

Towards a fully NNLO Monte Carlo generator for low energy e^+e^- data into hadrons and leptons

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This work is in collaboration w/ Pau Petit Rosas
F. Ignatov, T. Teubner and G. Venanzoni

ICHEP 2024, Prague
Top Quark and Electroweak Physics
July 19, 2024

This talk: I plan to provide an overview of the status of the Monte Carlo event generator at NNLO, with particular emphasis on the KLOE-2 experiment.

Introduction

RadioMonteCarLow → Strong 2020 initiative

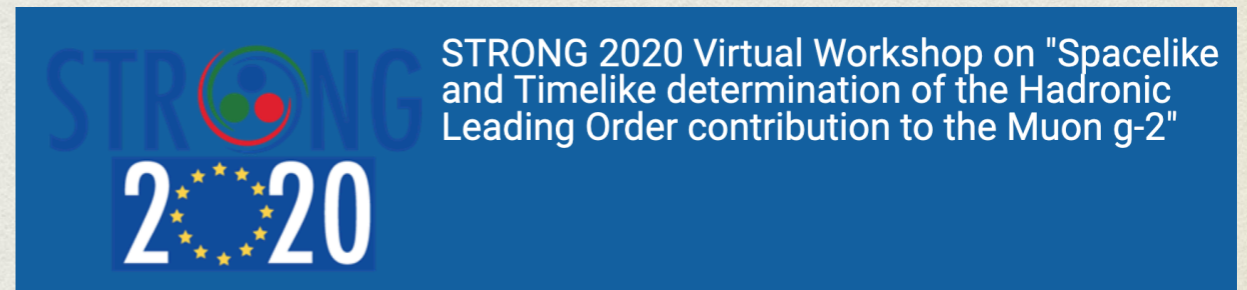
Eur. Phys. J. C (2010) 66: 585–686
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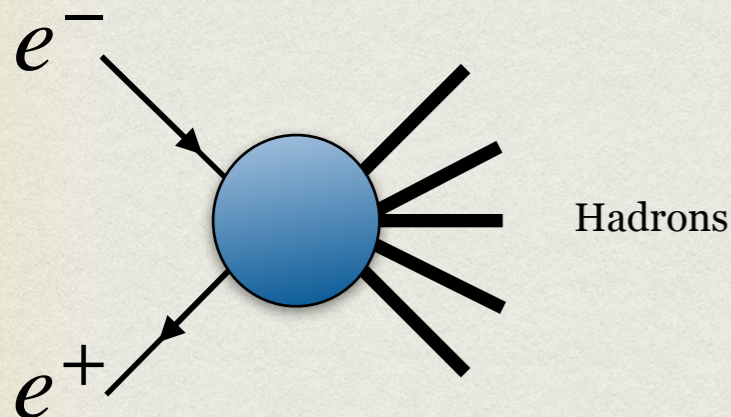
Review

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies



<https://agenda.infn.it/event/28089/>
<https://indico.psi.ch/event/13707/overview>



- Compute scattering amplitudes at low energies
- Perform Monte Carlo integration
- Provide theoretical predictions

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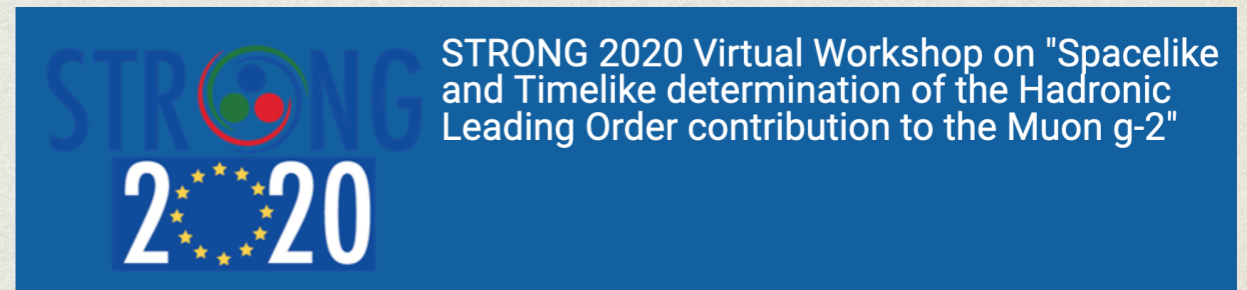
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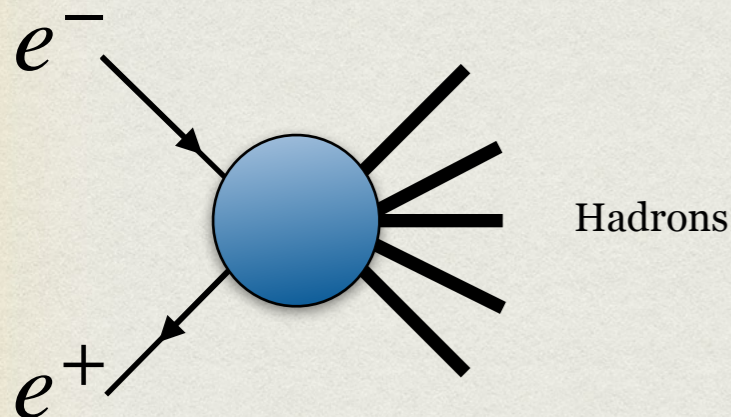
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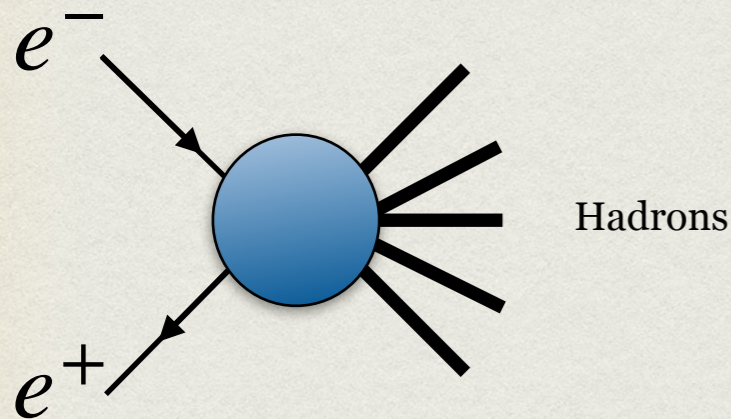
Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

Coordinated by A. Kupsc, A. Signer, Y. Ulrich, G. Venanzoni



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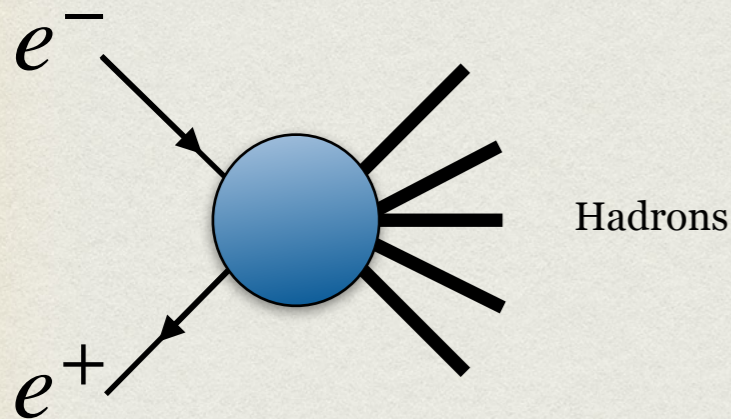
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- Scrutinise in detail form factor insertion in pion processes
- Unravel crucial differences between MC generators
- Fixed order vs resummation

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Today's focus :: Phokhara beyond NLO

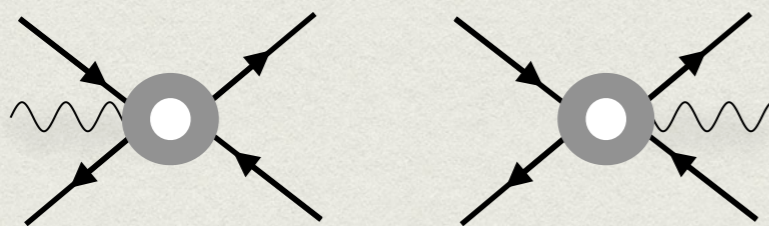
Current Status of Phokhara 10.0

(<https://looptreeduality.csic.es/phokhara/>)

LO & NLO contribution to radiative return processes

ISR, FSR & Mixed

$$e^+e^- \rightarrow \mu^+\mu^-\gamma$$



+ real radiations

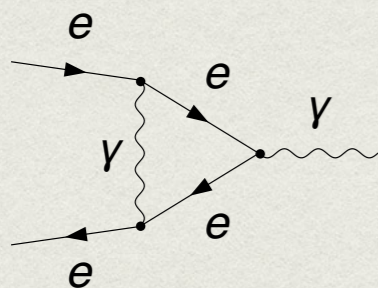
$$e^+e^- \rightarrow \pi^+\pi^-\gamma$$



Use of FsQED

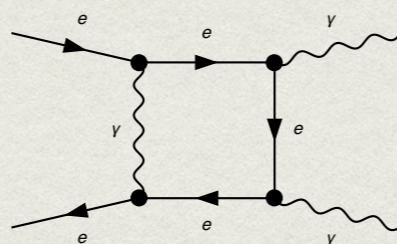
Three gauge invariant groups

$$f^+f^- \rightarrow \gamma^* \rightarrow F^+F^- + \gamma$$



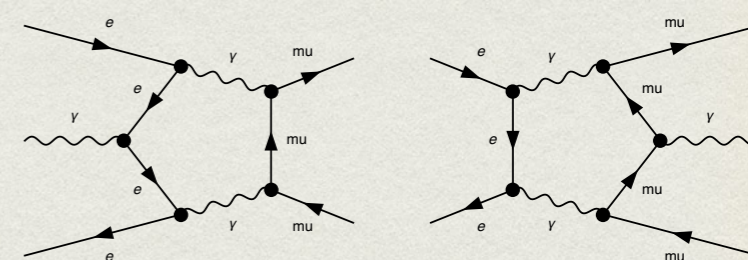
Easy (m_f^2, s)

$$f^+f^- \rightarrow \gamma\gamma^* \rightarrow F^+F^-$$



Normal (s, t, m_e^2, q^2)

$$f^+f^- \rightarrow F^+F^- \gamma$$

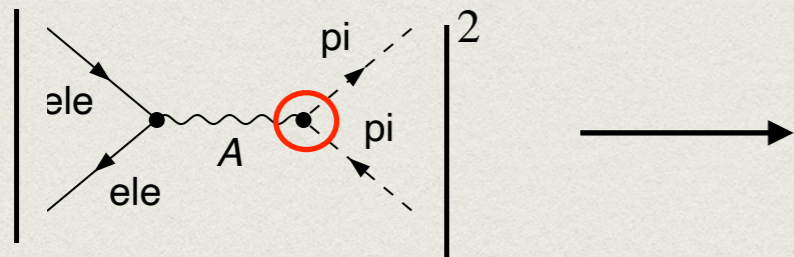


Hard ($s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_f^2, m_F^2$)

Pion processes

$e^+e^- \rightarrow \pi^+\pi^-$ (ISR)

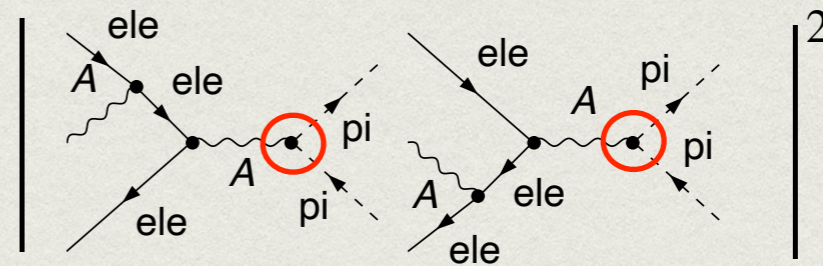
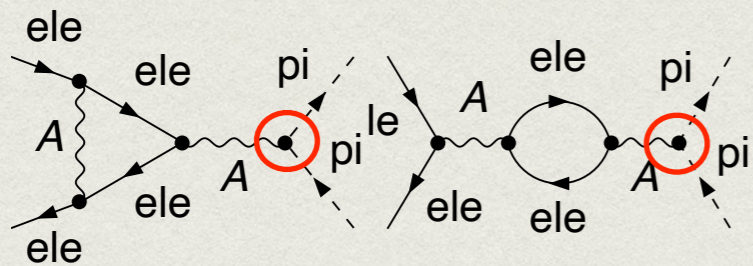
Differential cross section in the Center-of-mass frame



$$\frac{d\sigma_0}{d\cos\theta} = \frac{\pi\alpha^2}{4s} \beta^3 (1 - \cos^2\theta) \left| F_\pi(s) \right|^2$$

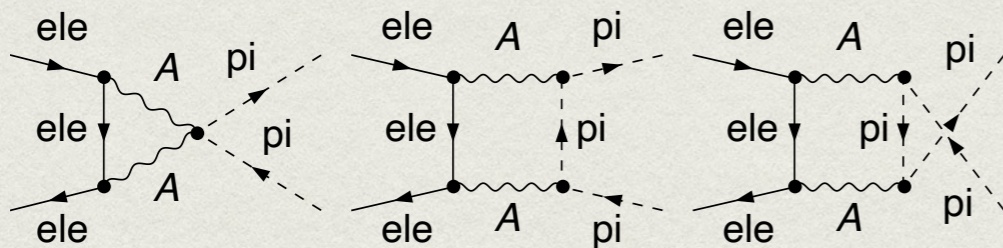
sQED FF

QED corrections



$F \times$ sQED

FF corrections :: $\mathcal{O}(F(q_i^2)F(q_j^2))$



Include Form factor according to the IR prediction from lower orders

Validation @ NLO

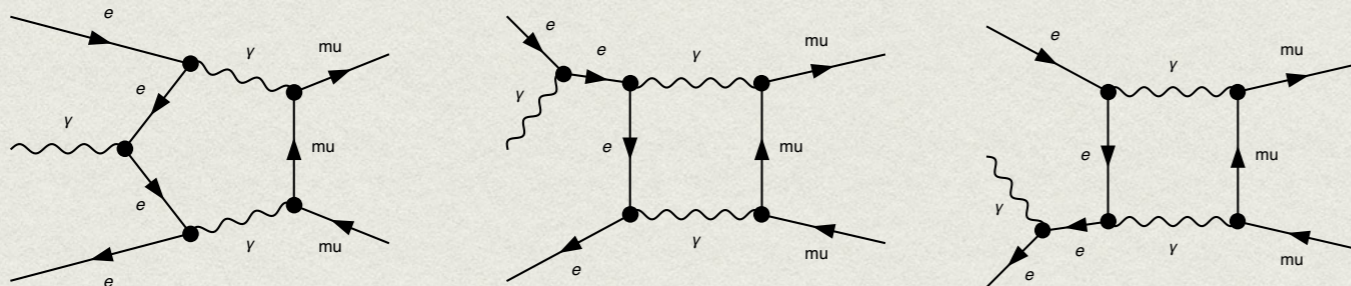
[Petit Rosas, WJT (work in progress)]

- Dimensionally regulated one-loop amplitude ($D = 4 - 2\epsilon$)

$$A^{(1)}(f^+ f^- \rightarrow F^+ F^- \gamma) = \frac{c_{-1}}{\epsilon} + c_0$$

given by Phokhara

- Organisation of virtual amplitudes



+ crossings & FSR

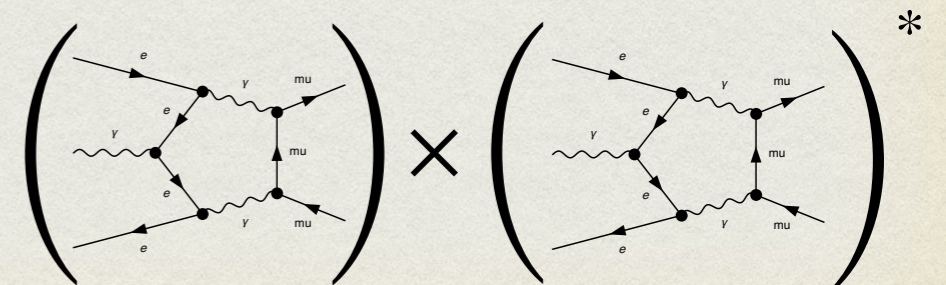
Automated tools in amplitudes' methodology ::

* `qgraph/FeynArts`, `Form/FeynCalc` (construction of amplitudes)

* `AMFlow` (numerical evaluation of amplitudes)

- Analytic evaluation of Feynman integrals

$$d\vec{J} = \epsilon d\tilde{A}\vec{J}$$



Validation @ NLO

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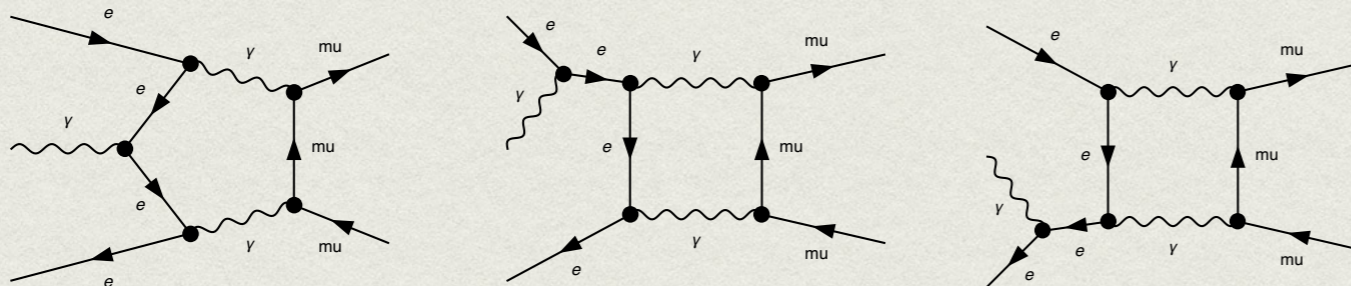
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$$A^{(1)}(f^+ f^- \rightarrow F^+ F^- \gamma) = \frac{c_{-1}}{\epsilon} + c_0 + c_1 \epsilon + c_1 \epsilon^2$$

given by Phokhara

first results

- Organisation of virtual amplitudes



+ crossings & FSR

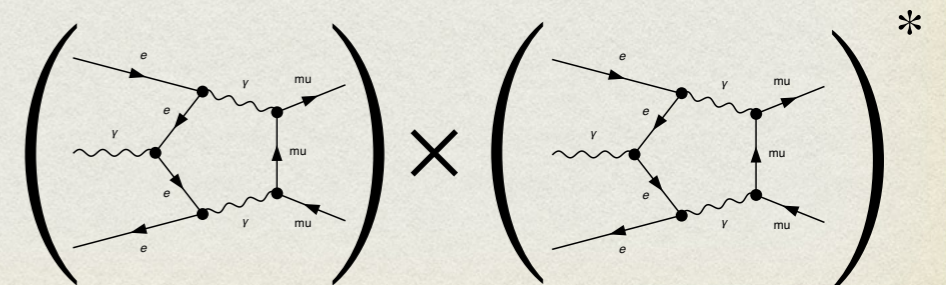
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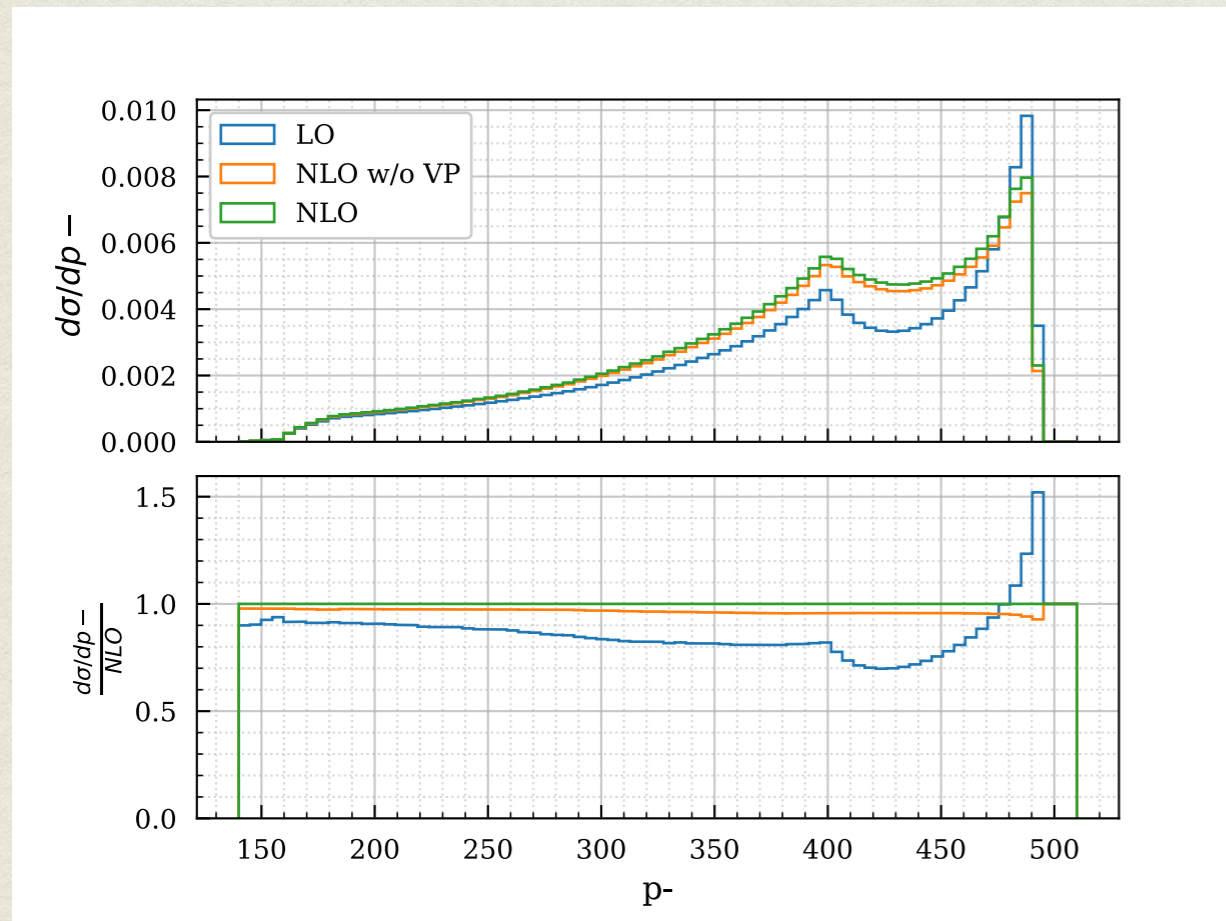
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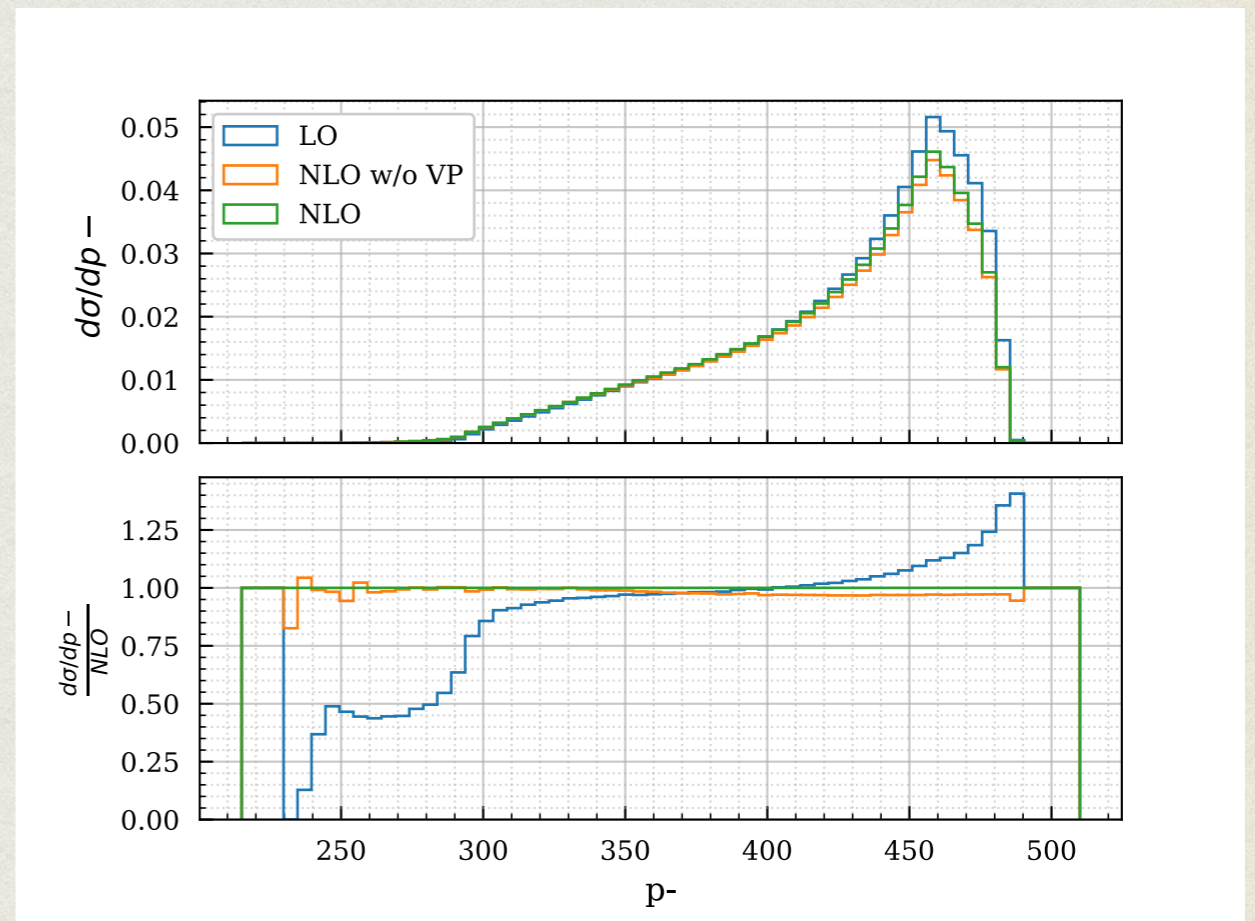
Validation @ NLO

[Courtesy of Petit Rosas]

Photon emission at large angle



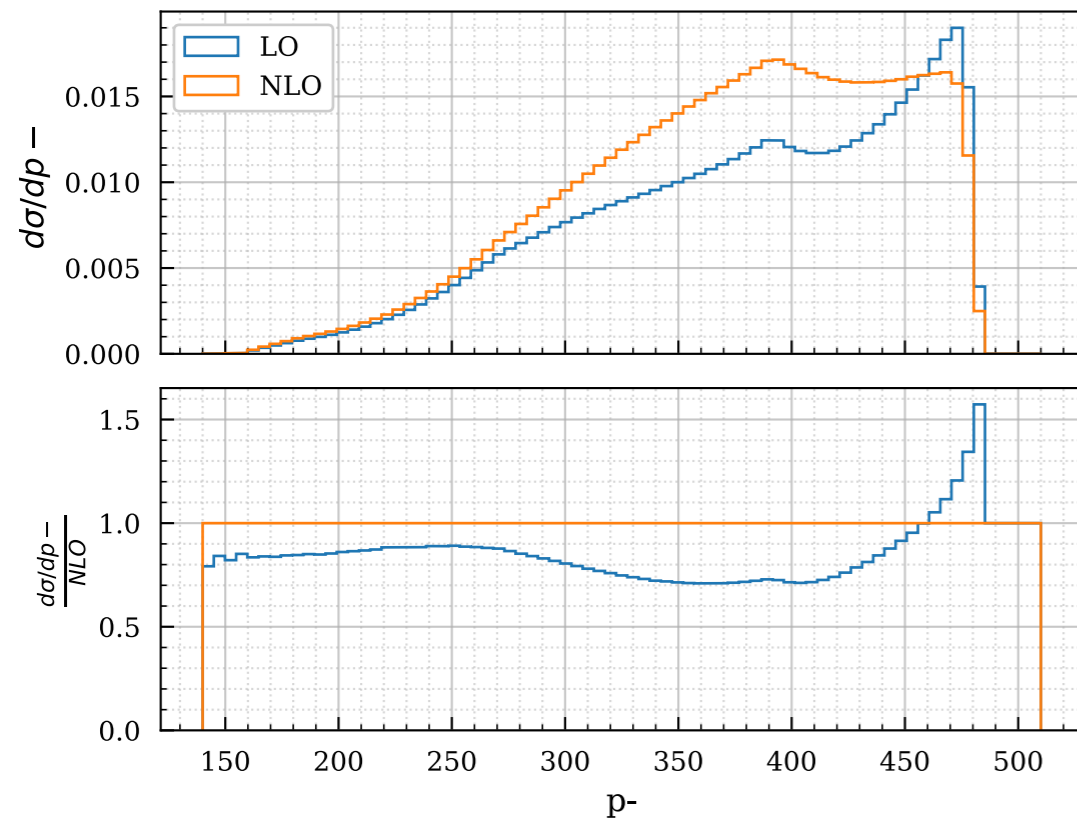
Photon emission at small angle



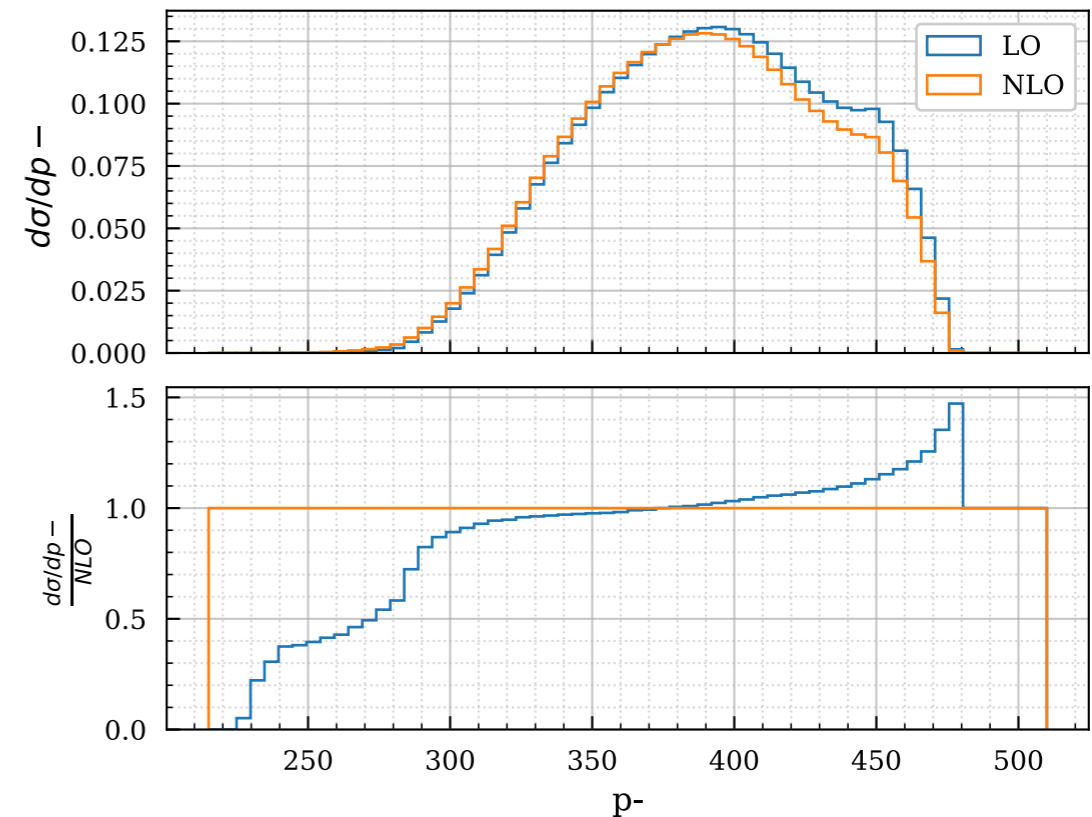
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[Courtesy of Petit Rosas]

Photon emission at large angle



Photon emission at small angle



From $F \times sQED$ to $FsQED$

- introduce a parameterisation of the form factor

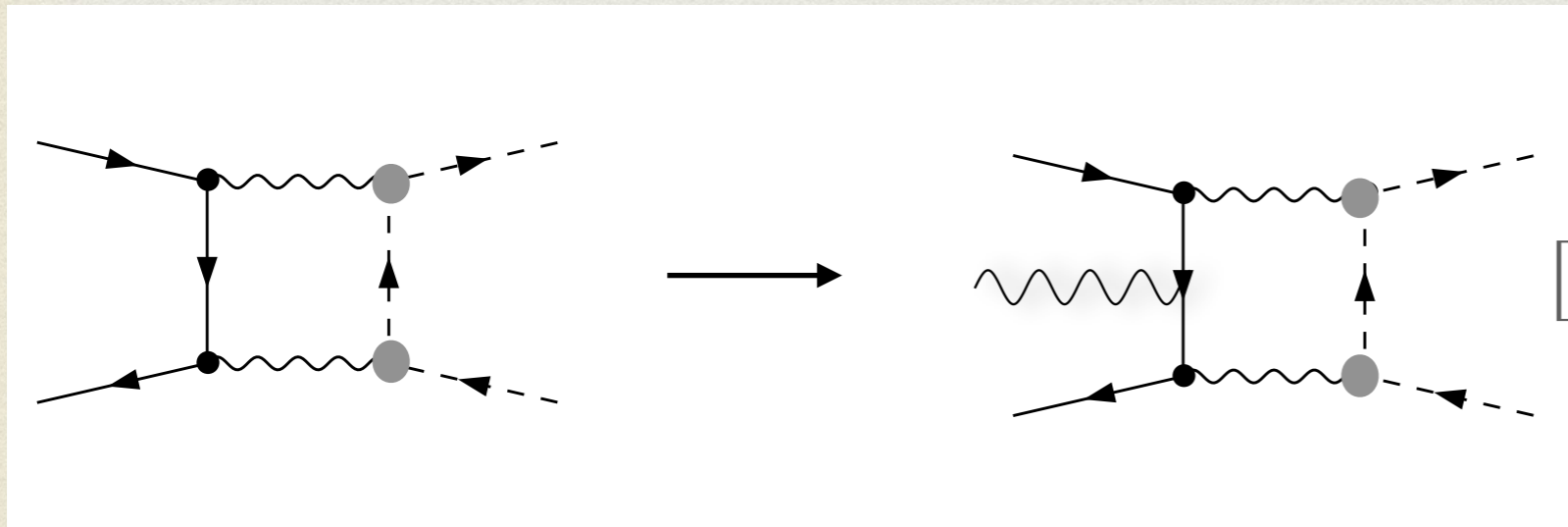
$$F(q^2) = \sum_{v=1}^n a_v \frac{\Lambda_n}{\Lambda_n - q^2}, \quad \text{with } \sum_{v=1}^n a_v = 1.$$

Include Form factor according to the IR prediction from lower orders

- Known results on forward-backward asymmetry of $e^+e^- \rightarrow \pi^+\pi^-$

[Lee, Ignatov (2022)]

[Colangelo, Hoferichter, Monnard, Ruiz de Elvira (2022)]



$$[F(s) - F(q_1^2)F(q_2^2) + F(s)]$$

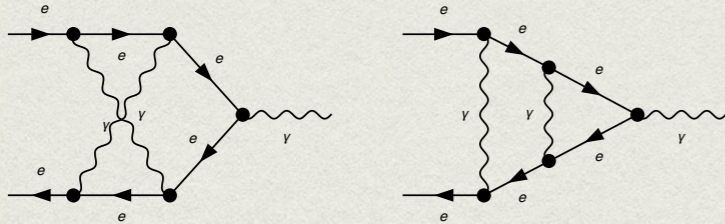
δ_{FF}^V :: UV and IR finite ?
Is IR structure unchanged

Progress on NNLO

[Petit Rosas, WJT (work in progress)]

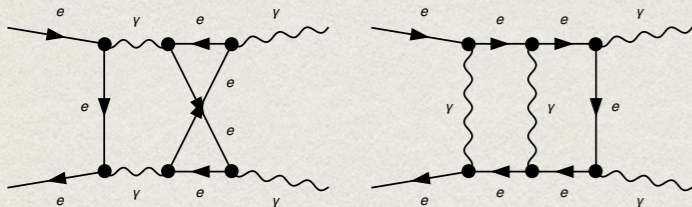
Two-loop pieces

$$f^+ f^- \rightarrow \gamma^* \rightarrow F^+ F^- + \gamma$$



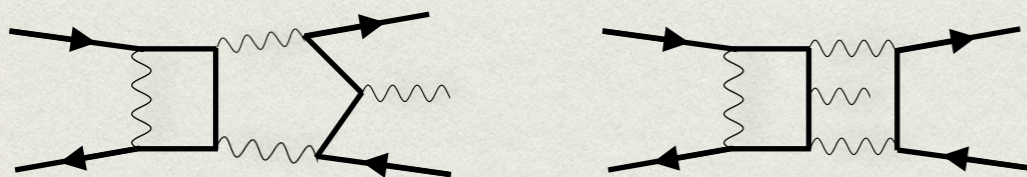
Form factors computed long time ago
 Revisited in [2106.13179](#) [hep-ph]
 Can be implemented in Phokhara framework
 But not of relevant interest

$$f^+ f^- \rightarrow \gamma \gamma^* \rightarrow F^+ F^-$$



Needs to be carefully analysed
 Can we find a “nice” form factor $q_e^a q_m^b$?
 Use of numerical tools Diffexp/SeaSyde

$$f^+ f^- \rightarrow F^+ F^- \gamma$$



Very challenging!!!
 Fully numerical framework

Conclusions & Outlook

We have reached:

- First improvements in the Phokhara generator
- Validation of NLO theoretical predictions for radiative processes
- Scrutinised in detail form factor insertion within Phokhara
- Unravel crucial differences between MC generators (Strong2020)

We are working on:

- Provide efficient approaches to combine numerics & analytics.
- Use novel mathematic insights in evaluation of Feynman integrals.
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