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New observables for testing Bell inequalities in W boson pair production

Based on Phys. Rev. D **109**, 036022
in collaboration with Qi Bi, Kun Cheng and Qing-Hong Cao

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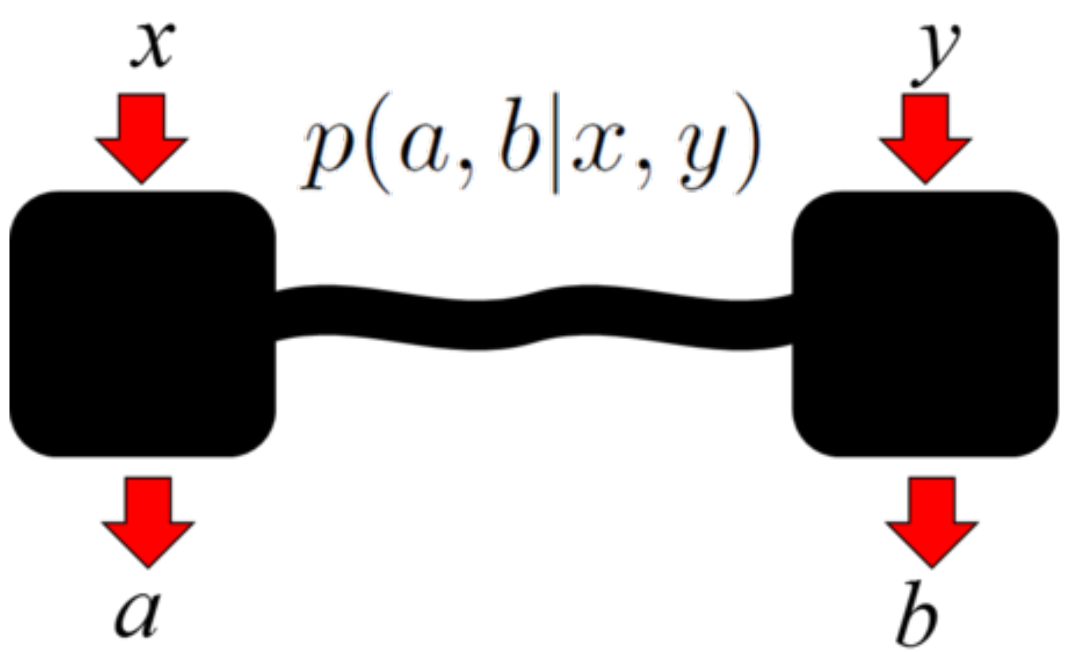
18-24 July · 2024 · Prague · Czech Republic

The different correlations

- A question of decomposition

"Alice"



"Bob"

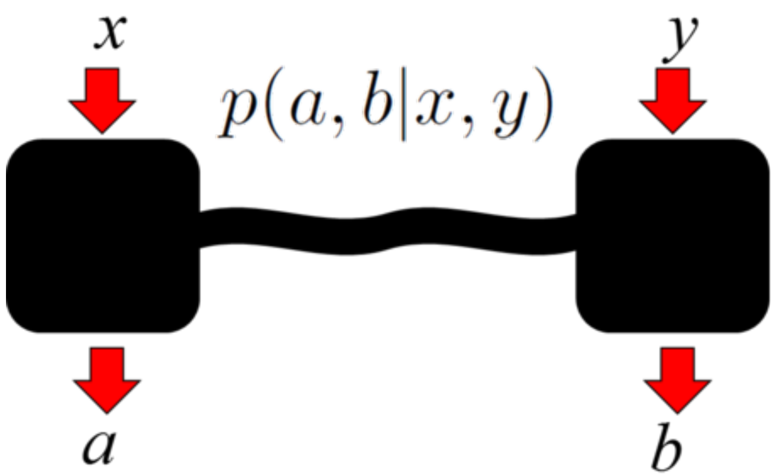
No-signaling correlations	**Local correlations**	**Quantum correlations**

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

The different correlations

"Alice"

No-signaling correlations



"Bob"

$$\sum_{b=1}^{\Delta} p(ab|xy) = \sum_{b=1}^{\Delta} p(ab|xy'), \quad \text{for all } \; a, x, y, y',$$

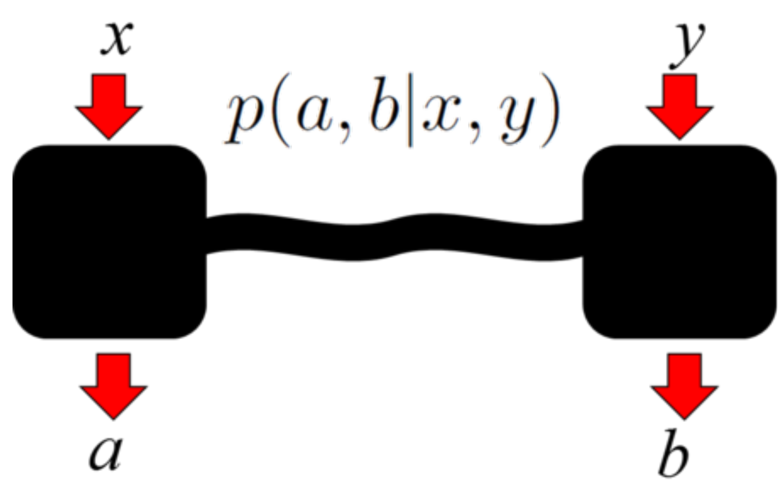
$$\sum_{a=1}^{\Delta} p(ab|xy) = \sum_{a=1}^{\Delta} p(ab|x'y), \quad \text{for all } \; b, y, x, x'.$$

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

The different correlations

“Alice”

Local correlations



“Bob”

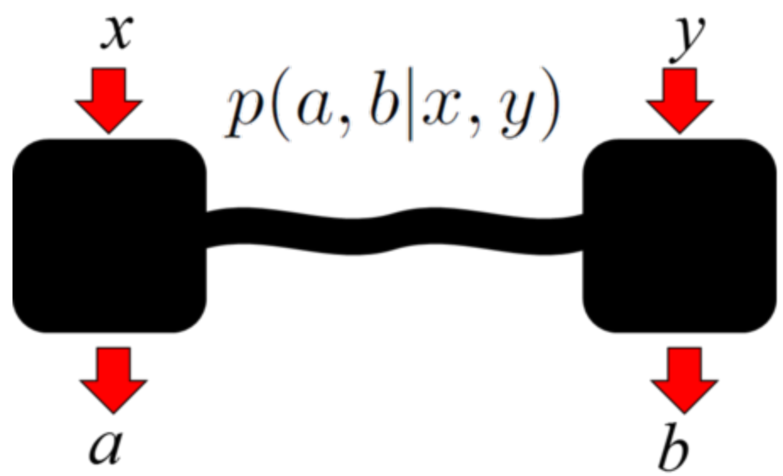
$$p(ab|xy) = \int_\Lambda d\lambda q(\lambda) p(a|x,\lambda) p(b|y,\lambda),$$

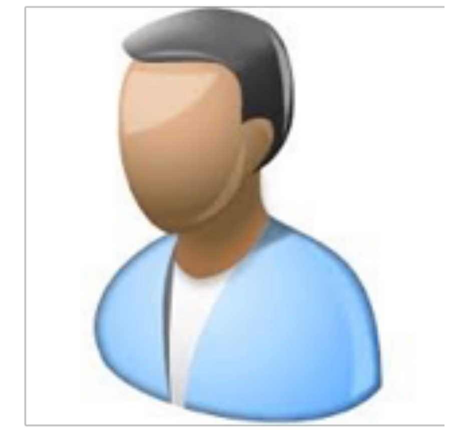
N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

The different correlations

Quantum correlations

“Alice”





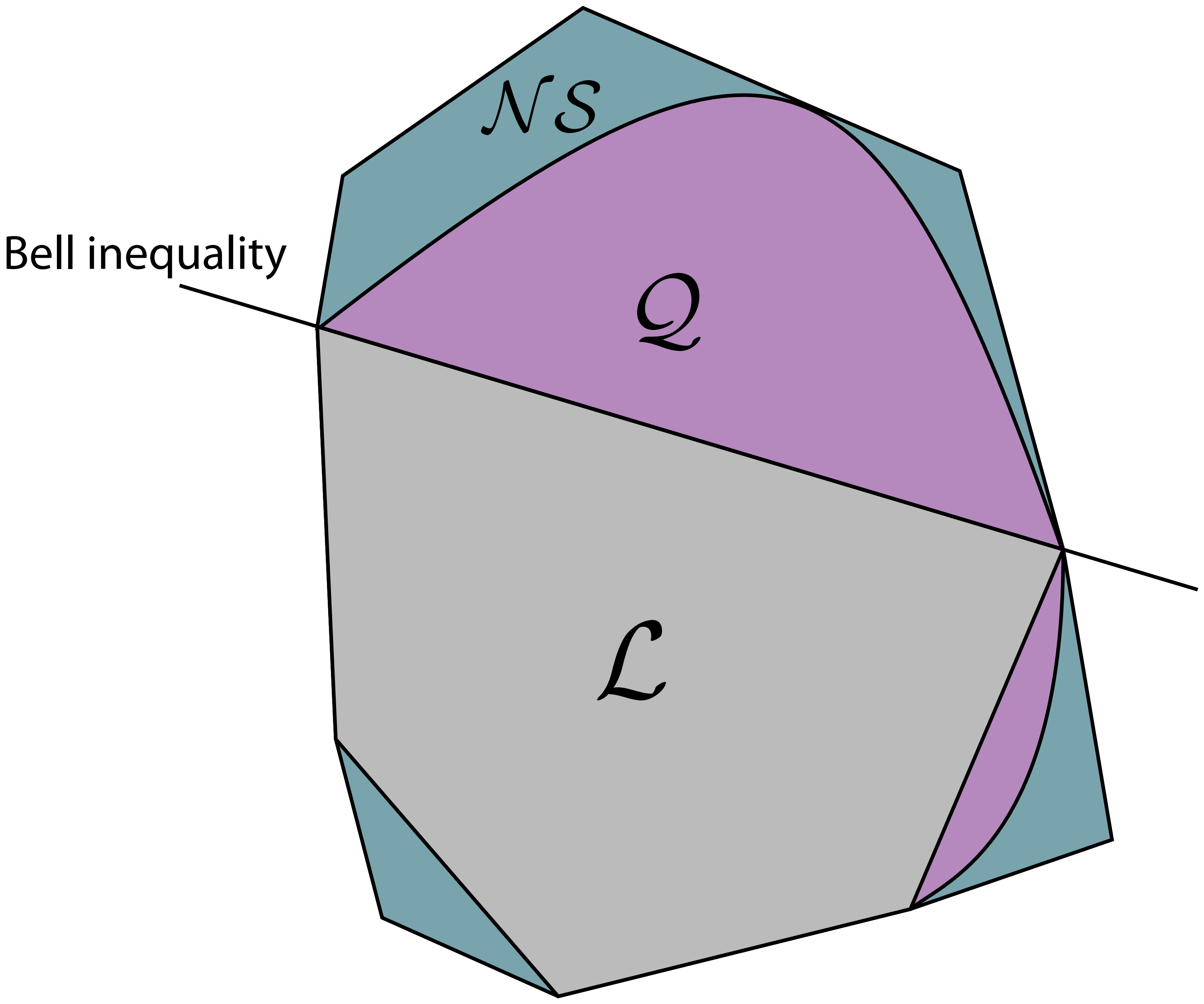
“Bob”

$$p(ab|xy) = \mathrm{tr}\left(\rho_{AB} M_{a|x} \otimes M_{b|y}\right),$$

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

The different correlations

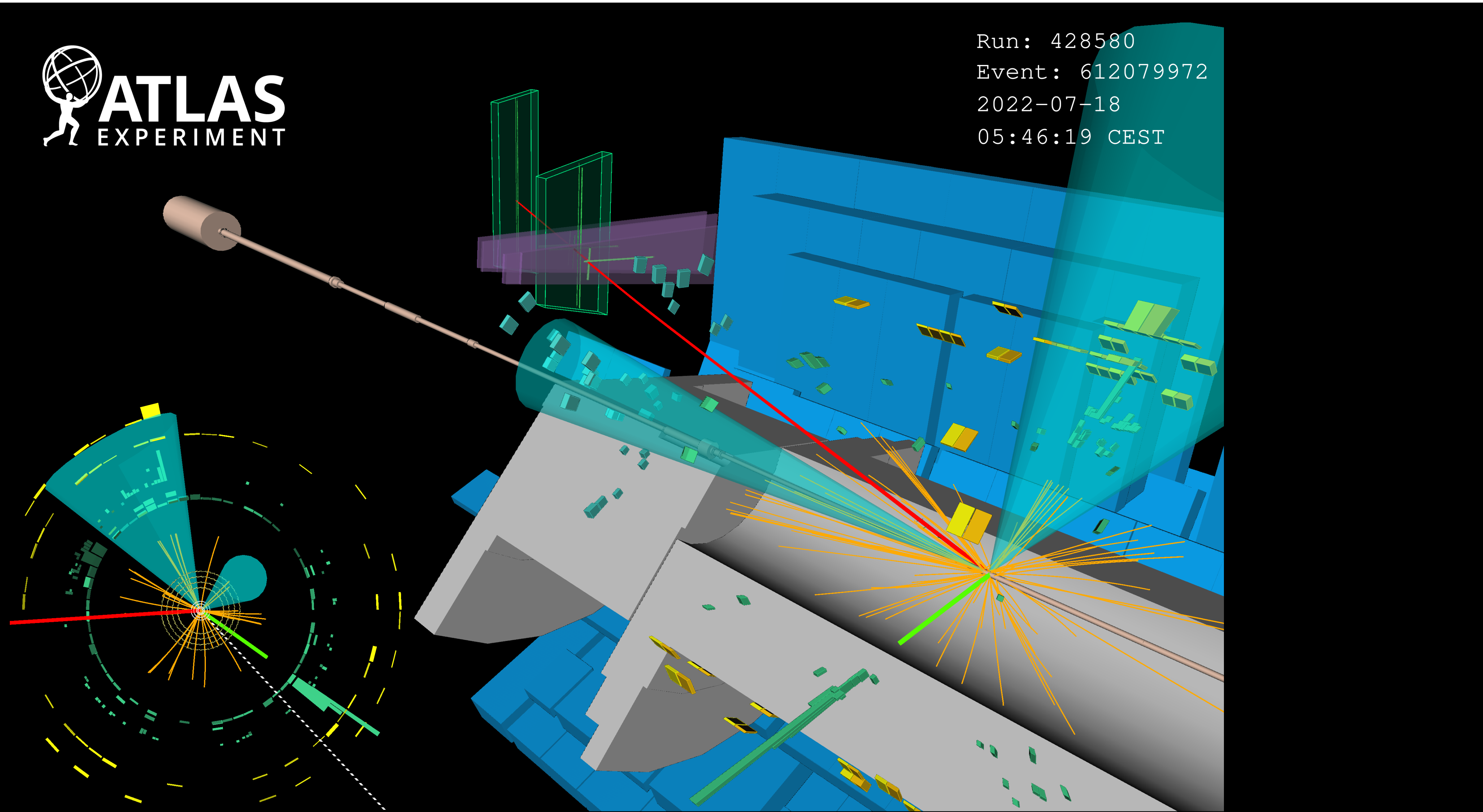
A visualization of the relation between these three kinds of correlation



N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

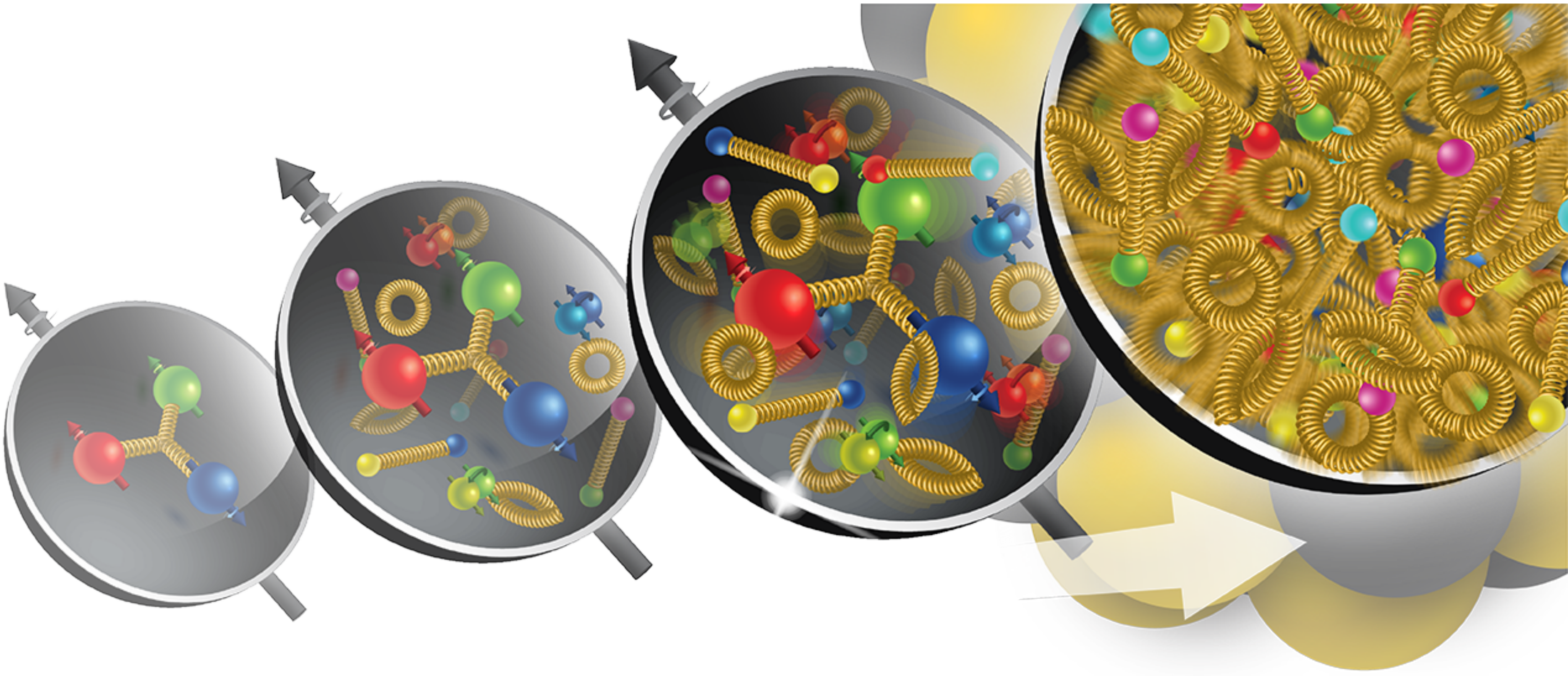
The Verification in EW scale

- The most popular topic: $t\bar{t}$ production at the LHC.
- Why?



The Verification at the EW scale

- It is not easy, why?



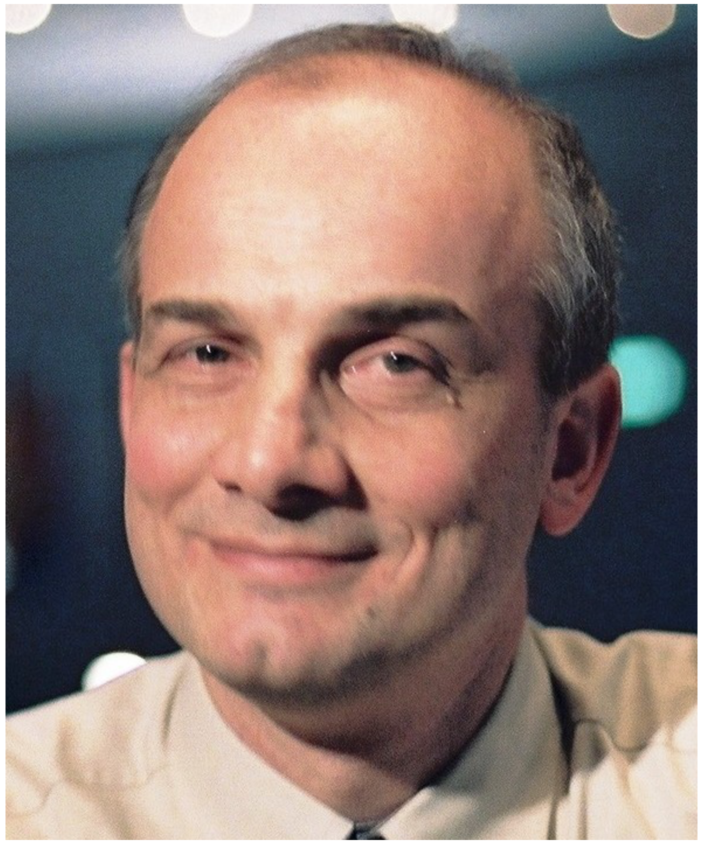
The Verification at the EW scale

"A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique!"

$$\rho_{AB} \overset{?}{=} \sum_{i=1}^{N} p_i \rho_A^{(i)} \otimes \rho_B^{(i)}, \quad \left(\sum_{i}^{N} p_i = 1,\, p_i > 0 \right)$$

"Finally, we prove that the weak membership problem for the convex set of separable normalized bipartite density matrices is ***NP-HARD***."

——Leonid Gurvits

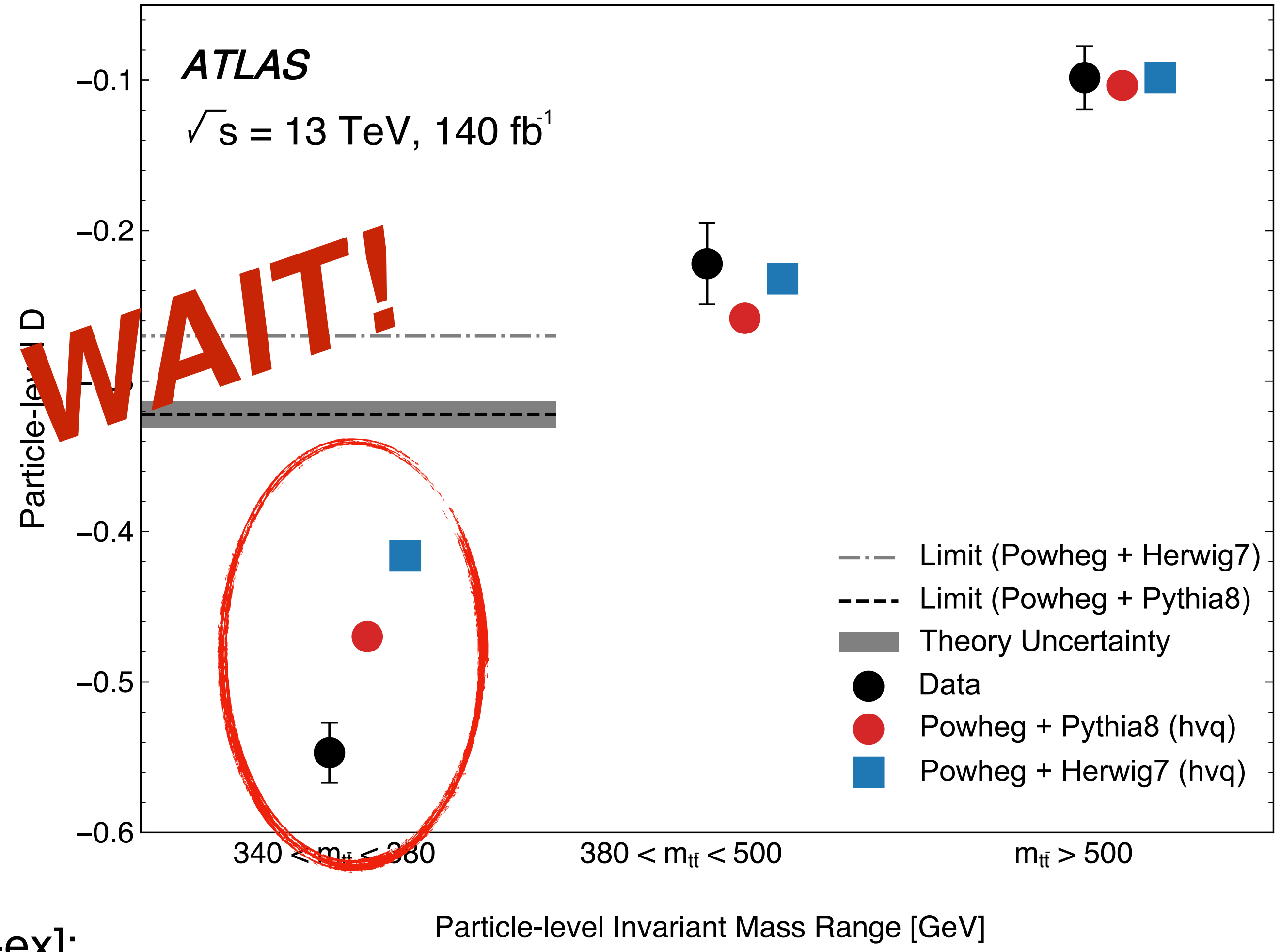


L. Gurvits, *"Classical Deterministic Complexity of Edmonds' Problem and Quantum Entanglement"*, 2003

The Verification at the EW scale

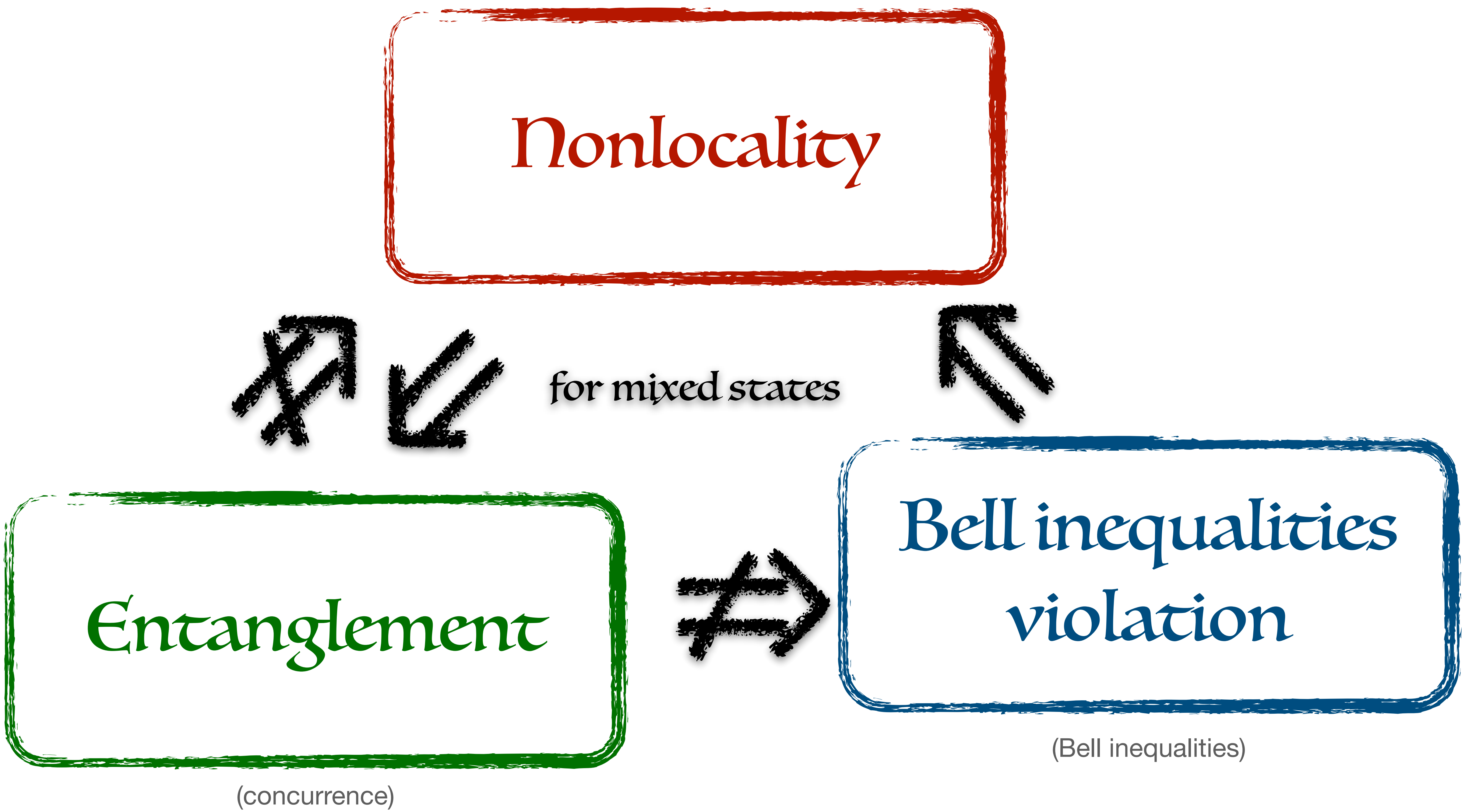
- For 2×2 and 2×3 system, it is solved by Horodecki et al 1995 (concurrence).
- The result from the ATLAS collaboration.

$$D \equiv -3\langle \cos\varphi(\ell_t^+ \ell_{\bar{t}}^-)\rangle$$



ATLAS Collaboration, arXiv:2311.07288[hep-ex];
Y. Afik and J. R. M. de Nova, Eur. Phys. J. Plus **136** (2021) 907.

The Verification at the EW scale



WW production at Higgs factory

- The initial state is a mixed state

➡ (Generalized) Bell inequality as a test

Observables: $\hat{A}_1$ (Alice 1), $\hat{A}_2$ (Alice 2), $\hat{B}_1$ (Bob 1), $\hat{B}_2$ (Bob 2)

The results of measurement $\in \mathbb{Z}_3$

$$\max_{\hat{A}_1,\hat{A}_2,\hat{B}_1,\hat{B}_2} \mathcal{I}_3(\hat{A}_1, \hat{A}_2; \hat{B}_1, \hat{B}_2) > 2$$

$$\begin{aligned}\mathcal{I}_3 \equiv +&\left[P\left(A_1 = B_1\right) + P\left(B_1 = A_2 + 1\right)\right.\\ &\left.+ P\left(A_2 = B_2\right) + P\left(B_2 = A_1\right)\right]\\ -&\left[P\left(A_1 = B_1 - 1\right) + P\left(B_1 = A_2\right)\right.\\ &\left.+ P\left(A_2 = B_2 - 1\right) + P\left(B_2 = A_1 - 1\right)\right]\end{aligned}$$

Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality

D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

WW production at Higgs factory

- The initial state is a mixed state
  - ➡ (Generalized) Bell inequality
- 9-dim but not 4-dim Hilbert space.

3x3>2x2

- QuNit vs. qubit?
  - ➡ “the results for large N are shown to be more resistant to noise with a suitable choice of the observables”

D. Kaszlikowski, P. Gnaciński, M. Żukowski, W. Miklaszewski, A. Zeilinger, Phys. Rev. Lett. **85**, 4418 (2000);
T. Durt, D. Kaszlikowski, M. Żukowski, Phys. Rev. A **64**, 024101 (2001);
J.-L. Chen, D. Kaszlikowski, L. C. Kwek, C. H. Oh, M. Żukowski, Phys. Rev. A **64**, 052109 (2001);
D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

WW production at Higgs factory

- The density matrix (some technical details…)

$$\hat{\rho}_{WW} \propto \mathcal{M}(e^+e^- \to W^+W^-)\hat{\rho}_{e^+e^-}\mathcal{M}(e^+e^- \to W^+W^-)^\dagger$$

$$\hat{\rho}_W = \frac{1}{3}\hat{I}_3 + d^i\hat{S}_i + q^{ij}\hat{S}_{\{ij\}}, \ \ i,j = 1,2,3$$

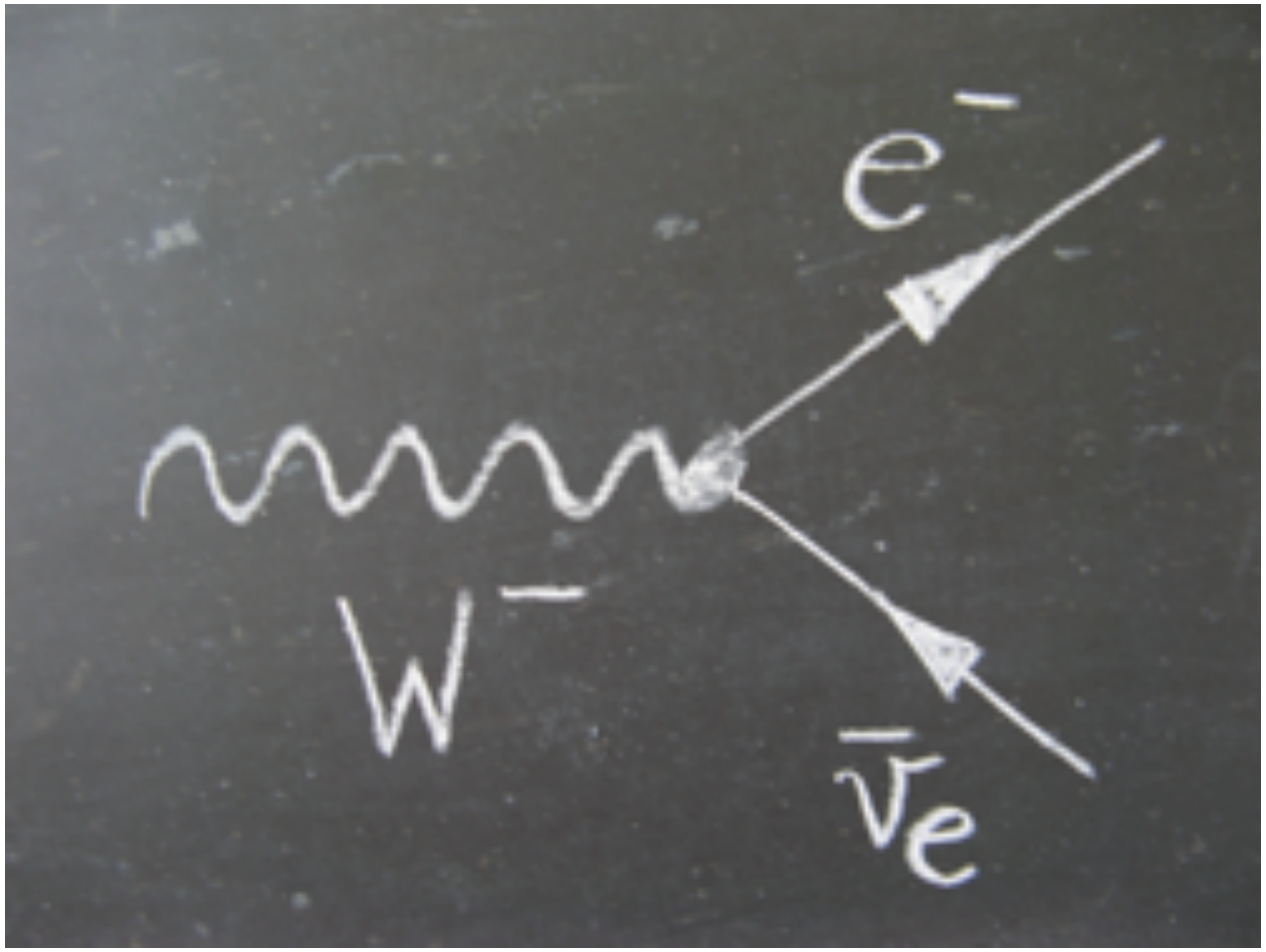
$$\begin{aligned}\hat{\rho}_{WW} \ = \ & \frac{1}{9}\hat{I}_9 + \frac{1}{3}d^i_+\hat{S}^+_i \otimes \hat{I}_3 + \frac{1}{3}d^i_-\hat{I}_3 \otimes \hat{S}^-_i \\ & + \frac{1}{3}q^{ij}_+\hat{S}^+_{\{ij\}} \otimes \hat{I}_3 + \frac{1}{3}q^{ij}_-\hat{I}_3 \otimes \hat{S}^-_{\{ij\}} \\ & + C^{ij}_d\hat{S}^+_i \otimes \hat{S}^-_j + C^{i,jk}_{d,q}\hat{S}^+_i \otimes \hat{S}^-_{\{jk\}} \\ & + C^{ij,k}_{q,d}\hat{S}^+_{\{ij\}} \otimes \hat{S}^-_k + C^{ij,k\ell}_q\hat{S}^+_{\{ij\}} \otimes \hat{S}^-_{\{k\ell\}}\end{aligned}$$

WW production at Higgs factory

- The density matrix (some technical details…) is "easy" to calculate
- $\beta \to 0$?
- *s*-channel is *p*-wave and suppressed by a factor of β.
- *t*-channel is purely left-handed current so that the initial state is selected by the (weak) interaction to be a pure state $|e_L^-\rangle \otimes |e_L^+\rangle$.

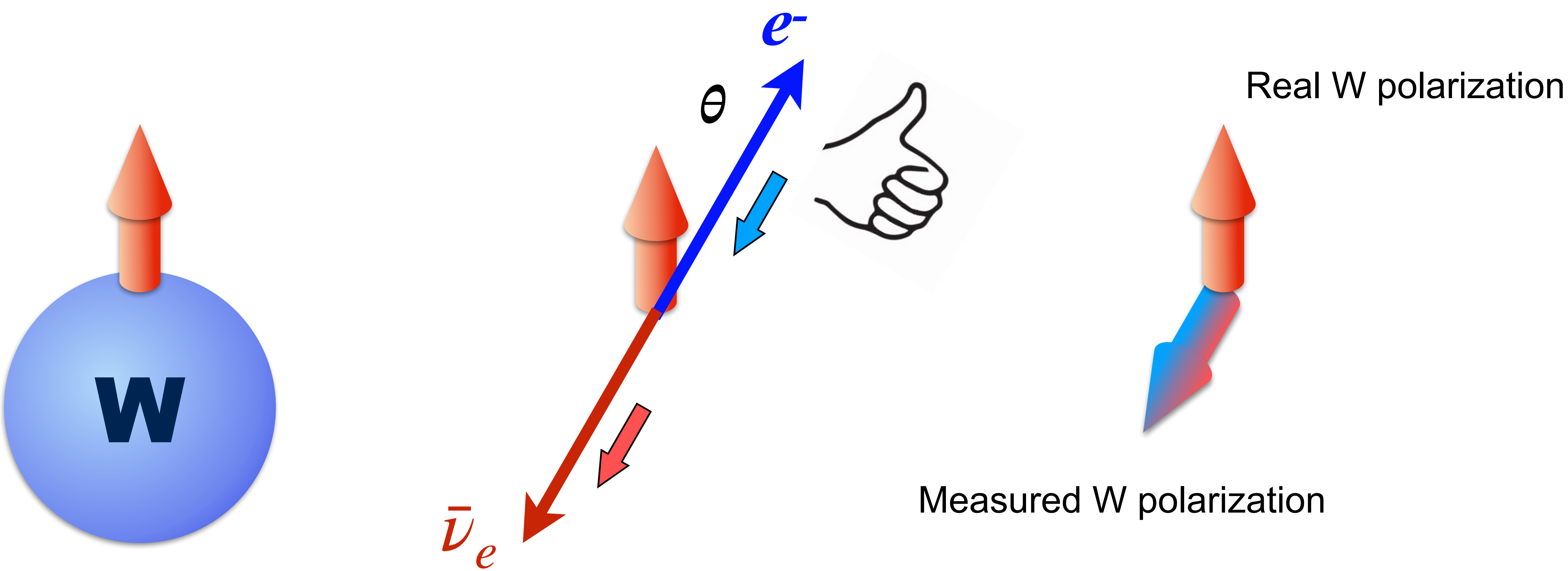
WW production at Higgs factory

- How to measure it at Higgs factory???
- “Measuring” the polarization direction of the W boson.



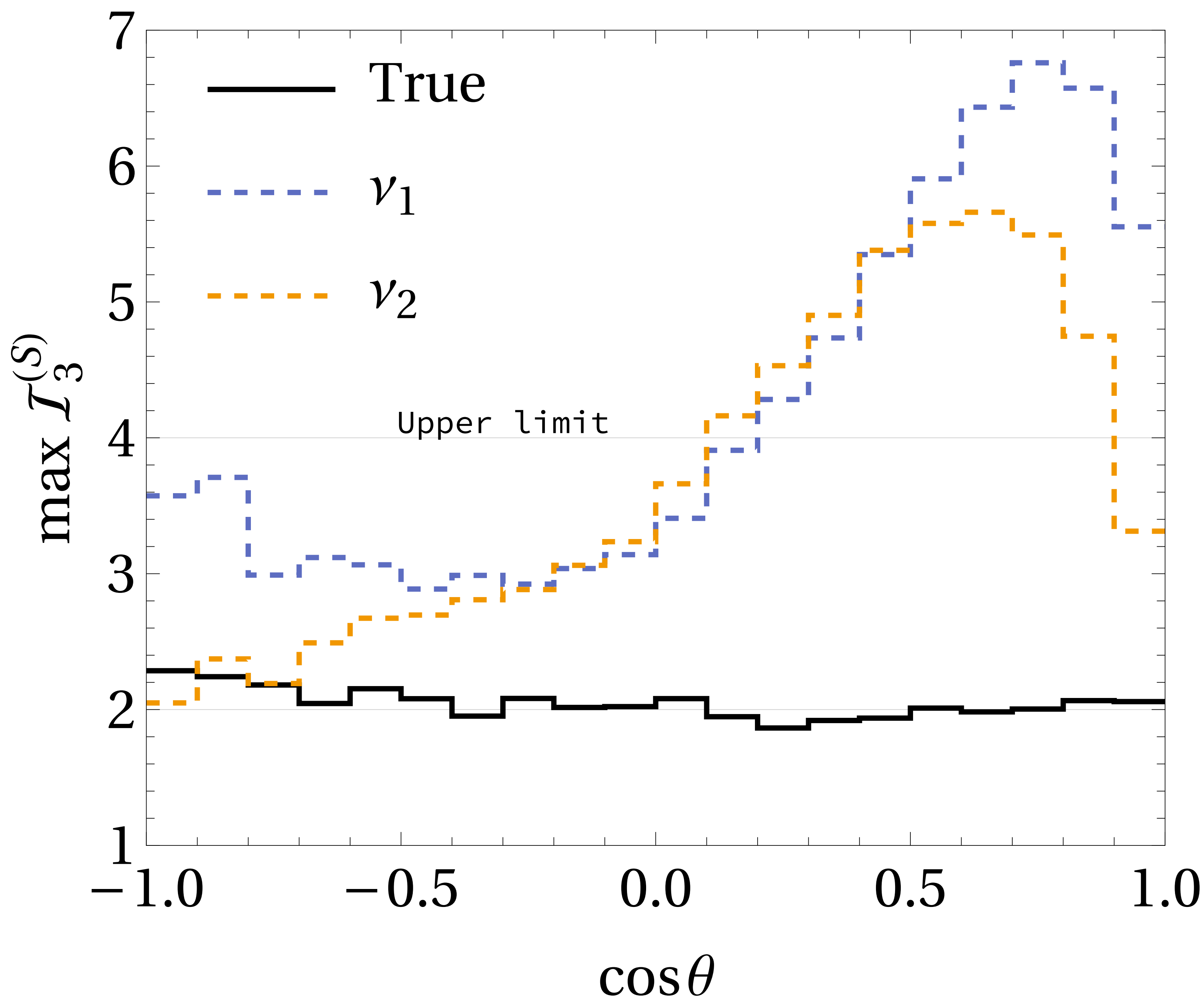
WW production at Higgs factory

- How to measure it at Higgs factory???
- “Measuring” the polarization direction of the W boson.



WW production at Higgs factory

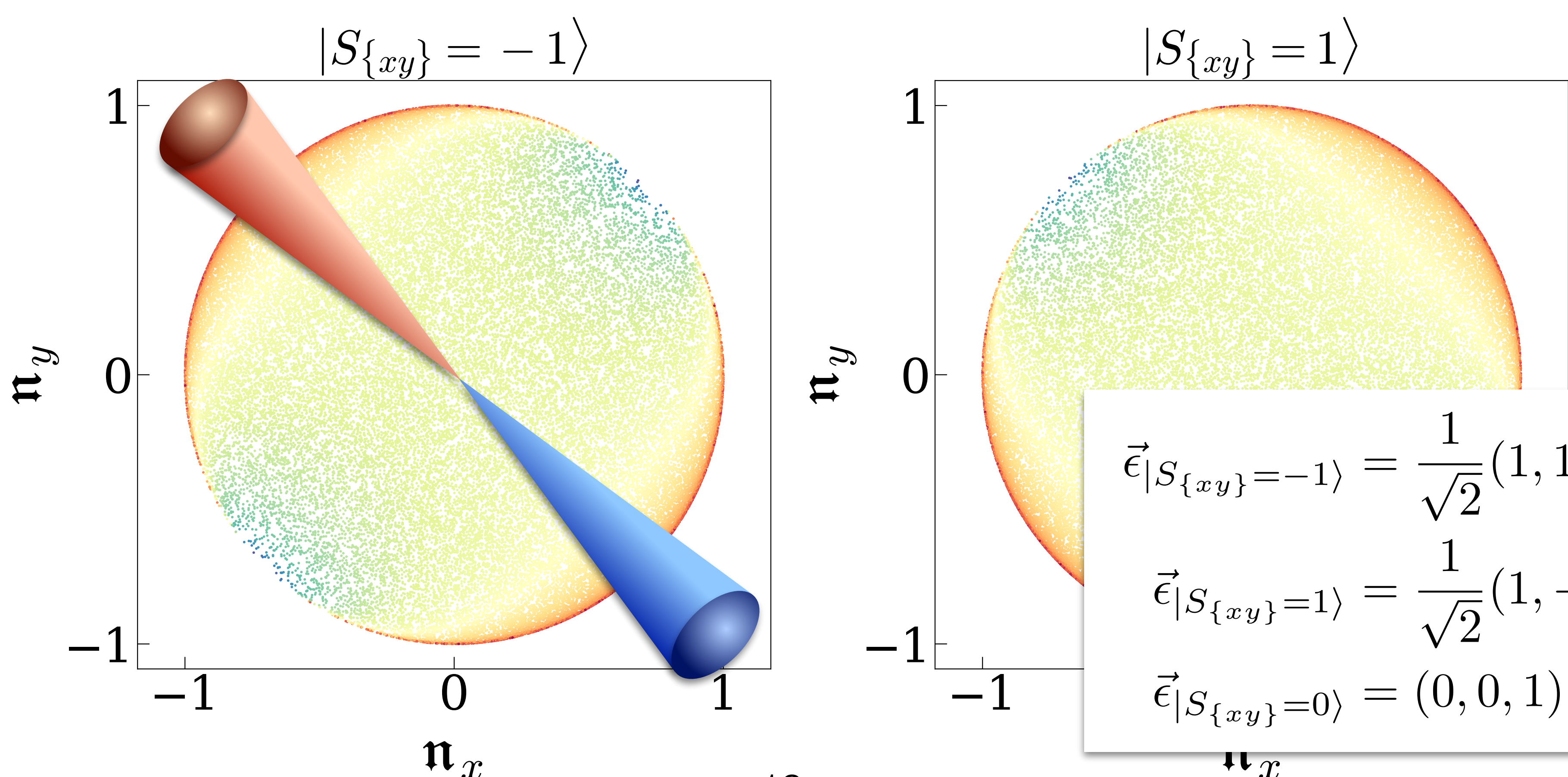
- Collider phenomenology



WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.
- Circular polarization → linear polarization.

$$\hat{\Pi}_{\mathbf{n}} = \hat{I}_3 - \hat{S}_{\mathbf{n}}^2$$



$$\vec{\epsilon}_{|S_{\{xy\}}=-1\rangle} = \frac{1}{\sqrt{2}}(1,1,0),$$
$$\vec{\epsilon}_{|S_{\{xy\}}=1\rangle} = \frac{1}{\sqrt{2}}(1,-1,0),$$
$$\vec{\epsilon}_{|S_{\{xy\}}=0\rangle} = (0,0,1),$$

WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.
- Circular polarization → linear polarization.

$$\mathcal{I}_3(\hat{S}_{\vec{a}_1}, \hat{S}_{\vec{a}_2}; \hat{S}_{\{x_3y_3\}}, \hat{S}_{\{x_4y_4\}}) \equiv + \left[P(S_{\vec{a}_1} = S_{\{x_3y_3\}}) + P(S_{\{x_3y_3\}} = S_{\vec{a}_2} + 1) + P(S_{\vec{a}_2} = S_{\{x_4y_4\}}) + P(S_{\{x_4y_4\}} = S_{\vec{a}_1})\right]$$
$$- \left[P(S_{\vec{a}_1} = S_{\{x_3y_3\}} - 1) + P(S_{\{x_3y_3\}} = S_{\vec{a}_2}) + P(S_{\vec{a}_2} = S_{\{x_4y_4\}} - 1) + P(S_{\{x_4y_4\}} = S_{\vec{a}_1} - 1)\right]$$

WW production at Higgs factory

- Calculating the generalized Bell observable

$$P(S_{\vec{a}_1} = S_{\{x_3y_3\}}) = \sum_{\lambda=-1}^{1} \mathrm{Tr}\left[\hat{\rho}_{WW}\hat{\Pi}_{|S_{\vec{a}_1}=\lambda, S_{\{x_3y_3\}}=\lambda\rangle}\right]$$

$$= \mathrm{Tr}\left[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=-1)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=-1)\right]$$

$$+\,\mathrm{Tr}\left[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=1)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=1)\right]$$

$$+\,\mathrm{Tr}\left[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=0)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=0)\right]$$

$$= \; 1 - 2q^{-}_{ij}\epsilon_{3i}\epsilon_{3j} - 2C^{dq}_{i,jk}a_{1i}(\epsilon_{1j}\epsilon_{1k} - \epsilon_{2j}\epsilon_{2k})$$

$$+\,2C^{q}_{ij,kl}a_{1i}a_{1j}(-\epsilon_{1k}\epsilon_{1l} - \epsilon_{2k}\epsilon_{2l} + 2\epsilon_{3k}\epsilon_{3l}).$$

$$\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=-1) = \frac{1}{2}(-\hat{S}_{\vec{a}_1} + \hat{S}^2_{\vec{a}_1}),$$

$$\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=1) = \frac{1}{2}(\hat{S}_{\vec{a}_1} + \hat{S}^2_{\vec{a}_1}),$$

$$\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=0) = \hat{I}_3 - \hat{S}^2_{\vec{a}_1}.$$

$$\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=-1) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_1}, \quad \vec{\epsilon}_1 = \frac{\hat{x}_3+\hat{y}_3}{\sqrt{2}},$$

$$\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=1) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_2}, \quad \vec{\epsilon}_2 = \frac{\hat{x}_3-\hat{y}_3}{\sqrt{2}},$$

$$\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=0) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_3}, \quad \vec{\epsilon}_3 = \hat{x}_3\times\hat{y}_3.$$

WW production at Higgs factory

- Calculating the generalized Bell observable

$$
\begin{aligned}
&\mathcal{I}_3(\hat{S}_{\vec{a}_1}, \hat{S}_{\vec{a}_2}; \hat{S}_{\{x_3 y_3\}}, \hat{S}_{\{x_4 y_4\}}) \\
&= 2q^-_{ij}(\omega_{1i}\omega_{1j} + \omega_{2i}\omega_{2j} - 2\omega_{3i}\omega_{3j}) \\
&\quad + 2C^{dq}_{i,jk} a_{1i}(2\epsilon_{1j}\epsilon_{1k} - \epsilon_{2j}\epsilon_{2k} - \epsilon_{3j}\epsilon_{3k} + \omega_{1j}\omega_{1k} \\
&\qquad - 2\omega_{2j}\omega_{2k} + \omega_{3j}\omega_{3k}) \\
&\quad + 2C^{dq}_{i,jk} a_{2i}(-2\epsilon_{1j}\epsilon_{1k} + \epsilon_{2j}\epsilon_{2k} + \epsilon_{3j}\epsilon_{3k} + 2\omega_{1j}\omega_{1k} \\
&\qquad - \omega_{2j}\omega_{2k} - \omega_{3j}\omega_{3k}) \\
&\quad + 6C^{q}_{ij,kl} a_{1i}a_{1j}(-\epsilon_{2k}\epsilon_{2l} + \epsilon_{3k}\epsilon_{3l} - \omega_{1k}\omega_{1l} + \omega_{3k}\omega_{3l}) \\
&\quad + 6C^{q}_{ij,kl} a_{2i}a_{2j}(\epsilon_{2k}\epsilon_{2l} - \epsilon_{3k}\epsilon_{3l} - \omega_{2k}\omega_{2l} + \omega_{3k}\omega_{3l})
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathfrak{n}^\pm_i \rangle &= d^\pm_i, \\
\langle \mathfrak{q}^\pm_{ij} \rangle &= \frac{2}{5} q^\pm_{ij}, \\
\langle \mathfrak{n}^+_i \mathfrak{n}^-_j \rangle &= C^d_{ij}, \\
\langle \mathfrak{q}^+_{ij} \mathfrak{q}^-_{kl} \rangle &= \frac{4}{25} C^q_{ij,kl}, \\
\left\langle \mathfrak{n}^+_i \mathfrak{q}^-_{jk} \right\rangle &= \frac{2}{5} C^{dq}_{i,jk}, \\
\langle \mathfrak{q}^+_{ij} \mathfrak{n}^-_k \rangle &= \frac{2}{5} C^{qd}_{ij,k}.
\end{aligned}
$$

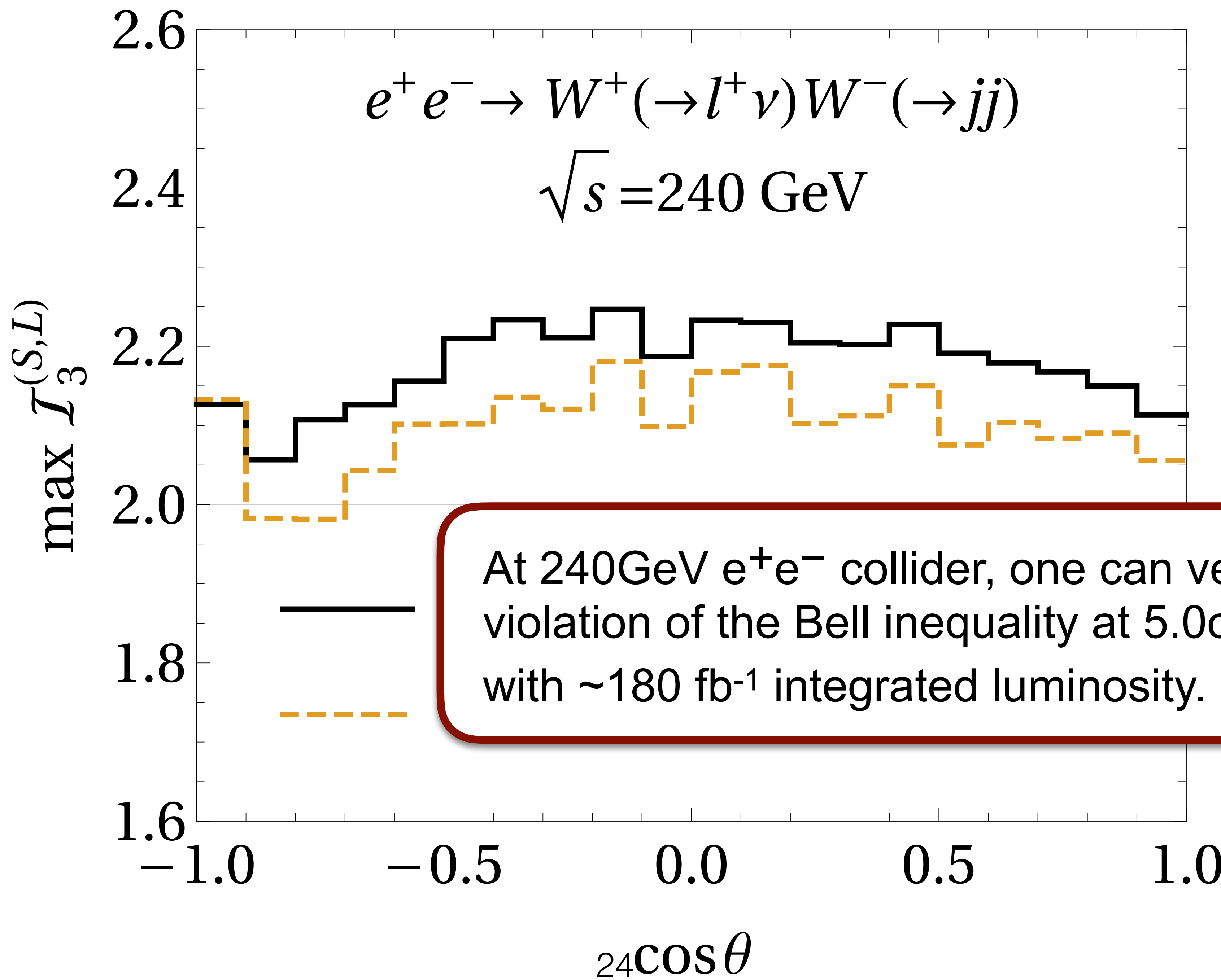
- We need to choose the directions according the coefficients to maximize the generalized Bell observable.

WW production at Higgs factory

- Some details ($e^+e^- \to W^+W^- \to \ell^\pm \nu jj$)
  - 240GeV electron-positron collider
  - (LO) MadGraph5_aMC@NLO+Pythia8+FastJet
  - 2 Exclusive jets with Durham algorithm ($E_j > 5\text{GeV}, |\eta_j| < 3.5$)
  - One isolated charged lepton ($e^\pm, \mu^\pm$) ($E_\ell > 15\text{GeV},\ |\cos\theta_\ell| < 0.98$)
  - Missing energy ($\cos\theta_{\ell\nu} < 0.2$)
  - Reconstructed *W* mass ($|m_{jj} - m_W| < 20\text{GeV}$)

WW production at Higgs factory

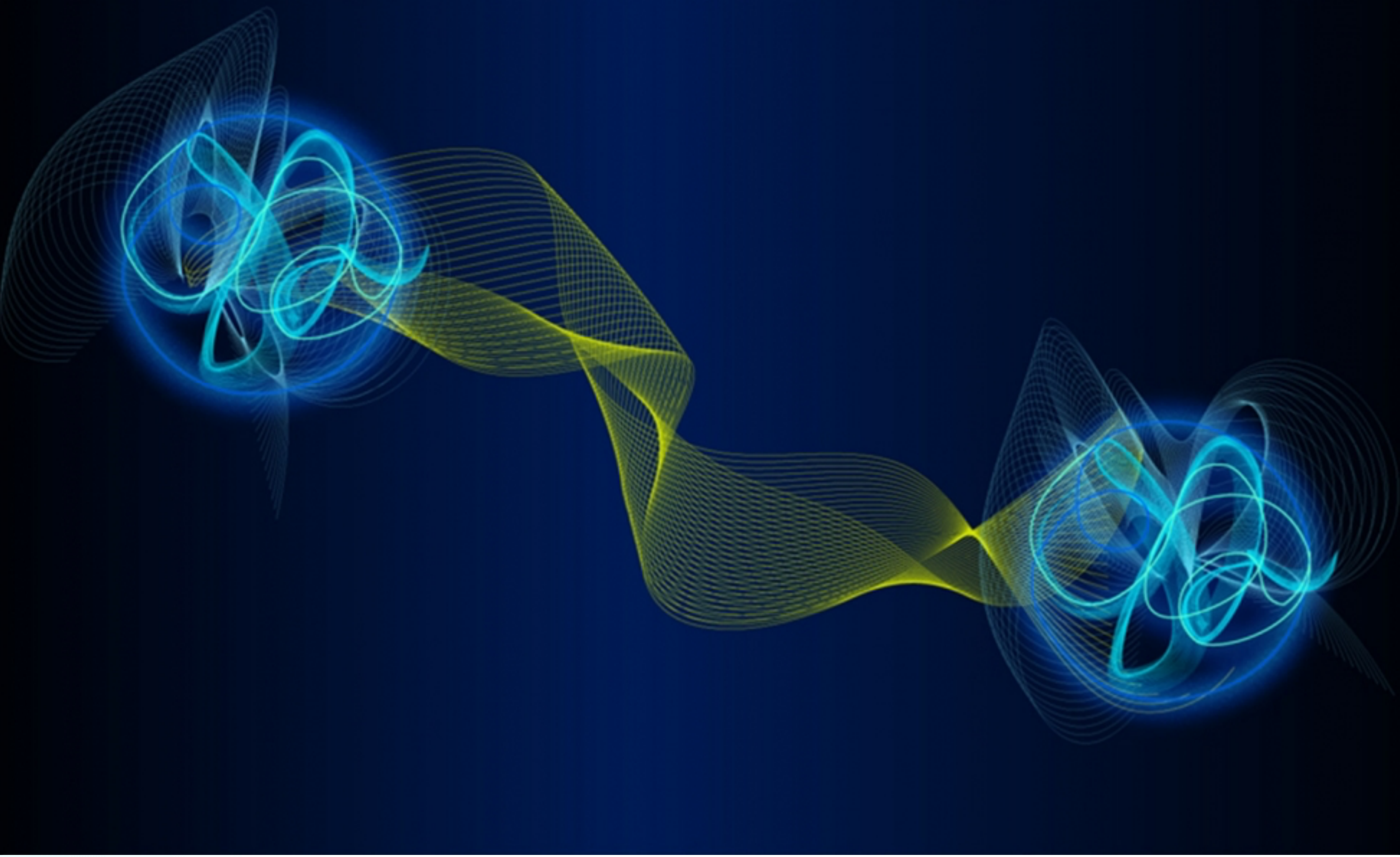
- The result



At 240GeV e^+e^- collider, one can verify the violation of the Bell inequality at 5.0σ significance with ~180 fb^{-1} integrated luminosity.

Conclusion and Discussion

- We provide a realistic approach to test Bell inequalities in W pair systems using a new set of Bell observables based on measuring the linear polarization of W bosons.
- Our observables depend on only part of the density matrix that can be correctly measured in the semi-leptonic decay mode of W.



Thank you!