare propagator

$$\Delta_{\mu\nu}^{Z^0} = -\frac{i}{p^2 - M_Z^2 + i\epsilon} \left( \eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2} \right) - \frac{p_\mu p_\nu}{M_Z^2} \frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon}, \quad \epsilon \rightarrow 0^+$$