Looking for the solution to Hierarchy Problem in Top Quark Physics

with Andreas Bally, Florian Goertz, based on 2211.17254 , 2309.00072...

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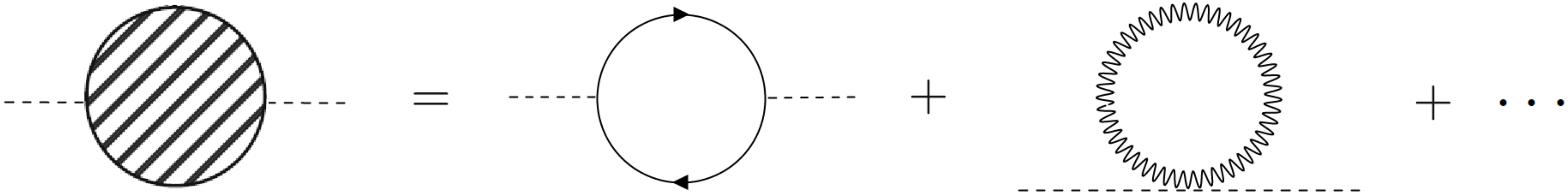
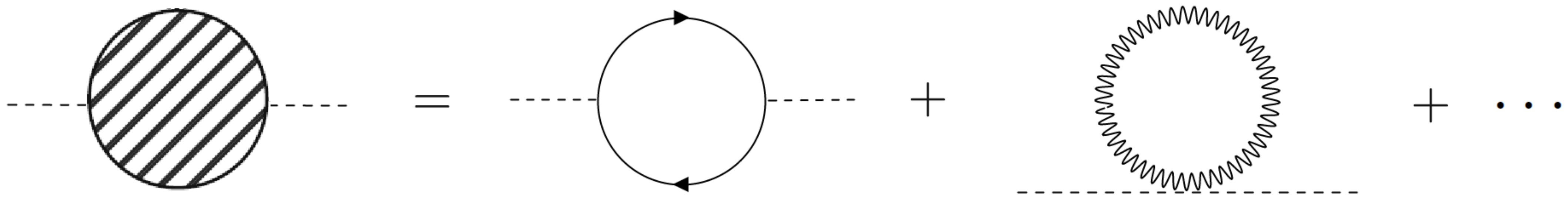
The Hierarchy Problem



$$\delta m_h^2 = -\frac{3}{8\pi^2} y_t^2 \Lambda_t^2 + \frac{9}{64\pi^2} g_W^2 \Lambda_g^2 + \cdots$$

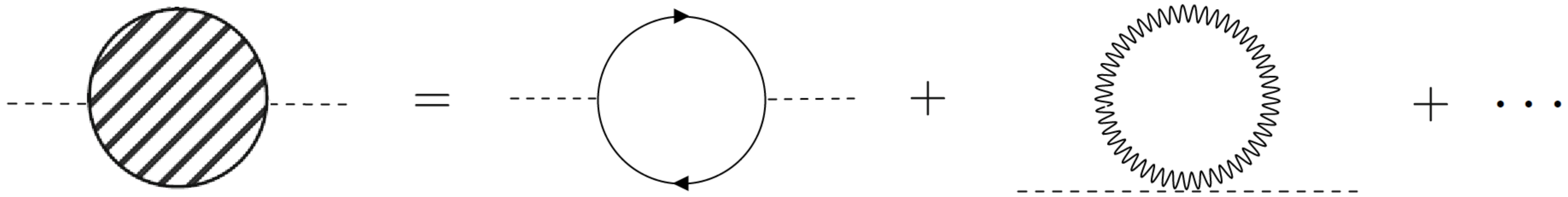
The Naturalness Principle



$$\delta m_h^2 = -\frac{3}{8\pi^2} y_t^2 \Lambda_t^2 + \frac{9}{64\pi^2} g_W^2 \Lambda_g^2 + \cdots$$

New Physics at $\Lambda_t \sim 500$ GeV $\Lambda_g \sim 1200$ GeV

The Naturalness Principle

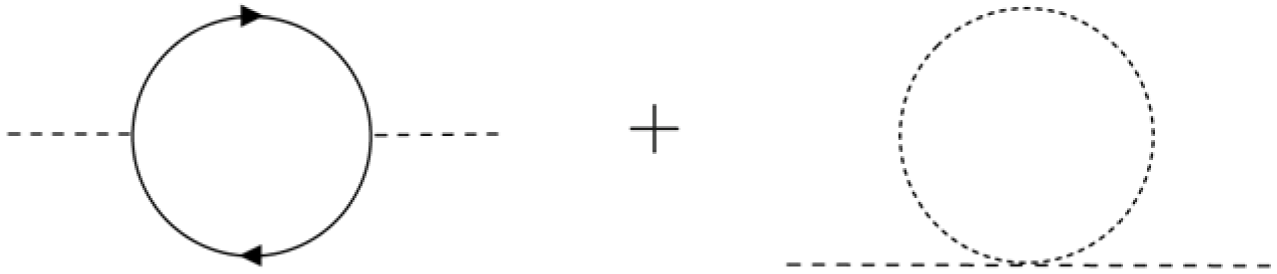


$$\delta m_h^2 = -\frac{3}{8\pi^2} y_t^2 \Lambda_t^2 + \frac{9}{64\pi^2} g_W^2 \Lambda_g^2 + \cdots$$

New Physics at $\Lambda_t \sim 500$ GeV $\Lambda_g \sim 1200$ GeV

The lightest new d.o.f.!!

Top partner solutions

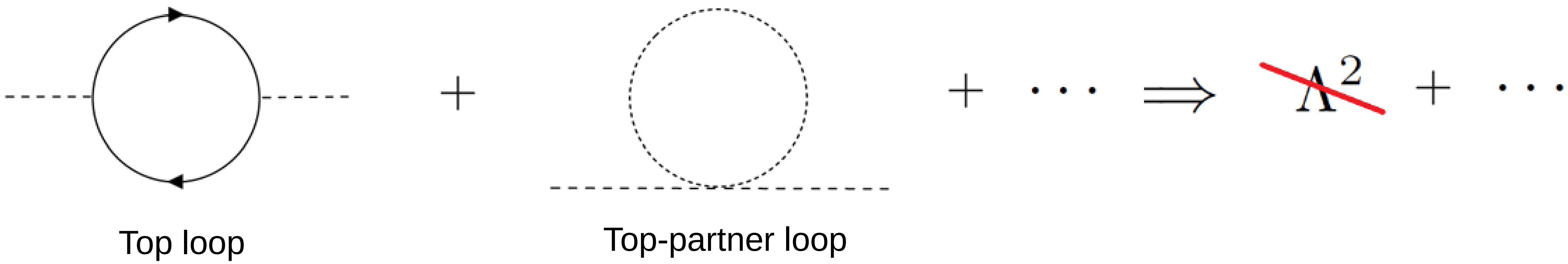


$$+ \cdots \Longrightarrow \cancel{\Lambda^2} + \cdots$$

Top loop   Top-partner loop

- The cancellation is guaranteed by Symmetry, ex: SUSY, shift sym. (CHM) …

Top partner solutions



- The cancellation is guaranteed by Symmetry, ex: SUSY, shift sym. (CHM) …
- The Higgs mass term is still generated due to the difference between

$$\delta m_h^2|_{\text{top}} + \delta m_h^2|_{\text{top partner}} \sim -\frac{3}{8\pi^2} y_t^2 M_T^2 \ln\left(\frac{\Lambda^2}{M_T^2}\right)$$

- Naturalness principle suggests top partners with masses $M_T \approx 500$ GeV

Problems with Colored top partners

- Absence of colored top partners up to 1.2 TeV

  $\Rightarrow\ \sim 10\%$ fine tuning (even worse for large log factor)

Quantum #	**Scalar**	**Fermion**
QCD x EW	SUSY	CHM / RS

$\Rightarrow$ Colored top partners

Alternative to Colored top partners

- Absence of colored top partners up to 1.2 TeV

  $\Rightarrow$ $\sim 10\%$ fine tuning (even worse for large log factor)

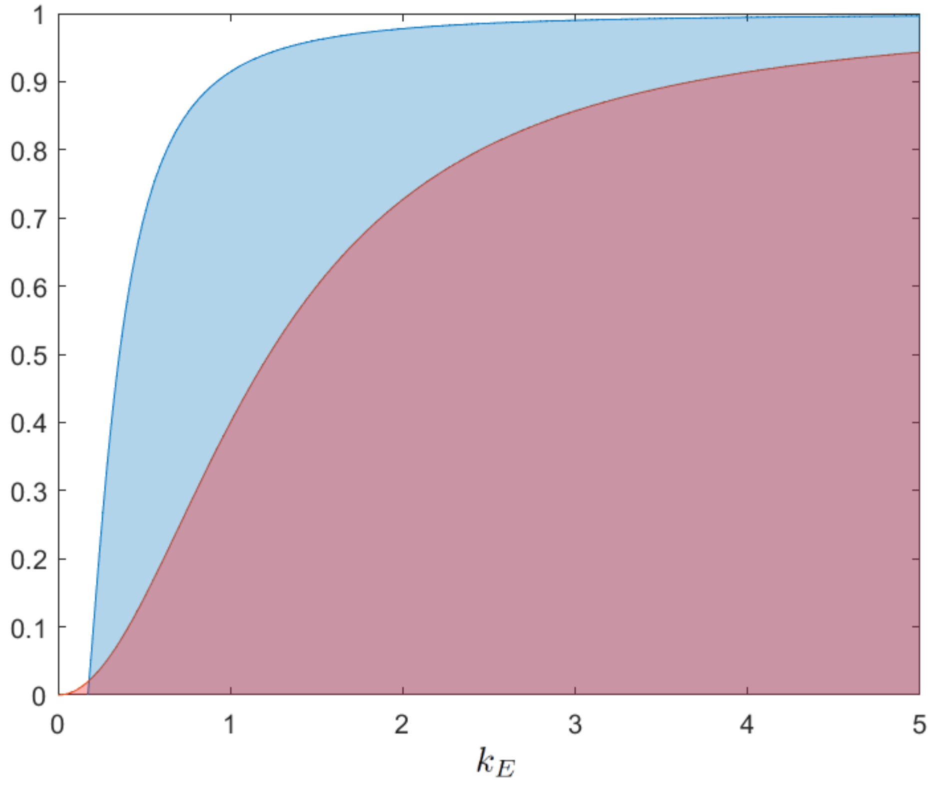
Quantum #	Scalar	Fermion
QCD x EW	SUSY	CHM / RS
Neutral x EW	Folded SUSY	Quirky Little Higgs
Neutral x Neutral	Tripled Top Hyperbolic Higgs	Twin Higgs

$\Rightarrow$ Colorless top partners

Table borrowed from Chris Verhaaren

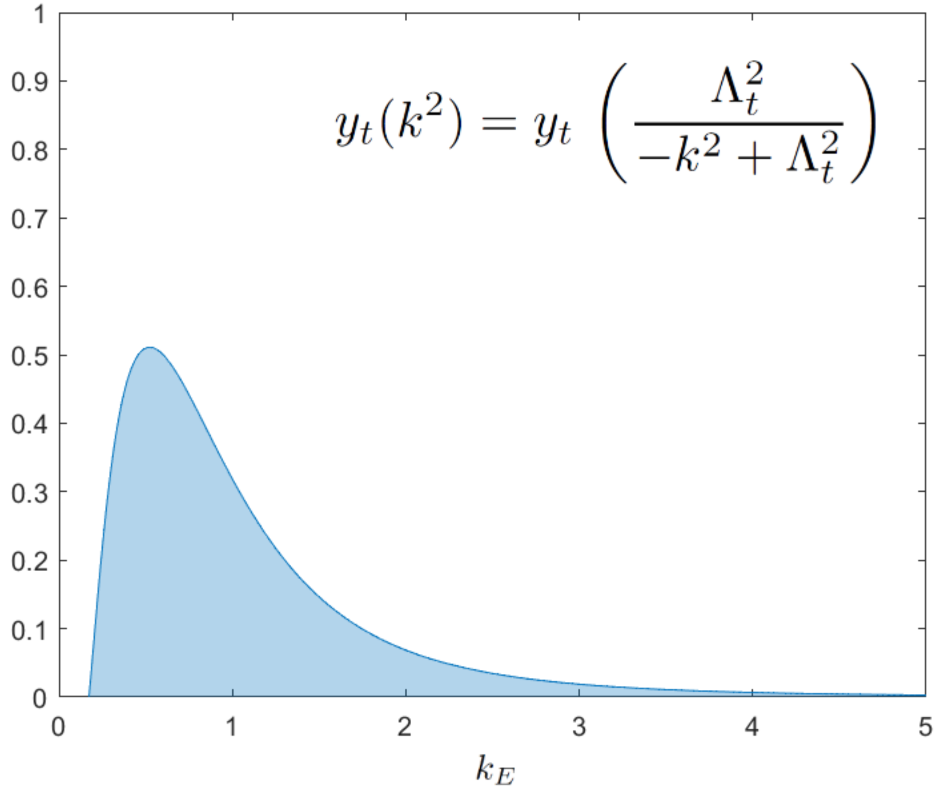
Alternative to Top-partner scenarios

- Cancellation (take $M_T = 1.2$ TeV)



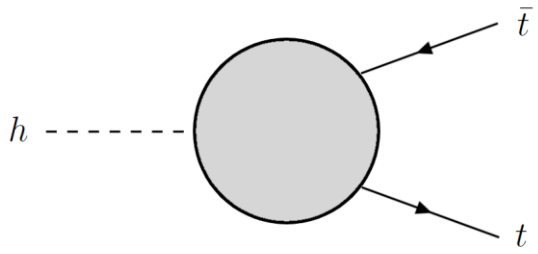
$$\delta m_h^2|_{\text{top}} + \delta m_h^2|_{\text{top partner}} \sim -\frac{3}{8\pi^2} y_t^2 M_T^2 \ln\left(\frac{\Lambda^2}{M_T^2}\right)$$

- Reduction (take $\Lambda_t = 1.2$ TeV)



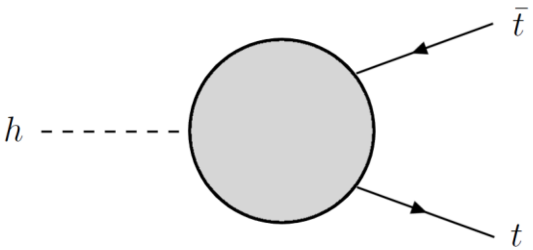
$$\delta m_h^2|_{\text{top}} \sim -i\, 2N_c \int \frac{d^4k}{(2\pi)^4} y_t^2(k^2) \frac{k^2 + m_t^2}{(k^2 - m_t^2)^2} \sim -\frac{3}{8\pi^2} y_t^2 \Lambda_t^2$$

Zoom in the Top Yukawa vertex



$$y_t = y_t(k^2)$$

Zoom in the Top Yukawa vertex

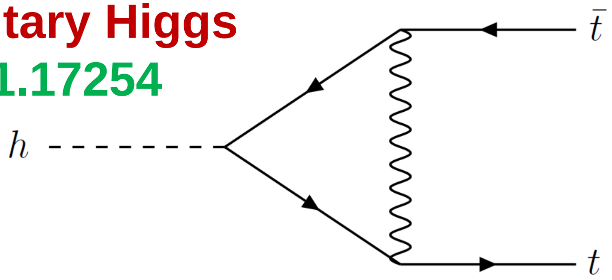


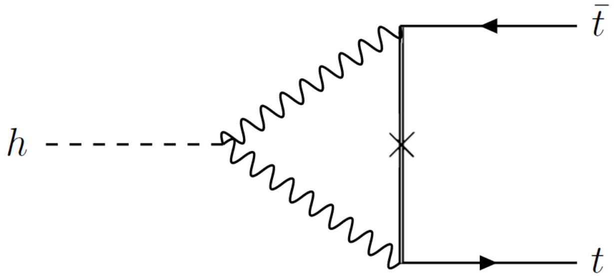
$\Rightarrow$

$$y_t = y_t(k^2)$$

Elementary Higgs

2211.17254





(1) large y_t running

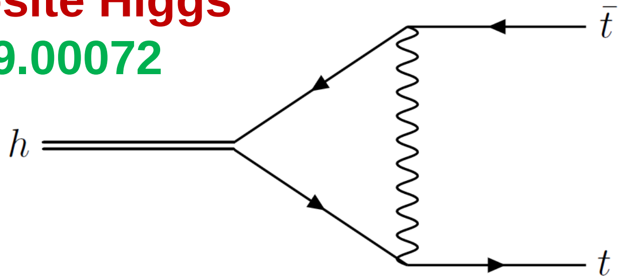
new top-philic bosons
with strong coupling

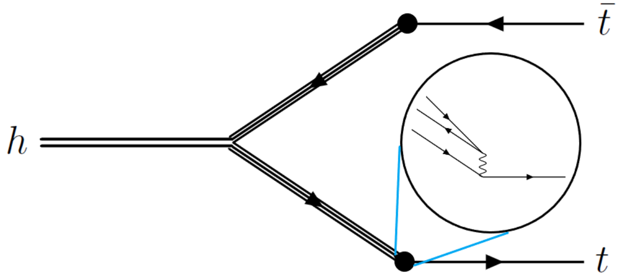
(2) radiative y_t generation

bosons and VL fermions
with strong coupling

Composite Higgs

2309.00072



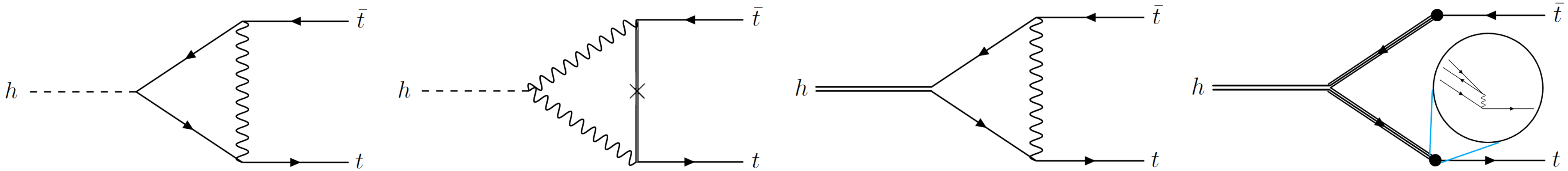


(3) y_t from from four-fermion interations

extended-hypercolor bosons with weak coupling

Direct searches

- First, we need to select a model



- For each theory, there comes different d.o.f. and difficulties

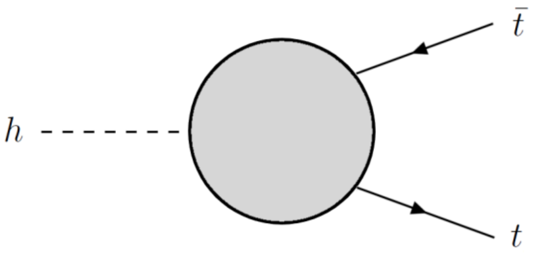
  (1) **top-philic bosons** with strong coupling : broad resonances with $\Gamma/M \gg 10\%$

  (2) **bosons and VL fermions** with strong coupling, diverse quantum number and spectrum

  (3) **extended-hypercolor bosons** with diverse quantum number including hypercolor

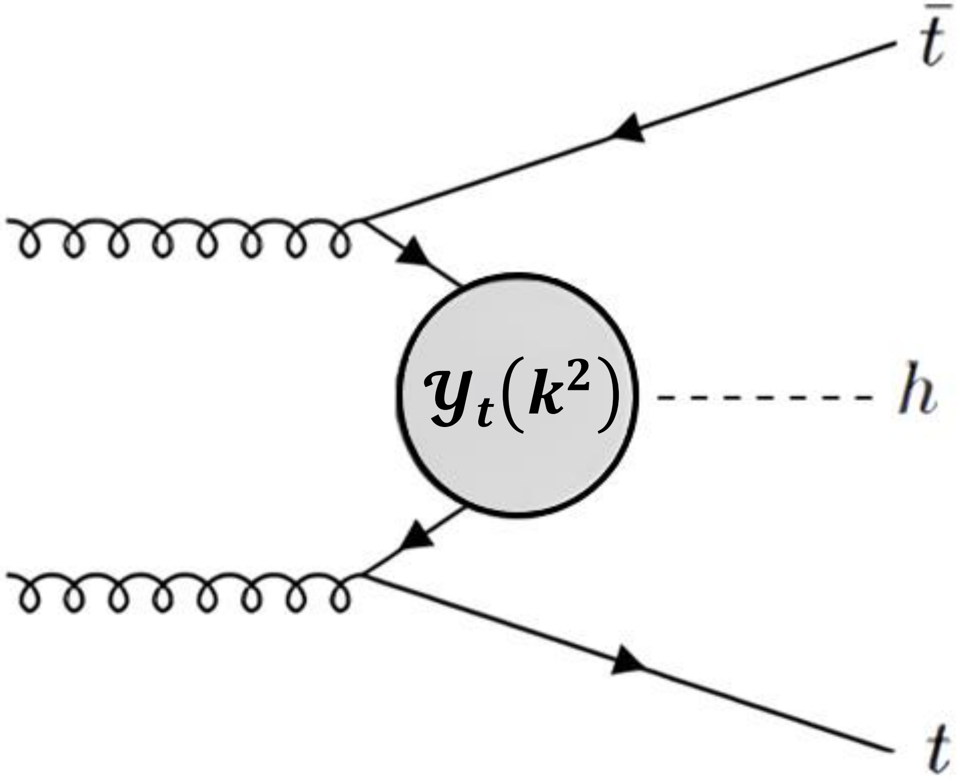
$\Rightarrow$ **Searches are challenging, suffering from strong couplings and model-dependence**

Direct test of the idea!



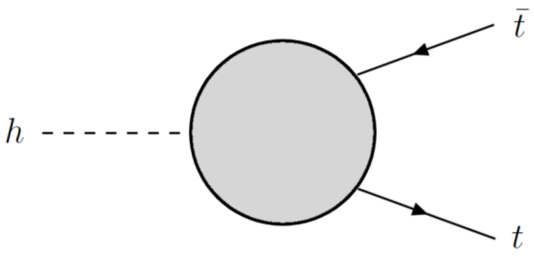
$$y_t = y_t(k^2)$$

$\Rightarrow$



$t\bar{t}h$ differential cross section

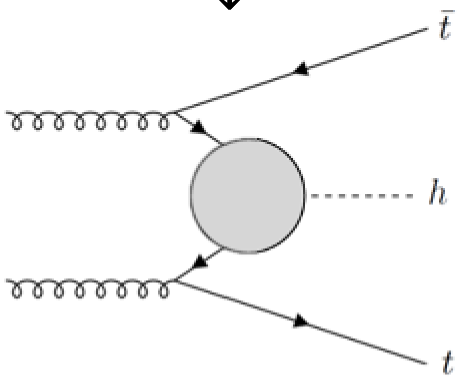
Running of the top Yukawa coupling



$$y_t = y_t(k^2)$$

Top Yukawa at high scales

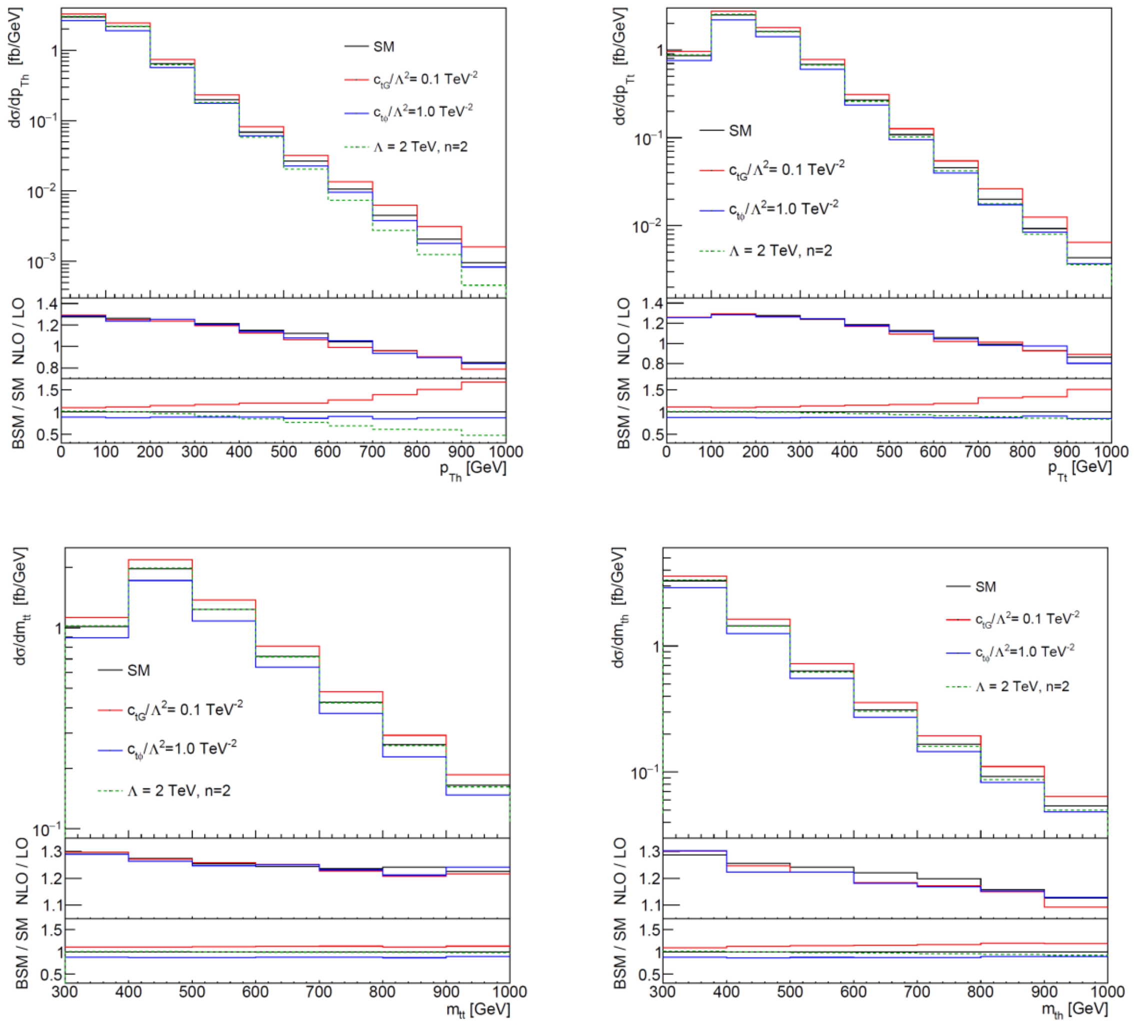
↓



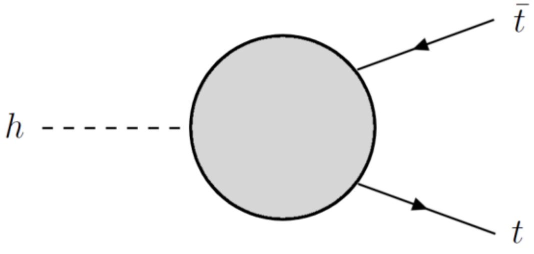
$t\bar{t}h$ differential xs

T. Han et al
2106.00018

⇒



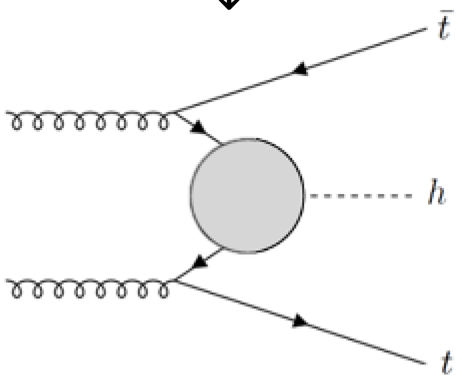
Running of the top Yukawa coupling



$$y_t = y_t(k^2)$$

Top Yukawa at high scales

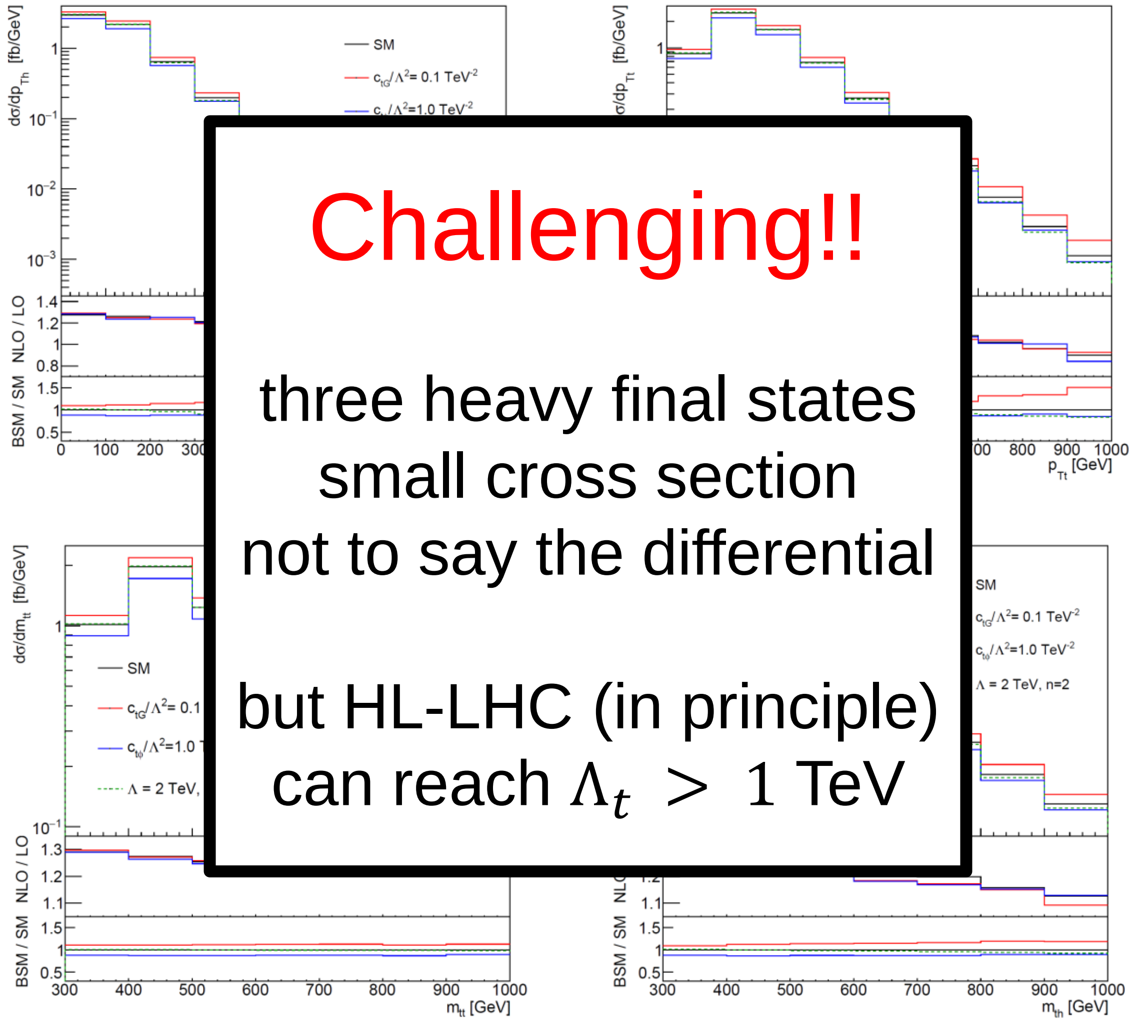
↓



$t\bar{t}h$ differential xs

T. Han et al
2106.00018

$\Rightarrow$

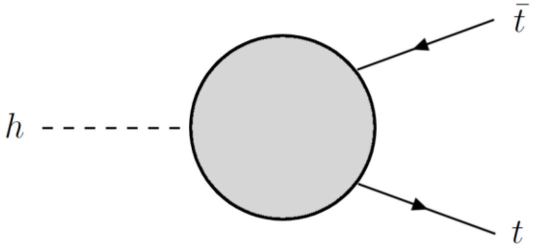


Challenging!!

three heavy final states
small cross section
not to say the differential

but HL-LHC (in principle)
can reach $\Lambda_t > 1$ TeV

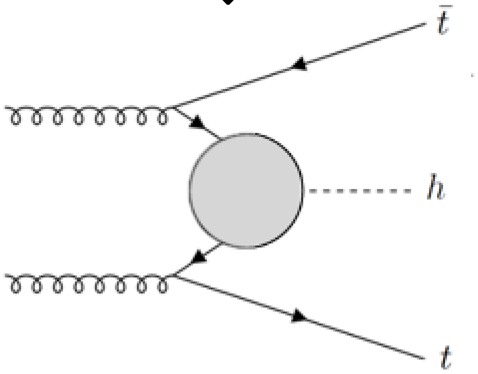
More Indirect searches



$$y_t = y_t(k^2)$$

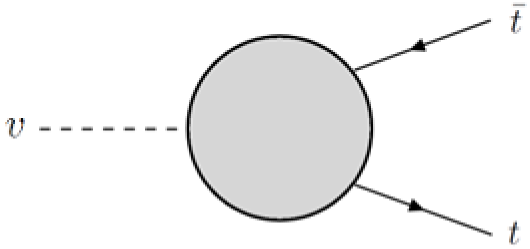
Top Yukawa at high scales

↓



$t\bar{t}h$ differential xs

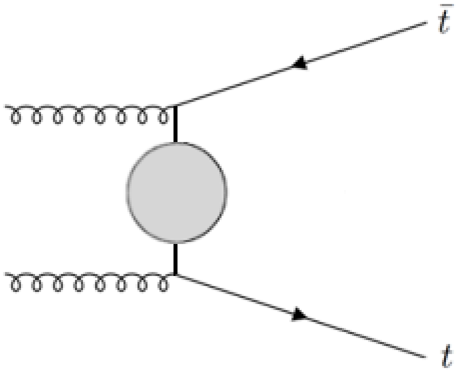
⇒



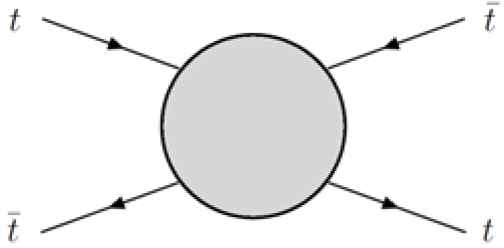
$$m_t = m_t(k^2)$$

Top mass at high scales

↓



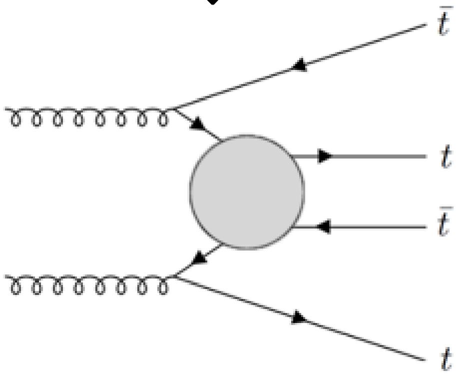
$t\bar{t}$ differential xs



$$\sim C/\Lambda_t^2$$

Top-philic new interactions

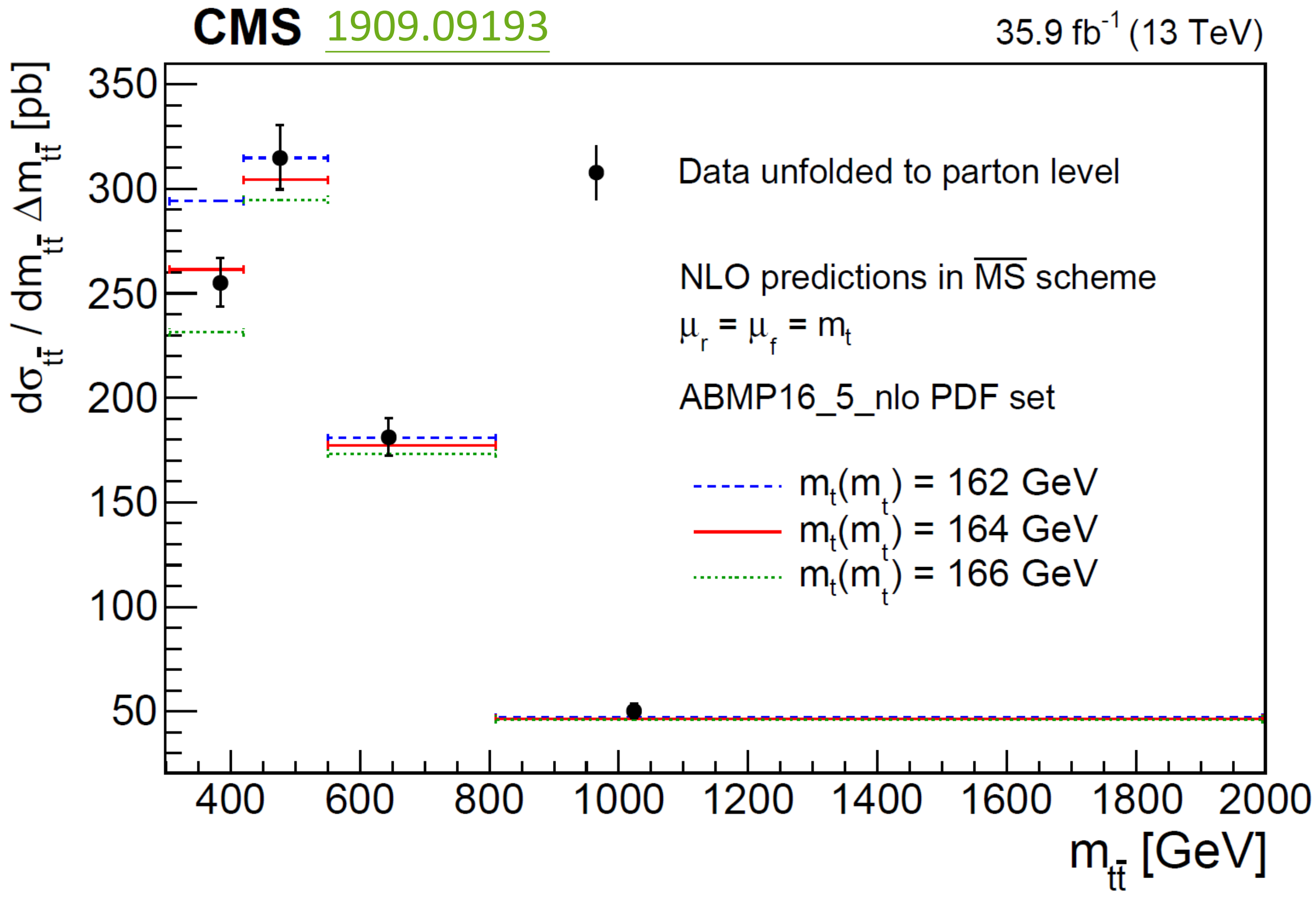
↓



$t\bar{t}t\bar{t}$ (4top) xs

Running of the top quark mass

- Nontrivial running m_t at highs scale will affect the $t\bar{t}$ differential cross section

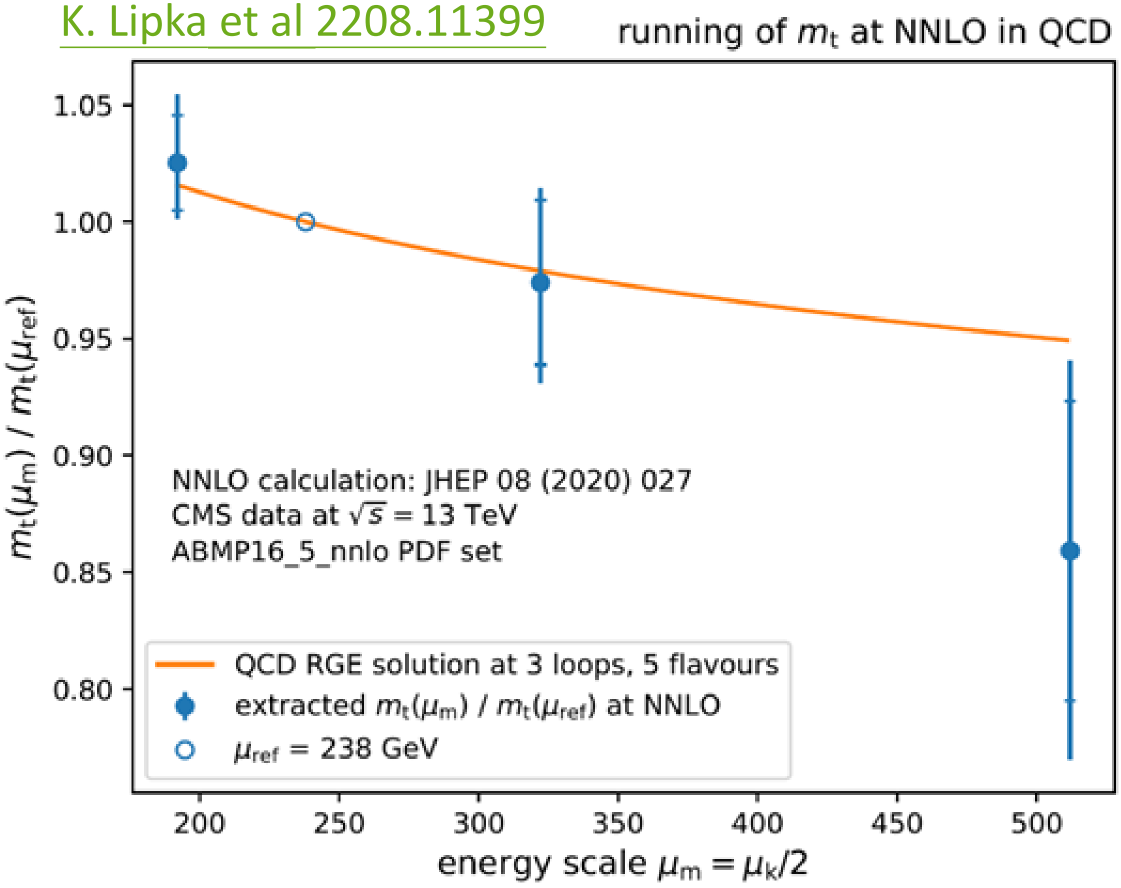


First measurement of top mass at high scales !
(using 2016 Run 2 data)

Running of the top quark mass

- Nontrivial running m_t at high scales will affect the $t\bar{t}$ differential cross section

K. Lipka et al 2208.11399



Assuming $m_t(\mu_m) = m_{t,\mathrm{SM}}(\mu_m)\left(\frac{\Lambda_t^2}{\mu_m^2+\Lambda_t^2}\right)$

we can already put a constraint on Λ_t as

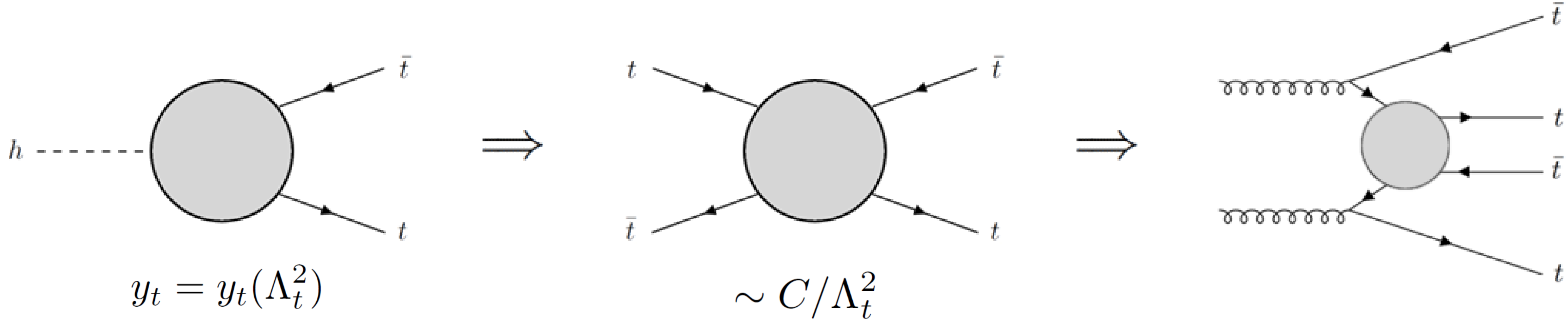
95% CL bound : $\Lambda_t \gtrsim$ 700 GeV

Relevant parameter spaces will be tested in **LHC Run 3 and HL-LHC** !

More precise $t\bar{t}$ differential cross section calculation is needed !

Running mass of the heaviest particle !!

Four top quark cross section



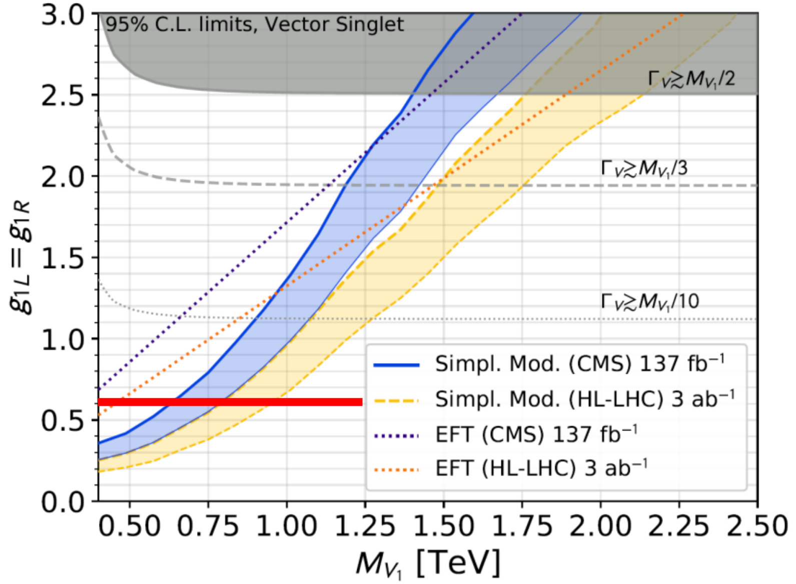
- Standard Model prediction: $13.4^{+1.0}_{-1.8}$ fb

  including NLL' (arXiv: 2212.03259)
- ATLAS with 139 fb^{-1} : $22.5^{+6.6}_{-5.6}$ fb
- CMS with 138 fb^{-1} : $17.7^{+4.4}_{-4.0}$ fb

  → $\sigma_{t\bar{t}t\bar{t}} < 36\ (27)$ fb at 95% C.L.

Four-top could be the first hint !?

F. Maltoni et al
2104.09512
$\Rightarrow$
the old analysis based on xs of $12.6^{+5.8}_{-5.2}$ fb



Summary

- Top quarks play the most important role in the Hierarchy Problem
  - Traditionally, top partners are introduced to **cancel** the top-loop contribution
  - Alternative: **modify the running** of y_t to lower down the top-loop contribution
  - What should show up at $\Lambda_t \approx 500$ GeV : **Top partner** → **New top-philic d.o.f.**

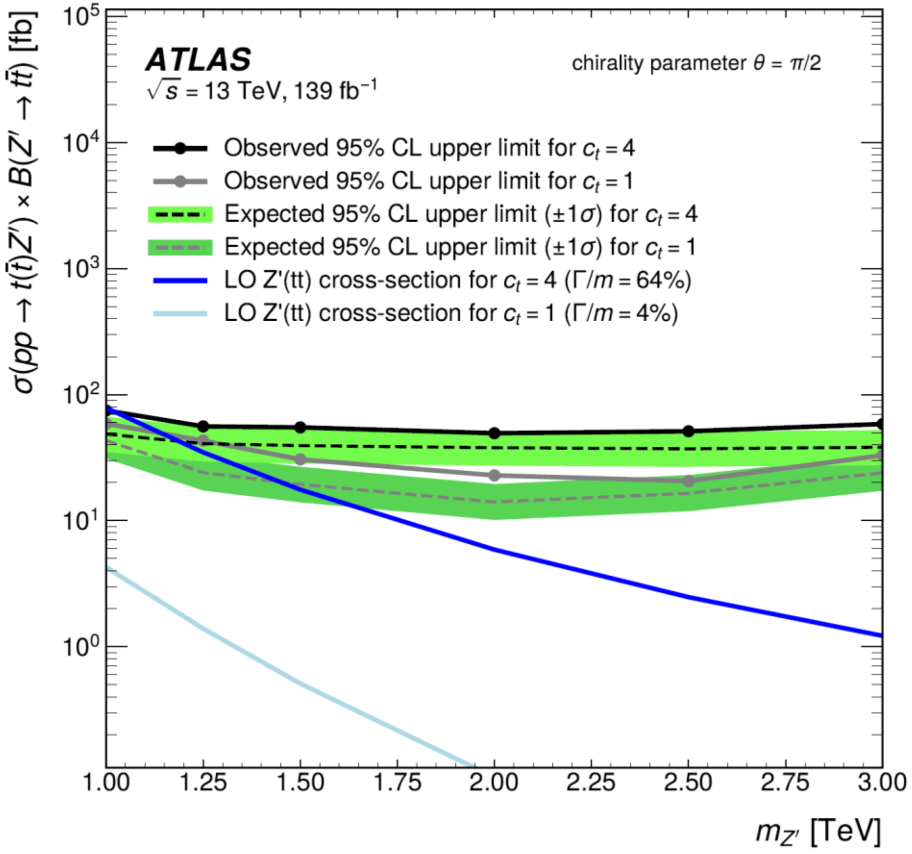
- Phenomenology of the alternative scenario
  - Hard to perform direct searches due to strong couplings and requirements of UV models
  - Common phenomenology of modified top Yukawa coupling
    - Top Yukawa at high scales : $t\bar{t}h$ **differential cross section**
    - Top mass at high scales : $t\bar{t}$ **differential cross section**
    - Top-philic new interactions : $t\bar{t}t\bar{t}$ **cross section**

Solutions to Hierarchy Problem might still be hidden in Top Physics !!

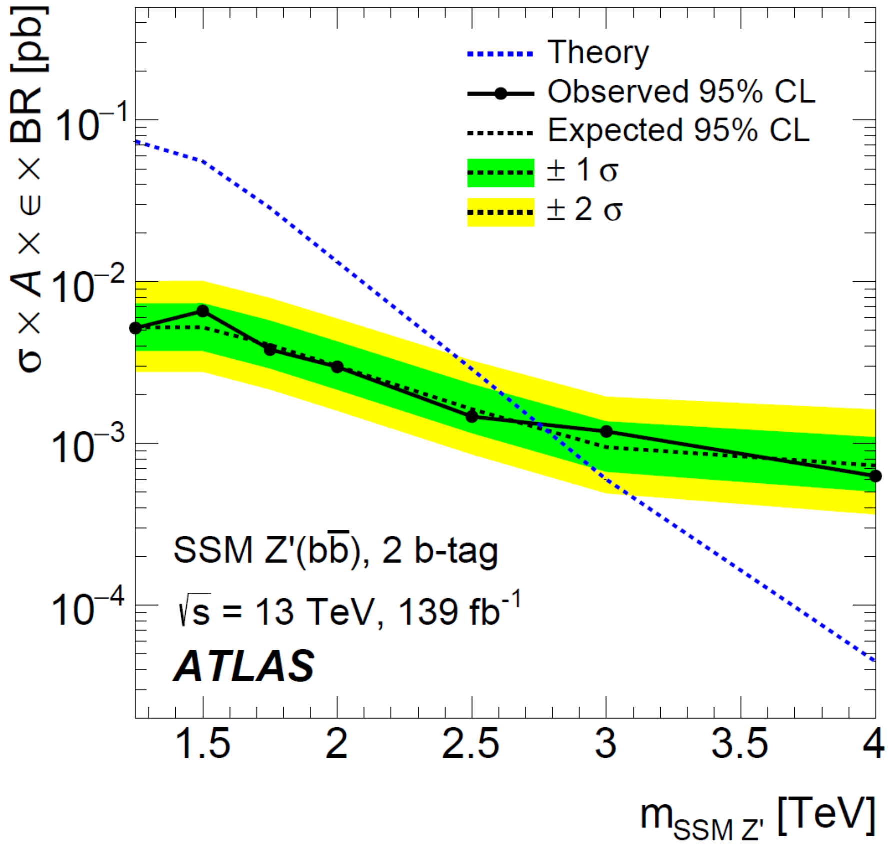
Back up

Direct searches – top-philic Z' boson

- Only couple to t_R (in linear case)
- Process: $t_R \bar{t}_R \to Z' \to t_R \bar{t}_R$ 2304.01678



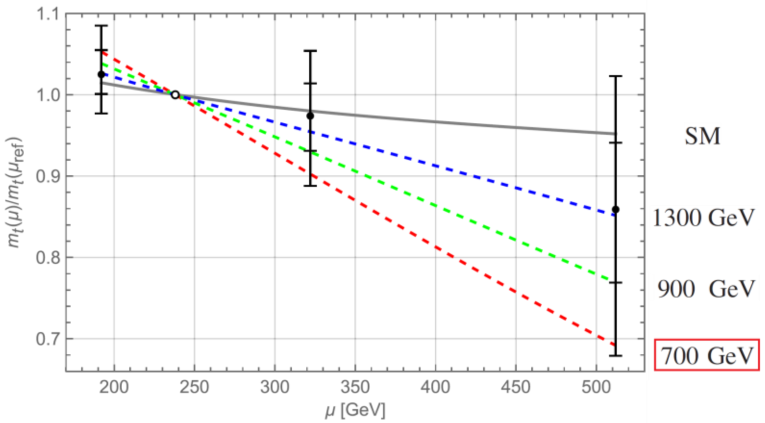
- Couple to $q_L = (t_L, b_L)$ (in bilinear case)
- Process: $b_L \bar{b}_L \to Z' \to b_L \bar{b}_L$ 1910.08447



Naturalness and the Higgs potential

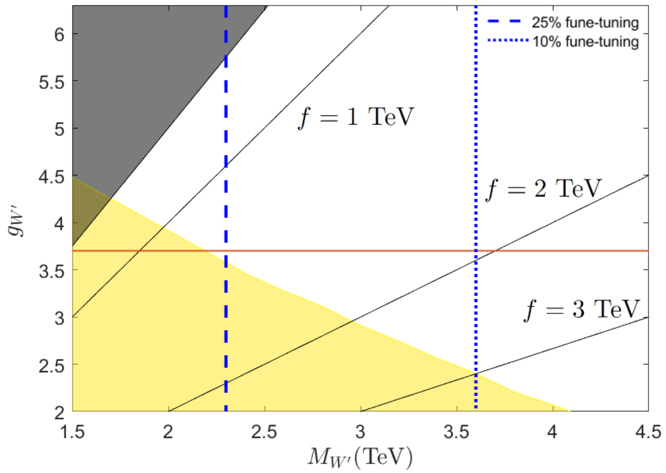
2311.17169

- The Higgs potential is generated mainly from the top loop and gauge loop given by



Top-philic EHC boson

Third-gen.-philic Heavy SU(2)′ boson



$$\begin{cases} m_H^2 = 0^2 - \dfrac{3}{8\pi^2} y_t^2 M_E^2 + \dfrac{9}{64\pi^2} g_W^2 M_{W'}^2 \ln \dfrac{\Lambda^2}{M_{W'}^2} = -(89\ \text{GeV})^2 \\ \lambda = 0 + \dfrac{3\, y_t^4}{16\pi^2} \ln \dfrac{M_E^2}{m_t^2} + \dfrac{9\, g_W^2 g_{W'}^2}{512\pi^2} \ln \dfrac{\Lambda^2}{M_{W'}^2} + \dfrac{c\, g_W^2}{4} = 0.13 \end{cases}$$

~ 0.05 when $M_E = 1$ TeV

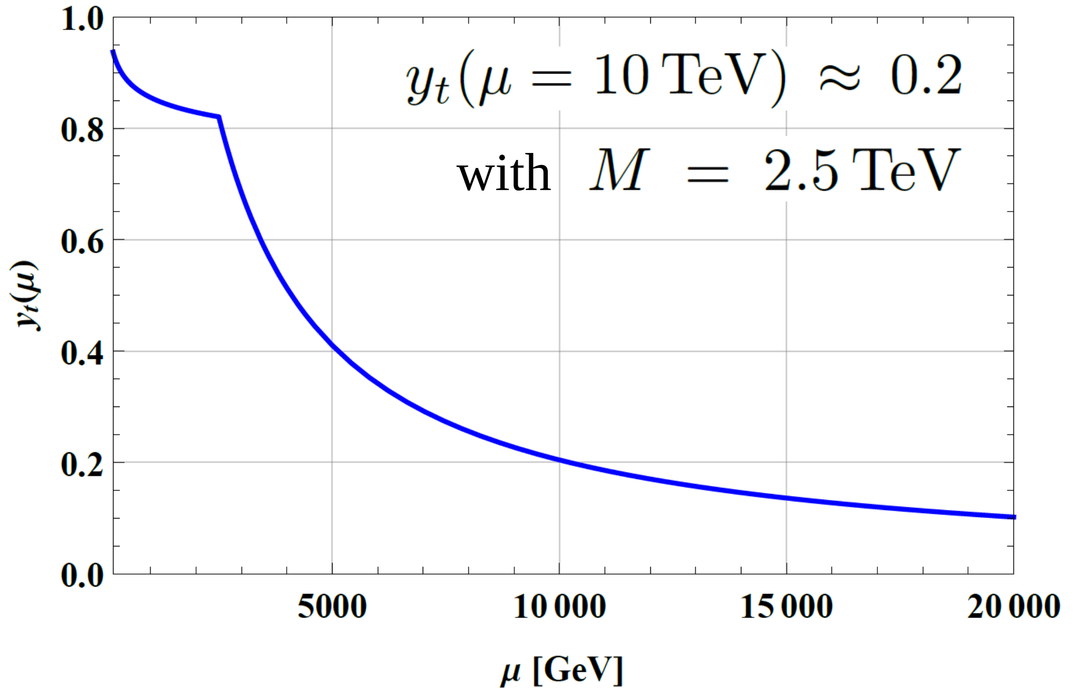
~ 0.02 when $g_{W'} = 4$ (broad W')

$\sim 0.1\, c$

Large y_t running

2211.17254

Strongly interacting top-philic boson

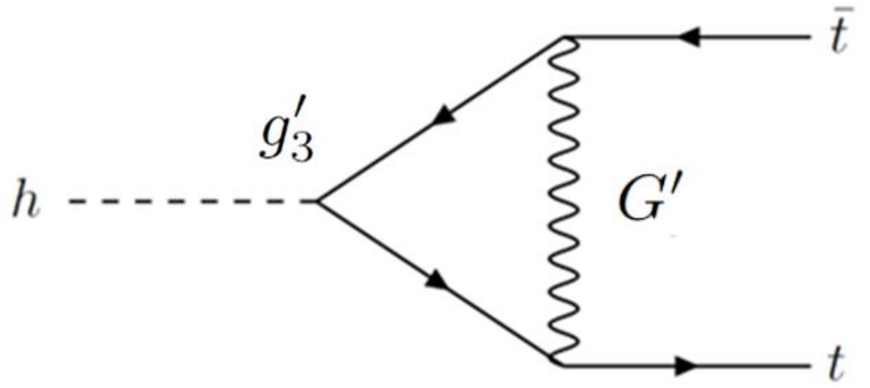


$$\frac{d\, y_t(\mu)}{d \ln \mu} = \frac{y_t(\mu)}{16\pi^2}\left(\frac{9}{2}\, y_t^2(\mu) - \frac{3(N^2-1)}{N} g_N^2 \right)$$

Top-loop becomes subleading with $\Lambda_{\text{NP}} = 10\,\text{TeV}$

$\Longrightarrow$

Ex: Heavy gluons (Coloron)



with the $SU(3)$ coupling $g_3' \sim 4.5$

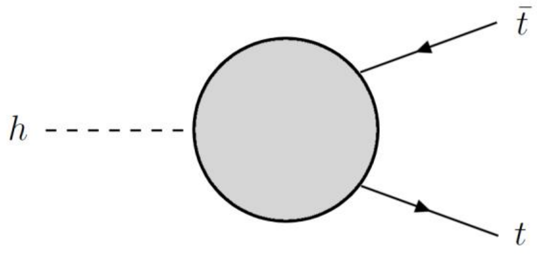
Direct consequences:

- Bound state of top-anti-top with the mass around $M = 2.5$ TeV
- Enhancement in 4t cross section!!

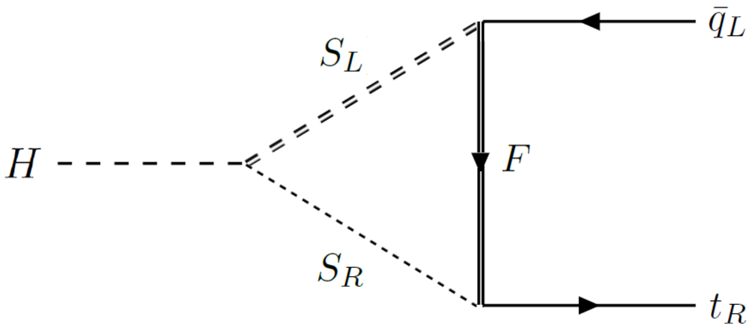
Radiative y_t generation

2211.17254

Top Yukawa arise from Dim-six operator



$\Rightarrow$



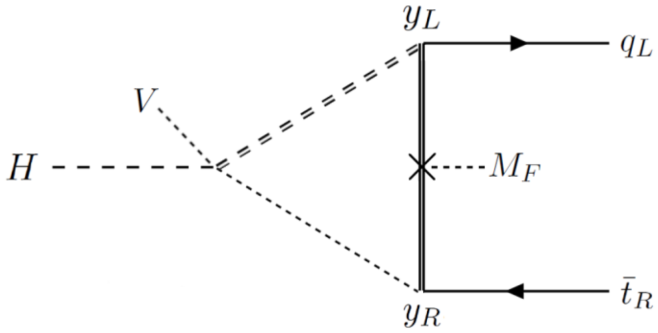
Minimal setup:

EW doublet scalar S_L

EW singlet scalar S_R

EW singlet fermion F

- The diagram introduces a dim-6 operator



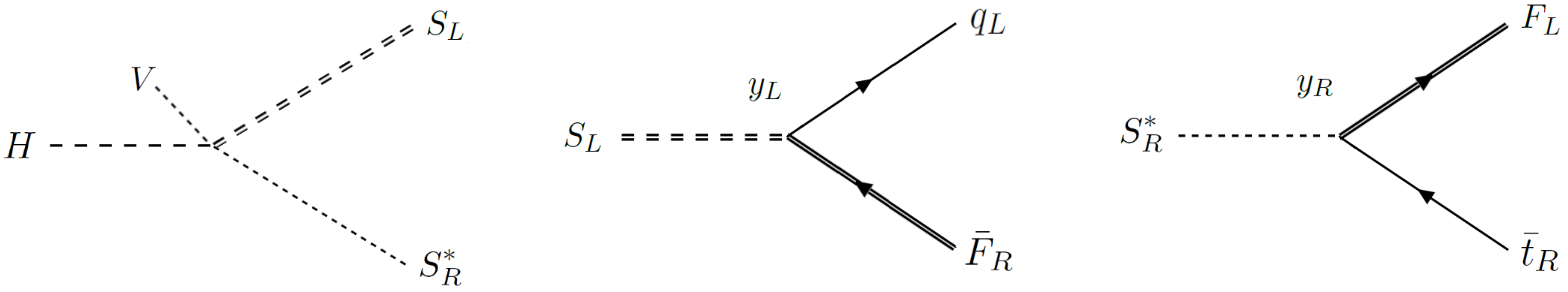
$$\sim \frac{y_L y_R}{16\pi^2} \frac{(H S_V \bar{q}_L S_F t_R)}{M^2} \Longrightarrow \boxed{\frac{y_L y_R}{16\pi^2} \frac{V M_F}{M^2}} (\bar{q}_L H t_R)$$

$= y_t \sim 1$

from strong dynamics!!

Simplified Scalar Model

- At least three vertices are required



or written in Lagrangian

$$\mathcal{L}_{\text{int}} = -V S_R S_L^\dagger H - y_L \bar{q}_L S_L F_R - y_R \bar{t}_R S_R F_L + \text{h.c.} \ ,$$

where S_L is a doublet, S_R is a singlet, and F is a singlet vector-like fermion.

- Mass terms are also required

$$\mathcal{L}_{\text{mass}} = -M_L^2 |S_L|^2 - M_R^2 |S_R|^2 - M_F \bar{F}_L F_R + \text{h.c.} \ .$$

Simplified Scalar Model

- Focus on the neutral scalar components

$$\mathcal{L}_{\text{neutral}} = |\partial s_L|^2 + |\partial s_R|^2 - M_L^2|s_L|^2 - M_R^2|s_R|^2 - V\langle H\rangle(s_L^* s_R + s_R^* s_L)$$
$$= |\partial s_h|^2 + |\partial s_\ell|^2 - M_s^2|s_h|^2 - m_s^2|s_\ell|^2$$

where the mass eigenstates are given by

$$\begin{pmatrix} s_L \\ s_R \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} s_{\text{heavy}} \\ s_{\text{light}} \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} s_h \\ s_\ell \end{pmatrix}$$

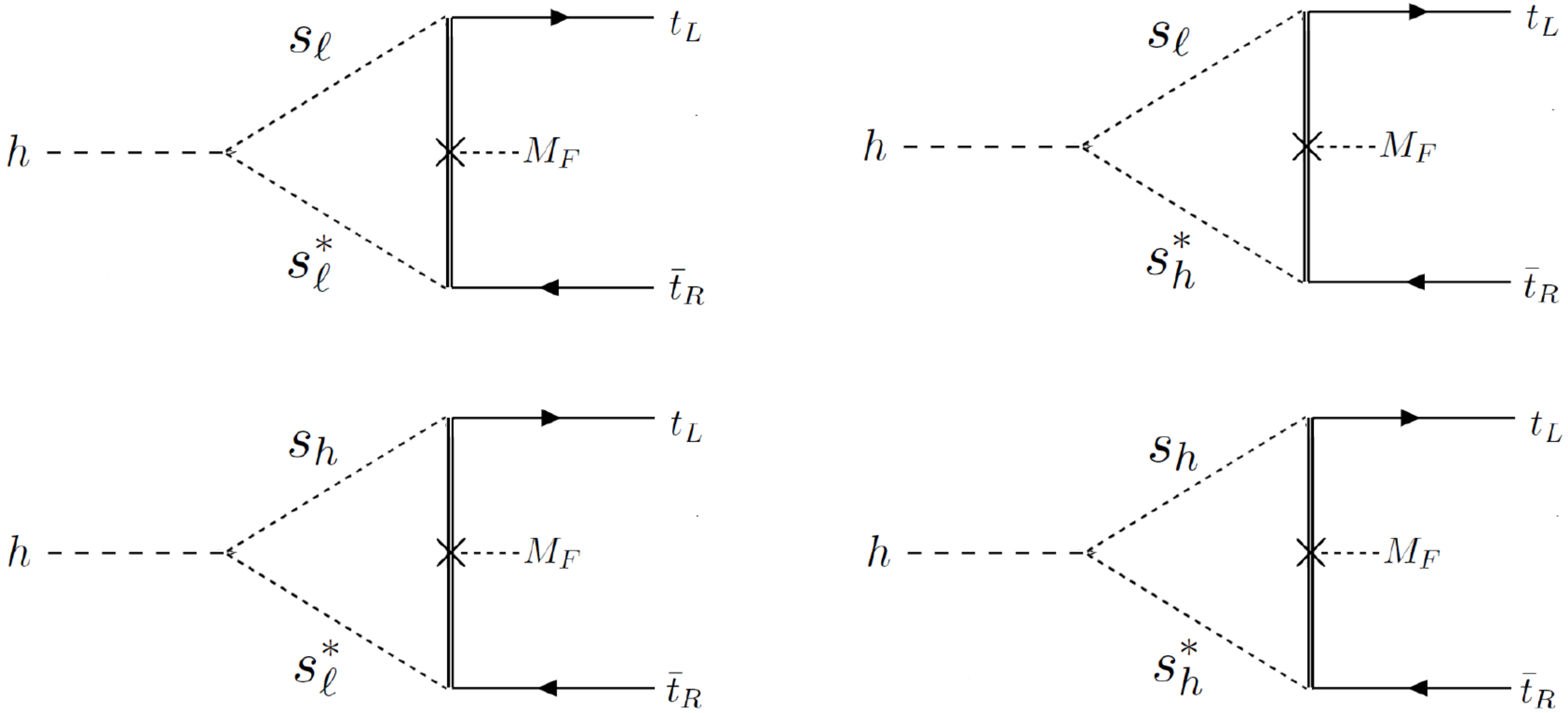
- The interaction terms also become

$$\mathcal{L}_{\text{trilinear}} = -\sqrt{2}\,V c_\beta s_\beta\, h|s_h|^2 + \sqrt{2}\,V c_\beta s_\beta\, h|s_\ell|^2 - \frac{V(c_\beta^2 - s_\beta^2)}{\sqrt{2}}\, h s_h^* s_\ell + \text{h.c.}$$

$$\mathcal{L}_{\text{fermion}} = -\left(y_L c_\beta\, \bar{t}_L s_h F_R + y_R s_\beta\, \bar{t}_R s_h F_L\right) - \left(-y_L s_\beta\, \bar{t}_L s_\ell F_R + y_R c_\beta\, \bar{t}_R s_\ell F_L\right) + \text{h.c.}$$

Generate the Top Yukawa coupling

- The original one-loop diagram is decomposed to the four diagrams below



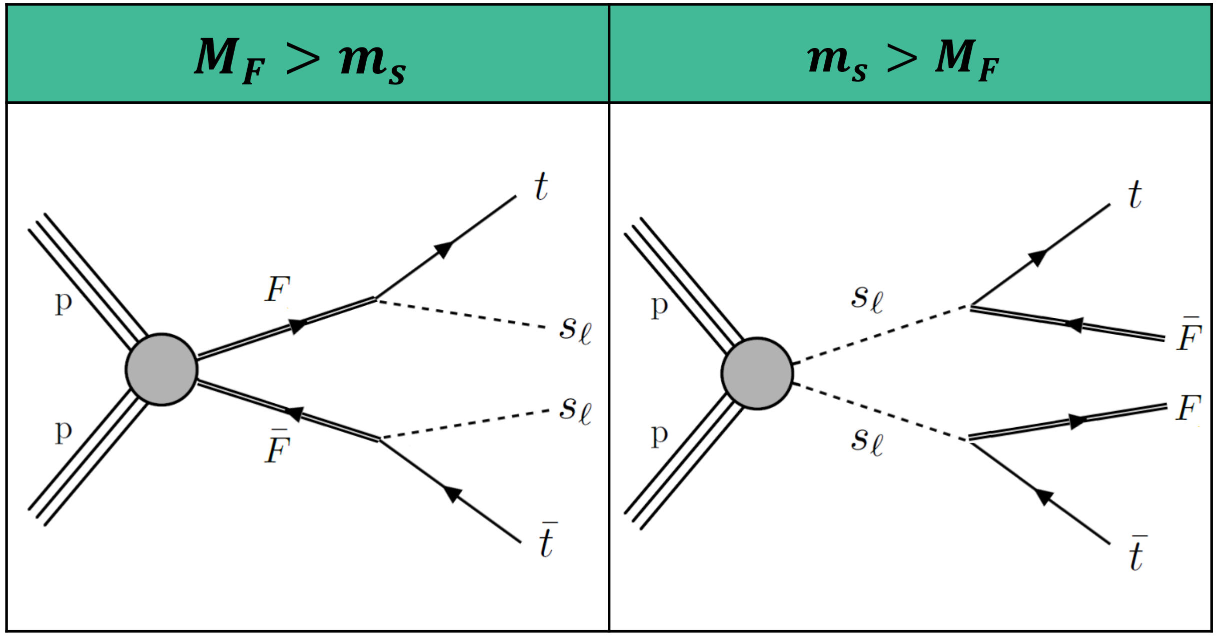
$$\Longrightarrow \quad y_t = V y_L y_R \left((c_\beta^2 - s_\beta^2)^2 \int [s_\ell, s_h, F] + 2\, c_\beta^2 s_\beta^2 \int [s_\ell, s_\ell, F] + 2\, c_\beta^2 s_\beta^2 \int [s_h, s_h, F] \right)$$

Diverse phenomenology

- Phenomenology are determined by the lightest scalar s_ℓ and vector-like fermion F
- The quantum number and the spectrum of the new d.o.f. are not determined
- They can have diverse **"Quantum number"** and **"Spectrum"**

Scalar	Fermion
$(1, 0)$	$(3, +2/3)$
$(1, -1)$	$(3, -1/3)$
$(3, -1/3)$	$(1, -1)$
$(3, +2/3)$	$(1, 0)$

$\Longrightarrow$

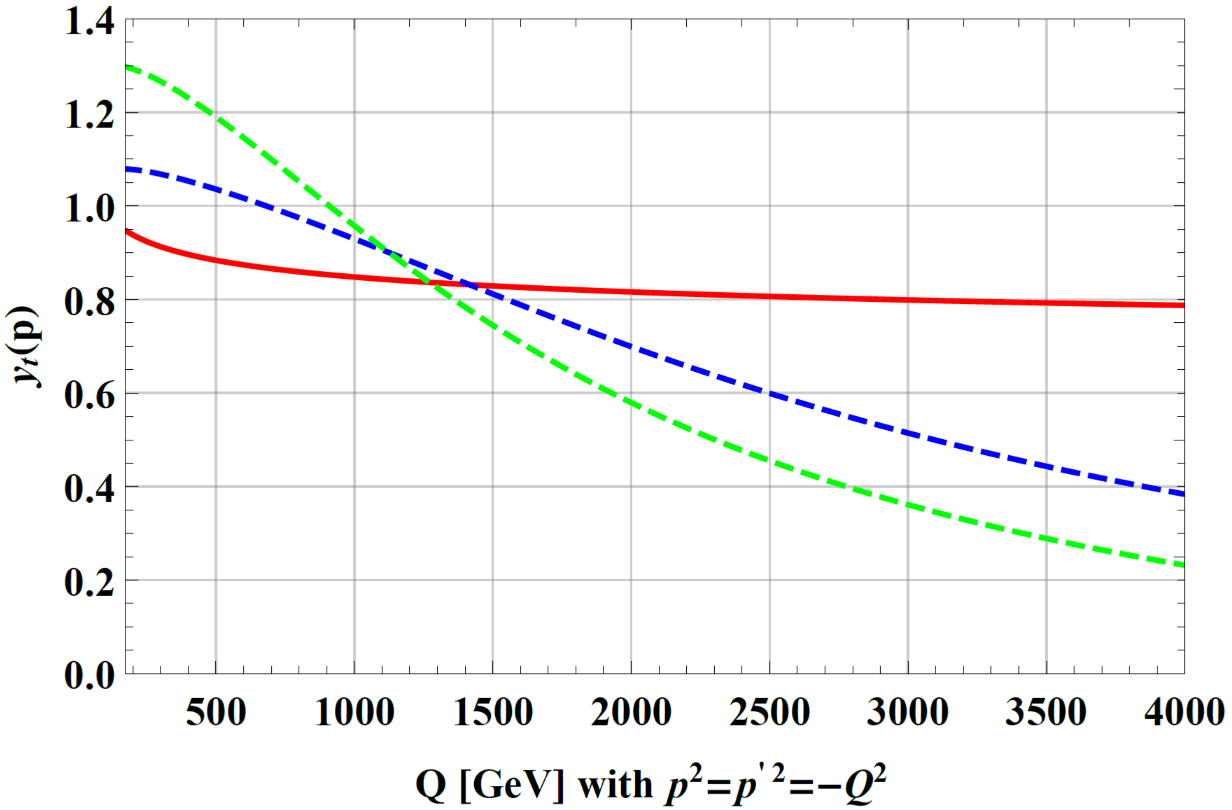


Warning: they might be broad resonances which are not under current search strategy.

Top Yukawa from low scale to high scale

$M_F \sim 1550$ GeV, $m_s \sim 600$ GeV, $M_s \sim 1400$ GeV (BM1, blue)
$M_F \sim 850$ GeV, $m_s \sim 450$ GeV, $M_s \sim 1300$ GeV (BM2, green)

Two benchmarks are calculated and compared with SM running (red)



yi.chung@mpi-hd.mpg.de

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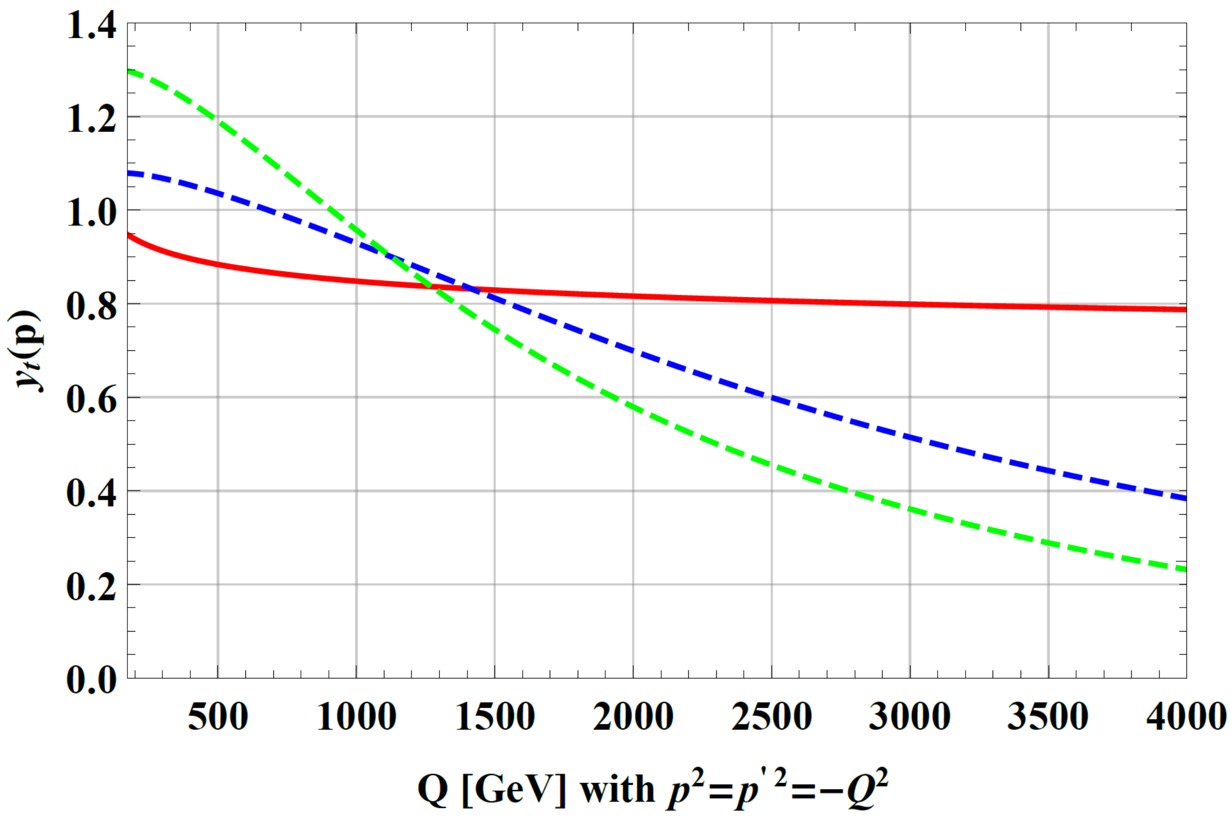


- The value of y_t is normalized according to the correct top mass
- Larger y_t due to additional diagrams with extra Higgs insertion, which lead to

$$\mathcal{L}_{\text{top}} = c_6 \left(\bar{q}_L H t_R\right) + c_{6+4n} \left(H^\dagger H\right)^n \left(\bar{q}_L H t_R\right)$$

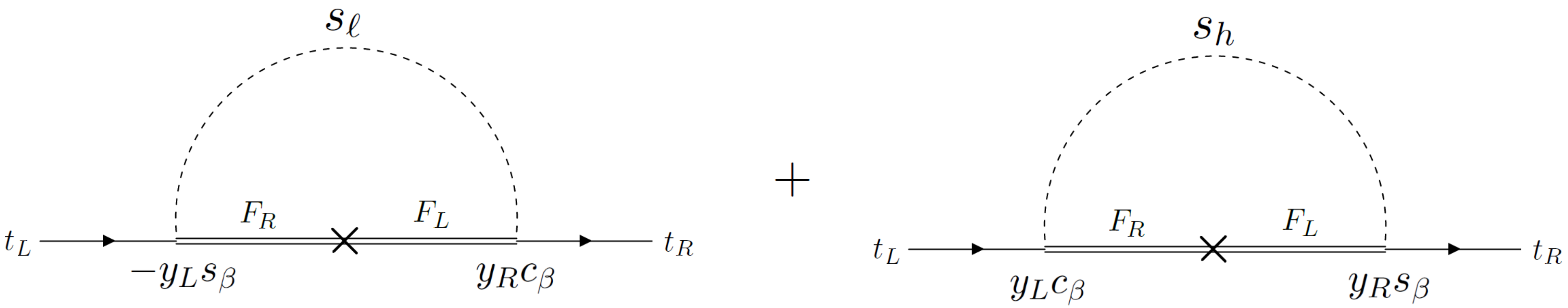
- Main Constraint: top Yukawa measurement

$$\kappa_t \equiv \frac{y_t}{y_t^{\text{SM}}} = 1 + \mathcal{O}\left(\frac{V^2 v^2}{M^4}\right)$$

with current bound $\mathbf{0.7} < \boldsymbol{\kappa_t} < \mathbf{1.1}$ at 95% CL
(likely be weaker considering off-shell effect)

Running of the top quark mass

- The top quark mass is generated through



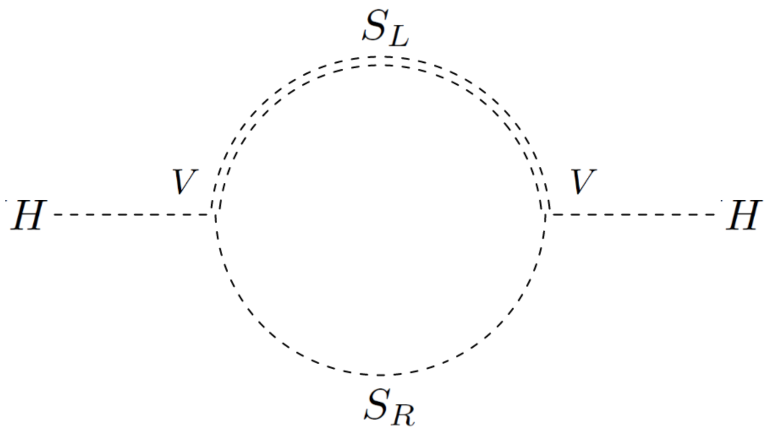
$$- y_L y_R c_\beta s_\beta \int [s_\ell, F] \qquad\qquad y_L y_R c_\beta s_\beta \int [s_h, F]$$

- The top quark mass m_t is radiatively generated in the intermediate scale

  → Nontrivial running m_t at the high scale which will affect the $t\bar{t}$ cross section

Additional contribution

- The trilinear couplings between the Higgs and scalars will introduce a new loop

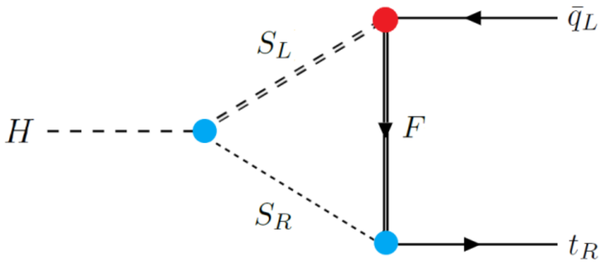


$$\Longrightarrow \quad \Delta m_H^2\big|_{\text{scalar}} \sim \frac{1}{16\pi^2} V^2 \ln\left(\frac{\Lambda_{\text{NP}}^2}{M^2}\right)$$

- This loop is however logarithmically sensitive to NP and will not reintroduce a HP
- Assuming a low-scale UV completion, the correction leads to 7% tuning in both benchmarks, which is at the same order as the top-quark tuning. Therefore, the new scalar loops do not worsen the tuning.

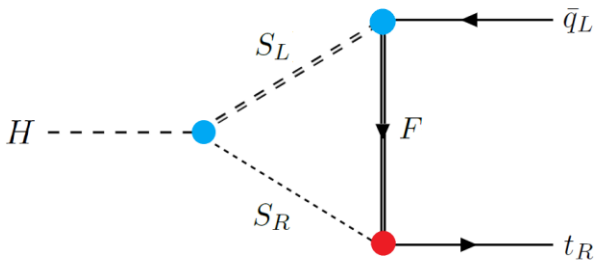
Top Yukawa from strong dynamics

- If y_t comes from pure strong dynamics, then even at one-loop level we expect $y_t \sim 4\pi$
- A suppression $\boldsymbol{\varepsilon}$ is required between the strong and weak sector to get $y_t \sim 1$
- Three possibilities



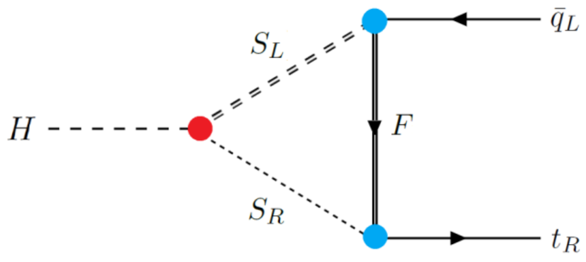
Strong sec. S_L, S_R, F H, t_R

Weak sec. q_L



Strong sec. S_L, S_R, F H, q_L

Weak sec. t_R



Strong sec. S_L, S_R, F q_L, t_R

Weak sec. H

small M without large κ_t

Strongly coupled UV theory

- A Top seesaw-like model based on $SU(3)_L \times SU(2)_R$ global symmetry with bound states

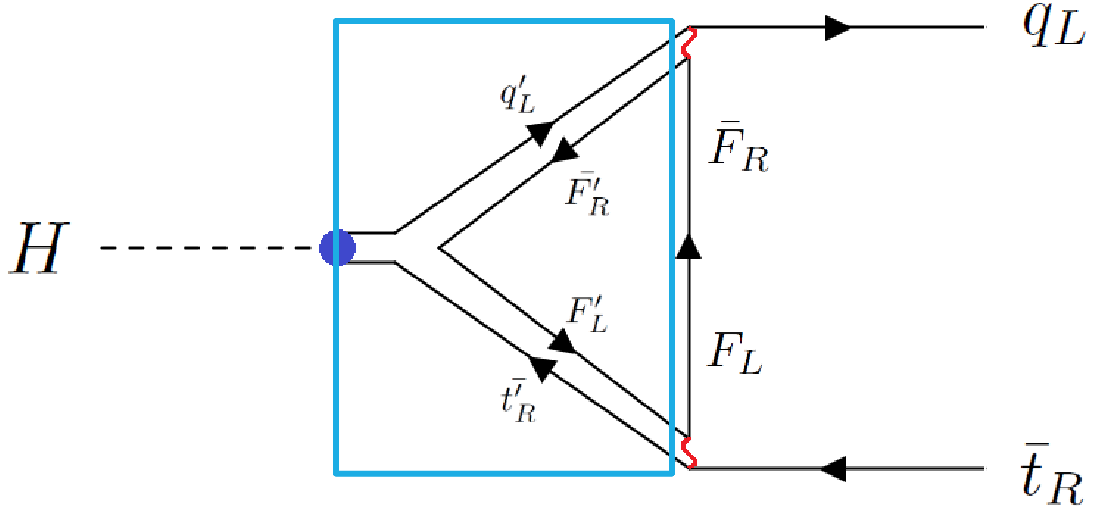
Weak sector:

$$H,\, Q_L = \begin{pmatrix} F_L \\ t_L \\ b_L \end{pmatrix}, \quad Q_R = \begin{pmatrix} F_R \\ t_R \end{pmatrix}$$

Strong sector:

$$\Phi,\, Q'_L = \begin{pmatrix} F'_L \\ t'_L \\ b'_L \end{pmatrix}, \quad Q'_R = \begin{pmatrix} F'_R \\ t'_R \end{pmatrix}$$

$$\Phi = \bar{Q}'_R Q'_L = \begin{pmatrix} S_V^* & S_R^* \\ S_L & S_H \end{pmatrix}$$



$\Longrightarrow$

$$\mathcal{L}_\Phi = |\partial\Phi|^2 - \tilde{M}(\mu)^2|\Phi|^2 - \tilde{\lambda}(\mu)|\Phi|^4 - \tilde{y}(\mu)\,\bar{Q}'_L\,\Phi\,Q'_R$$

$$\supset\; 2\,\tilde{\lambda}\,\langle S_V\rangle\,(S_R S_L^\dagger S_H) - \tilde{y}\,\bar{q}'_L S_L F'_R - \tilde{y}\,\bar{t}'_R S_R F'_L$$

$$\tilde{\lambda}(\mu) = \frac{16\pi^2}{NC},\; \tilde{y}(\mu) = \frac{4\pi}{\sqrt{NC}},\; C = \ln\left(\frac{M'^2}{\mu^2}\right)$$

y_t from four-fermion interaction

2309.00072

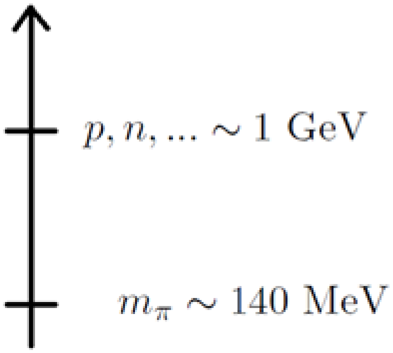
Fundamental Composite Higgs Models

- Chiral symmetry breaking

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

which gives three massless NG bosons, i.e. pions!!

However, the symmetry is broken by quark masses and EM interactions, and we get massive pions.



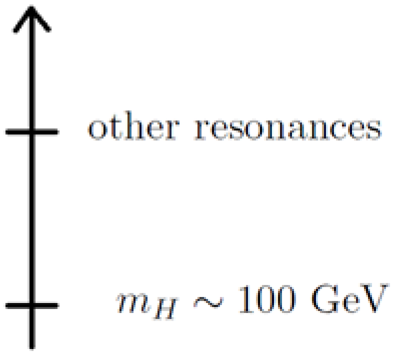
Quark with QCD

- (Some global) symmetry breaking with a scale $f_{HC} \sim 1\,\text{TeV}$

$$\mathcal{G} \to \mathcal{H} \ni SU(2)_L \times U(1)_Y$$

which gives (at least) four NG bosons as **Higgs doublet!!**

The symmetry can be broken by different interactions (usually by electroweak interaction and Yukawa interaction) and give us the nontrivial Higgs potential.



Hyperfermion with Hypercolor (HC)

Fundamental Composite Higgs Models

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$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

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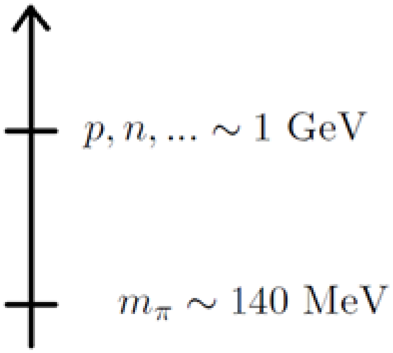
However, the symmetry is b ... s, and we get massive pions.

$$H = \bar{\psi}\psi$$



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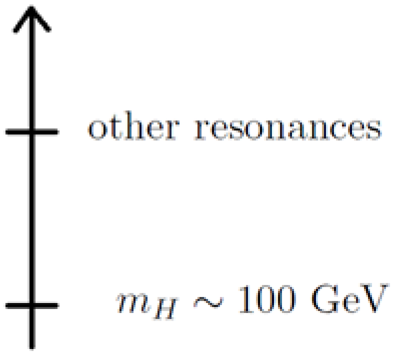
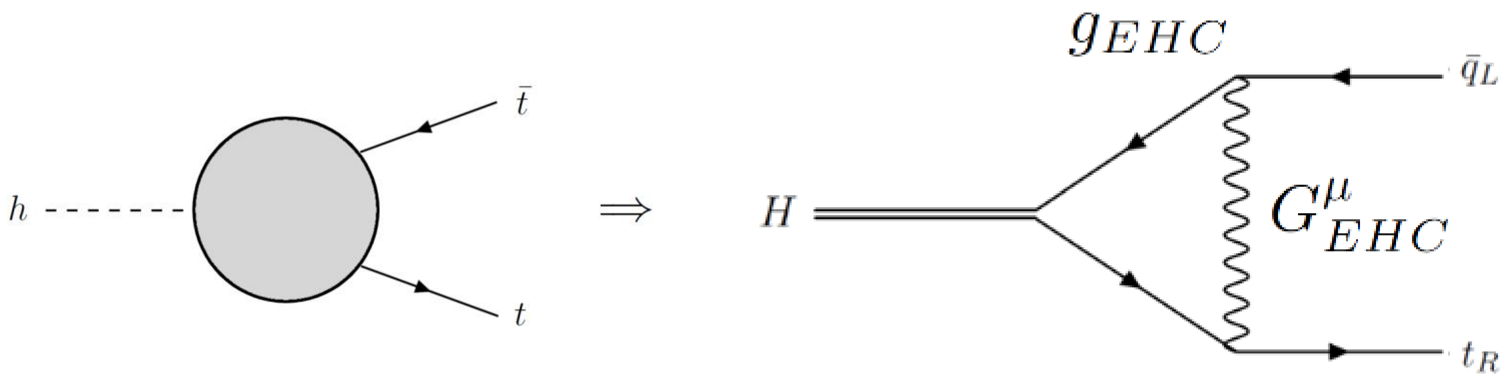


Hyperfermion with Hypercolor (HC)

y_t from four-fermion - bilinear



Composite Higgs with
Extended Hypercolor
$M_{EHC} = g_{EHC} f_{EHC}$
f_{EHC} : the scale of EHC

- Top Yukawa in CHM can originate from connecting the strong sector to a SM bilinear

$$\mathcal{L}_{\rm EHC} = g_{EHC} G^{\mu}_{EHC} (\bar{q}_L \gamma_\mu \psi_L + \bar{\psi}_R \gamma_\mu t_R) \quad \rightarrow \quad \mathcal{L}_{\rm eff} \supset \frac{g^2_{EHC}}{M^2_{EHC}} (\bar{\psi}_R \psi_L)(\bar{q}_L t_R)$$

- Naive dimensional analysis in CHM gives $\bar{\psi}_R \psi_L \sim (4\pi f^2_{HC}) H$ and thus

$$y_t \sim \frac{g^2_{EHC}}{M^2_{EHC}} \cdot 4\pi f^2_{HC} \sim 4\pi \left(\frac{f_{HC}}{f_{EHC}} \right)^2 \sim 1 \quad \Rightarrow \quad f_{EHC} \sim 3.5 \times f_{HC} \gtrsim 3 \text{ TeV}$$

- The cutoff of top loop is determined by $M_{EHC} = g_{EHC} f_{EHC}$, which requires weak g_{EHC}

Extend the gauge group in new direction

- To have a light EHC boson, we need it to be hypercolorless which require new approach.

➢ Traditional approach $\mathcal{G}_{HC} \times \mathcal{G}_{SM} \subset \mathcal{G}_E$

$$E_\mu = (N, \bar{3}, 1)$$

$$SU(N+3)_E \to SU(N)_{HC} \times SU(3)_C$$

$$\begin{aligned} Q_L &= (N+3, 2) \\ Q_R &= (N+3, 1) \end{aligned} \longrightarrow \begin{aligned} \psi_L &= (N, 1, 2), \quad & q_L &= (1, 3, 2) \\ \psi_R &= (N, 1, 1), \quad & t_R &= (1, 3, 1) \end{aligned}$$

➢ New approach $\mathcal{G}_{HC} \times (\mathcal{G}_{HF} \times \mathcal{G}_{SM} \subset \mathcal{G}_E)$

hypercolorless

$$E_\mu = (1, \bar{3}, 1)$$

$$SU(3)_{HC} \times SU(4)_{EC} \to SU(3)_{HC} \times SU(3)_{EC} \to SU(3)_C$$

$$Q_L = (3, 4, 2) \to (3, 3, 2) + (3, 1, 2) \to (6, 2) + (\bar{3}, 2) + (3, 2)$$

$$Q_R = (3, 4, 1) \to (3, 3, 1) + (3, 1, 1) \to (6, 1) + (\bar{3}, 1) + (3, 1)$$

A concrete model

- Gauge group

$$\mathcal{G}_E = SU(3)_{HC} \times SU(4)_{EC} \times SU(2)_W \times U(1)_X$$

$$g_H \qquad g_E \sim g_s \qquad g_X \sim g_Y$$

- Fermion content

$$Q_L = (3, 4, 2, 1/24), \quad Q_R = (3, 4, 1, 13/24)$$

- Several steps are required

A. The $\mathcal{G}_E \to \mathcal{G}_{SM}$ breaking at the scale $f_E \sim 1.7$ TeV

B. Composite Higgs formation at the scale $f \sim 1$ TeV

C. Generation of top Yukawa coupling at the scale M_E

Step A: Tumbling with exotic fermions

- To realize the first breaking dynamically, we need exotic fermions as sextet under $SU(3)_{HC}$

$$Q_L = (3, 4, 2, 1/24) \ , \quad F_R = (6, 1, 2, 0)$$

- The most attractive channel under $SU(3)_{HC}$ is RH 6 combined with some of LH 3

$$\bar{F}_R Q_L = \bar{\mathbf{6}}\,\mathbf{3} = (\bar{3}, 4, 1, 1/24) = \frac{f_E}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

which will break $\mathcal{G}_E \to \mathcal{G}_{SM}$. The fermion content then deconstruct to

$$Q_L \to \ f_L = (6, 2)_0, \ \psi_L = (\bar{3}, 2)_0, \ q_L = (3, 2)_{\frac{1}{6}}$$

$$Q_R \to f_R = (6, 1)_{\frac{1}{2}}, \ \psi_R = (\bar{3}, 1)_{\frac{1}{2}}, \ t_R = (3, 1)_{\frac{2}{3}}$$

- The exotics F will mix with f of Q forming a massive Dirac color sextet fermion

Step B: CHM by broken hypercolor

- After the first breaking, the $SU(3)_{HC}$ is broken and there is no further condensate.

$$g_c = \sqrt{8\pi^2/3} \sim 5.1$$ is critical coupling derived from NJL

- However, there is another $SU(3)_{EC}$ which allows tilting mechanism to happen

$$g_\psi^2 \sim g_H^2 + g_E^2 > g_c^2 \,, \quad g_t^2 \sim g_H^2 < g_c^2$$

- For a realistic CHM, additional hyperfermion is required with

$$\Psi_1 = (\bar{3}, 2)_0, \; \Psi_2 = (\bar{3}, 1)_{\frac{1}{2}}, \; \Psi_3 = (\bar{3}, 1)_{-\frac{1}{2}}$$

which results in a SU(4) × SU(4) / SU(4) FCHM with 15 pNGBs including Higgs

Step C: Top Yukawa from the E_μ boson

- In this model, we can then generate the top Yukawa coupling with a value given by

$$y_t \sim \frac{1}{v}\frac{g_E^2}{M_E^2}\langle\bar{\psi}_R\psi_L\rangle_{HC} \sim \left(\frac{f}{f_E}\right)^2 Y_S \quad \text{,where} \quad Y_S \sim \frac{4\pi}{\sqrt{N_{HC}\ln(\Lambda^2/M_\psi^2)}}$$

- Taking $Y_S = 3$, we get the relation among the scale for the observed top Yukawa as

$$f_E \sim \sqrt{Y_S} \times f \sim 1.7 \times f$$

PS It is derived from bottom-up and will be more attractive if one get derive from top down.

- With $g_E \sim g_s \sim 1$ and $f_E \sim 1.7$ TeV, the mass of $\boldsymbol{E_\mu}$ boson is given by

$$M_E = \frac{1}{2} g_E f_E \sim \boxed{0.9\ \text{TeV}}$$

The desired light cutoff !!

Breaking pattern

$$SU(3)_{HC} \times SU(4)_{EC} \times SU(2)_W \times U(1)_X$$

$$\downarrow \quad \bar{F}_R Q_L = \bar{\mathbf{6}}\,\mathbf{3} \text{ condensate with } f_E \sim 1.7 \text{ TeV}$$

$$SU(3)_C \times SU(2)_W \times U(1)_Y + \text{massive } G',\ E_\mu,\ Z',\ \text{fermion sextet } F$$

$$\downarrow \quad \bar{\psi}_R \psi_L = \bar{\mathbf{3}}\,\mathbf{3} \text{ condensate with } f \sim 1 \text{ TeV}$$

$$SU(3)_C \times SU(2)_W \times U(1)_Y + \text{composite Higgs}$$

$$\downarrow \quad \text{intergrating out } E_\mu \text{ with } M_E \sim 0.9 \text{ TeV}$$

$$SU(3)_C \times SU(2)_W \times U(1)_Y + \text{composite Higgs} + \text{"Top Yukawa"}$$

The overall spectrum

$$M_{G'} = \frac{1}{\sqrt{2}}\sqrt{g_H^2 + g_E^2} f_E \sim 6 \text{ TeV}$$ **Coloron**

$$M_F \sim Y_s f_E \sim 5 \text{ TeV}$$ **Sextet fermion**

$$M_\psi \sim Y_s f \sim 3 \text{ TeV}$$ **Triplet fermion**

Hypercolored Particles @ few TeV

$$M_E = \frac{1}{2} g_E f_E \sim 0.9 \text{ TeV}$$ **E_μ boson**

$$M_{Z'} = \frac{1}{\sqrt{8}}\sqrt{c^2 g_X^2 + g_E^2} f_E \sim 0.6 \text{ TeV}$$ **Z' boson**

Hypercolorless Particles @ sub TeV