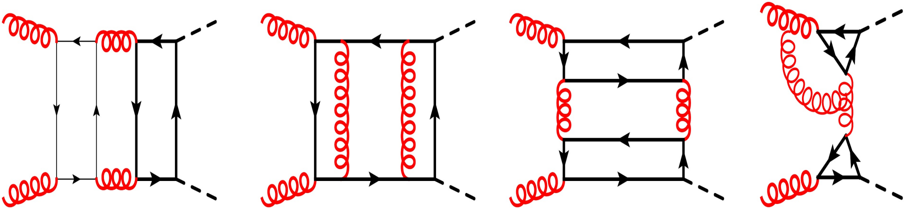
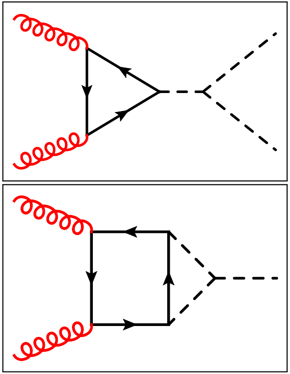
Towards NNLO QCD corrections to Higgs boson pair production

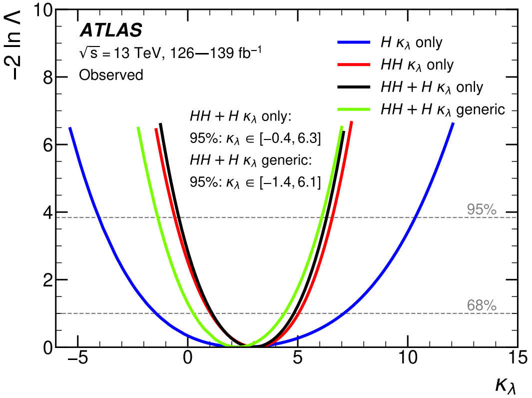
ICHEP, Parague, July 18, 2024

Matthias Steinhauser | TTP KIT



λ from H and HH production



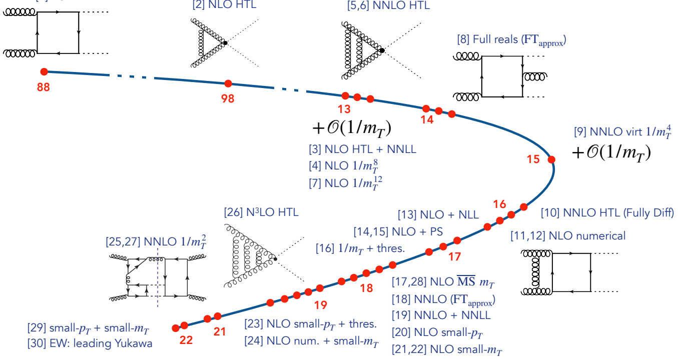


SM: $\lambda_{\rm HHH}^{\rm SM} = \frac{m_H^2}{2v^2} \approx 0.13$

$\kappa_\lambda = \frac{\lambda_{\rm HHH}}{\lambda_{\rm HHH}^{\rm SM}}$

HH: Theory History

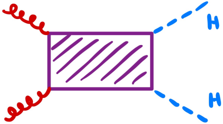
(Slide design inspired by: G. Salam)



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22;

[slide from Stephen Jones]

Analytic expansions



(s, t, m_t, m_H)

- Numeric calculations [pySecDec, Heinrich,...]
- Analytic calculations [Duhr,...,Tancredi,...]
- Analytic expansions:
  - large-mass expansion: $m_t^2 \gg s, t, \ldots$
    "simple": vacuum integrals and massless integrals, known
  - high energy: $m_t^2 \ll s, t, \ldots$
    involved asymptotic expansion
    complicated MIs
  - $t \to 0$ ($p_T \to 0$)
    (often) Taylor expansion

High energy expansion and Padé improvement

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$$\mathcal{A} = \sum_{j}^{N} \sum_{i,k} c_{ijk} \epsilon^i \left(\frac{m_t}{\sqrt{s}} \right)^j \log^k \left(\frac{m_t^2}{s} \right)$$

$N = \mathcal{O}(100)$

$$\sum_{k=0}^{N} c_k m_t^k \quad \to \quad \frac{a_0 + \cdots + a_r {m_t}^r}{1 + b_1 + \cdots + b_s m_t^s} \qquad r + s = N$$

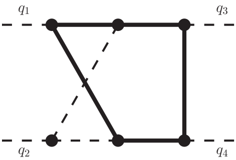
➪ For each phase-space point $(\sqrt{s}, p_T)$:
prediction of central value and corresponding uncertainty

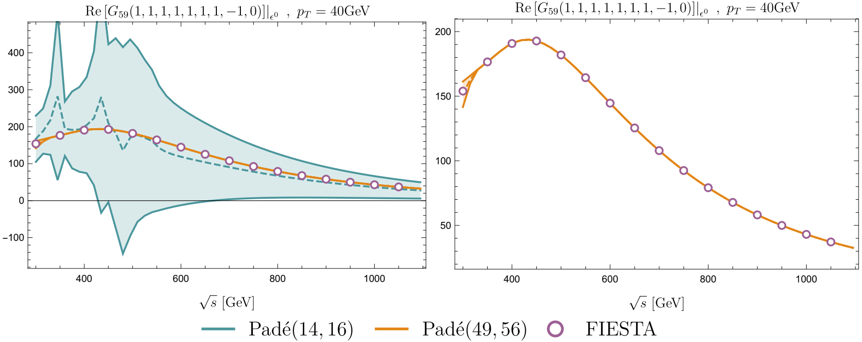
$p_T^2 = (tu - m_H^4)/s, s + t + u = 2m_H^2$

High energy expansion ⊕ PA

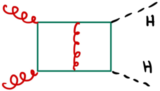
- expansion up to $(m_t^2)^{56}$
- construct PAs with input for $(N_{\min}, N_{\max})$
  (for each phase-space point)

PA is a precision tool

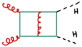




$t \to 0$ expansion



[Bonciani,Degrassi,Giardino,Gröber'18]
[Bellafronte,Degrassi,Giardino,Gröber,Vitti'22; ...]
[Davies,Mishima,Schönwald,Steinhauser'23]

- forward scattering kinematics
- Taylor expansion in t ➪  $= f(s/m_t^2)$
- compute $f(s/m_t^2)$ with "expand and match" [Fael,Lange,Schönwald,Steinhauser'21'22]

“Expand and match” [Fael,Lange,Schönwald,Steinhauser’21’22]

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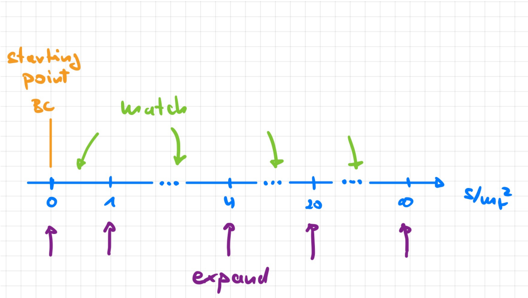
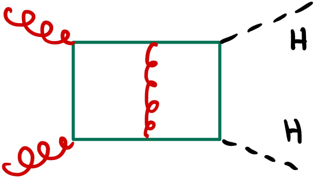
Consruct overlapping expansions using differential equations for the master integrals



Expansion of (unknown) function $f(s/m_t^2)$ around properly chosen s/m_t^2 values with precise numerical coefficients

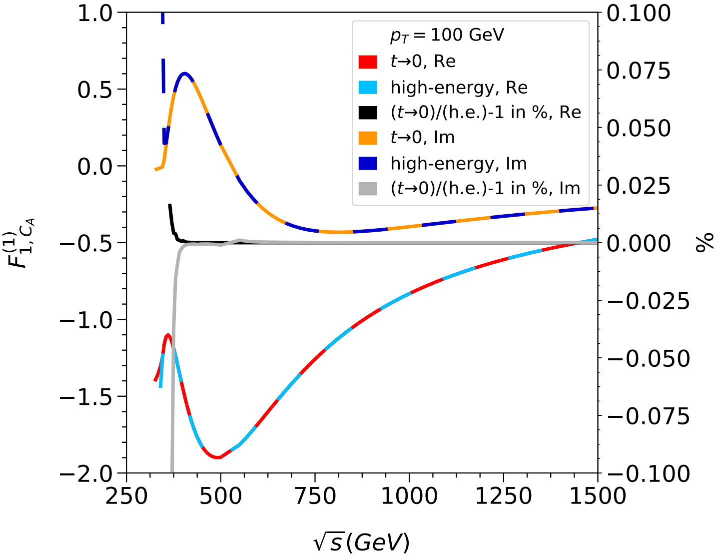
Similar approaches: [Blümlein,Czakon,Hidding,Laporta,Lee,Liu,Smirnov,...]

Back to $gg \to hh$ @ NLO



$$\left\{ \begin{array}{l} \text{high-energy expansion} \\ t \to 0 \text{ expansion} \end{array} \right.$$

Combine: $t \to 0$ and h.e. at 2 loops



[Davies,Mishima,Schönwald,Steinhauser'23]

$\mathcal{V}_{\text{fin}}$: virtual NLO QCD corrections

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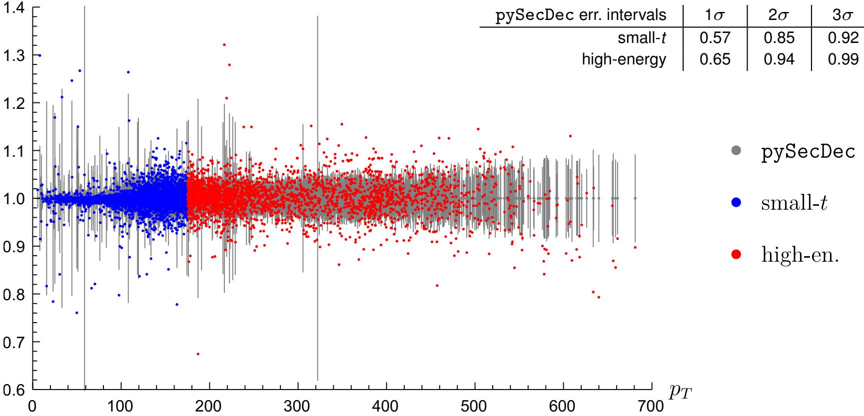
Comparison to “pySecDec”

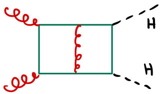
pySecDec err. intervals	1σ	2σ	3σ
small-t	0.57	0.85	0.92
high-energy	0.65	0.94	0.99



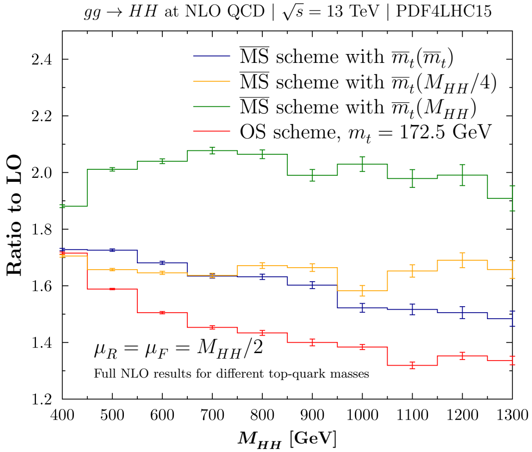
https://github.com/mppmu/hhgrid

[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke’16]

$gg \to HH$ at NLO

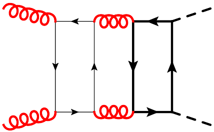


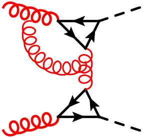
[Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira'20]
[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]

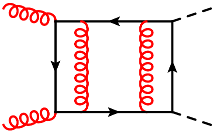


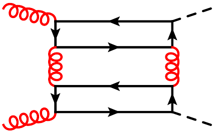
➪ Need for NNLO

Can we go to 3 loops?









[Davies,Schönwald,Steinhauser'23]

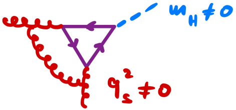
[Davies,Schönwald,
Steinhauser,Vitti'24]

in progress

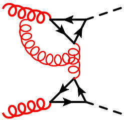
in progress

NNLO 1PR

off-shell $gg^\star H$ vertex at 2 loops



➪



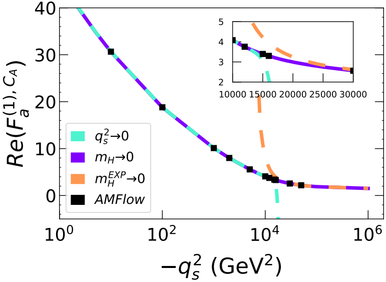
2 asymptotic expansions

1. $m_H^2 \ll q_s^2, m_t^2$
2. $q_s^2 \ll m_H^2, m_t^2$

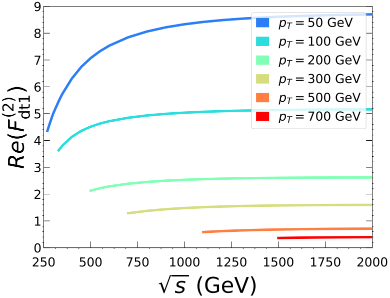
➪ cover whole PS for

$$\{s, t; m_H, m_t\}$$

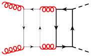
$gg^\star H$ form factor

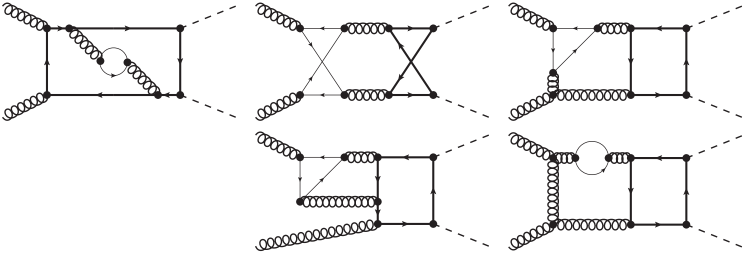


$gg \to HH$ form factor



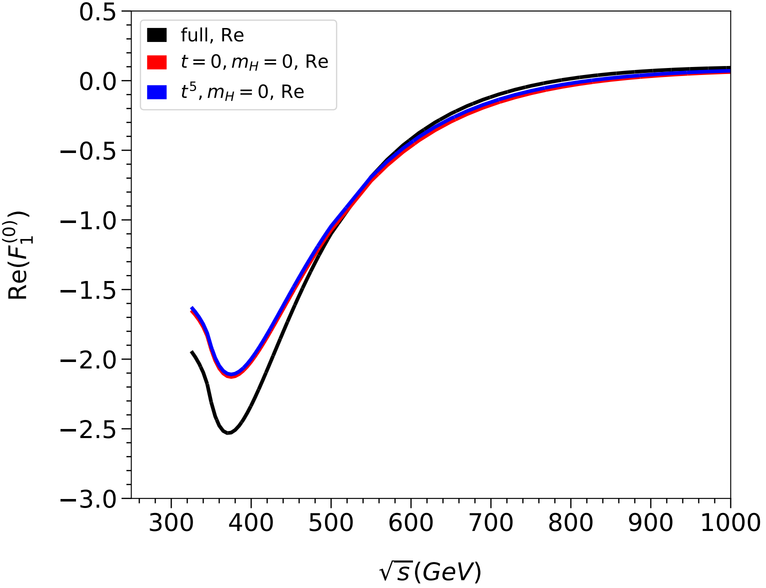
[Davies,Schönwald,Steinhauser,Vitti'24]

Fermionic corrections: 



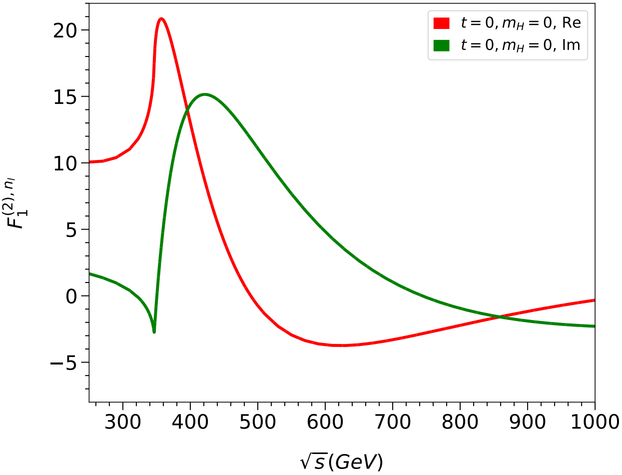
- $t = 0$, $m_H = 0$
- 176 MIs Kira: [Klappert,Lange,Maierhöfer,Usovitsch'20]
- useful software: `tapir` [Gerlach,Herren,Lang'23] `LiteRed` [Lee], `LIMIT` [Herren], `Feynson` [Magerya]
- reduction to MIs: about 1 week for most complicated family

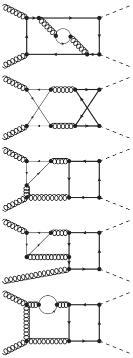
1-loop result, $p_T = 100$ GeV



3-loop n_l for $t = 0, m_H = 0$

[Davies,Schönwald,Steinhauser'23]

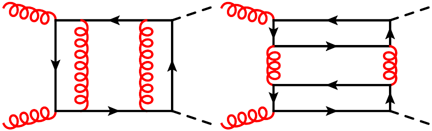




At threshold: NLO: $v \log v, v^2 \log v, v^3 \log^2 v, \ldots$

$v = \sqrt{1 - 4m_t^2/s}$ NNLO: $v \log^2 v, v^2 \log^2 v, v^3 \log^3 v, \ldots$

Challenges beyond the fermionic corrections



- 203 integral families
- IBP reduction
  - hardest job: 41 days, $>$ 2 TB RAM; took several attempts
  - 33.000 MIs accross all families
- Minimization of MIs
  - cannot be done with `Kira`
  - Apply `FIRE`'s `FindRule` to all 2.600.000 input integrals
    + additional test reductions with `FIRE`
    ➪ 1561 MIs
- Solve differential equation system
  BC: large-m_t limit ✓
- cut in t-channel

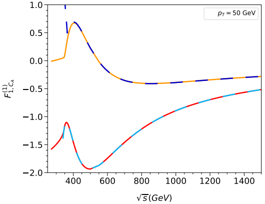
Conclusions

- Higgs boson pair production:
  experimentally challenging
  theoretically complicated
- Analytic expansions:
  Combine large-m_t, high-energy, $t \to 0$ expansions
  Analytic and semi-analytic expressions ➪ fast and flexible
- QCD: 2 loops ✓
  3 loop $gg \to HH$: light-fermion and 1-particle reducible ✓
- Apply techniques to $gg \to ZH$, $gg \to ZZ$, $gg \to Hg$, ...
  (with massive m_t, m_H, m_Z)

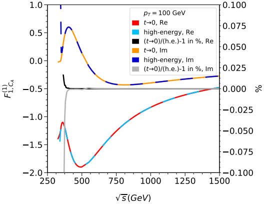
Backup

Combine: $t \to 0$ and h.e. at 2 loops

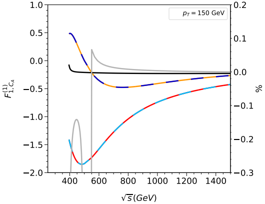
$p_T = 50$ GeV

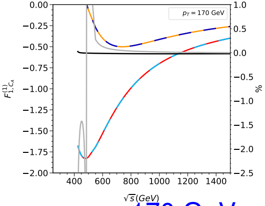


$p_T = 100$ GeV

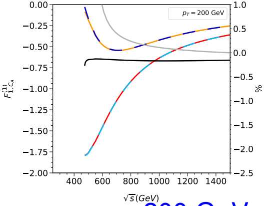


$p_T = 150$ GeV

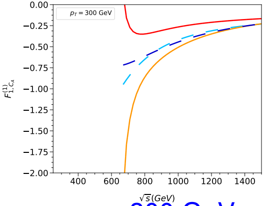




$p_T = 170$ GeV



$p_T = 200$ GeV



$p_T = 300$ GeV

[Davies,Mishima,Schönwald,Steinhauser'23]